# Graphs – an introduction

## an introduction

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## Graphs?

- a **graph** is mainly a formalism enabling the representation of **relations** linking various **entities**
- relations: social relations, geographical proximity, common points, etc.
- advantage of using such a representation: erase the semantics and keep the **structure** formed by these relations
- graph: a set of entities called **nodes** or **vertices** (one vertex, several vertices), and a set of **links** between them: **edges** (non directed links) and **arcs** (directed links).
- we associate entities to nodes/vertices and relations to edges/arcs.

# Graphs: basics

- a graph is defined by a set of vertices $V$ and a set of edges/links/arcs $E$
- denoted $G = (V, E)$ usually $|V| = n$ and $|E| = m$
- $n$ is called the order of the graph
- Most of the time during this course we will deal with **1-graphs**: graphs for which any two vertices can be linked by at most one edge (or one arc in each direction)
- Other types of graphs not covered by this course:
  - multi-graphs are graphs in which two vertices can be linked by more than one edge
    for instance $k$-graphs: at most $k$ edges can link two vertices
  - hyper-graphs are graphs for which one edge can link more than two vertices (e.g. chemical reactions)
  - multilayer graphs and networks are multi-graphs that can be seen as layers of graphs each layer corresponding to a specific semantic (geographic data may be represented as multilayer graphs)

## Networks $\longleftrightarrow$ Graphs

### Network

- a set of **elements** connected by **links**
- networks are structures traversed by flows

### Graph

- a set of **nodes/vertices** connected by **edges** or **arcs** (directed edges)

  $\Longrightarrow$ graph is a relevant formalism for representing relations between entities
- thus, *good candidates for modeling networks*

# Networks are everywhere

- urban transportation network: buses, metro lines
- rail network: trains
- internet
- social network
- (tele)communication networks: GSM, 4G, Wifi
- energy network: power grid
- sewer/wastewater network
- human/animal/insect societies

## Network?

### What is a network?

- a set of **elements** (homogeneous or not) connected by **links**
- a network is a structure allowing flow circulation
- flows:
  - exchange of ideas, money
  - transport of people, goods, energy, information
  - diffusion of molecules, news, emotions
- flows sometimes need vectors (car, ship, mosquito, plane, human, etc.)

→ Some Examples

# Maritime Networks

colors: density of ships along maritime roads

- within these networks we can find: ships, harbors, containers, shipping companies, crews, maritime roads, raw materials, manufactured goods
- topic of interest $\leftrightarrow$ elements to consider

# Air Traffic Network

- within these networks we can find: planes, airports, passengers, companies, crews, goods

# Web (Internet)

- within these networks we can find: web pages, servers (including databases), routers, queries, switches, communication protocols

# Urban Networks

London city

Paris metro lines

- different types of networks: rails networks, metro networks, street networks, country-scale road networks
- within urban street networks we can find: streets, cars, buses, crossroads, people, pedestrians, buildings (school, factories, residential districts), fires (red lights, semaphores)

# Social Networks

- within social networks we can find: web pages, messages, pictures, videos, users, links between elements (like, follow, friend)

# Mobile/Sensor Ad Hoc Networks

- An **ad hoc** network is a spontaneous network made or communicating devices.
- Such networks need neither infrastructure, nor control, nor supervision.
- If some elements are mobile they are called mobile ad hoc networks a.k.a. **MANETs**

- Within mobile/sensor ad hoc networks we can find: classical computers, smartphones, sensors.

# Human and Animal Societies

- Within human and/or animal societies we can find: living beings.

# Graphs: basics

## Given a Graph $G = (V, E)$

- given an edge $e = \{v_i, v_j\}$, we say that: e = {u,v} = {v,u}
  - $v_i$ is **adjacent** to $v_j$,
  - $v_i$ is **incident** to $e$ and
  - $v_i$ and $v_j$ are **neighbors**
  - $v_i$ and $v_j$ are the endpoints of $e$
- the **degree** of a vertex is the number of its neighbors
- a trail is a sequence of vertices $p = (v_0, v_1, v_2, \ldots, v_k)$ such that $\forall i (v_i, v_{i+1}) \in E$
- a **path** is a trail such that $\forall i$, $v_i$ appears only once in $p$
- a **cycle** is a closed path (the first and last vertices are the same)
- a **connected component** is a set of vertices $cc = \{v_0, v_1, \ldots, v_n\}$ such that $\forall v_i, v_j \in cc$, $\exists p$ a path containing both vertices
- $G = (V, E)$ is a **connected graph** if $\forall v_i, v_j \in V$, $\exists p$ a path between $v_i$ and $v_j$

GraphStream viewer
Connected Components
Nodes
Edges
degree(D) = 5
D
A
A path between
vertices A and B
B
C
B and C are neighbors

# Directed Graphs: basics

## Given a directed Graph $G = (V, A)$

- an **arc** is an edge with an orientation
- given an arc $a = (v_i, v_j)$, we say that $v_i$ is the tail (source) of $a$, while $v_j$ is the head (destination) of $a$

  a = (v,u) != (u,v)
- $G = (V, A)$ is a **strongly connected graph** if $\forall v_i, v_j \in V$, $\exists p$ a directed path starting in $v_i$ and ending in $v_j$

# Networks: Graphs + values on nodes/edges/arcs

## Network $N = (V, A, f, g)$

- a **network**[a] is a graph (directed or not) and two functions $f$ and $g$ such that:

  to each $v_i \in V$ is associated a value $a_i = f(v_i)$

  to each $e_{ij} \in E$ (or $A$) is associated a value $b_{ij} = g(e_{ij})$

[a] graph theory terminology

# Some particular Graphs