

FUN with Complexity: Walking through Doors is Hard, even without Staircases

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Super Mario Galaxy 2 is PSPACE-Hard

Super Mario Galaxy is PSPACE-Hard

Super Mario Odessy is PSPACE-Hard

PSPACE-Complexity

A given problem requires at most a polynomial amount of memory in relation to the input, to be solved \Leftrightarrow The problem is in PSPACE

SAT

$$x_1 \wedge x_2 \vee \neg x_3$$

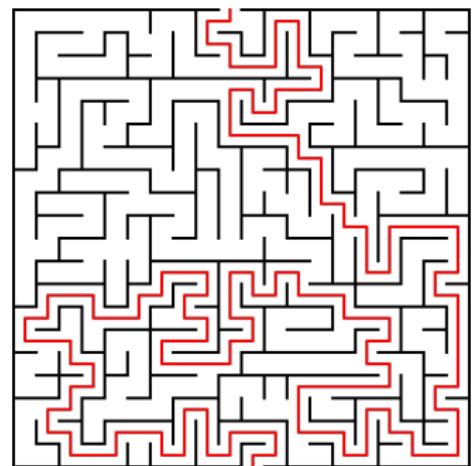
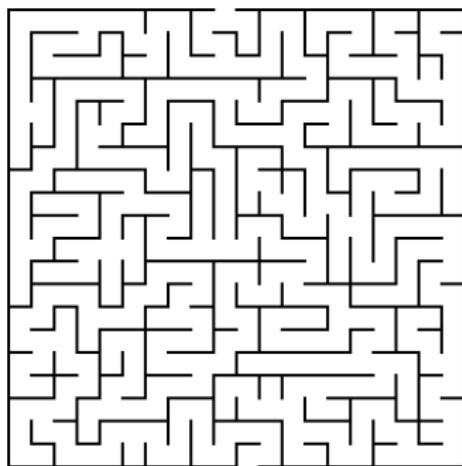
Quantified SAT

$$\forall x_1 \exists x_2 : x_1 \wedge x_2 \vee \neg x_3$$

1-Player Motion Planning

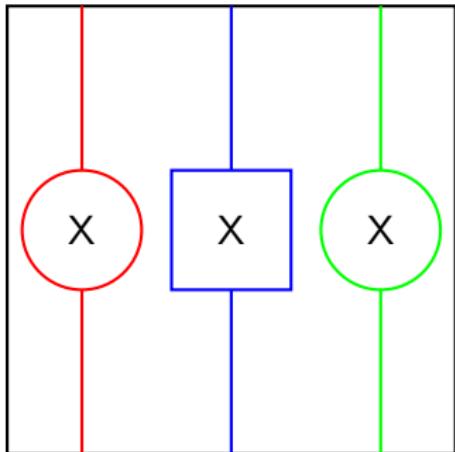
Given: Environment, Agent, Goal

Question: Is the goal achievable

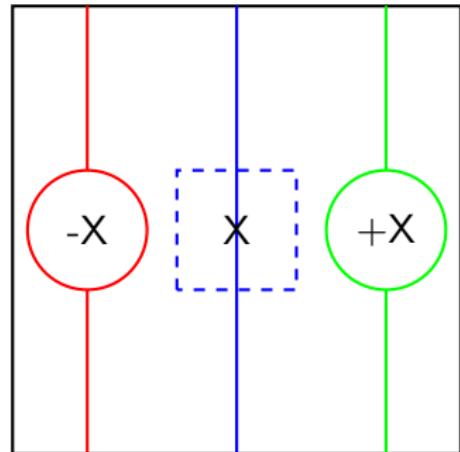


Basic Door Device

Close Traverse Open



Close Traverse Open



PSPACE-hardness of doors

Theorem

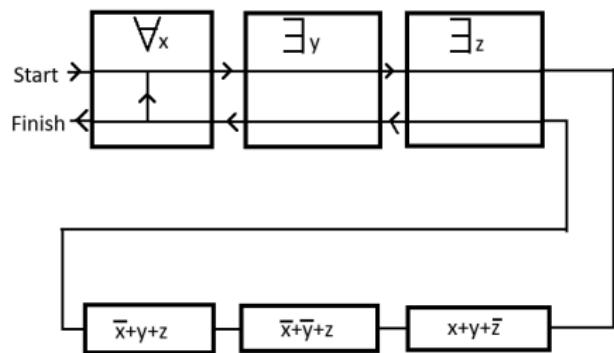
*If a game features **door devices** which each are controlled by an **open** and a **close** pressure plate and the agent has to navigate from entrance to exit, then the game is **PSPACE-hard***

PSPACE-hardness of doors - Proof

True Quantified SAT

$$\begin{aligned} \forall x \exists y \exists z : & (\bar{x} \vee y \vee z) \wedge \\ & (\bar{x} \vee \bar{y} \vee z) \wedge (\vee x \vee y \vee \bar{z}) \end{aligned}$$

1-Player Motion Planning

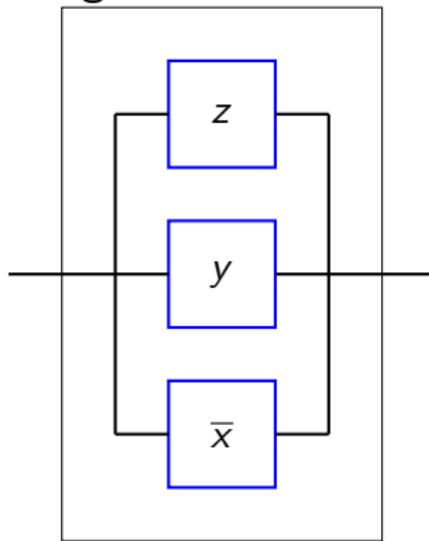


PSPACE-hardness of doors - Proof

Clause

$$(\bar{x} \vee y \vee z)$$

Gadget

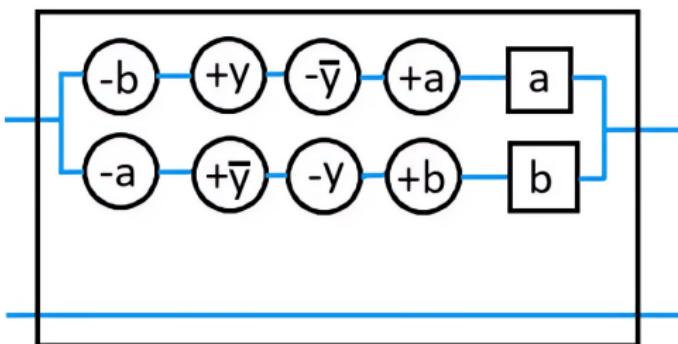


PSPACE-hardness of doors - Proof

Exists-Quantor

$\exists y$

Gadget

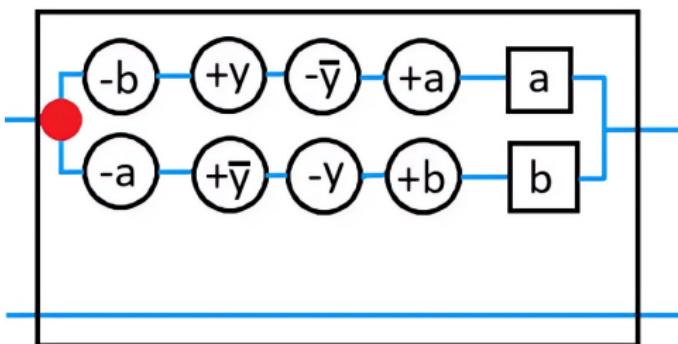


PSPACE-hardness of doors - Proof

Exists-Quantor

$\exists y$

Gadget

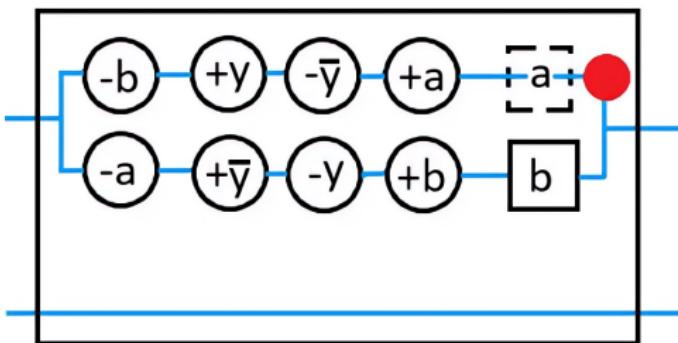


PSPACE-hardness of doors - Proof

Exists-Quantor

$\exists y$

Gadget

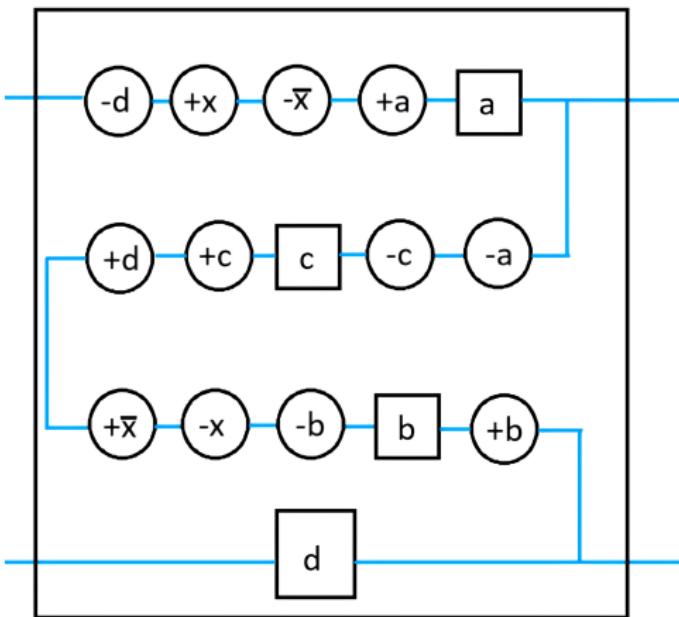


PSPACE-hardness of doors - Proof

All-Quantor

$\forall x$

Gadget

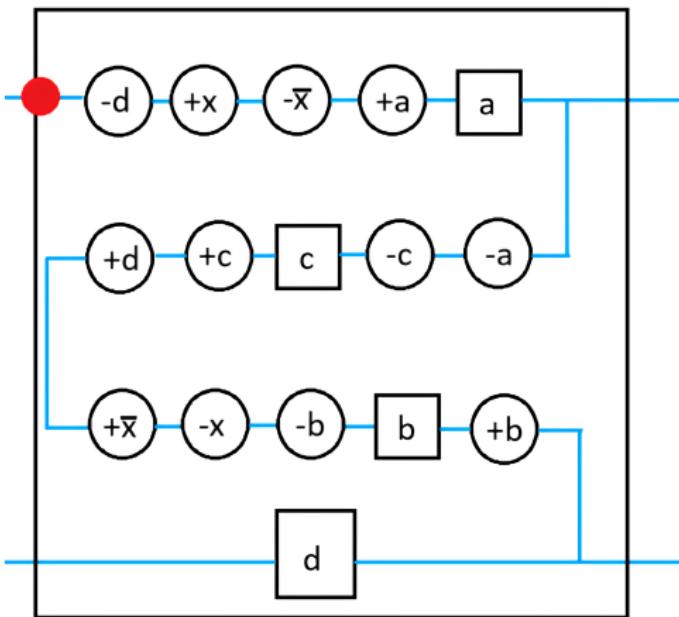


PSPACE-hardness of doors - Proof

All-Quantor

$\forall x$

Gadget

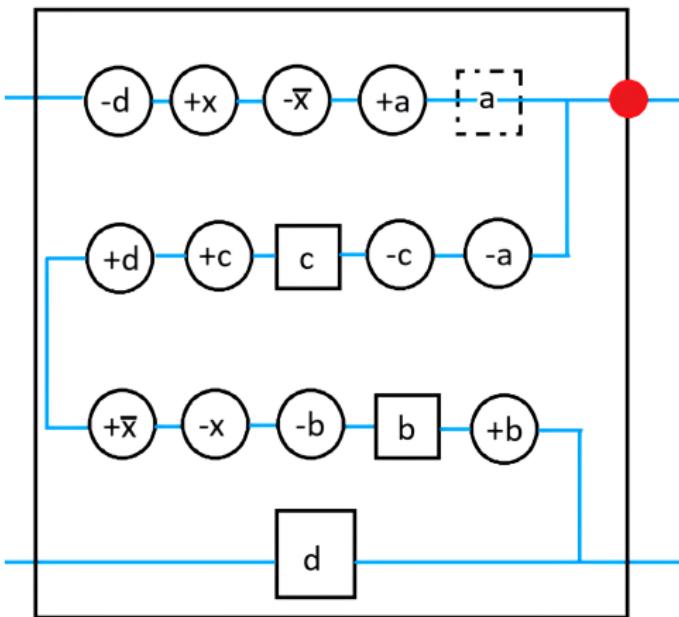


PSPACE-hardness of doors - Proof

All-Quantor

$\forall x$

Gadget

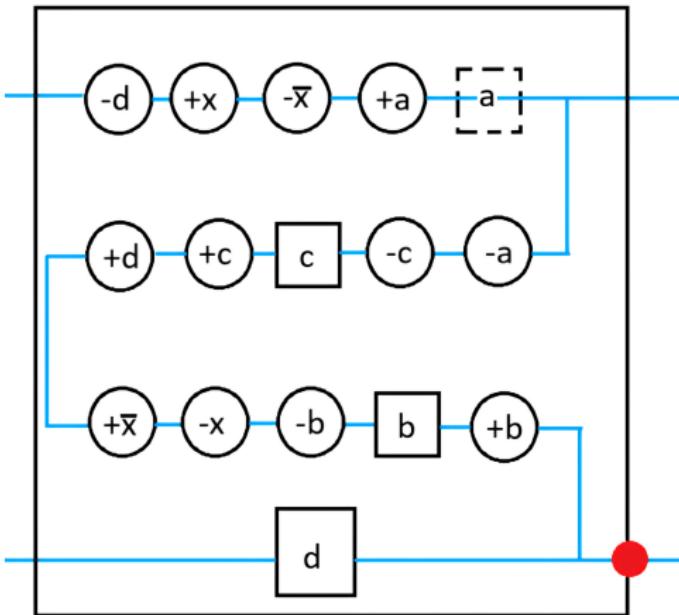


PSPACE-hardness of doors - Proof

All-Quantor

$\forall x$

Gadget

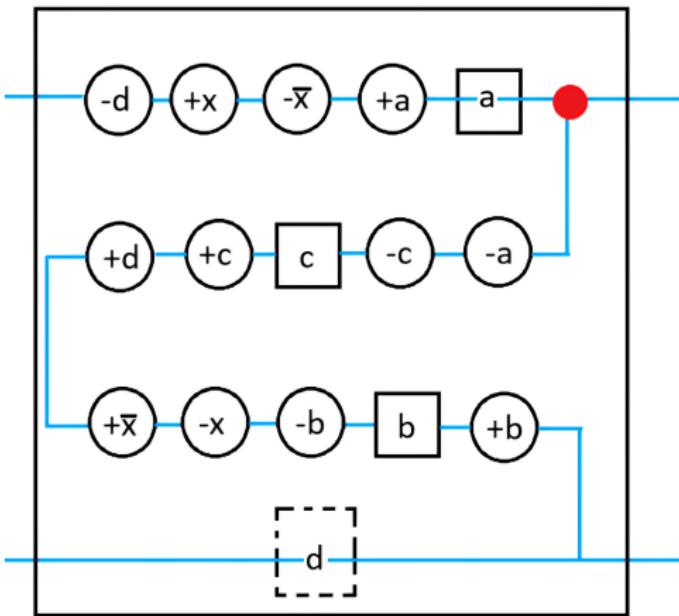


PSPACE-hardness of doors - Proof

All-Quantor

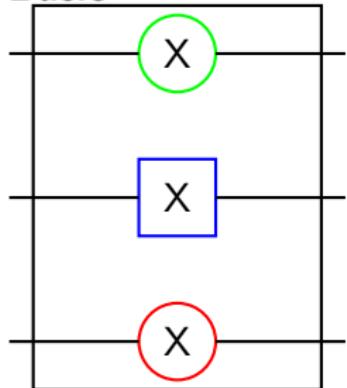
$\forall x$

Gadget

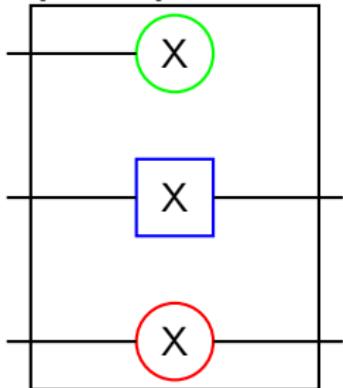


Door Device - Variants

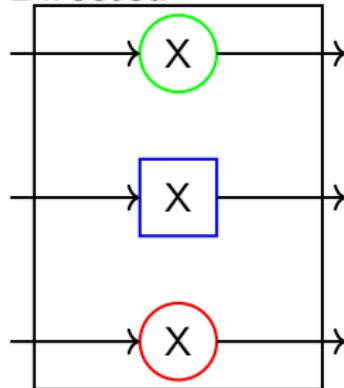
Basic



Open-Optional



Directed



Open pressure plate:



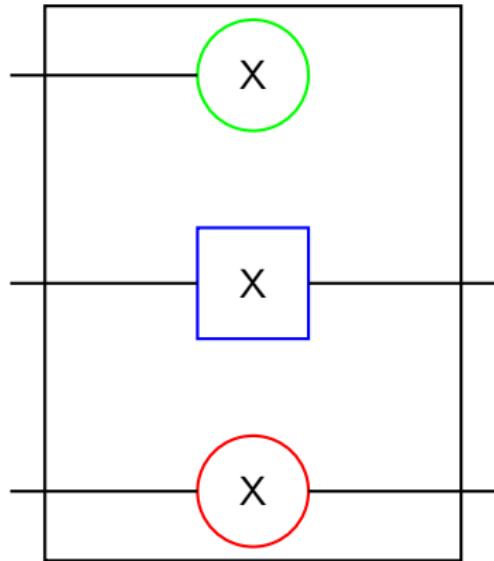
Close pressure plate:



Door:



PSpace-Hardness - Open optimal door



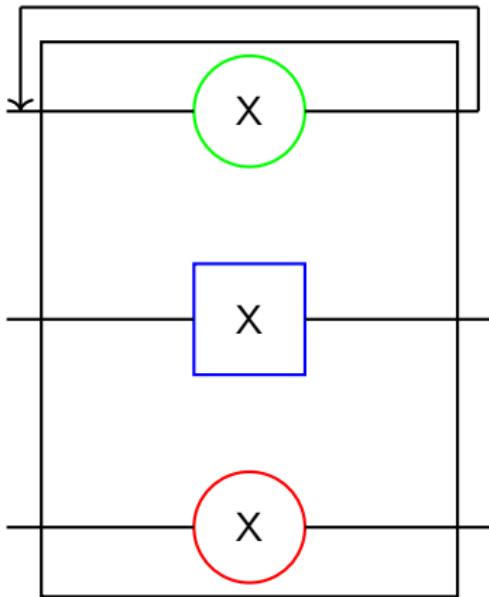
Open pressure plate:



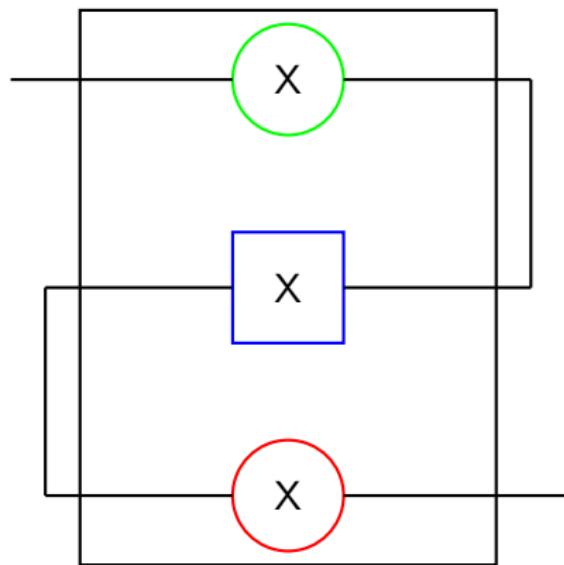
Close pressure plate:



Door:

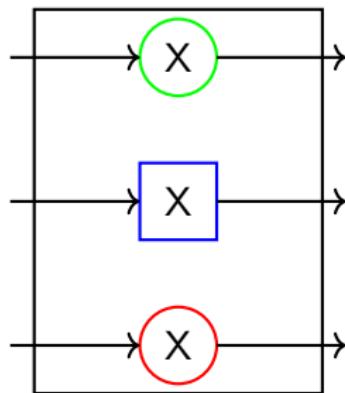


The Diode



Open pressure plate: ● Close pressure plate: ● Door: ■

PSpace-Hardness - Directed Door



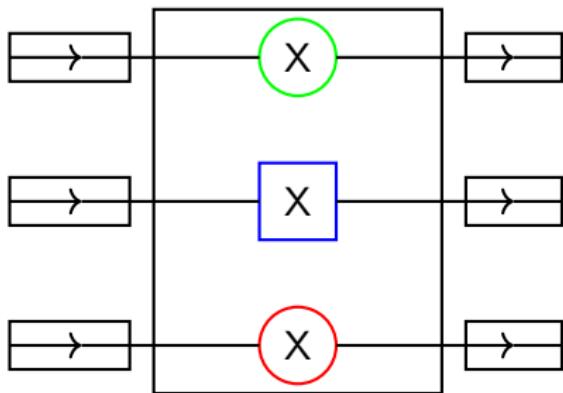
Open pressure plate:



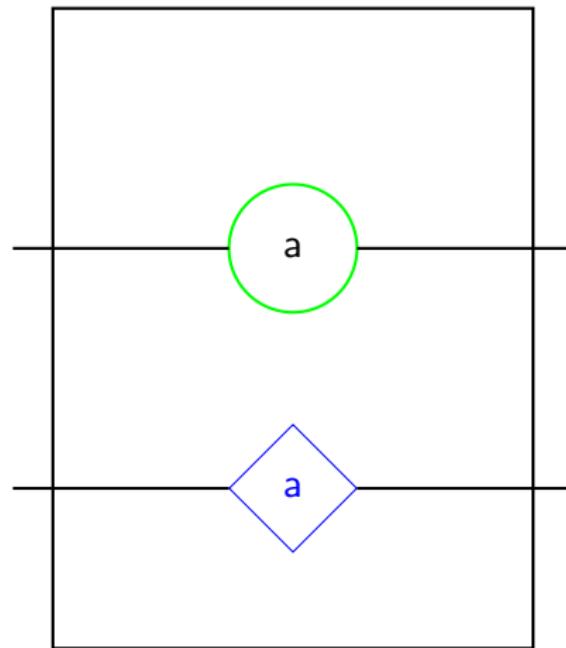
Close pressure plate:



Door:

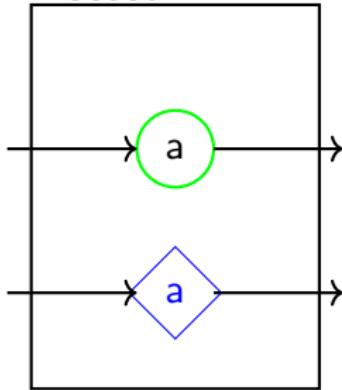


Self closing doors

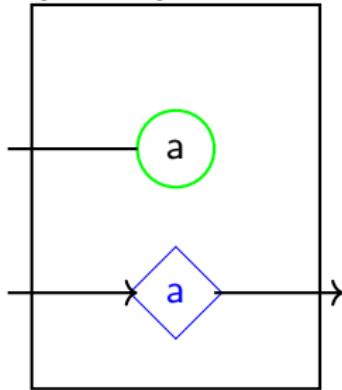


Self closing doors - Variants

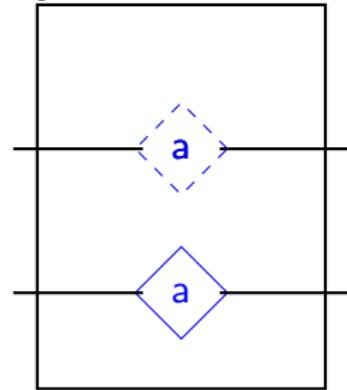
Directed



Open-Optionals



Symmetric



Open pressure plate:

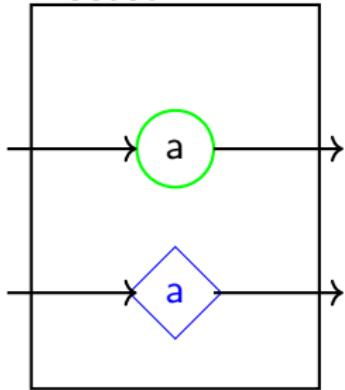


Self closing door:

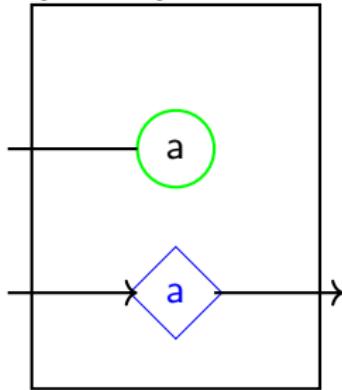


Self closing doors - Variants

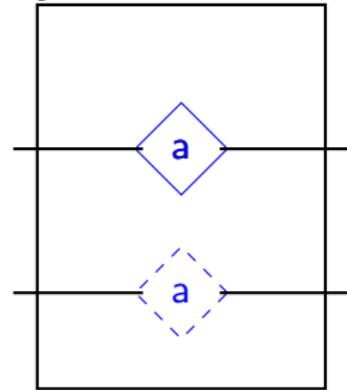
Directed



Open-Optionals



Symmetric



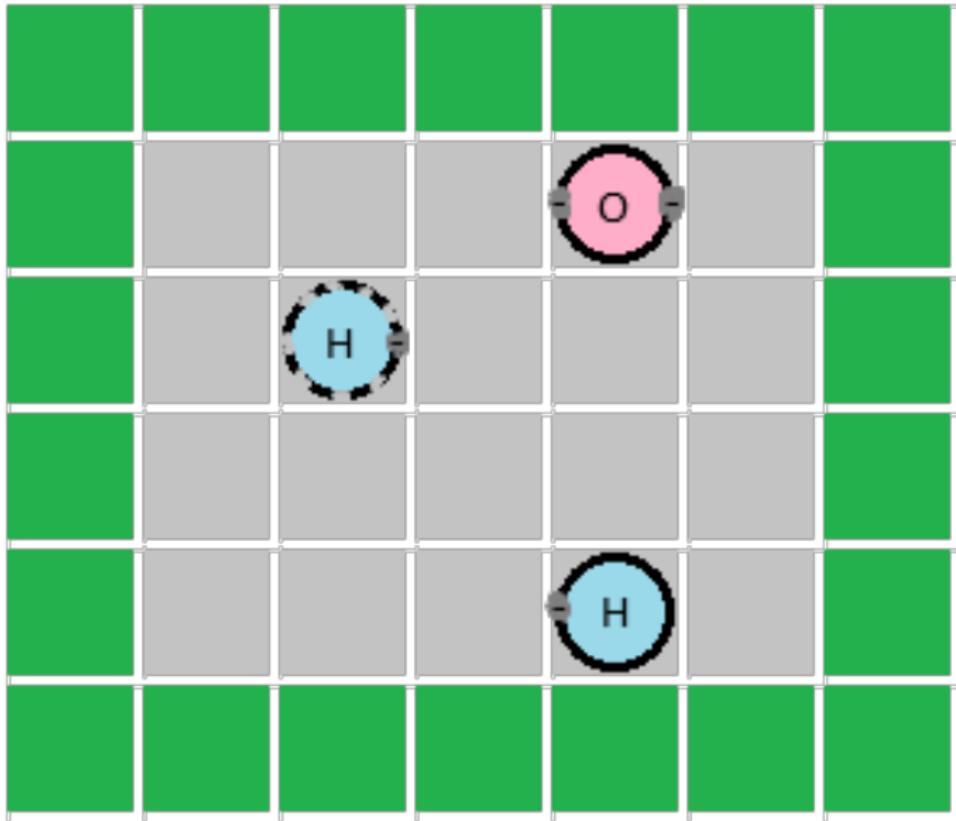
Open pressure plate:



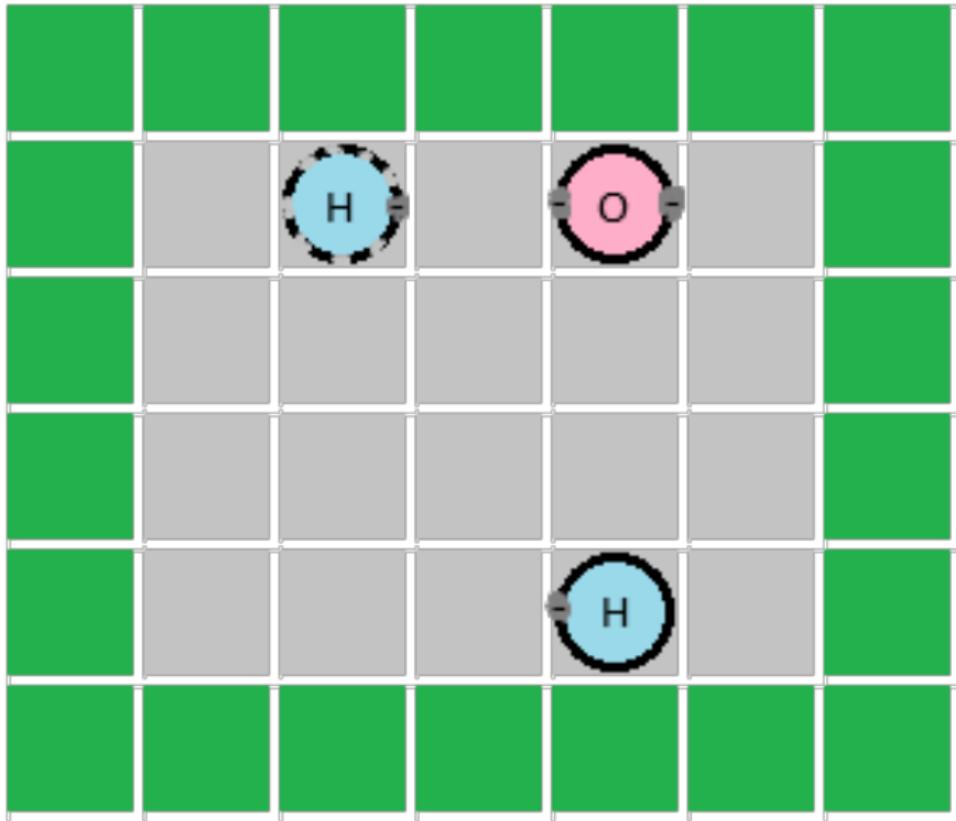
Self closing door:



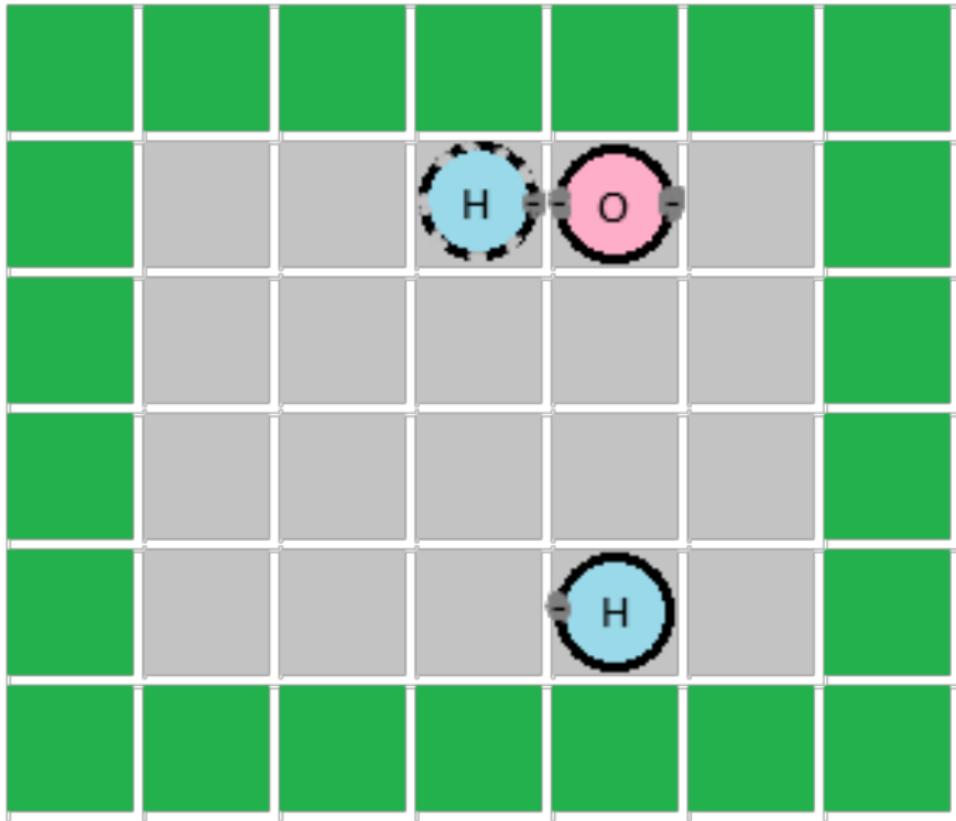
Application - Sokobond



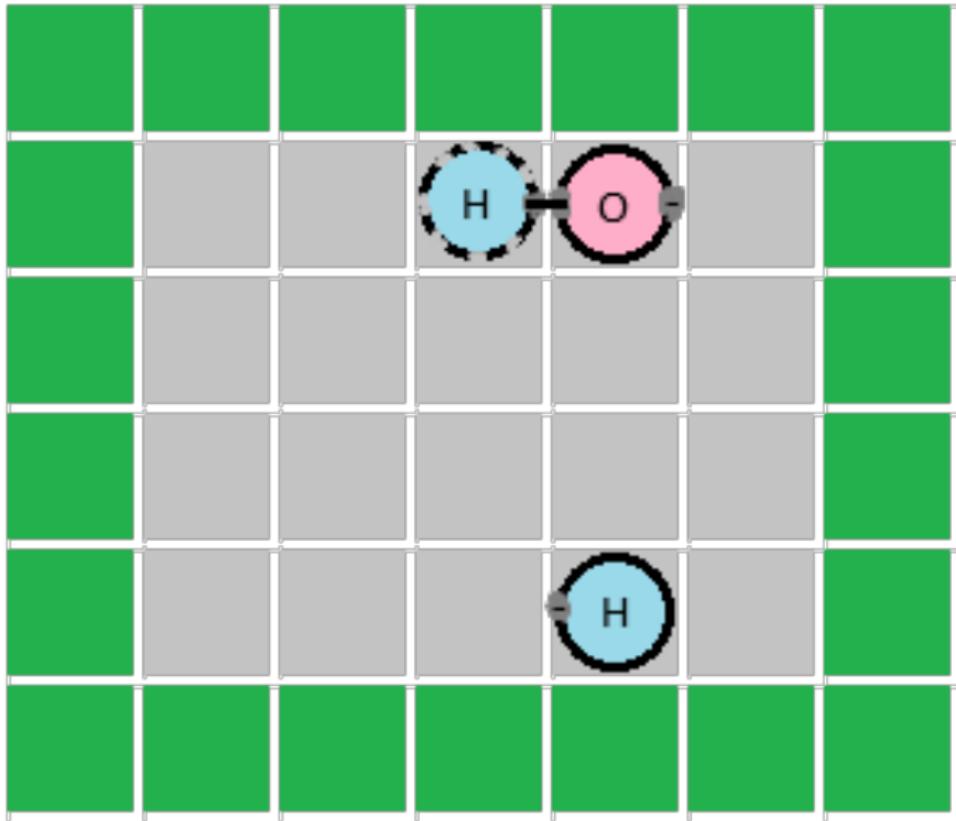
Application - Sokobond



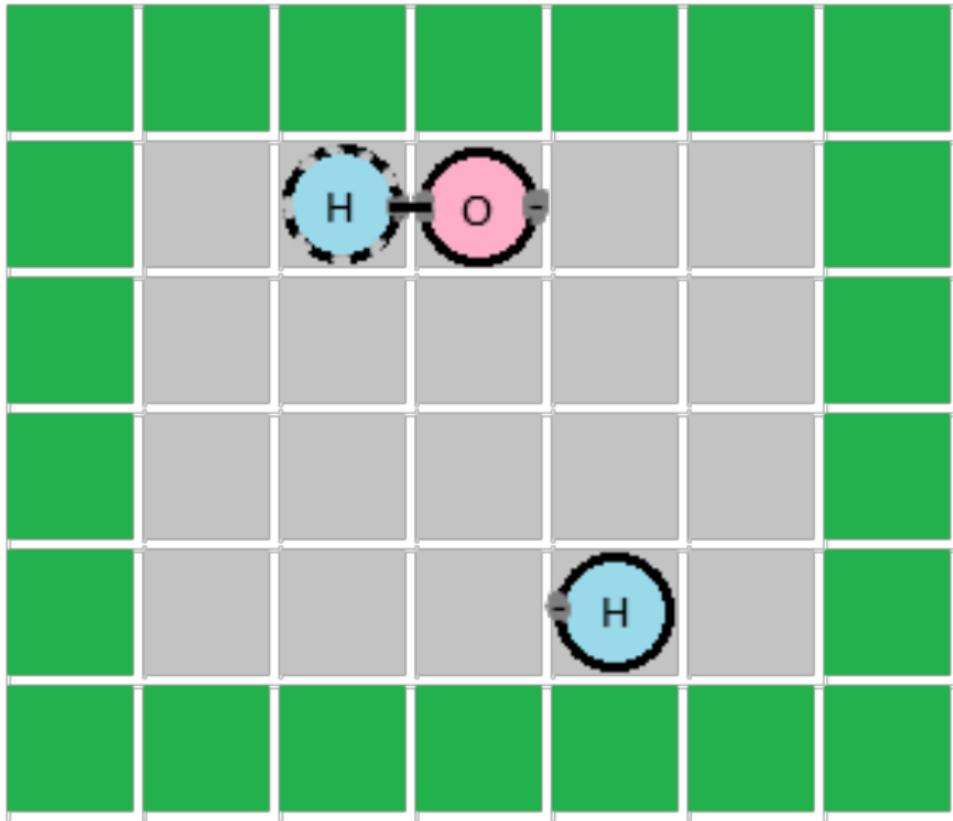
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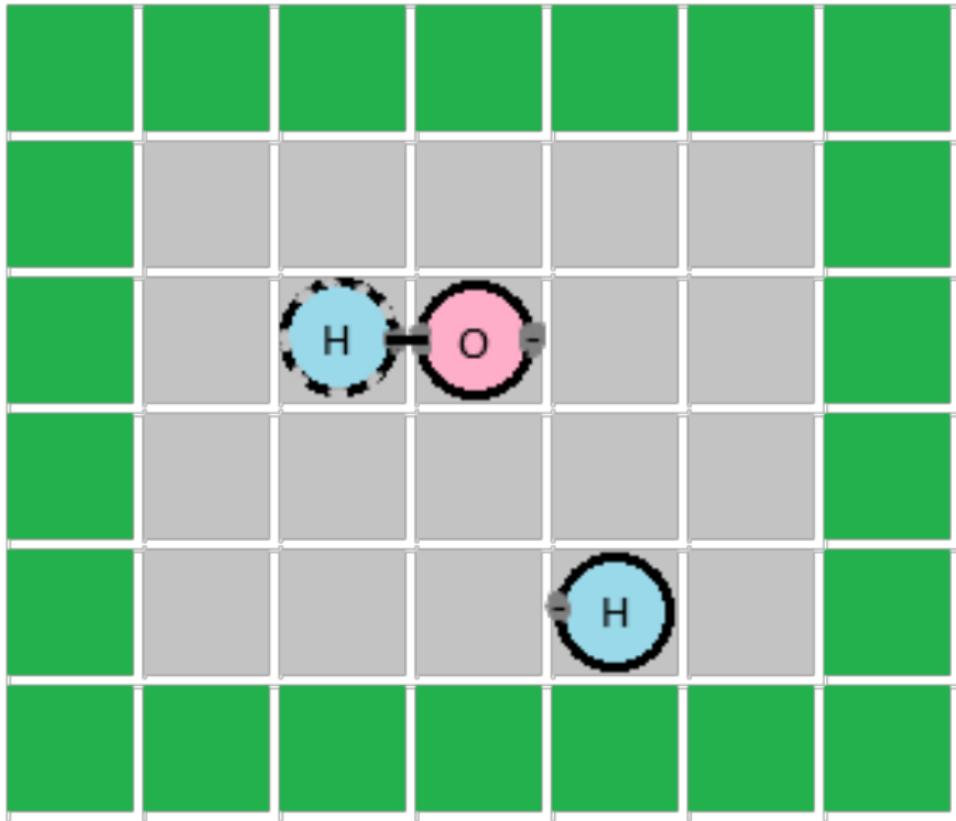
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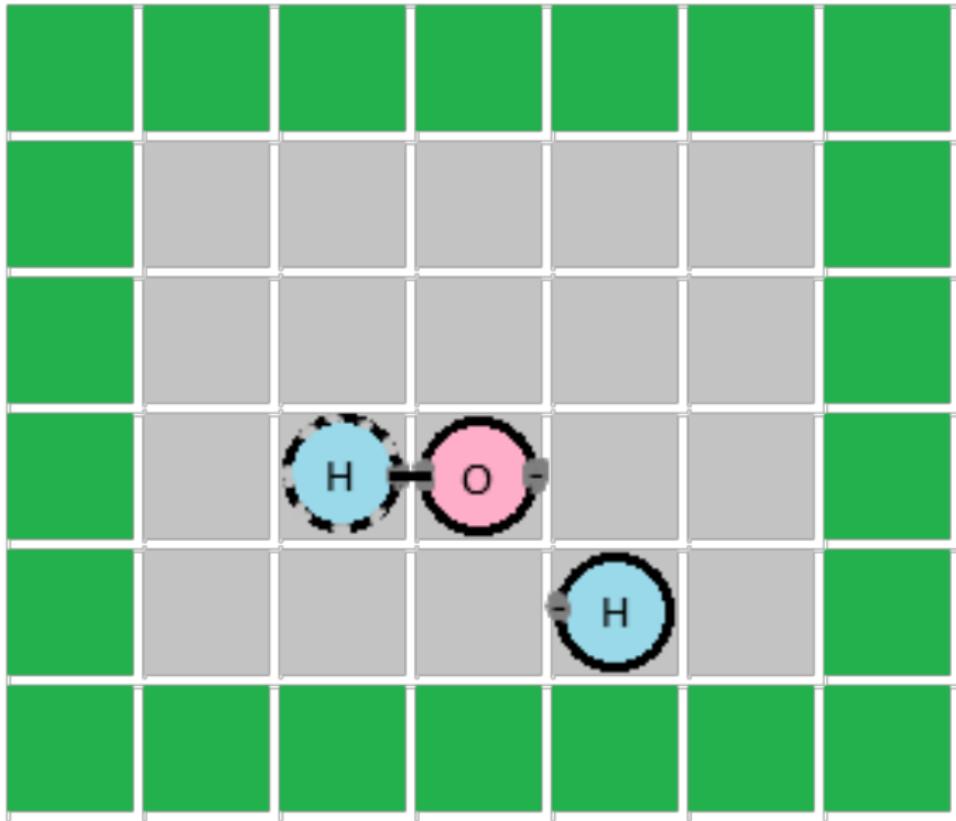
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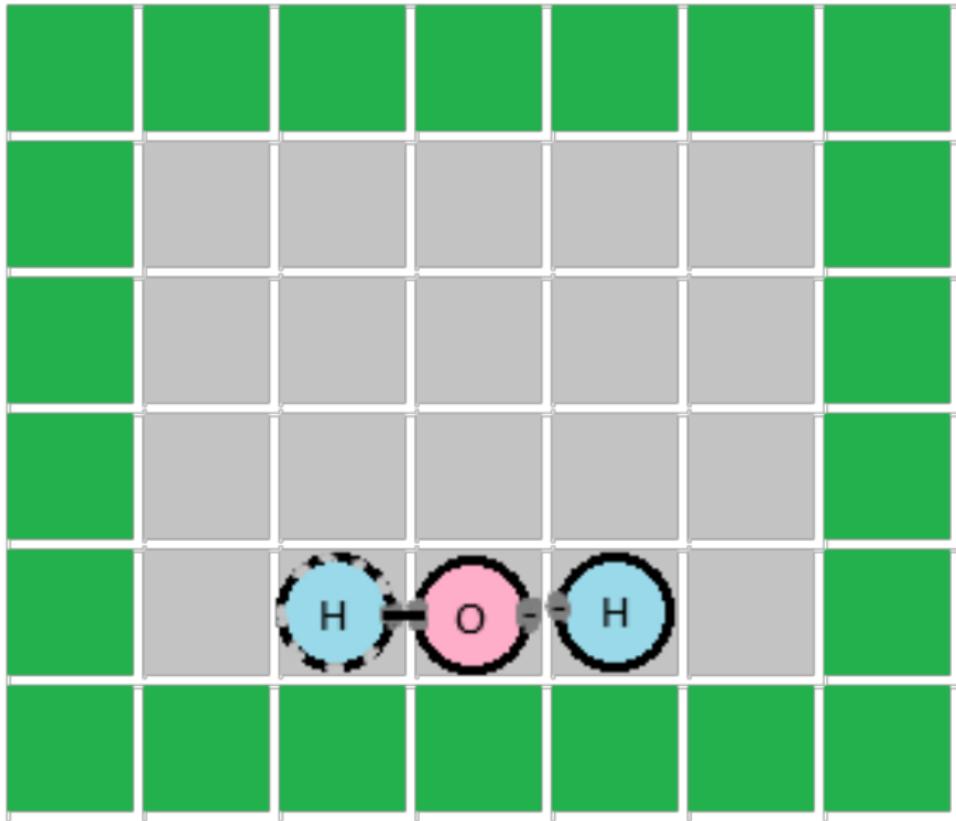
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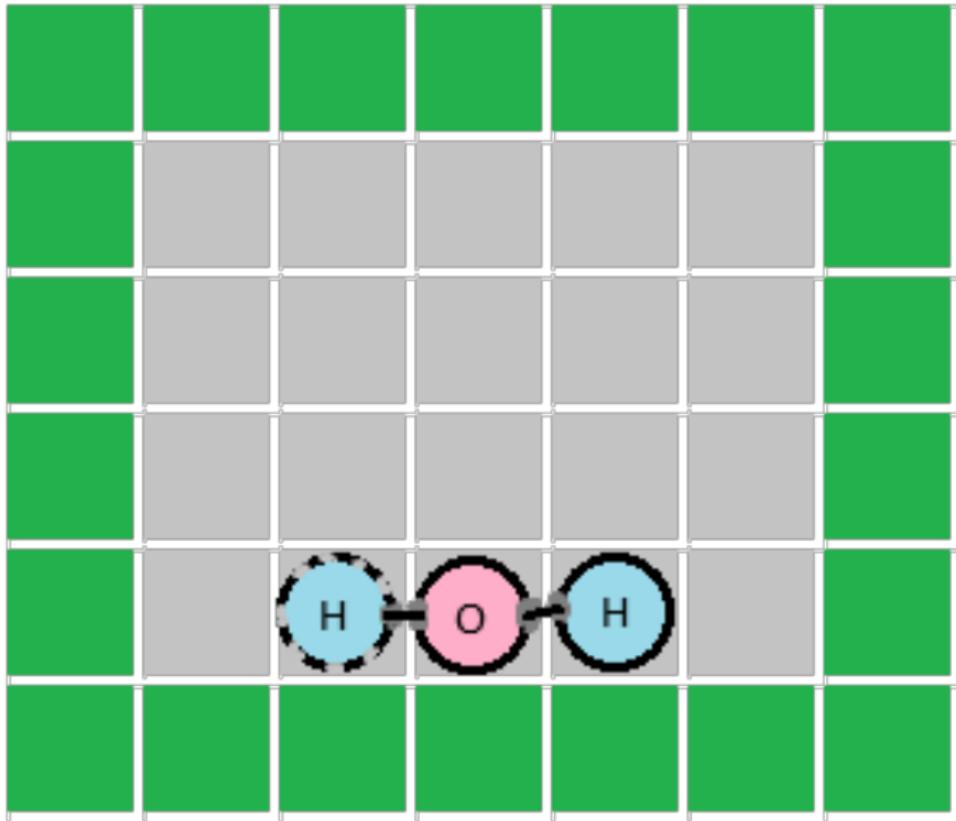
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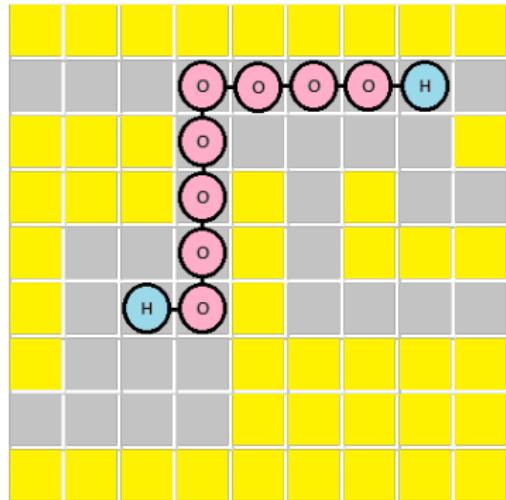
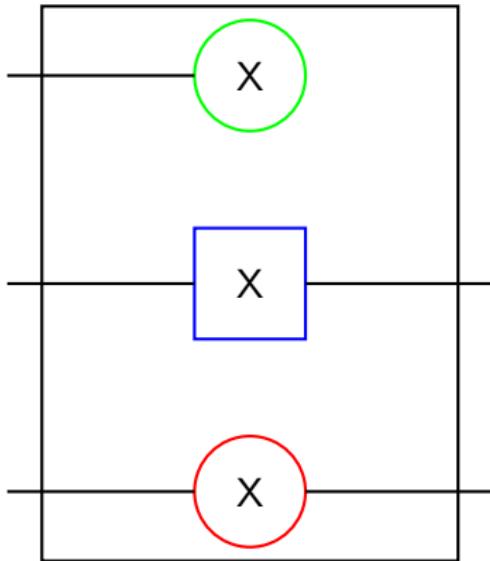
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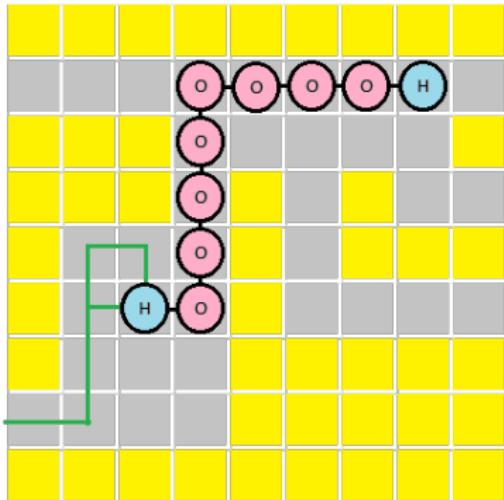
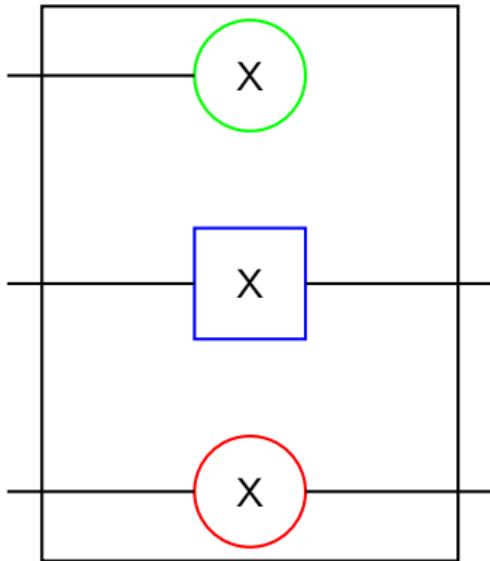
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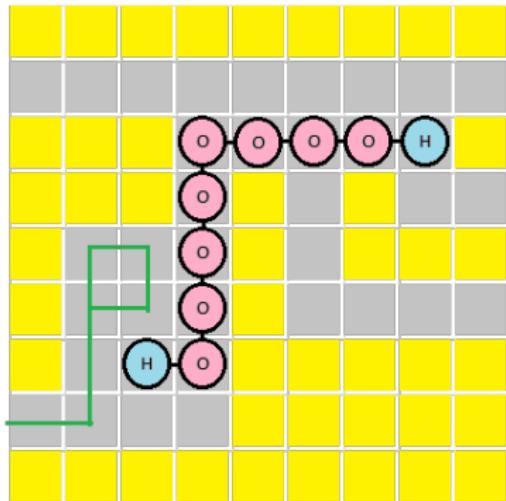
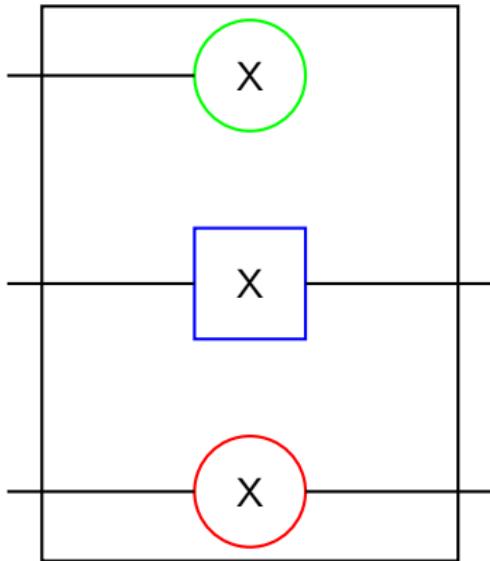
Sokobond is PSpace-Hard



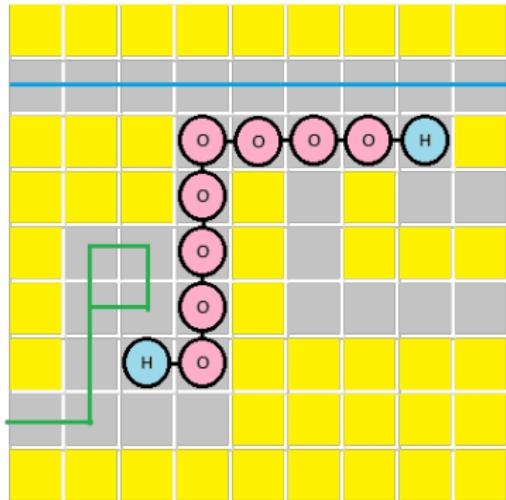
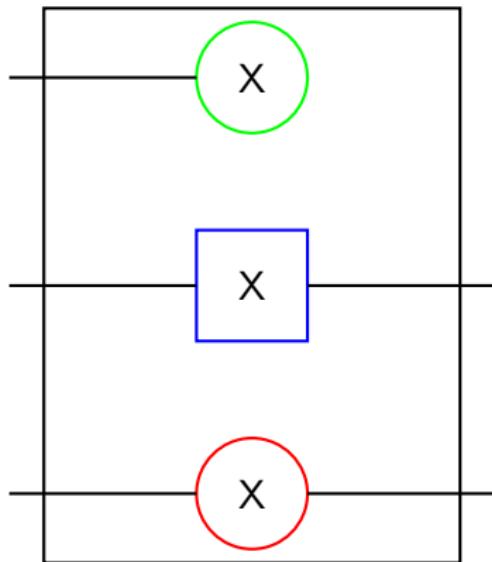
Sokobond is PSpace-Hard



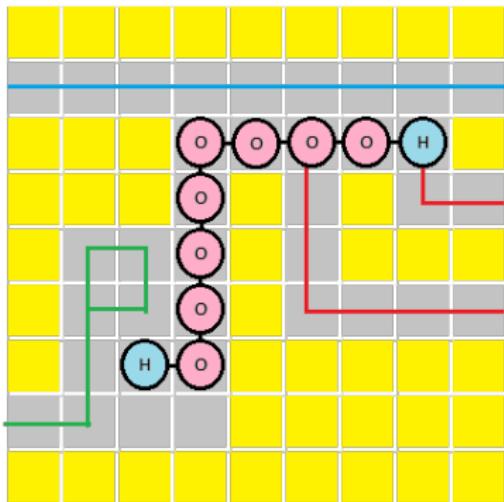
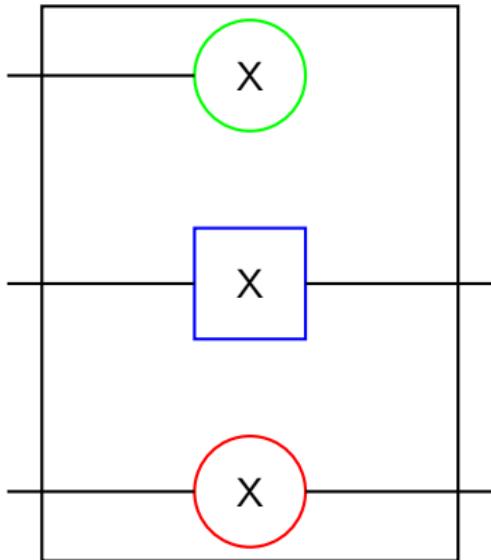
Sokobond is PSpace-Hard



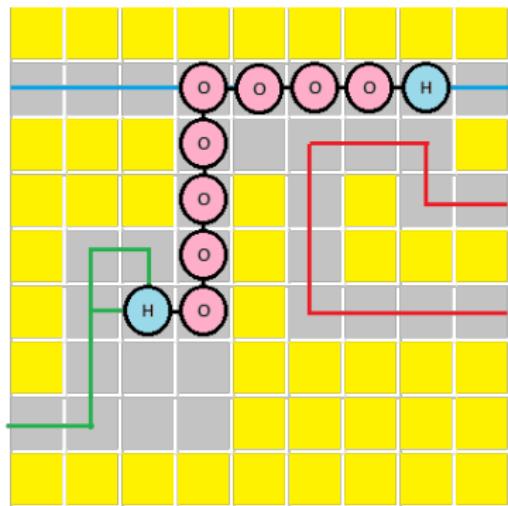
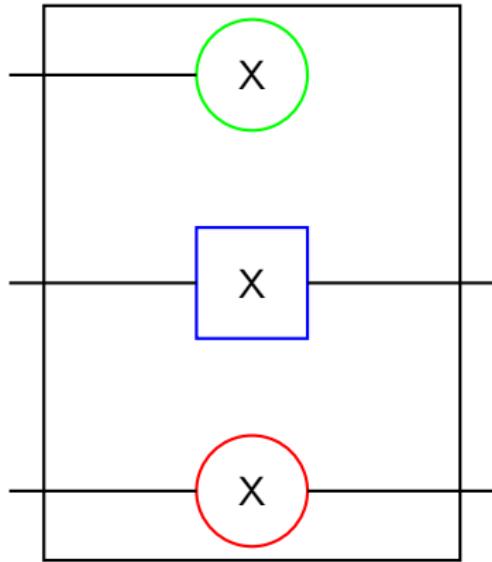
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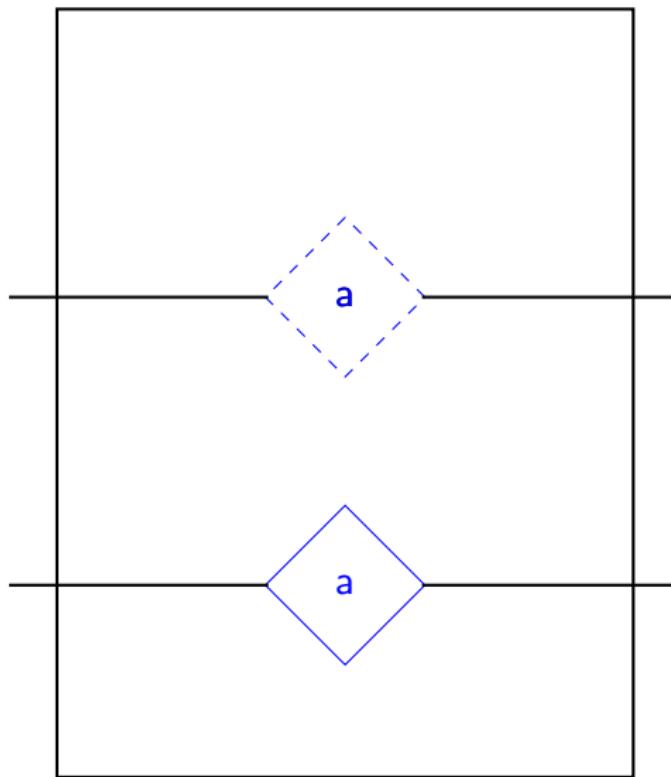
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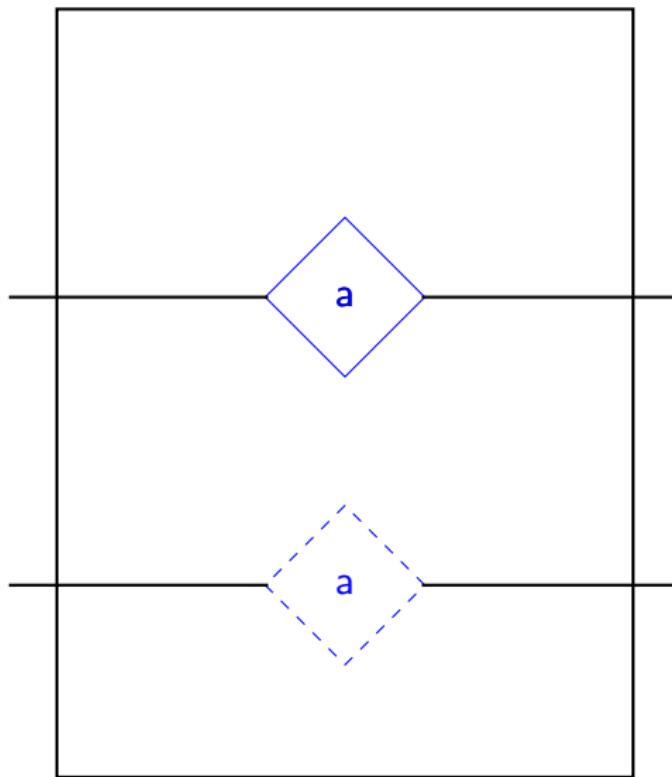
Sokobond is PSpace-Hard



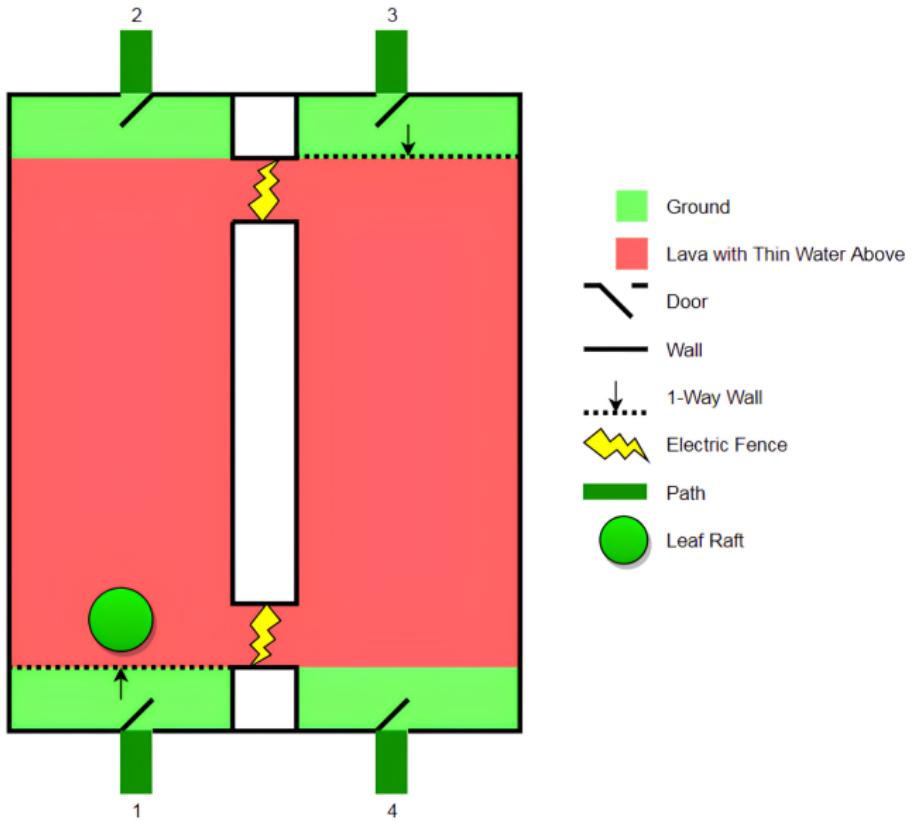
Reminder - Symmetric self closing door



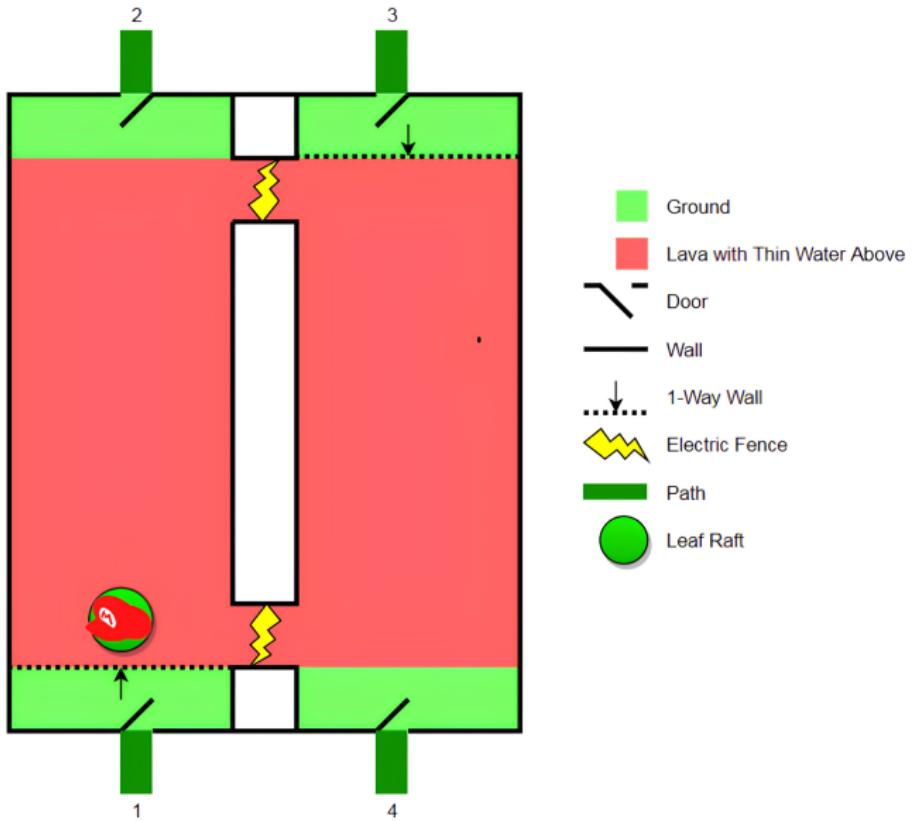
Reminder - Symmetric self closing door



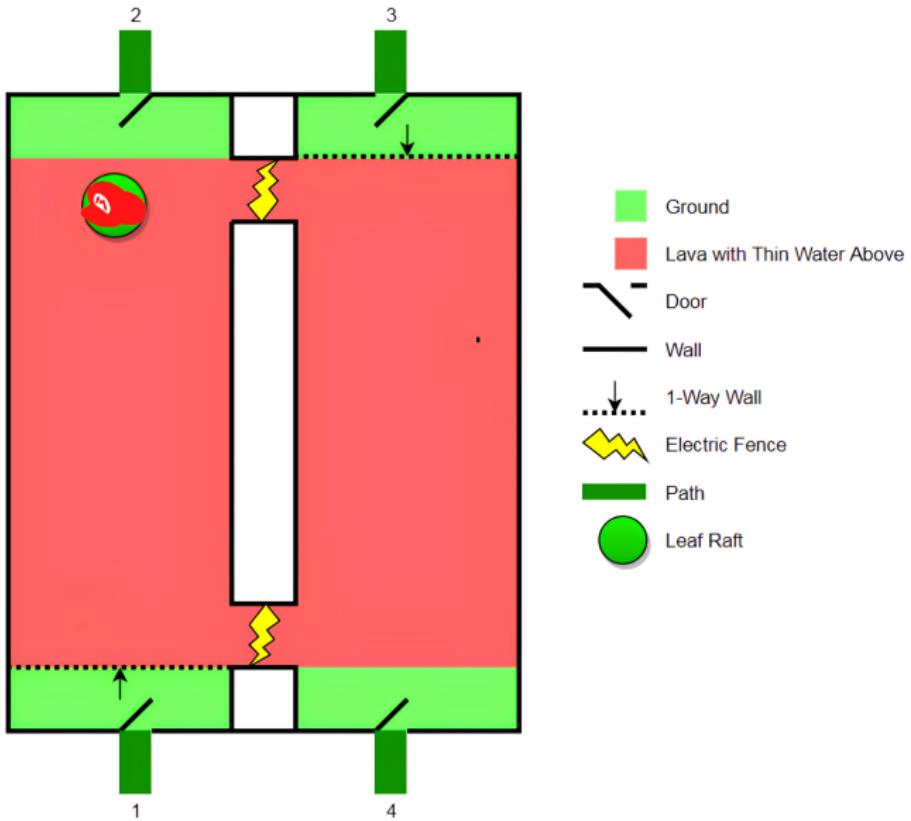
Super Mario Galaxy 2 is PSpace-hard



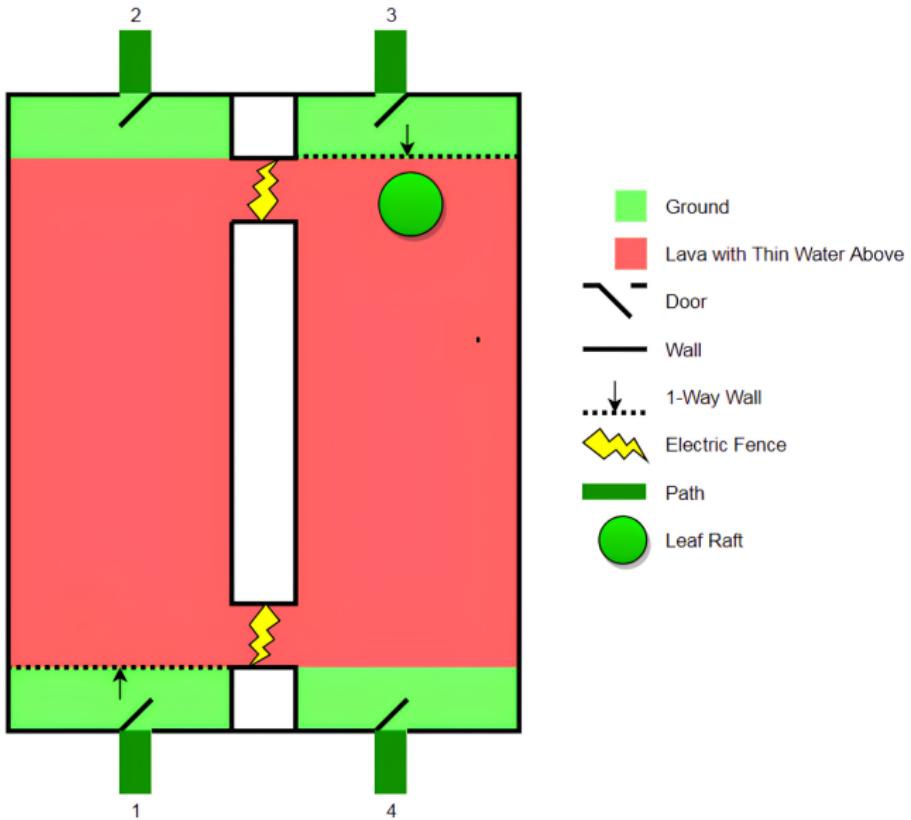
Super Mario Galaxy 2 is PSpace-hard



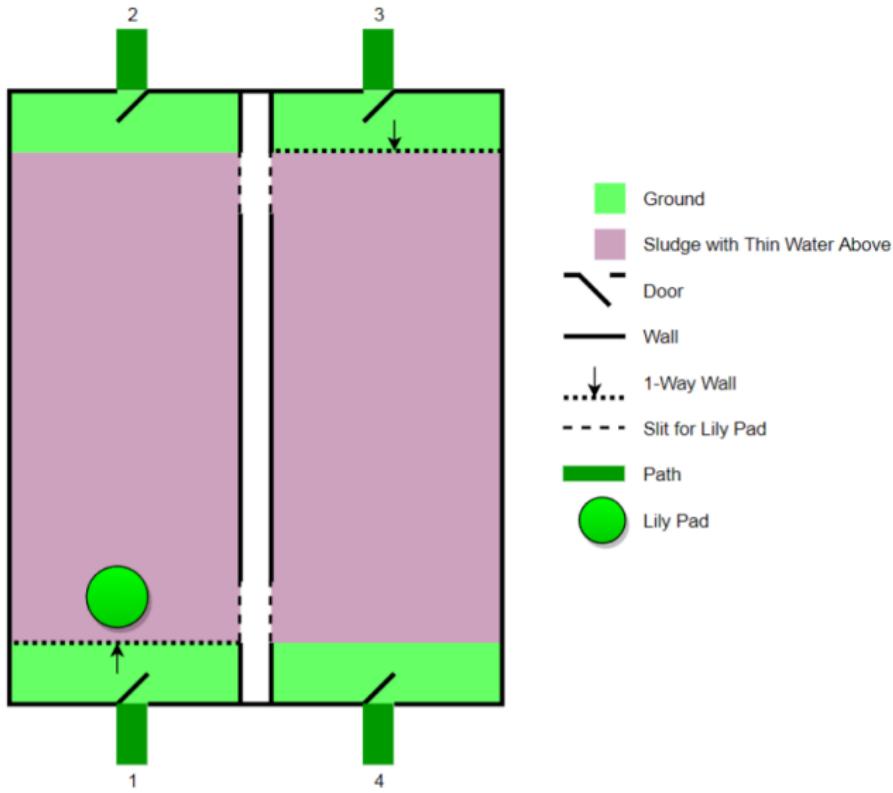
Super Mario Galaxy 2 is PSpace-hard



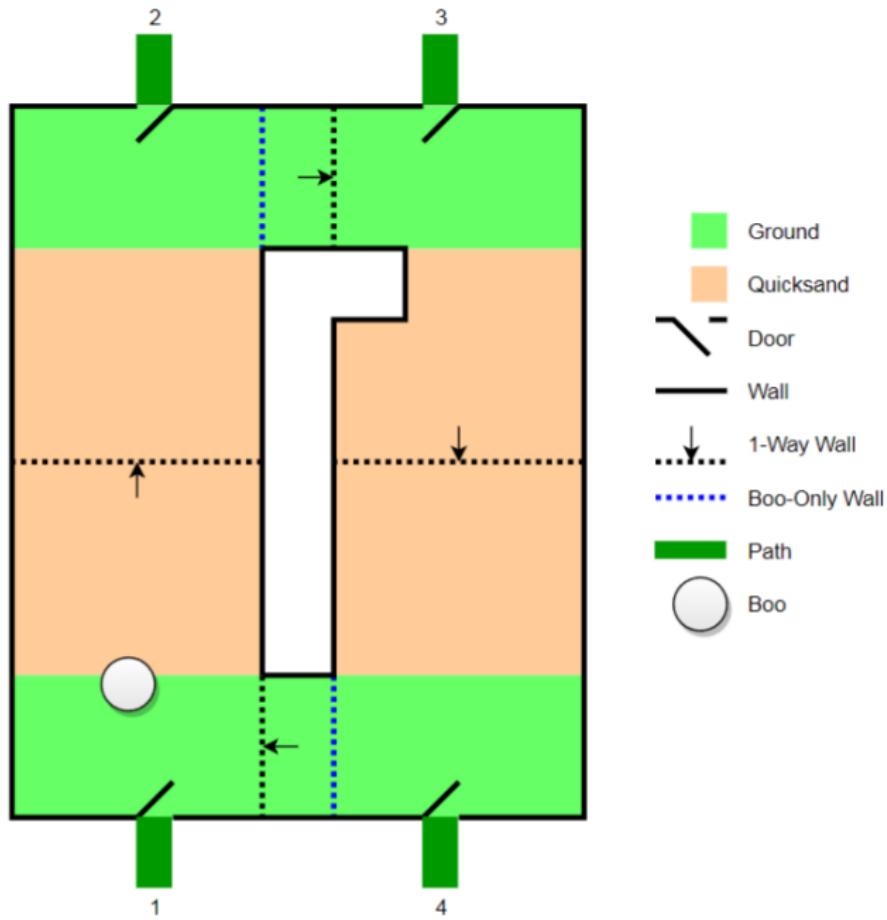
Super Mario Galaxy 2 is PSpace-hard



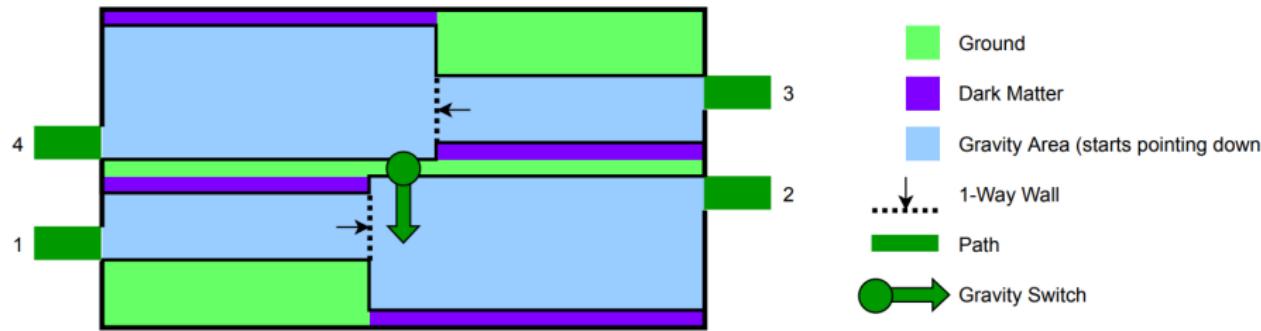
Parralelities



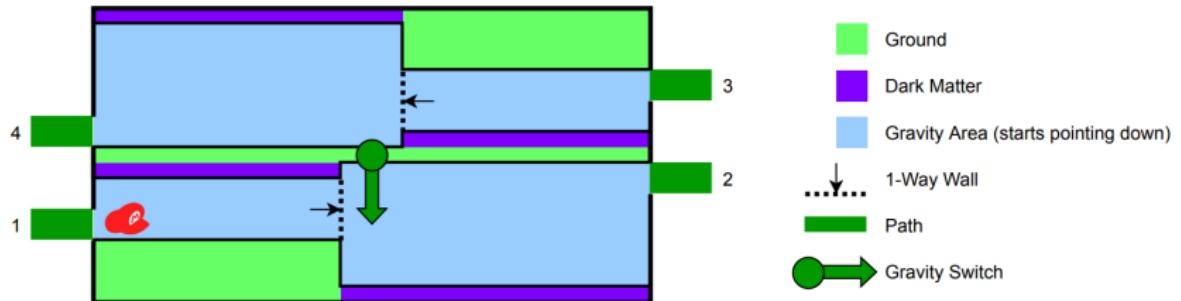
Parralelities



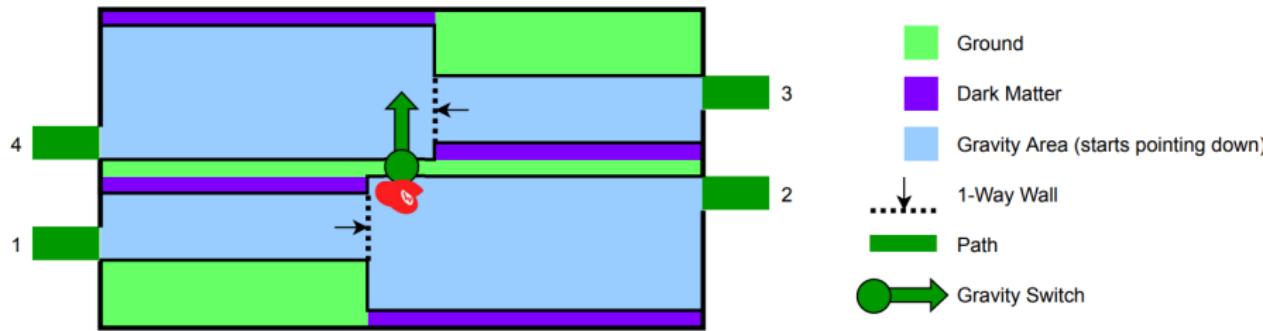
Super Mario Galaxy is PSpace-hard



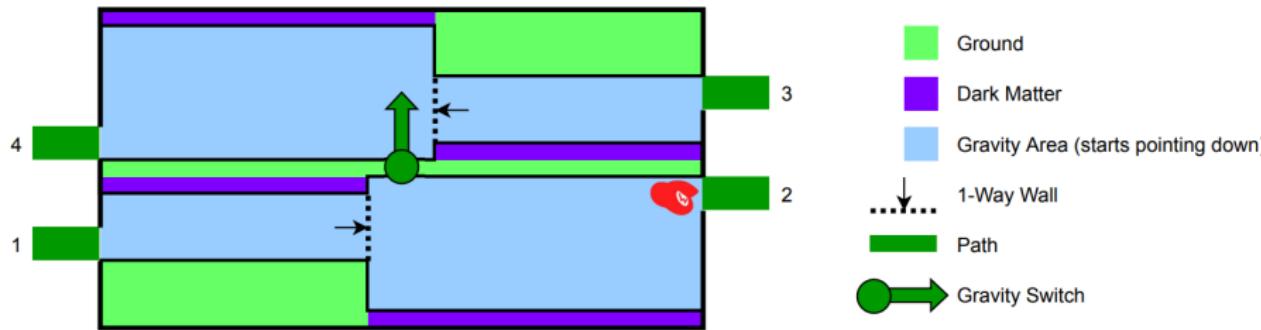
Super Mario Galaxy is PSpace-hard



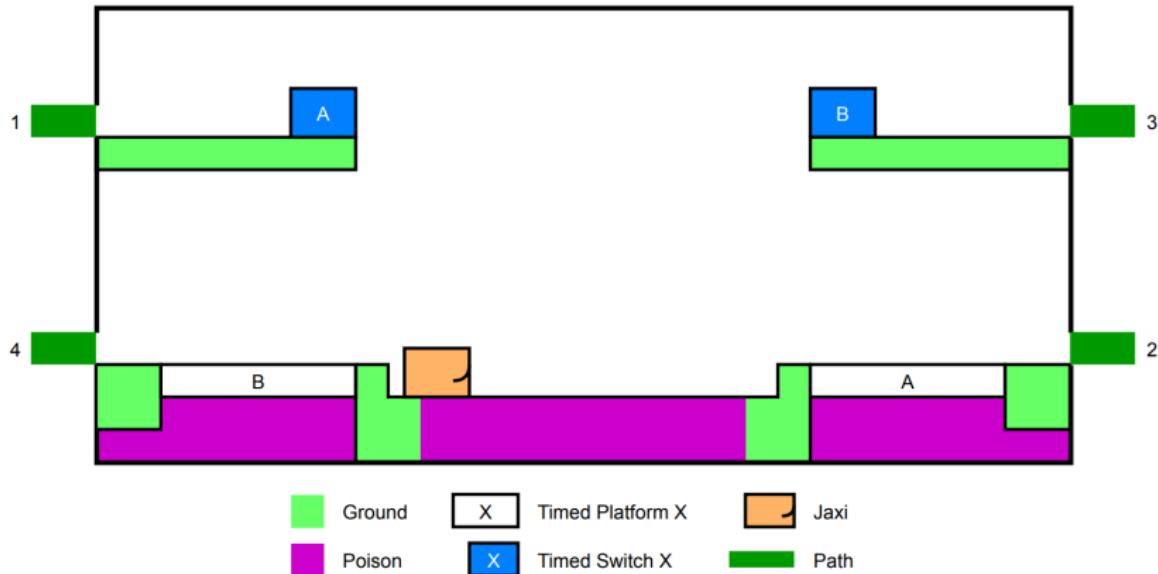
Super Mario Galaxy is PSpace-hard



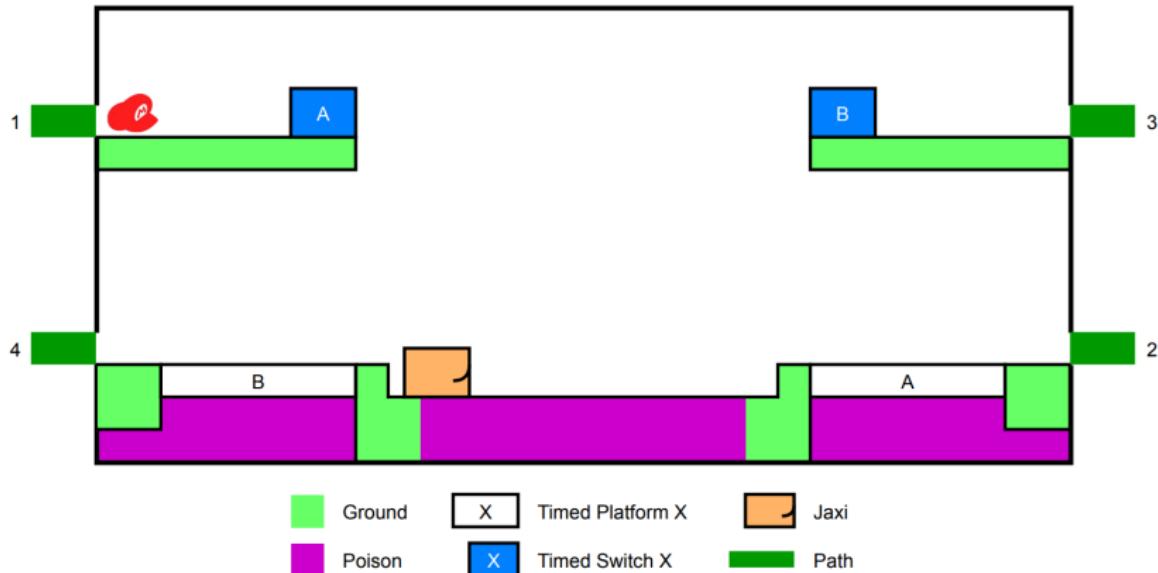
Super Mario Galaxy is PSpace-hard



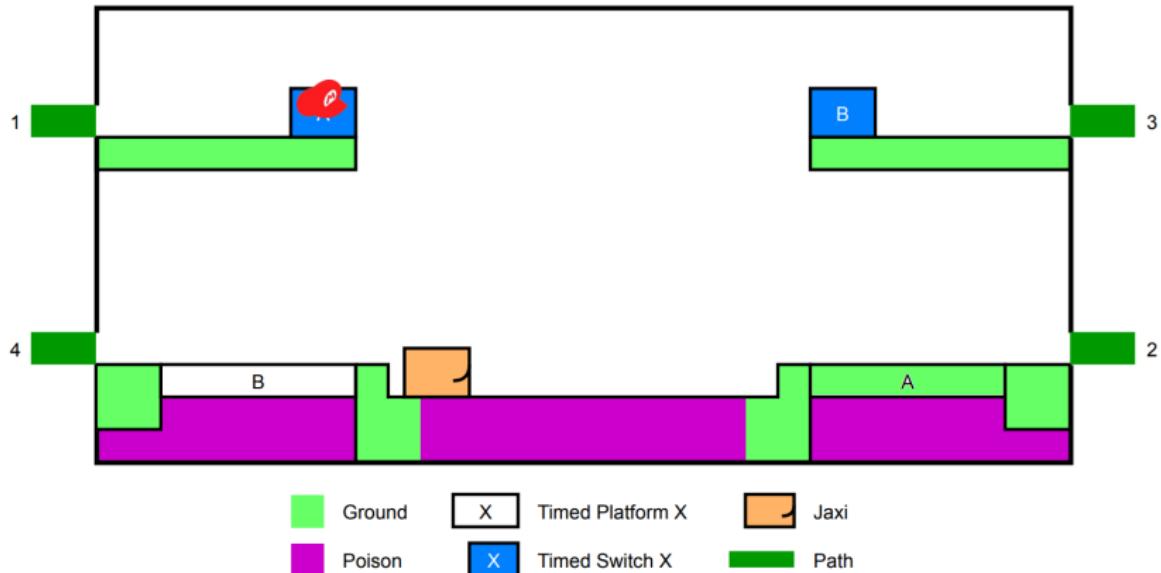
Super Mario Odyssey is PSpace-hard



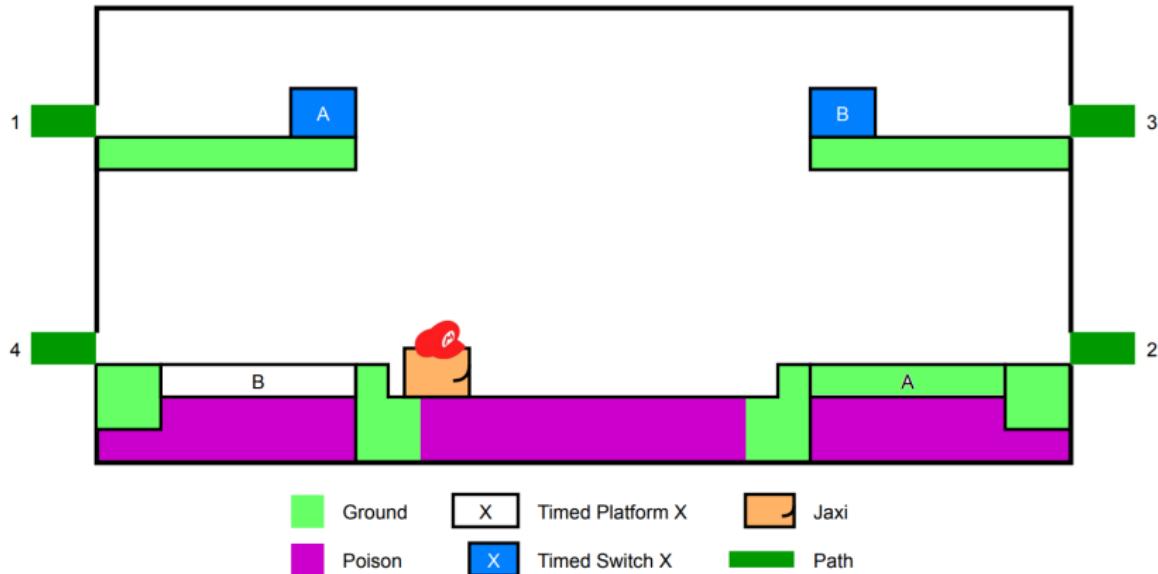
Super Mario Odyssey is PSpace-hard



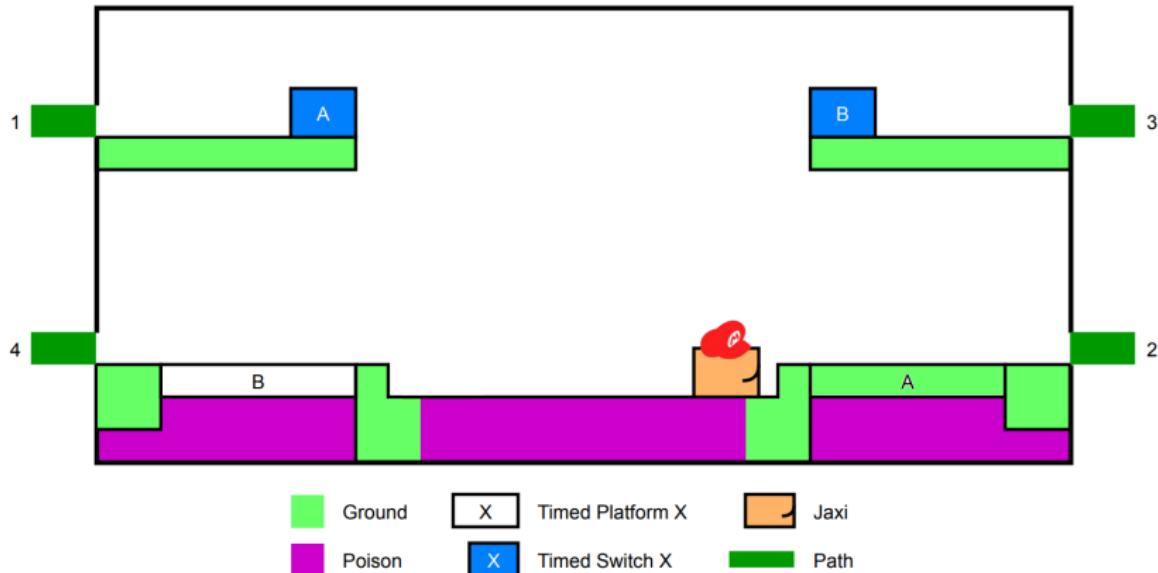
Super Mario Odyssey is PSpace-hard



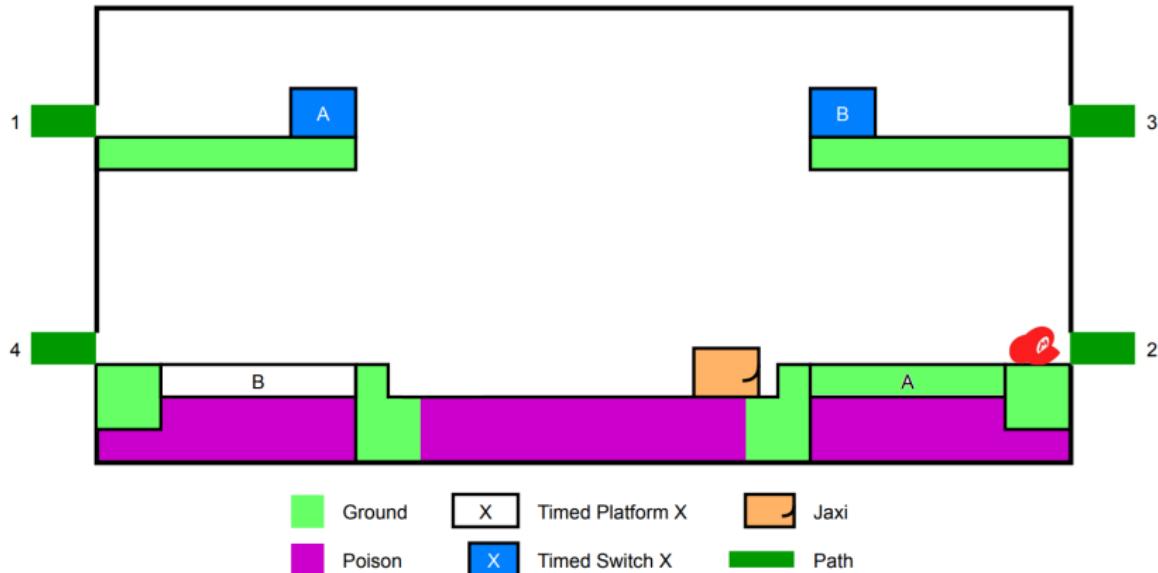
Super Mario Odyssey is PSpace-hard



Super Mario Odyssey is PSpace-hard



Super Mario Odyssey is PSpace-hard



Super Mario Odyssey is PSpace-hard

