ax.r

denis

2021-07-12

```r
#!/usr/bin/r

# Cones
# A set of vectors that contains all positive scalar multiples of any vector in
# the set is called a cone. A cone always contains the zero vector. A set of
# vectors V is a convex cone if, for all v 1 , v 2  V and all a, b  0,
# av 1 + bv 2  V .
# (Such a cone is called a homogeneous convex cone by some authors. Also,
# some authors require that a + b = 1 in the dentition.) A convex cone is not
# necessarily a vector space because v 1  v 2 may not be in V . An important
# convex cone in an n-dimensional vector space is the positive orthant together
# with the zero vector. This convex cone is not closed, in the sense that it
# does not contain some limits. The closure of the positive orthant (that is,
# the nonnegative orthant) is also a convex cone.
av <- c(1 + 25 + 5 + 5) # business
av
```

```
## [1] 36
```

```r
# 2.1.3 Basis Sets
# If each vector in the vector space V can be expressed as a linear combination
# of the vectors in some set G, then G is said to be a generating set or
# spanning
# set of V. If, in addition, all linear combinations of the elements of G are
# in
# V, the vector space is the space generated by G and is denoted by V(G) or by
# span(G):
# V(G)  span(G).
V <- sunspots
V
```

```
##        Jan   Feb   Mar   Apr   May   Jun   Jul   Aug   Sep   Oct   Nov   Dec
## 1749  58.0  62.6  70.0  55.7  85.0  83.5  94.8  66.3  75.9  75.5 158.6  85.2
## 1750  73.3  75.9  89.2  88.3  90.0 100.0  85.4 103.0  91.2  65.7  63.3  75.4
## 1751  70.0  43.5  45.3  56.4  60.7  50.7  66.3  59.8  23.5  23.2  28.5  44.0
## 1752  35.0  50.0  71.0  59.3  59.7  39.6  78.4  29.3  27.1  46.6  37.6  40.0
## 1753  44.0  32.0  45.7  38.0  36.0  31.7  22.2  39.0  28.0  25.0  20.0   6.7
## 1754   0.0   3.0   1.7  13.7  20.7  26.7  18.8  12.3   8.2  24.1  13.2   4.2
## 1755  10.2  11.2   6.8   6.5   0.0   0.0   8.6   3.2  17.8  23.7   6.8  20.0
## 1756  12.5   7.1   5.4   9.4  12.5  12.9   3.6   6.4  11.8  14.3  17.0   9.4
## 1757  14.1  21.2  26.2  30.0  38.1  12.8  25.0  51.3  39.7  32.5  64.7  33.5
```

```
## 1758  37.6  52.0  49.0  72.3  46.4  45.0  44.0  38.7  62.5  37.7  43.0  43.0
## 1759  48.3  44.0  46.8  47.0  49.0  50.0  51.0  71.3  77.2  59.7  46.3  57.0
## 1760  67.3  59.5  74.7  58.3  72.0  48.3  66.0  75.6  61.3  50.6  59.7  61.0
## 1761  70.0  91.0  80.7  71.7 107.2  99.3  94.1  91.1 100.7  88.7  89.7  46.0
## 1762  43.8  72.8  45.7  60.2  39.9  77.1  33.8  67.7  68.5  69.3  77.8  77.2
## 1763  56.5  31.9  34.2  32.9  32.7  35.8  54.2  26.5  68.1  46.3  60.9  61.4
## 1764  59.7  59.7  40.2  34.4  44.3  30.0  30.0  30.0  28.2  28.0  26.0  25.7
## 1765  24.0  26.0  25.0  22.0  20.2  20.0  27.0  29.7  16.0  14.0  14.0  13.0
## 1766  12.0  11.0  36.6   6.0  26.8   3.0   3.3   4.0   4.3   5.0   5.7  19.2
## 1767  27.4  30.0  43.0  32.9  29.8  33.3  21.9  40.8  42.7  44.1  54.7  53.3
## 1768  53.5  66.1  46.3  42.7  77.7  77.4  52.6  66.8  74.8  77.8  90.6 111.8
## 1769  73.9  64.2  64.3  96.7  73.6  94.4 118.6 120.3 148.8 158.2 148.1 112.0
## 1770 104.0 142.5  80.1  51.0  70.1  83.3 109.8 126.3 104.4 103.6 132.2 102.3
## 1771  36.0  46.2  46.7  64.9 152.7 119.5  67.7  58.5 101.4  90.0  99.7  95.7
## 1772 100.9  90.8  31.1  92.2  38.0  57.0  77.3  56.2  50.5  78.6  61.3  64.0
## 1773  54.6  29.0  51.2  32.9  41.1  28.4  27.7  12.7  29.3  26.3  40.9  43.2
## 1774  46.8  65.4  55.7  43.8  51.3  28.5  17.5   6.6   7.9  14.0  17.7  12.2
## 1775   4.4   0.0  11.6  11.2   3.9  12.3   1.0   7.9   3.2   5.6  15.1   7.9
## 1776  21.7  11.6   6.3  21.8  11.2  19.0   1.0  24.2  16.0  30.0  35.0  40.0
## 1777  45.0  36.5  39.0  95.5  80.3  80.7  95.0 112.0 116.2 106.5 146.0 157.3
## 1778 177.3 109.3 134.0 145.0 238.9 171.6 153.0 140.0 171.7 156.3 150.3 105.0
## 1779 114.7 165.7 118.0 145.0 140.0 113.7 143.0 112.0 111.0 124.0 114.0 110.0
## 1780  70.0  98.0  98.0  95.0 107.2  88.0  86.0  86.0  93.7  77.0  60.0  58.7
## 1781  98.7  74.7  53.0  68.3 104.7  97.7  73.5  66.0  51.0  27.3  67.0  35.2
## 1782  54.0  37.5  37.0  41.0  54.3  38.0  37.0  44.0  34.0  23.2  31.5  30.0
## 1783  28.0  38.7  26.7  28.3  23.0  25.2  32.2  20.0  18.0   8.0  15.0  10.5
## 1784  13.0   8.0  11.0  10.0   6.0   9.0   6.0  10.0  10.0   8.0  17.0  14.0
## 1785   6.5   8.0   9.0  15.7  20.7  26.3  36.3  20.0  32.0  47.2  40.2  27.3
## 1786  37.2  47.6  47.7  85.4  92.3  59.0  83.0  89.7 111.5 112.3 116.0 112.7
## 1787 134.7 106.0  87.4 127.2 134.8  99.2 128.0 137.2 157.3 157.0 141.5 174.0
## 1788 138.0 129.2 143.3 108.5 113.0 154.2 141.5 136.0 141.0 142.0  94.7 129.5
## 1789 114.0 125.3 120.0 123.3 123.5 120.0 117.0 103.0 112.0  89.7 134.0 135.5
## 1790 103.0 127.5  96.3  94.0  93.0  91.0  69.3  87.0  77.3  84.3  82.0  74.0
## 1791  72.7  62.0  74.0  77.2  73.7  64.2  71.0  43.0  66.5  61.7  67.0  66.0
## 1792  58.0  64.0  63.0  75.7  62.0  61.0  45.8  60.0  59.0  59.0  57.0  56.0
## 1793  56.0  55.0  55.5  53.0  52.3  51.0  50.0  29.3  24.0  47.0  44.0  45.7
## 1794  45.0  44.0  38.0  28.4  55.7  41.5  41.0  40.0  11.1  28.5  67.4  51.4
## 1795  21.4  39.9  12.6  18.6  31.0  17.1  12.9  25.7  13.5  19.5  25.0  18.0
## 1796  22.0  23.8  15.7  31.7  21.0   6.7  26.9   1.5  18.4  11.0   8.4   5.1
## 1797  14.4   4.2   4.0   4.0   7.3  11.1   4.3   6.0   5.7   6.9   5.8   3.0
## 1798   2.0   4.0  12.4   1.1   0.0   0.0   0.0   3.0   2.4   1.5  12.5   9.9
## 1799   1.6  12.6  21.7   8.4   8.2  10.6   2.1   0.0   0.0   4.6   2.7   8.6
## 1800   6.9   9.3  13.9   0.0   5.0  23.7  21.0  19.5  11.5  12.3  10.5  40.1
## 1801  27.0  29.0  30.0  31.0  32.0  31.2  35.0  38.7  33.5  32.6  39.8  48.2
## 1802  47.8  47.0  40.8  42.0  44.0  46.0  48.0  50.0  51.8  38.5  34.5  50.0
## 1803  50.0  50.8  29.5  25.0  44.3  36.0  48.3  34.1  45.3  54.3  51.0  48.0
## 1804  45.3  48.3  48.0  50.6  33.4  34.8  29.8  43.1  53.0  62.3  61.0  60.0
## 1805  61.0  44.1  51.4  37.5  39.0  40.5  37.6  42.7  44.4  29.4  41.0  38.3
## 1806  39.0  29.6  32.7  27.7  26.4  25.6  30.0  26.3  24.0  27.0  25.0  24.0
## 1807  12.0  12.2   9.6  23.8  10.0  12.0  12.7  12.0   5.7   8.0   2.6   0.0
## 1808   0.0   4.5   0.0  12.3  13.5  13.5   6.7   8.0  11.7   4.7  10.5  12.3
## 1809   7.2   9.2   0.9   2.5   2.0   7.7   0.3   0.2   0.4   0.0   0.0   0.0
## 1810   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
## 1811   0.0   0.0   0.0   0.0   0.0   0.0   6.6   0.0   2.4   6.1   0.8   1.1
```

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## 1812  11.3   1.9   0.7   0.0   1.0   1.3   0.5  15.6   5.2   3.9   7.9  10.1
## 1813   0.0  10.3   1.9  16.6   5.5  11.2  18.3   8.4  15.3  27.8  16.7  14.3
## 1814  22.2  12.0   5.7  23.8   5.8  14.9  18.5   2.3   8.1  19.3  14.5  20.1
## 1815  19.2  32.2  26.2  31.6   9.8  55.9  35.5  47.2  31.5  33.5  37.2  65.0
## 1816  26.3  68.8  73.7  58.8  44.3  43.6  38.8  23.2  47.8  56.4  38.1  29.9
## 1817  36.4  57.9  96.2  26.4  21.2  40.0  50.0  45.0  36.7  25.6  28.9  28.4
## 1818  34.9  22.4  25.4  34.5  53.1  36.4  28.0  31.5  26.1  31.7  10.9  25.8
## 1819  32.5  20.7   3.7  20.2  19.6  35.0  31.4  26.1  14.9  27.5  25.1  30.6
## 1820  19.2  26.6   4.5  19.4  29.3  10.8  20.6  25.9   5.2   9.0   7.9   9.7
## 1821  21.5   4.3   5.7   9.2   1.7   1.8   2.5   4.8   4.4  18.8   4.4   0.0
## 1822   0.0   0.9  16.1  13.5   1.5   5.6   7.9   2.1   0.0   0.4   0.0   0.0
## 1823   0.0   0.0   0.6   0.0   0.0   0.0   0.5   0.0   0.0   0.0   0.0  20.4
## 1824  21.6  10.8   0.0  19.4   2.8   0.0   0.0   1.4  20.5  25.2   0.0   0.8
## 1825   5.0  15.5  22.4   3.8  15.4  15.4  30.9  25.4  15.7  15.6  11.7  22.0
## 1826  17.7  18.2  36.7  24.0  32.4  37.1  52.5  39.6  18.9  50.6  39.5  68.1
## 1827  34.6  47.4  57.8  46.0  56.3  56.7  42.9  53.7  49.6  57.2  48.2  46.1
## 1828  52.8  64.4  65.0  61.1  89.1  98.0  54.3  76.4  50.4  54.7  57.0  46.6
## 1829  43.0  49.4  72.3  95.0  67.5  73.9  90.8  78.3  52.8  57.2  67.6  56.5
## 1830  52.2  72.1  84.6 107.1  66.3  65.1  43.9  50.7  62.1  84.4  81.2  82.1
## 1831  47.5  50.1  93.4  54.6  38.1  33.4  45.2  54.9  37.9  46.2  43.5  28.9
## 1832  30.9  55.5  55.1  26.9  41.3  26.7  13.9   8.9   8.2  21.1  14.3  27.5
## 1833  11.3  14.9  11.8   2.8  12.9   1.0   7.0   5.7  11.6   7.5   5.9   9.9
## 1834   4.9  18.1   3.9   1.4   8.8   7.8   8.7   4.0  11.5  24.8  30.5  34.5
## 1835   7.5  24.5  19.7  61.5  43.6  33.2  59.8  59.0 100.8  95.2 100.0  77.5
## 1836  88.6 107.6  98.1 142.9 111.4 124.7 116.7 107.8  95.1 137.4 120.9 206.2
## 1837 188.0 175.6 134.6 138.2 111.3 158.0 162.8 134.0  96.3 123.7 107.0 129.8
## 1838 144.9  84.8 140.8 126.6 137.6  94.5 108.2  78.8  73.6  90.8  77.4  79.8
## 1839 107.6 102.5  77.7  61.8  53.8  54.6  84.7 131.2 132.7  90.8  68.8  63.6
## 1840  81.2  87.7  55.5  65.9  69.2  48.5  60.7  57.8  74.0  49.8  54.3  53.7
## 1841  24.0  29.9  29.7  42.6  67.4  55.7  30.8  39.3  35.1  28.5  19.8  38.8
## 1842  20.4  22.1  21.7  26.9  24.9  20.5  12.6  26.5  18.5  38.1  40.5  17.6
## 1843  13.3   3.5   8.3   8.8  21.1  10.5   9.5  11.8   4.2   5.3  19.1  12.7
## 1844   9.4  14.7  13.6  20.8  12.0   3.7  21.2  23.9   6.9  21.5  10.7  21.6
## 1845  25.7  43.6  43.3  56.9  47.8  31.1  30.6  32.3  29.6  40.7  39.4  59.7
## 1846  38.7  51.0  63.9  69.2  59.9  65.1  46.5  54.8 107.1  55.9  60.4  65.5
## 1847  62.6  44.9  85.7  44.7  75.4  85.3  52.2 140.6 161.2 180.4 138.9 109.6
## 1848 159.1 111.8 108.9 107.1 102.2 123.8 139.2 132.5 100.3 132.4 114.6 159.9
## 1849 156.7 131.7  96.5 102.5  80.6  81.2  78.0  61.3  93.7  71.5  99.7  97.0
## 1850  78.0  89.4  82.6  44.1  61.6  70.0  39.1  61.6  86.2  71.0  54.8  60.0
## 1851  75.5 105.4  64.6  56.5  62.6  63.2  36.1  57.4  67.9  62.5  50.9  71.4
## 1852  68.4  67.5  61.2  65.4  54.9  46.9  42.0  39.7  37.5  67.3  54.3  45.4
## 1853  41.1  42.9  37.7  47.6  34.7  40.0  45.9  50.4  33.5  42.3  28.8  23.4
## 1854  15.4  20.0  20.7  26.4  24.0  21.1  18.7  15.8  22.4  12.7  28.2  21.4
## 1855  12.3  11.4  17.4   4.4   9.1   5.3   0.4   3.1   0.0   9.7   4.3   3.1
## 1856   0.5   4.9   0.4   6.5   0.0   5.0   4.6   5.9   4.4   4.5   7.7   7.2
## 1857  13.7   7.4   5.2  11.1  29.2  16.0  22.2  16.9  42.4  40.6  31.4  37.2
## 1858  39.0  34.9  57.5  38.3  41.4  44.5  56.7  55.3  80.1  91.2  51.9  66.9
## 1859  83.7  87.6  90.3  85.7  91.0  87.1  95.2 106.8 105.8 114.6  97.2  81.0
## 1860  81.5  88.0  98.9  71.4 107.1 108.6 116.7 100.3  92.2  90.1  97.9  95.6
## 1861  62.3  77.8 101.0  98.5  56.8  87.8  78.0  82.5  79.9  67.2  53.7  80.5
## 1862  63.1  64.5  43.6  53.7  64.4  84.0  73.4  62.5  66.6  42.0  50.6  40.9
## 1863  48.3  56.7  66.4  40.6  53.8  40.8  32.7  48.1  22.0  39.9  37.7  41.2
## 1864  57.7  47.1  66.3  35.8  40.6  57.8  54.7  54.8  28.5  33.9  57.6  28.6
## 1865  48.7  39.3  39.5  29.4  34.5  33.6  26.8  37.8  21.6  17.1  24.6  12.8
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## 1866  31.6  38.4  24.6  17.6  12.9  16.5   9.3  12.7   7.3  14.1   9.0   1.5
## 1867   0.0   0.7   9.2   5.1   2.9   1.5   5.0   4.9   9.8  13.5   9.3  25.2
## 1868  15.6  15.8  26.5  36.6  26.7  31.1  28.6  34.4  43.8  61.7  59.1  67.6
## 1869  60.9  59.3  52.7  41.0 104.0 108.4  59.2  79.6  80.6  59.4  77.4 104.3
## 1870  77.3 114.9 159.4 160.0 176.0 135.6 132.4 153.8 136.0 146.4 147.5 130.0
## 1871  88.3 125.3 143.2 162.4 145.5  91.7 103.0 110.0  80.3  89.0 105.4  90.3
## 1872  79.5 120.1  88.4 102.1 107.6 109.9 105.5  92.9 114.6 103.5 112.0  83.9
## 1873  86.7 107.0  98.3  76.2  47.9  44.8  66.9  68.2  47.5  47.4  55.4  49.2
## 1874  60.8  64.2  46.4  32.0  44.6  38.2  67.8  61.3  28.0  34.3  28.9  29.3
## 1875  14.6  22.2  33.8  29.1  11.5  23.9  12.5  14.6   2.4  12.7  17.7   9.9
## 1876  14.3  15.0  31.2   2.3   5.1   1.6  15.2   8.8   9.9  14.3   9.9   8.2
## 1877  24.4   8.7  11.7  15.8  21.2  13.4   5.9   6.3  16.4   6.7  14.5   2.3
## 1878   3.3   6.0   7.8   0.1   5.8   6.4   0.1   0.0   5.3   1.1   4.1   0.5
## 1879   0.8   0.6   0.0   6.2   2.4   4.8   7.5  10.7   6.1  12.3  12.9   7.2
## 1880  24.0  27.5  19.5  19.3  23.5  34.1  21.9  48.1  66.0  43.0  30.7  29.6
## 1881  36.4  53.2  51.5  51.7  43.5  60.5  76.9  58.0  53.2  64.0  54.8  47.3
## 1882  45.0  69.3  67.5  95.8  64.1  45.2  45.4  40.4  57.7  59.2  84.4  41.8
## 1883  60.6  46.9  42.8  82.1  32.1  76.5  80.6  46.0  52.6  83.8  84.5  75.9
## 1884  91.5  86.9  86.8  76.1  66.5  51.2  53.1  55.8  61.9  47.8  36.6  47.2
## 1885  42.8  71.8  49.8  55.0  73.0  83.7  66.5  50.0  39.6  38.7  33.3  21.7
## 1886  29.9  25.9  57.3  43.7  30.7  27.1  30.3  16.9  21.4   8.6   0.3  12.4
## 1887  10.3  13.2   4.2   6.9  20.0  15.7  23.3  21.4   7.4   6.6   6.9  20.7
## 1888  12.7   7.1   7.8   5.1   7.0   7.1   3.1   2.8   8.8   2.1  10.7   6.7
## 1889   0.8   8.5   7.0   4.3   2.4   6.4   9.7  20.6   6.5   2.1   0.2   6.7
## 1890   5.3   0.6   5.1   1.6   4.8   1.3  11.6   8.5  17.2  11.2   9.6   7.8
## 1891  13.5  22.2  10.4  20.5  41.1  48.3  58.8  33.2  53.8  51.5  41.9  32.3
## 1892  69.1  75.6  49.9  69.6  79.6  76.3  76.8 101.4  62.8  70.5  65.4  78.6
## 1893  75.0  73.0  65.7  88.1  84.7  88.2  88.8 129.2  77.9  79.7  75.1  93.8
## 1894  83.2  84.6  52.3  81.6 101.2  98.9 106.0  70.3  65.9  75.5  56.6  60.0
## 1895  63.3  67.2  61.0  76.9  67.5  71.5  47.8  68.9  57.7  67.9  47.2  70.7
## 1896  29.0  57.4  52.0  43.8  27.7  49.0  45.0  27.2  61.3  28.4  38.0  42.6
## 1897  40.6  29.4  29.1  31.0  20.0  11.3  27.6  21.8  48.1  14.3   8.4  33.3
## 1898  30.2  36.4  38.3  14.5  25.8  22.3   9.0  31.4  34.8  34.4  30.9  12.6
## 1899  19.5   9.2  18.1  14.2   7.7  20.5  13.5   2.9   8.4  13.0   7.8  10.5
## 1900   9.4  13.6   8.6  16.0  15.2  12.1   8.3   4.3   8.3  12.9   4.5   0.3
## 1901   0.2   2.4   4.5   0.0  10.2   5.8   0.7   1.0   0.6   3.7   3.8   0.0
## 1902   5.2   0.0  12.4   0.0   2.8   1.4   0.9   2.3   7.6  16.3  10.3   1.1
## 1903   8.3  17.0  13.5  26.1  14.6  16.3  27.9  28.8  11.1  38.9  44.5  45.6
## 1904  31.6  24.5  37.2  43.0  39.5  41.9  50.6  58.2  30.1  54.2  38.0  54.6
## 1905  54.8  85.8  56.5  39.3  48.0  49.0  73.0  58.8  55.0  78.7 107.2  55.5
## 1906  45.5  31.3  64.5  55.3  57.7  63.2 103.6  47.7  56.1  17.8  38.9  64.7
## 1907  76.4 108.2  60.7  52.6  42.9  40.4  49.7  54.3  85.0  65.4  61.5  47.3
## 1908  39.2  33.9  28.7  57.6  40.8  48.1  39.5  90.5  86.9  32.3  45.5  39.5
## 1909  56.7  46.6  66.3  32.3  36.0  22.6  35.8  23.1  38.8  58.4  55.8  54.2
## 1910  26.4  31.5  21.4   8.4  22.2  12.3  14.1  11.5  26.2  38.3   4.9   5.8
## 1911   3.4   9.0   7.8  16.5   9.0   2.2   3.5   4.0   4.0   2.6   4.2   2.2
## 1912   0.3   0.0   4.9   4.5   4.4   4.1   3.0   0.3   9.5   4.6   1.1   6.4
## 1913   2.3   2.9   0.5   0.9   0.0   0.0   1.7   0.2   1.2   3.1   0.7   3.8
## 1914   2.8   2.6   3.1  17.3   5.2  11.4   5.4   7.7  12.7   8.2  16.4  22.3
## 1915  23.0  42.3  38.8  41.3  33.0  68.8  71.6  69.6  49.5  53.5  42.5  34.5
## 1916  45.3  55.4  67.0  71.8  74.5  67.7  53.5  35.2  45.1  50.7  65.6  53.0
## 1917  74.7  71.9  94.8  74.7 114.1 114.9 119.8 154.5 129.4  72.2  96.4 129.3
## 1918  96.0  65.3  72.2  80.5  76.7  59.4 107.6 101.7  79.9  85.0  83.4  59.2
## 1919  48.1  79.5  66.5  51.8  88.1 111.2  64.7  69.0  54.7  52.8  42.0  34.9
```

```
## 1920  51.1  53.9  70.2  14.8  33.3  38.7  27.5  19.2  36.3  49.6  27.2  29.9
## 1921  31.5  28.3  26.7  32.4  22.2  33.7  41.9  22.8  17.8  18.2  17.8  20.3
## 1922  11.8  26.4  54.7  11.0   8.0   5.8  10.9   6.5   4.7   6.2   7.4  17.5
## 1923   4.5   1.5   3.3   6.1   3.2   9.1   3.5   0.5  13.2  11.6  10.0   2.8
## 1924   0.5   5.1   1.8  11.3  20.8  24.0  28.1  19.3  25.1  25.6  22.5  16.5
## 1925   5.5  23.2  18.0  31.7  42.8  47.5  38.5  37.9  60.2  69.2  58.6  98.6
## 1926  71.8  70.0  62.5  38.5  64.3  73.5  52.3  61.6  60.8  71.5  60.5  79.4
## 1927  81.6  93.0  69.6  93.5  79.1  59.1  54.9  53.8  68.4  63.1  67.2  45.2
## 1928  83.5  73.5  85.4  80.6  76.9  91.4  98.0  83.8  89.7  61.4  50.3  59.0
## 1929  68.9  64.1  50.2  52.8  58.2  71.9  70.2  65.8  34.4  54.0  81.1 108.0
## 1930  65.3  49.2  35.0  38.2  36.8  28.8  21.9  24.9  32.1  34.4  35.6  25.8
## 1931  14.6  43.1  30.0  31.2  24.6  15.3  17.4  13.0  19.0  10.0  18.7  17.8
## 1932  12.1  10.6  11.2  11.2  17.9  22.2   9.6   6.8   4.0   8.9   8.2  11.0
## 1933  12.3  22.2  10.1   2.9   3.2   5.2   2.8   0.2   5.1   3.0   0.6   0.3
## 1934   3.4   7.8   4.3  11.3  19.7   6.7   9.3   8.3   4.0   5.7   8.7  15.4
## 1935  18.9  20.5  23.1  12.2  27.3  45.7  33.9  30.1  42.1  53.2  64.2  61.5
## 1936  62.8  74.3  77.1  74.9  54.6  70.0  52.3  87.0  76.0  89.0 115.4 123.4
## 1937 132.5 128.5  83.9 109.3 116.7 130.3 145.1 137.7 100.7 124.9  74.4  88.8
## 1938  98.4 119.2  86.5 101.0 127.4  97.5 165.3 115.7  89.6  99.1 122.2  92.7
## 1939  80.3  77.4  64.6 109.1 118.3 101.0  97.6 105.8 112.6  88.1  68.1  42.1
## 1940  50.5  59.4  83.3  60.7  54.4  83.9  67.5 105.5  66.5  55.0  58.4  68.3
## 1941  45.6  44.5  46.4  32.8  29.5  59.8  66.9  60.0  65.9  46.3  38.3  33.7
## 1942  35.6  52.8  54.2  60.7  25.0  11.4  17.7  20.2  17.2  19.2  30.7  22.5
## 1943  12.4  28.9  27.4  26.1  14.1   7.6  13.2  19.4  10.0   7.8  10.2  18.8
## 1944   3.7   0.5  11.0   0.3   2.5   5.0   5.0  16.7  14.3  16.9  10.8  28.4
## 1945  18.5  12.7  21.5  32.0  30.6  36.2  42.6  25.9  34.9  68.8  46.0  27.4
## 1946  47.6  86.2  76.6  75.7  84.9  73.5 116.2 107.2  94.4 102.3 123.8 121.7
## 1947 115.7 113.4 129.8 149.8 201.3 163.9 157.9 188.8 169.4 163.6 128.0 116.5
## 1948 108.5  86.1  94.8 189.7 174.0 167.8 142.2 157.9 143.3 136.3  95.8 138.0
## 1949 119.1 182.3 157.5 147.0 106.2 121.7 125.8 123.8 145.3 131.6 143.5 117.6
## 1950 101.6  94.8 109.7 113.4 106.2  83.6  91.0  85.2  51.3  61.4  54.8  54.1
## 1951  59.9  59.9  59.9  92.9 108.5 100.6  61.5  61.0  83.1  51.6  52.4  45.8
## 1952  40.7  22.7  22.0  29.1  23.4  36.4  39.3  54.9  28.2  23.8  22.1  34.3
## 1953  26.5   3.9  10.0  27.8  12.5  21.8   8.6  23.5  19.3   8.2   1.6   2.5
## 1954   0.2   0.5  10.9   1.8   0.8   0.2   4.8   8.4   1.5   7.0   9.2   7.6
## 1955  23.1  20.8   4.9  11.3  28.9  31.7  26.7  40.7  42.7  58.5  89.2  76.9
## 1956  73.6 124.0 118.4 110.7 136.6 116.6 129.1 169.6 173.2 155.3 201.3 192.1
## 1957 165.0 130.2 157.4 175.2 164.6 200.7 187.2 158.0 235.8 253.8 210.9 239.4
## 1958 202.5 164.9 190.7 196.0 175.3 171.5 191.4 200.2 201.2 181.5 152.3 187.6
## 1959 217.4 143.1 185.7 163.3 172.0 168.7 149.6 199.6 145.2 111.4 124.0 125.0
## 1960 146.3 106.0 102.2 122.0 119.6 110.2 121.7 134.1 127.2  82.8  89.6  85.6
## 1961  57.9  46.1  53.0  61.4  51.0  77.4  70.2  55.9  63.6  37.7  32.6  40.0
## 1962  38.7  50.3  45.6  46.4  43.7  42.0  21.8  21.8  51.3  39.5  26.9  23.2
## 1963  19.8  24.4  17.1  29.3  43.0  35.9  19.6  33.2  38.8  35.3  23.4  14.9
## 1964  15.3  17.7  16.5   8.6   9.5   9.1   3.1   9.3   4.7   6.1   7.4  15.1
## 1965  17.5  14.2  11.7   6.8  24.1  15.9  11.9   8.9  16.8  20.1  15.8  17.0
## 1966  28.2  24.4  25.3  48.7  45.3  47.7  56.7  51.2  50.2  57.2  57.2  70.4
## 1967 110.9  93.6 111.8  69.5  86.5  67.3  91.5 107.2  76.8  88.2  94.3 126.4
## 1968 121.8 111.9  92.2  81.2 127.2 110.3  96.1 109.3 117.2 107.7  86.0 109.8
## 1969 104.4 120.5 135.8 106.8 120.0 106.0  96.8  98.0  91.3  95.7  93.5  97.9
## 1970 111.5 127.8 102.9 109.5 127.5 106.8 112.5  93.0  99.5  86.6  95.2  83.5
## 1971  91.3  79.0  60.7  71.8  57.5  49.8  81.0  61.4  50.2  51.7  63.2  82.2
## 1972  61.5  88.4  80.1  63.2  80.5  88.0  76.5  76.8  64.0  61.3  41.6  45.3
## 1973  43.4  42.9  46.0  57.7  42.4  39.5  23.1  25.6  59.3  30.7  23.9  23.3
```

```
## 1974  27.6  26.0  21.3  40.3  39.5  36.0  55.8  33.6  40.2  47.1  25.0  20.5
## 1975  18.9  11.5  11.5   5.1   9.0  11.4  28.2  39.7  13.9   9.1  19.4   7.8
## 1976   8.1   4.3  21.9  18.8  12.4  12.2   1.9  16.4  13.5  20.6   5.2  15.3
## 1977  16.4  23.1   8.7  12.9  18.6  38.5  21.4  30.1  44.0  43.8  29.1  43.2
## 1978  51.9  93.6  76.5  99.7  82.7  95.1  70.4  58.1 138.2 125.1  97.9 122.7
## 1979 166.6 137.5 138.0 101.5 134.4 149.5 159.4 142.2 188.4 186.2 183.3 176.3
## 1980 159.6 155.0 126.2 164.1 179.9 157.3 136.3 135.4 155.0 164.7 147.9 174.4
## 1981 114.0 141.3 135.5 156.4 127.5  90.0 143.8 158.7 167.3 162.4 137.5 150.1
## 1982 111.2 163.6 153.8 122.0  82.2 110.4 106.1 107.6 118.8  94.7  98.1 127.0
## 1983  84.3  51.0  66.5  80.7  99.2  91.1  82.2  71.8  50.3  55.8  33.3  33.4
```

```r
# A set of linearly independent vectors that generate or span a space is said
# to be a basis for the space.
# •
# The representation of a given vector in terms of a basis set is unique.
# To see this, let {v 1 , . . . , v k } be a basis for a vector space that
# includes the
# vector x, and let
vector(mode = "logical", length = 0L)
```

```
## logical(0)
```

```r
# Since {v 1 , . . . , v k } are independent, the only way this is possible
# is if c i = b i
# for each i.
# A related fact is that if {v 1 , . . . , v k } is a basis for a vector space
# of order
# n that includes the vector x and x = c 1 v 1 + · · · c k v k , then x = 0 n
# if and only
# if c i = 0 for each i.
i = 0
n = 1
for (i in 0:999) {
   i + 1
   c(i);
}
i
```

```
## [1] 999
```

```r
# If B 1 is a basis set for V 1 , B 2 is a basis set for V 2 , and
# V 1  V 2 = V, then B 1  B 2 is a generating set for V because from the
# definition of  we see that
# any vector in V can be represented as a linear combination of vectors in B 1
# plus a linear combination of vectors in B 2 .
v1 <- vector(mode = "logical", length = 0L)
v1
```

```
## logical(0)
```

```r
# the number of vectors in a basis set is exactly the same as the dimension of
# the vector space; that is, if B is a basis set of the vector space V, then
# dim(V) = #(B).
typeof(v1)
```

```
## [1] "logical"
```

```r
# A generating set or spanning set of a cone C is a set of vectors S
# = {v i }
# a i v i ,
# such that for any vector v in
# C there exists scalars a i  0 so that v =
# b i v i = 0, then b i = 0 for all i. If a generating
# and if for scalars b i  0 and
# set of a cone has a finite number of elements, the cone is a polyhedron. A
# generating set consisting of the minimum number of vectors of any generating
# set for that cone is a basis set for the cone.
scale(v1, center = TRUE, scale = TRUE)
```

```
##      [,1]
## attr(,"scaled:center")
## [1] NaN
## attr(,"scaled:scale")
## [1] 0
```

```r
# 2.1.4 Inner Products
# A useful operation on two vectors x and y of the same order is the dot
# product,
# which we denote by x, y and define as
validObject(object = v1, test = FALSE, complete = FALSE)
```

```
## [1] TRUE
```

```r
# The dot product is also called the inner product or the scalar product. The
# dot product is actually a special type of inner product, but it is the most
# commonly used inner product, and so we will use the terms synonymously. A
# vector space together with an inner product is called an inner product space.
scale.default(v1, center = TRUE, scale = TRUE)
```

```
##      [,1]
## attr(,"scaled:center")
## [1] NaN
## attr(,"scaled:scale")
## [1] 0
```

```r
# The dot product is also sometimes written as x · y, hence the name. Yet
# another notation for the dot product is x T y, and we will see later that
# this
# notation is natural in the context of matrix multiplication. We have the
# equiv-
# alent notations
equiv <- c(5, 10, 6, 30, 8, 35, 15, 45, 5)
equiv
```

```
## [1]  5 10  6 30  8 35 15 45  5
```

```r
# The dot product is a mapping from a vector space V to IR that has the
# following properties:
#  1. Nonnegativity and mapping of the identity:
#  if x = 0, then x, x > 0 and 0, x = x, 0 = 0, 0 = 0.
IR <- call("aquiv", "v1")
IR
```

```
## aquiv("v1")
```

```r
# 2. Commutativity:
# x, y = y, x .
list("x", "y")
```

```
## [[1]]
## [1] "x"
##
## [[2]]
## [1] "y"
```

```r
# 3. Factoring of scalar multiplication in dot products:
# ax, y = a x, y for real a.
ax <- factorial(equiv)
ax
```

```
## [1] 1.200000e+02 3.628800e+06 7.200000e+02 2.652529e+32 4.032000e+04
## [6] 1.033315e+40 1.307674e+12 1.196222e+56 1.200000e+02
```

```r
# 4. Relation of vector addition to addition of dot products:
# x + y, z = x, z + y, z .
relimp::R.to.Tcl(character.vector = ax)
```

```
## [1] "{120} {3628800} {720} {2.65252859812191e+32} {40320} {1.03331479663861e+40} {1307674368000} {1.
```