

FUN with Complexity: Walking through Doors is Hard, even without Staircases

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Content

Theory

- PSPACE-Complexity

- 1-Player Motion Planning

- Basic Door Device

- PSPACE-hardness of doors

PSPACE-Complexity

A given problem requires a polynomial amount of memory in relation to the input, to be solved \Leftrightarrow The problem is in PSPACE

SAT

$$x_1 \wedge x_2 \vee \neg x_3$$

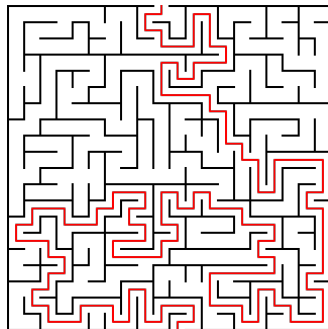
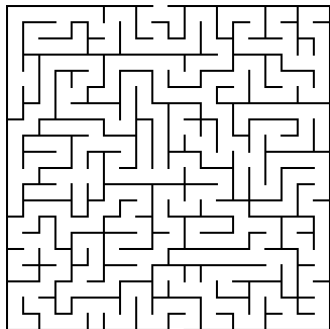
Quantified SAT

$$\forall x_1 \exists x_2 : x_1 \wedge x_2 \vee \neg x_3$$

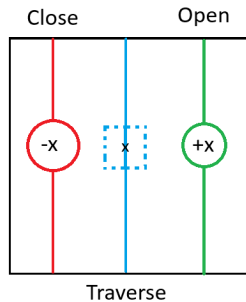
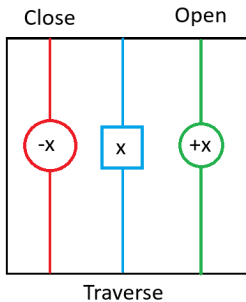
1-PlayerMotionPlaning

Given: Enviroment, Agent, Goal

Question: Is the goal achivable



Basic Door Device



PSPACE-hardness of doors

Theorem

*If a game features **door devices** which each are controlled by an **open** and a **close** **pressure plate** and the agent has to navigate from entrance to exit, then the game is **PSPACE-hard***

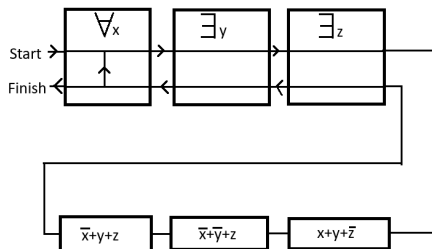
PSPACE-hardness of doors - Proof

True Quantified SAT

$$\forall x \exists y \exists z : (\bar{x} \vee y \vee z) \wedge$$

$$(\bar{x} \vee \bar{y} \vee z) \wedge (\forall x \vee y \vee \bar{z})$$

1-Player Motion Planning

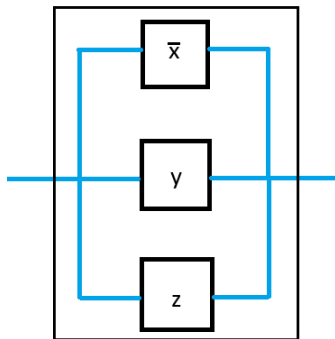


PSPACE-hardness of doors - Proof

Clause

$$(\bar{x} \vee y \vee z)$$

Gadget

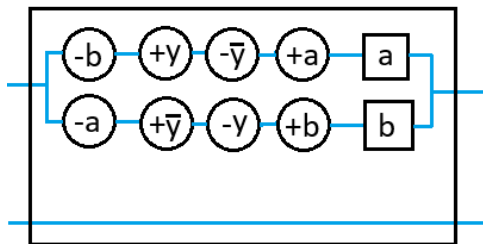


PSPACE-hardness of doors - Proof

Exists-Quantor

$\exists y$

Gadget

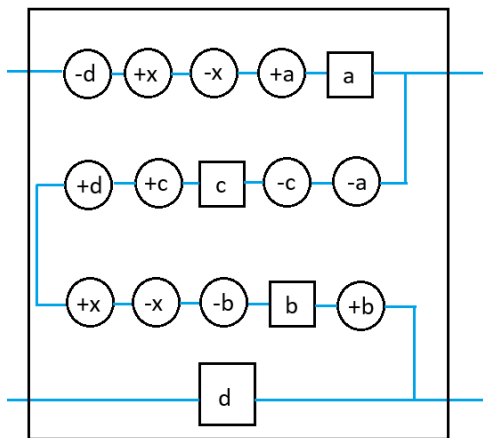


PSPACE-hardness of doors - Proof

All-Quantor

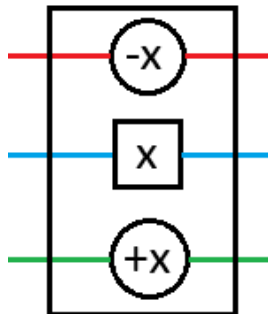
$\forall x$

Gadget

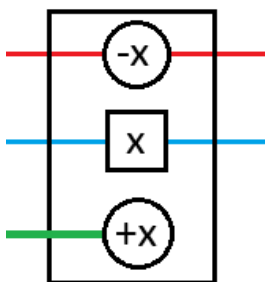


Door Device - Variants

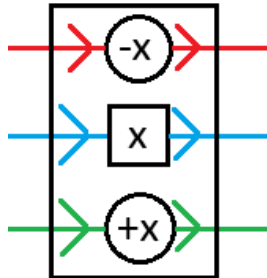
Basic



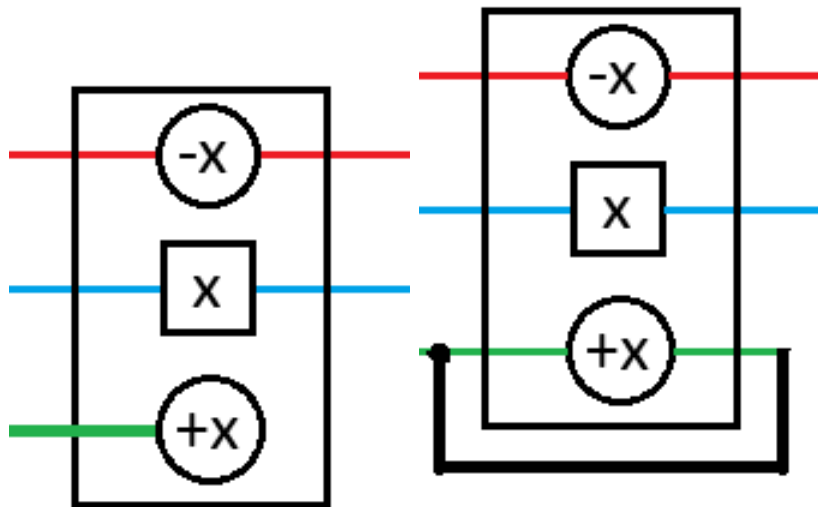
Open-Optional



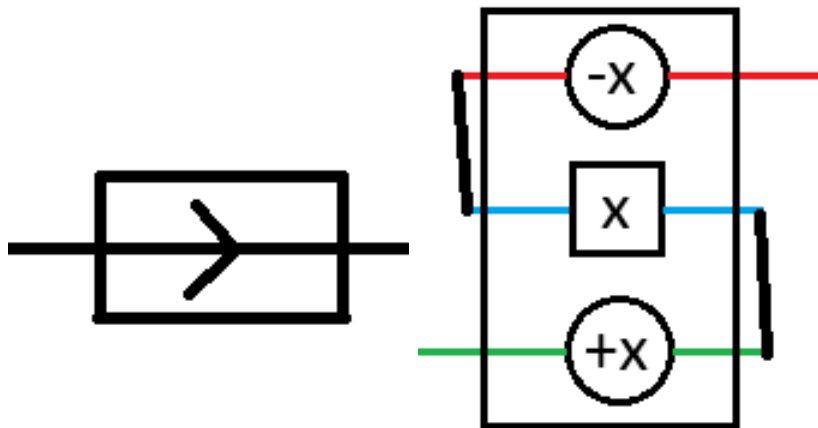
Directed



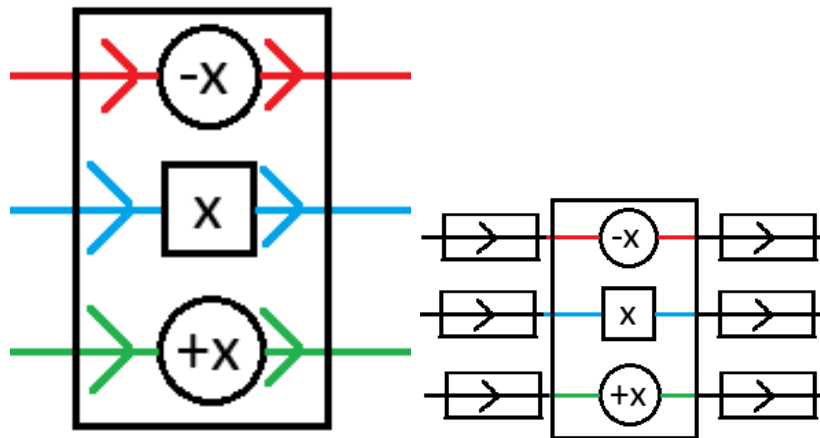
PSpace-Hardness - Open optimal door



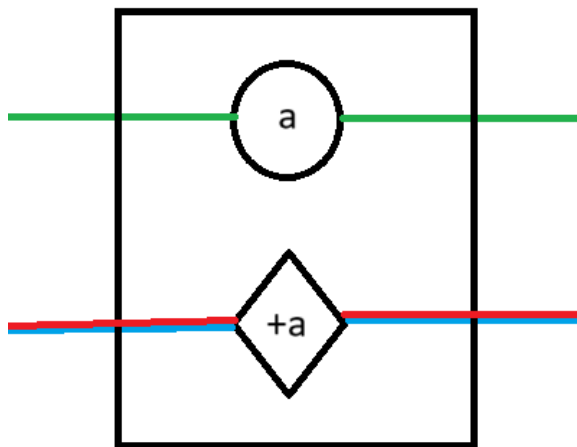
The Diode



PSpace-Hardness - Directed Door

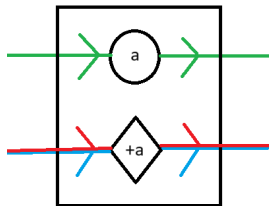


Self closing doors

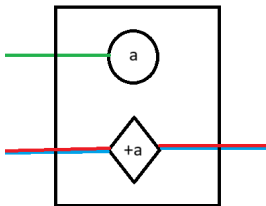


Self closing doors - Variants

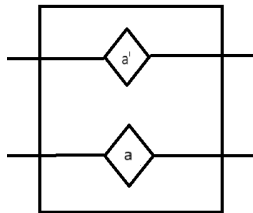
Directed



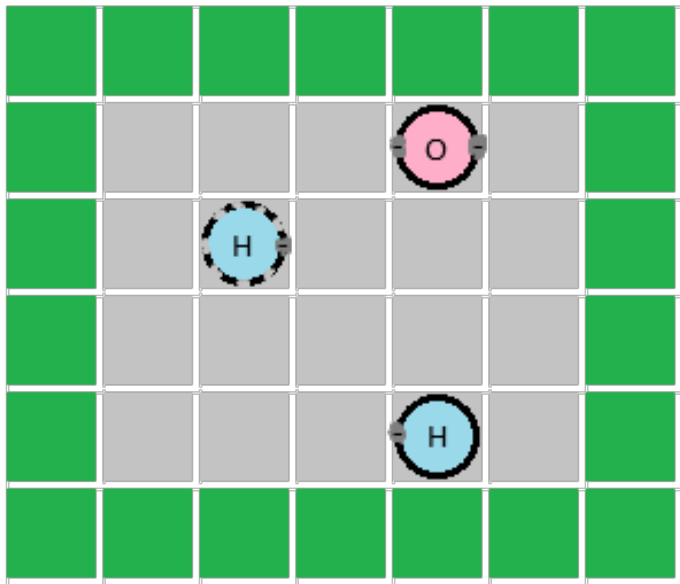
Open-Optionals



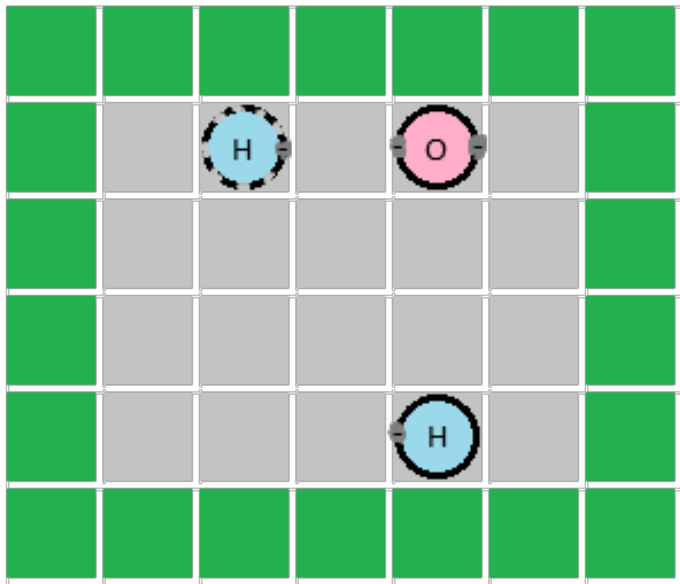
Symetric



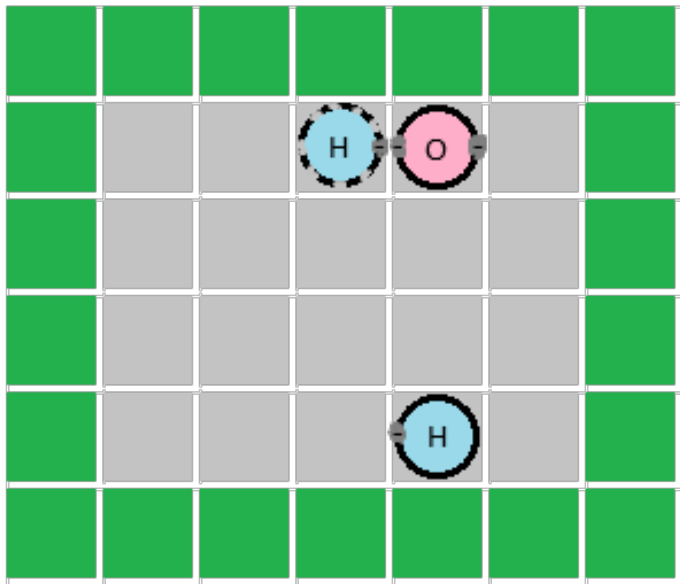
Application - Sokobond



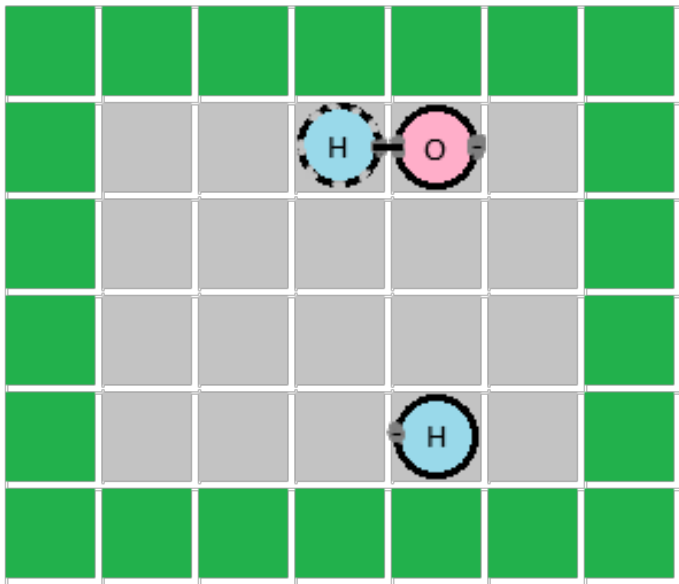
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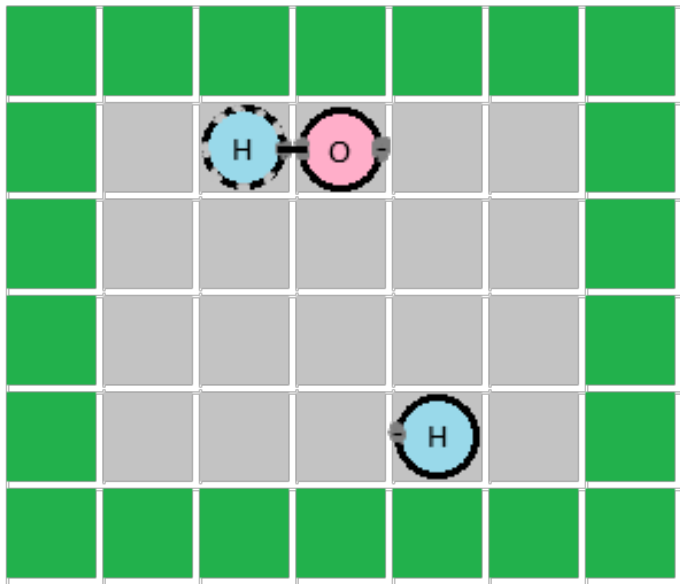
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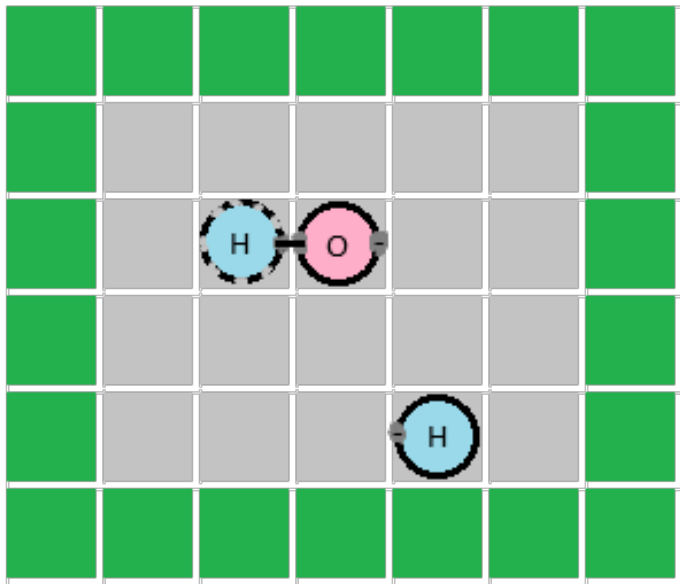
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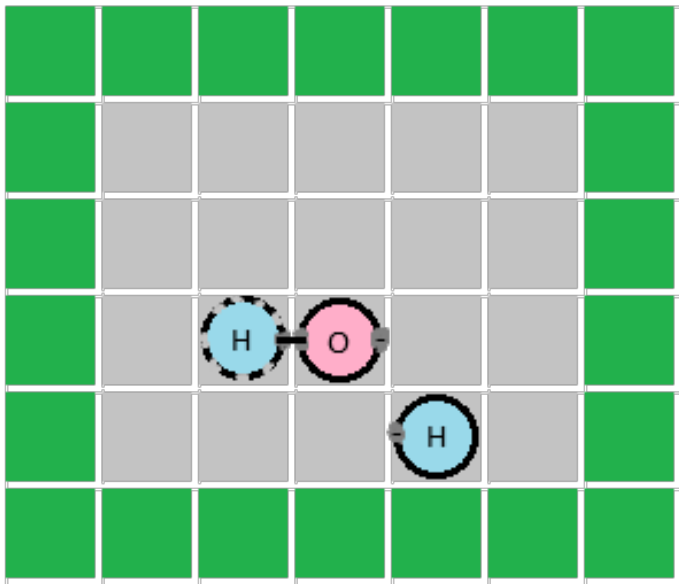
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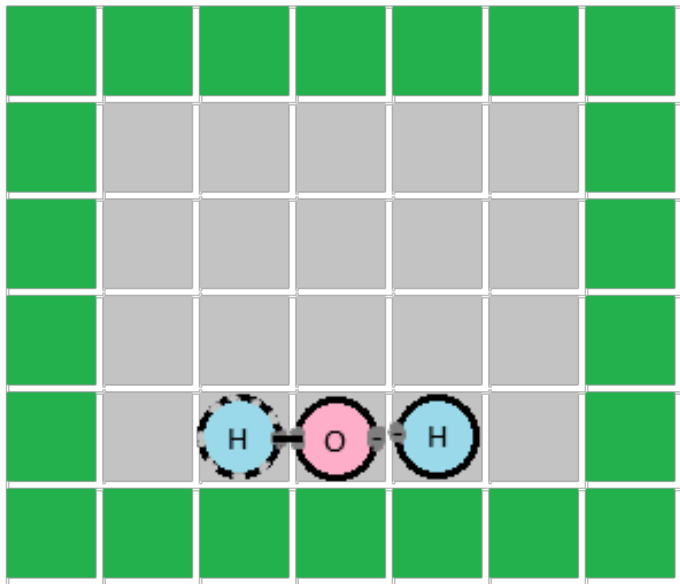
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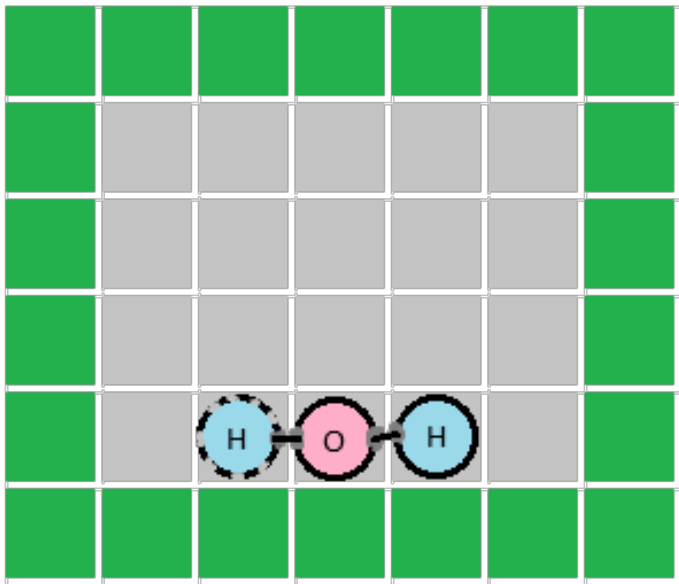
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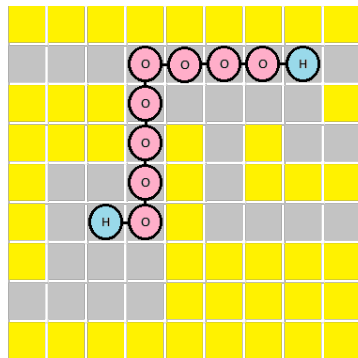
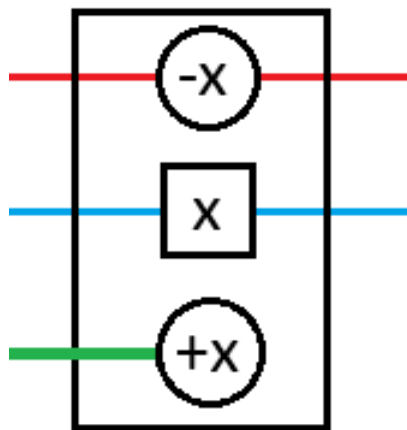
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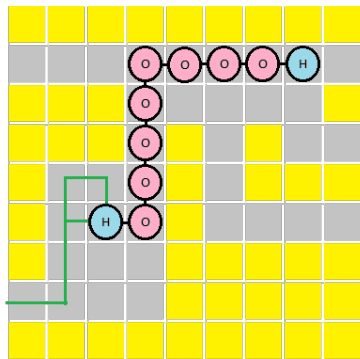
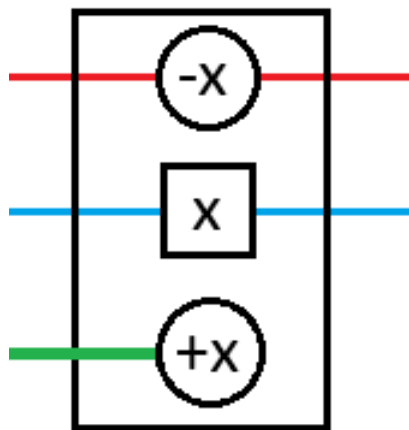
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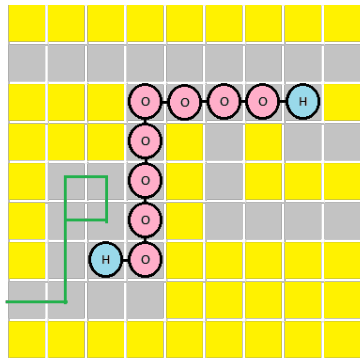
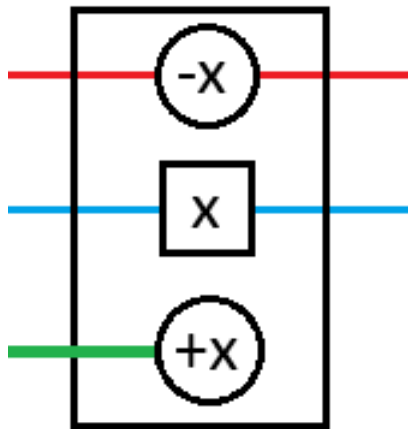
Sokobond is PSpace-Hard



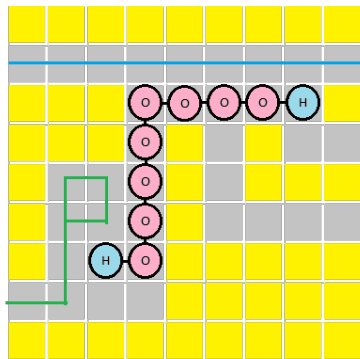
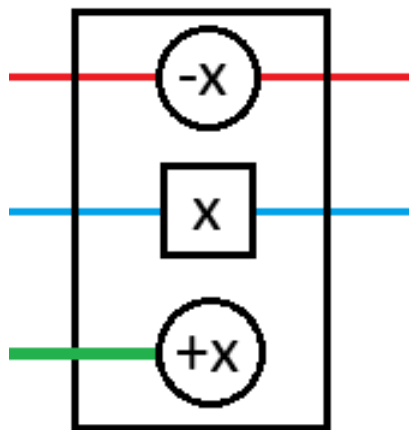
Sokobond is PSpace-Hard



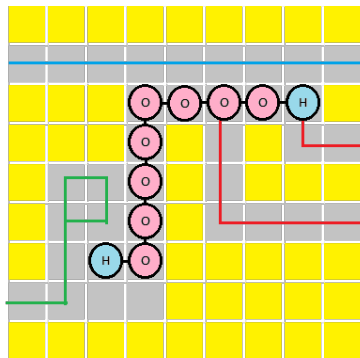
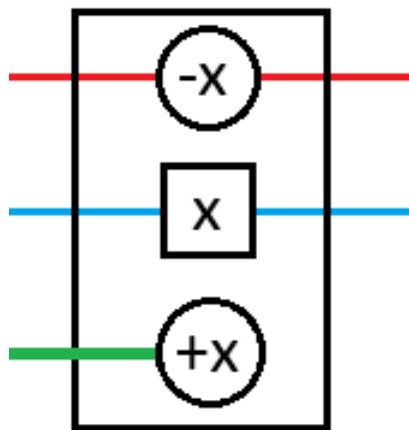
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