FUN with Complexity: Walking through Doors is Hard, even without Staircases

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Content

PSPACE-Complexity

A given problem requires a polynomial amount of memory in relation to the input, to be solved $\Leftrightarrow$ The problem is in PSPACE

SAT

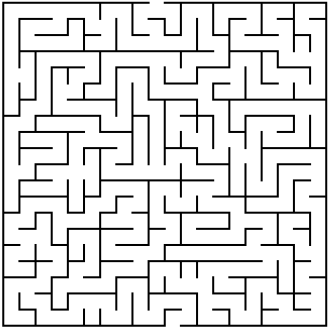
$$x_1 \wedge x_2 \vee \neg x_3$$

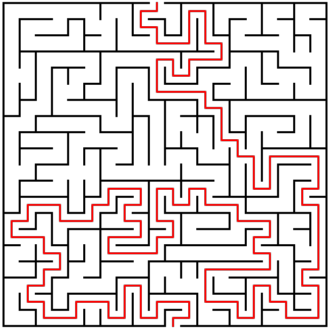
Quantified SAT

$$\forall x_1 \exists x_2 : x_1 \wedge x_2 \vee \neg x_3$$

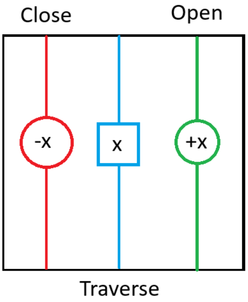
1-PlayerMotionPlaning

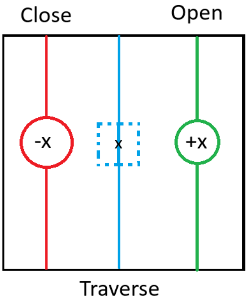
Given: Enviroment, Agent, Goal
Question: Is the goal achivable





Basic Door Device





PSPACE-hardness of doors

Theorem

If a game features **door devices** *which each are controled by an open and a close* **preasure plate** *and the agent has to navigate from entrance to exit, then the game is* **PSPACE-hard**

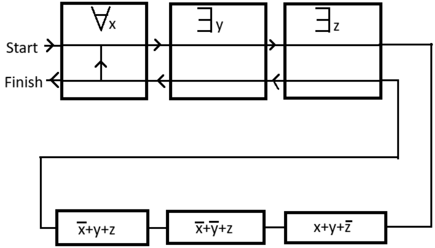
PSPACE-hardness of doors - Proof

True Quantified SAT

$$\forall x \exists y \exists z : (\overline{x} \vee y \vee z) \wedge$$

$$(\overline{x} \vee \overline{y} \vee z) \wedge (\vee x \vee y \vee \overline{z})$$

1-Player Motion Planning

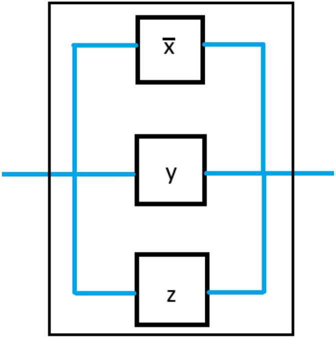


PSPACE-hardness of doors - Proof

Clause

$(\overline{x} \vee y \vee z)$

Gadget

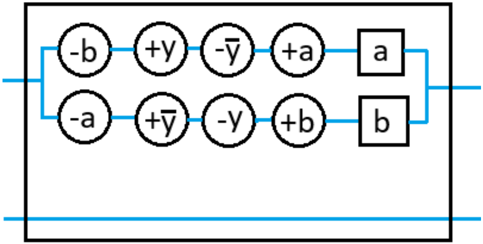


PSPACE-hardness of doors - Proof

Exists-Quantor

$$\exists y$$

Gadget

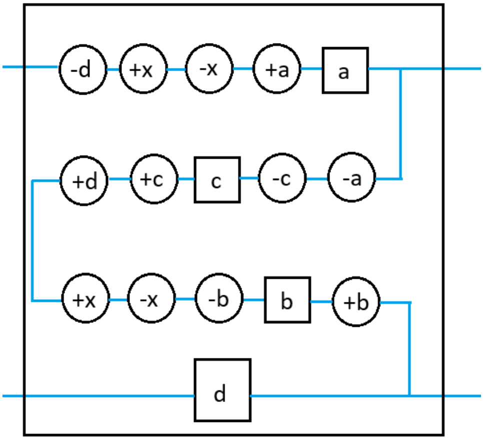


PSPACE-hardness of doors - Proof

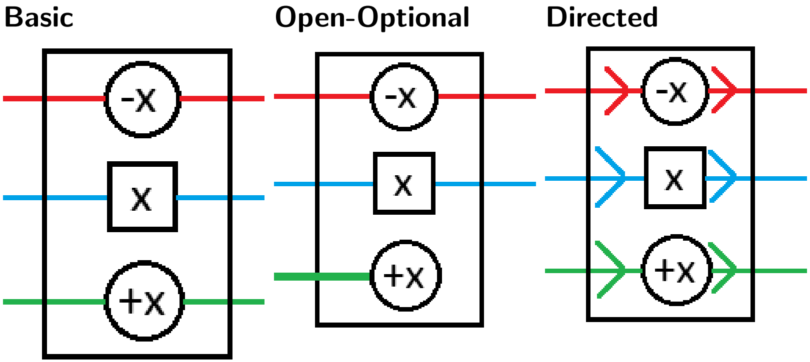
All-Quantor

$\forall x$

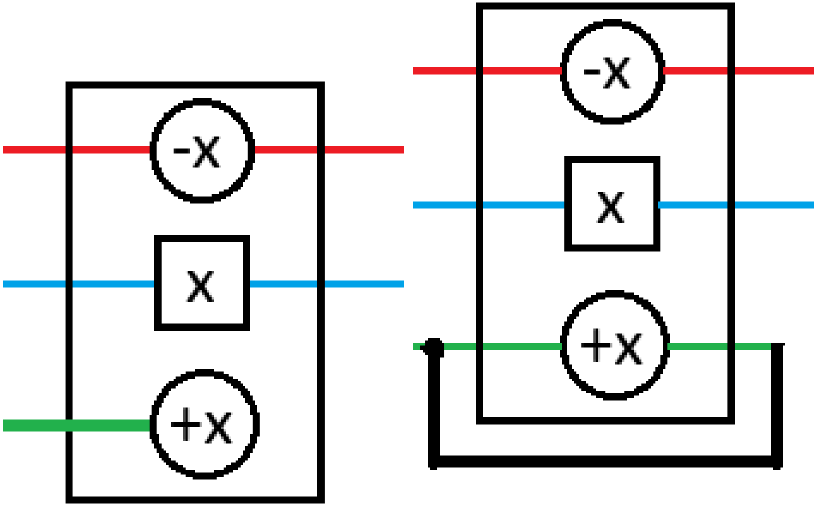
Gadget



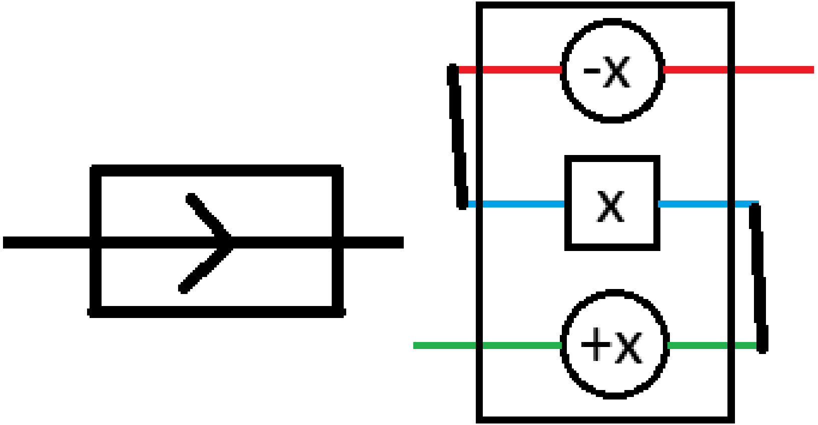
Door Device - Variants



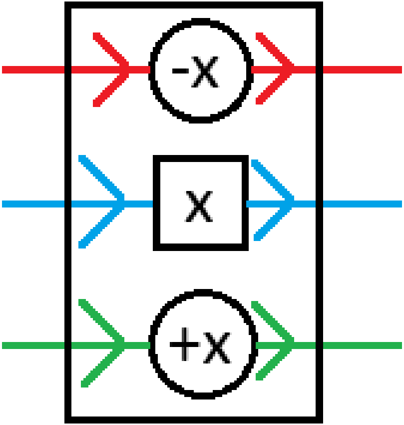
PSpace-Hardness - Open optinal door

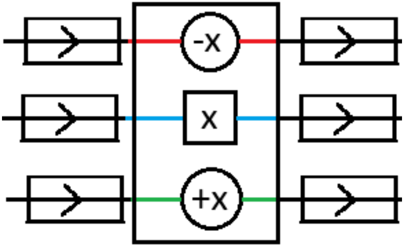


The Diode

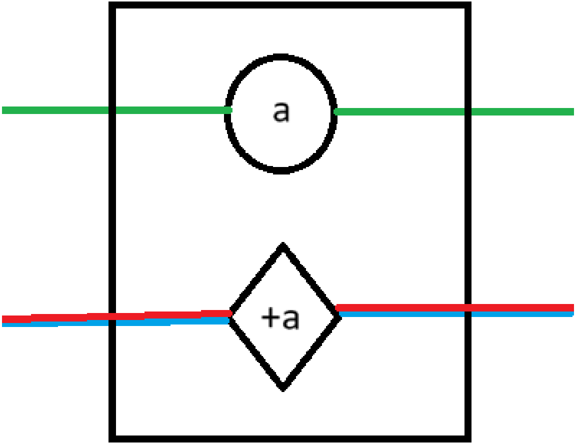


PSpace-Hardness - Directed Door





Self closing doors



Self closing doors - Variants

Directed **Open-Optionals** **Symetric**