FUN with Complexity: Walking through Doors is Hard, even without Staircases

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Content

PSPACE-Complexity

A given problem requires at most a polynomial amount of memory in relation to the input, to be solved $\Leftrightarrow$ The problem is in PSPACE

SAT

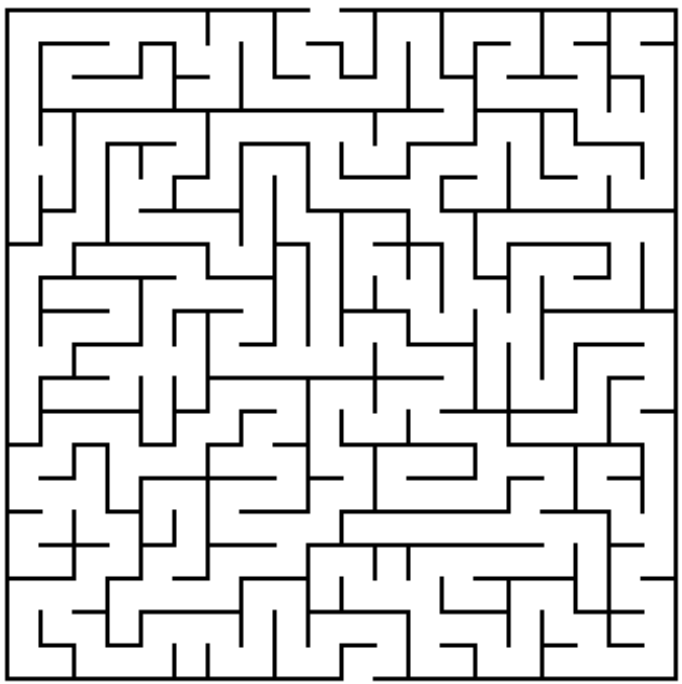
$$x_1 \wedge x_2 \vee \neg x_3$$

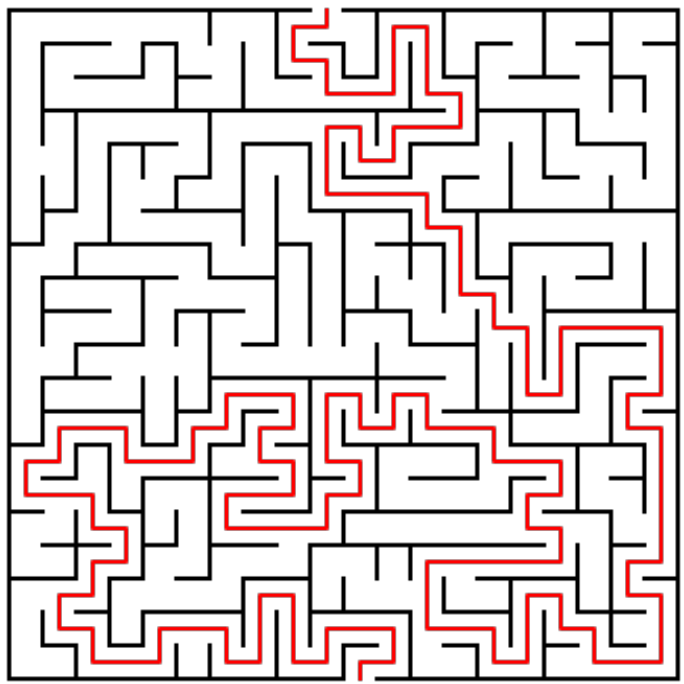
Quantified SAT

$$\forall x_1 \exists x_2 : x_1 \wedge x_2 \vee \neg x_3$$

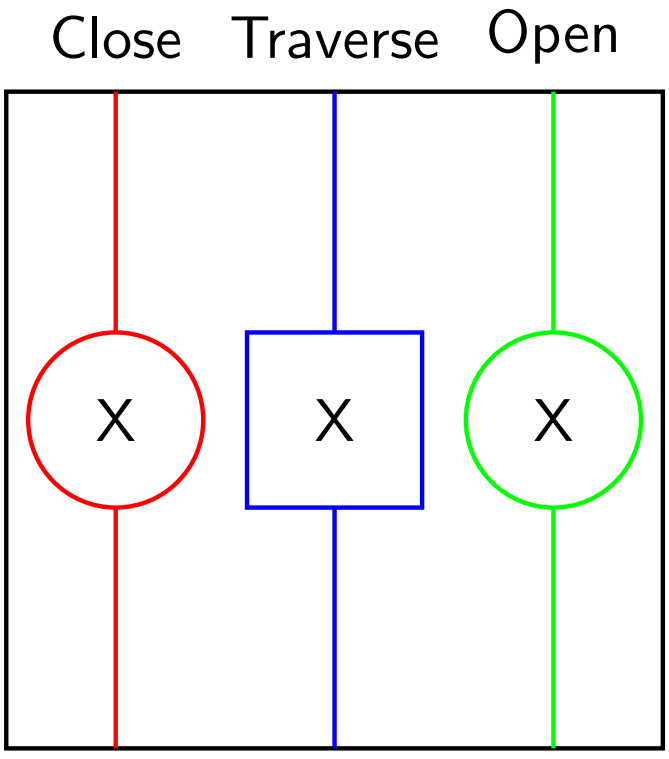
1-PlayerMotionPlaning

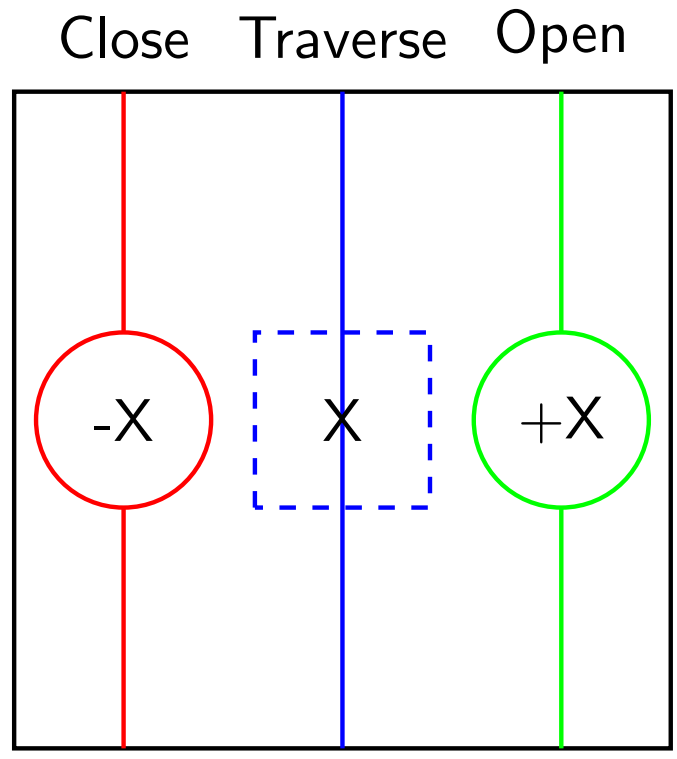
Given: Enviroment, Agent, Goal
Question: Is the goal achivable





Basic Door Device





PSPACE-hardness of doors

Theorem

If a game features **door devices** *which each are controled by an open and a close* **preasure plate** *and the agent has to navigate from entrance to exit, then the game is* **PSPACE-hard**

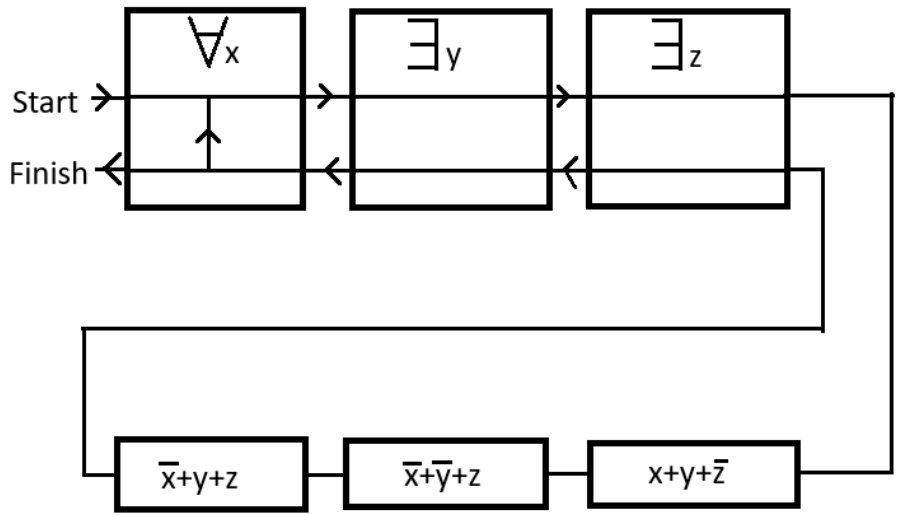
PSPACE-hardness of doors - Proof

True Quantified SAT

$$\forall x \exists y \exists z : (\overline{x} \vee y \vee z) \wedge$$

$$(\overline{x} \vee \overline{y} \vee z) \wedge (\vee x \vee y \vee \overline{z})$$

1-Player Motion Planning

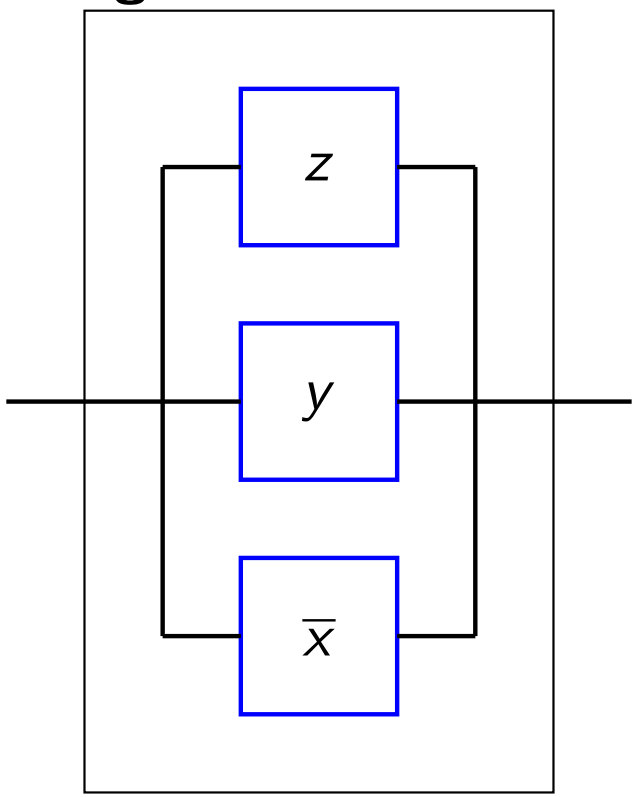


PSPACE-hardness of doors - Proof

Clause

$$(\overline{x} \vee y \vee z)$$

Gadget



PSPACE-hardness of doors - Proof

Exists-Quantor

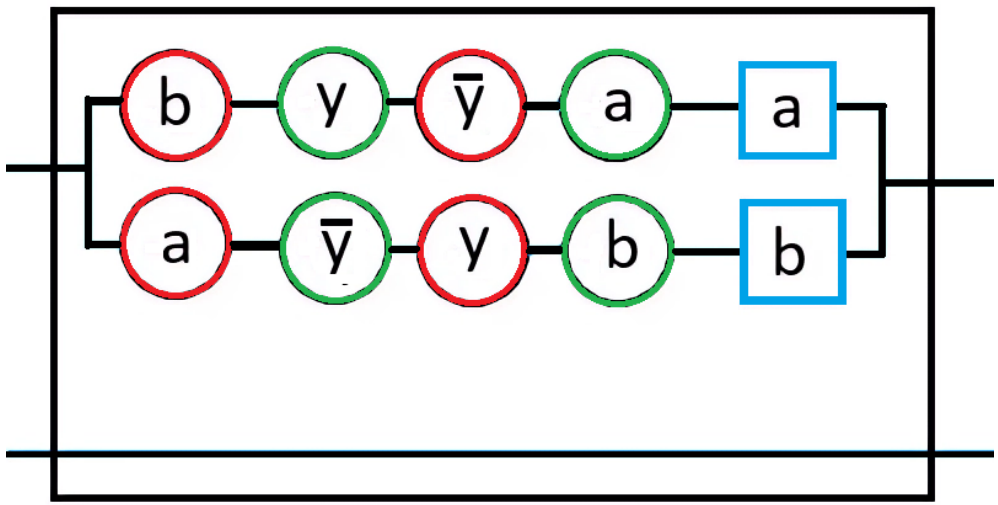
$\exists y$

Open preasure plate:

Close preasure plate:

Door:

Gadget



PSPACE-hardness of doors - Proof

All-Quantor

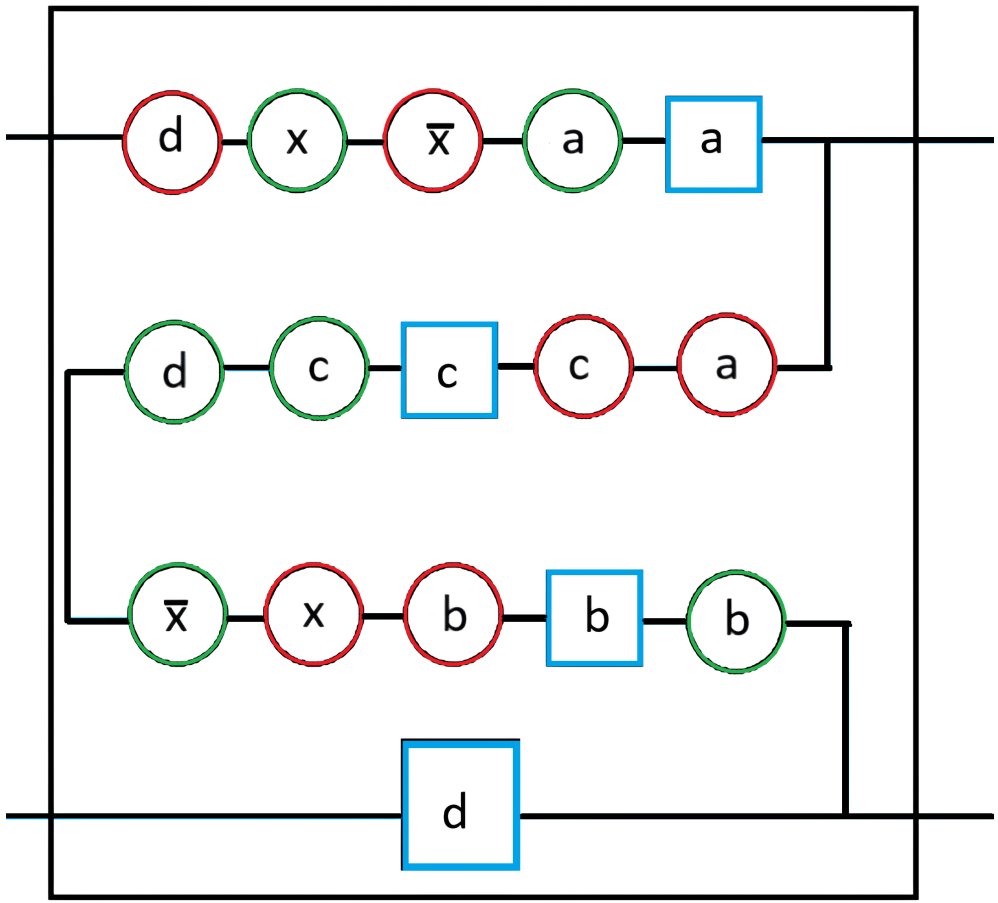
$\forall x$

Open preasure plate:

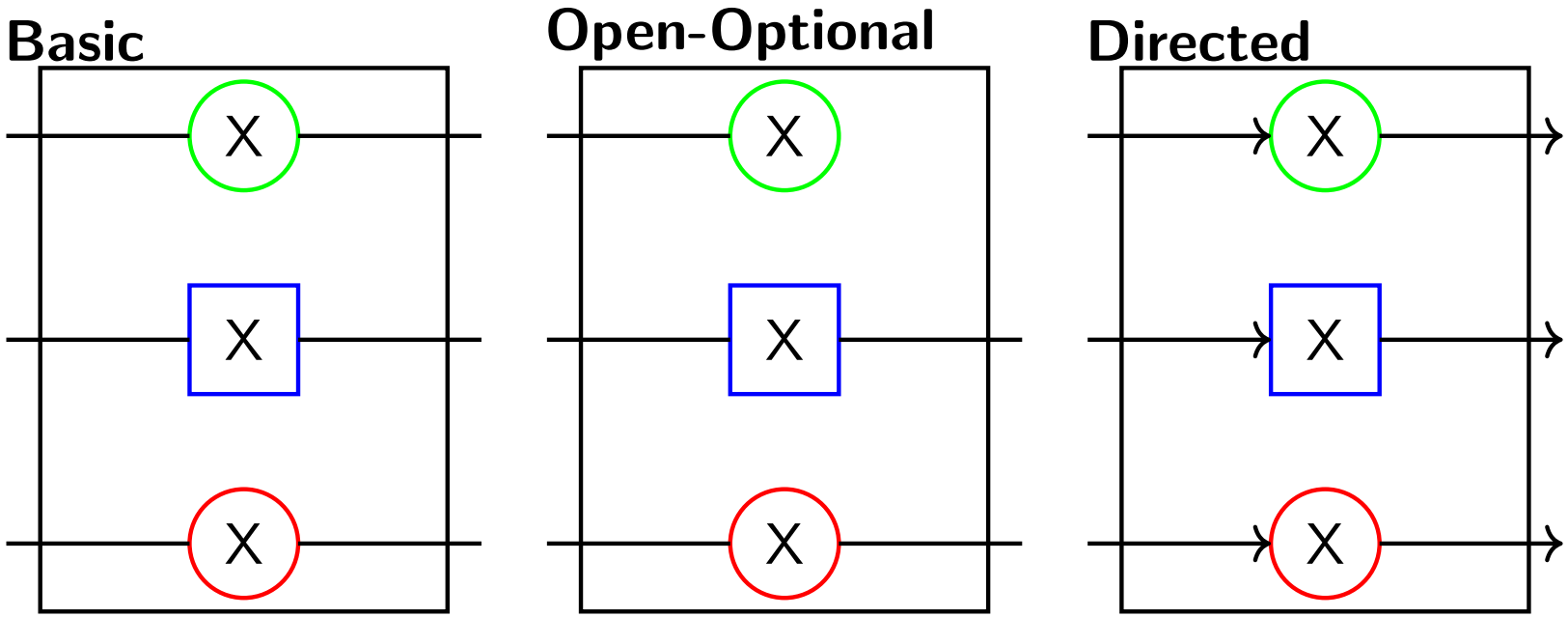
Close preasure plate:

Door:

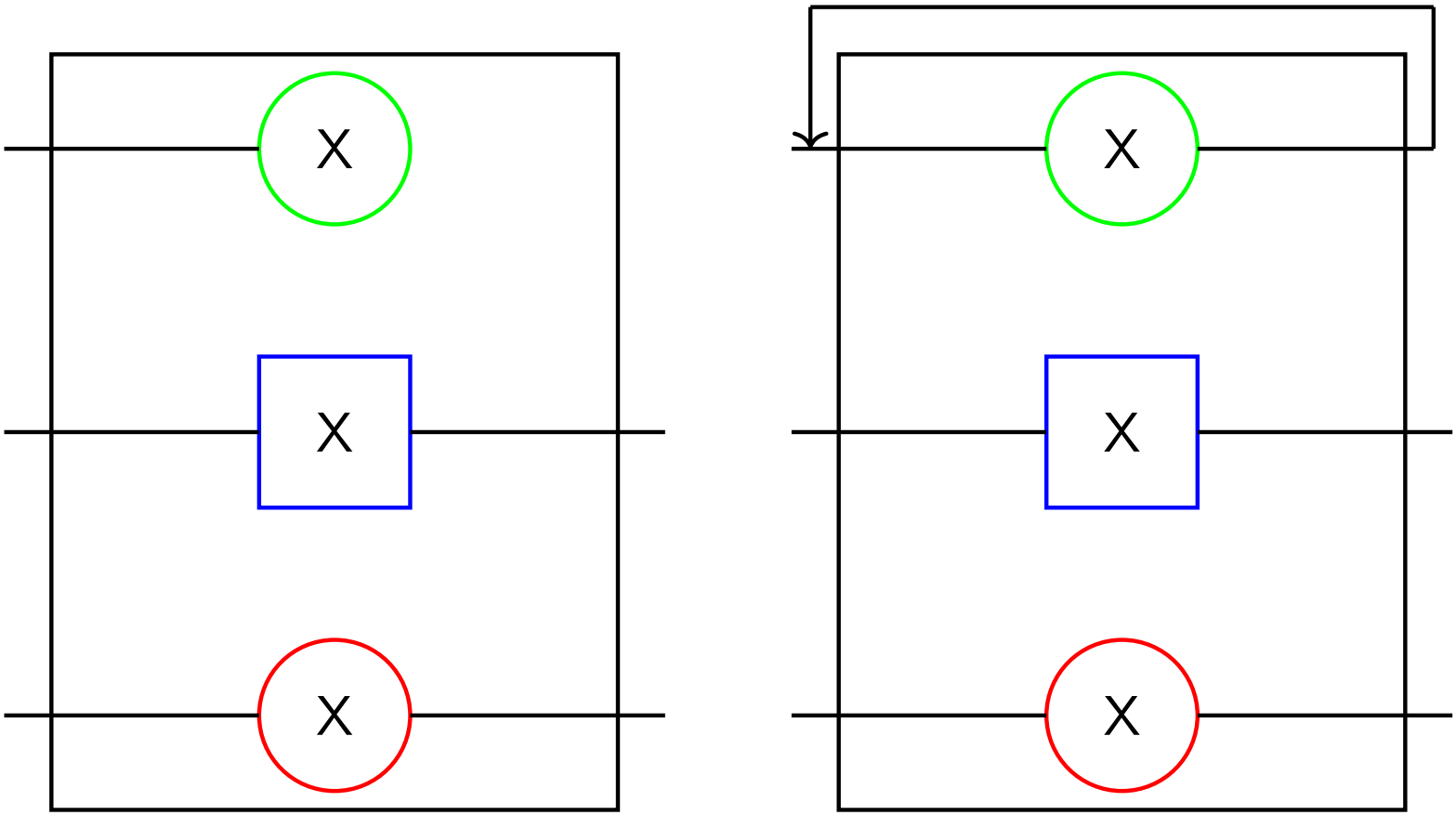
Gadget



Door Device - Variants

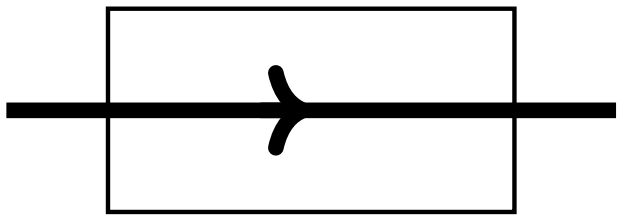


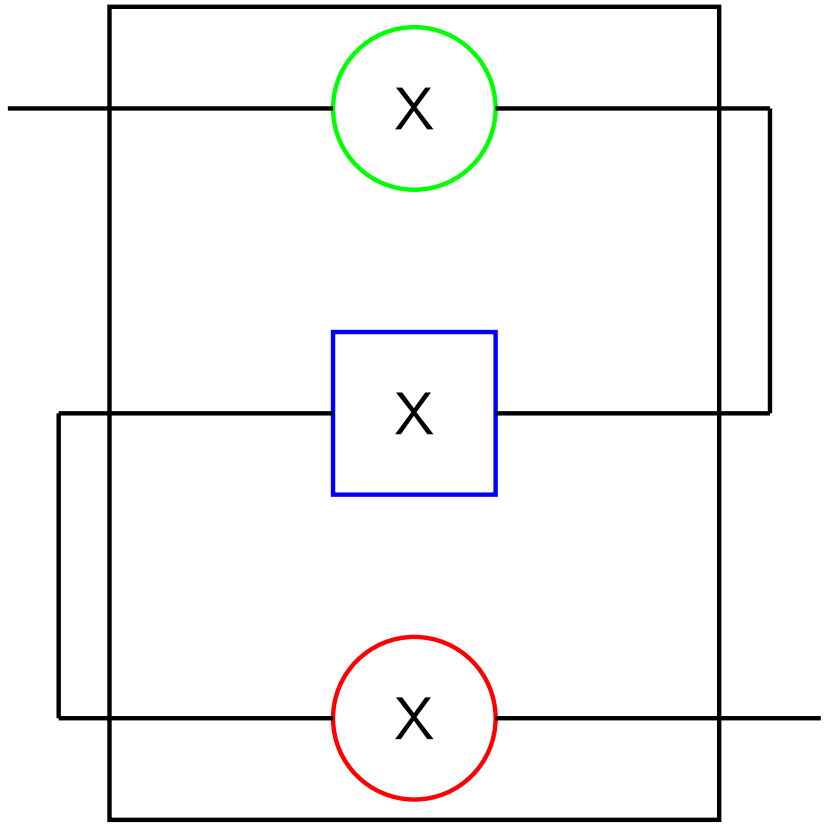
PSpace-Hardness - Open optinal door



Open preasure plate: Close preasure plate: Door:

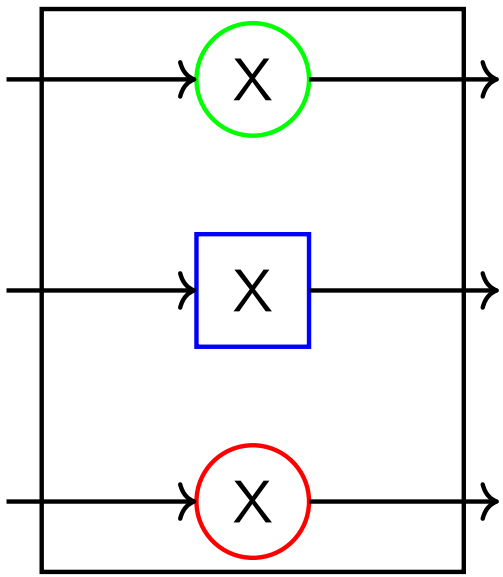
The Diode

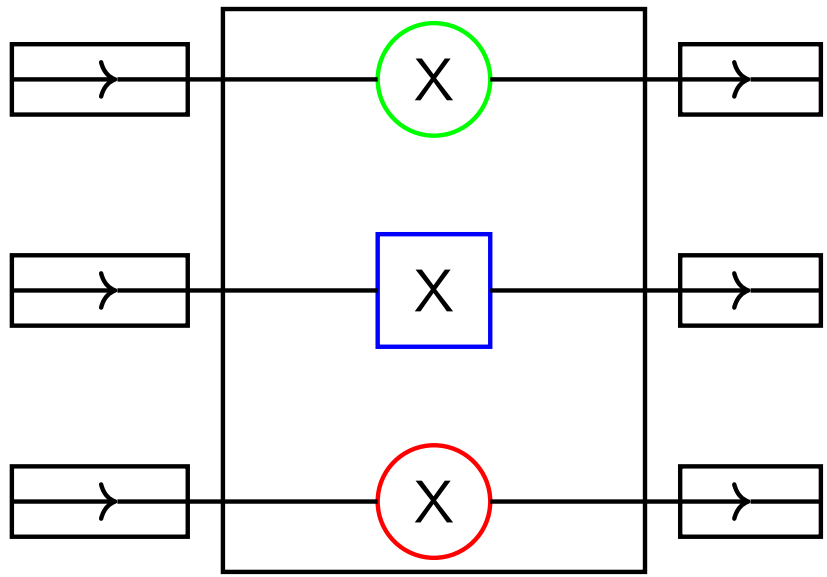




Open preasure plate: Close preasure plate: Door:

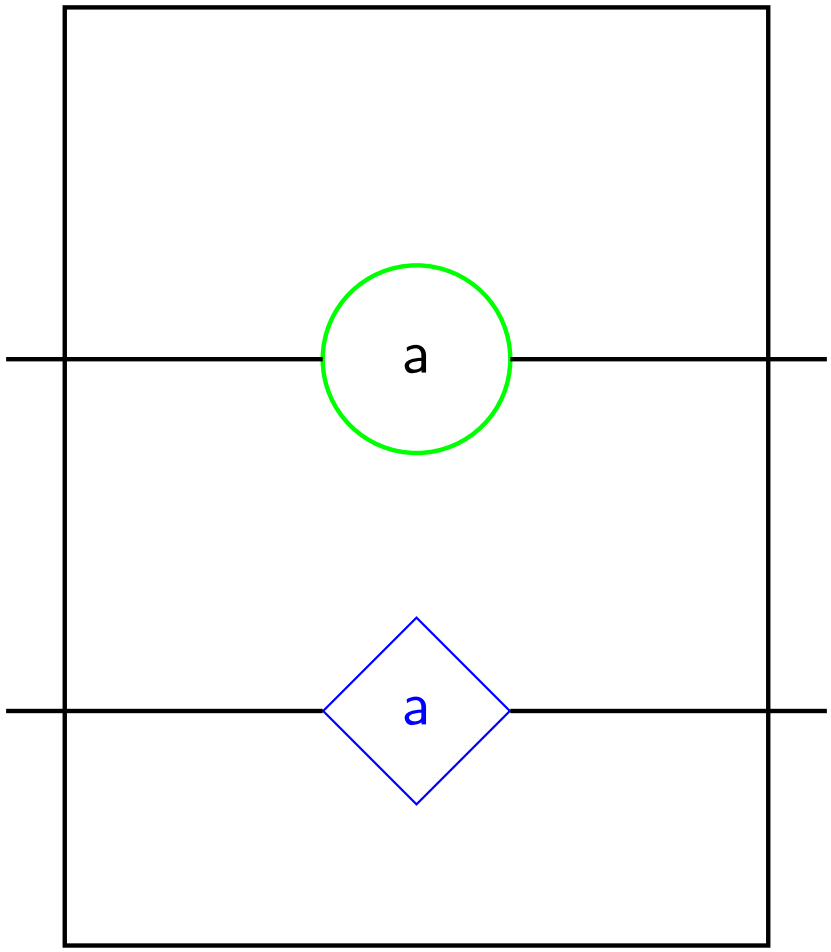
PSpace-Hardness - Directed Door



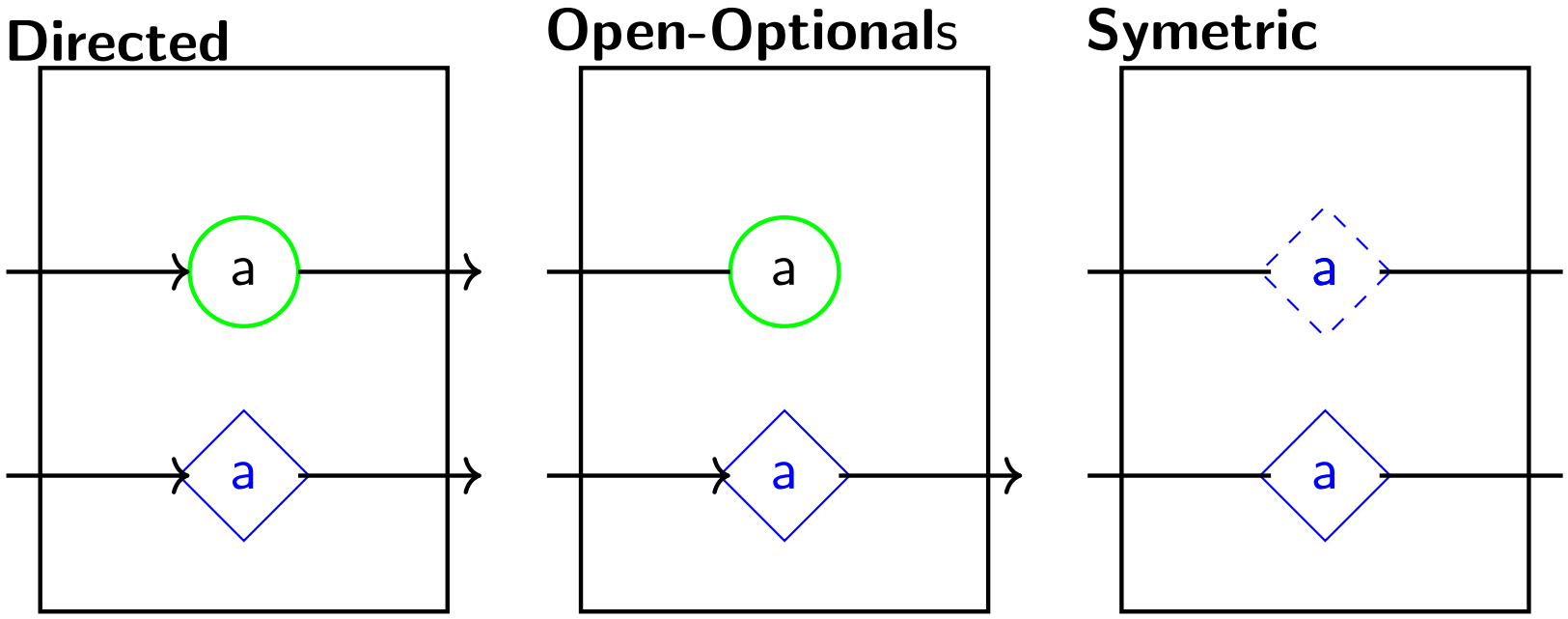


Open preasure plate: Close preasure plate: Door:

Self closing doors

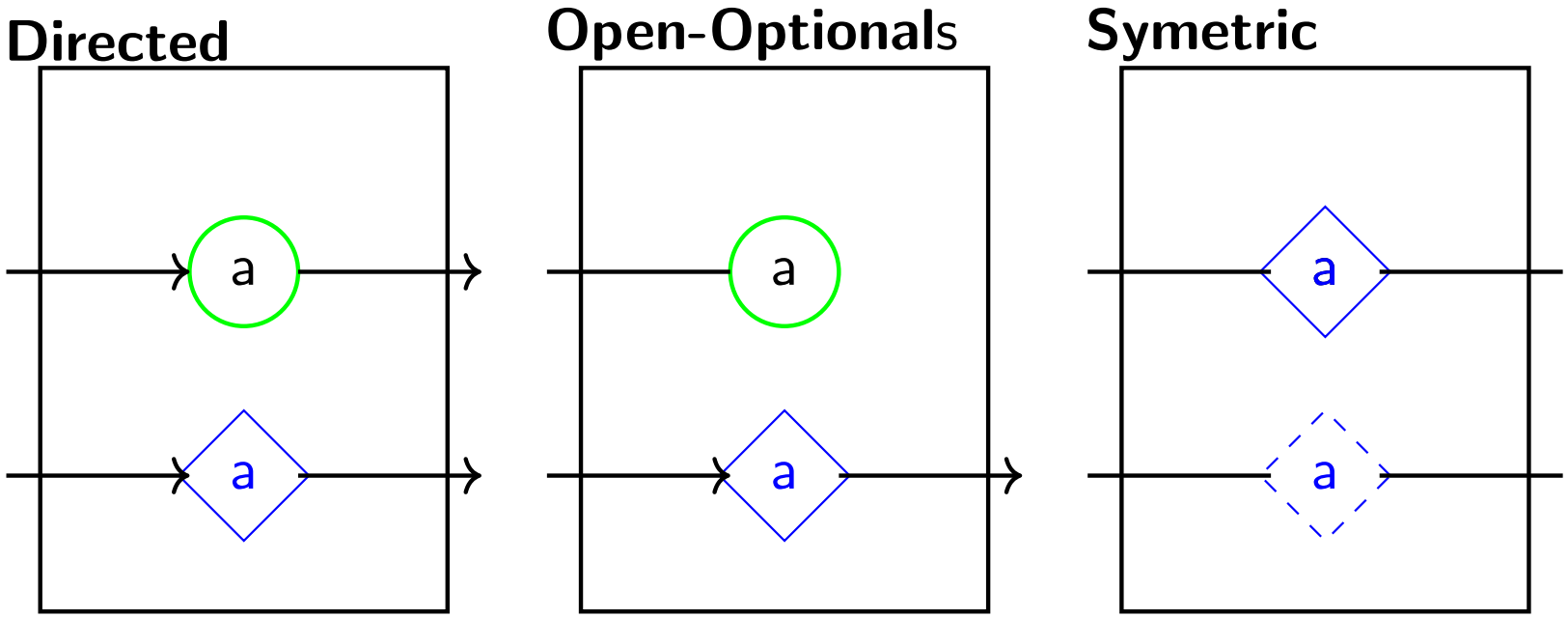


Self closing doors - Variants



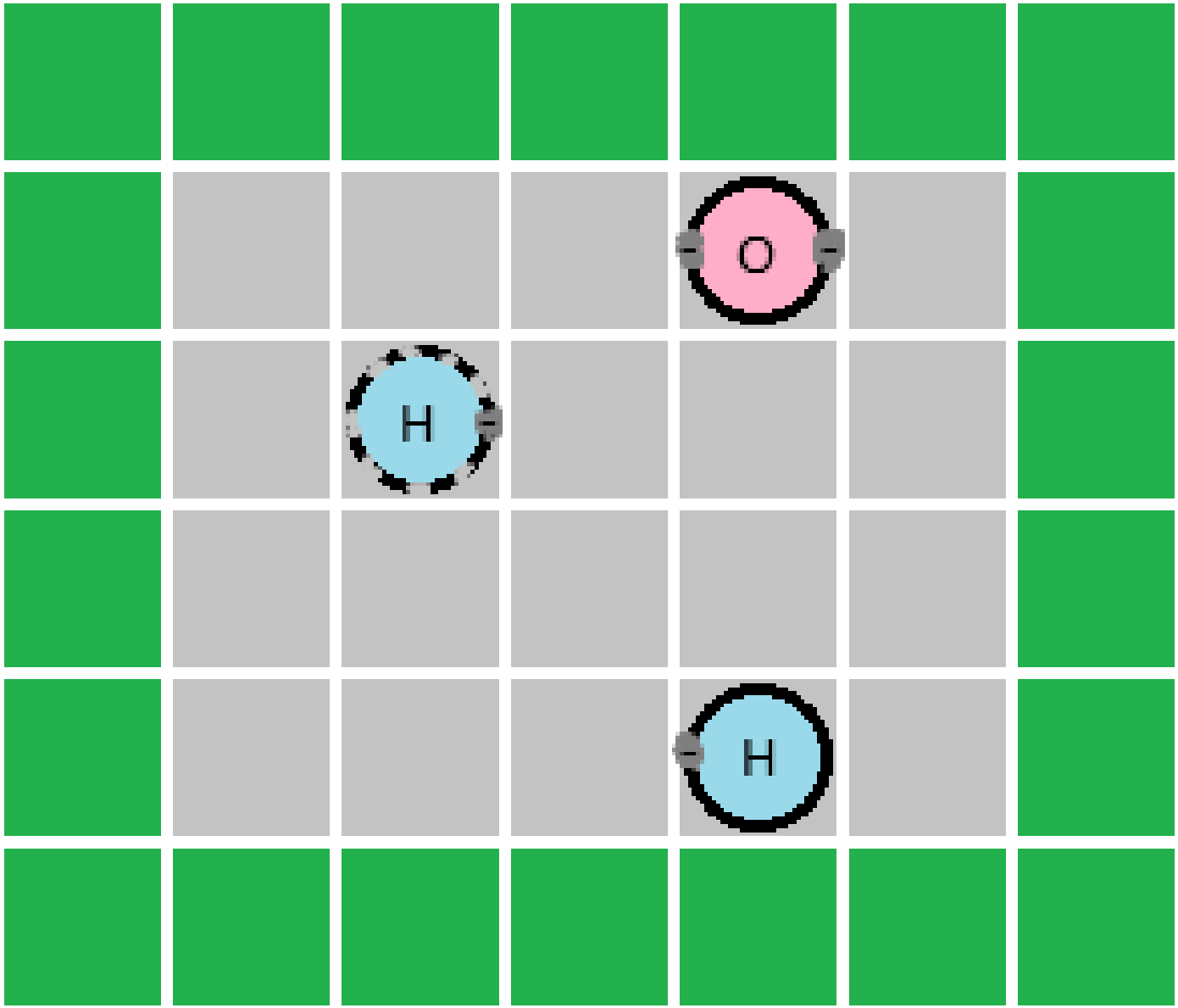
Open preasure plate: Self closing door:

Self closing doors - Variants



Open preasure plate: Self closing door:

Application - Sokobond



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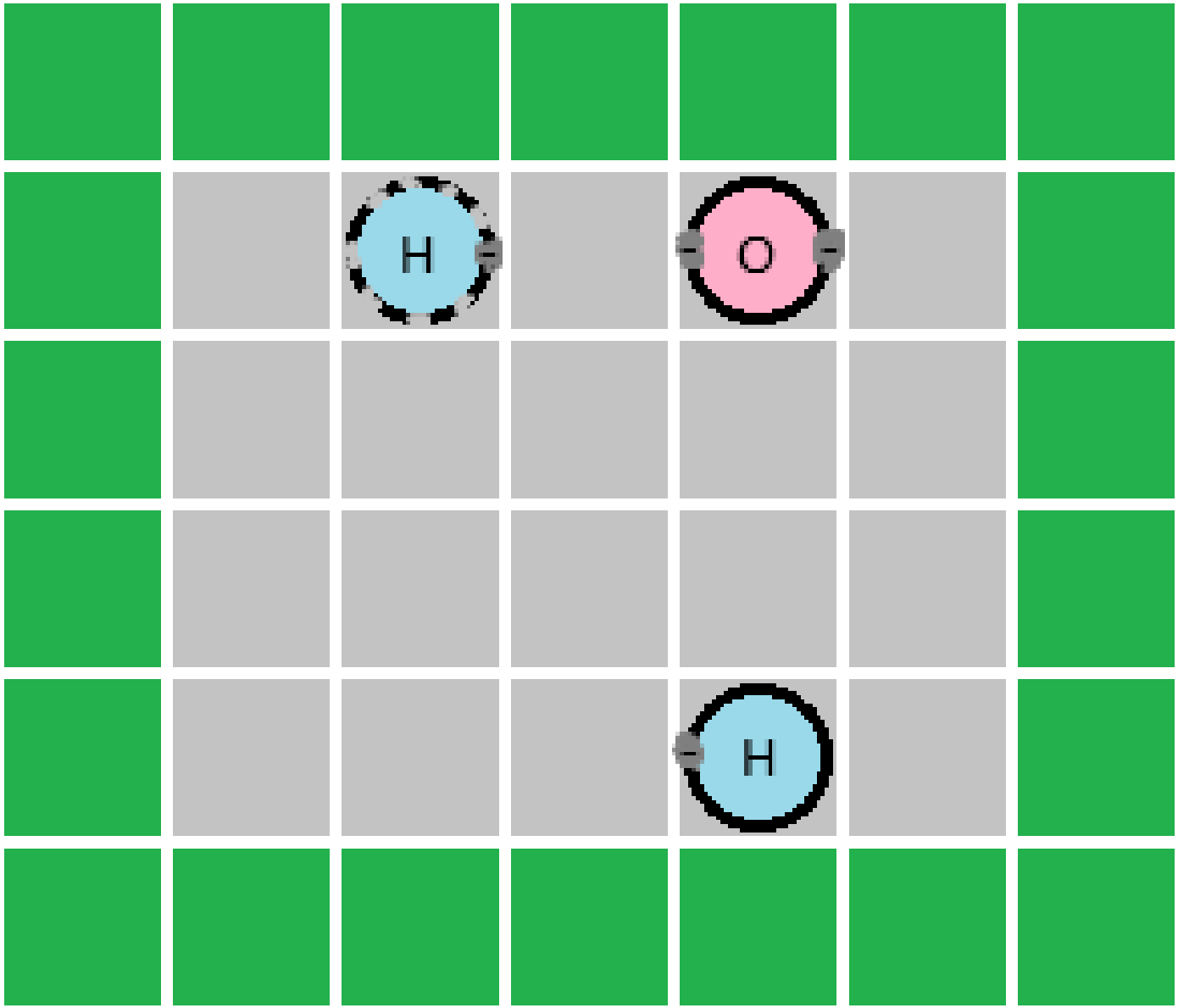

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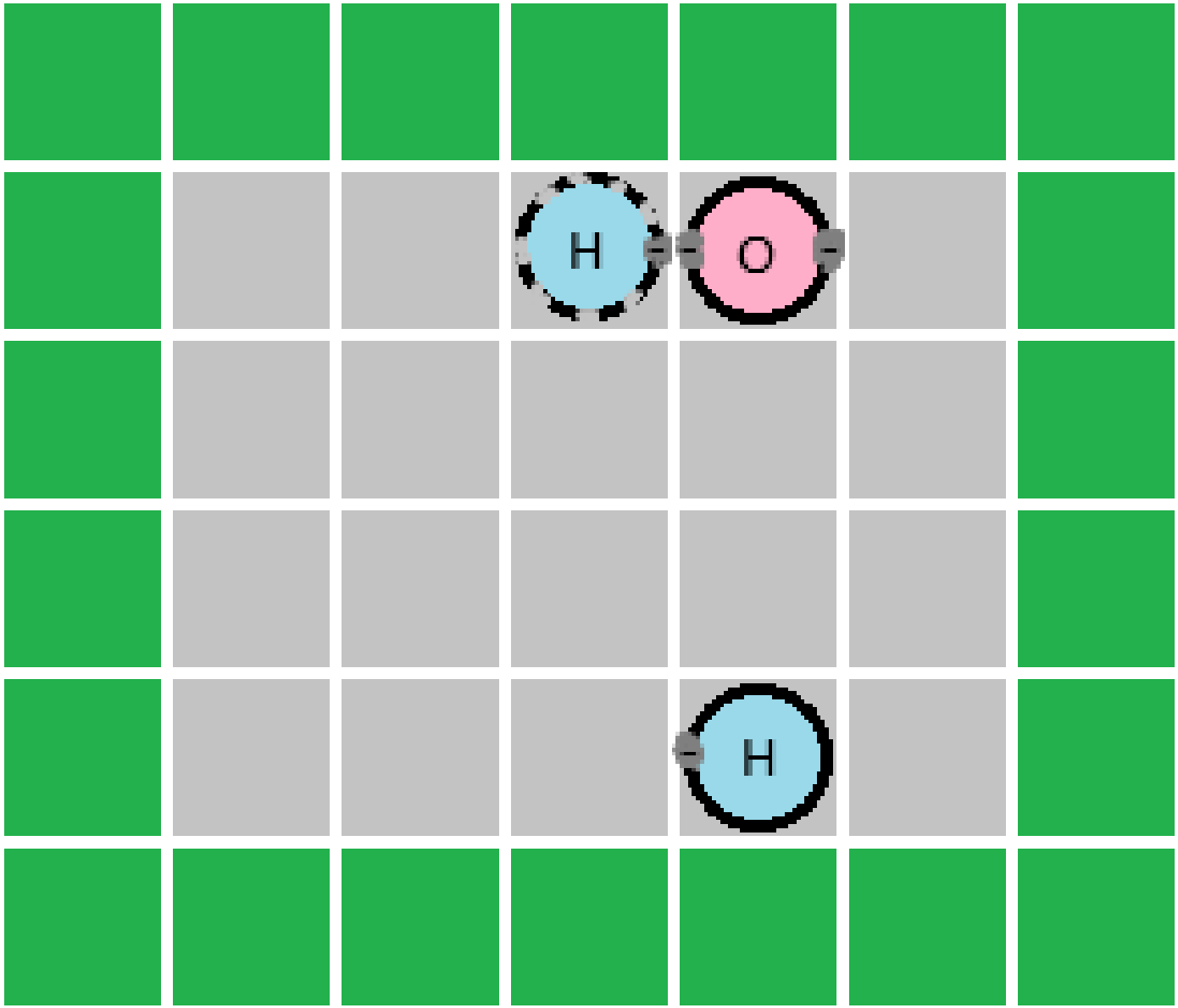

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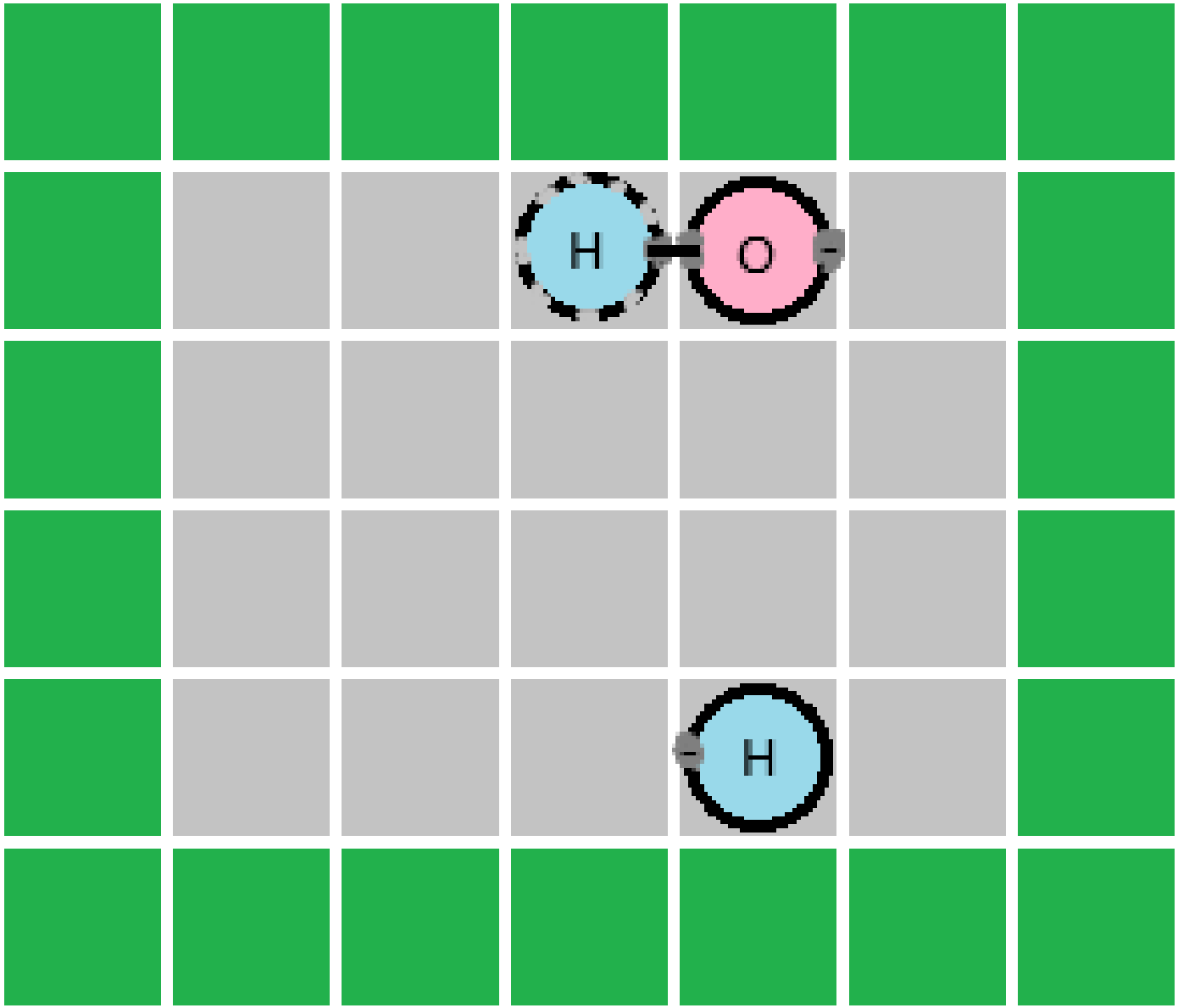

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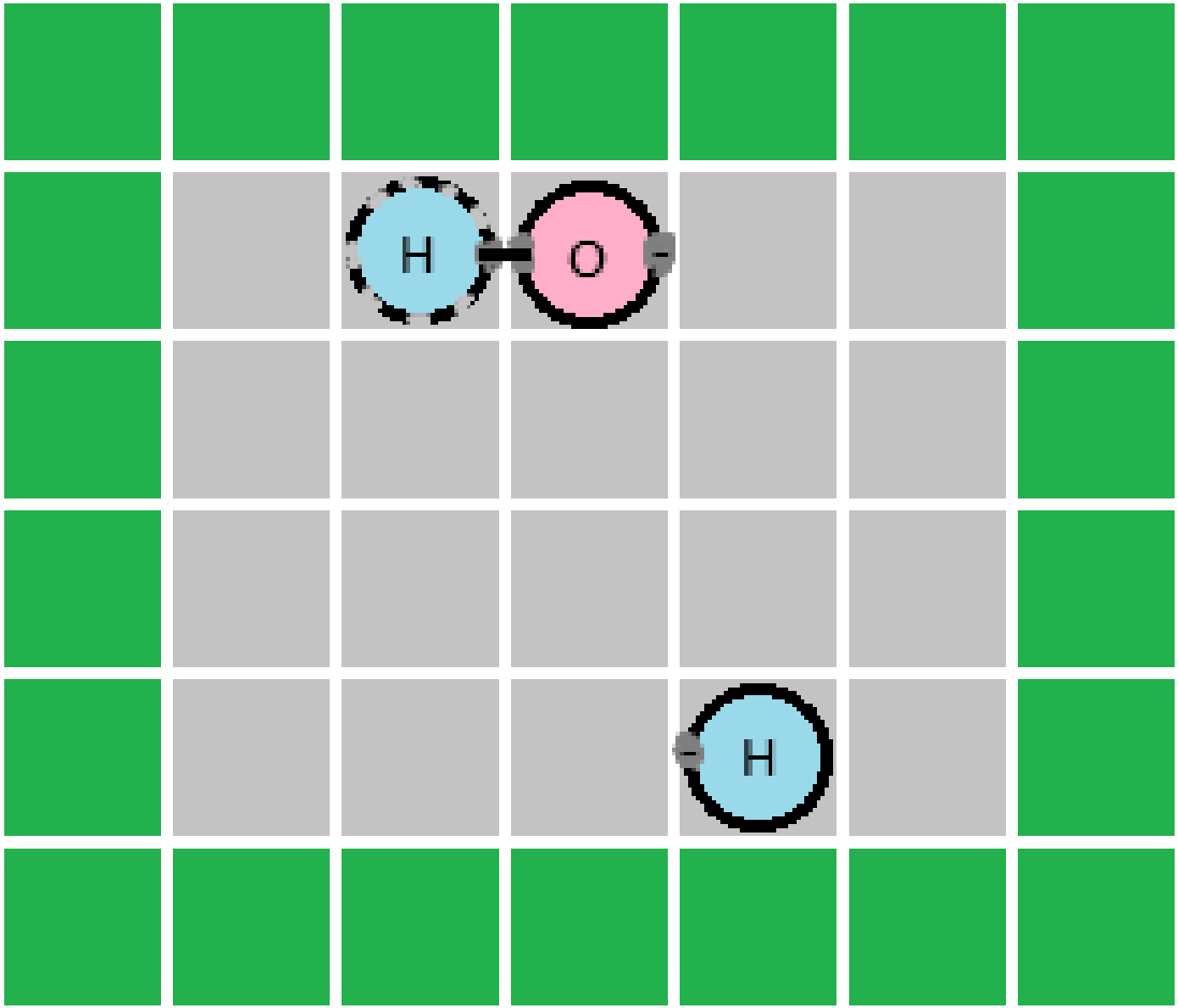

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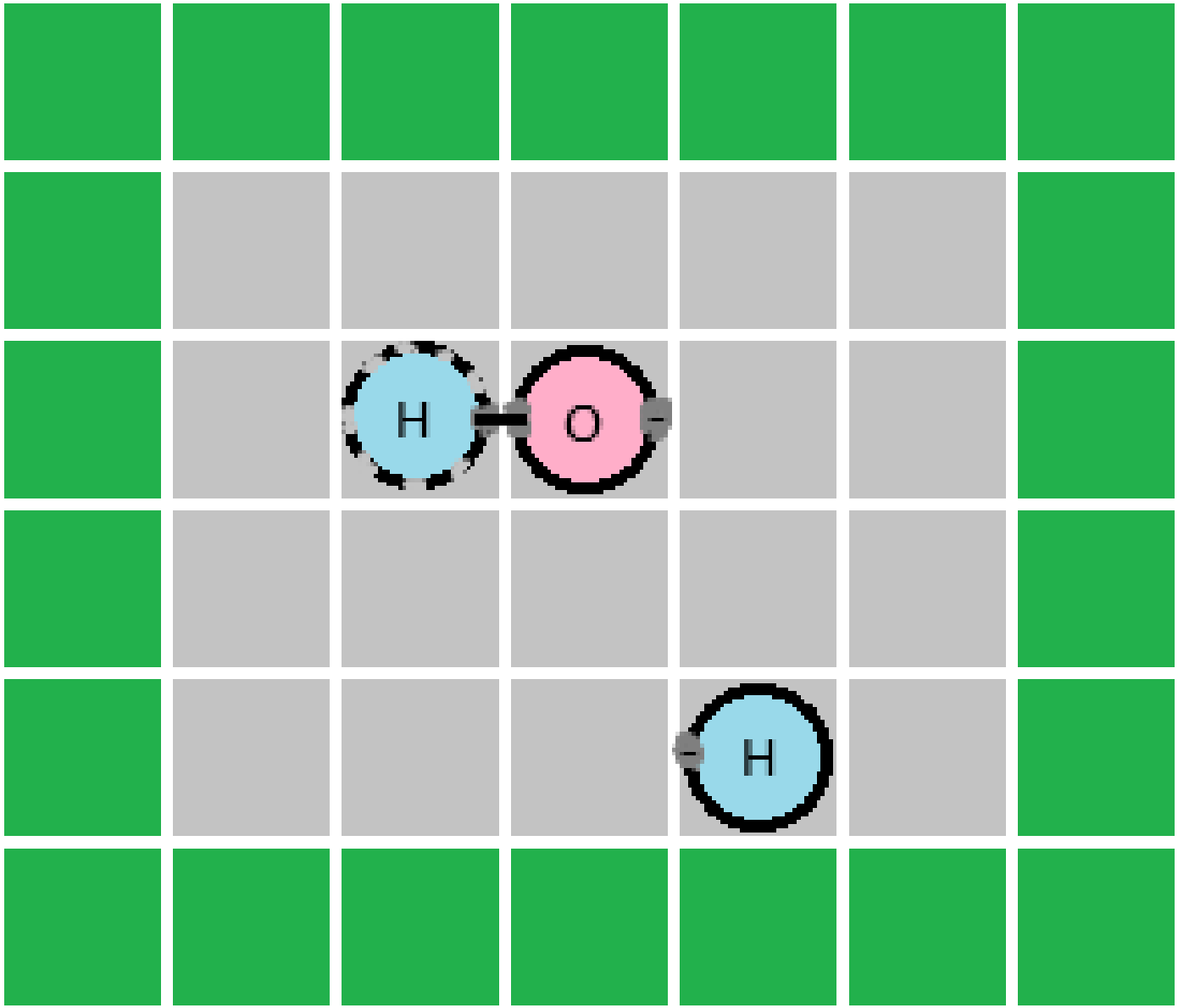

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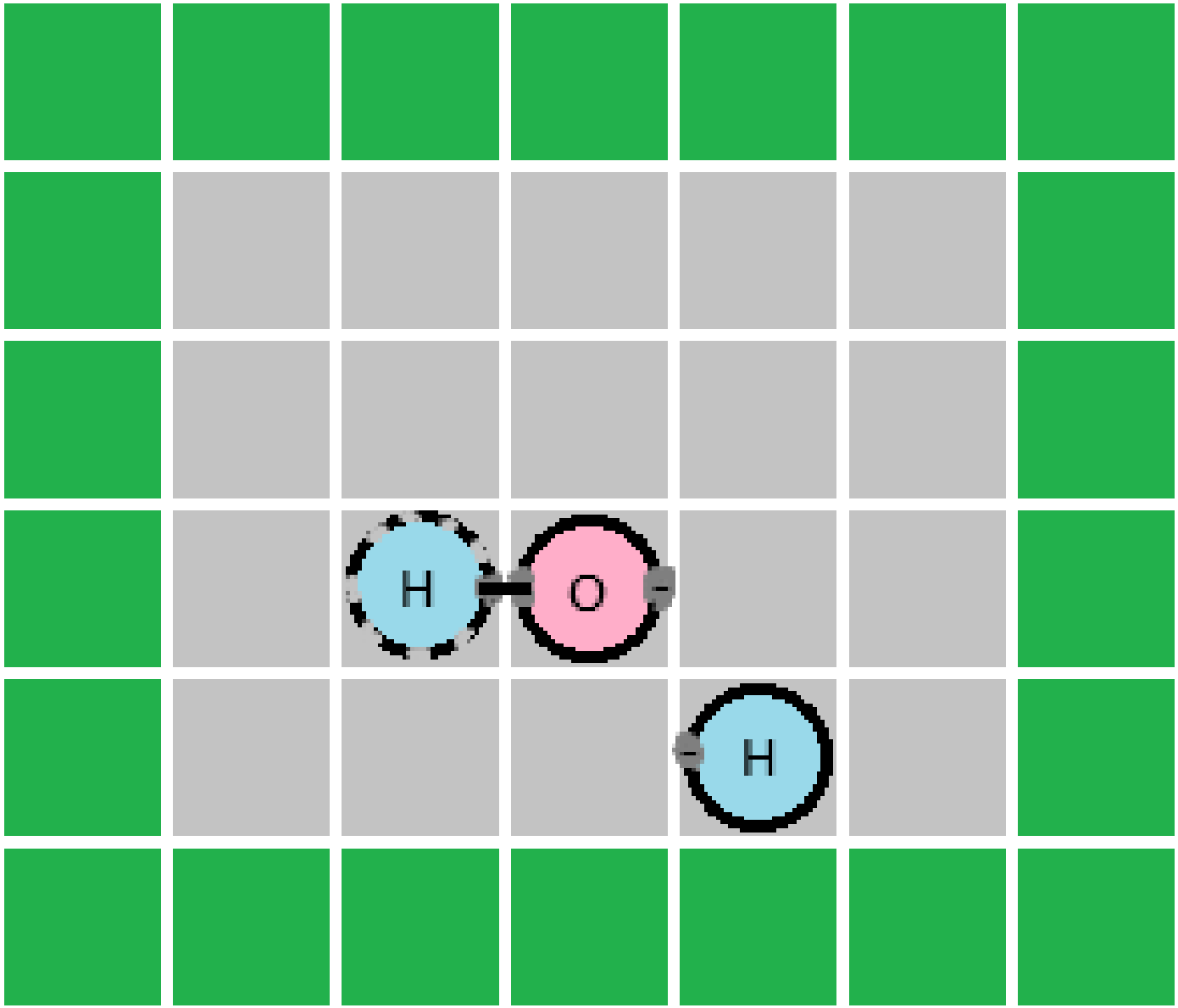

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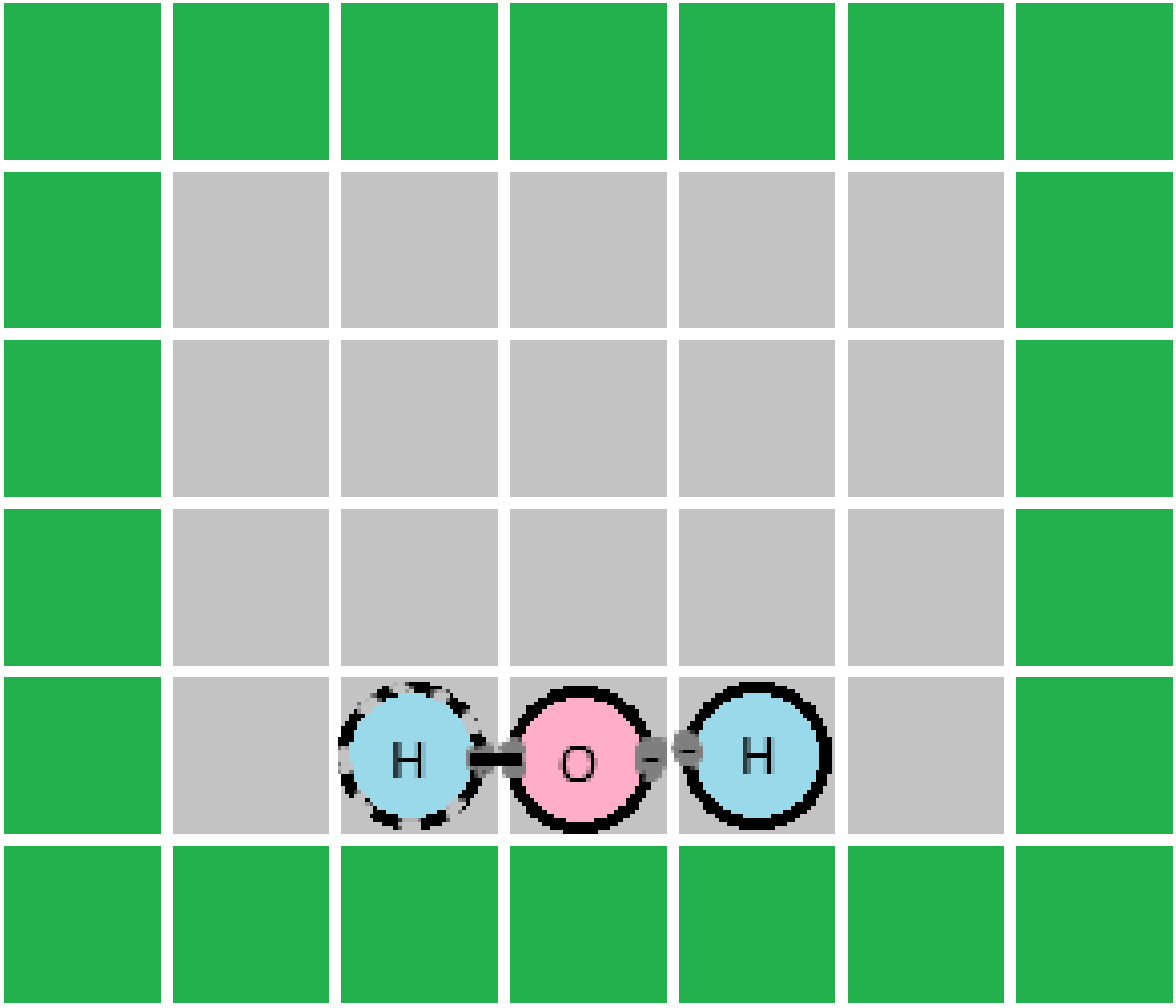

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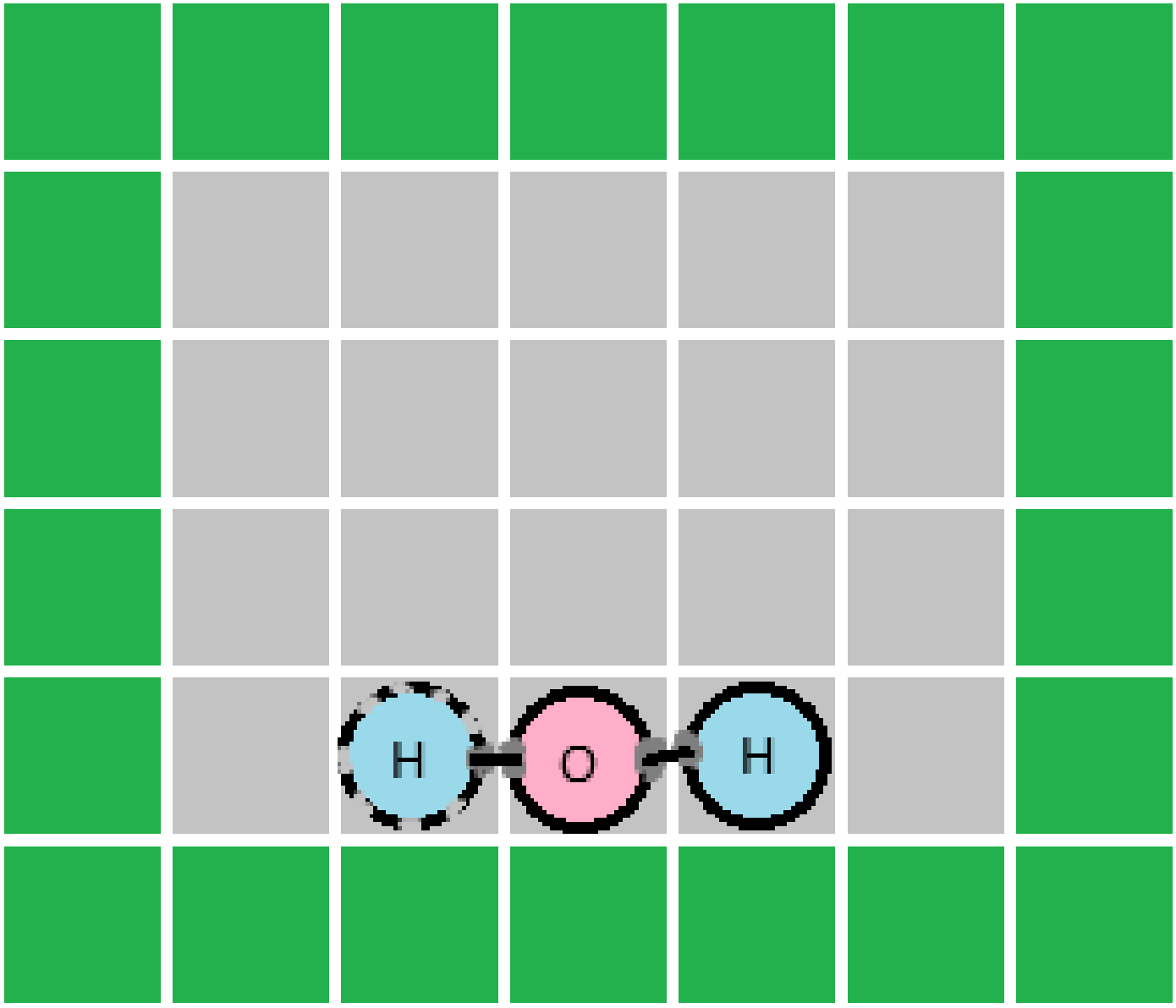
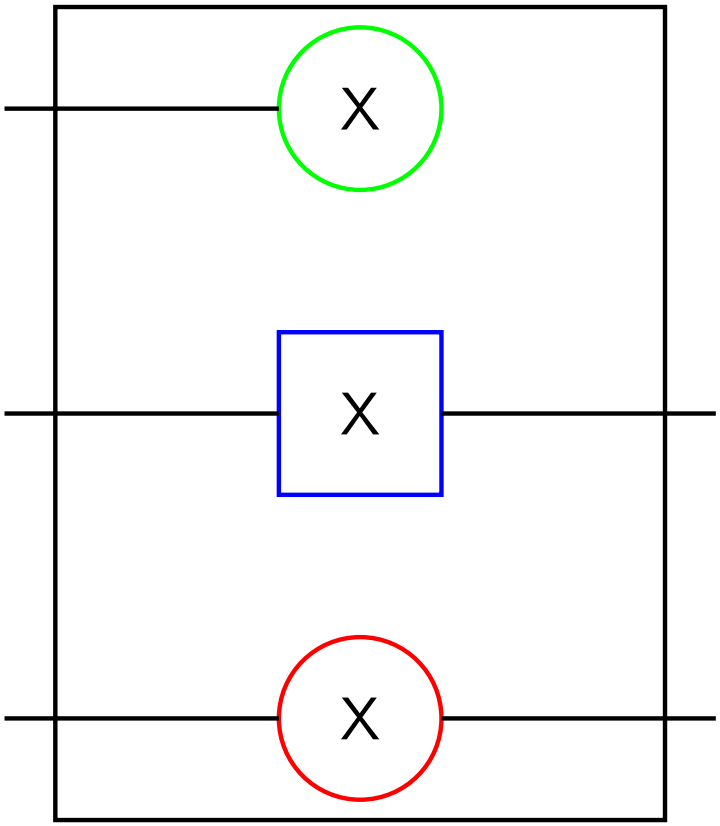
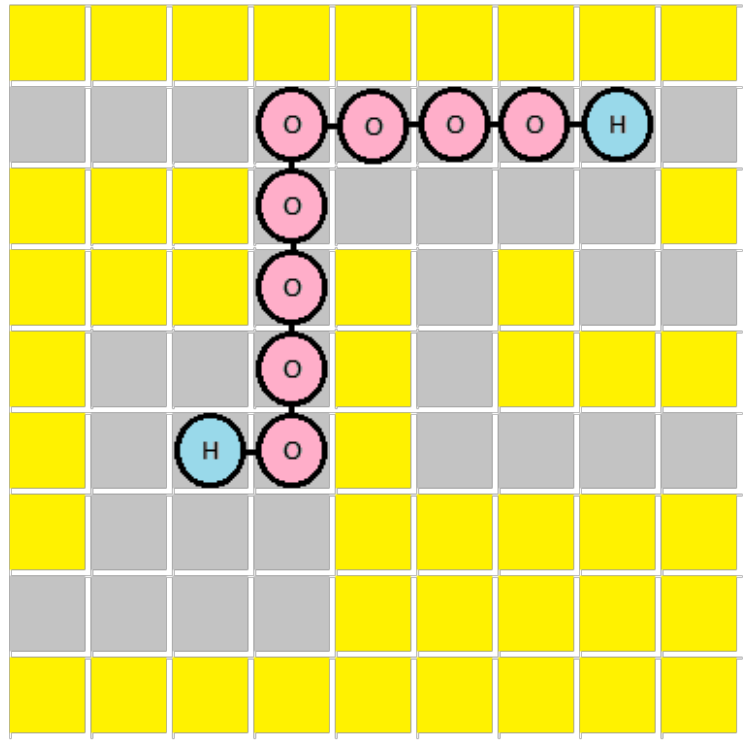


Sokobond is PSpace-Hard





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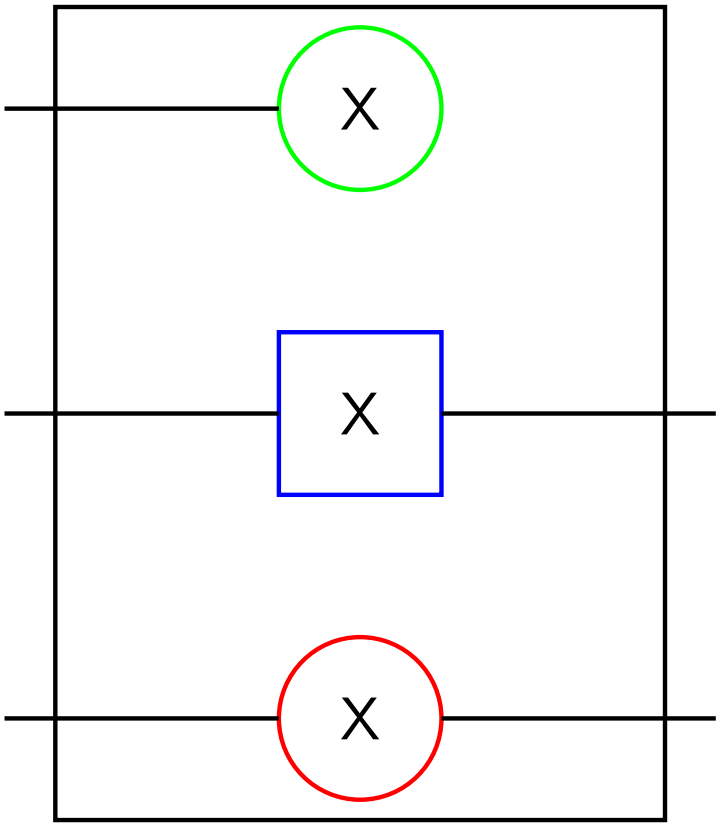

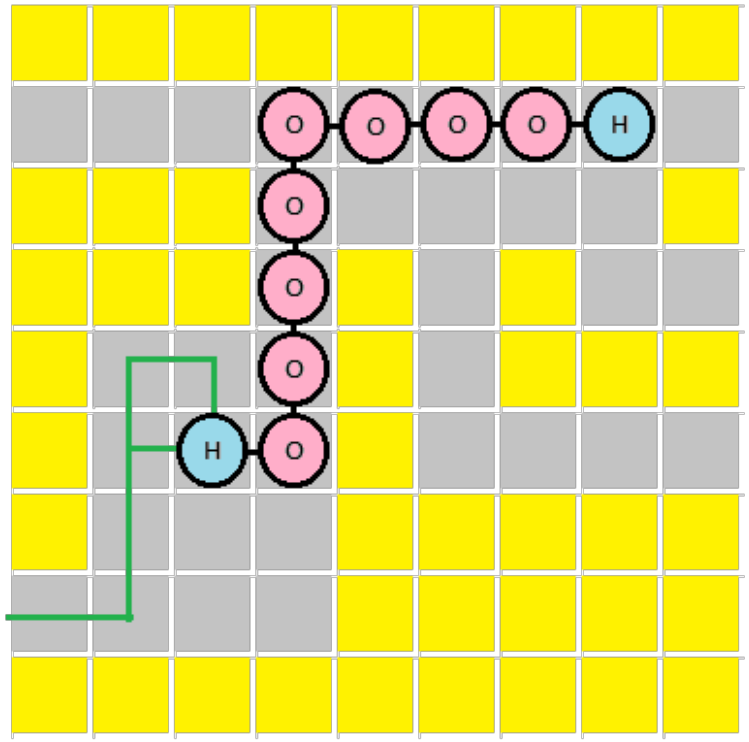

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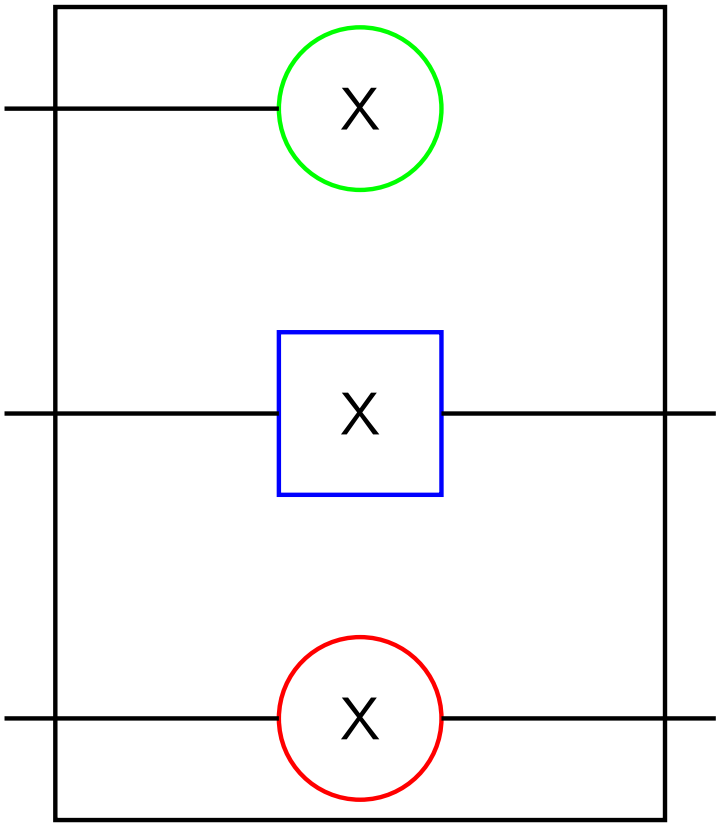

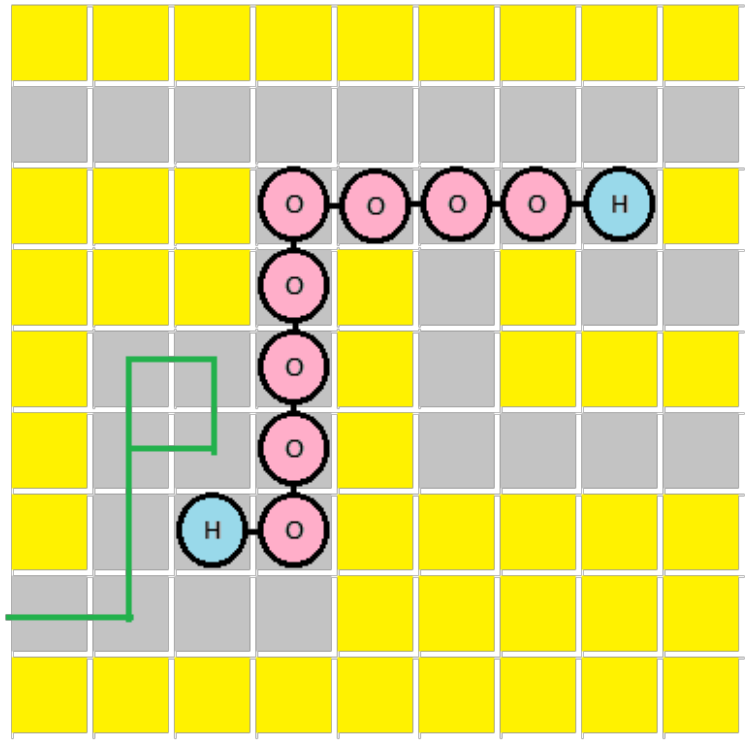

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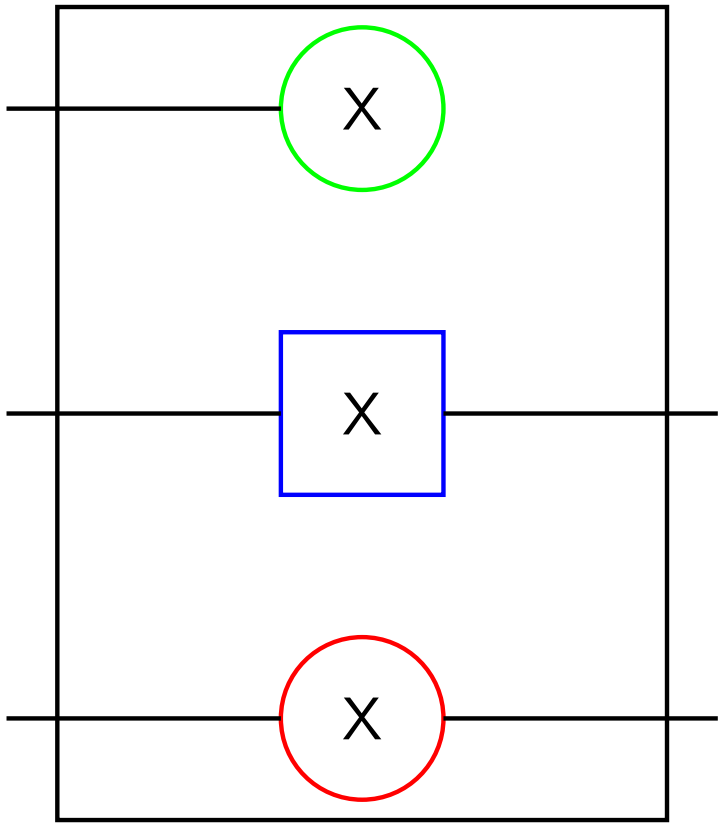

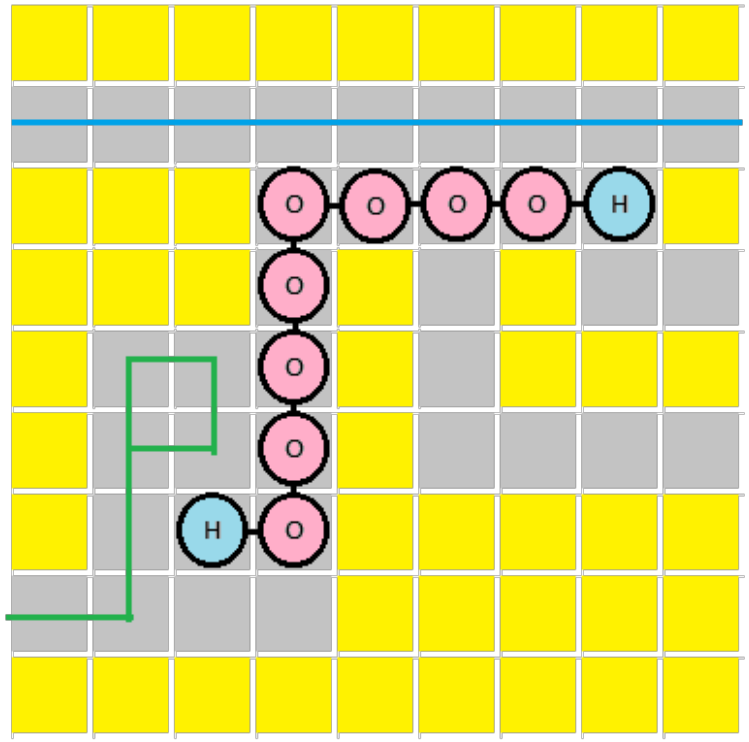

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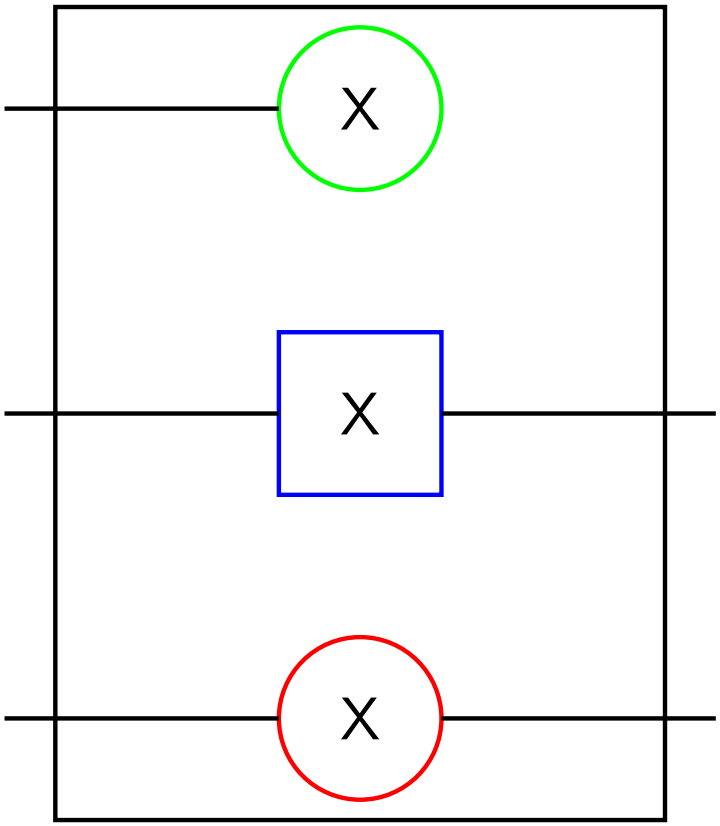

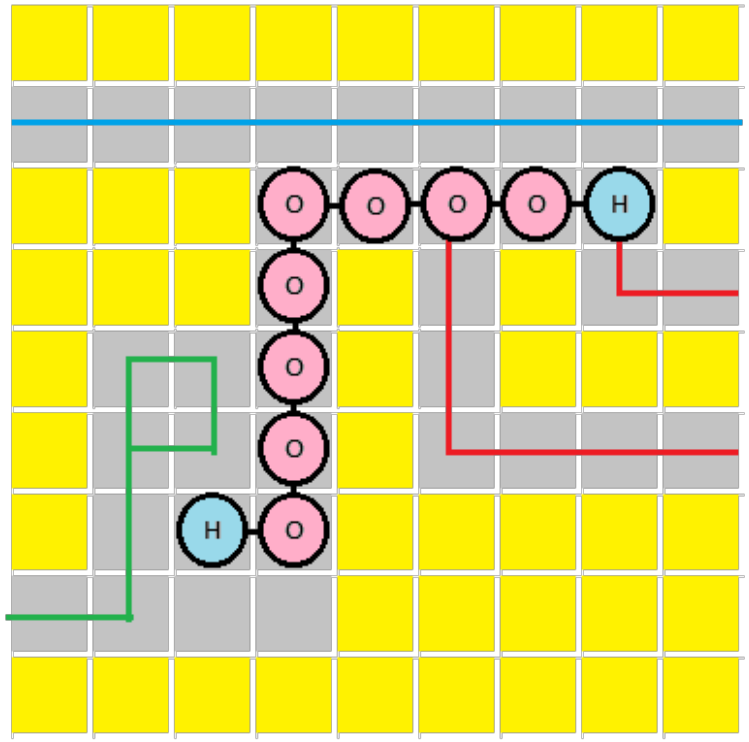

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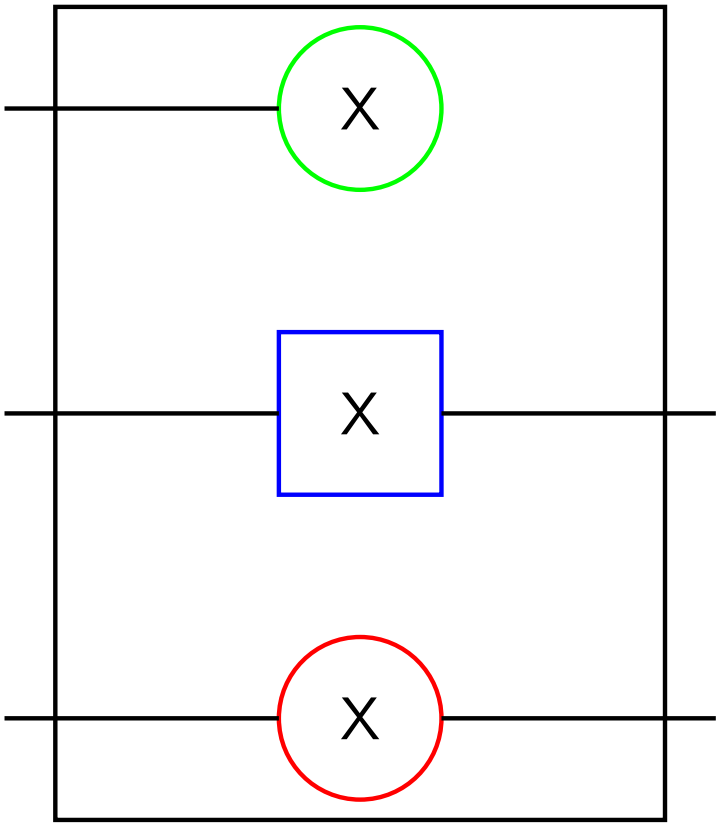
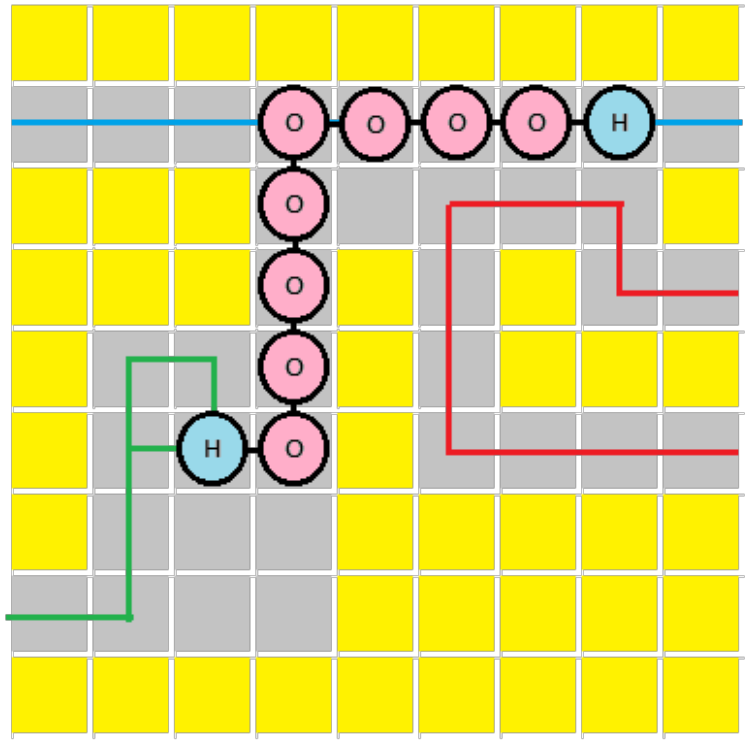
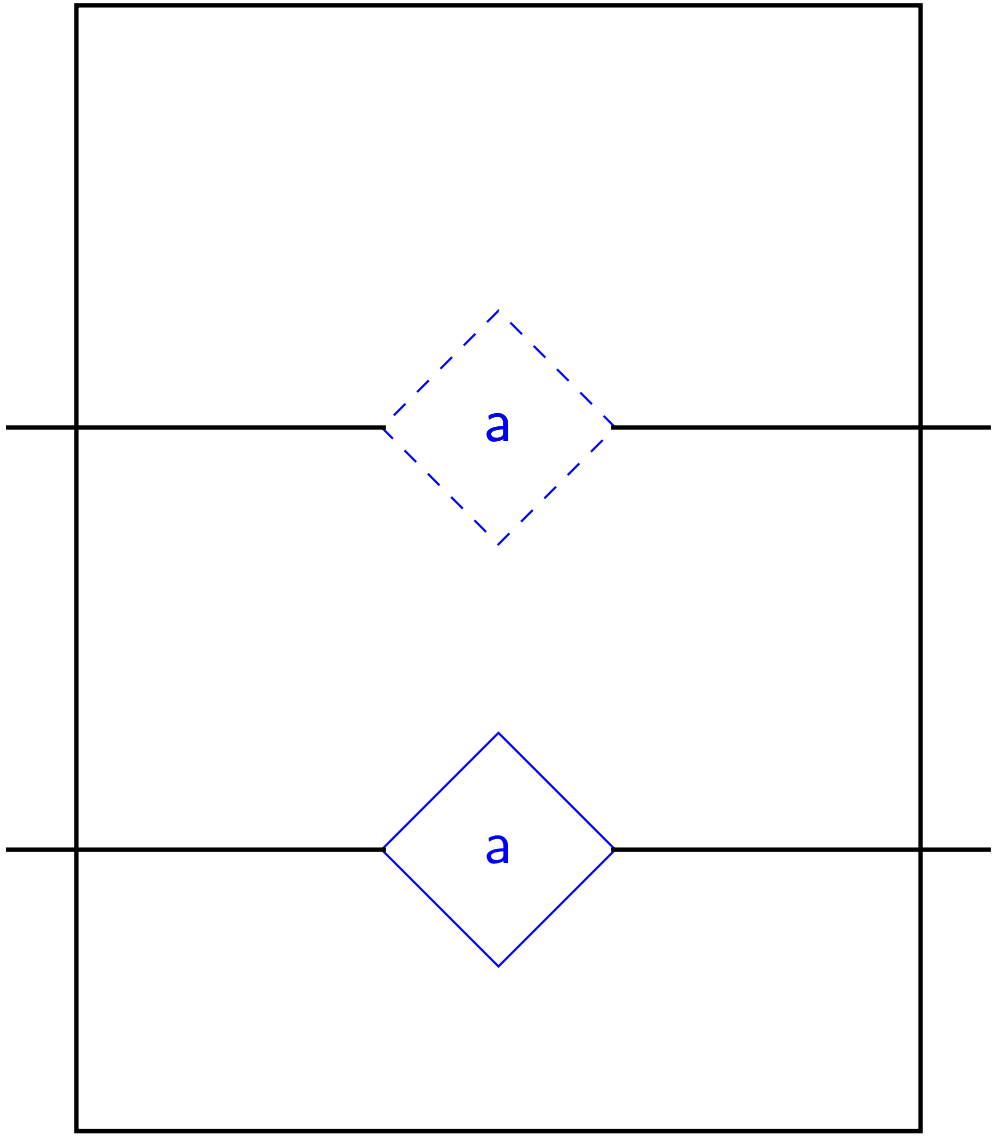




Reminder - Symetric self closing door



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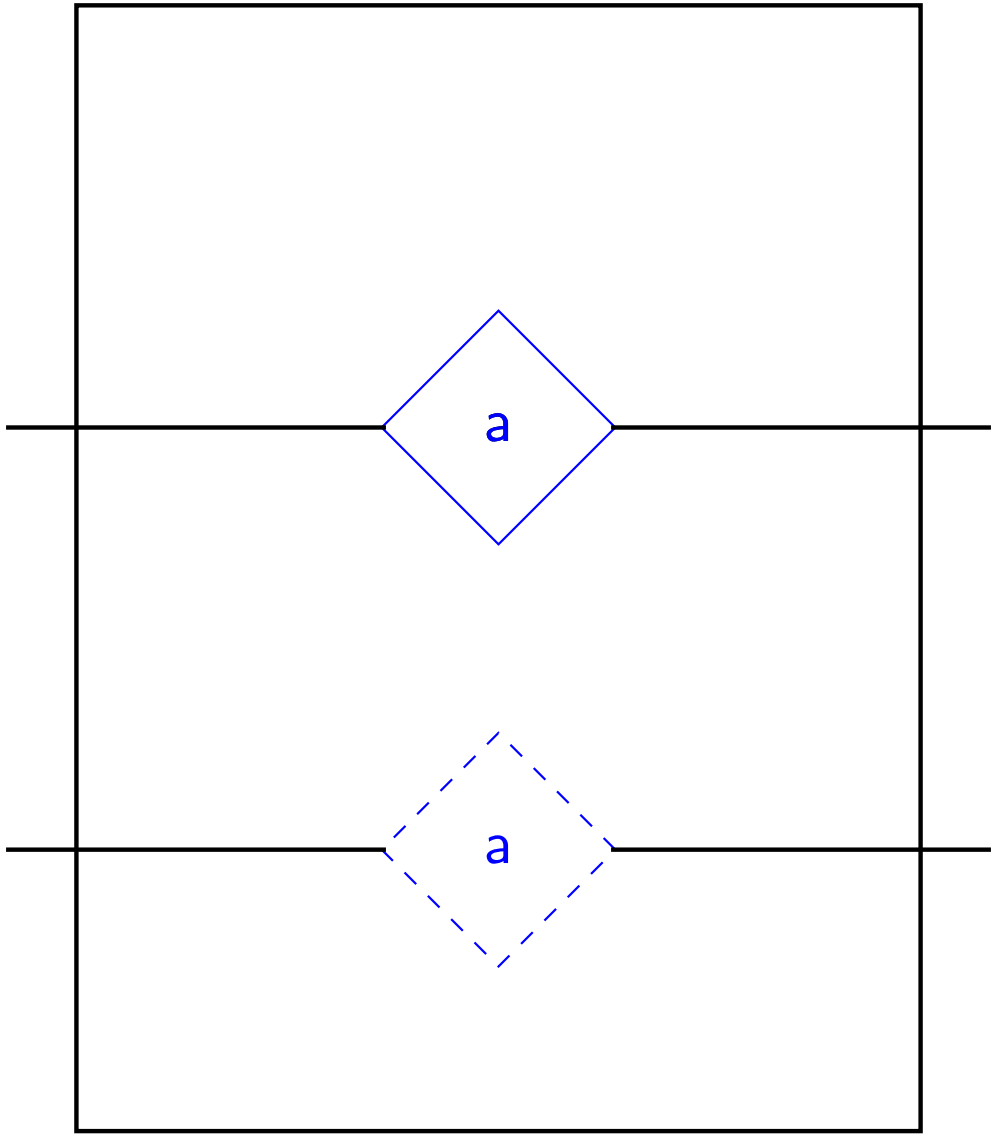
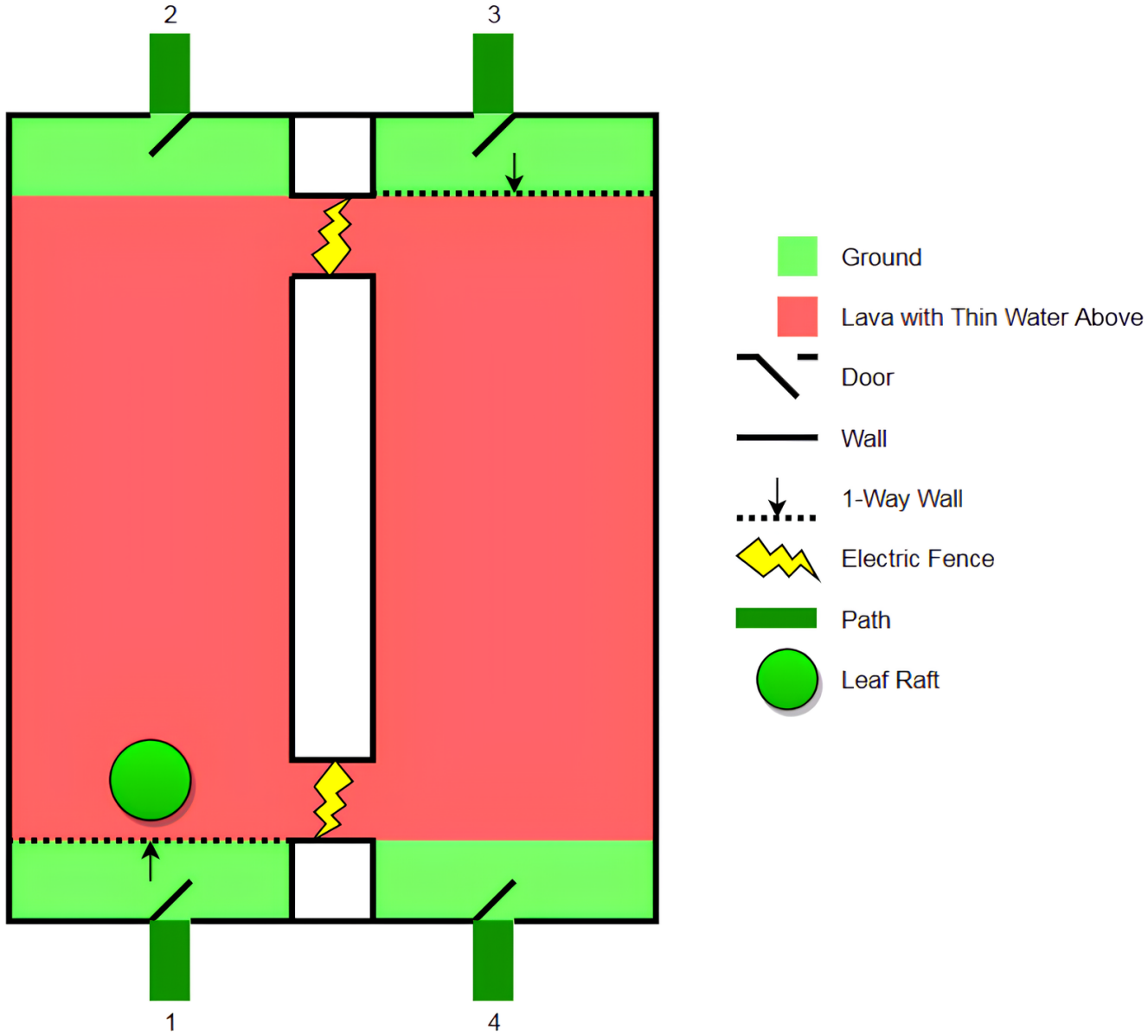


Super Mario Galaxy 2 is PSpace-hard



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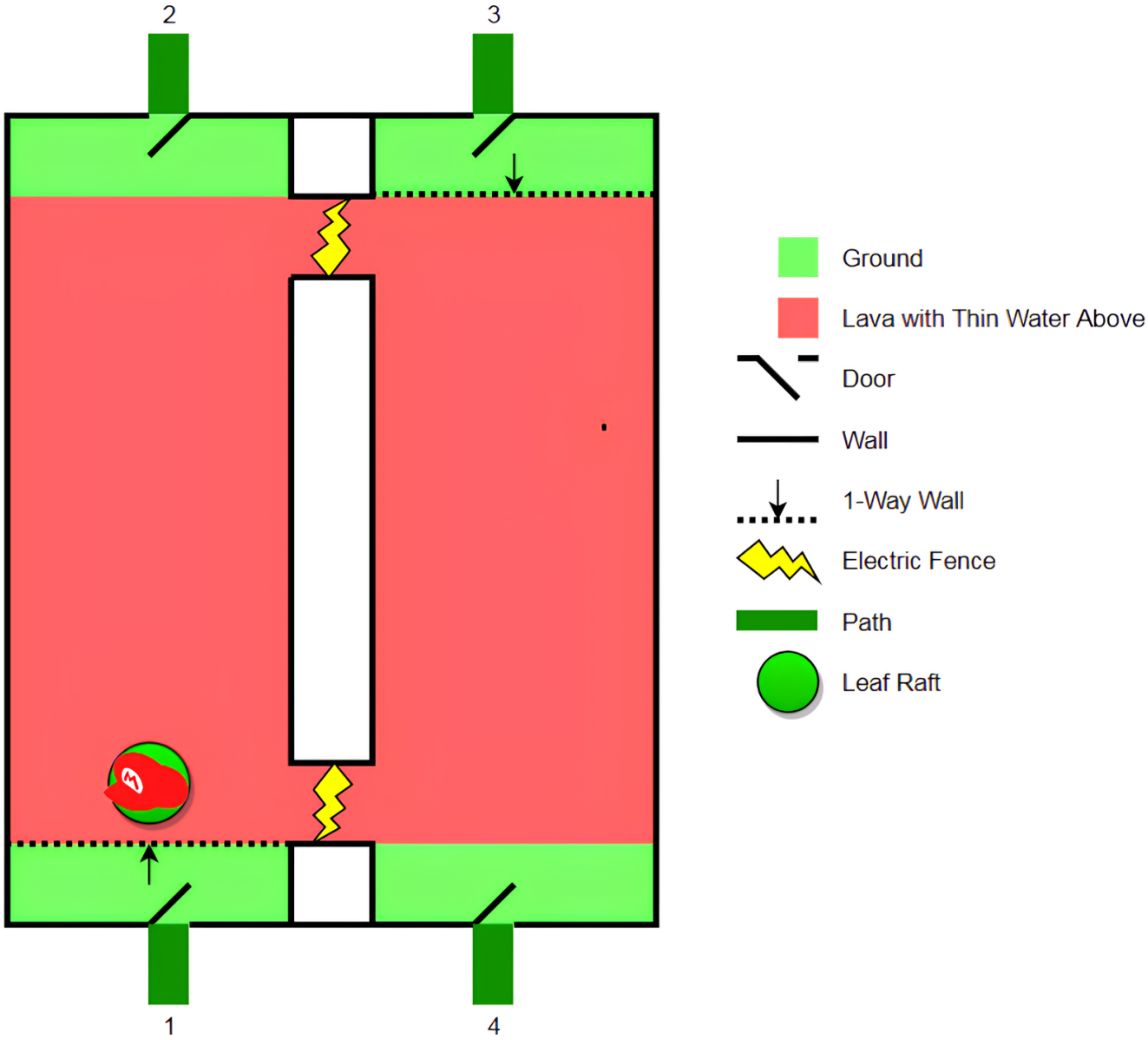

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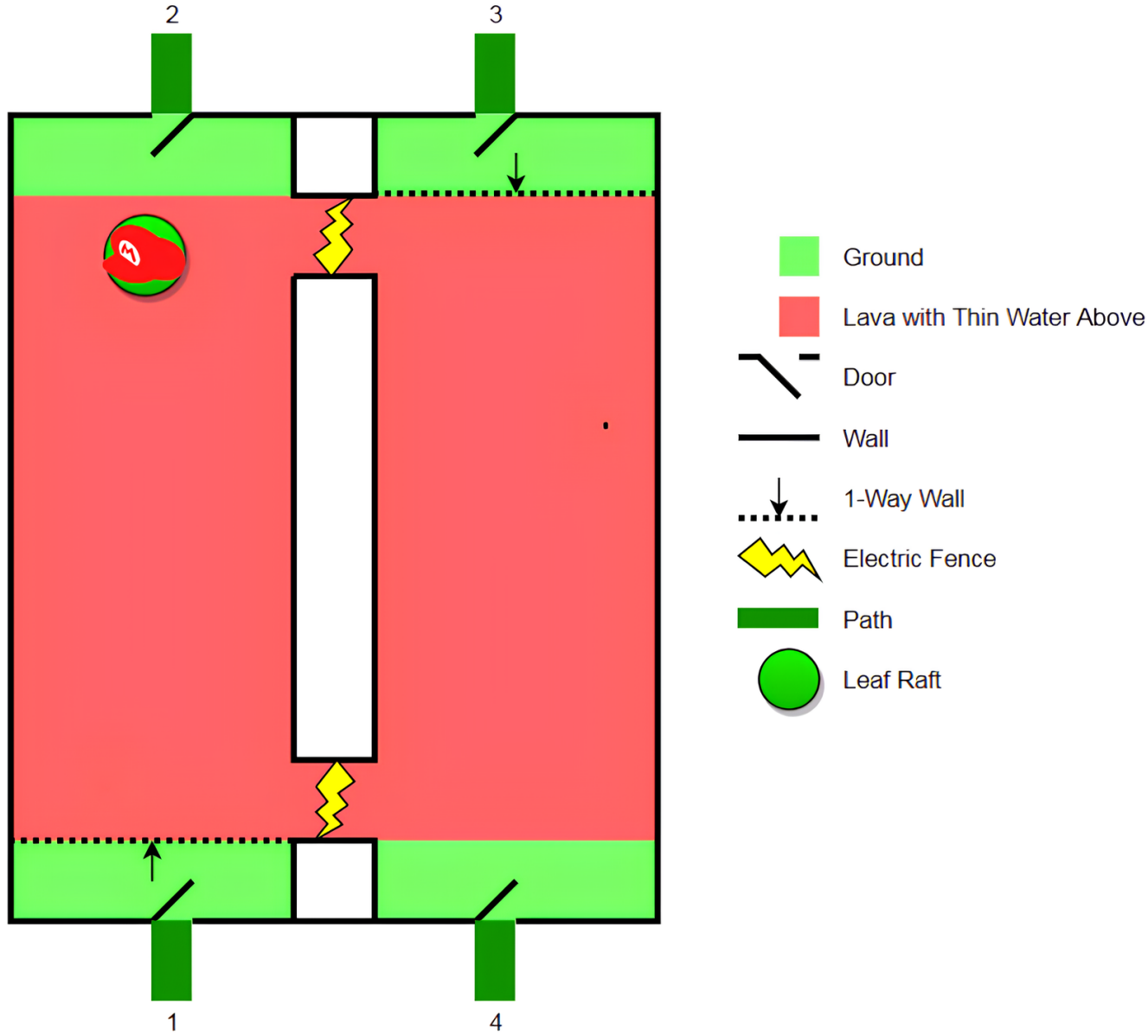

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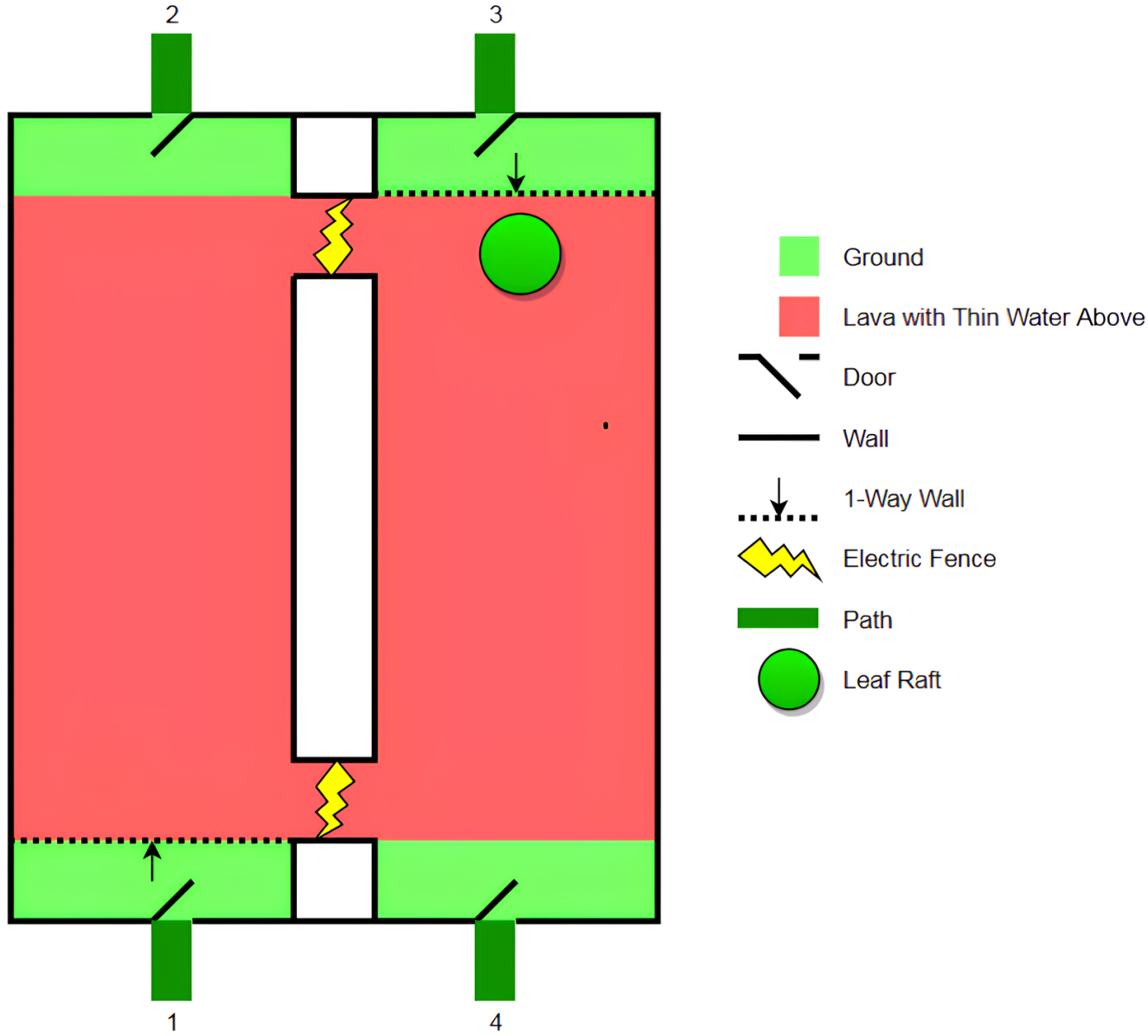
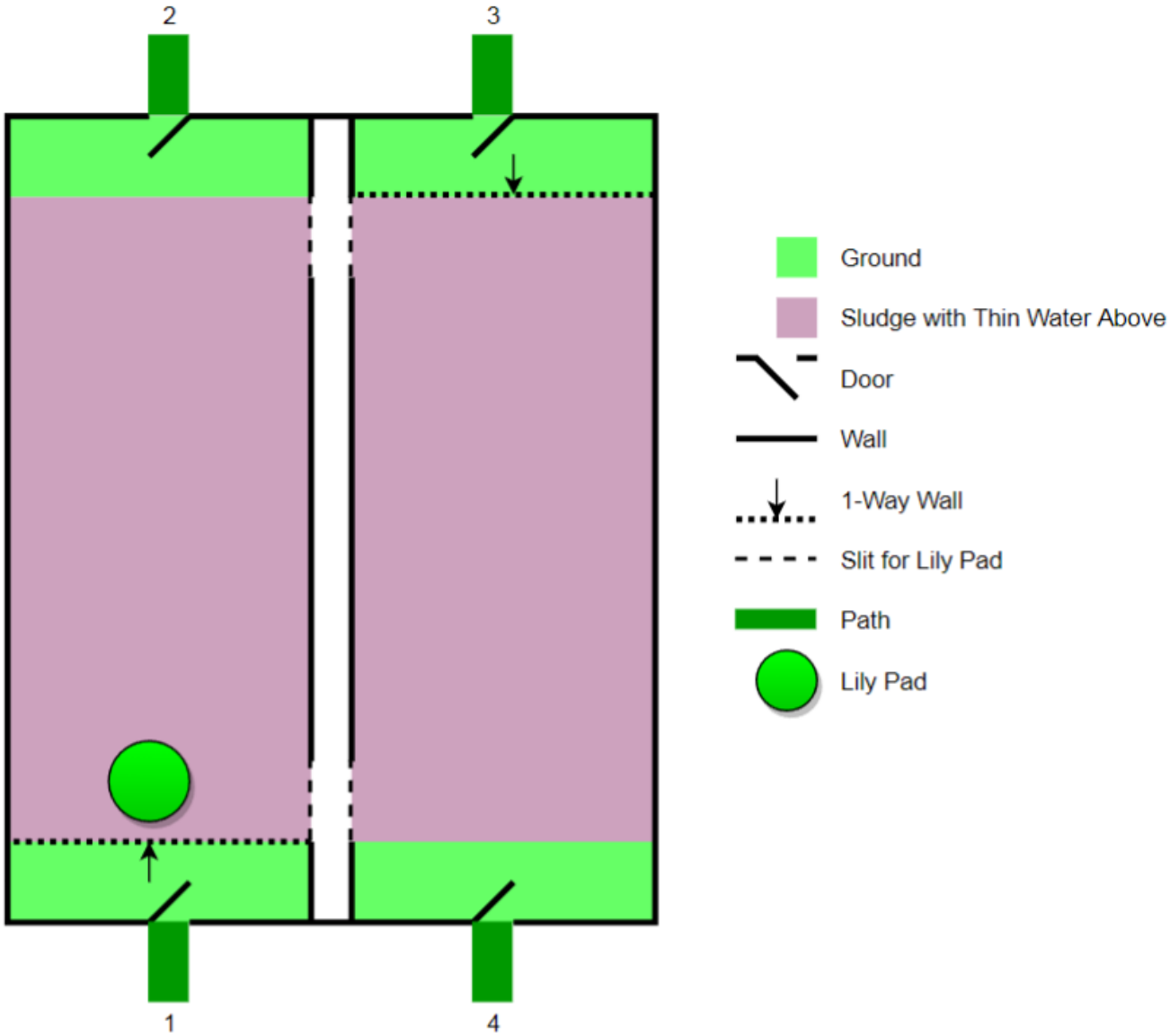
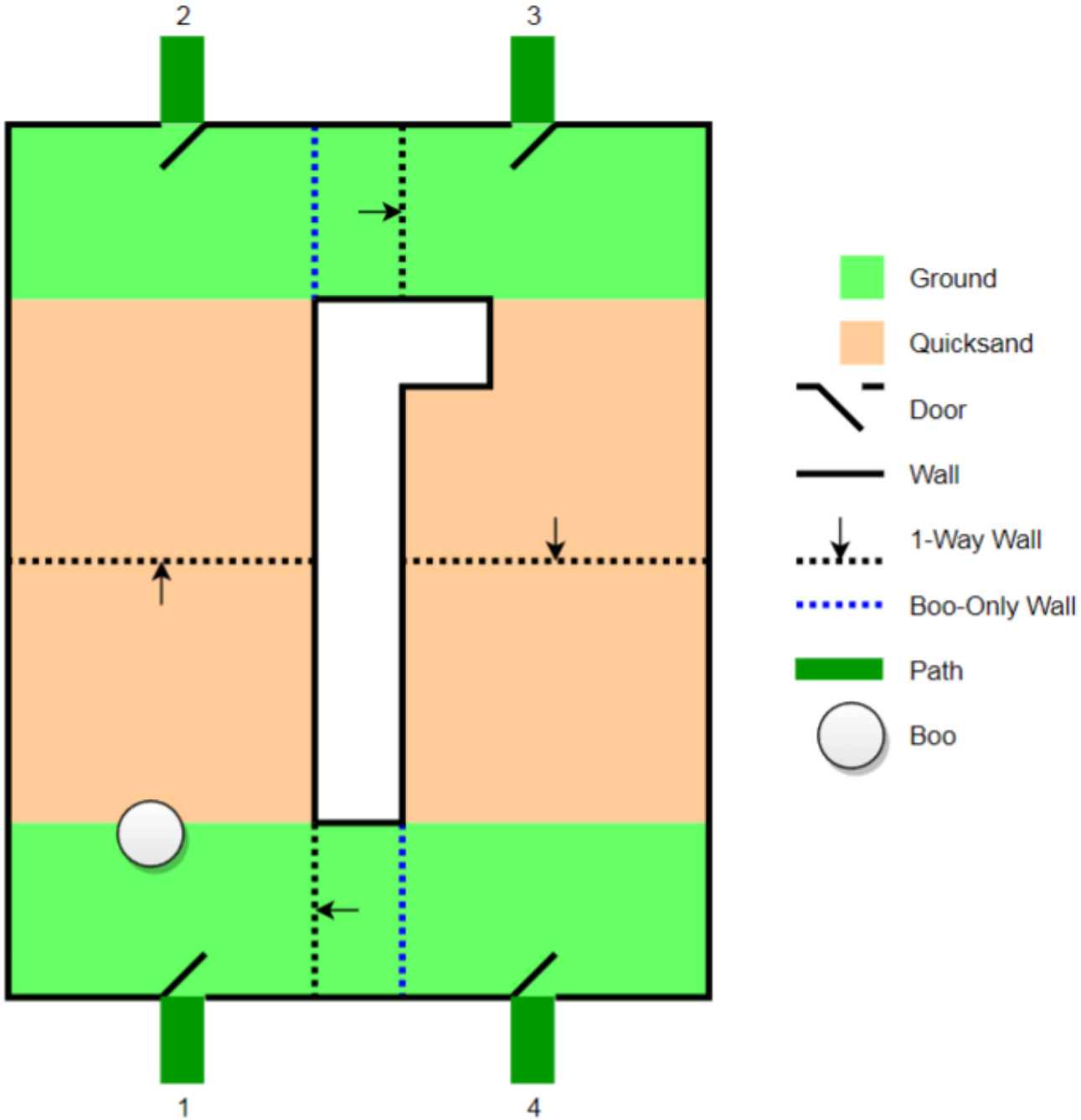


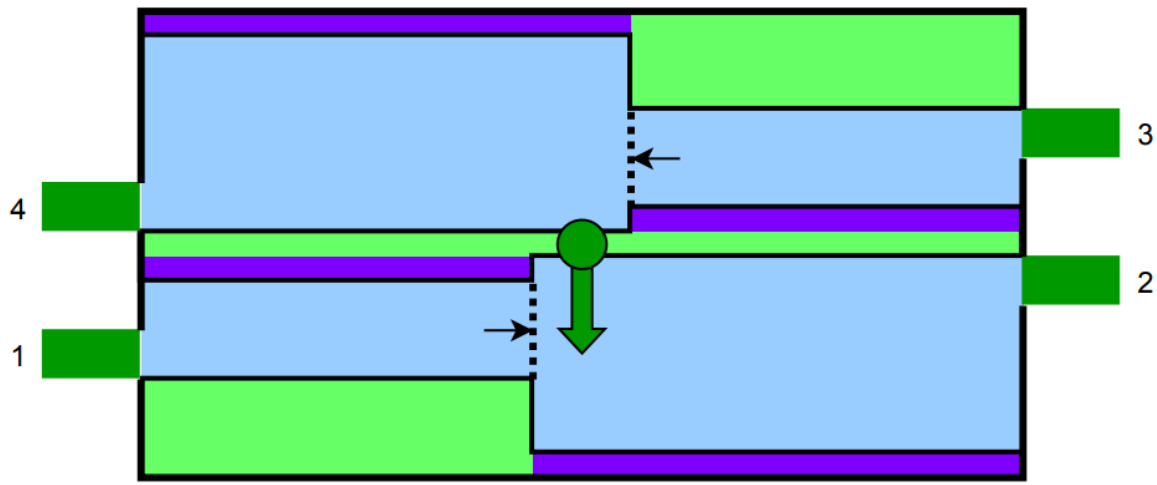
Similar constructs



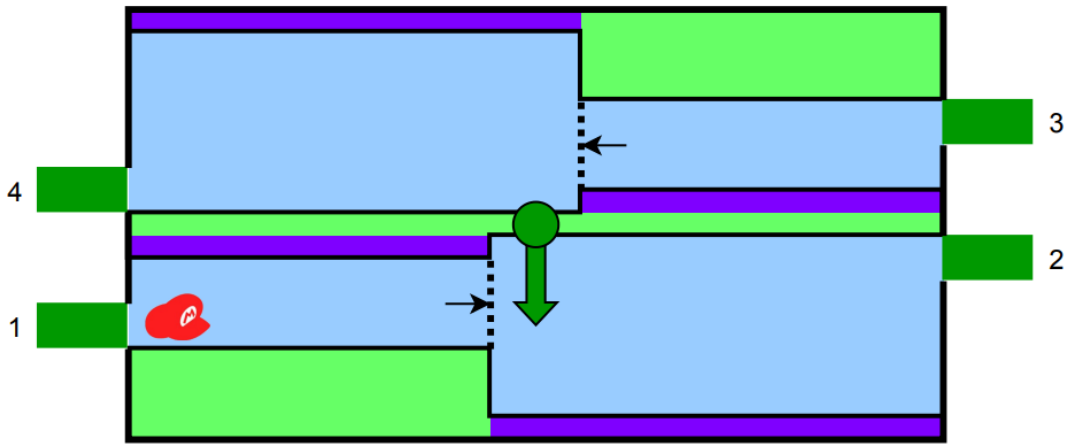
Similar constructs

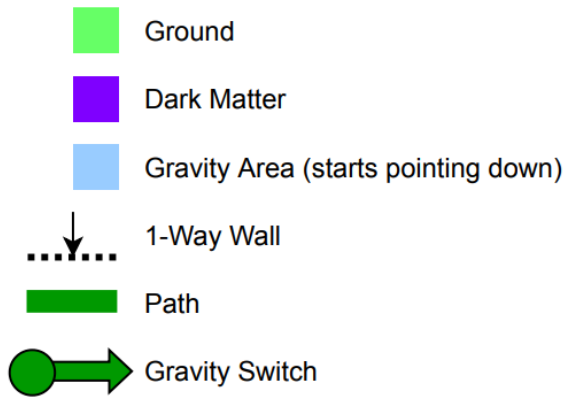


Super Mario Galaxy is PSpace-hard

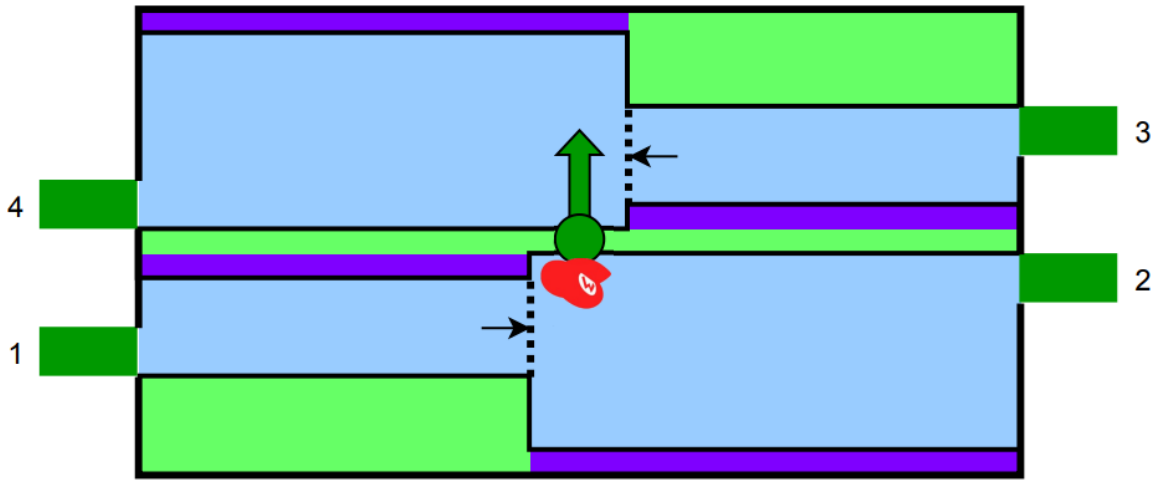


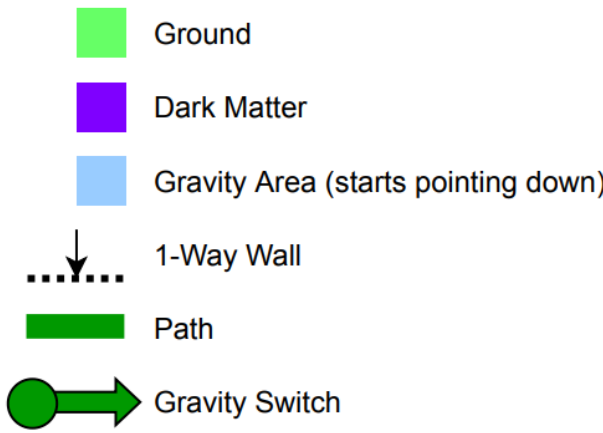
Super Mario Galaxy is PSpace-hard





Super Mario Galaxy is PSpace-hard





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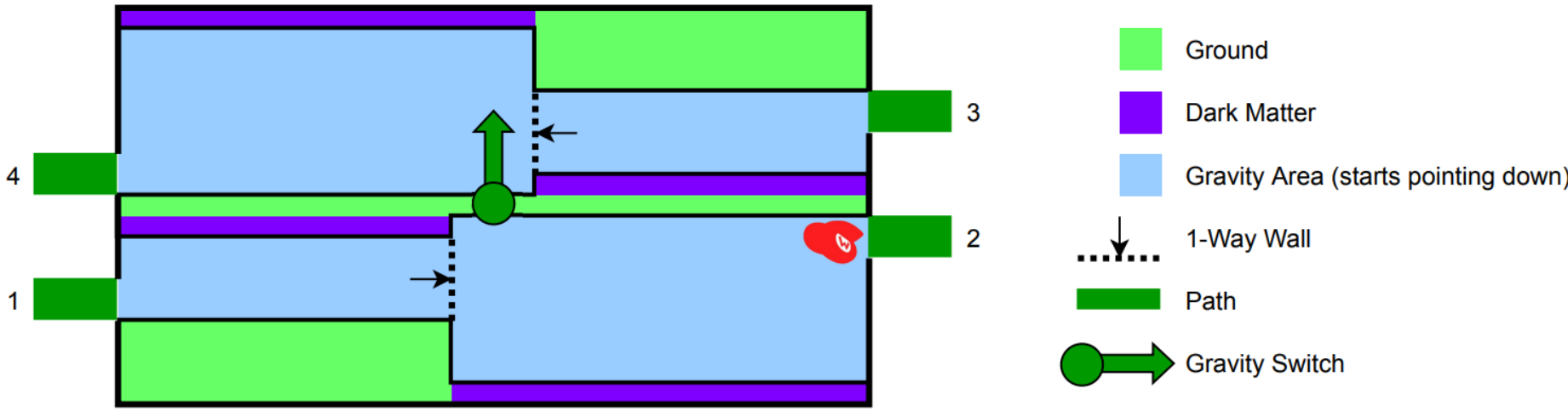
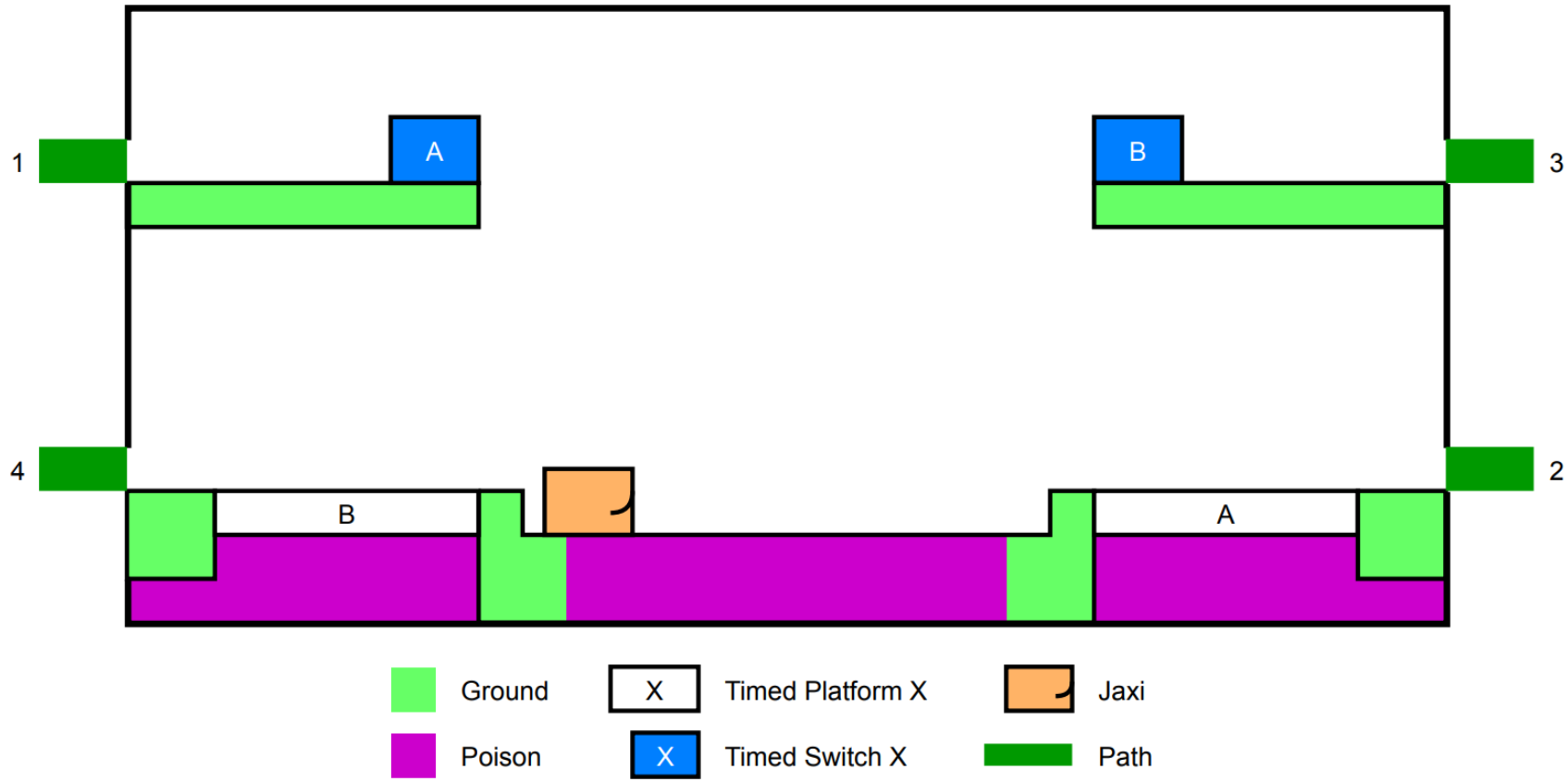
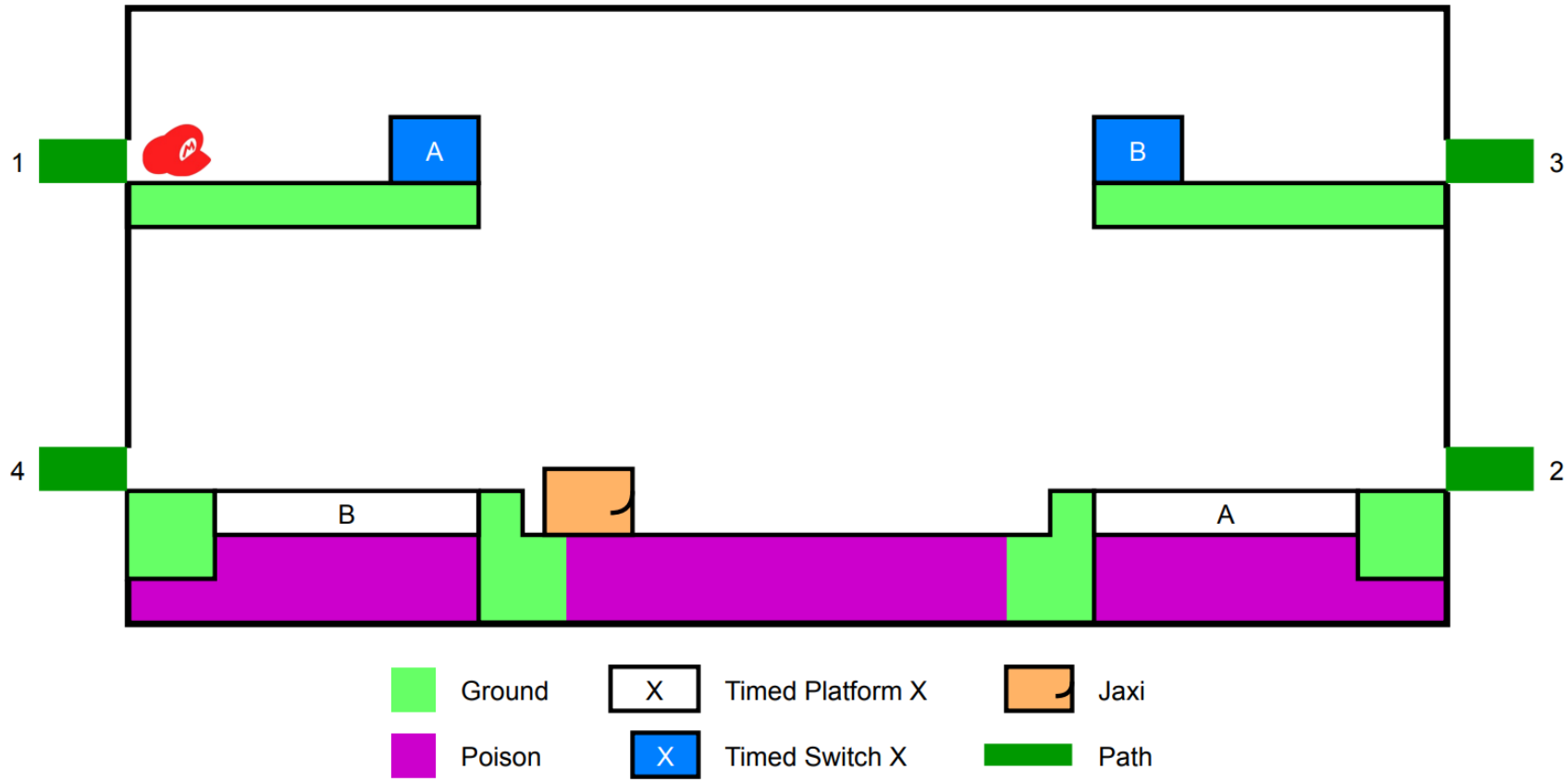


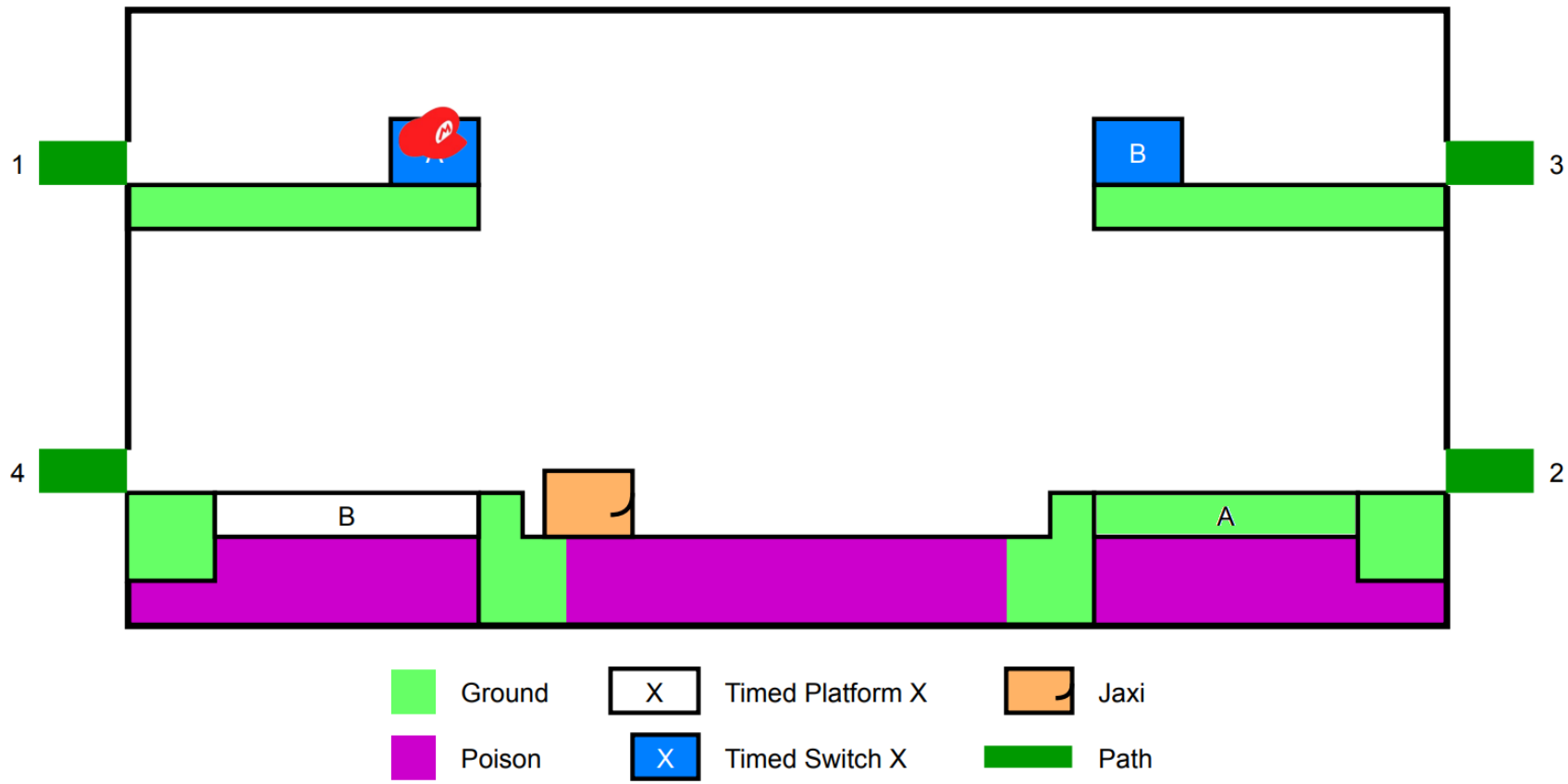
Super Mario Odessy is PSpace-hard



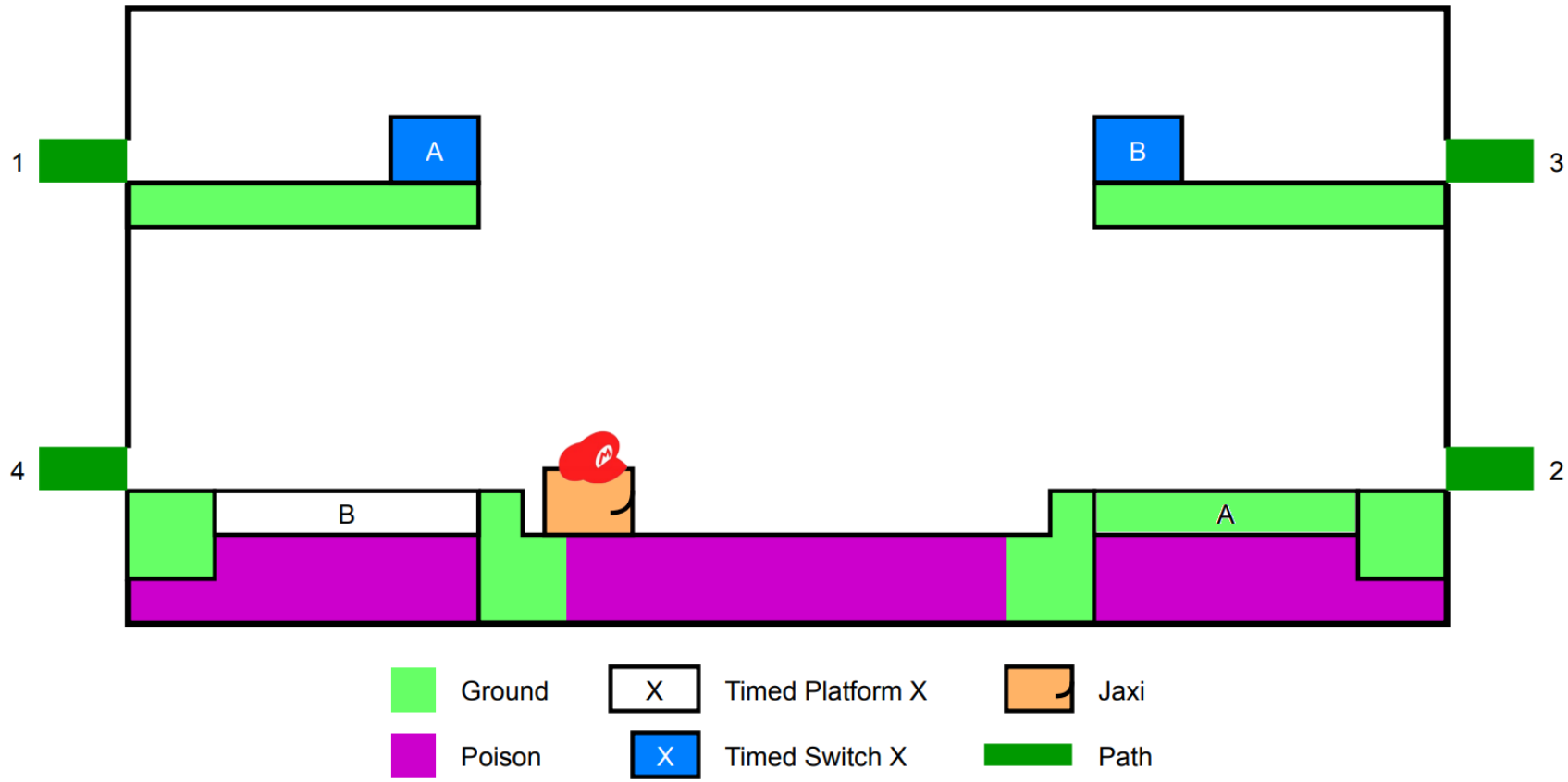
Super Mario Odessy is PSpace-hard



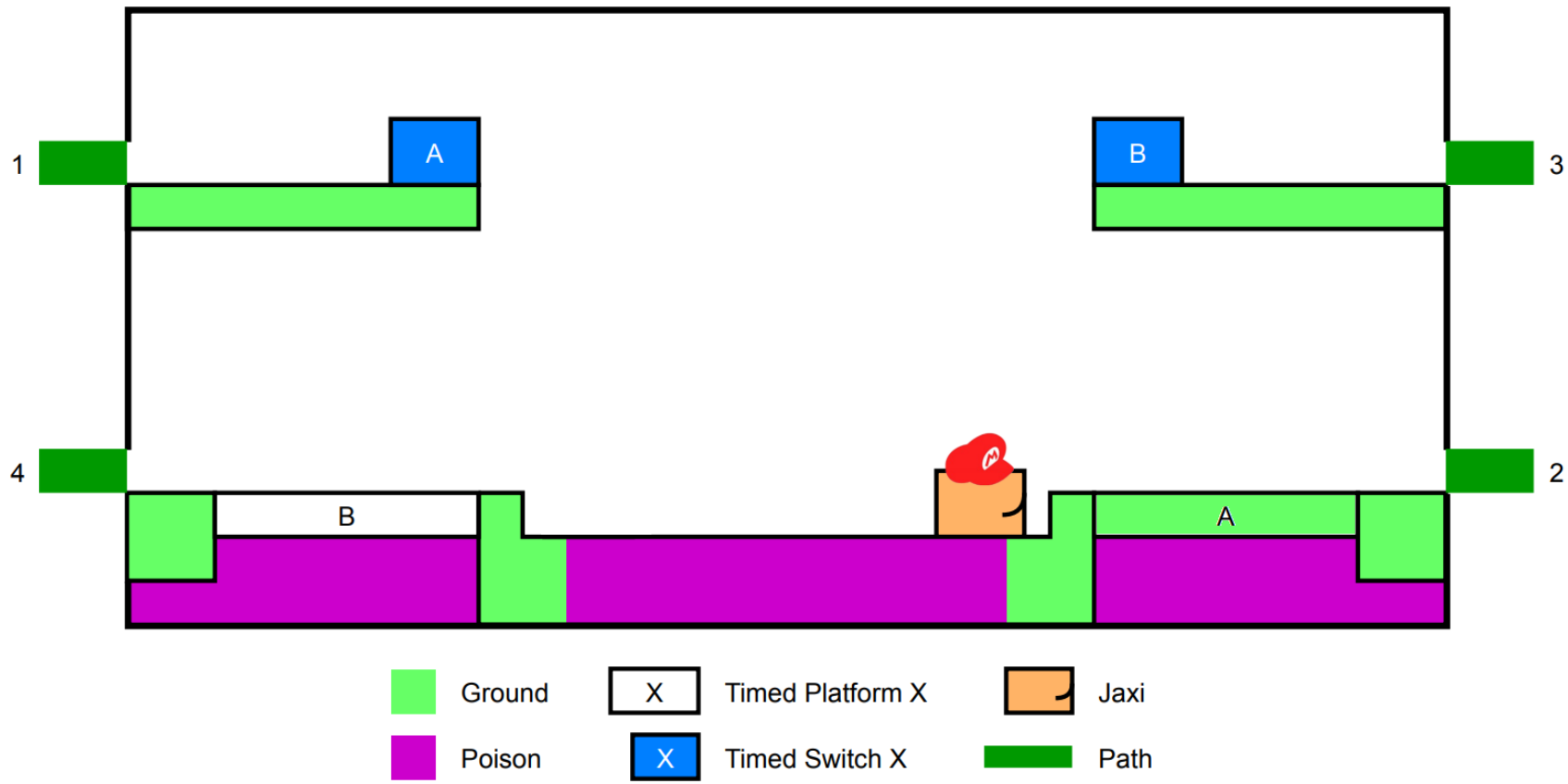
Super Mario Odessy is PSpace-hard



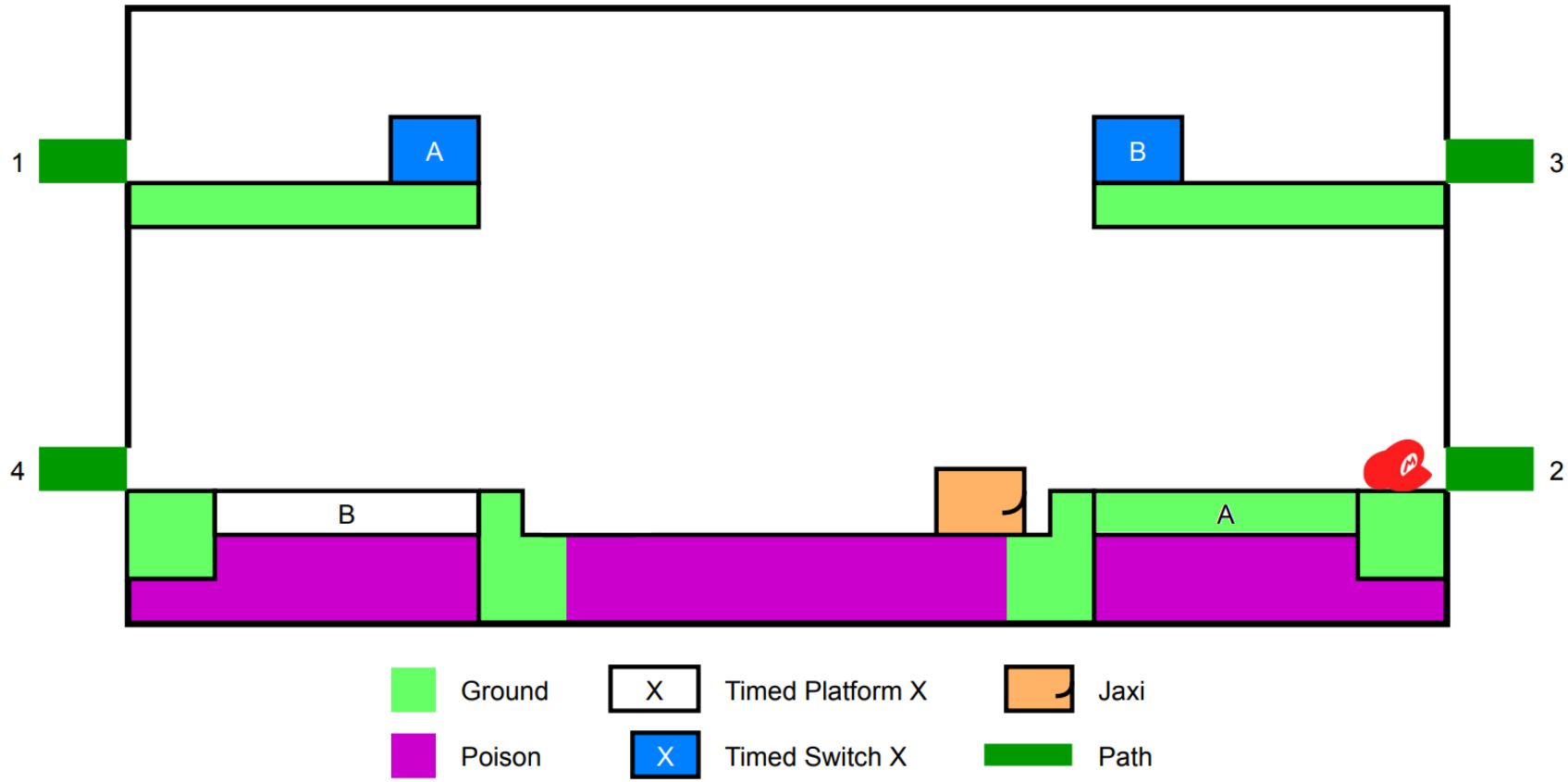
Super Mario Odessy is PSpace-hard



Super Mario Odessy is PSpace-hard



Super Mario Odessy is PSpace-hard



Super Mario Odessy is PSpace-hard