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A hidden Markov model of credit quality

Małgorzata W. Korolkiewicz [a,*], Robert J. Elliott [b]

[a] *School of Mathematics and Statistics, University of South Australia, City West Campus, GPO Box 2471, Adelaide SA 5001, Australia*
[b] *Haskayne School of Business, University of Calgary, Canada*

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ABSTRACT

This paper presents a hidden Markov model of credit quality dynamics, and highlights the use of filtering-based estimation methods for models of this kind. We suppose that the Markov chain governing the 'true' credit quality evolution is hidden in 'noisy' or incomplete observations represented by posted credit ratings. Parameters of the model, namely credit transition probabilities, are estimated using the EM algorithm. Filtering methods provide recursive updates of optimal estimates so the model is 'self-calibrating'. The estimation procedure is illustrated with an application to a data set of Standard & Poor's credit ratings.

1. Introduction

The rating of borrowers is a widespread practice aimed at summarizing quantitative and qualitative information about a borrower's credit quality. It is performed by major credit rating agencies such as Moody's and & Poor's, but also internally by banks and other financial institutions. Agency rating information is made public while the details of the rating process remain undisclosed. After the assignment of the initial credit rating, reviews are performed either periodically or based on market events. If warranted, these reviews result in a rating change signifying improvement (upgrade) or deterioration (downgrade) in a borrower's creditworthiness. With the recent Basel II accord, there is renewed interest in efficiency of credit ratings as indicators of credit quality and in models of their dynamics (Basel Committee on Banking Supervision, 2000, 2001, 2005). These dynamics are typically summarized in a transition matrix, where each entry represents a probability

* Corresponding author. Tel.: +61 8 8302 0237; fax: +61 8 8302 0765.
E-mail address: malgorzata.korolkiewicz@unisa.edu.au (M.W. Korolkiewicz).

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of migration from one credit rating class to another. New regulatory requirements provide an added incentive to ensure that rating systems produce estimates of transition probabilities that are up-to-date and accurate.

Given the method of presenting credit quality information adopted by the credit industry, it seems natural to consider a Markov chain representation of credit rating dynamics. The Markov property then implies that the rating process should have no memory of its past behaviour so that predictions made based on the current rating are as good as predictions based on the whole history of ratings. In other words, current credit rating should accurately represent current credit quality. However, non-Markov effects in rating transitions, in the form of 'rating momentum' or 'rating drift' for downgrades, were identified in Lando and Skødeberg (2002). Further evidence that credit ratings do not fully reflect available information can be found in Altman and Kao (1992) and Delianedis and Geske (2003). As pointed out in Löffler (2004a, b), and also in Carey and Hrycay (2001), for major rating agencies these violations of information efficiency could be the result of some of the agencies' rating policies, namely rating through-the-cycle and avoiding rating reversals, employed to promote greater rating stability.

In this paper, we propose a hidden Markov model (HMM) of credit quality dynamics which takes it into account that published credit ratings may not always accurately reflect 'true' creditworthiness. We suppose that the 'true' credit quality evolution can be described by a Markov chain but that we do not observe this Markov chain directly. Rather, it is hidden in noisy or incomplete observations represented by the posted credit ratings. Our model is formulated in discrete time, with a Markov chain of 'true' credit quality observed in martingale noise. The model produces a probability distribution of 'true' credit quality given the observed history of credit ratings as well as estimates of parameters, namely transition probabilities. We obtain not only the estimates of the transition probabilities for the Markov chain of 'true' credit quality, but also the probabilities that a particular rating is observed for each level of 'true' credit quality.

HMMs are considered in Wendin and McNeil (2006), where it is supposed that credit ratings are subject to both observed and unobserved systematic risk. Estimation of parameters for HMMs by standard maximum likelihood (ML) techniques is quite involved (see for example Shumway and Stoffer, 2006) and so in Wendin and McNeil (2006) Bayesian computational techniques are used for model inference. In our case, we avoid the complications of traditional ML estimation by applying hidden Markov filtering and estimation techniques described in Elliott et al. (1995). Filtering methods provide a continual, recursive up-date of optimal estimates so the model is 'self-calibrating' in contrast to the static model fitting of ML.

Another very attractive feature of the filtering approach is that parameters are easily estimated by a filtering version of the EM (expectation maximization) algorithm and reference probability methods that greatly simplify calculations. The EM algorithm has the appealing property that the likelihood function is always improved after each iteration. The filter-based EM procedure retains the well-established statistical properties of the EM algorithm while reducing memory costs and thus allowing faster computation (see Elliott and Krishnamurthy, 1999; Krishnamurthy and Chung, 2006). Our application of these methods to modelling credit quality appears new.

The methodology can be applied to internal rating systems, or to rating data collected by external agencies, such as Moody's and Standard & Poor's. Here we illustrate this methodology by analyzing a data set of Standard & Poor's annual issuer credit ratings extracted from the COMPUSTAT database.

The paper is organized as follows. Section 2 describes the dynamics of the Markov chain and observations. The reference probability and the forward filter for the 'true' credit rating process are also introduced. The section concludes with recursive formulae for updating the parameters of the model. The rating data and the application results are discussed in Section 3.

2. Hidden Markov model

Our goal is to estimate, from published credit ratings, the evolution of 'true' credit quality over time. We take 'true' credit quality to be represented by a Markov chain $\{X_k\}_{k\in\mathbb{N}}$ with state space $S = \{1, 2, \ldots, N\}$, where each state represent a different credit quality category. We suppose that the transition matrix of X is $A=(a_{ji})_{1\leqslant i,j\leqslant N}$. Without loss of generality, we identify the elements of S with the standard unit vectors $\{e_1,e_2,\dots,e_N\}$, $e_i=(0,\dots,0,1,0,\dots,0)'\in\mathbb{R}^N$ and define an (i,j)-th entry of A as $a_{ji}:=\mathrm{P}(X_{k+1}=e_j|X_k=e_i)$. The dynamics of X are then given by $X_{k+1}=AX_k+V_{k+1},\quad k=0,1,\dots,$ where V_{k+1} is a martingale increment with $\mathrm{E}[V_{k+1}|\mathscr{F}_k]=0\in\mathbb{R}^N$ and $\{\mathscr{F}_k\}$ is a complete filtration generated by X.

Suppose we do not observe X directly. Rather, we observe a process Y with the state space $\{f_1,f_2,\dots,f_M\}$, $f_j=(0,\dots,0,1,0,\dots,0)'\in\mathbb{R}^M$. Define a new matrix $C=(c_{ji})$, where $c_{ji}=\mathrm{P}(Y_k=f_j|X_k=e_i),\quad 1\leqslant i\leqslant N,\ 1\leqslant j\leqslant M$. Then $\mathrm{E}[Y_k|X_k]=CX_k$ and if we define $W_k:=Y_k-CX_k$, we have $\mathrm{E}[W_k|\mathscr{G}_{k-1}\vee\{X_k\}]=0\in\mathbb{R}^M$ so W_k is a martingale increment. The dynamics of Y can then be written as $Y_k=CX_k+W_k$. Here $\{\mathscr{G}_k\}$ is the complete filtration generated by both processes X and Y. In our context, the observation process Y is represented by posted credit ratings.

We are in a situation analogous to the HMMs discussed in Chapter 2 of Elliott et al. (1995). The difference here is that we are assuming zero-delay between the observation process Y and the signal process X. In other words, we suppose that the current credit rating contains information about current credit quality, although this information may not be fully revealed due to 'noise' present in the rating system. This is reasonable given that in general, an obligor is assigned a credit rating based on an assessment of their current creditworthiness. Note also that the matrix C has an important interpretation in our model when credit *quality* categories are taken to be the same as credit *rating* categories, i.e. when $N=M$ and $f_i=e_i$ for all i. The elements of C give probabilities of observing a particular credit rating given each credit quality category, so when $C=I$, we have $\mathrm{P}(Y_k=e_i|X_k=e_i)=1$. This implies that $Y_k=X_k$ and, from its definition, $W_k=0$. Hence when $C=I$, a current credit rating correctly reveals current credit quality. Otherwise, state-to-observation probabilities in C serve as indicators of departures from Markov chain dynamics within the rating system.

In summary, our model for the Markov Chain X of 'true' credit quality hidden in martingale noise is as follows:

Hidden Markov Model (HMM). *Under a probability measure* P,

$$X_{k+1}=AX_k+V_{k+1}\quad (\textit{signal equation, ' true' credit quality}), \tag{1}$$
$$Y_k=CX_k+W_k\quad (\textit{observation equation, posted rating}), \tag{2}$$

A and C are matrices of transition probabilities whose entries satisfy $\sum_{j=1}^N a_{ji}=1$, $a_{ji}\geqslant 0$, $\sum_{j=1}^M c_{ji}=1$, $c_{ji}\geqslant 0$.

V_k *and* W_k *are martingale increments satisfying*

$$\mathrm{E}[V_{k+1}|\mathscr{F}_k]=0,\quad Var\,V_k=\mathrm{diag}\,(Ap_{k-1})-A\,\mathrm{diag}\,(p_{k-1})A', \tag{3}$$
$$\mathrm{E}[W_{k+1}|\mathscr{G}_k\vee\{X_{k+1}\}]=0,\quad Var\,W_k=\mathrm{diag}\,(Cp_k)-C\,\mathrm{diag}\,(p_k)C', \tag{4}$$

where $p_k=(p_1,\dots,p_N)'=\mathrm{E}[X_k]$, *with* $p_{k+1}=Ap_k=A^{k+1}p_0\in\mathbb{R}^N$.

Our processes are defined on a probability space $(\Omega,\mathscr{F},\mathrm{P})$. However, following Elliott et al. (1995), the analysis takes place under a probability measure $\bar{\mathrm{P}}$ on $(\Omega,\mathscr{F})$ such that under $\bar{\mathrm{P}}$, $\{Y_k\}$ is a sequence of i.i.d. uniform variables. Further, under $\bar{\mathrm{P}}$, X is a Markov chain independent of Y, with transition matrix $A=(a_{ji})$. Suppose $C=(c_{ji}),\quad 1\leqslant i\leqslant N,\ 1\leqslant j\leqslant M$, is a matrix with $c_{ji}\geqslant 0$, and $\sum_{j=1}^M c_{ji}=1$. Define $\bar{\lambda}_l=M\sum_{j=1}^M\langle CX_l,f_j\rangle\langle Y_l,f_j\rangle$ and $\bar{\Lambda}_k=\prod_{l=1}^k\bar{\lambda}_l$. The probability measure P is defined by putting $\mathrm{dP}/\mathrm{d}\bar{\mathrm{P}}|_{\mathscr{G}_k}=\bar{\Lambda}_k$. It can be shown, as in Elliott et al. (1995) that under P, we still have $X_{k+1}=AX_k+V_{k+1}$ but $Y_k=CX_k+W_k$. Hence under P, the observation process Y is given by 'real world' dynamics. However, it is mathematically easier to work under measure $\bar{\mathrm{P}}$. The measure change thus described is fundamental in facilitating easy derivation of all filters required in estimating the model.

Suppose we observe $Y_0,\dots,Y_k$, and we wish to estimate $X_0,\dots,X_k$. The best (mean-square) estimate of X_k given $\mathscr{Y}_k=\sigma\{Y_0,\dots,Y_k\}$ is $\mathrm{E}[X_k|\mathscr{Y}_k]\in\mathbb{R}^N$. A version of Bayes' Theorem (see Elliott et al., 1995) implies that

$$\mathrm{E}[X_k|\mathscr{Y}_k]=\frac{\bar{\mathrm{E}}[\bar{\Lambda}_kX_k|\mathscr{Y}_k]}{\bar{\mathrm{E}}[\bar{\Lambda}_k|\mathscr{Y}_k]}. \tag{5}$$

As in Elliott et al. (1995), write $B(Y_{k+1})$ for the diagonal matrix with entries

$$M\left(\sum_{j=1}^{M} c_{ji}\langle Y_{k+1}, f_j\rangle\right).$$

It can then be shown that the dynamics of $q_k := \bar{\mathrm{E}}[\bar{\Lambda}_k X_k | \mathscr{Y}_k]$ are given by

$$q_{k+1} = B(Y_{k+1})Aq_k. \tag{6}$$

Given the parameters of the model, namely matrices A and C, the distribution of X_k given information in $\mathscr{Y}_k$ is then $\mathrm{E}[X_k|\mathscr{Y}_k] = q_k/\langle q_k, \mathbf{1}\rangle$, where $\langle\, , \rangle$ denotes the scalar product in $\mathbb{R}^N$ and $\mathbf{1} = (1, 1, \ldots, 1)' \in \mathbb{R}^N$.

To estimate parameters of the model, matrices A and C, we need estimates of the following processes:

$$J_k^{ij} = \sum_{n=1}^{k} \langle X_{n-1}, e_i\rangle\langle X_n, e_j\rangle, \quad 1 \leqslant i,j \leqslant N, \tag{7}$$

$$O_k^i = \sum_{n=1}^{k} \langle X_{n-1}, e_i\rangle, \quad 1 \leqslant i \leqslant N, \tag{8}$$

$$T_k^{ij} = \sum_{n=0}^{k} \langle X_n, e_i\rangle\langle Y_n, f_j\rangle, \quad 1 \leqslant i \leqslant N, \;\; 1 \leqslant j \leqslant M. \tag{9}$$

The above processes represent, respectively, the number of jumps from e_i to e_j in time k, the occupation time in e_i, and the number of transitions from state e_i to observation f_j. As for the state of Markov chain, for $H = J, O, T$ we wish to estimate

$$\mathrm{E}[H_k|\mathscr{Y}_k] = \frac{\bar{\mathrm{E}}[\bar{\Lambda}_k H_k|\mathscr{Y}_k]}{\bar{\mathrm{E}}[\bar{\Lambda}_k|\mathscr{Y}_k]}. \tag{10}$$

However, as noted in Elliott et al. (1995), we work with $\sigma(HX)_k = \bar{\mathrm{E}}[\bar{\Lambda}_k H_k X_k|\mathscr{Y}_k]$ rather than $\sigma(H)_k = \bar{\mathrm{E}}[\bar{\Lambda}_k H_k|\mathscr{Y}_k]$, in order to obtain closed form recursions. For the zero-delay case considered here, these recursions are

$$\sigma(J^{ij}X)_{k+1} = B(Y_{k+1})A\sigma(J^{ij}X)_k + \left(M\sum_{s=1}^{M} c_{sj}\langle Y_{k+1}, f_s\rangle\right)\langle q_k, e_i\rangle a_{ji} e_j, \tag{11}$$

$$\sigma(O^i X)_{k+1} = B(Y_{k+1})A\sigma(O^i X)_k + \langle q_k, e_i\rangle B(Y_{k+1})Ae_i, \tag{12}$$

$$\sigma(T^{ij}X)_{k+1} = B(Y_{k+1})A\sigma(T^{ij}X)_k + Mc_{ji}\langle Y_{k+1}, f_j\rangle\langle Aq_k, e_i\rangle e_i. \tag{13}$$

Since $\sigma(H)_k = \bar{\mathrm{E}}[\bar{\Lambda}_k H_k|\mathscr{Y}_k] = \bar{\mathrm{E}}[\bar{\Lambda}_k H_k\langle X_k, \mathbf{1}\rangle|\mathscr{Y}_k] = \langle \bar{\mathrm{E}}[\bar{\Lambda}_k H_k X_k|\mathscr{Y}_k], \mathbf{1}\rangle = \langle \sigma(HX)_k, \mathbf{1}\rangle$, summing the components in expressions (11)–(13) gives recursions for $\sigma(J^{ij})_k$, $\sigma(O^i)_k$ and $\sigma(T^{ij})_k$. The normalizing factor is again $\langle q_k, \mathbf{1}\rangle$.

Expressions (11)–(13) can be used to estimate the parameters of the model. Using the filter-based EM algorithm as described in Elliott et al. (1995), the transition probabilities in matrix A are estimated as

$$\hat{a}_{ji} = \frac{\sigma(J^{ij})_k}{\sigma(O^i)_k}, \tag{14}$$

and the probabilities in matrix C as

$$\hat{c}_{ji} = \frac{\sigma(T^{ij})_k}{\sigma(O^i)_k + (M\sum_{s=1}^{M} c_{si}\langle Y_k, f_s\rangle)\langle Aq_{k-1}, e_i\rangle}. \tag{15}$$

The revised parameters $\hat{a}_{ji}$ and $\hat{c}_{ji}$ give new probability measures for the model. New parameters and new data allow for reestimation of quantities $\sigma(J^{ij})_k$, $\sigma(O^i)_k$ and $\sigma(T^{ij})_k$ as well as the Markov chain representing 'true' credit quality, so the model is 'self-calibrating'.

3. Application

The HMM described in Section 2 was applied to a data set of Standard & Poor's credit ratings. Description of the data and implementation results are given below.

3.1. Data description

Our analysis takes advantage of the Standard & Poor's COMPUSTAT database, which contains rating histories for 1301 obligors over the period 1985–1999 (Standard & Poor's, 2000). The universe of obligors is mainly large U.S. and Canadian corporate institutions. The obligors include industrials, utilities, insurance companies, banks and other financial institutions and real estate companies. Prior to September 1, 1998, the company level rating is taken to be the highest issue level rating that the company has on its senior secured debt. When a company does not have senior secured debt issues, the implied senior rating is used. As of September 1, 1998, all ratings in the COMPUSTAT data set are Standard & Poor's issuer credit ratings. The COMPUSTAT database provides annual ratings. Every year each of the rated obligors is assigned to one of the Standard & Poor's 7 rating categories, ranging from *AAA* (highest rating) to *CCC* (lowest rating) as well as *D* (payment in default) and the *NR* (not rated) state.

We have a total of 19,515 firm-years in our sample. However, only 34% of those observations are 'non-zero', i.e. correspond to a firm with one of the eight rating labels *AAA* to *D* in a given year. The remaining 66% of observations represent the so-called *NR* (not rated) status. The distribution of ratings is shown in Fig. 1. Approximately 1% of the observed 'non-zero' ratings are *AAA* and 2% are recorded as defaults. The 'median' rating is *BB*, the highest non-investment-grade rating. The most common rating is *B*, two rating categories above default, which accounts for 25.5% of the 'non-zero' observations.

Our main objective is to utilize as many rating transitions as possible. However, most firms experience few rating changes and when ratings do change, they are confined to a few consecutive rating categories. The many transitions to *NR* also pose a problem, since few obligors in our data set have been consistently assigned a rating in the considered time period from 1985 to 1999. For example, there are only 84 firms in the data set rated every year from 1988 to 1998.

As discussed in the literature (see for example Altman, 1998; Bangia et al., 2002), transitions to the *NR* (not rated) status can occur for a number of reasons. The majority of rating withdrawals occur due to an early call, when a firm's only outstanding issue is paid off or when its debt issuance program matures, but they may be due to other reasons as well, such as insufficient information to

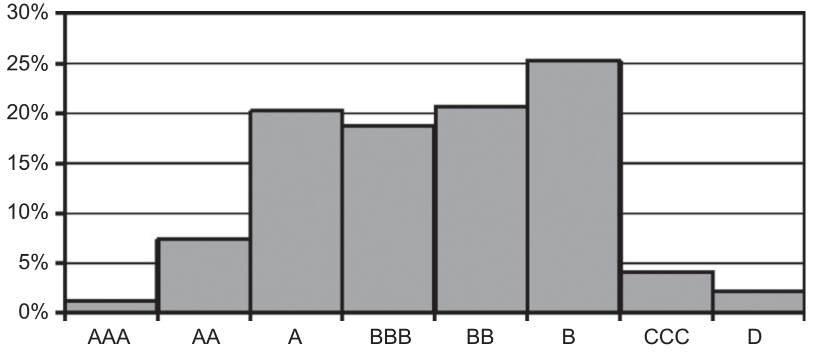

Fig. 1. Distribution of ratings among Standard & Poor's rated firms, 1985–1999.

rate the bond. Unfortunately, the details of individual transitions to *NR* in our data set are not known. In particular, we do not know whether a deterioration of credit quality known only to the obligor has led the issuer to decide to bypass an agency rating. The industry standard generally calls for removing transitions to *NR* from the data set by distributing probability transitions to *NR* among other states. In our case, we follow Lando and Skødeberg (2002) and retain the *NR* category in our data set contrary to the industry standard, in order to make full use of sample information. As a result, we have 15 years of rating history for all 1,301 obligors.

3.2. Implementation

For this illustration of our methodology, we suppose that both credit quality and credit rating categories range from *AAA* to *CCC*, as well as default *D* and *NR*, and identify each with a unit vector in $\mathbb{R}^9$.

Since individual firms generally experience few rating changes and changes that do occur are to neighbouring categories, parameter estimation is performed for an aggregate of firms in the data set. Quantities needed, namely estimates of processes J, O and T, are first filtered from observations individually for each issuer using recursions (11)–(13) and then aggregated to obtain estimates of parameters $\hat{a}_{ji}$ and $\hat{c}_{ji}$ using the following analogues of expressions (14) and (15):

$$\hat{a}_{ji} = \frac{\sum_{l=1}^{L} \sigma(J^{ij})_k^l}{\sum_{l=1}^{L} \sigma(O^i)_k^l}, \tag{14$'$}$$

$$\hat{c}_{ji} = \frac{\sum_{l=1}^{L} \sigma(T^{ij})_k^l}{\sum_{l=1}^{L} (\sigma(O^i)_k^l + (M \sum_{s=1}^{M} c_{si} \langle Y_k^l, f_s \rangle) \langle Aq_{k-1}^l, e_i \rangle)}, \tag{15$'$}$$

where L is the total number for firms in the sample. Our parameter estimation method can thus be termed a *filter-based cohort approach*. Since the estimates are directly computed from the recursive equations (11)–(13), the filtering-based EM estimation considered here is very easy to implement.

The initial values of the model parameters are as follows. Matrix A was taken to be the transition matrix in Table 3 in Lando and Skødeberg (2002) given that it explicitly includes transitions to and from the *NR* category. This matrix is given in Appendix A. Initial matrix C is given in Appendix B. For the initial matrix C, the probability of the observed rating agreeing with the 'true' rating is assumed to be 0.5 for all rating categories *AAA* to *D*. The probability of the observed rating being one category higher or one category lower than the 'true' credit quality is 0.2 for categories *AA* to *CCC* (0.4 for lower than *AAA* and higher than *D*). It is also assumed that any credit quality can be posted as *NR* with probability 0.1. Finally, the diagonal entry corresponding to the *NR* category is set as 1.0.

Given the relatively short time period, parameter estimates were updated with the arrival of every new observation for the 1,301 firms in the data set, for the total of 15 estimation steps. Estimated parameters of the model, namely matrices $\hat{A}$ and $\hat{C}$, are given in Appendix C, together with $Var\, V_k$ and $Var\, W_k$.[1] Repetition of the estimation procedures ensures that the model and estimates improve with each iteration by construction, which is demonstrated by the rapidly decreasing estimates of the entries in both $Var\, V_k$ and $Var\, W_k$. It is worth noting that there is generally more uncertainty regarding estimates of transitions to *NR* than any other category. The least uncertainty seems to be associated with parameter estimates for categories *BBB*, *BB* and *B*, which account for over 60% of 'non-zero' ratings in the data set (see Fig. 1).

3.3. Discussion of results

Consider first the final estimate of the transition matrix A. Entries above the diagonal correspond to rating upgrades and those below the diagonal to rating downgrades. We see that non-zero

[1] Note that in our HMM model, $Var\, V_k$ and $Var\, W_k$ are obtained via a straightforward matrix calculation after estimates of the transition matrices A and C have been obtained.

transition probabilities are concentrated and highest on the diagonal and the second largest probability is in the last row, indicating that obligors generally either maintain their rating or enter the *NR* (not rated) category. The next largest entry is usually on the off-diagonal. This confirms the observation that rating agencies usually do not change a company's rating by more than one category at a time. Downgrades seem to be more common than upgrades for *AA*, *A* and *CCC*. For firms rated *BBB* and *BB*, an upgrade appears to be as likely as a downgrade. For firms rated *B*, an upgrade is twice as likely as a downgrade. Finally, for firms rated *CCC*, an upgrade seems more likely than a downgrade to the default state. The probability of transition to the *NR* category is highest for firms rated *BBB* (30%), *BB* (34%) and *B* (38%), compared with 16% for *AAA*. Finally, obligors in the *NR* category remain in that category with probability virtually 1. Hence many firms disappear from the data set not just as a result of bankruptcy, but also because of maturing debt followed by no new issues and even possibly because of concerns over credit quality, or opting for no rating in anticipation of unfavourable rating assessment. Note that compared to the estimates in Lando and Skødeberg (2002) shown in Appendix A and used as a starting point for our estimation method, probabilities on the diagonal are much lower while probabilities of transition to *NR* in the last row are much higher. Another notable difference is in the probability of an upgrade from *CCC* to *B*, which in our case is double the Lando and Skødeberg estimate.

Recall that in general matrix C describes the relationship between the signal process X and the observation process Y. In particular, non-zero entries above (below) the diagonal indicate that the observed rating may be higher (lower) than the 'true' credit quality. Recall also that when $C = I$, processes X (the signal) and Y (the observations) are identical, i.e. there is no 'noise' in the system. In our case, the highest probabilities are either on the main diagonal or in the last row, reflecting the many *NR* observations in our data set. For the investment-grade categories *AAA* to *BBB* there is 50–60% chance that the obligor is posted as *NR*, compared to 30–40% for the speculative grade *BB* to *CCC*. *B* appears to be the category with the highest probability (54%) that the posted rating reflects the 'true' credit quality. For *AAA*, the observed rating is either *AA* (estimated probability 40%) or *NR* (estimated probability 60%), indicating that rating agencies may be very reluctant to upgrade obligors to *AAA*. If the posted rating differs from 'true' credit quality, for *A* and *BB* it is more likely to be lower. In contrast, for *BBB* and especially *B*, the posted rating is more likely to be higher. For the near-default category *CCC*, our estimates suggest that the posted rating is either *B* (probability 54%), *NR* (probability 31%) or *D* (probability 15%). Finally, an estimated 93% of defaults could be posted as *NR*. While these observations indicate possible 'non-Markov' effects, longer rating histories may be required to accurately capture credit rating dynamics.

Estimates of state-to-observation transition probabilities in matrix C suggest differences in the experiences of firms rated investment-grade (*BBB* or higher) and speculative-grade (*BB* or lower). A corporation which can issue higher rated debt usually receives better financing terms. Further, as a matter of policy or law, some institutional investors can only purchase investment-grade bonds. Hence it is often crucial for a borrower to maintain an investment-grade rating. We have therefore reclassified all firms in the sample as *IG* (investment grade), *SG* (speculative grade), *D* or *NR* and then applied the HMM algorithm to the new data set.

The results presented in Appendix D for the modified sample confirm that investment-grade firms do generally hold on to their status, but there is an estimated 9% probability of downgrade to speculative-grade status. However, for speculative-grade firms, the probability of upgrade to investment-grade status is lower (estimated probability of 4%). Speculative-grade firms tend to maintain their status or disappear from the data set because of either default or withdrawn rating. The probability of transition to the *NR* status is higher for speculative-grade obligors. The final estimated matrix C indicates that rating agencies may be reluctant to upgrade firms to investment-grade status. We have an estimated probability of 30% that the observed rating is speculative-grade when the 'true' credit quality is investment-grade, while the probability in the opposite direction is estimated at 20%. Moreover, speculative-grade firms are as likely to disappear to the *NR* category as they are to be upgraded to the investment-grade status. For the default category *D*, the estimates suggest that it is most likely to be posted as *NR* (87%) or *SG* (13%). However, we conclude as before that longer rating histories and further analysis may be required to verify our results.

4. Conclusion

We have proposed a hidden Markov model for the evolution of credit quality in discrete time with credit ratings as 'noisy' observations. Filtering-based estimation techniques of Hidden Markov models were applied to obtain the best estimate of the Markov chain representing 'true' credit quality and estimates of parameters, namely probabilities of migration from one credit quality category to another, and probabilities of observing a credit rating different from 'true' credit quality. The model was applied to a data set of Standard & Poor's issuer ratings and our preliminary results agree with qualitative observations made in the literature regarding credit rating systems.

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Appendix A. Standard & Poor's transition matrix with *NR*

Source: Table 3 in Lando and Skødeberg (2002). The one-year transition matrix estimated from continuous data over the period 1988–1998.

	AAA	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*	*D*	*NR*
AAA	0.900	0.006	0.000	0.000	0.000	0.000	0.003	0.000	0.000
AA	0.064	0.879	0.013	0.002	0.001	0.002	0.000	0.000	0.000
A	0.009	0.076	0.894	0.048	0.009	0.002	0.005	0.000	0.000
BBB	0.001	0.006	0.044	0.848	0.087	0.008	0.002	0.000	0.001
BB	0.002	0.001	0.005	0.039	0.730	0.064	0.022	0.000	0.001
B	0.000	0.000	0.001	0.006	0.061	0.673	0.067	0.000	0.000
CCC	0.000	0.000	0.000	0.001	0.008	0.053	0.382	0.000	0.000
D	0.000	0.000	0.000	0.001	0.007	0.044	0.376	1.000	0.004
NR	0.025	0.032	0.042	0.055	0.097	0.152	0.143	0.000	0.994

Recall from Section 2 that an entry of the transition matrix is defined as $a_{ji} = \mathrm{P}(X_{k+1} = e_j | X_k = e_i)$. Hence for matrix A, probabilities above the diagonal correspond to rating upgrades, and those below the diagonal to rating downgrades.

Appendix B. Initial matrix *C*

	AAA	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*	*D*	*NR*
AAA	0.500	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.400	0.500	0.200	0.000	0.000	0.000	0.000	0.000	0.000
A	0.000	0.200	0.500	0.200	0.000	0.000	0.000	0.000	0.000
BBB	0.000	0.000	0.200	0.500	0.200	0.000	0.000	0.000	0.000
BB	0.000	0.000	0.000	0.200	0.500	0.200	0.000	0.000	0.000
B	0.000	0.000	0.000	0.000	0.200	0.500	0.200	0.000	0.000
CCC	0.000	0.000	0.000	0.000	0.000	0.200	0.500	0.400	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.500	0.000
NR	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	1.000

Recall from Section 2 that an entry of the matrix C is defined as $c_{ji} = \mathrm{P}(Y_k = f_j | X_k = e_i)$. Hence for matrix C, entries above the diagonal correspond to the probability that the observed rating is higher that the 'true' credit quality. A non-zero entry below the diagonal means that the observed rating is lower than the 'true' credit quality. The final row indicates for each 'true' credit quality category the probability of being posted as *NR*.

Appendix C. HMM implementation results with nine rating categories

Estimation step 1.
Estimated matrix A:

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	0.763	0.004	0.000	0.000	0.000	0.000	0.001	0.000	0.000
AA	0.068	0.750	0.009	0.002	0.001	0.001	0.000	0.000	0.000
A	0.011	0.077	0.769	0.042	0.007	0.002	0.004	0.000	0.000
BBB	0.001	0.005	0.035	0.683	0.062	0.005	0.001	0.000	0.000
BB	0.002	0.001	0.004	0.031	0.513	0.040	0.016	0.000	0.000
B	0.000	0.000	0.001	0.005	0.040	0.394	0.046	0.000	0.000
CCC	0.000	0.000	0.000	0.000	0.004	0.023	0.194	0.000	0.000
D	0.000	0.000	0.000	0.000	0.003	0.017	0.169	1.000	0.001
NR	0.155	0.162	0.182	0.237	0.371	0.518	0.568	0.000	0.999

Aggregate variance matrix $Var\,V_k$:

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	14.420	−9.098	−1.306	−0.110	−0.271	−0.048	−0.139	−0.150	−3.312
AA	−9.098	26.777	−11.438	−1.110	−0.265	−0.284	−0.036	−0.046	−4.490
A	−1.306	−11.438	34.154	−11.689	−1.878	−0.555	−0.354	−0.345	−6.576
BBB	−0.110	−1.110	−11.689	38.568	−14.105	−2.383	−0.364	−0.327	−8.493
BB	−0.271	−0.265	−1.878	−14.105	46.850	−12.966	−2.576	−2.269	−12.520
B	−0.048	−0.284	−0.555	−2.383	−12.966	50.364	−8.999	−8.005	−17.124
CCC	−0.139	−0.036	−0.354	−0.364	−2.576	−8.999	42.795	−21.132	−9.196
D	−0.150	−0.046	−0.345	−0.327	−2.269	−8.005	−21.132	41.723	−9.449
NR	−3.312	−4.490	−6.576	−8.493	−12.520	−17.124	−9.196	−9.449	71.160

Estimated matrix C:

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	0.030	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.232	0.233	0.078	0.000	0.000	0.000	0.000	0.000	0.000
A	0.000	0.164	0.345	0.149	0.000	0.000	0.000	0.000	0.000
BBB	0.000	0.000	0.078	0.212	0.086	0.000	0.000	0.000	0.000
BB	0.000	0.000	0.000	0.100	0.254	0.108	0.000	0.000	0.000
B	0.000	0.000	0.000	0.000	0.113	0.301	0.162	0.000	0.000
CCC	0.000	0.000	0.000	0.000	0.000	0.007	0.023	0.021	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.033	0.094	0.000
NR	0.738	0.593	0.498	0.539	0.547	0.584	0.782	0.886	1.000

Aggregate variance matrix $Var\,W_k$:

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	26.606	−40.181	−5.561	0.000	0.000	0.000	0.000	0.000	−9.350
AA	−40.181	58.878	−29.002	−6.040	0.000	0.000	0.000	0.000	−15.227
A	−5.561	−29.002	49.547	−29.486	−5.755	0.000	0.000	0.000	−13.208
BBB	0.000	−6.040	−29.486	48.079	−26.871	−4.994	0.000	0.000	−12.710
BB	0.000	0.000	−5.755	−26.871	43.279	−24.196	−4.685	0.000	−11.462
B	0.000	0.000	0.000	−4.994	−24.196	41.786	−18.148	−2.574	−9.640
CCC	0.000	0.000	0.000	0.000	−4.685	−18.148	1.708	−47.836	−13.840
D	0.000	0.000	0.000	0.000	0.000	−2.574	−47.836	61.737	−11.637
NR	−9.350	−15.227	−13.208	−12.710	−11.462	−9.640	−13.840	−11.637	97.074

Estimation step 5.
Estimated matrix A:

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	0.703	0.005	0.000	0.000	0.000	0.000	0.001	0.000	0.000
AA	0.043	0.614	0.008	0.001	0.001	0.001	0.000	0.000	0.000

A	0.006	0.048	0.551	0.024	0.003	0.001	0.001	0.000	0.000
BBB	0.000	0.004	0.031	0.503	0.037	0.003	0.001	0.000	0.000
BB	0.002	0.001	0.004	0.026	0.361	0.024	0.008	0.000	0.000
B	0.000	0.000	0.001	0.005	0.034	0.280	0.029	0.000	0.000
CCC	0.000	0.000	0.000	0.001	0.005	0.022	0.166	0.000	0.000
D	0.000	0.000	0.000	0.001	0.003	0.017	0.152	1.000	0.000
NR	0.246	0.328	0.405	0.440	0.557	0.653	0.642	0.000	0.999

Aggregate variance matrix $Var\, V_k$:

	AAA	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*	*D*	*NR*
AAA	11.323	−1.798	−0.233	−0.019	−0.060	−0.004	0.000	−0.004	−9.206
AA	−1.798	16.368	−2.067	−0.193	−0.027	−0.015	0.000	−0.004	−12.264
A	−0.233	−2.067	20.751	−1.687	−0.185	−0.044	−0.008	−0.008	−16.519
BBB	−0.019	−0.193	−1.687	13.423	−0.782	−0.126	−0.017	−0.015	−10.585
BB	−0.060	−0.027	−0.185	−0.782	5.369	−0.247	−0.033	−0.025	−4.010
B	−0.004	−0.015	−0.044	−0.126	−0.247	2.359	−0.056	−0.044	−1.824
CCC	0.000	0.000	−0.008	−0.017	−0.033	−0.056	0.467	−0.038	−0.315
D	−0.004	−0.004	−0.008	−0.015	−0.025	−0.044	−0.038	0.743	−0.605
NR	−9.206	−12.264	−16.519	−10.585	−4.010	−1.824	−0.315	−0.605	55.327

Estimated matrix C:

	AAA	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*	*D*	*NR*
AAA	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.013	0.020	0.005	0.000	0.000	0.000	0.000	0.000	0.000
A	0.000	0.026	0.084	0.026	0.000	0.000	0.000	0.000	0.000
BBB	0.000	0.000	0.015	0.047	0.020	0.000	0.000	0.000	0.000
BB	0.000	0.000	0.000	0.026	0.089	0.034	0.000	0.000	0.000
B	0.000	0.000	0.000	0.000	0.035	0.114	0.049	0.000	0.000
CCC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.012	0.011	0.000
NR	0.987	0.955	0.895	0.901	0.856	0.851	0.939	0.989	1.000

Aggregate variance matrix $Var\, W_k$:

	AAA	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*	*D*	*NR*
AAA	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	−0.001
AA	0.000	1.205	−0.036	−0.003	0.000	0.000	0.000	0.000	−1.379
A	0.000	−0.036	4.202	−0.081	−0.014	0.000	0.000	0.000	−4.762
BBB	0.000	−0.003	−0.081	1.618	−0.038	−0.005	0.000	0.000	−1.659
BB	0.000	0.000	−0.014	−0.038	1.114	−0.030	0.000	0.000	−1.145
B	0.000	0.000	0.000	−0.005	−0.030	0.504	0.000	0.000	−0.499
CCC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	−0.001	−0.048
D	0.000	0.000	0.000	0.000	0.000	0.000	−0.001	2.667	−2.670
NR	−0.001	−1.379	−4.762	−1.659	−1.145	−0.499	−0.048	−2.670	12.163

Estimation step 10.
Estimated matrix A:

	AAA	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*	*D*	*NR*
AAA	0.700	0.005	0.000	0.000	0.000	0.000	0.001	0.000	0.000
AA	0.043	0.611	0.008	0.001	0.000	0.001	0.000	0.000	0.000
A	0.006	0.049	0.556	0.024	0.003	0.001	0.002	0.000	0.000
BBB	0.000	0.004	0.031	0.510	0.038	0.003	0.001	0.000	0.000
BB	0.002	0.001	0.004	0.027	0.378	0.025	0.009	0.000	0.000
B	0.000	0.000	0.001	0.005	0.035	0.298	0.032	0.000	0.000
CCC	0.000	0.000	0.000	0.001	0.005	0.022	0.171	0.000	0.000
D	0.000	0.000	0.000	0.001	0.003	0.016	0.148	1.000	0.000
NR	0.250	0.330	0.400	0.431	0.538	0.634	0.636	0.000	1.000

Aggregate variance matrix $Var\,V_k$:

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	2.008	−0.303	−0.038	−0.003	−0.011	−0.001	0.000	−0.001	−1.653
AA	−0.303	2.239	−0.253	−0.023	−0.003	−0.002	0.000	−0.001	−1.655
A	−0.038	−0.253	1.988	−0.138	−0.015	−0.004	−0.001	−0.001	−1.539
BBB	−0.003	−0.023	−0.138	0.972	−0.047	−0.008	−0.001	−0.001	−0.751
BB	−0.011	−0.003	−0.015	−0.047	0.332	−0.010	−0.001	−0.001	−0.244
B	−0.001	−0.002	−0.004	−0.008	−0.010	0.107	−0.002	−0.001	−0.081
CCC	0.000	0.000	−0.001	−0.001	−0.001	−0.002	0.013	−0.001	−0.008
D	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001	0.416	−0.411
NR	−1.653	−1.655	−1.539	−0.751	−0.244	−0.081	−0.008	−0.411	6.341

Estimated matrix C:

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.021	0.030	0.007	0.000	0.000	0.000	0.000	0.000	0.000
A	0.000	0.039	0.125	0.040	0.000	0.000	0.000	0.000	0.000
BBB	0.000	0.000	0.023	0.073	0.032	0.000	0.000	0.000	0.000
BB	0.000	0.000	0.000	0.036	0.127	0.050	0.000	0.000	0.000
B	0.000	0.000	0.000	0.000	0.051	0.171	0.079	0.000	0.000
CCC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.020	0.009	0.000
NR	0.980	0.931	0.845	0.851	0.790	0.779	0.900	0.991	1.000

Aggregate variance matrix $Var\,W_k$:

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.000	0.239	−0.007	−0.001	0.000	0.000	0.000	0.000	−0.258
A	0.000	−0.007	0.531	−0.012	−0.002	0.000	0.000	0.000	−0.578
BBB	0.000	−0.001	−0.012	0.174	−0.005	−0.001	0.000	0.000	−0.171
BB	0.000	0.000	−0.002	−0.005	0.086	−0.003	0.000	0.000	−0.090
B	0.000	0.000	0.000	−0.001	−0.003	0.031	0.000	0.000	−0.032
CCC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	−0.025
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.956	−1.960
NR	0.000	−0.258	−0.578	−0.171	−0.090	−0.032	−0.025	−1.960	3.113

Estimation step 15.
Estimated matrix A:

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	0.736	0.004	0.000	0.000	0.000	0.000	0.001	0.000	0.000
AA	0.079	0.691	0.008	0.001	0.000	0.001	0.000	0.000	0.000
A	0.017	0.077	0.672	0.030	0.004	0.001	0.007	0.000	0.000
BBB	0.001	0.006	0.035	0.625	0.052	0.005	0.003	0.000	0.000
BB	0.005	0.001	0.004	0.034	0.552	0.048	0.043	0.000	0.000
B	0.000	0.001	0.001	0.005	0.047	0.529	0.135	0.000	0.000
CCC	0.000	0.000	0.000	0.001	0.004	0.024	0.335	0.000	0.000
D	0.000	0.000	0.000	0.000	0.002	0.013	0.203	1.000	0.000
NR	0.161	0.220	0.279	0.304	0.338	0.379	0.274	0.000	1.000

Aggregate variance matrix $Var\,V_k$:

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	0.338	−0.071	−0.013	−0.001	−0.004	0.000	0.000	0.000	−0.250
AA	−0.071	0.359	−0.058	−0.005	−0.001	0.000	0.000	0.000	−0.224
A	−0.013	−0.058	0.308	−0.026	−0.003	−0.001	0.000	0.000	−0.208
BBB	−0.001	−0.005	−0.026	0.170	−0.012	−0.002	0.000	0.000	−0.125
BB	−0.004	−0.001	−0.003	−0.012	0.131	−0.005	−0.001	0.000	−0.106
B	0.000	0.000	−0.001	−0.002	−0.005	0.049	−0.001	−0.001	−0.040

CCC	0.000	0.000	0.000	0.000	−0.001	−0.001	0.003	0.000	−0.001
D	0.000	0.000	0.000	0.000	0.000	−0.001	0.000	0.399	−0.397
NR	−0.250	−0.224	−0.208	−0.125	−0.106	−0.040	−0.001	−0.397	1.351

Estimated matrix C:

	AAA	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*	*D*	*NR*
AAA	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.398	0.187	0.023	0.000	0.000	0.000	0.000	0.000	0.000
A	0.000	0.246	0.390	0.127	0.000	0.000	0.000	0.000	0.000
BBB	0.000	0.000	0.072	0.232	0.096	0.000	0.000	0.000	0.000
BB	0.000	0.000	0.000	0.115	0.382	0.168	0.000	0.000	0.000
B	0.000	0.000	0.000	0.000	0.144	0.536	0.542	0.000	0.000
CCC	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.001	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.149	0.068	0.000
NR	0.602	0.567	0.515	0.526	0.378	0.296	0.305	0.931	1.000

Aggregate variance matrix $Var\, W_k$:

	AAA	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*	*D*	*NR*
AAA	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.000	0.283	−0.028	−0.001	0.000	0.000	0.000	0.000	−0.270
A	0.000	−0.028	0.271	−0.022	−0.004	0.000	0.000	0.000	−0.261
BBB	0.000	−0.001	−0.022	0.079	−0.012	−0.002	0.000	0.000	−0.067
BB	0.000	0.000	−0.004	−0.012	0.043	−0.012	0.000	0.000	−0.048
B	0.000	0.000	0.000	−0.002	−0.012	0.023	0.000	0.000	−0.022
CCC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	−0.006	−0.127
D	0.000	0.000	0.000	0.000	0.000	0.000	−0.006	9.929	−9.941
NR	0.000	−0.270	−0.261	−0.067	−0.048	−0.022	−0.127	−9.941	10.736

Appendix D. HMM implementation results with four rating categories

Estimation step 1:

Estimated matrix A					Estimated matrix C				
	IG	*SG*	*D*	*NR*		*IG*	*SG*	*D*	*NR*
IG	0.678	0.035	0.000	0.002	*IG*	0.457	0.229	0.000	0.000
SG	0.068	0.304	0.000	0.001	*SG*	0.243	0.380	0.422	0.000
D	0.000	0.040	1.000	0.000	*D*	0.000	0.016	0.055	0.000
NR	0.255	0.622	0.000	0.997	*NR*	0.301	0.376	0.523	1.000

Estimation step 5:

Estimated matrix A					Estimated matrix C				
	IG	*SG*	*D*	*NR*		*IG*	*SG*	*D*	*NR*
IG	0.345	0.010	0.000	0.000	*IG*	0.249	0.066	0.000	0.000
SG	0.065	0.165	0.000	0.000	*SG*	0.154	0.221	0.231	0.000
D	0.000	0.008	1.000	0.000	*D*	0.000	0.008	0.007	0.000
NR	0.591	0.818	0.000	0.999	*NR*	0.597	0.705	0.762	1.000

Estimation step 10:

Estimated matrix A					Estimated matrix C				
	IG	*SG*	*D*	*NR*		*IG*	*SG*	*D*	*NR*
IG	0.387	0.012	0.000	0.000	*IG*	0.310	0.094	0.000	0.000
SG	0.069	0.191	0.000	0.000	*SG*	0.193	0.292	0.124	0.000
D	0.000	0.004	1.000	0.000	*D*	0.000	0.011	0.004	0.000
NR	0.544	0.794	0.000	1.000	*NR*	0.498	0.603	0.872	1.000

Estimation step 15:

Estimated matrix *A*					Estimated matrix *C*				
	IG	*SG*	*D*	*NR*		*IG*	*SG*	*D*	*NR*
IG	0.601	0.037	0.000	0.000	*IG*	0.490	0.197	0.000	0.000
SG	0.089	0.490	0.000	0.001	*SG*	0.296	0.588	0.129	0.000
D	0.000	0.006	1.000	0.000	*D*	0.000	0.023	0.005	0.000
NR	0.310	0.467	0.000	0.999	*NR*	0.215	0.192	0.866	1.000

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