

$$\int \frac{\cos x dx}{1 + \sqrt{\sin x}}$$

נגדיר $\sin x = t^2$ ובהתאם $\cos x dx = 2t dt$.

$$\begin{aligned} \int \frac{\cos x dx}{1 + \sqrt{\sin x}} &= \int \frac{2t dt}{1 + t} \\ &= 2 \int 1 - \frac{1}{1 + t} dt \\ &= 2t - 2 \ln |1 + t| + C \\ &= 2\sqrt{\sin x} - 2 \ln |1 + \sqrt{\sin x}| + C \end{aligned}$$

נגזור את הביטוי כדי לוודא את עצמנו

$$\begin{aligned} \left(2\sqrt{\sin x} - 2 \ln |1 + \sqrt{\sin x}| \right)' &= \left(2\sqrt{\sin x} - 2 \ln |1 + \sqrt{\sin x}| \right)' \\ &= 2 \frac{\cos x}{2\sqrt{\sin x}} - 2 \frac{\frac{\cos x}{2\sqrt{\sin x}}}{1 + \sqrt{\sin x}} \\ &= \frac{\cos x}{\sqrt{\sin x}} - \frac{\frac{\cos x}{\sqrt{\sin x}}}{1 + \sqrt{\sin x}} \\ &= \frac{\cos x(\sin x + \sqrt{\sin x}) - \cos x \sqrt{\sin x}}{\sin x \sqrt{\sin x} + \sin x} \\ &= \frac{\cos x \cancel{\sin x}}{\cancel{\sin x} \sqrt{\sin x} + \cancel{\sin x}} \\ &= \frac{\cos x}{\sqrt{\sin x} + 1} \end{aligned}$$