Template for \LaTeX proof

Prove by induction that the following statement is true for all positive integers.

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \tag{1}$$

Proof.

Base step – Show that eq. (19) holds for the base case, when n = 1, if correct we move on to Inductive step.

$$\sum_{i=1}^{n} i^2 = \frac{1(1+1)(2*1+1)}{6} = \frac{2*(3)}{6} = 1, i^2 = 1^2 = 1 \tag{2}$$

$$\dots \tag{3}$$

Inductive step – First, assume that statement is true for all k.

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6} \tag{4}$$

$$\dots \tag{5}$$

Second, prove that the statement also holds true for $p+1$.

$$\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6} = \frac{2k^3+9k^2+13k+6}{6} \tag{6}$$

$$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + k^2 + 2k + 1 \tag{7}$$

$$= \frac{k(k+1)(2k+1)}{6} + \frac{6k^2+12k+6}{6} \tag{8}$$

$$= \frac{k(k+1)(2k+1)+6k^2+12k+6}{6} = \frac{2k^3+3k^2+k+6k^2+12k+6}{6} \tag{9}$$

$$= \frac{2k^3 + 9k^2 + 13k + 6}{6} \tag{10}$$

$$\ldots \tag{11}$$

Conclusion – And we are done! We have proven by induction that the statement in eq. (19) is indeed true.

□

Prove by induction that the following statement is true for all positive integers.

$$\sum_{j=1}^{n}(2j-1) = n^2 \tag{12}$$

Proof.

Base step – Show that eq. (19) holds for the base case, when n = 1, if correct we move on to Inductive step.

$$\sum_{j=1}^{n}(2j-1) = n^2 = 1^2 = 1, (2 * 1 - 1) \tag{13}$$

$$\ldots \tag{14}$$

Inductive step – First, assume that statement is true for all k.

$$\sum_{j=1}^{k}(2j-1) = k^2 \tag{15}$$

$$\ldots \tag{16}$$

Second, prove that the statement also holds true for $p + 1$.

$$\sum_{j=1}^{k+1}(2j-1) = \sum_{j=1}^{k+1}(k+1)^2 \tag{17}$$

$$\sum_{j=1}^{k+1}(2j-1) = \sum_{j=1}^{k}(2j-1) + (2(k+1)-1) = k^2 + (2k+1) \tag{18}$$

$$= k^2 + 2k + 1 = (k+1)^2 \tag{19}$$

$$\ldots \tag{20}$$

Conclusion – And we are done! We have proven by induction that the statement in eq. (19) is indeed true.

□