FUN with Complexity: Walking through Doors is Hard, even without Staircases

Manuel Frohn

RWTH Aachen

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Content

PSPACE-Complexity

A given problem requires at most a polynomial amount of memory in relation to the input, to be solved $\Leftrightarrow$ The problem is in PSPACE
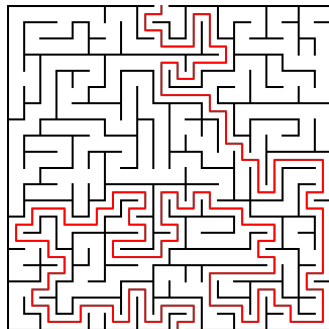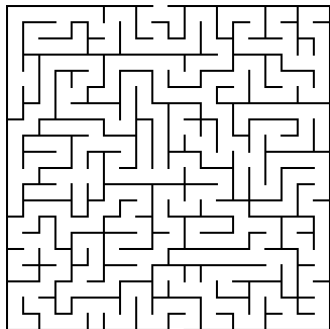
SAT

$x_1 \wedge x_2 \vee \neg x_3$

Quantified SAT

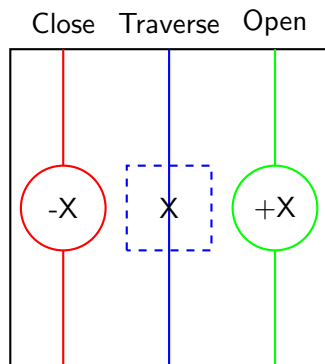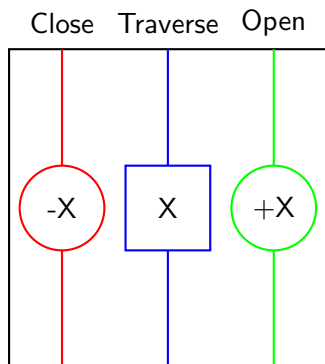$\forall x_1 \exists x_2 : x_1 \wedge x_2 \vee \neg x_3$

1-PlayerMotionPlaning

Given: Enviroment, Agent, Goal
Question: Is the goal achivable

Basic Door Device

PSPACE-hardness of doors

Theorem

If a game features **door devices** *which each are controled by an* *open* *and a* *close* **preasure plate** *and the agent has to navigate from entrance to exit, then the game is* **PSPACE-hard**
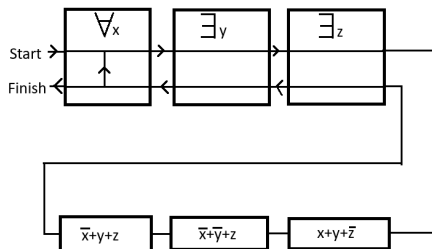
PSPACE-hardness of doors - Proof

True Quantified SAT

$$\forall x \exists y \exists z : (\overline{x} \lor y \lor z) \land$$

$$(\overline{x} \lor \overline{y} \lor z) \land (\lor x \lor y \lor \overline{z})$$

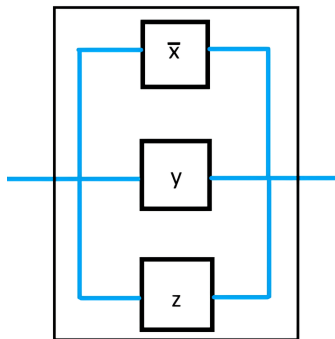**1-Player   Motion   Planning**

PSPACE-hardness of doors - Proof

Clause

$$(\overline{x} \lor y \lor z)$$

Gadget

PSPACE-hardness of doors - Proof
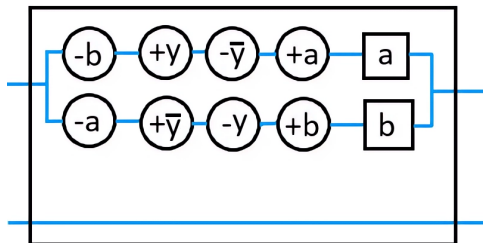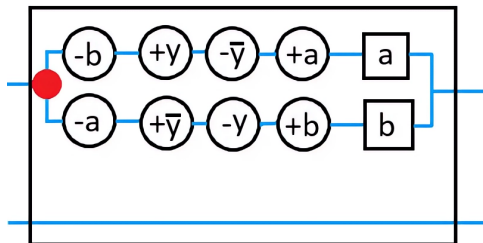
Exists-Quantor

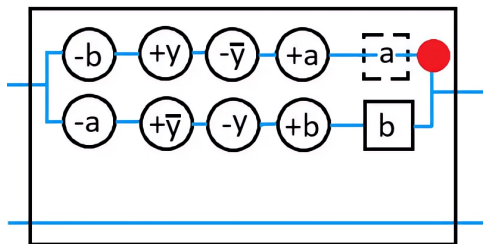$$\exists y$$

Gadget

PSPACE-hardness of doors - Proof

Exists-Quantor

$$\exists y$$

Gadget

PSPACE-hardness of doors - Proof

Exists-Quantor

$$\exists y$$

Gadget

PSPACE-hardness of doors - Proof
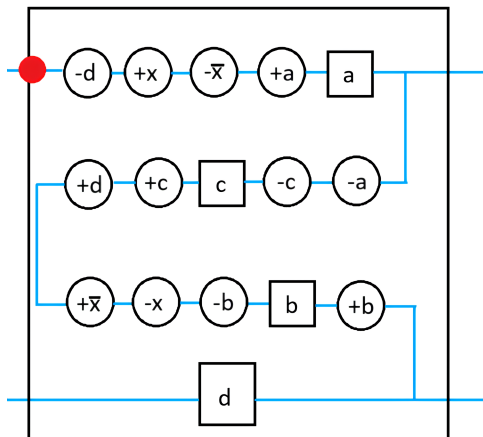
All-Quantor

$\forall x$

Gadget

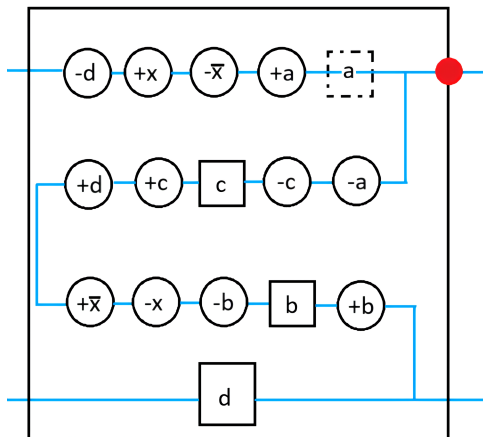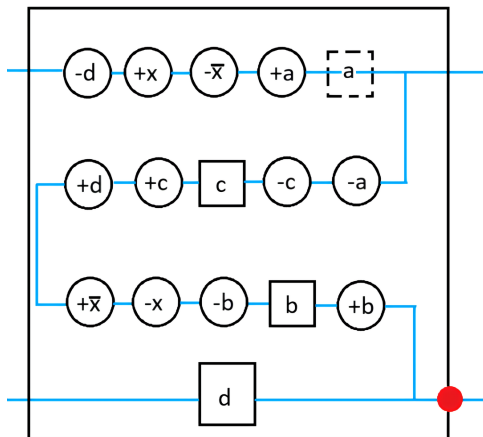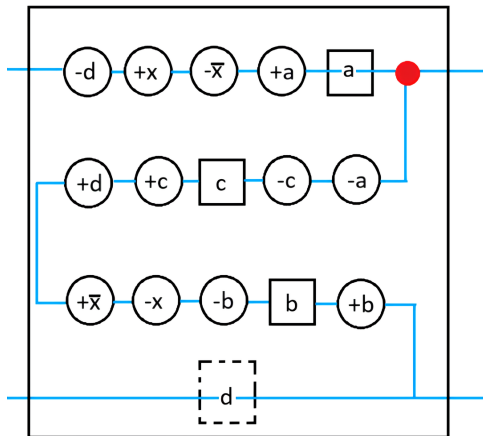# PSPACE-hardness of doors - Proof
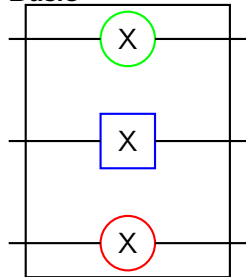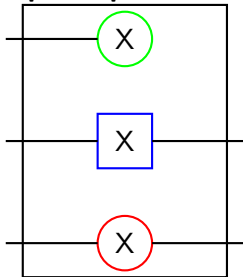
All-Quantor

$\forall x$

Gadget

PSPACE-hardness of doors - Proof

All-Quantor

$\forall x$

Gadget

PSPACE-hardness of doors - Proof

All-Quantor

$\forall x$
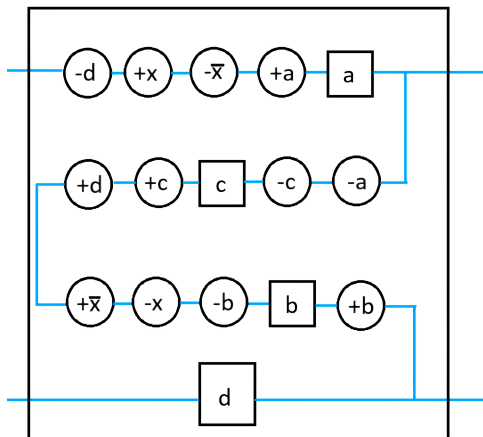
Gadget

PSPACE-hardness of doors - Proof

All-Quantor

$$\forall x$$

Gadget

Door Device - Variants

Basic

Open-Optional

Directed
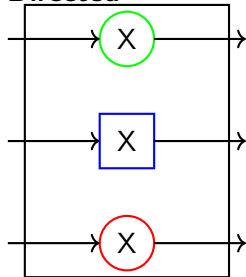
Open preasure plate: ● Close preasure plate: ● Door: ■
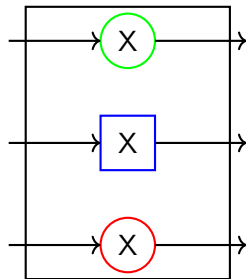
PSpace-Hardness - Open optinal door
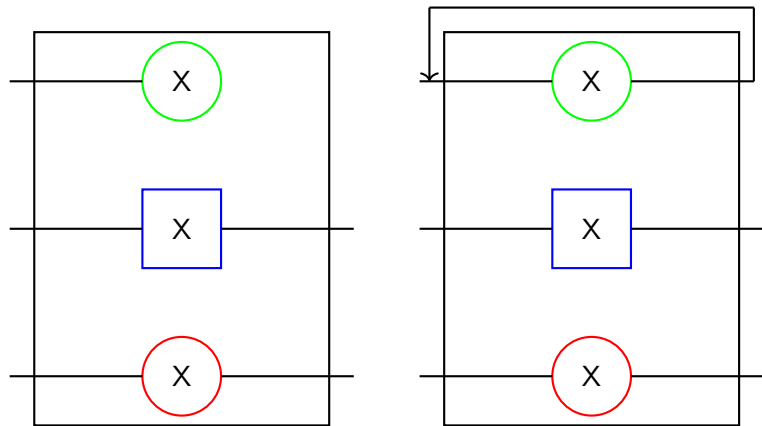
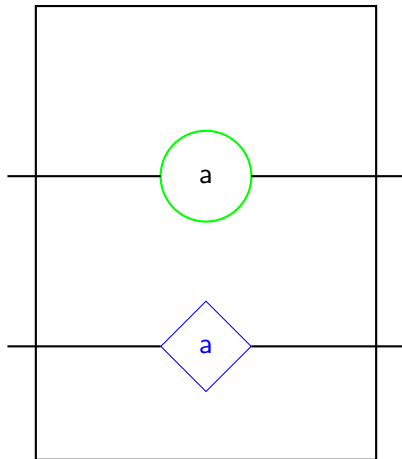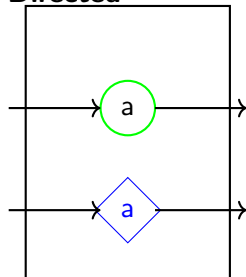Open preasure plate: 🟢 Close preasure plate: 🔴 Door: 🟦

The Diode
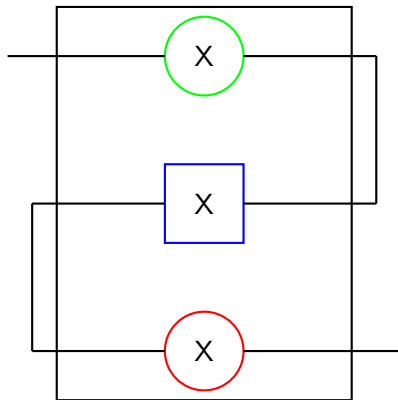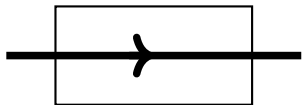
Open preasure plate: 🟢 Close preasure plate: 🔴 Door: 🟦

PSpace-Hardness - Directed Door
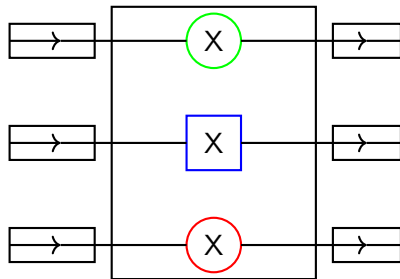
Open preasure plate: 🟢 Close preasure plate: 🔴 Door: 🟦

Self closing doors

Open preasure plate: ⬤ Self closing door: ◆

Self closing doors - Variants

Self closing doors - Variants

Directed **Open-Optional**s **Symetric**
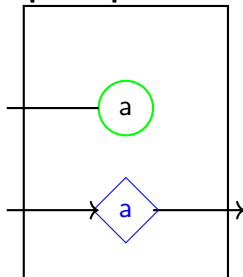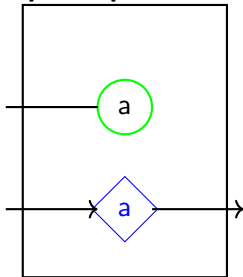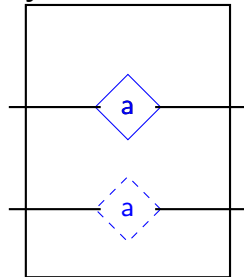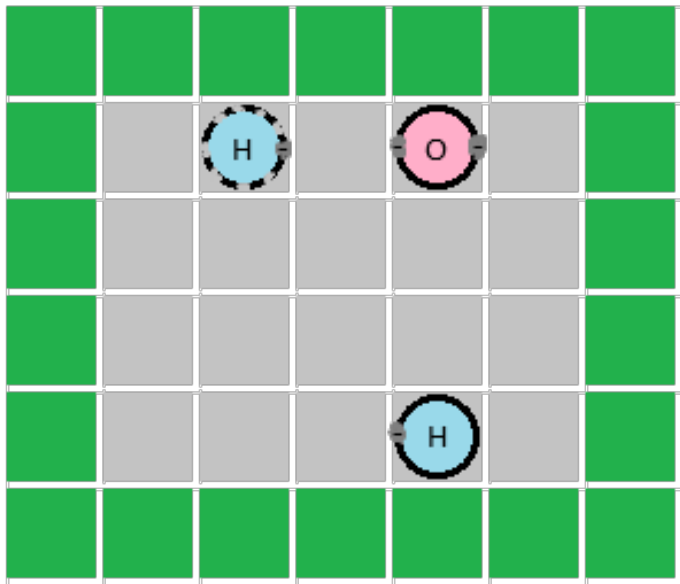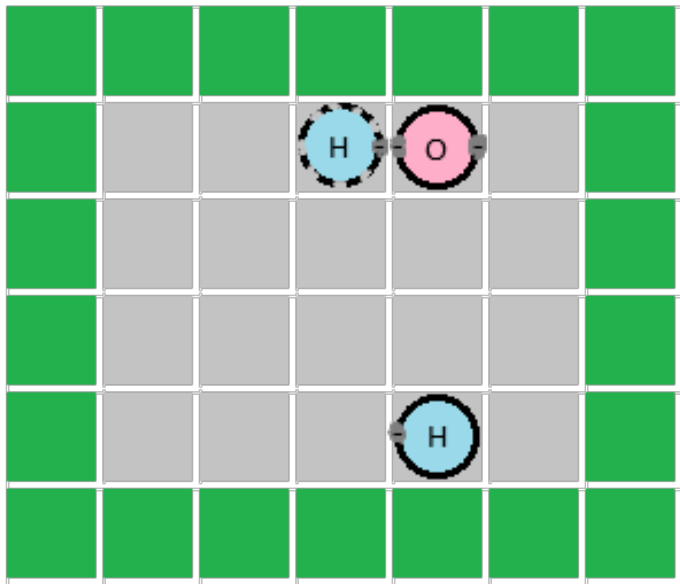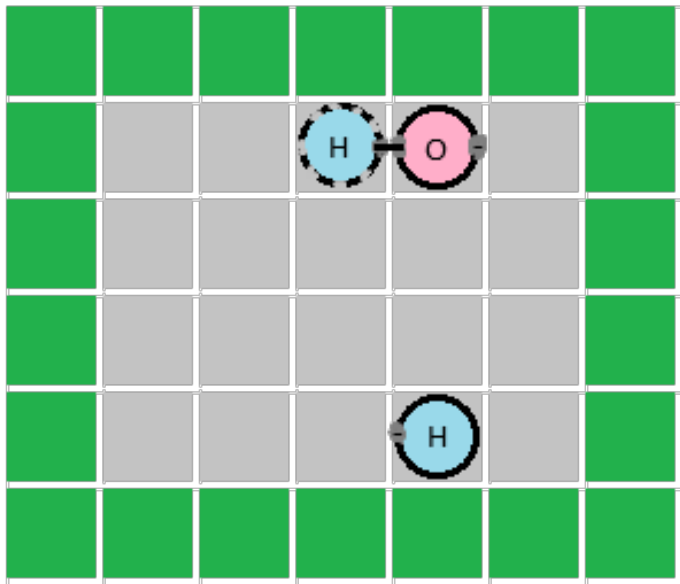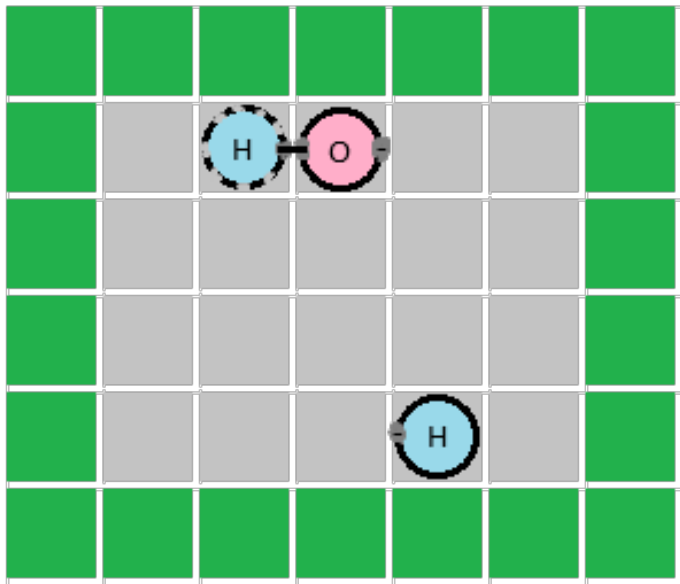
Open preasure plate: ● Self closing door: ◆

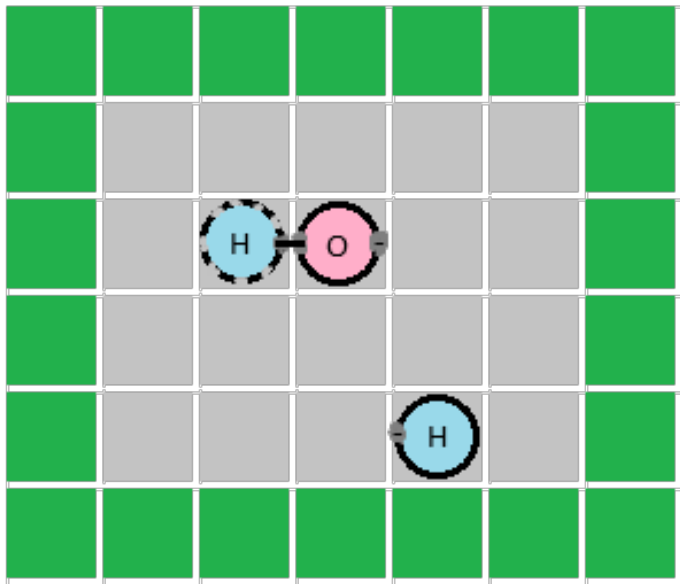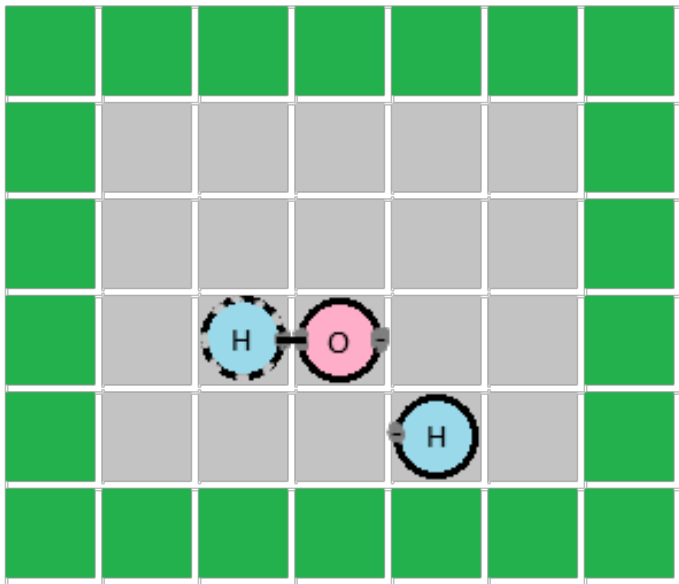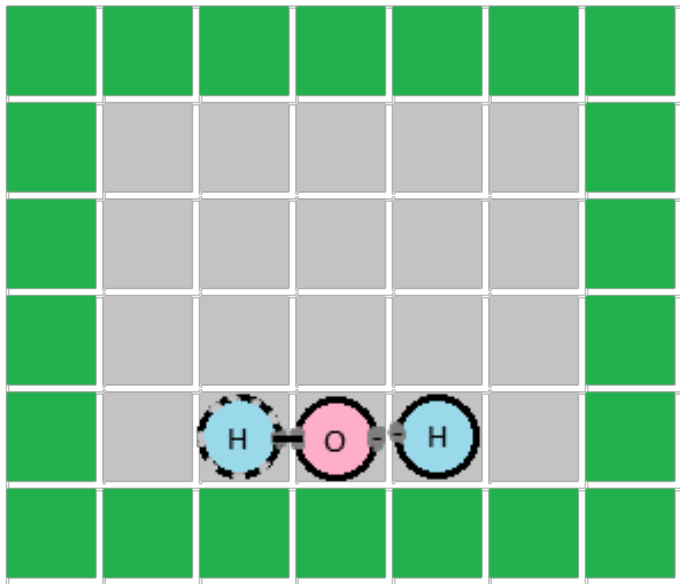# Application - Sokobond
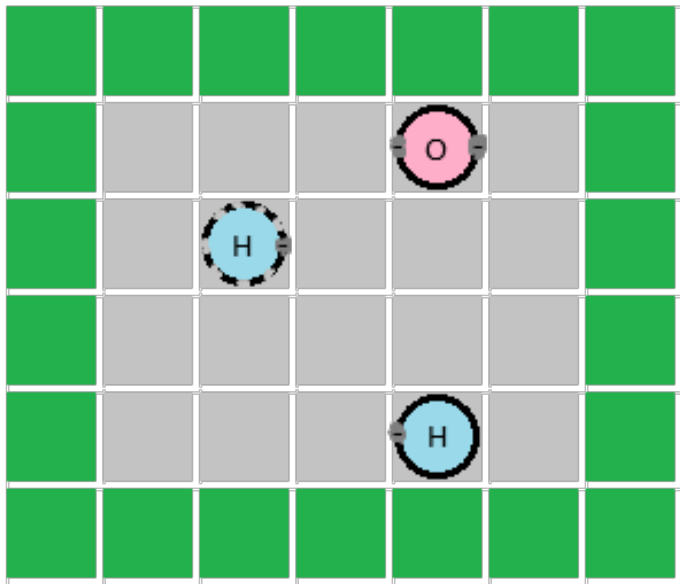
Application - Sokobond

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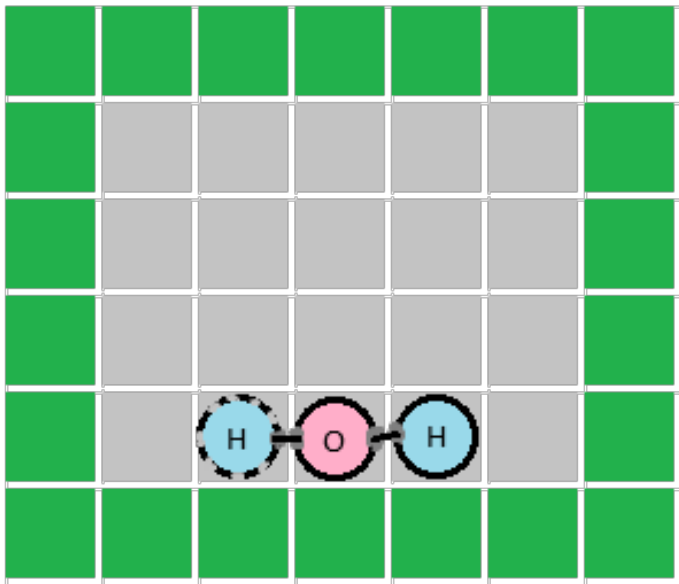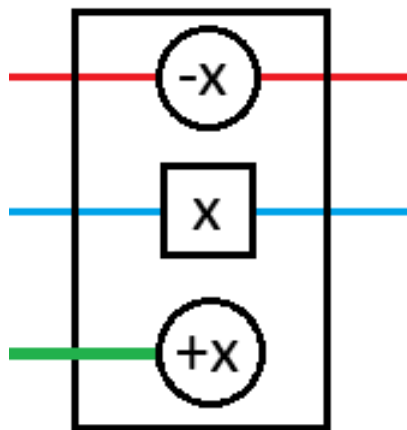
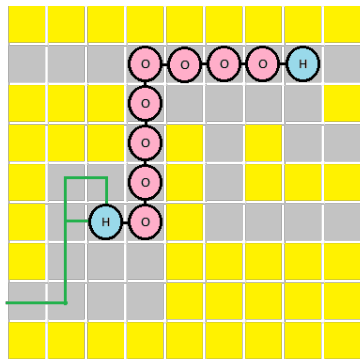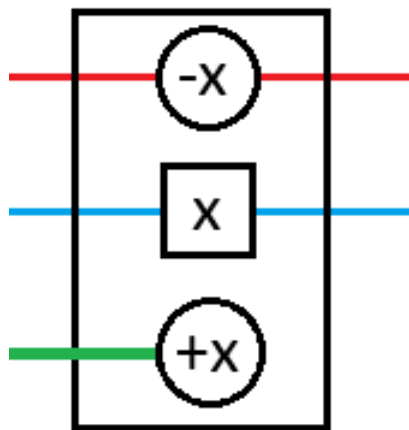# Application - Sokobond
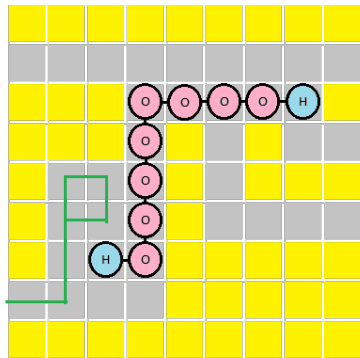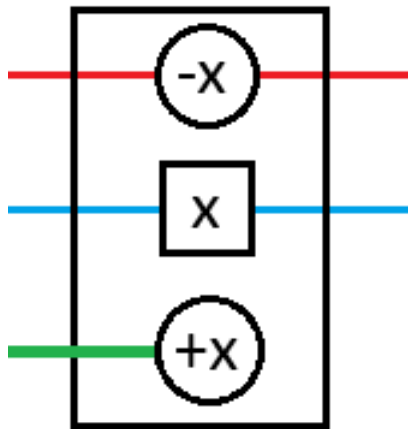
Application - Sokobond
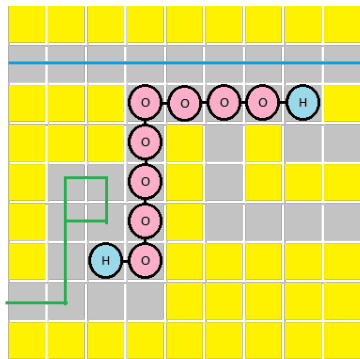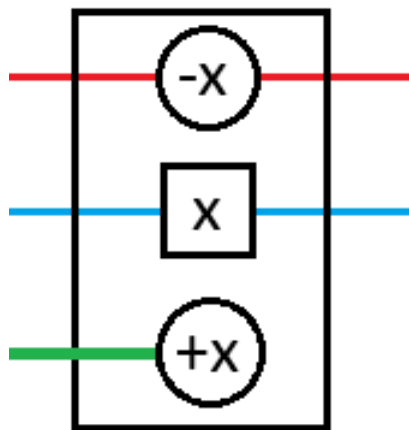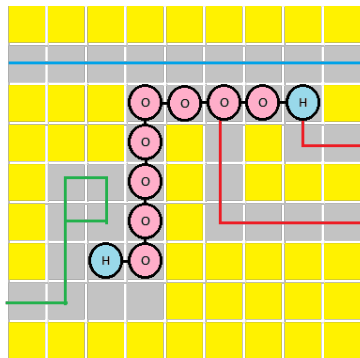
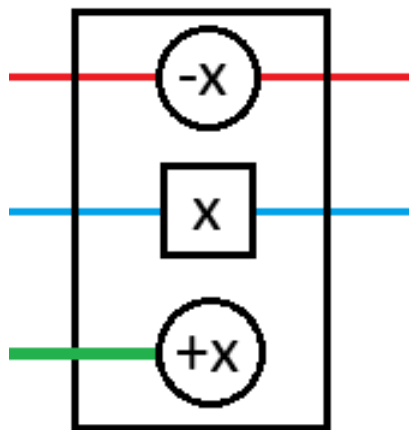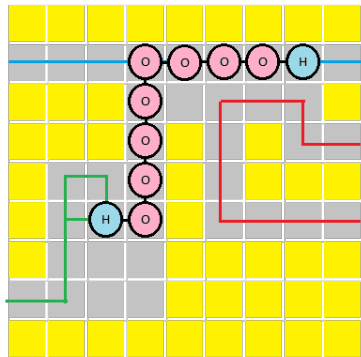# Application - Sokobond

Application - Sokobond

Sokobond is PSpace-Hard
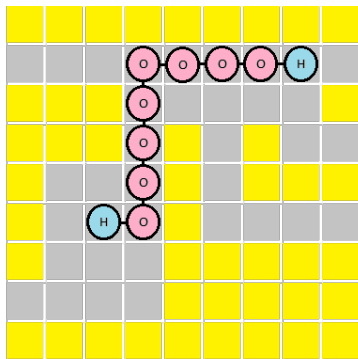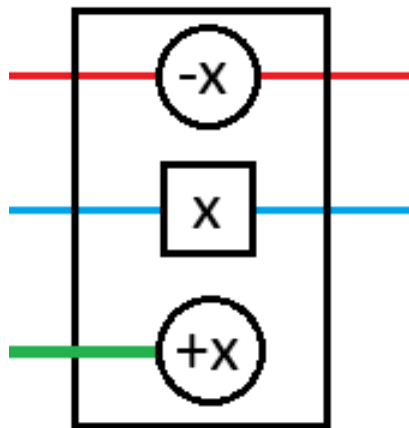
Sokobond is PSpace-Hard
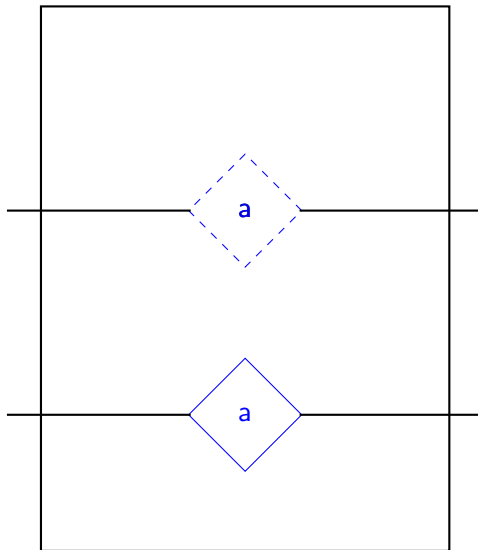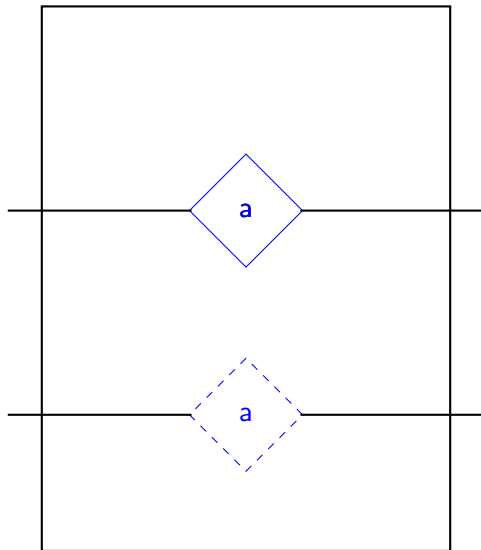
Sokobond is PSpace-Hard

Sokobond is PSpace-Hard

Sokobond is PSpace-Hard

Sokobond is PSpace-Hard

Reminder - Symetric self closing door
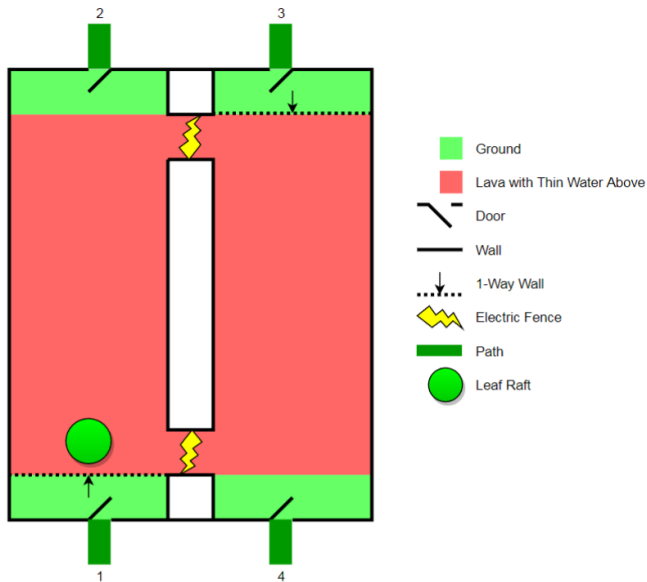
Reminder - Symetric self closing door

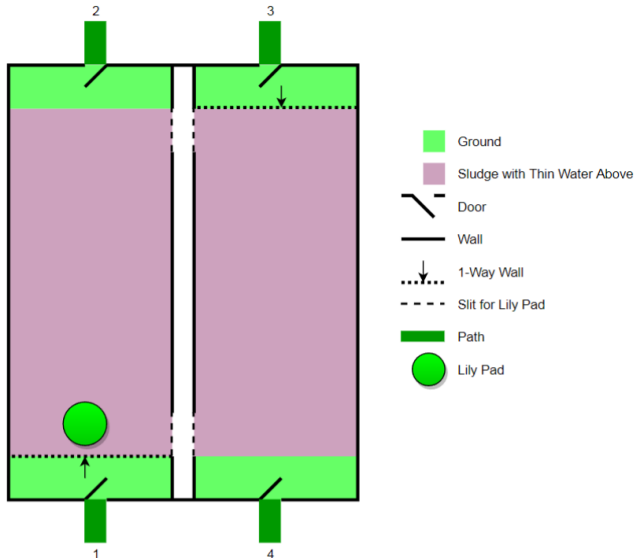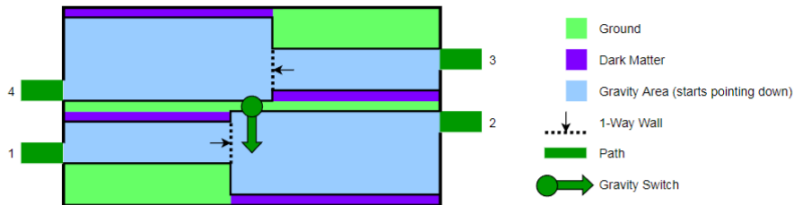# Super Mario Galaxy 2 is PSpace-hard

Legend:
- Ground
- Lava with Thin Water Above
- Door
- Wall
- 1-Way Wall
- Electric Fence
- Path
- Leaf Raft

Parralelities

Parralelities
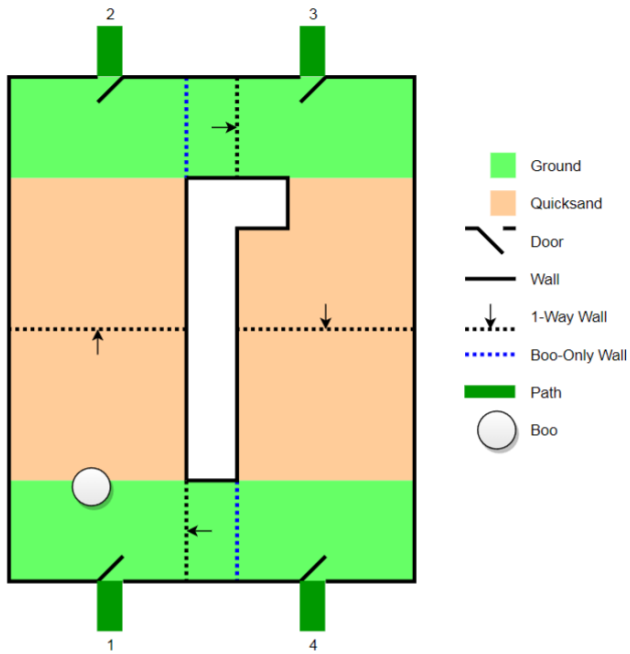
Super Mario Galaxy is PSpace-hard

Ground

Dark Matter

Gravity Area (starts pointing down)

1-Way Wall

Path

Gravity Switch

Super Mario Odessy is PSpace-hard