Graphs Introduction – Exercises

1 Modeling

1.1 Exercise 1

Given the following set of relations:

- A, B, C and D composed a full-connected graph.
- E and F are both linked to B
- F is also linked to G and H
- H, I and J are linked with each other

If we represent this set by a graph, what is its order? Draw the graph.

1. Is the graph connected?
2. What are the neighbors of vertex B?
3. What is the average degree of the graph (sum of the degrees divided by the number of vertices)?
4. How many edges should be removed for obtaining a tree? Which are these edges?
5. What happen if vertex F was removed?

1.2 Exercise 2

- According to you, the relation ”can divide” results in a directed or non directed graph?
- Build this graph for the vertices 1, 2, 3 and 4.

1.3 Exercise 3

- A non increasing series of integer is graphic is there exist a non directed graph in which the degrees of its vertices correspond to this series. For instance, the triangle (3 vertices linked with each other) correspond to the series (2,2,2).
- Can you show that series $(3, 3, 2, 1, 1)$, $(3, 3, 2, 2)$ and $(4, 2, 1, 1, 1, 1)$ are graphics?
- Is the series $(5, 4, 3, 1, 1, 1, 1)$ graphic?

1.4 Exercise 4

We are interested in non directed graph in which all vertices have a degree equal to 3.

- Do graphs of orders 4, 5, 6 or 7 exist?
- What can you deduce from that?

1.5 Exercise 5

In a graph, the sum of degrees is equal to twice the number of edges. More formally, given $G = (V, E)$ a graph, if $d(v)$ represents the degree of vertex v, then:

$$\sum_{v \in V} d(v) = 2Card(E)$$

where $Card(E)$ is the number of edges.

- What can you deduce about the number of vertices with an odd degree?

2 Enumeration

2.1 Exercise 6

Given a full-connected graph of order n, how many edges has this graph?

2.2 Exercise 7

- Draw all non directed graphs connected or not connected of order 1, 2, 3 and 4.
- For each of them try to name its topology.

  $\rightarrow$ Beware, some of them may have several topologies...

3 Algorithms

For the algorithms you can use these methods:

- `createGraph()` that returns g an empty instance of a graph.
- `g.addNode(v)` where v is the identifier of a vertex.
- `g.addEdge(`v_i`,`v_j`)` that adds a new edge between vertices v_i and v_j
- $v \leftarrow$ `g.getRandomVertex()` v is randomly chosen within the set of vertices of graph g

3.1 Exercise 8: Ring Generation

Propose an algorithm that generates a ring of order n

3.2 Exercise 9: Tree Generation

Propose an algorithm that generates a tree of order equal to n

4 Additional Exercises

4.1 Exercise 10

- are the following graphs regular ?
  - a ring
  - a grid
  - a star
  - a full connected graph
  - a bipartite graph
- How can we modify a grid graph with a von Neumann neighborhood such that it becomes regular? Can you try with a 3×3 von-Neumann grid?

4.2 Exercise 11

Propose a definition of a star graph without using the word ”degree”.

4.3 Exercise 12

1. Given a von-Neumann grid of size $n \times n$, how many edges has this graph?
2. Given a Moore grid of size $n \times n$, how many edges has this graph?