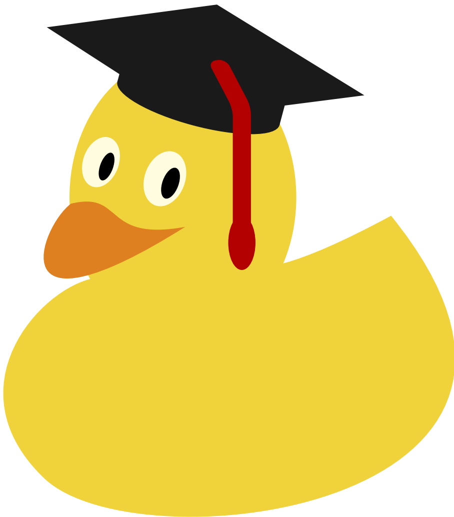
פתרון ממ"ן 12 – חשבון אינפיניטסימלי 2 (20475)

18 ביולי 2023



שאלה 1

נחשב את האינטגרלים הבאים

סעיף א'

$$
\begin{aligned}
\int x^3(1-3x^2)^{10}dx &= \int x^3\left(\sum_{k=0}^{10}\binom{10}{k}(-3x^2)^k\right)dx \\
&= \int\left(\sum_{k=0}^{10}\binom{10}{k}(-3)^k x^{2k+3}\right)dx \\
&= C+\sum_{k=0}^{10}\binom{10}{k}\frac{1}{2k+4}(-3)^k x^{2k+4} \\
&= C+x^4\sum_{k=0}^{10}\binom{10}{k}\frac{(-3)^k}{2k+4}x^{2k}
\end{aligned}
$$

סעיף ב'

$$
\begin{aligned}
\int \frac{xe^{\arcsin x}}{\sqrt{1-x^2}}dx &= \int \frac{\sin\arcsin(x)e^{\arcsin x}}{\sqrt{1-x^2}}dx \\
&= \left.\int \sin(t)e^t dt\right|_{dt=\frac{dx}{\sqrt{1-x^2}}}^{t=\arcsin x} \\
&= -\cos(t)e^t+\int\cos(t)e^t dt && \text{אינטגרציה בחלקים} \\
&= -\cos(t)e^t+\sin(x)e^t-\int\sin(t)e^t dt && \text{אינטגרציה בחלקים}
\end{aligned}
$$

$$
\begin{aligned}
&2\int\sin(t)e^t dt=\sin(x)-\cos(t)e^t \\
&\int \frac{xe^{\arcsin x}}{\sqrt{1-x^2}}dx=\frac{1}{2}\left(x-\sqrt{1-x^2}e^{\arcsin x}\right)+C
\end{aligned}
$$

סעיף ג'

$$
\begin{aligned}
\int(x-1)^2e^{2x}dx &= \frac{1}{2}e^{2x}(x-1)^2-\int\frac{1}{2}e^{2x}(2x-2)dx \\
&= \frac{1}{2}e^{2x}(x-1)^2+\int e^{2x}dx-\int e^{2x}xdx \\
&= \frac{1}{2}e^{2x}(x-1)^2+\frac{1}{2}e^{2x}-\frac{1}{2}e^{2x}x+\int\frac{1}{2}e^{2x}dx \\
&= \frac{1}{2}e^{2x}(x^2-3x+2)+\frac{1}{4}e^{2x} \\
&= \frac{1}{2}e^{2x}(x^2-3x+2\frac{1}{2})+C
\end{aligned}
$$

סעיף ד'

$$\int_{-2}^{-1}\frac{4x+1}{\sqrt{(x^2+4x+5)^3}}dx=\left|\begin{matrix}x=t+2\\dx=dt\end{matrix}\right|$$
$$=\int_{-4}^{-3}\frac{4t-7}{\sqrt{(t^2+1)^3}}dt$$
$$=\left|\begin{matrix}t=\tan u\\dt=\frac{du}{\cos^2 u}\end{matrix}\right|$$
$$=\int_{\arctan -4}^{\arctan -3}\frac{4\tan u-7}{\cos^2 u\sqrt{(\tan^2 u+1)^3}}du$$
$$=\int_{\arctan -4}^{\arctan -3}\frac{4\tan u-7}{\cos^2 u\sqrt{(\frac{1}{\cos^2 u})^3}}du$$
$$=\int_{\arctan -4}^{\arctan -3}\frac{4\tan u-7}{\cos^2 u\cos^{-3} u}du$$
$$=\int_{\arctan -4}^{\arctan -3}(4\tan u-7)\cos u\,du$$
$$=\int_{\arctan -4}^{\arctan -3}4\sin u-7\cos u\,du$$
$$=-4\cos u-7\sin u|_{\arctan -4}^{\arctan -3}$$
$$=\frac{-4x}{\sqrt{x^2+1}}-\frac{7}{\sqrt{x^2+1}}\Big|_{-4}^{-3}$$
$$=\frac{-4(-3)}{\sqrt{(-3)^2+1}}-\frac{7}{\sqrt{(-3)^2+1}}+\frac{4(-4)}{\sqrt{(-4)^2+1}}+\frac{7}{\sqrt{(-4)^2+1}}$$

סעיף ה'

$$\int_{-\ln 2}^{\ln 2}e^{|x|}(1+x^3+\sin x)dx=\int_{-\ln 2}^{\ln 2}e^{|x|}(x^3+\sin x)dx+\int_{-\ln 2}^{\ln 2}e^{|x|}dx$$

נגדיר

$$f(x)=e^{|x|}(1+\sin x)$$

מחישוב עולה כי

$$f(-x)=e^{|-x|}(-x^3-\sin x)=-f(x)$$

ולכן

$$\int_{-\ln 2}^{\ln 2}f(x)dx=\int_{-\ln 2}^{0}f(x)dx+\int_{0}^{\ln 2}f(x)dx=-\int_{0}^{\ln 2}f(x)dx+\int_{0}^{\ln 2}f(x)dx=0$$

עוד נראה כי

$$\int_{-\ln 2}^{\ln 2}e^{|x|}dx=\int_{-\ln 2}^{0}e^{|x|}dx+\int_{0}^{\ln 2}e^{|x|}dx=\int_{-\ln 2}^{0}e^{-x}dx+\int_{0}^{\ln 2}e^{x}dx=2\int_{0}^{\ln 2}e^{x}dx=2(e^{\ln 2}-1)=4$$