Module 6. Bivariate analysis: quantitative—quantitative

Data Science

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2024–2025

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Learning goals

- Determine the equation of the regression line and plot it;
- Calculate the covariance *Cov*, the correlation coefficient *R* and the coefficient of determination R^2
- Interpret these values using the correct terms;
- Visualization

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Bivariate analysis: overview

| Independent	Dependent	Test/Metric
Qualitative	Qualitative	χ^2-test Cramér's V
Qualitative	Quantitative	two-sample t-test Cohen's d
Quantitative	Quantitative	— Regression, correlation

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Data visualization

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Data visualization

To visualize quantitative data, we use a *scatter plot*

- X-axis: independent variable
- Y-axis: dependent variable
- Each point corresponds to an observation

Data visualization

Scatterplot
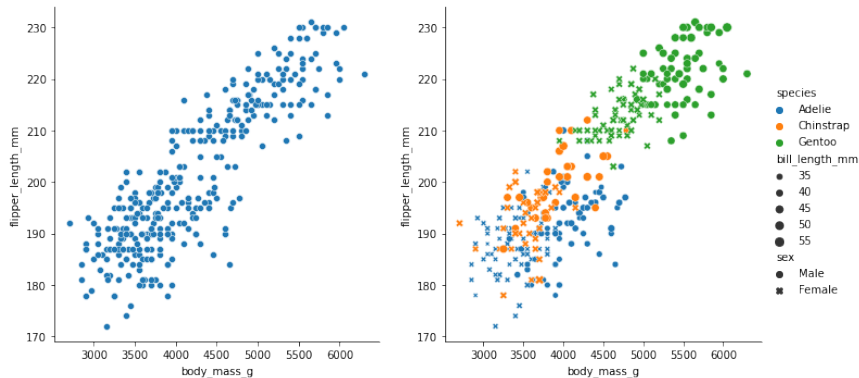
Source: Horst A., et al. (2020) palmerpenguins: Palmer Archipelago (Antarctica) penguin data,
https://allisonhorst.github.io/palmerpenguins/

Linear regression

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Linear Regression

With **regression** we will try to find a **consistent** and **systematic** relationship between two quantitative variables.

1. **Monotonic:** consistent direction of the relationship between the two variables: increasing or decreasing
2. **Non-monotonic:** value of dependent variable changes systematically with value of independent variable, but the direction is not consistent
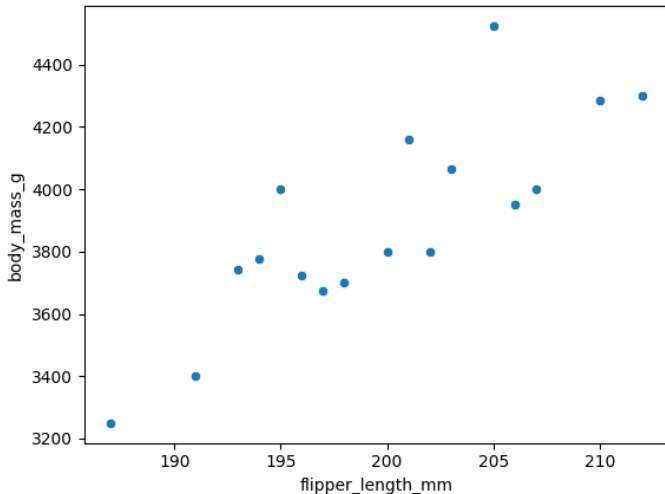
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Linear Regression

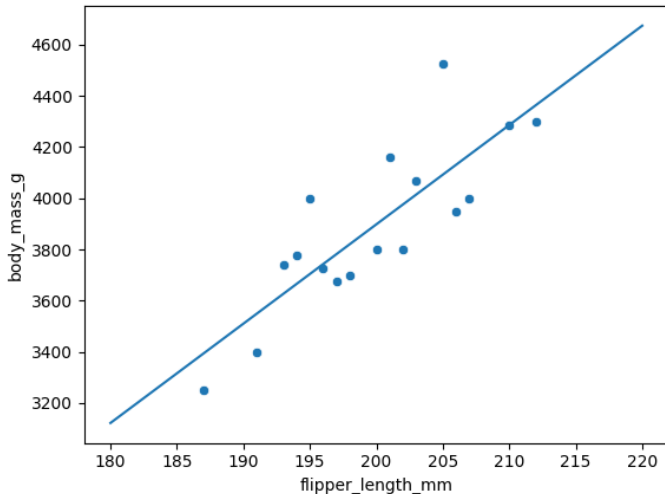A *linear* relationship between an independent and dependent variable.

Characteristics:
- Presence: is there a relationship?
- Direction: increasing or decreasing?
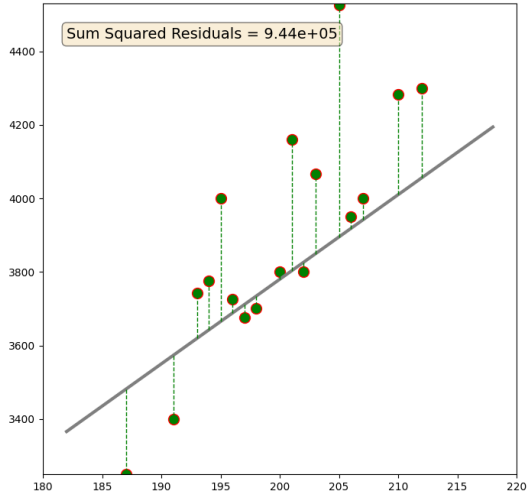- Strength of the relationship: strong, moderate, weak, nonexistent, ...
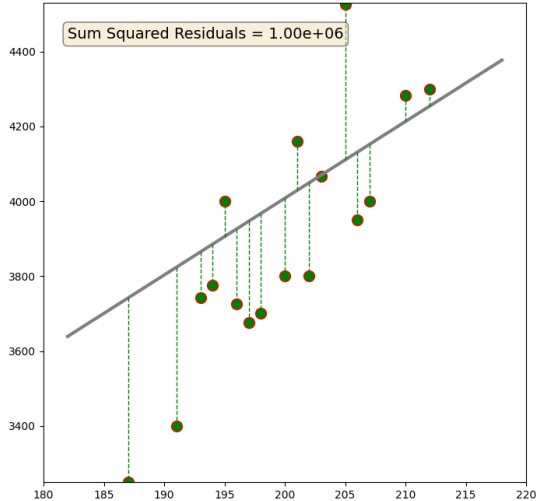
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Linear Regression

Linear Regression

Method of least squares

Method of least squares

Method of least squares

	x	y	(x-x̄)	(y-ȳ)	(x-x̄)(y-ȳ)	(x-x̄)²
0	187.0	3250.000000	-12.823529	-641.274510	8223.402537	164.442907
1	191.0	3400.000000	-8.823529	-491.274510	4334.775087	77.854671
2	193.0	3741.666667	-6.823529	-149.607843	1020.853518	46.560554
3	194.0	3775.000000	-5.823529	-116.274510	677.128028	33.913495
4	195.0	4000.000000	-4.823529	108.725490	-524.440600	23.266436
5	196.0	3725.000000	-3.823529	-166.274510	635.755479	14.619377
6	197.0	3675.000000	-2.823529	-216.274510	610.657439	7.972318
7	198.0	3700.000000	-1.823529	-191.274510	348.794694	3.325260

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Method of least squares

Equation

The regression line has the following equation:

$$\hat{y} = \beta_1 x + \beta_0$$

with:

$$\beta_1 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x - \overline{x})^2} \approx \frac{29375.49}{756.47} \approx 38.83$$

$$\beta_0 = \overline{y} - \beta_1\overline{x} \approx 3891.27 - 38.83 \times 199.82 \approx -3868.33$$

$$\hat{y} = 38.83x - 3868.33$$

Note: $\hat{y}$ indicates "an estimation for y"

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Covariance

Covariance

Covariance

Covariance is a measure that indicates whether a relationship between two variables is increasing or decreasing.

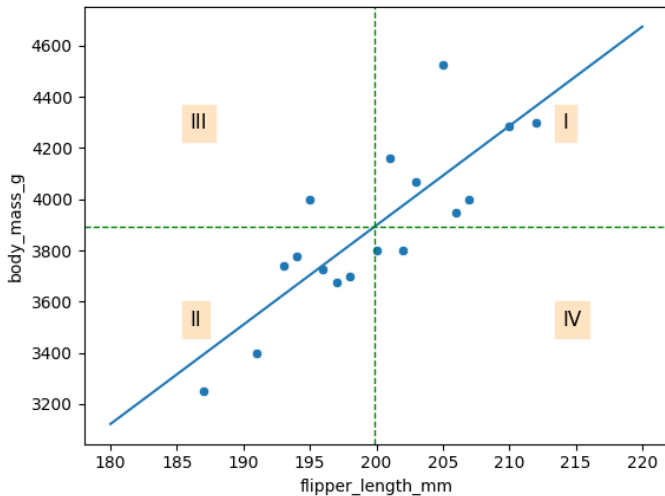$$\text{Cov}(X, Y) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})$$

Cov > 0: increasing
Cov ≈ 0: no relationship
Cov < 0: decreasing
Note Covariance of population (denominator n)
vs. sample (denominator $n-1$)

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Covariance

Covariance

$$Cov(X, Y) \approx \frac{29375.49}{17 - 1} \approx 1835.97$$

- $Cov > 0 \Rightarrow$ increasing relationship
- What if body mass was expressed in kg instead of g?

Covariance has limited use as a measure of the relationship between two variables.

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Pearson's correlation coefficient

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Pearson correlation coefficient

Pearson's Correlation Coefficient

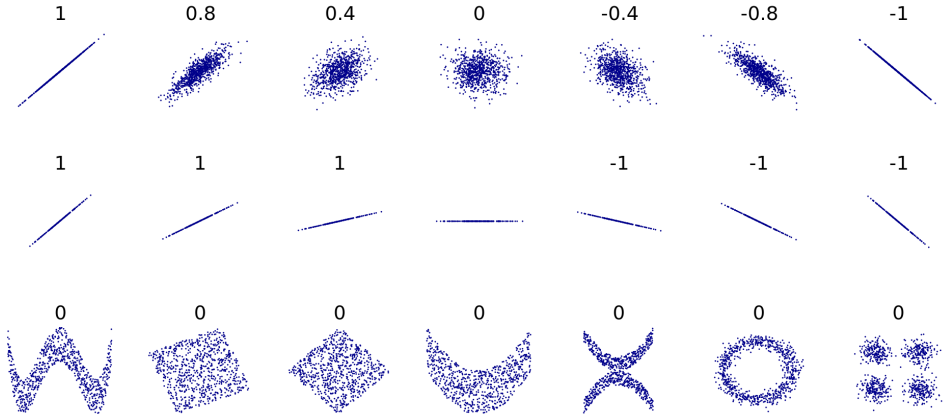**Pearson's product-moment correlation coefficient** R **is a measure for the strength of a linear correlation between** x **and** y

$$R = \frac{Cov(X, Y)}{\sigma_x \sigma_y} \tag{1}$$

$$= \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}} \tag{2}$$

$$R \in [-1, +1]$$

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Correlation coefficient

Some datasets and their R-value

Coefficient of determination

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Coefficient of determination

> **Coefficient of determination**
>
> The **coefficient of determination** R^2 **explains the percentage of the variance of the observed values relative to the regression line.**

R^2: percentage variance observations explained by the regression line
$1 - R^2$: percentage variance observations *not* explained by regression

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Interpretation of R and R^2 values

$|R|$	R^2	Explained variance	Interpretation
< 0.3	< 0.1	< 10%	very weak
0.3 – 0.5	0.1 – 0.25	10 – 25%	weak
0.5 – 0.7	0.25 – 0.5	25 – 50%	moderate
0.7 – 0.85	0.5 – 0.75	50 – 75%	strong
0.85 – 0.95	0.75 – 0.9	75 – 90%	very strong
> 0.95	> 0.9	> 90%	exceptional(!)

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Strength of relationship

Example chinstrap penguins

$$Cov(X, Y) \approx \frac{29375.49}{17 - 1} \approx 1835.97$$

$$R = \frac{Cov(X, Y)}{\sigma_x \sigma_y}$$

$$\approx \frac{1835.968}{6.876 \times 322.935} \approx 0.827$$

$$R^2 \approx 0.827^2 \approx 0.684$$

Conclusion: There is a strong linear and increasing relationship between flipper length and body mass of male chinstrap penguins. 68.4% of the variance in body mass can be explained by the variance in flipper length.

Considerations

- The correlation coefficient only looks at the relationship between **two variables**. Interactions with other variables are not considered.
- The correlation coefficient explicitly does **not** assume a **causal** relationship.
- Pearson's correlation coefficient only expresses **linear** relationships.

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