Timing the Big 4

I fixed the issue with colgen's stopping prematurely (deleted a piece of extraneous code; see "Finding the Column Generation Error.pdf"). I next fixed an issue with calculating mu_it (see "A Subtlety on the Mu Dual Variable.pdf"). Colgen has an additional intrigue.

There's a constraint $\sum_q u_i^q z^q \ ? \ 1$ that expresses that exactly one vehicle go to job i.

If I use == on the constraint, I get a different set of answers than if I use >=! Neither is unilaterally better than the other - see "The Mystery of Signing.pdf" for the details!

We split CG and PT testing based on this criterion. In the below table, DF lasts until 29 (it takes too long). Dark red means wrong answer without route splitting; light red, with.

n	DF	SP	CG >=	CG ==	PT >=	PT ==	DF Obj	SP Obj	CG >=O	CG ==O	PT >=O	PT ==O
10	2.40	0.44	1.53	1.2	1.09	1.18	2'327	2'262				
11	2.18	0.01	0.06	0.04	0.03	0.05	2'727	2'658				
12	3.38	0.01	0.14	0.04	0.23	0.04	2'795	2'717				
13	4.76	0.01	0.12	0.05	0.06	0.04	3'070	2'987				
14	7.86	0.01	0.14	0.07	0.09	0.06	3'124	3'035				
15	13.05	0.04	0.24	0.12	0.13	0.34	2'643	2'539				
16	17.60	0.09	0.35	0.17	0.17	0.2	2'677	2'566				
17	19.65	0.08	6.31	0.21	0.21	0.2	3'127	3'012				
18	22.73	0.12	0.25	0.25	0.47	0.21	3'135	3'014				
19	27.38	0.21	0.49	0.47	0.45	0.62	3'163	3'036				
20	39.91	0.65	1.51	1.24	1.2	0.85	3'156	3'028	3'077		3'077	
21	94.06	0.83	1.71	1.76	1.26	1.66	3'402	3'257				
22	75.20	1.03	2.0	1.65	1.58	1.28	3'512	3'364	3'383		3'383	
23	98.52	1.48	4.27	2.83	2.71	1.57	3'548	3'399				
24	120.74	1.76	2.82	3.14	1.83	2.63	3'601	3'445	3'518		3'518	
25	212.77	4.34	7.14	6.92	4.27	4.32	3'023	2'845				
26	206.89	5.38	7.91	9.48	3.42	6.79	3'043	2'856				
27	381.21	6.46	8.32	8.07	5.18	4.96	3'060	2'867				2'876
28	271.59	9.00	10.86	14.56	4.6	7.14	3'135	2'933	2'949		2'949	
29	591.43	12.96	27.12	17.15	9.44	13.64	3'206	3'003		3'019		

At 30 and above an emergent property is more common: As we saw in the Julia cutting stock example, there is a chance of non-integer z-values. This is disturbing because there is no canonical way to resolve the issue. The objective for certain CG cases doesn't match up with the SP objective because the route-splitting necessary for using CG (0 <= z <= 1 instead of z Bin) causes a different result to be calculated. *Observe importantly that this is NOT equivalent to having more than "one" vehicle at a job!* These issues are put in light red; dark red indicates wrong answer without route-splitting.

We cannot verify the correctness of n=36 and above, but we indicate route-splitting when it exists with purple bold.

n	SP	CG >=	CG ==	PT >=	PT ==	SP Obj	CG >=O	CG ==O	PT >=O	PT ==O
30	22.75	38.89	35.91	9.73	8.08	3'282				**3'438**
31	36.11	38.59	37.52	8.20	11.56	3'284	**3'522**	**3'359**	**3'522**	**3'359**
32	91.77	81.42	89.17	7.18	13.01	3'292	**3'356**	**3'345**	**3'356**	**3'345**
33	118.64	103.20	107.68	11.87	19.05	3'406	**3'740**	**3'443**	**3'684**	**3'443**
34	161.89	176.68	188.12	18.15	22.88	3'649	**3'691**	**3'703**		**3'703**
35	582.22	440.58	469.23	21.07	28.25	3'314				
36				26.27	24.50				3'335	2'224
37				20.42	27.36				**3'756**	**3'773**
38				28.78	29.80				3'967	3'967
39				30.73	31.74				**3'979**	3'971
40				85.78	86.60				3'784	3'848
41				132.06	148.88				3'857	3'857
42				100.80	75.56				3'966	3'966
43				108.10	98.31				3'970	4'034
44				93.39	163.44				4'083	4'050
45				319.69	214.19				3'295	**3'399**
46				190.89	215.83					3'498
47				246.88	375.16				Did not record (DNR)	3'570
48				406.48	487.11					3'581
49				346.19	510.54					3'621
50				1075.16	1566.34					DNR

Time Taken with DF, SP, CG, PT
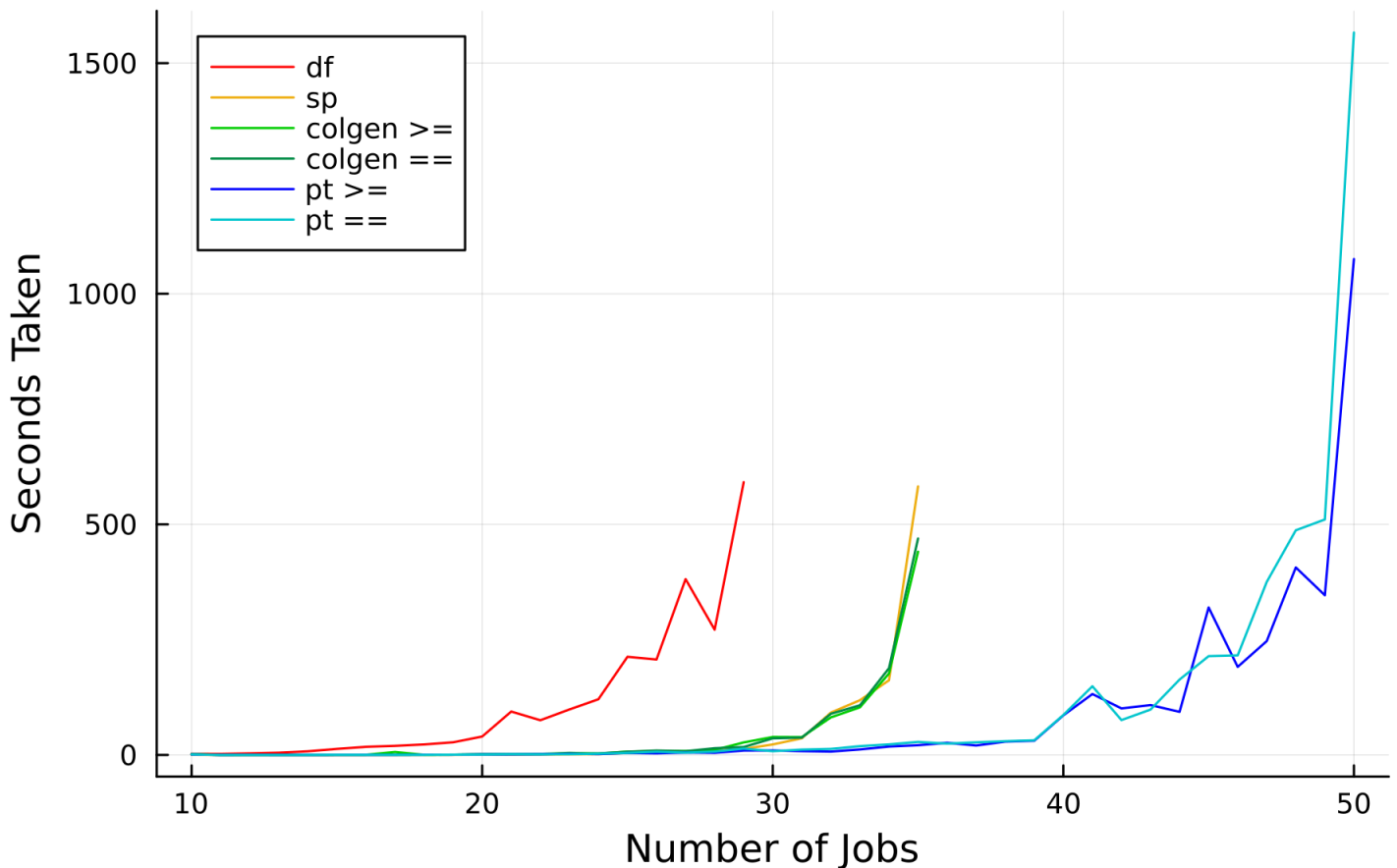
There is a substantial improvement offered by pruning. Once I revised the issues with my colgen, CG's runtime was brought level to SP's runtime—the magic number for both of them was 35. However, pruning continued to perform well, hitting 49 with under 10 minutes' runtime, and managing to hit 50 with 18 minutes (using >= on the constraint discussed above) and 26 minutes (using == instead).

Either way, the result at n = 50 has been attained; it also improves on the 2h 22m tabulated in the ICT presentation sent to me in January 2023.

The wrong values of CG and PT for n <= 35 are still of concern, despite constituting an improvement from before the two colgen issues (premature termination and mu_it calculation fix) were fixed. Even more troubling, neither >= nor == on the constraint was a watertight colgen across all 4 scenarios.

Once again, here's a list of guaranteed-correct routes.

10	7,8; 5,6; 1,2; 3,4; 9,10
11	5,6; 7,2; 9,10; 1,11,8; 3,4
12	11,12; 3,4; 5,6; 1,2; 7,8; 9,10
13	7,8; 9,10; 5,6; 13; 1,2; 3,4; 11,12
14	11,12; 13,14; 7,8; 5,6; 9,10; 1,2; 3,4
15	7,8,9; 1,2,3; 10,11,12; 13,14,15; 4,5,6
16	7,8,9; 13,14,15; 16,4,5,6; 10,11,12; 1,2,3
17	7,8,9; 10,11,12; 4,5,6; 13,14,15; 16,17; 1,2,3
18	13,14,15; 10,11,12; 1,2,3; 4,5,6; 16,17,18; 7,8,9
19	16,17,18; 13,14,15; 4,5,6; 10,11,12; 19,7,8,9; 1,2,3
20	4,19,5,20; 18,11,12,13,14; 15,16,17,6,7; 1,2,3; 8,9,10
21	21,11,12,13,14; 8,9,10; 18,19,20; 15,16,17; 4,5,6,7; 1,2,3
22	8,9,10; 21,22,3; 4,19,5,6,7; 1,2,13,14; 15,16,17; 18,11,12,20
23	21,22,23,3; 1,2,13,14; 15,16,17; 4,19,5,6,7; 18,11,12,20; 8,9,10
24	18,11,12,13,14; 15,16,17,6,7; 1,2,3; 4,19,5,20; 21,22,23,24; 8,9,10
25	16,17,18,19,13,14; 1,2,3; 24,25; 20,21,22,23; 4,5,6,7; 8,9,10; 11,12,15
26	24,25,26; 11,12,15; 20,21,22,23; 4,5,6,7; 16,17,18,19,13,14; 8,9,10; 1,2,3
27	16,17,18,19,13,14; 24,25,26,27; 1,2,3; 20,21,22,23; 11,12,15; 8,9,10; 4,5,6,7
28	20,21,22,23; 11,12,15; 24,25,26,27,28; 4,5,6,7; 1,2,3; 16,17,18,19,13,14; 8,9,10
29	16,17,18,19,13,14; 24,25,26,27,28; 1,29,2,3; 8,9,10; 20,21,22,23; 4,5,6,7; 11,12,15
30	13,14,15,16,17,18; 5,6,7,8,9; 28,29; 23,24,25,26; 1,2,3,4; 27,10,11,12,30; 19,20,21,22
31	27,10,11,12,30,31; 1,2,3,4; 23,24,25,26; 13,14,15,16,17,18; 28,29; 5,6,7,8,9; 19,20,21,22
32	5,6,7,8,9; 27,10,11,12,30,32; 13,14,15,16,17,18; 1,2,3,4; 19,20,21,22; 28,29,31; 23,24,25,26
33	27,10,11,12,30,32; 19,20,21,22; 33,28,29,31; 5,6,7,8,9; 1,2,3,4; 23,24,25,26; 13,14,15,16,17,18
34	23,5,24,25,26; 19,20,21,22; 27,10,11,12,30,32; 1,2,3,4; 28,29,31; 33,34,6,7,8,9; 13,14,15,16,17,18
35	5,6,7,8,9; 25,26,27; 10,11,12,13; 1,2,3,4; 21,22,23,24,28; 14,15,16,17,18,19,20; 29,30,31,32,33,34,35