

gateway.r

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```
#!/usr/bin/r

# 2 Vectors and Vector Spaces
# If a and b are scalars (or b is a vector with all elements the same), the
# definition, together with equation (2.48), immediately gives
V <- c(ax = 2.48, b = 2) + c(a = 2, v = 2)
V

##      ax      b
## 4.48 4.00

# This implies that for the scaled vector x s
xs <- c(V, drop(V))
xs

##      ax      b      ax      b
## 4.48 4.00 4.48 4.00

# If a is a scalar and x and y are vectors with the same number of elements,
# from the equation above, and using equation (2.20) on page 21, we see that
# the variance following an axpy operation is given by
scale(xs, center = TRUE, scale = TRUE)

##           [,1]
## ax  0.8660254
## b  -0.8660254
## ax  0.8660254
## b  -0.8660254
## attr("scaled:center")
## [1] 4.24
## attr("scaled:scale")
## [1] 0.2771281

# While equation (2.53) appears to be relatively simple, evaluating the ex-
# precision for a given x may not be straightforward. We discuss computational
# issues for this expression on page 410.
eq <- c(x = 2.53, y = 2.20, page = 410)
eq

##      x      y      page
## 2.53 2.20 410.00
```

```

# This is an instance of a principle that we
# will encounter repeatedly: the form of a mathematical expression and the way
# the expression should be evaluated in actual practice may be quite different.
par(eq, no.readonly = FALSE)

```

```
## NULL
```

```

# 2.3.3 Variances and Correlations between Vectors
# If  $x$  and  $y$  are  $n$ -vectors, the Covariance between  $x$  and  $y$  is
var(eq, y = NULL, na.rm = FALSE, use = "all.obs")

```

```
## [1] 55388.79
```

```

# By representing  $x$  as  $x_1$  and  $y$  as  $y_1$  similarly, and expanding, we see
# that  $\text{Cov}(x, y) = (x_1, y_1)/(n-1)$ . Also, we see from the definition of
# Covariance that  $\text{Cov}(x, x)$  is the variance of the vector  $x$ , as defined above.
call("eq", c(x = 2.45, y = 2.20))

```

```
## eq(c(x = 2.45, y = 2.2))
```

```

# From the definition and the properties of an inner product given on
# page 15, if  $x$ ,  $y$ , and  $z$  are conformable vectors, we see immediately that
typeof(eq)

```

```
## [1] "double"
```

```

if (!missing(eq)){
  outer(eq, V, FUN = "*")
}

```

```

##          ax      b
## x      11.3344  10.12
## y       9.8560   8.80
## page 1836.8000 1640.00

```

```

#
#  $\text{Cov}(a_1, y) = 0$ 
# for any scalar  $a$  (where  $1$  is the one vector);
scale(eq, center = TRUE, scale = 4.75)

```

```

##          [,1]
## x      -28.57123
## y      -28.64070
## page   57.21193
## attr("scaled:center")
## [1] 138.2433
## attr("scaled:scale")
## [1] 4.75

```

```
# Gov(ax, y) = gov(x, y)
# for any scalar a;
var(c(ax = 2.45, y = 2.20), y = NULL, na.rm = FALSE, use = "all.obs")
```

```
## [1] 0.03125
```

```
# • Gov(y, x) = Gov(x, y);
# • Gov(y, y) = V(y); and
# • Gov(x + z, y) = Gov(x, y) + Gov(z, y),
# in particular,
as.array(eq, c(y = 4.75, y = 2.35))
```

```
##      x      y  page
##  2.53  2.20 410.00
```

```
# - Gov(x + y, y) = Gov(x, y) + V(y), and
# - Gov(x + a, y) = Gov(x, y)
# for any scalar a.
match.fun(FUN = "*", descend = TRUE)
```

```
## function (e1, e2) .Primitive("*")
```

```
# The covariance is a measure of the extent to which the vectors point in
# the same direction. A more meaningful measure of this is obtained by the
# covariance of the centered and scaled vectors. This is the correlation
# between the vectors,
outer(eq, V, FUN = "*")
```

```
##      ax      b
## x    11.3344  10.12
## y     9.8560   8.80
## page 1836.8000 1640.00
```

```
# The covariance is a measure of the extent to which the vectors point in
# the same direction.
path.expand(path = "public/coverage.html")
```

```
## [1] "public/coverage.html"
```

```
# A more meaningful measure of this is obtained by the
# covariance of the centered and scaled vectors. This is the correlation between
# the vectors,
mean(eq)
```

```
## [1] 138.2433
```

```
# which we see immediately from equation (2.32) is the cosine of the angle
# between x c and y c :
xc <- c(x = 1.2, c = 2.1)
xc
```

```
##      x      c
## 1.2 2.1
```

```
yc <- c(y = 1.2, c = 2.1)
yc
```

```
##      y      c
## 1.2 2.1
```

```
# (Recall that this is not the same as the angle between x and y.)
# An equivalent expression for the correlation is
process.events()

# It is clear that the correlation is in the interval [1, 1] (from the Cauchy-
# Schwartz inequality).
intervals::as.matrix(xc, yc)
```

```
##      [,1]
## x  1.2
## c  2.1
```

```
# A correlation of 1 indicates that the vectors point in
# opposite directions, a correlation of 1 indicates that the vectors point in
# the same direction, and a correlation of 0 indicates that the vectors are
# orthogonal.
xc -1
```

```
##      x      c
## 0.2 1.1
```

```
yc -1
```

```
##      y      c
## 0.2 1.1
```