# FUN with Complexity: Walking through Doors is Hard, even without Staircases

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# Content

# PSPACE-Complexity

A given problem requires a polynomial amount of memory in relation to the input, to be solved $\Leftrightarrow$ The problem is in PSPACE

**SAT**

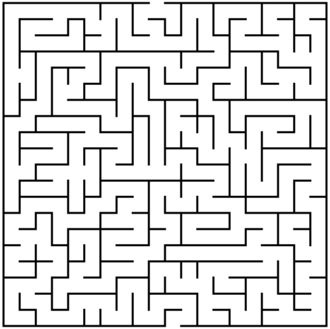
$$x_1 \wedge x_2 \vee \neg x_3$$

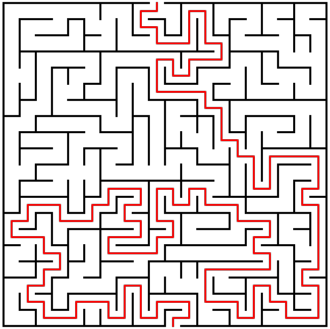
**Quantified SAT**

$$\forall x_1 \exists x_2 : x_1 \wedge x_2 \vee \neg x_3$$

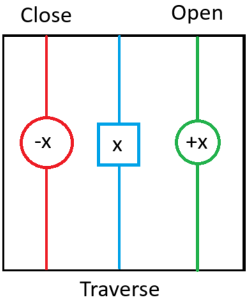
# 1-PlayerMotionPlaning

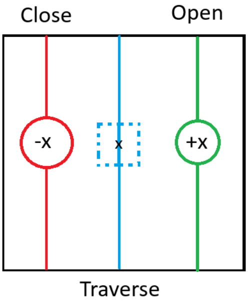
Given: Enviroment, Agent, Goal
Question: Is the goal achivable





# Basic Door Device



# PSPACE-hardness of doors

### Theorem

*If a game features* **door devices** *which each are controled by an open and a close* **preasure plate** *and the agent has to navigate from entrance to exit, then the game is* **PSPACE-hard**

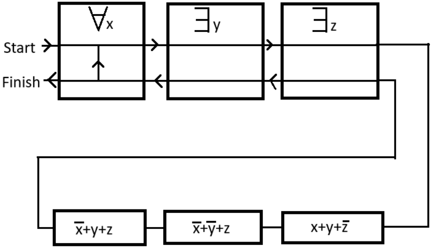
# PSPACE-hardness of doors - Proof

**True Quantified SAT**

$$\forall x \exists y \exists z : (\overline{x} \vee y \vee z) \wedge$$

$$(\overline{x} \vee \overline{y} \vee z) \wedge (\vee x \vee y \vee \overline{z})$$

**1-Player Motion Planning**

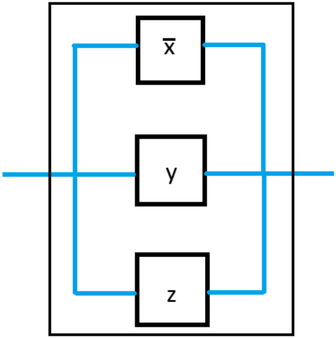


# PSPACE-hardness of doors - Proof

**Clause**

$$(\overline{x} \vee y \vee z)$$

**Gadget**

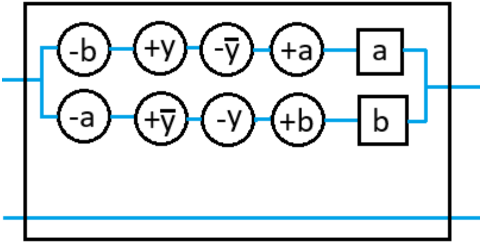


# PSPACE-hardness of doors - Proof

**Exists-Quantor**

$$\exists y$$

**Gadget**



# PSPACE-hardness of doors - Proof

**Exists-Quantor**

$\exists y$

**Gadget**