

# FUN with Complexity: Walking through Doors is Hard, even without Staircases

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# Content

## Theory

- PSPACE-Complexity
- 1-PlayerMotionPlaning
- Basic Door Device
- PSPACE-hardness of doors
- Door Device Variants

## Application

- Socobond is PSPACE-Hard
- Super Mario Galaxy 2 is PSPACE-Hard
- Super Mario Galaxy is PSPACE-Hard
- Super Mario Odessy is PSPACE-Hard

# PSPACE-Complexity

A given problem requires at most a polynomial amount of memory in relation to the input, to be solved  $\Leftrightarrow$  The problem is in PSPACE

## SAT

$$x_1 \wedge x_2 \vee \neg x_3$$

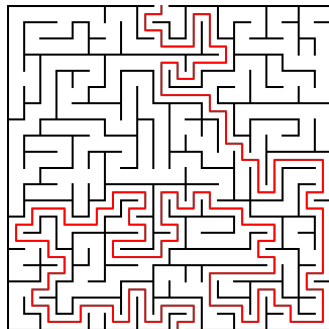
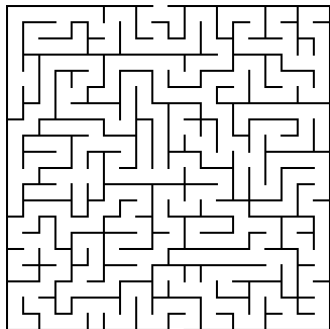
## Quantified SAT

$$\forall x_1 \exists x_2 : x_1 \wedge x_2 \vee \neg x_3$$

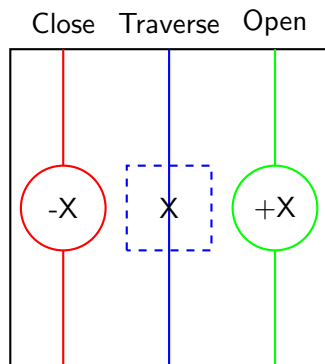
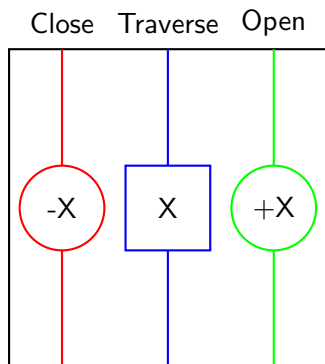
# 1-PlayerMotionPlaning

Given: Enviroment, Agent, Goal

Question: Is the goal achivable



# Basic Door Device



# PSPACE-hardness of doors

## Theorem

*If a game features **door devices** which each are controlled by an **open** and a **close** **pressure plate** and the agent has to navigate from entrance to exit, then the game is **PSPACE-hard***

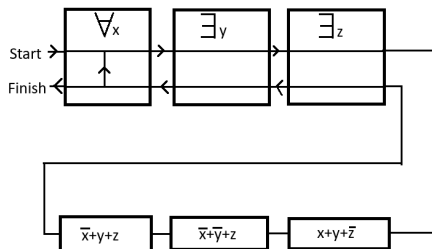
# PSPACE-hardness of doors - Proof

## True Quantified SAT

$$\forall x \exists y \exists z : (\bar{x} \vee y \vee z) \wedge$$

$$(\bar{x} \vee \bar{y} \vee z) \wedge (\forall x \vee y \vee \bar{z})$$

## 1-Player Motion Planning

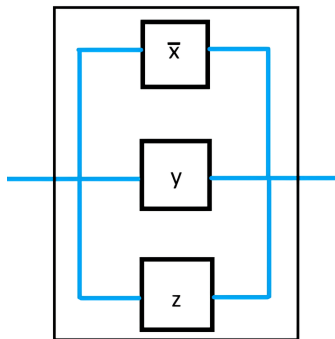


# PSPACE-hardness of doors - Proof

**Clause**

$$(\bar{x} \vee y \vee z)$$

**Gadget**



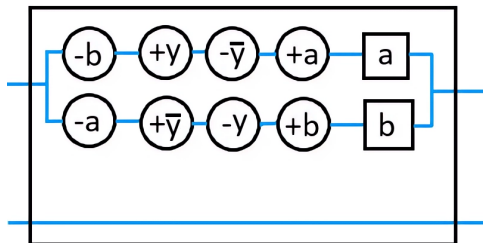


# PSPACE-hardness of doors - Proof

**Exists-Quantor**

$\exists y$

**Gadget**

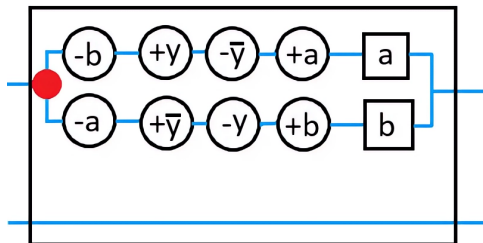


# PSPACE-hardness of doors - Proof

**Exists-Quantor**

$\exists y$

**Gadget**

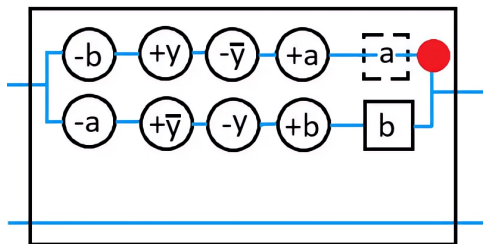


# PSPACE-hardness of doors - Proof

**Exists-Quantor**

$\exists y$

**Gadget**

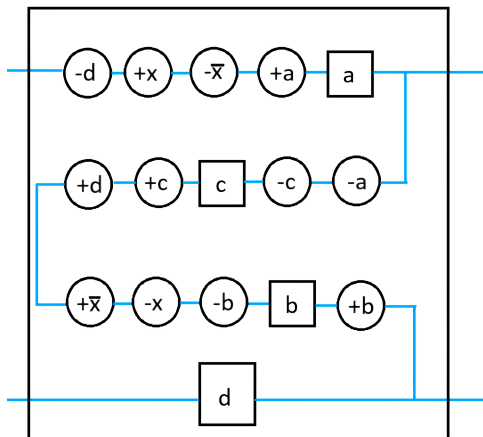


# PSPACE-hardness of doors - Proof

**All-Quantor**

$\forall x$

**Gadget**

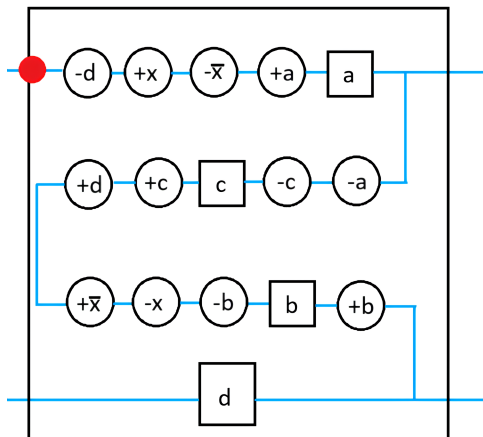


# PSPACE-hardness of doors - Proof

All-Quantor

$\forall x$

Gadget

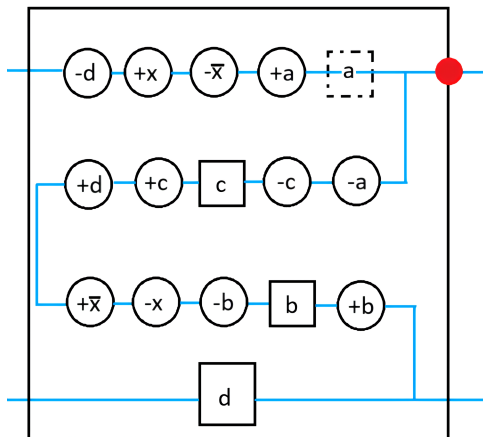


# PSPACE-hardness of doors - Proof

All-Quantor

$\forall x$

Gadget

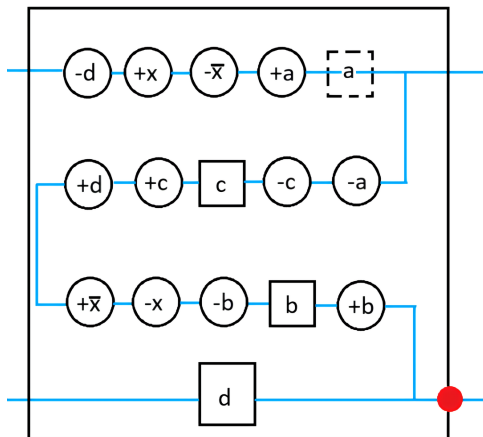


# PSPACE-hardness of doors - Proof

All-Quantor

$\forall x$

Gadget

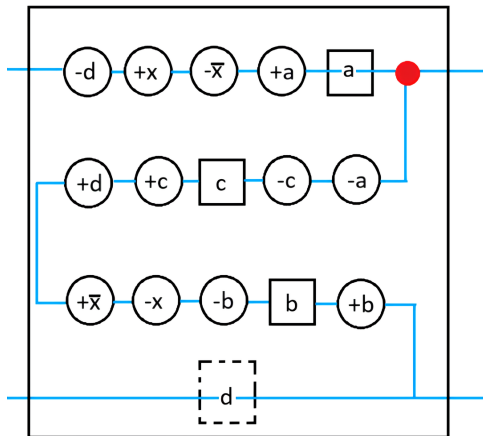


# PSPACE-hardness of doors - Proof

All-Quantor

$\forall x$

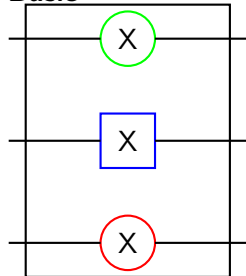
Gadget



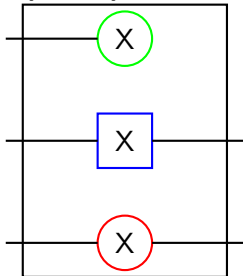


# Door Device - Variants

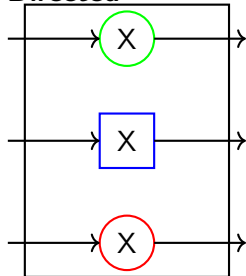
**Basic**



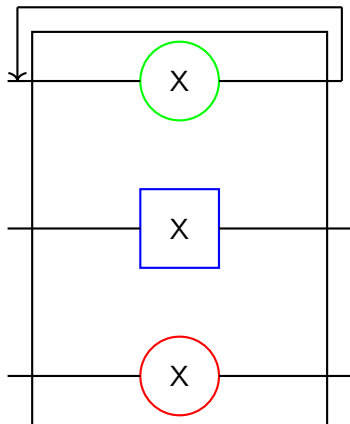
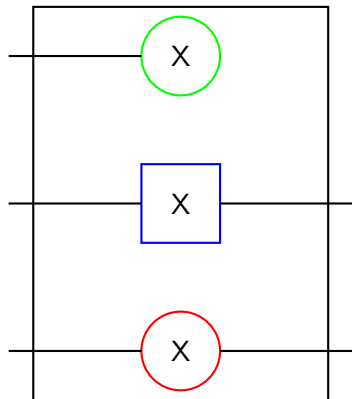
**Open-Optional**



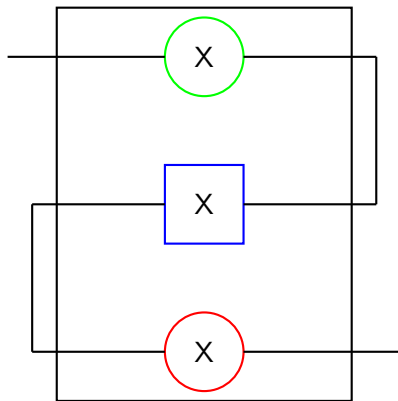
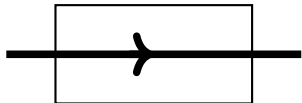
**Directed**



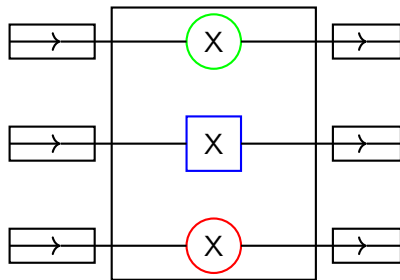
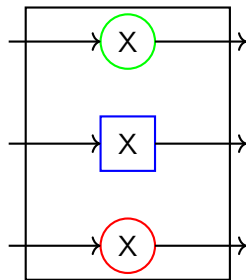
## PSpace-Hardness - Open optimal door



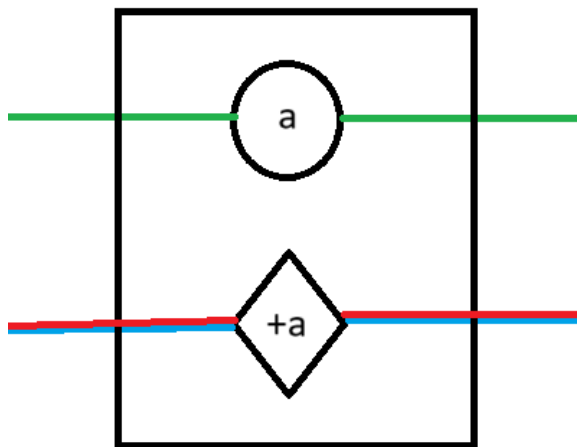
# The Diode



## PSpace-Hardness - Directed Door

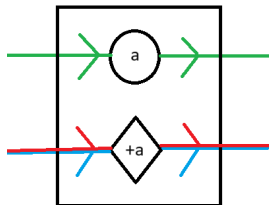


## Self closing doors

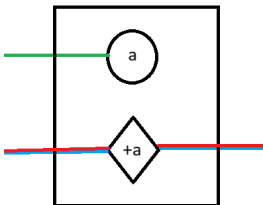


# Self closing doors - Variants

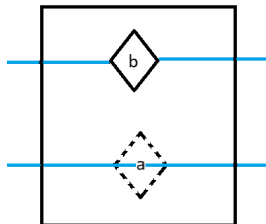
**Directed**



**Open-Optionals**

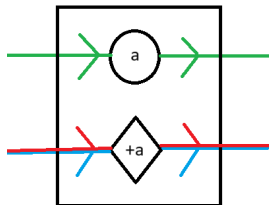


**Symetric**

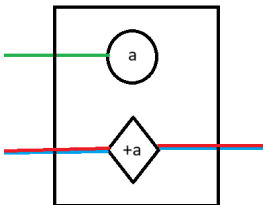


# Self closing doors - Variants

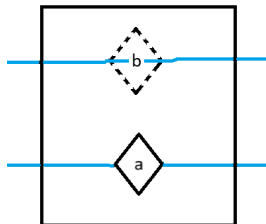
**Directed**



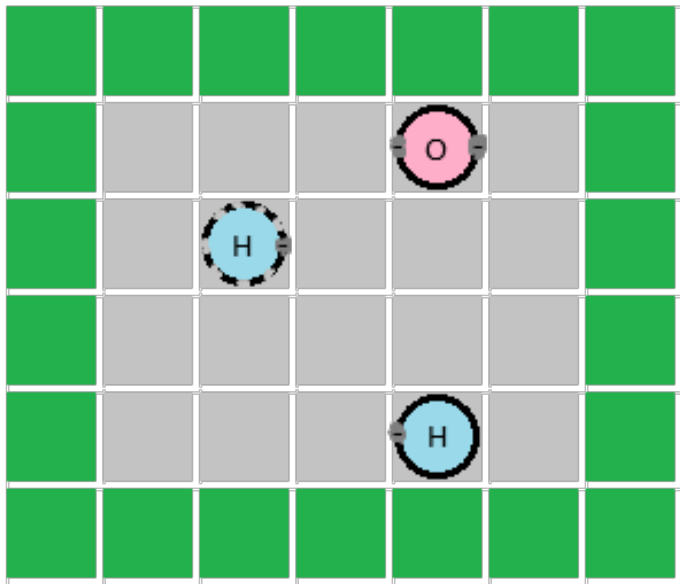
**Open-Optionals**



**Symetric**

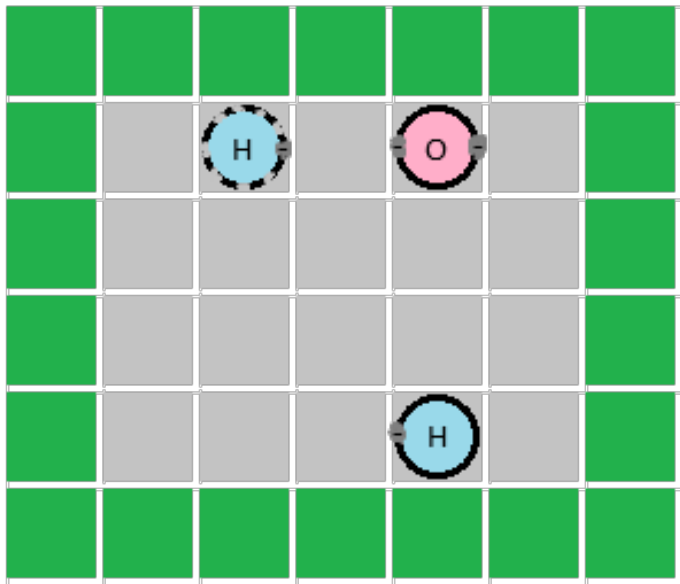


# Application - Sokobond

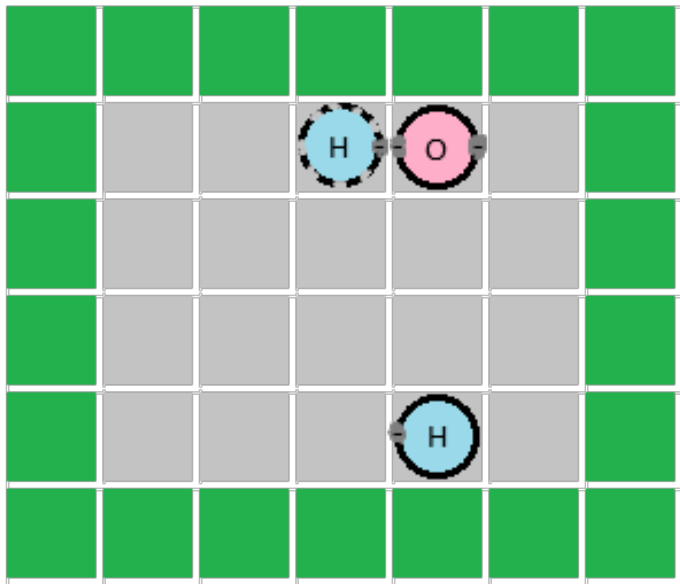




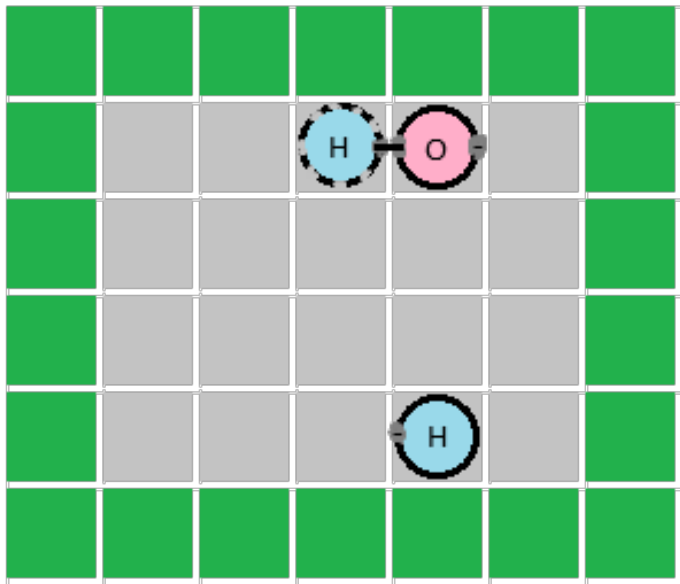
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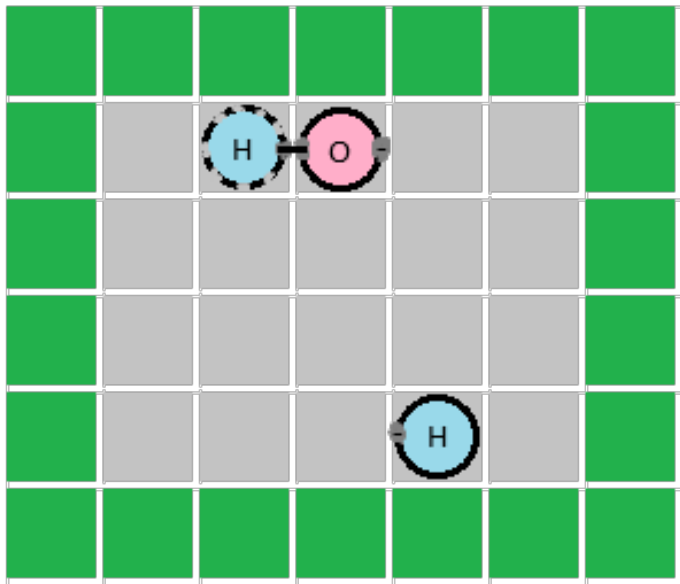
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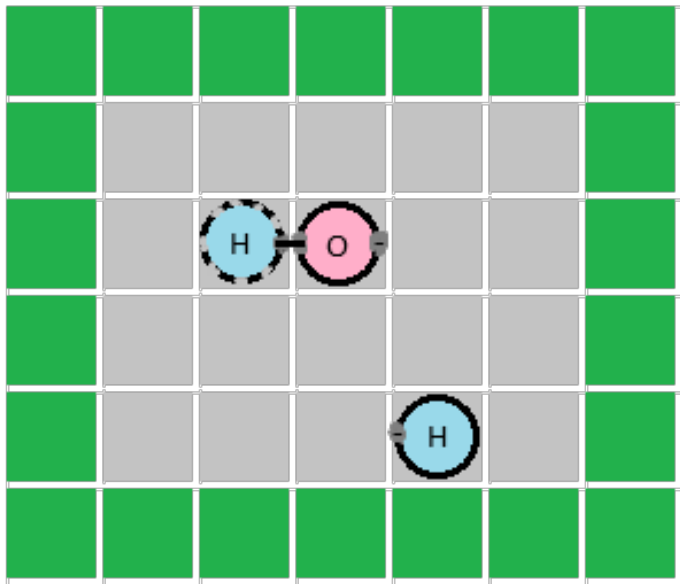
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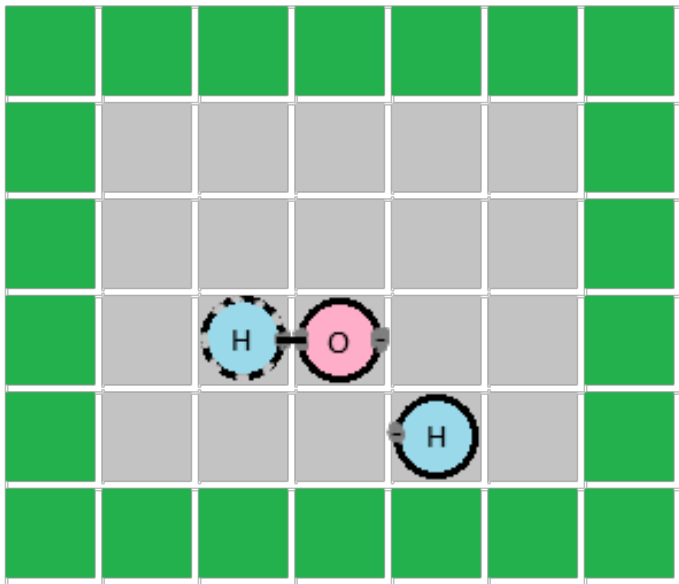
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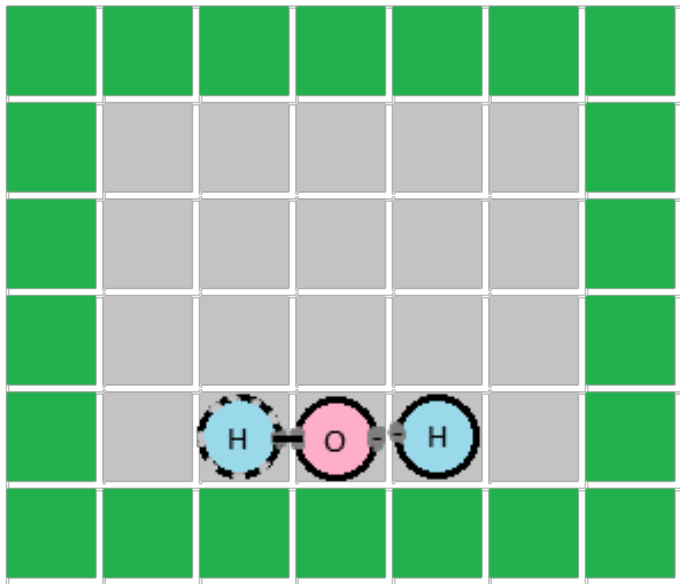
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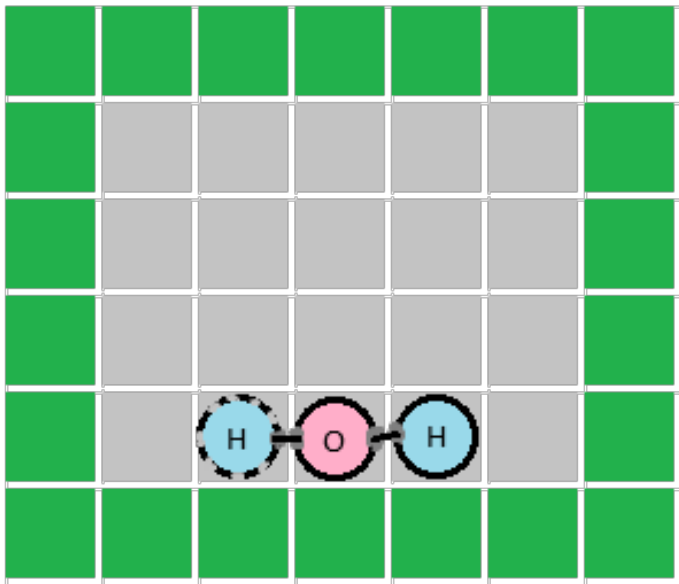
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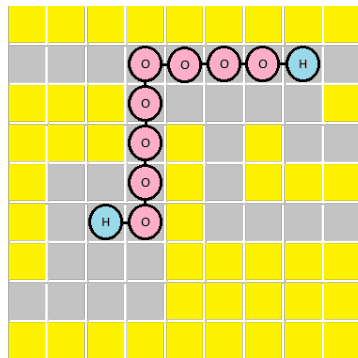
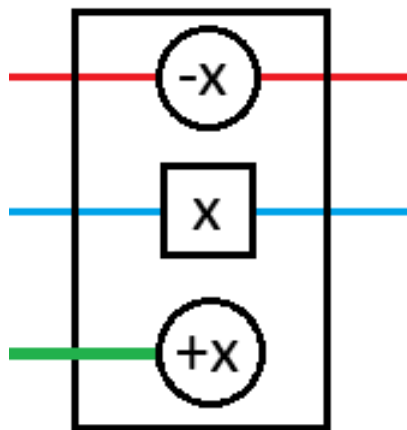


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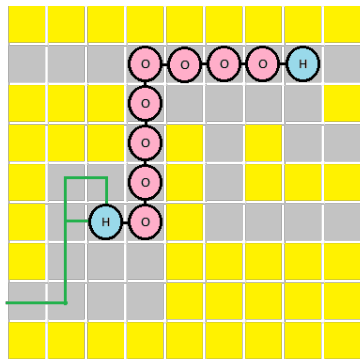
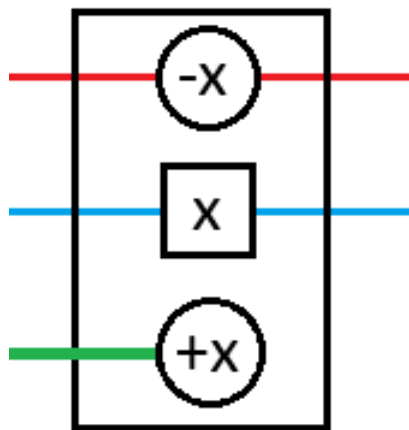




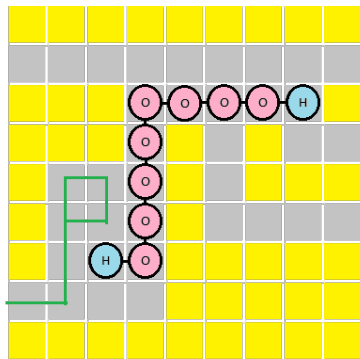
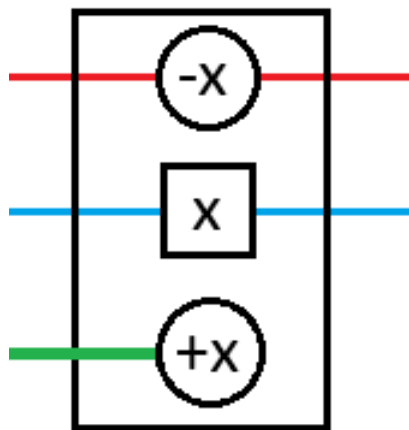
# Sokobond is PSpace-Hard



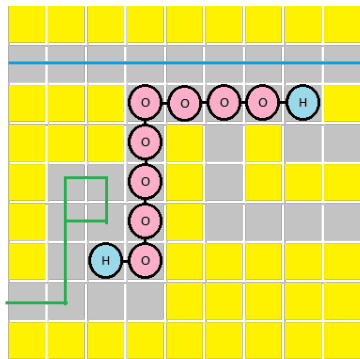
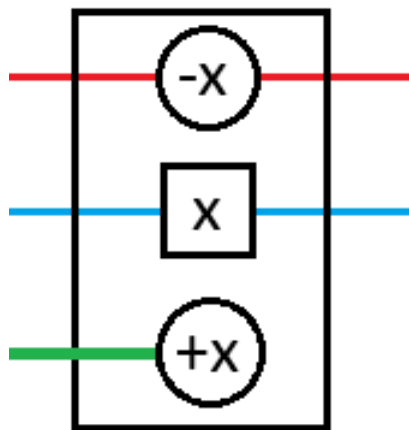
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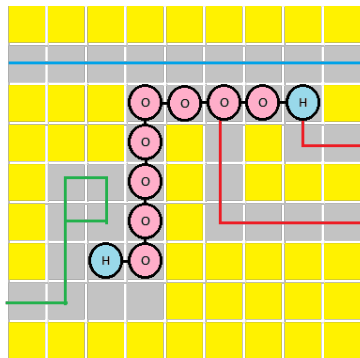
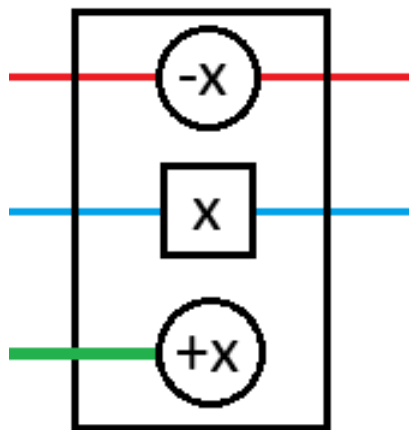
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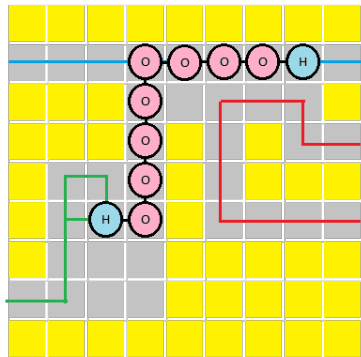
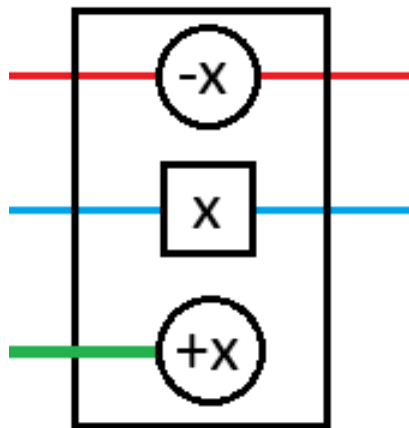
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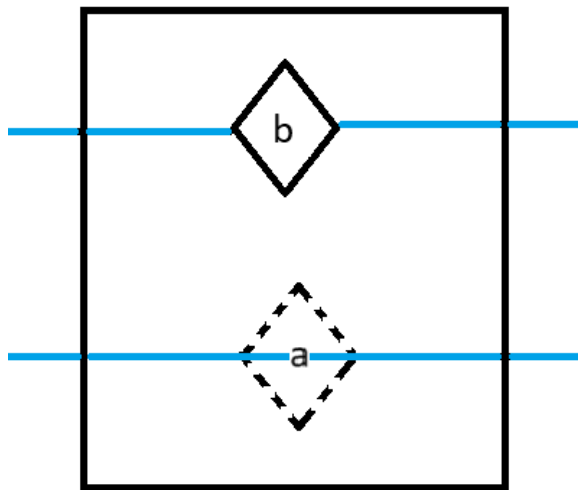
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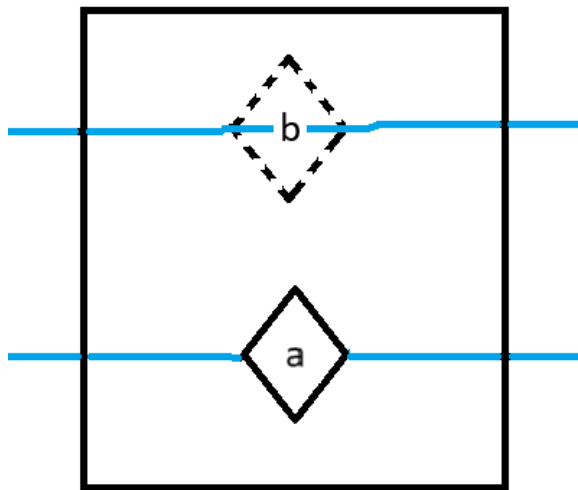
# Sokobond is PSpace-Hard



## Reminder - Symetric self closing door

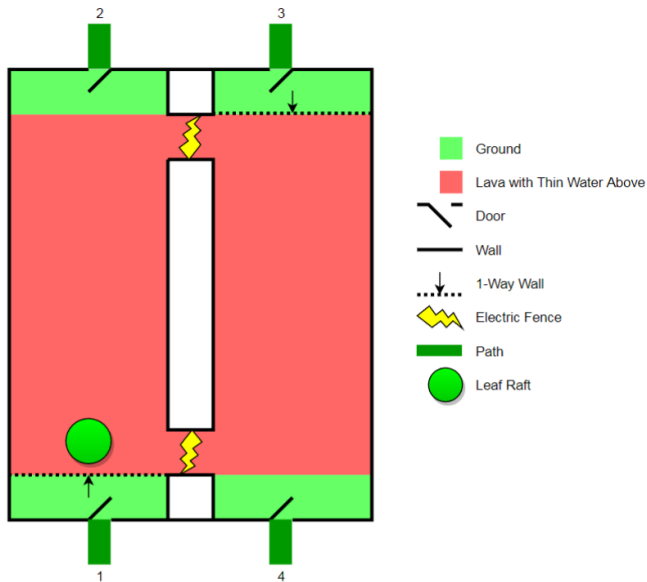


## Reminder - Symetric self closing door

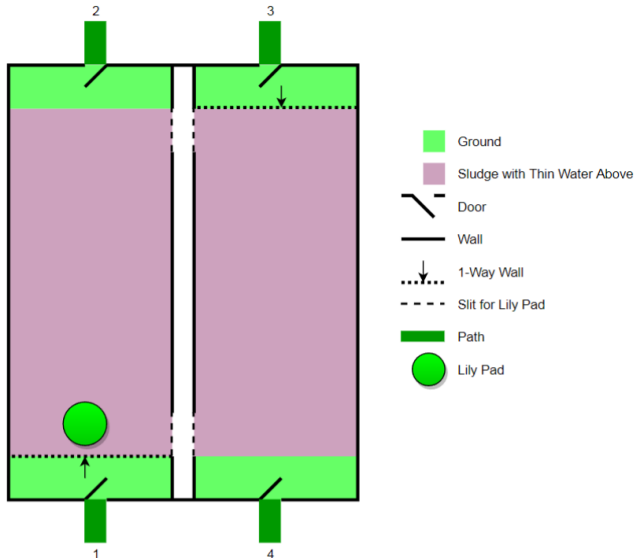




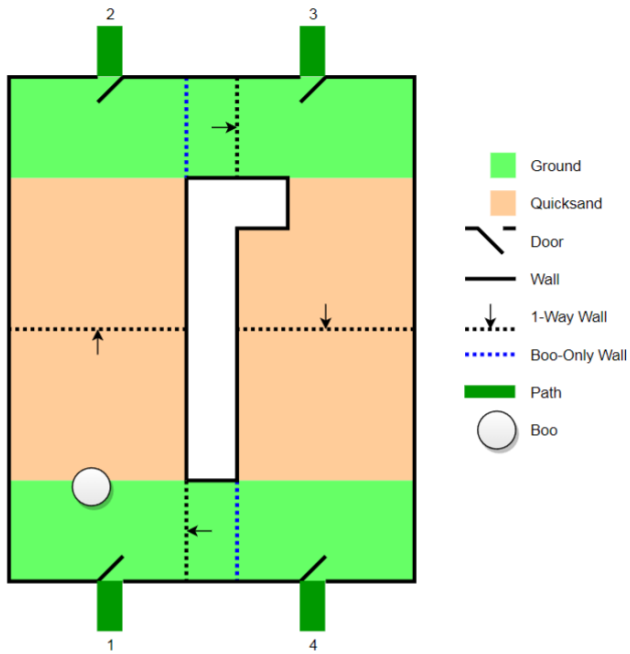
# Super Mario Galaxy 2 is PSpace-hard



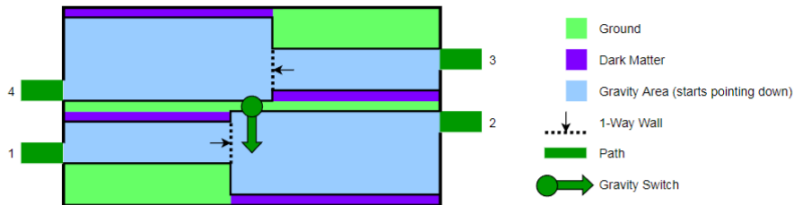
# Parrallelities



# Parralelities



# Super Mario Galaxy is PSpace-hard



# Super Mario Odyssey is PSpace-hard

