

## Subproblem LSA - 2-d Node List / 3-d Space-Time Map

When I first brainstormed an algorithm using graph traversal that would solve the subproblem, I imagined we needed a grid of  $i, t$  nodes rather than just location nodes, as we were able to use in the set-partitioning without column generation (i.e. enumerating all the routes). I said so because otherwise, how could we represent  $t$ ? But after thinking about it I observed the following:

1. We have a parameter  $T$  which we use in the LSA which can capture 'time of entry into last node so far', so we might be able to pull off the subproblem LSA with just this entry to add all the  $\mu_{it}$  from the reduced cost  $C^q - \sum_{i \in \mathcal{J}} \pi_i u_i^q - \rho - \sum_{i \in \mathcal{J}} \sum_{t \in \mathcal{T}} \mu_{it} \delta_{it}^q$ ;
2. If it is true that in  $C^q - \sum_{i \in \mathcal{J}} \pi_i u_i^q - \rho - \sum_{i \in \mathcal{J}} \sum_{t \in \mathcal{T}} \mu_{it} \delta_{it}^q$  we only actually subtract the  $\mu_{it}$  once\*; namely, when we enter node  $i$  at the special time  $t$  and NOT the subsequent  $t$ 's for which we stay at node  $i$  but do not actually move;
3. A 1-d node list / 2-d location map would be more time-efficient than the  $i, t$  grid because it would involve far less nodes (whether the bottleneck is in the NDom check or the generation and modification of new-node-searches I will determine soon);
4. If it is true that we actually have to subtract the  $\mu_{it}$  multiple times\*, then given our parameter  $T$ , we would be forced to 'retroactively' update all the  $\mu_{it}$ 's because you'd only know when you leave that node by updating the value of  $T$  when you actually stayed at the old node. In particular, this is given by: start index old\_value\_of\_  $T$ , end index new\_value\_of\_  $T$  minus transportation\_time\_from\_old\_node\_to\_new\_node. It is at least doable, but clumsy.

\* Do we subtract  $\mu_{it}$  once for every  $t$  at which we are at node  $i$ , or only when we enter node  $i$ ?

I've created and tested a 1-d node list / 2-d location map subproblem LSA and it seems to work fine for the first iteration - it clearly generates multiple feasible routes with negative reduced cost. I'd like to experiment and create a 2-d node list / 3-d space-time map to see how it works! It may actually be a benefit to do so for two reasons:

1. An  $i, t$  grid reduces the clumsiness of working with time as a variable; instead, time is embedded within the grid itself;
2. In the ICT document, section 9 "Attempt at a new version" expresses interest in solving the subproblem in which a route embodies not just the location, but also whether work is performed there or not. My  $i, t$ -grid solution will come one step closer to making this happen. It's not exactly this solution, because I don't explicitly tell you when to work there, but it makes all the times obvious.

The only true change of this 3d LSA is that our nodes are actually a 2d matrix, not a 1d list. The first index is location, ranging from 1 (depot) to 2 to  $n+1$  (jobs) to  $n+2$  (depot end). The second is time, ranging from 1 (start) to the end of time. The parameters, strategy, etc. are the same. It's just that now, nodes will involve two parameters rather than one. This is more a coding exercise and being careful than anything else.