# Module 2. Univariate statistics

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GENT

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## **Learning Goals**

- Descriptive statistics
- Central tendency and dispersion for each measurement level
- Know formulas, being able to calculate
- Suitable visualization techniques for each measurement level



# Central Tendency and Dispersion



# How tall are my friends?

### Remember our superheroes:





## **Measure of Central Tendency**

What value is representative of the entire group?



## Mean or Average

#### Arithmetic mean

The arithmetic mean (notation:  $\overline{x}$ ) is the sum of all values divided by the number of values

$$\overline{x} = \frac{1}{n} \sum_{x=1}^{n} x_i$$

<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>X</i> <sub>3</sub>	X <sub>4</sub>	<i>X</i> <sub>5</sub>
141	198	143	201	184



## Mean or Average

- Q1 What happens if Ant-Man shrinks to a size of 10 cm?
- Q2 The arithmetic mean of 15 numbers is 12. What number should be added to get a mean of 13?





### Median

#### Median

To find the median, sort all values and pick the middle number

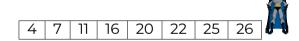
- Odd number of values: no problem
- Even number of values: average of the middle two

<i>X</i> <sub>1</sub>	$X_2$	<i>X</i> <sub>3</sub>	X <sub>4</sub>	<i>x</i> <sub>5</sub>
141	198	143	201	184



### Median

- Q1 What happens if Ant-Man shrinks to a size of 10 cm?
- Q2 What is the median of the number of people saved by Batman during the last eight years?



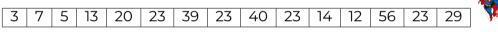


### Mode

#### Mode

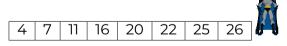
The mode is the value that appears most often in a dataset.

Number of people saved by Superman during the last 15 years:





Number of people saved by Batman during the last 8 years:





## **Measures of Dispersion**

How large are the differences within the group?



## Range

### Range

The range of a dataset is the absolute value of the difference between the highest and the lowest value.

<i>x</i> <sub>1</sub>	<i>X</i> <sub>2</sub>	<i>X</i> <sub>3</sub>	<i>X</i> <sub>4</sub>	<i>X</i> <sub>5</sub>
141	198	143	201	184



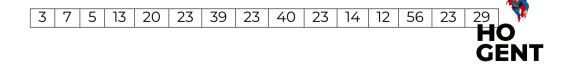


## **Quartiles**

### **Quartiles**

The quartiles of a sorted set of numbers are the three values that divide the set into 4 equally large subsets. Notation:  $Q_1$ ,  $Q_2$ ,  $Q_3$ 

Number of people saved by Superman during the last 15 years:



## **Calculating Quartiles**

- Different software programs have slightly different ways of calculating quartiles.
- The following method is easy to perform by hand. Start by sorting the values.
  - o When n is odd.
    - The median  $(Q_2)$  is the middle value (as before).
    - Leave out the median.  ${\it Q}_{1}$  is the median of the first half,  ${\it Q}_{3}$  is the median of the second half.
  - o When n is even.
    - The median  $(Q_2)$  is the average of the two middle values.
    - $-Q_1$  is the median of the first half,  $Q_3$  is the median of the second half.



## Interquartile Range (IQR)

### Interquartile Range

The interquartile range is the difference between the third and first quartile  $|Q_3 - Q_1|$ .



### Variance and Standard Deviation

#### Variance

The variance ( $s^2$  or  $\sigma^2$ ) is the mean squared difference between the values of a data set and the arithmetic mean.

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2$$

#### Standard deviation

The standard deviation (s or  $\sigma$ ) is the square root of the variance

<i>X</i> <sub>1</sub>	<i>X</i> <sub>2</sub>	<i>X</i> <sub>3</sub>	X <sub>4</sub>	<i>X</i> <sub>5</sub>
141	198	143	201	184



• Can the standard deviation be negative?



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- What effect do outliers have on the standard deviation?
- What is the unit of the standard deviation (in relation to the unit of the variable)?
- How do you interpret the standard deviation combined with the average?



Why n-1 in the denominator and not n? You can prove the reason for the change mathematically, but we will investigate it empirically

See Python example code in demo-analysis-1-var.ipynb





This news item reports on high prices for houses and flats. Do the numbers give a good idea of the situation?

### Remember!

### Providing only a center value is never sufficient!

- What is the dispersion?
- How is the data distributed? Normal distribution?
- Is the group sufficiently homogeneous?



# **Central Tendency and Dispersion: Summary**

Measurement Level	Center	Spread Distribution
Qualitative	Mode	_
Quantitative	Average/Mean Median	Variance, Standard Deviation Range, Interquartile Range



## **Summary of Symbols**

	Population	Sample
number of elements	N	n
average or mean	μ	$\overline{x}$
variance	$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$	$s^2 = \frac{\sum (x_i - \overline{x})^2}{n - 1}$
standard deviation	σ	S



## **Data visualisation**



# **Chart type overview**

Measurement level	Chart type
Qualitative	Bar chart
Quantitative	Boxplot Histogram Density plot

See Python-example code in demo-analysis-1-var.ipynb



### **Pie Chart**

### Attention!

Avoid using a pie chart!

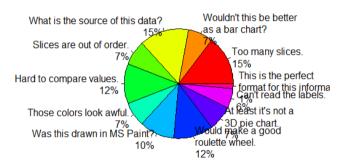
### Disadvantages:

- Comparing angles is harder than comparing length
- Unusable for data with many categories



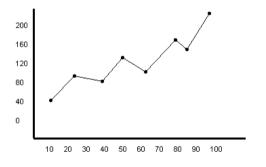
### **Pie Chart**

#### What people are saying about your pie chart





## **Interpretation of Charts**



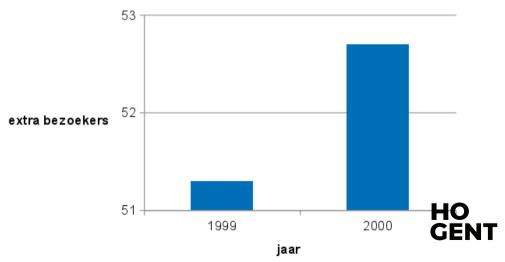
#### Tips:

- Label the axes
- Add a clear title
- Name the unit (and, if necessary, order of magnitude)
- Add a label that clarifies the chart



## **Data distortion**

= misrepresenting data so that invalid conclusions are drawn



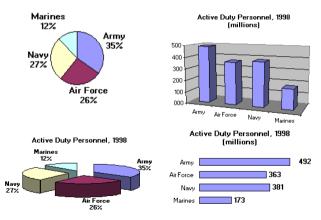
### **Data distortion**



### **Data distraction**

- Avoid bells and whistles
- Minimize "ink to data" ratio

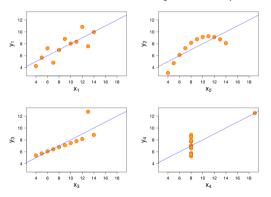
Active Duty Personnel, 1998



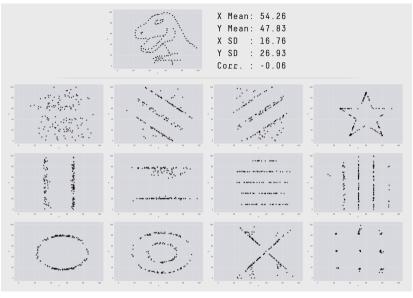


# The importance of data visualization

**Anscombe's Quartet** are four completely different datasets with the same measurements of central tendency and dispersion.







"The Datasaurus Dozen" (Source: https://www.autodeskresearch.com/publications/samestats)