

Template for L^AT_EXproof

Prove by induction that the following statement is true for all positive integers.

$$f(n) = x^{n/2} * x^{(n+1)/2} = x^n \quad (1)$$

Proof.

Base step – Show that eq. (8) holds for the base case, when $n = 5$ and $n = 6$, if correct we move on to Inductive step. All numbers are integers and if after division we end up with a remainder it is round down to closest integer.

$$f(5) = x^{5/2} * x^{(5+1)/2} = (\text{floor}5/2)x^2 * x^3 = x^5 \quad (2)$$

$$f(6) = x^{6/2} * x^{(6+1)/2} = x^3 * (\text{floor}7/2)x^3 = x^6 \quad (3)$$

$$\dots \quad (4)$$

Inductive step – First, assume that statement is true for all k .

$$f(k) = x^{k/2} * x^{(k+1)/2} = x^k \quad (5)$$

$$\dots \quad (6)$$

Second, prove that the statement also holds true for $p + 1$.

$$f(k + 1) = x^{(k+1)/2} * x^{((k+1)+1)/2} = x^{k+1} \quad (7)$$

$$f(k + 1) = x^{(k+1)/2} * x^{((k+1)+1)/2} = x^{\frac{k+1+k+1+1}{2}} = x^{\frac{2k+3}{2}} = x^{k+3/2} = \text{floor}(3/2)x^{k+1} \quad (8)$$

$$\dots \quad (9)$$

Conclusion – And we are done! We have proven by induction that the statement in eq. (8) is indeed true. □