onevector.r

denis

2021-07-13

```r
#!/usr/bin/r

# 22
# 2 Vectors and Vector Spaces
# is a normalized vector. Normalized vectors are sometimes referred to as "unit
# vectors", although we will generally reserve this term for a special kind of
# normalized vector (see page 12). A normalized vector is also sometimes
# referred o as a "normal vector". I use "normalized vector" for a vector such
# as x in equation (2.21) and use the latter phrase to denote a vector that is
# orthogonal to a subspace.
norms <- vector(mode = "logical", length = 0L)
norms
```

```
## logical(0)
```

```r
# 2.1.7 Metrics and Distances
# It is often useful to consider how far apart two vectors are; that is, the
# "distance" between them. A reasonable distance measure would have to satisfy
# certain requirements, such as being a nonnegative real number. A function
# that maps any two objects in a set S to IR is called a metric on S if, for all
# x, y, and z in S, it satisfies the following three conditions:
Matrix::as.matrix(norms)
```

```
##      [,1]
```

```r
# These conditions correspond in an intuitive manner to the properties we ex-
# pect of a distance between objects.
object.size(norms)
```

```
## 48 bytes
```

```r
# Metrics Induced by Norms
# If subtraction and a norm are defined for the elements of S, the most common
# way of forming a metric is by using the norm. If · is a norm, we can verify
# that
Matrix::as.array(norms)
```

```
## logical(0)
```

```r
# is a metric by using the properties of a norm to establish the three
# properties of a metric above (Exercise 2.7).
c(norms, sigSlots = c("target", "defined"))
```

```
## sigSlots1 sigSlots2
##  "target" "defined"
```

```r
# The general inner products, norms, and metrics defined above are relevant
# in a wide range of applications. The sets on which they are defined can consist
# of various types of objects. In the context of real vectors, the most common
# inner product is the dot product; the most common norm is the Euclidean
# norm that arises from the dot product; and the most common metric is the
# one defined by the Euclidean norm, called the Euclidean distance.
c(norms)
```

```
## logical(0)
```

```r
# 2.1.8 Orthogonal Vectors and Orthogonal Vector Spaces
# Two vectors v 1 and v 2 such that
# v 1 , v 2 = 0
# (2.23)
# are said to be orthogonal, and this condition is denoted by v 1  v 2 .
# (Some-
# times we exclude the zero vector from this definition, but it is not important
orthogonal <- c(v = 1, v = 2, v = 3)
orthogonal
```

```
## v v v
## 1 2 3
```

```r
# 2.1 Operations on Vectors
# 23
# to do so.) Normalized vectors that are all orthogonal to each other are called
# orthonormal vectors. (If the elements of the vectors are from the field of com-
# plex numbers, orthogonality and normality are defined in terms of the dot
# products of a vector with a complex conjugate of a vector.)
options("orthogonal")
```

```
## $orthogonal
## NULL
```

```r
# A set of nonzero vectors that are mutually orthogonal are necessarily lin-
# early independent. To see this, we show it for any two orthogonal vectors
# and then indicate the pattern that extends to three or more vectors. Sup-
# pose v 1 and v 2 are nonzero and are orthogonal; that is, v 1 , v 2 = 0. We
# see
# immediately that if there is a scalar a such that v 1 = av 2 , then a must be
# nonzero and we have a contradiction because v 1 , v 2 = a v 1 , v 1 = 0. For
# three
# mutually orthogonal vectors, v 1 , v 2 , and v 3 , we consider
# v 1 = av 2 + bv 3 for a
```

```r
 #or b nonzero, and arrive at the same contradiction.
set.seed(c(v = 1, v = 2, v = 3))
orthogonal + sin(2)
```

```
##        v        v        v
## 1.909297 2.909297 3.909297
```

```r
# Two vector spaces V 1 and V 2 are said to be orthogonal, written V 1  V 2 ,
# if each vector in one is orthogonal to every vector in the other.
# If V 1  V 2 and
# V 1  V 2 = IR n , then V 2 is called the orthogonal complement of V 1 ,
# and this is
# written as V 2 = V 1  . More generally, if V 1  V 2 and V 1  V 2 = V, t
# hen V 2 is
# called the orthogonal complement of V 1 with respect to V. This is obviously
# a symmetric relationship; if V 2 is the orthogonal complement of V 1 , then
# V 1
# is the orthogonal complement of V 2 .
vec <- c(orthogonal, norms)
vec
```

```
## v v v
## 1 2 3
```

```r
# If B 1 is a basis set for V 1 , B 2 is a basis set for V 2 , and V 2 is the
# orthogonal
# complement of V 1 with respect to V, then B 1  B 2 is a basis set for V.
# It is
# a basis set because since V 1 and V 2 are orthogonal, it must be the case that
# B 1  B 2 = .
# If V 1  V, V 2  V, V 1  V 2 , and dim(V 1 ) + dim(V 2 ) = dim(V), then
# V 1  V 2 = V;
dim(norms)
```

```
## NULL
```

```r
# that is, V 2 is the orthogonal complement of V 1 . We see this by first letting
# B 1 and B 2 be bases for V 1 and V 2 . Now V 1  V 2 implies that B 1  B
# 2 =  and dim(V 1 ) + dim(V 2 ) = dim(V) implies #(B 1 ) + #(B 2 ) = #(B),
# for any basis set B for V; hence, B 1  B 2 is a basis set for V.
vb <- c(b = 1, b = 2, v = 1, v = 2)
dim(vb)
```

```
## NULL
```

```r
vb
```

```
## b b v v
## 1 2 1 2
```

```r
# 2.1.9 The "One Vector"
# Another often useful vector is the vector with all elements equal to 1. We
# call this the "one vector" and denote it by 1 or by 1 n . The one vector can
# be used in the representation of the sum of the elements in a vector:
c("https://tidyselect.r-lib.org/reference/faq-selection-context.html",
       vars = NULL)
```

```
##
## "https://tidyselect.r-lib.org/reference/faq-selection-context.html"
```

```r
# The one vector is also called the "summing vector".
summary(.data = "dplyr", vb, groups = NULL)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     1.0     1.0     1.5     1.5     2.0     2.0
```