

# Brushless permanent-magnet motor drives

The brushless PM DC motor is extremely common in a wide variety of low-power applications such as computers and office machinery. As magnet properties improve and control electronics become more sophisticated, the brushless PM motor expands its application potential to higher power levels

by Tim Miller

## Permanent magnets

Permanent magnets provide a motor with life-long excitation. The only cost is the initial cost, which is buried in the cost of the motor. It ranges from a few pence for small ferrite motors, to several pounds for rare-earth motors. Broadly speaking, the primary determinants of magnet cost are the power density (or torque per unit volume) of the motor; the operating temperature range; and the severity of the operational duty of the magnet.

### Power density

For maximum power density the product of the electric and magnetic loadings of the motor must be as high as possible. The electric loading is limited not only by thermal factors, but also by the demagnetising effect on the magnet. A high electric loading necessitates a long magnet length in the direction of magnetisation, to prevent demagnetisation. It also requires a high coercivity, and this may lead to the more expensive grades of material (such as 2-17 cobalt-samarium, for example).

The magnetic loading, or air-gap flux, is directly proportional to the remanent flux density of the magnet, and is nearly proportional to its pole-face area. A high power density thus requires the largest possible magnet volume (length times pole area) to be fitted to the rotor (see Fig. 1). With ferrite magnets the limit on the magnet volume is often the geometrical limit on the volume of the rotor itself, and the highest power densities cannot be obtained with these magnets. With rare-earth or other high-energy magnets, the cost of the magnet may be the limiting factor.

The air-gap flux density is limited by saturation of the stator teeth. Excessive saturation absorbs too much excitation MMF (requiring a disproportionate increase in magnet volume); or causes excessive heating due to hysteresis and eddy currents. For this reason there is an upper limit to the usable energy of a permanent magnet. With a straight demagnetisation characteristic throughout the second quadrant and a recoil

permeability of unity (see Fig. 2), the maximum energy product  $(BH)_{max}$  is given by

$$(BH)_{max} = \frac{B_r^2}{4\mu_0} \text{ joule/m}^3$$

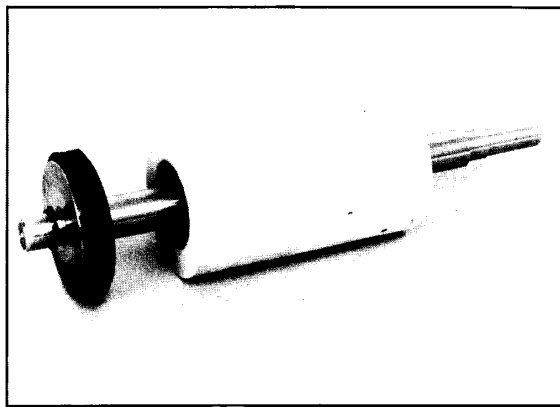
Assuming that the stator teeth saturate at 1.8 T and that the tooth width is half the tooth pitch, the maximum air-gap flux density cannot be much above 0.9 T and is usually lower than this. Therefore, there will be little to gain from a magnet with a remanent flux density above about 1 or 1.2 T, implying that the highest usable energy product is about 300 kJ/m<sup>3</sup> (equivalent to 35-40 MGOe). At 100°C, such characteristics are barely within the range of the best available neodymium-iron-boron or rare-earth magnets. According to this argument, which has been made before in connection with line-start motors,<sup>1</sup> it is just as important to develop magnet materials with 'moderate' properties and low cost as it is to develop 'super magnets' regardless of cost. The long awaited material with cobalt-samarium properties at ferrite prices is unfortunately still awaited, although progress is being made with neodymium-iron-boron.

### Operating temperature range

Fig. 2 shows the demagnetisation curves for several common motor magnets at 25 and 125°C. Because of the degradation in the remanent flux density and in the coercive force, the choice of material and the magnet volume must often be determined with reference to the highest operating temperature. Fortunately brushless motors have very low rotor losses. The stator is easily cooled because of the fine slot structure and the proximity of the outside air. Consequently the magnet can run fairly cool (often below 100°C), and it is further protected by its own thermal mass and that of the rest of the motor. The short-time thermal overload capability of the electronic controller would normally be less than that of the motor, providing a further margin of protection against magnet over-temperature.

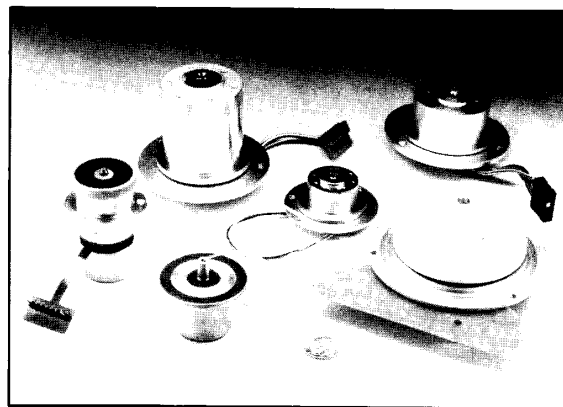
### Severity of operational duty

Magnets can be demagnetised by fault



**1A Rotor of brushless PM motor with bonded magnet ring for Hall-effect commutation sensor [Courtesy: Walter Jones & Co. (Engineers) Ltd.]**

**1B Selection of computer-peripheral brushless PM motors with internal stators. The permanent magnets are mounted on the inside of the rotor hub and rotate with it. The laser-scanner motor (bottom right) is integrally mounted on the driver circuit card [Courtesy: Synektron Corporation, Portland, Oregon, USA]**



currents such as short-circuit currents produced by inverter faults. In brushless motors with electronic control the problem is generally limited by the protective measures taken in the inverter and the control. (In line-start AC PM motors the problems are more severe.<sup>2)</sup> With an over-running load, or where two motors are coupled to a single load, short-circuited turns or windings can be troublesome because of drag torque and potential overheating of the stator. But, by the same token, the dynamic braking is usually excellent with a short-circuit applied to the motor terminals. As is often the case, characteristics that are desirable for one application are undesirable for another. The design must accommodate all the factors that stress the magnet, not only electromagnetic but thermal and mechanical as well.

#### Why permanent-magnet motors?

Because of the natural laws of electromagnetic scaling there is an 'excitation penalty' associated with small motors.<sup>6</sup> As the geometrical size is decreased, the cross-sectional area available for copper conductors decreases faster than the need for MMF. The per-unit copper losses increase and the efficiency decreases. The loss-free excitation provided by permanent magnets is therefore of increasing value as the motor size is reduced. In larger motors magnets can help achieve very high efficiency. But in larger motors the excitation penalty is small and the magnet cost becomes prohibitive. It is therefore rare to find PM motors rated much larger than a few kilowatts.

There is no hard-and-fast power level below which permanent-magnet excitation becomes advantageous, but it is possible to examine the excitation penalty in ways which indicate roughly where the breakpoint lies, and why. For a given level of excitation the choice can be made between magnets or copper windings operating at a current density  $J$  (in the copper).

It can be shown that the ratio of magnet volume  $V_m$  to the volume  $V_c$  of copper required to produce the same fundamental air-gap flux density  $B_1$  is

$$\frac{V_m}{V_c} = \frac{\mu_0 \mu_{rec} J B_1 D}{8 k_1 B_r^2 p^2 \gamma (1 - \gamma)}$$

where  $D$  is the stator diameter,  $B_r$  is the

remanent flux density of the magnet;  $p$  is the number of pole pairs; and  $k_1$  is the ratio of  $B_1$  to the maximum air-gap flux density produced over the pole arc by the magnet when the motor is on no load.  $\mu_{rec}$  is the relative recoil permeability of the magnet. The parameter  $\gamma$  is the ratio of the actual flux density in the magnet at no load to the remanent flux density, and a value of 0.8 is chosen for illustration.

Consider two four-pole motors with  $B_1 = 0.7$  T and  $k_1 = 1.1$ . The PM motor has rare-earth magnets with  $B_r = 0.8$  T and  $\mu_{rec} = 1.05$ . The electrically excited motor has  $J = 4$  A/mm<sup>2</sup>, giving

$$\frac{V_m}{V_c} \approx \frac{D}{1000}$$

where  $D$  is measured in millimetres. This means that, for motors less than 1000 mm in diameter, the magnet volume is less than the volume of copper needed for excitation in a separate field winding. Unfortunately the cost per unit volume of high-energy magnets at this level is of the order of 25 times that of copper. For the magnet cost to be less than the cost of the copper in a separate field winding, the motor diameter must therefore be less than  $1000/25$ , i.e. only 40 mm. This result suggests that the technical potential of high-energy magnets is offset by their very high cost in all but the smallest motors. In very small motors a smaller value should be used for the current density  $J$ ; with  $J = 2.5$  A/mm<sup>2</sup> the diameter for equal cost would be increased from 40 mm to 64 mm (2.5 in). In general, high-energy magnets can only be justified where there is a special premium on efficiency or compactness. Of course, this argument is simplistic, and ignores factors such as process and manufacturing costs and many others; but it provides a basic physical understanding of the application potential of magnets, and the effects of scale.

Motors magnetised with ceramic magnets must settle for a lower air-gap flux density. Using values of  $J = 4$  A/mm<sup>2</sup>;  $B_1 = 0.3$  T;  $\mu_{rec} = 1$ ;  $B_r = 0.35$  T, the result is

$$\frac{V_m}{V_c} = \frac{D}{460}$$

For motors of less than 460 mm stator diameter the magnet volume indicated is less than the volume of copper in a separate field

winding. Ceramic magnets are much less expensive than high-energy magnets, the cost per unit volume being of the order of 0.6 times that of copper, so that the magnet cost will be less than the cost of field copper in motors of diameter less than 460/0.6, i.e. 767 mm. In practice, PM motors as large as this are relatively uncommon. With ferrite magnets the flux density is too low, and with rare-earth magnets the cost is too high.

If running costs are taken into account, the comparison between PM and electrically excited motors changes significantly. With the present cost of raw materials and the present kWh tariff, the kWh cost of electrical excitation would outstrip the raw-material cost of the copper in just a few months, assuming the motor runs at full excitation 24 h per day. Even when all the manufacturing costs are added up, the PM motor will eventually pay for itself in this way. How long it takes is a complicated calculation beyond the scope of this article.

### Torque and speed control

Fig. 3 summarises the operation of the motor. Fig. 3a depicts a three-phase motor with 12 slots (i.e. two slots per pole per phase). Fig. 3b shows the current flowing through phases a and b of the motor, supplied by switches 1 and 6 in the controller. Fig. 3c shows the spatial distribution of air-gap flux at the instant when the north-pole axis of the rotor is 90° out of alignment with phase a. The flux linkage of phase a is therefore zero, but the induced EMF is a maximum. This instant is indicated by a vertical line on Fig. 3d, which is the time diagram of the induced voltage and the corresponding phase current. Although Fig. 3 shows a two-pole motor, the relationships are identical in motors of any pole number if the angles are expressed in electrical degrees.

The motor shown in Fig. 3 is an inverted DC motor with the mechanical commutator replaced by the electronic controller. Using the 'BLv' and 'BLi' formulas for EMF and force, the induced voltage  $E$  and the torque  $T$  can be derived:

$$E = 2NBLr\omega \quad \text{volts/phase}$$

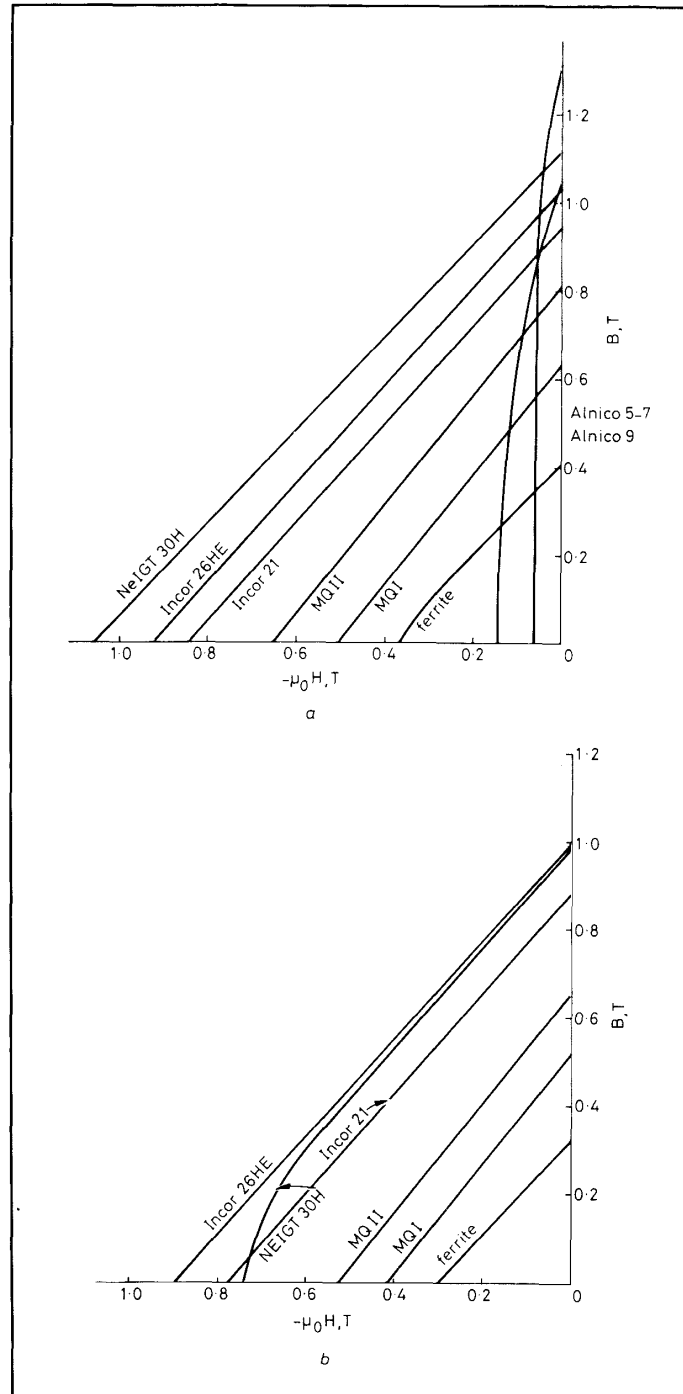
$$\text{and } T = 4NBLrl \quad \text{newton metres}$$

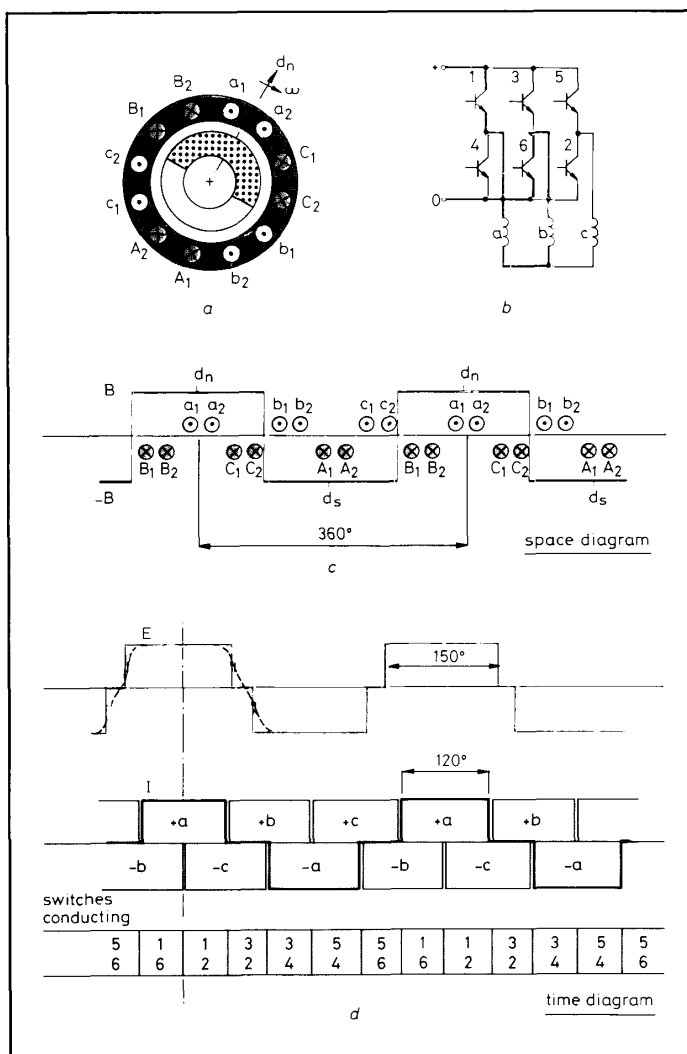
where  $N$  is the number of turns in series per phase;  $B$  is the air-gap flux density produced by the magnet;  $L$  is the active length;  $r$  is the rotor radius;  $l$  is the phase current; and  $\omega$  is the angular velocity. In Fig. 3 two phases are conducting at any time, the conduction angle being 120° electrical. The current waveform can be maintained approximately rectangular by voltage PWM, and there are several commercially available integrated circuits that perform both the commutation and the voltage PWM; some of these even provide facilities for speed feedback.<sup>6</sup>

The torque and EMF equations are identical in form to those of the PM commutator motor. Torque is proportional to the product of flux and current, while EMF is proportional to the product of flux and speed. If the phase resistance and leakage inductance are sufficiently small, and if other losses are

negligible, the EMF  $E$  is equal to the per-phase terminal voltage  $V/2$ . This results in a speed/torque characteristic of the form of Fig. 4. Control is effected through pulsewidth modulation of the voltage, and if a closely regulated constant speed is required it may well be necessary to add a tachogenerator and a speed feedback loop in addition to the position sensor used for commutation. Full torque can be maintained over a wide range of speed, and the maximum speed is determined by the driving voltage  $V$  available from the controller. Since the motor EMF is proportional to speed, this limit tends to be a sharp one,

2 Second-quadrant demagnetisation curves (B/H curves) of high-energy permanent magnet materials: (a) 25°C; (b) 125°C





**3 Principle of operation of idealised brushless PM motor: (a) two-pole motor schematic; (b) electronic controller (freewheel diodes omitted); (c) air-gap flux distribution at the instant shown in (a); (d) EMF and ideal phase-current waveforms**

and occurs as soon as the PWM duty cycle reaches 100%.

Further increases of speed can then be obtained by advancing the conduction angle, or by a combination of both these techniques.<sup>3</sup> Although the motor speed may be increased, it is difficult to maintain constant power: the torque tends to fall away rather rapidly.

#### Sinewave-fed motors

It is not possible to formulate a simple analytical expression describing the increase of speed above 'base' speed, the highest speed at which maximum torque can be obtained. Such a result is more easily obtained by simulation or experiment. However, such an expression can be derived for the synchronous form of PM motor from an analysis of its circle diagram. The brushless DC motor with magnets mounted on the rotor surface is similar to a synchronous AC motor with no 'saliency'. If the stator winding is distributed and short-pitched, and if the phase currents are sinusoidal, then the motor can be analysed using  $d, q$ -axis theory.

The limiting torque/speed characteristic of the sinewave-fed synchronous motor is shown in Fig. 5. At speeds below 'base' speed the

voltage  $V$  can be increased in proportion to the frequency, so that maximum current and maximum torque can always be obtained, even though the motor EMF  $E$  is increasing. At the base speed the controller voltage reaches its maximum. Now as the supply frequency increases the torque decreases until it reaches zero at some frequency  $k$  times the frequency at base speed. At this point it is still possible to get rated current into the motor; but if losses are neglected it is entirely in the direct axis and no torque is produced. The following expression for  $k$  illustrates the factors that control the high-speed operation:

$$k = \frac{1}{e - \sqrt{1 - e^2}}$$

where  $e$  is the per-unit EMF of the motor at base speed, i.e.  $E/V$ . Neglecting all losses, it can be shown that  $e = \cos \phi$ , the power factor at base speed, and that  $\sin \phi = x_s$ , the per-unit synchronous reactance. It is characteristic of this type of motor that  $x_s$  is quite small, giving a high power factor at base speed. In motors with ceramic magnets this results from the need for a long magnet length in the direction of magnetisation (to prevent demagnetisation). In motors with high-energy magnets  $x_s$  is small for a different reason: it is proportional to the ratio of electric loading to magnetic loading,<sup>4</sup> and this ratio tends to be quite low in such motors (correspondingly, the short-circuit current is high). If the per-unit synchronous reactance (which includes stator leakage) is, for instance,  $x_s = 0.15$ , the base-speed power factor is 0.99,  $e = 0.99$  per unit and  $k = 1.19$ , indicating a very limited capability to operate above base speed. In the case of the synchronous motor, this is the best that can be done; there is nothing to be gained from advancing the phase or duration of the current 'pulses', since these are sinewaves whose phase is already as far advanced as is possible with the available driving voltage.

Other forms of permanent-magnet AC and brushless motors are possible besides the surface-magnet motor, and some of these have magnetic saliency with a significant difference between the  $d$ - and  $q$ -axis reactances. In these motors there is an appreciable reluctance torque; and correspondingly the air-gap flux is not fixed solely by the magnet, but can be controlled to some extent by the magnitude of the stator current and its phase relative to the rotor position.<sup>10</sup> The speed range above base speed is then wider. A different type of motor with similar properties is shown in Fig. 6. This motor, which is similar to an inductor-type machine, has permanent magnets at each end of the rotor, but there is a significant component of reluctance torque, which helps to provide a wider range at constant power above base speed. The permanent magnets in this machine can be replaced by a simple field coil, which permits full control of the air-gap flux. The price paid for these operational advantages is a longer rotor, but in many applications this may be acceptable.

In these hybrid motors the reluctance and PM torques can only be satisfactorily combined

if the supply is sinusoidal and the windings more or less sinusoidally distributed. If the magnets are completely removed or demagnetised the resulting motor is a cageless synchronous reluctance motor.<sup>6-10</sup>

### Slotless motors

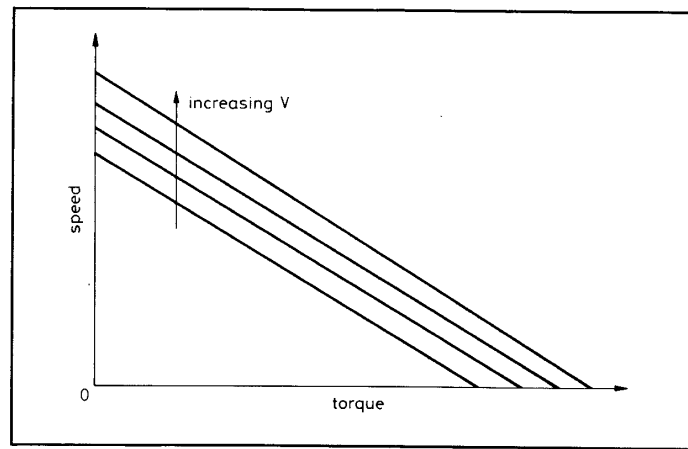
A pure synchronous machine produces no torque ripple, and the same is true of the ideal brushless DC motor with perfectly concentrated windings, rectangular air-gap flux distribution, and rectangular current waveforms. But neither of these machines can be realised perfectly in practice, and there is always a certain amount of torque ripple. Torque ripple in PM motors is a parasitic effect associated with departures from the ideal structure and the ideal control. These departures are inevitable in any practical motor.

In the last three or four years the availability of extremely high-energy neodymium-iron-boron magnets has re-awakened interest in the slotless motor, in which the stator teeth are completely removed and the resulting space is partially filled with additional copper. At least one such motor is manufactured commercially. The slotless construction permits an increase in rotor diameter within the same frame size, or alternatively an increase in electric loading without a corresponding increase in current density. The magnetic flux density at the stator winding is inevitably lessened, but the effect is not so drastic as might be expected. For a motor with an iron stator yoke and an iron rotor body the magnetic field and its harmonic components can be calculated by the methods of Reference 5. Considering the fundamental radial component of  $B$ , the value is greatest at the rotor surface (radius  $r$ ) and falls off with increasing radius to its smallest value just inside the stator yoke (radius  $R$ ). The ratio between the values of the fundamental radial component at these two radii is given by

$$b = \frac{2(r/R)^{p+1}}{1 + (r/R)^{2p}}$$

Consider a rotor of 40mm diameter with a high-energy magnet of remanent flux density 1.2T and thickness 5mm (Fig. 7). If the radial thickness of the stator winding is 5mm (including the air gap), then, for a four-pole magnet,  $b = 0.78$ . The magnet flux density will be about half the remanent flux density with these proportions, so that the radial flux density in the stator winding varies from about 0.6T near the bore to 0.47T just inside the stator yoke, giving a mean value of about 0.53T (fundamental). Given that the electric loading may be increased relative to that of a slotted stator, the power density should be roughly the same and possibly a little higher, since the stator-tooth iron losses are eliminated. This machine may well accept less expensive grades of lamination steel because of the absence of slotting and the relatively low flux density in the stator yoke. The reactance is also lessened by the elimination of slot-leakage effects, and the risk of demagnetisation is decreased.

In this type of motor the maximum usable



**4 Speed/torque characteristic of ideal brushless motor. If resistance, leakage inductance and losses are negligible, the curve is straight**

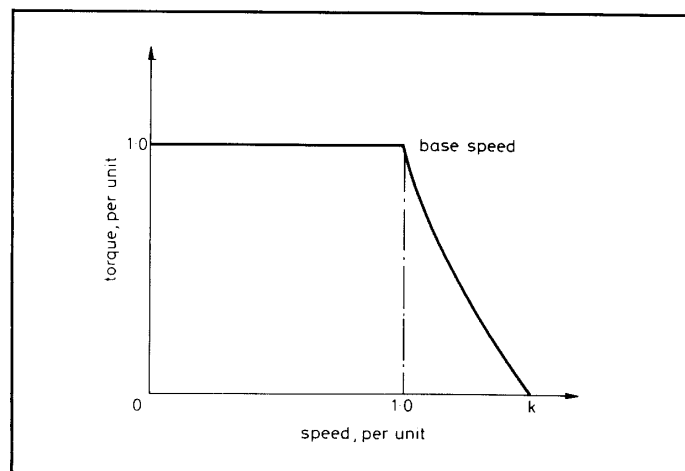
magnet energy is obviously higher than in a conventional slotted motor; indeed the concept would not be viable at all without magnets of high remanence and coercivity.

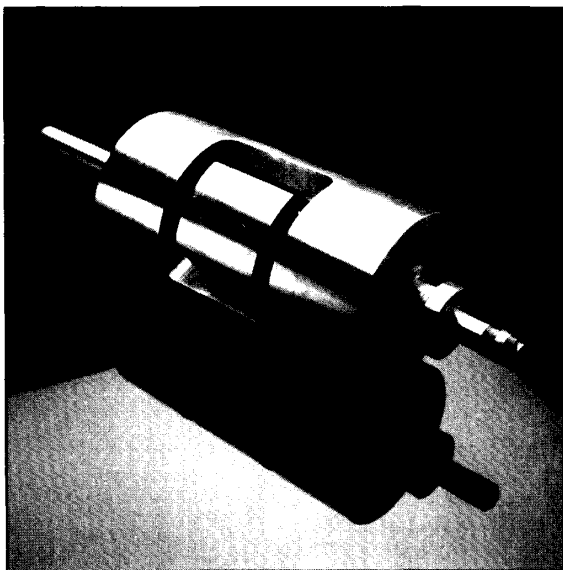
Once the stator teeth are removed, the conductors are no longer constrained to lie parallel to the axis. They may be skewed by a small amount to reduce torque ripple (which is already reduced by the elimination of cogging effects against the stator teeth). A further possibility is a completely helical winding such as that proposed for superconducting AC generators,<sup>11</sup> or as used in very small PM commutator motors. Because the helical winding has no end turns, its utilisation of copper is higher than the severe skew might suggest, and it should permit the design of a very compact motor.

### Conclusion

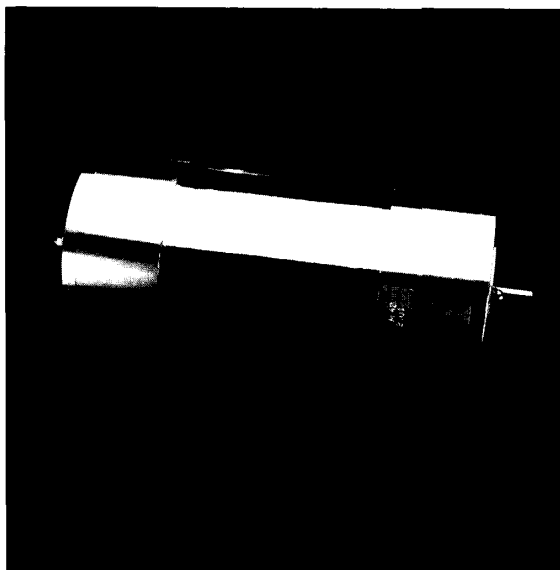
Permanent magnets are the natural means of exciting small motors. Without them the brushless DC motor would have to rely on a wound rotor, or purely on reluctance torque. In both cases, the performance would suffer and the control would tend to become more complex for the same operational flexibility. Ease of speed control, bidirectional operation, good dynamic braking and a simple commutation strategy are among the attractive features of these drives. As the controller costs become relatively less, and as application requirements for controllability and

**5 Idealised speed/torque curve of synchronous PM motor**

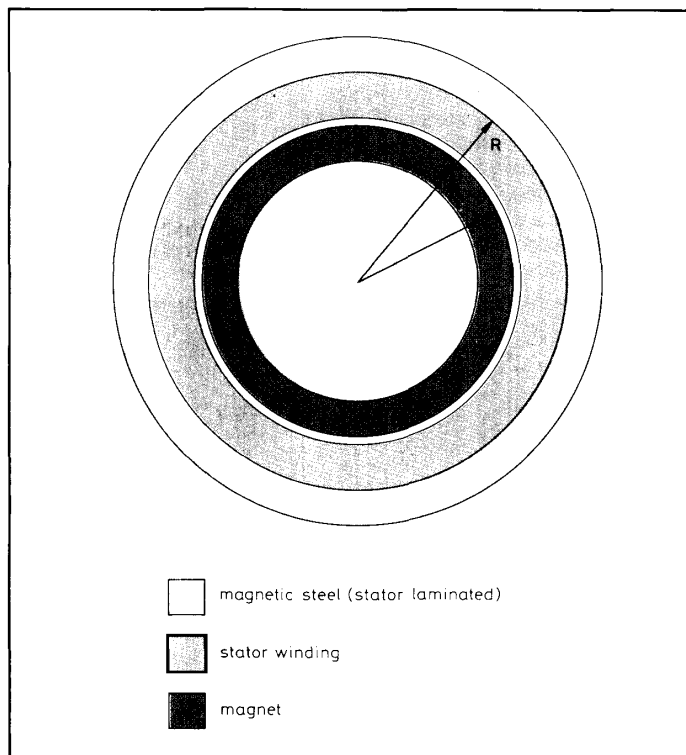




**6 Brushless variable-reluctance motor (VRM) with permanent-magnet excitation: (a) rotor; (b) complete motor**  
[Courtesy: Magnetics Research International, Fairfield, Iowa, USA]



**7 Slotless-stator brushless PM motor**



low maintenance increase, the brushless PM motor is certain to expand its applications over a wide range of industrial, aerospace and commercial areas.

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