We show that the scalar $1'U\alpha = 0$. Assumptions:

$$
\begin{aligned}
1'W &= 0, \\
G &= WW' = UDU', \\
U'U &= UU' = I.
\end{aligned}
$$

Then

$$
\begin{aligned}
1'WW' &= 0, \\
1'UDU' &= 0, \\
1'UD &= 0.
\end{aligned}
$$

Write

$$
\begin{aligned}
1'UD &= 1'\left[U_1 U_2 \ldots U_n\right] D \\
&= \left[m_1, m_2, \ldots, m_n\right] D \\
&= \left[m_1\lambda_1, m_2\lambda_2, \ldots, m_n\lambda_n\right] \\
&= 0, \qquad (1)
\end{aligned}
$$

where $m_i\lambda_i$ are scalars. The implication of (1) is that $m_1 = m_2 = \ldots = m_{n-1} = 0$, because D is a diagonal matrix of positive eigenvalues, except the last one which is equal to zero; that is $\lambda_n = 0$.

The vector α has elements $\alpha_i \sim N\left(0, \lambda_i \sigma_b^2\right)$, $i = 1, 2, \ldots, n-1$, with $\alpha_n \sim N\left(0, \lambda_n \sigma_b^2\right)$, $\lambda_n = 0$. Therefore $\alpha_n = 0$ a posteriori (with probability 1), and

$$
\begin{aligned}
1'U\alpha &= \left[m_1, m_2, \ldots, m_n\right]\left[\alpha_1, \alpha_2, \ldots, \alpha_n\right]' \\
&= \left[0, 0, \ldots, m_n\right]\left[\alpha_1, \alpha_2, \ldots, 0\right]' = 0.
\end{aligned}
$$