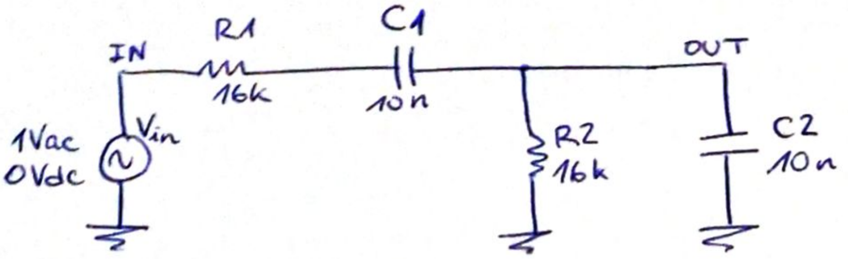
LAB FISE. PRÀCTICA 5 (ESTUDI PREVI)

1. Anàlisi del filtre passabanda passiu RC



EP1. $H(s)$?

$$H(s) = \frac{V_{out}}{V_{in}} \qquad \frac{V_{out} - V_{in}}{R + \frac{1}{Cs}} + \frac{V_{out}}{\frac{\frac{R}{Cs}}{R + \frac{1}{Cs}}} = 0$$

$$(V_{out} - V_{in})\frac{R}{Cs} + V_{out}\left(R + \frac{1}{Cs}\right)^2 = 0$$

$$V_{out}\left(\frac{R}{Cs} + \left(R + \frac{1}{Cs}\right)^2\right) = V_{in}\frac{R}{Cs}$$

$$H(s) = \frac{\frac{R}{Cs}}{\frac{R}{Cs} + \left(R + \frac{1}{Cs}\right)^2} = \frac{\frac{R}{Cs}}{\frac{R}{Cs} + \left(\frac{RCs+1}{Cs}\right)^2} = \frac{R}{R + \frac{(RCs+1)^2}{Cs}} =$$

$$= \frac{R}{R + \frac{R^2C^2s^2 + 2RCs + 1}{Cs}} = \frac{R \cdot Cs}{R^2C^2s^2 + 3RCs + 1} = \frac{RCs}{R^2C^2\left(s^2 + \frac{3s}{RC} + \frac{1}{R^2C^2}\right)} =$$

$$= \frac{1}{RC} \cdot \frac{s}{s^2 + \frac{3}{RC}s + \frac{1}{(RC)^2}}$$

$$H(s) = \frac{A\omega_0 s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

A constant
Q factor de qualitat
ω_0 freqüència angular central

EP2. A, ω_0, Q?

$$A\omega_0 = \frac{1}{RC} \Rightarrow A = \frac{1}{RC} \cdot \frac{1}{\omega_0} = \frac{1}{RC} \cdot RC \Rightarrow \boxed{A = 1}$$

$$\omega_0^2 = \left(\frac{1}{RC}\right)^2 \Rightarrow \boxed{\omega_0 = \frac{1}{RC}}$$

$$\frac{\omega_0}{Q} = \frac{3}{RC} \Rightarrow Q = \omega_0 \cdot \frac{RC}{3} = \frac{1}{RC} \cdot \frac{RC}{3} \Rightarrow \boxed{Q = \frac{1}{3}}$$

EP3. pols? $|H(j\omega_0)|$?

$$s^2 + \frac{\omega_0}{Q}s + \omega_0^2 = 0$$

$$s = \frac{-\frac{\omega_0}{Q} \pm \sqrt{\frac{\omega_0^2}{Q^2} - 4\omega_0^2}}{2} = \frac{-\frac{3}{RC} \pm \sqrt{\frac{9}{R^2C^2} - \frac{4}{R^2C^2}}}{2} =$$

$$= \frac{-\frac{3}{RC} \pm \frac{1}{RC}\sqrt{5}}{2} = \left.\frac{-3 \pm \sqrt{5}}{2RC}\right]$$ pols del circuit.

$$H(j\omega_0) = \frac{1}{RC} \cdot \frac{j/RC}{\left(\frac{j}{RC}\right)^2 + \frac{3}{RC}\cdot\frac{j}{RC} + \frac{1}{(RC)^2}} = \frac{1}{RC} \cdot \frac{j/RC}{-\frac{1}{(RC)^2} + \frac{3j}{(RC)^2} + \frac{1}{(RC)^2}} =$$

$$= \frac{j}{-1+3j+1} = \frac{j}{3j} = \frac{1}{3}$$

$$\boxed{|H(j\omega_0)| = \frac{1}{3}}$$

EP4. $R = 16k\Omega$, $C = 10nF$. A, Q, f_0 ?

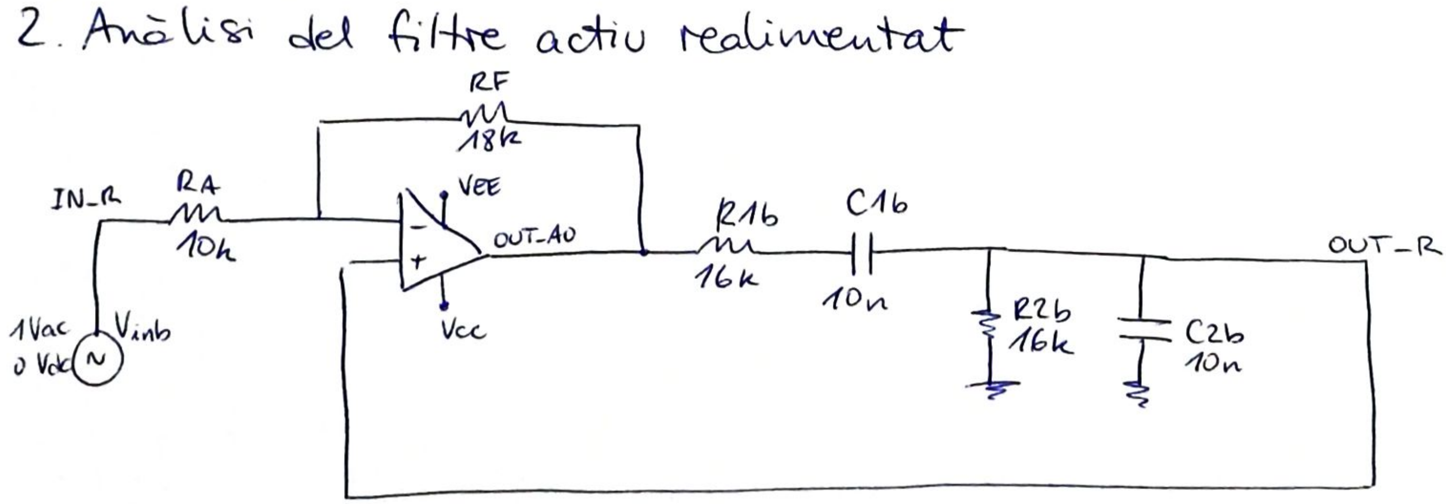
$\boxed{A = 1}$ $\boxed{Q = \frac{1}{3}}$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi RC} = \frac{1}{2\pi \cdot 16k \cdot 10n} = \frac{1}{2\pi \cdot 16 \cdot 10^3 \cdot 10 \cdot 10^{-9}}$$

$$\boxed{f_0 = 994{,}72\ Hz}$$

$(\omega_0 = 6250)$

2. Anàlisi del filtre actiu realimentat

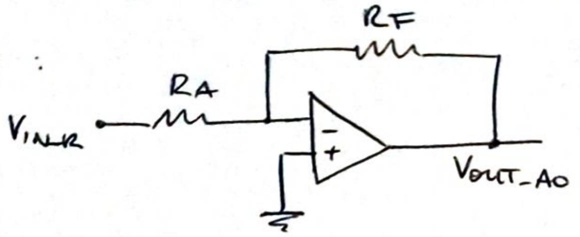


EP5. $\alpha = V_{out-AO} / V_{in-R}$ (quan $V_{OUT-R} = 0$)

$-f_R = V_{OUT-AO} / V_{OUT-R}$ (quan $V_{IN-R} = 0$)

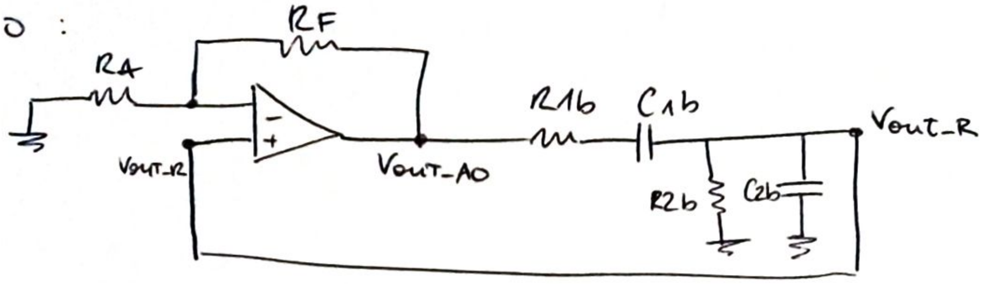
En funció de RA i RF!

$V_{OUT-R} = 0$:



$$\boxed{\alpha = -\frac{RF}{RA}}$$

$V_{IN-R} = 0$:



$$\frac{V_{OUT-R} - 0}{RA} + \frac{V_{OUT-R} - V_{OUT-AO}}{R_F} = 0$$

$$R_F \cdot V_{OUT-R} + V_{OUT-R} \cdot RA - V_{OUT-AO} \cdot RA = 0$$

$$V_{OUT-R}\,(RF + RA) = V_{OUT-AO} \cdot RA$$

$$\boxed{-f_R = \frac{RF + RA}{RA} = 1 + \frac{RF}{RA}}$$

EP6. Guany de llaç $T(s)$?
Tipus de realimentació ?

$$T(s) = H(s) \cdot f_R = -\frac{1}{RC} \cdot \frac{s}{s^2 + \frac{3}{RC}s + \frac{1}{(RC)^2}} \cdot \left(1 + \frac{RF}{RA}\right)$$

$$K = -\frac{1}{RC} \cdot \left(1 + \frac{RF}{RA}\right) \rightarrow \text{real positiva}$$

EP7. funció de transferència en llaç tancat $H_R(s) = \dfrac{V_{out-R}}{V_{in-R}}$?

$$H_R(s) = \frac{\alpha \cdot H(s)}{1 + H(s) \cdot f_R} = \frac{-\frac{RF}{RA} \cdot \frac{1}{RC} \cdot \frac{s}{s^2 + \frac{3}{RC}s + \frac{1}{(RC)^2}}}{1 - \frac{1}{RC} \cdot \frac{s}{s^2 + \frac{3}{RC}s + \frac{1}{(RC)^2}} \cdot \left(1 + \frac{RF}{RA}\right)} =$$

$$= \frac{-\frac{RF}{RA} \cdot \frac{1}{RC}\,s}{s^2 + \frac{3}{RC}s + \frac{1}{(RC)^2} - \frac{1}{RC}s \cdot \left(1 + \frac{RF}{RA}\right)} = \frac{-\frac{RF}{RA} \cdot \frac{1}{RC}\,s}{s^2 + \frac{3}{RC}s + \frac{1}{(RC)^2} - \frac{1}{RC}s - \frac{RF}{RA} \cdot \frac{1}{RC}s} =$$

$$\boxed{H_R(s) = \frac{-\frac{R_F}{R_A}\cdot\frac{1}{RC}s}{s^2+\left(\frac{2}{RC}-\frac{R_F}{R_A}\cdot\frac{1}{RC}\right)s+\frac{1}{(RC)^2}} = \frac{-\frac{R_F}{R_A}\cdot\frac{1}{RC}s}{s^2+\frac{2R_A-R_F}{RCR_A}s+\frac{1}{(RC)^2}}}$$

EP8. A, ω_0, Q ?

$$A\omega_0 = -\frac{R_F}{R_A}\cdot\frac{1}{RC} \Rightarrow \boxed{A = -\frac{R_F}{R_A}}$$

$$\omega_0^2 = \frac{1}{(RC)^2} \Rightarrow \boxed{\omega_0 = \frac{1}{RC}}$$

$$\frac{\omega_0}{Q} = \frac{2R_A-R_F}{RC\cdot R_A} \Rightarrow \boxed{Q = \frac{R_A}{2R_A-R_F}}$$

EP9. f_R per tal d'aconseguir $Q=5$ i $Q=10$?
$R_A = 10k\Omega$, R_F per que $Q=5$ i $Q=10$?

$$f_R = -\left(1+\frac{R_F}{R_A}\right)$$

$\boxed{Q=5}$:

$$Q = \frac{R_A}{2R_A-R_F} \Rightarrow 5 = \frac{10k}{2\cdot 10k - R_F} \Leftrightarrow 20k - R_F = 2k$$

$$\Leftrightarrow \boxed{R_F = 18k\Omega}$$

$$f_R = -\left(1+\frac{18}{10}\right) \Leftrightarrow \boxed{f_R = -2,8}$$

$\boxed{Q=10}$:

$$10 = \frac{10k}{2\cdot 10k - R_F} \Leftrightarrow 20k - R_F = 1k \Leftrightarrow \boxed{R_F = 19k\Omega}$$

$$f_R = -\left(1+\frac{19}{10}\right) \Leftrightarrow \boxed{f_R = -2,9}$$

EP10. pols complexes conjugats ? $|H_R(j\omega_0)|$? (per $Q=5$, $Q=10$)

$\boxed{Q=5}$:

$$s^2+\frac{\omega_0}{Q}s+\omega_0^2 = 0 \Leftrightarrow s = \frac{-\frac{\omega_0}{Q}\pm\sqrt{\frac{\omega_0^2}{Q^2}-4\omega_0^2}}{2}$$

$$s = \frac{-\frac{6250}{5}\pm\sqrt{\frac{6250^2}{5^2}-4\cdot 6250^2}}{2} = \frac{-1250\pm j\cdot 12437,34}{2} = \boxed{-625 \pm j\cdot 6218,7}$$

pols

$$H_R(s) = \frac{A\omega_0 s}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

$$H_R(j\omega_o) = \frac{A\omega_o \cdot j\omega_o}{j^2\omega_o^2 + \frac{\omega_o}{Q} j\omega_o + \omega_o^2} = \frac{Aj}{j^2 + j\frac{1}{Q} + 1} = \frac{Aj}{-1 + \frac{j}{Q} + 1} = A \cdot Q =$$

$$= -\frac{R_F}{R_A} \cdot Q = -\frac{18}{10} \cdot 5 = \boxed{-9}$$

$$\boxed{|H_R(j\omega_o)| = 9}$$

$\boxed{Q = 10}$:

$$s = \frac{-\frac{6250}{10} \pm \sqrt{\frac{6250^2}{10^2} - 4 \cdot 6250^2}}{2} = \boxed{-312{,}5 \pm j \cdot 6242{,}2} \text{ pols}$$

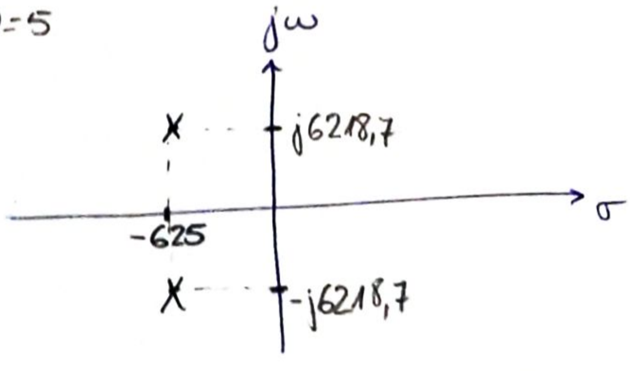
$$H_R(j\omega_o) = A \cdot Q = -\frac{R_F}{R_A} \cdot Q = \frac{-19}{10} \cdot 10 = -19$$

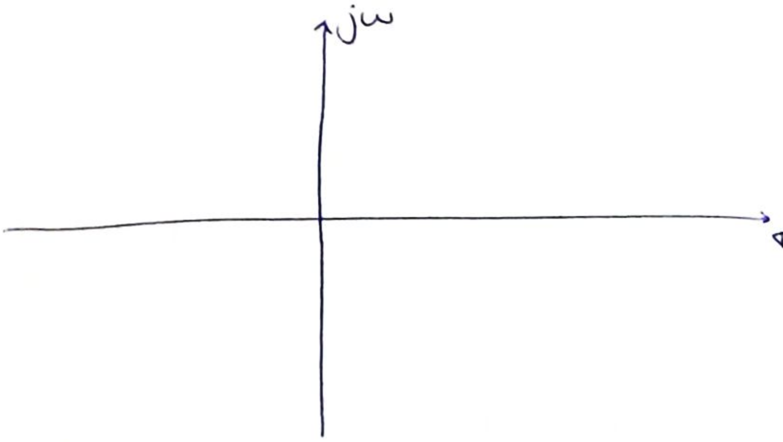
$$\boxed{|H_R(j\omega_o)| = 19}$$

3. Estabilitat de l'amplificador realimentat. Condició d'oscil·lació

EP11. Dibuixar Lloc Geomètric d'Arrels (LGA) atenent a la variació de R_F i situar sobre ell la posició dels pols per als casos del filtre passiu i el filtre actiu amb $Q=5$ i $Q=10$.

Q=5





EP12. f_R que fa el circuit estable?
$R_A = 10k\Omega$, R_F perquè el circuit oscil·li?

$$Re(\text{pols}) < 0 \Rightarrow \frac{-\omega_0}{2Q} < 0 \Rightarrow Q > 0$$ (condició d'estabilitat)

$$Q = \frac{R_A}{2R_A - R_F} > 0 \Rightarrow \frac{10k}{2\cdot 10k - R_F} > 0$$

$$f_R = -\left(1 + \frac{R_F}{R_A}\right) = -\left(1 + \frac{R_F}{10k}\right) \Leftrightarrow$$

$$-\frac{R_F}{10k} = f_R + 1 \Leftrightarrow -R_F = 10k\cdot f_R + 10k$$

$$Q = \frac{10k}{20k + 10k f_R + 10k} = \frac{1}{3 + f_R} > 0$$

$$\Leftrightarrow 3 + f_R > 0 \Leftrightarrow \boxed{f_R > -3}$$

$$Re(\text{pols}) = 0 \Rightarrow \frac{-\omega_0}{2Q} = 0$$

$$Q = \frac{R_A}{2R_A - R_F}$$

$$\frac{-\omega_0}{2R_A}\cdot(2R_A - R_F) = 0$$

$$-\omega_0 + \frac{\omega_0 R_F}{2R_A} = 0$$

$$\omega_0\left(-1 + \frac{R_F}{2R_A}\right) = 0$$

Si $\omega_0 \neq 0$:

$$\frac{R_F}{2R_A} - 1 = 0 \Leftrightarrow \frac{R_F}{2\cdot 10k} = 1 \Leftrightarrow \boxed{R_F = 20k\Omega}$$