UNESCO-IHE
(Delft, the Netherlands)

Exams (with answers) of module Transient Groundwater Flow from 2006 onwards

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Closed book reexamination Transient Flow 2015-2016

Question 1:

1) Explain what barometer efficiency (BE) is and how it physically works.
2) Explain in words what the charateristic (half) time of a groundwater system is. What does is say about the behavior of the system?
3) For which of the parameters L (system width), kD (transmissivity) and S_y (specific yield) would an increase make the characteristic system time smaller?
4) Explain whey in hydrological logic you think that this is the case.
5) If you see a close-up of two grains held together by a small amount of water at their point of contact. What then is the pressure in that water? Explain why that is so.

Question 2:

Consider an aquifer in direct contact with the ocean. The tide of the ocean has an amplitude $A = 1.0m$ and the cycle time is $T = 0.5d$ (one full tide in 12h). The aquifer is confined. It consists of two parts. The first part reaches from the ocean to 500 m inland, the second part is present at more than 500 m from the ocean. The first part of the aquifer has the following properties: transmissivity $kD = 900 m^2 / d$ and storage coefficient $S = 0.002$. The second part of the aquifer has the following properties: $kD = 1800 m^2 / d$ and storage coefficient $S = 0.001$. Because we consider the fluctuation of the head to be superposed on the mean head, we are only interested in the head s relative to the mean head at every location, that is, $s(x,t) = h(x,t) - \overline{h}(x)$. This head fluctuation s obeys following expression:

$$s = Ae^{-ax}\cos(\omega t - ax), \quad a = \sqrt{\frac{\omega S}{2kD}}, \ \omega = \frac{2\pi}{T}$$

Notice that the storage coefficient is capital S and the head relative to the mean head is lowercase s.
1) Explain the parameters in the expression and given their dimension
2) What is the amplitude of the groundwater head fluctuation, that is, the amplitude of s, in the aquifer at 500 m and at 1000 m from the ocean?
3) What is the delay of the head wave at 500 m and 1000 m relative to the ocean tide? Hint: compute the velocity of the tidal wave in the aquifer and then the time until the peak of the wave starting at the ocean reaches $x = 500$ m and $x = 1000$ m.

Question 3:

Consider a well in an infinite water table (phreatic) aquifer. Drawdowns are considered small compared to the thickness of the aquifer, so that $kD = 900\, m^2 / d$ may be considered constant. The specific yield, $S_y = 0.15$, is also constant. As there are no boundary conditions, the drawdown by the well follows the Theis equation. An approximation of which is

$$s = \frac{2.3Q}{4\pi kD} \log\left(\frac{2.25kDt}{r^2 S}\right)$$

1) Derive a mathematical expression for the so-called radius of influence, that is, the distance beyond which the drawdown can be neglected at a given time t after the well was first switched on.

2) As you can see, the drawdown is a logarithmic function in time. Derive a mathematical expression of the increase of the drawdown per log cycle, that is, between for instance *t*=6d and *t*=60d, or *t*=2d and *t*=20d.

3) Assume the well has been continuously pumping for time $t = t_1$, after which the extraction was stopped. What is the drawdown at distance r_0 at time is $t = t_1 + \Delta t$, where Δt is any time passed since t_1.

Exam Hydrogeology Module: Transient Groundwater Flow Course 2015/2016

Written exam held on Feb. 1, 2016

Question 1: (16 points)

1. Explain loading efficiency, LE
2. Explain the barometer efficiency, BE
3. What is the difference registered by a pressure gauge in a confined aquifer measuring absolute pressure, given on the one hand a uniform mass placed at ground surface of weight Δp N/m^2 and on the other hand a barometer increase of the same value of Δp N/m^2?
4. What is the origin of delayed yield?
5. In which case does the influence of tide reach further inland into an aquifer?
   1) The case with the higher or with the lower frequency?
   2) The case with the larger or the smaller transmissivity kD?
   3) The case with the larger or with the smaller storage coefficient S?
6. What is the difference between the situations with the wells that were studied by Theis and by Hantush?
7. Does the Theis case have a final equilibrium drawdown? Explain your answer.
   8. Does the Hantush case have a final, steady-state drawdown? Explain your answer.

Question 2: (14 points)

1. Explain what is the radius of influence of an extraction well in an aquifer of constant transmissivity and storage coefficient?
2. The simplified Theis solution is as follows:
   $$s(r,t) \approx \frac{Q}{4\pi kD} \ln\left(\frac{2.25kDt}{r^2 S}\right)$$
   From it derive an expression of the radius of influence.
3. Also show what is the drawdown difference per log cycle of time, that is, between time is t and time is $10t$.
4. Consider a well in a water table aquifer at 300 m from an impervious wall that reaches to the bottom of the aquifer. The aquifer has $kD = 600$ m^2/d and the specific yield of $S_y = 0.2$. The pumping rate is Q=1200 m2/d. Assume that the approximation of the Theis equation that is given in this question is applicable. Compute the head change of the groundwater at the wall closest to the well.

1 Question 3?

Exam Hydrogeology, Module Transient Groundwater Flow Course 2014/2015

Question 1:

1. What types of storage or storage coefficients are associated with transient groundwater flow? And explain short how they physically work.
2. Explain the relation between capillary rise and pore diameter
3. Explain the general shape of the moisture curve in the unsaturated zone. Describe where the water comes from when the water table is lowered.
4. Explain the difference between the loading efficiency (LE) and the barometer efficiency (BE)?
5. When you see animal holes in the field, like rabbit, rat and worm holes, how much do you think these holes may contribute to the infiltration of rainwater during and after showers, to what extent are the animals living in those holes affected by heavy rains, and , finally, what would it take to swim them out of their holes? Explain your answer from your insight in how water in the subsurface behaves.

Question 2:

Consider a confined aquifer in direct contact with the ocean in which the head fluctuates along with the tide of the ocean. The daily solar tide, with cycle time T=12 h or, equivalently, $T = 0.5$ d, has amplitude $A = 2.5$ m and the 4 weekly moon tide, with cycle time T=1/28 d, has amplitude A=1 m. The groundwater head in the aquifer relative to the mean value at time t and distance x from the ocean obeys to the following expression:

$$s = A\,e^{-ax}\cos(\omega t - ax), \quad a = \sqrt{\frac{\omega S}{2kD}}$$

If T is the time required for a complete cycle, then the angular velocity $\omega = 2\pi / T$. Further, kD and S are known to be 900 m^2/d and 0.001 respectively.

1. Explain the parameters in the expression and give their dimension.
2. What is the amplitude of the groundwater fluctuation due to both tides individually at 500 and 2000 m from the coast? So the twice-a-day tide amplitude at 500 m and at 2000 m and the 28-day tide amplitude at 500 m and 2000 m?
3. How much are the waves of both tides delayed at 500 m from the coast?
4. Over what distance does the maximum tide-induced amplitude in the groundwater declines by a factor of two in both cases?

Question 3:

A groundwater table rise after it was agitated by a sudden recharge N [m] will decay over the thereafter. For a system of bounded by two parallel water courses at L mutual distance, this decline after some time can be approximated by the following expression:

$$s = A\frac{4}{\pi}\cos\left(\pi\frac{x}{L}\right)\exp\left(-\left(\frac{\pi}{L}\right)^2\frac{kD}{S_y}t\right)$$

1. Describe the parameters and given their dimension.
2. Give an expression for the sudden rise *A* caused by a sudden recharge amount equal to N [m]:
3. Describe in a few words what this expression is and does, so what does its graph look like and how does it behave over time.
4. Give an expression of what can be called characteristic time of this system.
5. Derive an expression of the half time of this system.
6. Derive an expression for the discharge of this system.

Question 4:

A 300 m deep well in Jordan with borehole radius $r=0.25$ m was drilled in a limestone aquifer to serve a refugee camp. The well was recently test pumped during one day at a rate of $Q=60$ m3/h. The head at 0, 0.01, 0.1 and 1 d after the start of the pump was 100, 135, 147 and 159 m below ground surface respectively. The pump is installed at 200 m below ground surface.

Further assume:
The estimated specific yield of this aquifer is 0.01.
The unknown transmissivity is constant.
You may use the simplified expression of transient drawdown in an infinite aquifer

$$s = \frac{Q}{4\pi kD}\ln\left(\frac{2.25kDt}{r^2S_y}\right)$$

1. Estimate the transmissivity of this aquifer.
2. How much will be the drawdown after 3 years (1000 d)? Is the pump at 200 m below ground surface (i.e. 100 m below the initial water table) still deep enough to pump the water up?
3. Another well of equal size, depth and flow rate is planned at a second location in the camp at 2 km distance. How much will be the drawdown in each well after 3 years (1000 days) in this case? Assume that both wells pump for the same period. How deep should the pumps be installed to allow pumping both wells at he given rate for 3 years?

Re-exam Hydrogeology, Module Transient Groundwater Flow Course 2014/2015

Question 1

6. Explain what barometer efficiency (BE) is and how it physically works?
7.
8. When you see animal holes in the field, like rabbit, rat and worm holes, how much do you think these holes may contribute to the infiltration of rainwater during and after showers, to what extent are the animals living in those holes affected by heavy rains, and , finally, what would it take to swim them out of their holes? Explain your answer from your insight in how water in the subsurface behaves.

Question 2

Consider an aquifer in direct contact with the ocean. The tide of the ocean has an amplitude $A = 1.0$ m and cycle time is $T = 0.5$ d (one full tide in 12h).
The aquifer is confined. It consists of two parts. The first part reaches from the ocean to 500 m in land, the second part is present at more than 500 m from the ocean.
The first part of the aquifer has the following properties: transmissivity $kD = 900$ m^2/d and storage coefficient $S = 0.002$.
The second part of the aquifer has the following properties, $kD = 1800$ m^2/d and storage coefficient $S = 0.001$.
Because we consider the fluctuation of the head to be superposed on the mean head, we are only interested in the head s relative to the mean head at every location, that is $s(x,t) = h(x,t) - \overline{h}(x)$. This head fluctuation s obeys following expression:

$$s = A e^{-ax} \cos(\omega t - ax), \quad a = \sqrt{\frac{\omega S}{2kD}}, \quad \omega = \frac{2\pi}{T}$$

Notice that the storage coefficient is capital S and the head relative to the mean head is lower case s.

5. Explain the parameters in the expression and give their dimension.
6. What is the amplitude of the groundwater head fluctuation, that is, the amplitude of s, in the aquifer at 500 m and at 1000 m from the ocean?
7. What is the delay of the head wave at 500 and 1000 m relative to the ocean tide? Hint: compute the velocity of the tidal wave in the aquifer and then the time until the peak of the wave starting at the ocean reaches $x = 500$ m and $x = 1000$ m.

Question 3

Consider a well in an infinite water-table (phreatic) aquifer. Drawdowns are considered small compared to the thickness of the aquifer, so that $kD = 900$ m^2/d may be considered constant. The specific yield, $S_y = 0.15$, is also constant. As there are no boundary conditions, the drawdown by the well follows the Theis equation. An approximation of which is

$$s = \frac{2.3\,Q}{4\pi kD}\log\left(\frac{2.25kDt}{r^2 S_y}\right)$$

4. Derive a mathematical expression for the so-called radius of influence, that is, the distance beyond which the drawdown can be neglected at a given time t after the well was first switched on.
5. As you can see the drawdown is a logarithmic in time. Derive a mathematical expression for the increase of the drawdown per log cycle, that is between for instance *t*=6 days and *t*=60 days, or *t*=2 days and *t*=20 days.
6. Assume the well has been continuously pumping for time $t = t_1$, after which the extraction was stopped. What is the drawdown at distance r_0 at time $t = t_1 + \Delta t$, where Δt is any time passed since t_1.

Exam Hydrogeology, Module Transient Groundwater Flow Course 2013/2014

Question 1:

What types of storage or storage coefficients are associated with transient groundwater flow?

9. Explain how these types of storage physically work.
10. Explain the relation between capillary fringe and air entry pressure.
11. Explain the difference between the loading efficiency (LE) and the barometer efficiency (BE)?
12. Why does the specific yield of unconfined aquifers with a shallow groundwater table depend on the depth of the water table?
13. What is halftime when considering decay of a water mound between rivers? How would you describe it?
14. The halftime is determined by the parameters L (distance between the rivers), kD (transmissivity) and the storage coefficient S (or specific yield S_y in the case of unconfined flow). Give a formula for the half time and explain which parameters increase the halftime and which parameters decrease the halftime.

Question 2

Consider a confined aquifer in direct contact with the ocean in which the head fluctuates along with the tide of the ocean. The tide has an amplitude of $a = 2.5$ m. The groundwater head in the aquifer at time t and distance x from the ocean obeys to the following expression:

$$s = a\,e^{-ax}\cos\left(\omega t - ax\right), \quad a = \sqrt{\frac{\omega S}{2kD}}$$

The frequency f of the tide is one complete cycle per 24 hours, i.e. $f = 1$/d, with, of course, $\omega = 2\pi / f$.

We don't know the value of kD and S, but we have measured the amplitude of the groundwater head fluctuation at $x = 500$ m. This amplitude is 25 cm, one tenth of that of the ocean.

8. Explain the parameters in the expression and give their dimension.
9. Give an expression for the amplitude at distance x from the ocean.
10. With the given information, compute parameter a, and the diffusivity of the aquifer, i.e. the ratio kD / S .
11. Give an expression for the velocity of the wave of the groundwater-head in the subsurface? How much is this velocity?

Question 3

Consider an unconfined aquifer with conductivity $k=10$ m/d, a specific yield of $S_y=0.1$ and an initial thickness $h=20$ m. A well is located in this aquifer on each of the four corners of a square with sides of $L=200$ m. The wells start pumping at $t=0$. They pump with a rate of $Q=120$ m^3/d for 1d, after which they stop.

The drawdown according to Theis is

$$s=\frac{Q}{4\pi kD}W(u), \qquad u=\frac{r^2S}{4kDt}$$

The Theis well function is graphically given in Figure 1 below.

1. Compute the drawdown in the center of the square after $t=2$ d. You may neglect the change of the transmissivity caused by the change of the water depth in the aquifer.

Question 4

Given that the well function can be computed by the following infinite series

$$W(u)=-\gamma-\ln u+u-\frac{u^2}{2\times 2!}+\frac{u^3}{3\times 3!}-\frac{u^4}{4\times 4!}+\ldots$$

with $\gamma=0.577216$, and $u=r^2S/(4kDt)$

1. What would be a good approximation of the drawdown for small values of u? (Assume for instance that u<0.01). Notice for mathematical convenience, that $\gamma=\ln\left(e^\gamma\right)$.
2. How could you define the radius of influence of the drawdown? Use the formula for the drawdown from the previous question together with the approximation from the previous question.

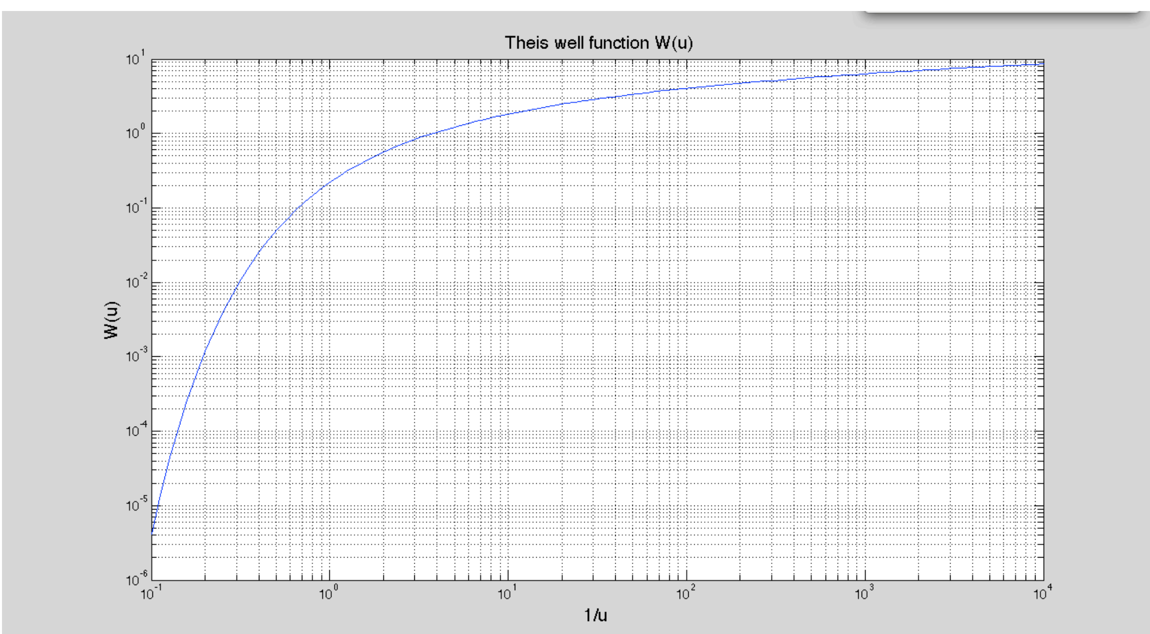


Figure 1: Theis' well function W(*u*) versus 1/*u*

Exam 2013

Only available in a separate pdf file, could not be transferred to Microsoft Word format

Exam 2012

Only available in a separate pdf file, could not be transferred to Microsoft Word format

Exam Transient Groundwater Flow, February 3, 2011

Time 1 hour

Question 1: Pressure in confined aquifer

A water level in a piezometer in a confined aquifer is affected if the weight on a ground surface is suddenly changed. Compare two situations a) Sudden change by a load placed on the ground, such as sand or flooding by water and b) Sudden increase of the barometric pressure.

Case a: --- a load is placed on ground surface

1. How does the water pressure change in the piezometer? (Up? Down? not?)
2. How does the head change in the piezometer? (Up? Down? Not?)

Case b: --- barometer pressure increased

3. How does the water pressure change in the piezometer? (Up? Down? Not?
4. How does the head change in the piezometer? (Up? Down? Not?)

General:

5. If there is a difference between the two cases, then why is that?

Question 2

The groundwater head variation in a confined aquifer due to a tidal wave at *x=0* can be expressed mathematically as follows

$$s(x,t) = \phi(x,t) - \phi_0 = A\exp(-\alpha x)\sin(\omega t - ax)$$

in which $\alpha = \sqrt{\frac{\omega S}{2kD}}$ and, of course, $\omega T = 2\pi$ with *T* the period of the wave.

1. What is the amplitude of the wave at distance x?
2. What is the velocity of the wave?
3. If the wave would be just observable in a piezometer at x=1000 m from the coast, then at what distance would the wave be just observable on another spot along the coast where the storage coefficient is be 100 times greater than at the current spot and the transmissivity is the same?
4. A tidal wave occurring daily is just observable in the aquifer at a distance of x=500 m from shore, where x=0. At what distance from the shore will a 14-day wave be just observable occurring due to the monthly moon cycle? Assume the same amplitude for both waves.

Question 3

In class we discussed the somewhat complicated solution by series expansion of the evolution of the head after a sudden rain shower of P [m] in a strip of land of width L [m] between parallel fixed-head boundaries with water level ϕ_0 [m]. We have seen that after some time t [d], only the first term matters, which is

$$s(x,t) = \phi(x,t) - \phi_0 = \frac{P}{S_y}\frac{4}{\pi}\cos\left(\pi\frac{x}{L}\right)\exp\left(-\pi^2\frac{kD}{L^2 S_y}t\right)$$

Whenever possible express your answers mathematically:

- **a** **What is the shape of the head?**
- **b** **What is the maximum head, take ϕ_0=0?**
- **c** **What would you consider the characteristic time of this system?**
- **d** **What would be the half-time of this groundwater system?**

Question 4

The solution by Theis is given by

$$s(r,t) = \phi_0 - \phi(r,t) = \frac{Q_0}{4\pi kD}W(u), \qquad u = \frac{r^2 S}{4kDt}$$

- **e** **What flow conditions are described by Theis' well solution?**
- **f** **What are its parameters and what are their dimensions?**

As you know, the function W(u) is the exponential integral, which may be written as a series expansion :

$$W(u) = -0.577216 - \ln(u) + u - \frac{u^2}{2\times 2!} + \frac{u^3}{3\times 3!} - \frac{u^4}{4\times 4!} + \ldots$$

- **g** **How can you mathematically approximate the Theis' solution for very small values of *u* given that $-0.577216 = \ln(0.5615)$?**
- **h** **With this approximation, mathematically give the difference between the drawdown obtained at time *t* and the drawdown at time 10*t* in a piezometer at some arbitrary distance *r* from the well.**

You may use the fact that $\ln(10) = 2.3$.

- **i** **Also give the difference between the drawdown in a piezometer at distance *r* from the well and in a piezometer at distance 10*r* from the well, both at the same time.**

Exam: Transient Groundwater Flow, Exam Feb 2010

Question 1:

1. What is liquefaction?
2. What is the difference between specific yield and elastic storage?
3. How does the specific yield change if an already shallow water table rises?
4. Why does this happen (make a sketch and explain)
5. Which of the two materials, gravel and fine sand, has the highest specific yield and why (assume both have the same porosity).

Question 2:

The head in an aquifer connected to the ocean fluctuates due to tide. This fluctuation is given by the following formula, in which s expresses the head variation caused by the tide as a function of time t and the inland distance from the shore x:

$$s(x,t) = A\exp(-\alpha x)\sin(\omega t - \alpha x)$$

With $\alpha = \sqrt{\dfrac{\omega S}{2kH}}$

1. What is the expression for the maximum head fluctuation as a function of x?
2. Sketch the head change s as a function of x at time t=0 and sketch also the envelope (maximum and minimum value of s as a function of x)
3. Which parameters increase the inland penetration of the tide and which parameters decrease this inland penetration?

Question 3:

Consider an extraction canal in direct contact with an aquifer of infinite extent. The aquifer has transmissivity kH=400 m2/d and specific yield Sy =0.1. As long as t<0, the head in the aquifer is everywhere 0 m (we take the initial water level as our reference level).

At time t=0 d, the water level in the canal suddenly changes to 2 m. Then, at time t=2 d, the water level in the canal suddenly changes back to its original value of 0 m and remains constant afterwards.

The head change and the head-change gradient are:

$$s = s_o \operatorname{erfc}\left(\sqrt{\frac{x^2 S}{4kDt}}\right) \text{ and } \frac{\partial s}{\partial x} = -s_o \sqrt{\frac{S}{\pi kDt}} \exp\left(-\frac{x^2 S}{4kDt}\right)$$

To obtain values for the **erfc** function, use the graph below.

Answer the following two questions.

1. Compute the head at x=100 m at t=3 d. Show the formula you use and include the dimension in your answer!

2. Compute the discharge at x=0 at t=3 d. Show the formula you use and include the dimension in your answer?

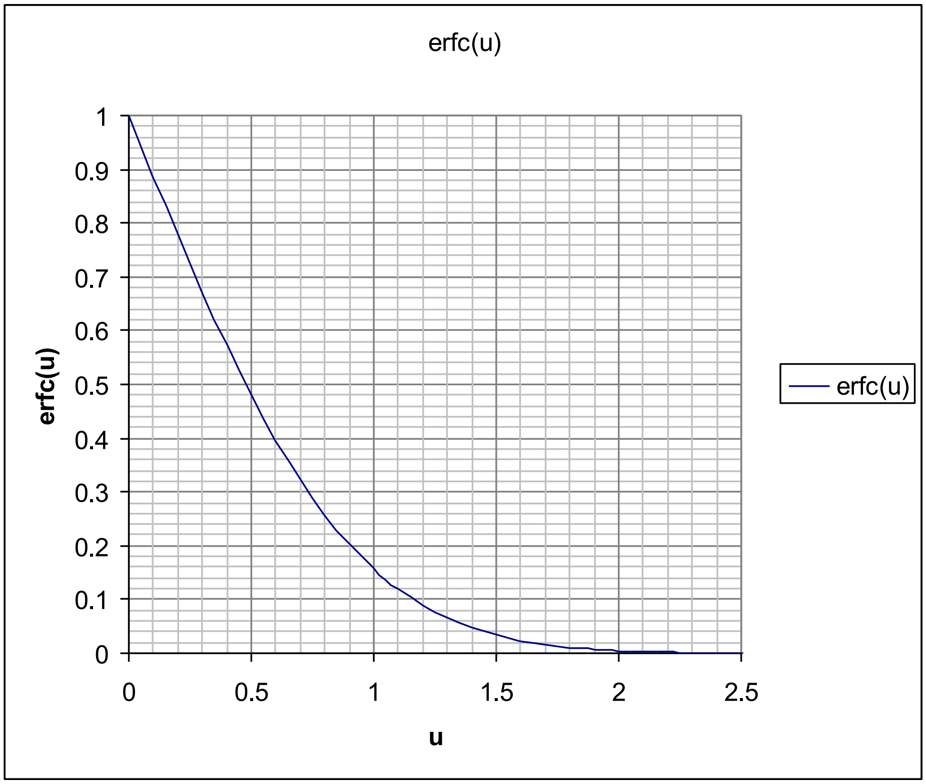


Figure 2: The function erfc(u)

Question 4:

Consider a well in a system of infinite extent which starts extracting at time t=0. We know that Theis' formula applies:

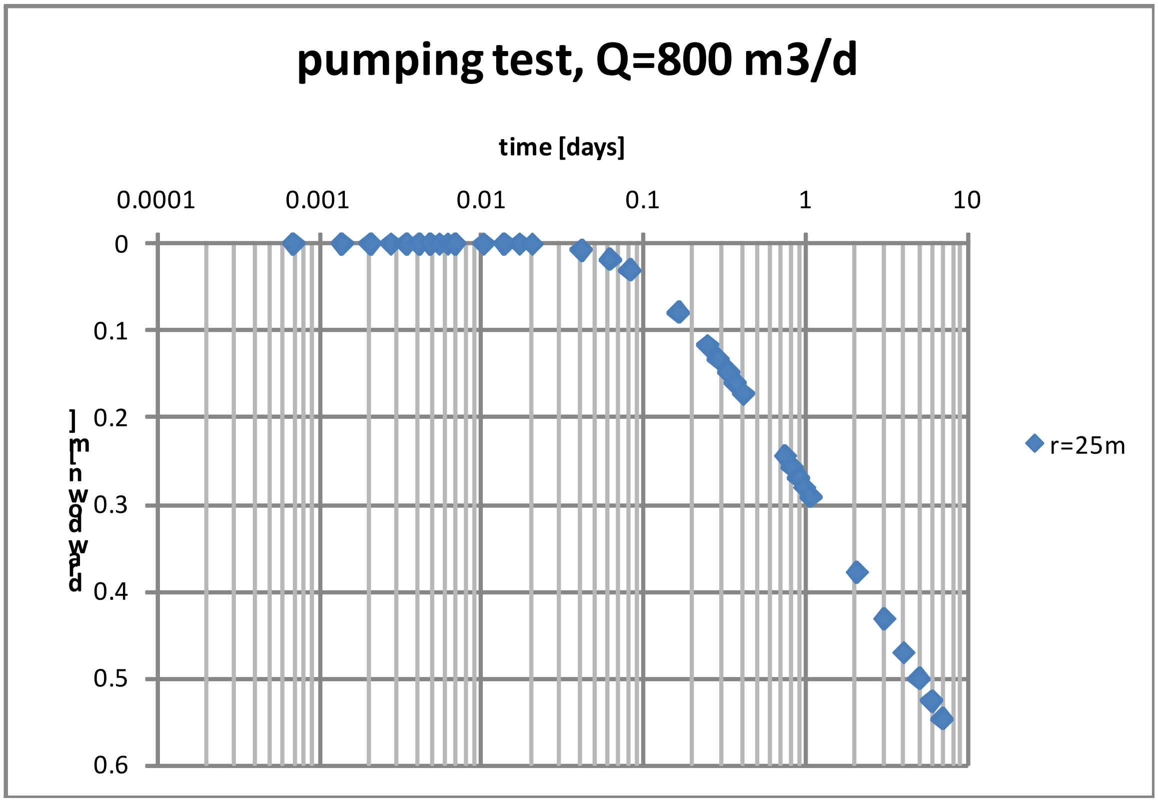
$$s(r,t) = \frac{Q}{4\pi kH} \mathrm{W}(u), \qquad u = \frac{r^2 S}{4kHt}$$

We also know that for small values of u, the well function, W(u), can be approximated by a straight line on log-t scale, which is given by:

$$\mathrm{W}(u) \approx 2.3\log\left(\frac{0.5625}{u}\right) = 2.3\log\left(\frac{2.25kHt}{r^2 S}\right)$$

Consider a pumping test on this well, starting the constant extraction Q=800 m^3/d at t=0. The drawdown is measured over a number of days at an observation well at 25 m distance. The measured drawdowns are shown in the figure below, which clearly reveals the straight portion of the drawdown that we expect from the expression above for large-enough values of time.

1. Using the straight line through the measured data, compute the transmissivity kH and the storage coefficientx S of this aquifer


pumping test, Q=800 m3/d
time [days]
0.0001
0.001
0.01
0.1
1
10
0
0.1
0.2
0.3
0.4
0.5
0.6
drawdown [m]
r=25m

Exam: Transient Groundwater Flow, Exam Feb 2009

Question 1:

1. What is the difference between specific yield and elastic storage
2. How does the specific yield change if an already shallow water table rises further and becomes even shallower?
3. Why does this happen (make a sketch and explain)
4. Which of the two materials, gravel and fine sand, has the highest specific yield and why (assume both have the same porosity).

Question 2:

1. What do we mean by Loading Efficiency (LE) and what do we mean by Barometric Efficiency (BE)?
2. What is the difference in terms of head change if we compare a loading on land surface with an equal increase of the barometer pressure? And why?

Question 3:

Tidal flow in a confined aquifer may be described mathematically by

$s = A e^{-\alpha x} \sin\left(\omega t - \alpha x\right)$, where $\alpha = \sqrt{\frac{\omega}{2}\frac{S}{kD}}$

1. What are the different quantities in these expressions and what are their dimensions?
2. By what expression is the envelope given (the envelope describes the maximum amplitude as a function of x?)
3. How does the envelope change if the frequency of the tide would double?
4. How will the envelope change if the transmissivity would be two times less and the storage coefficient 100 times less?

Question 4:

The picture below shows a strip of land of width L bounded by two canals. Both the strip and the canals run perpendicular to the paper (so the picture is a cross section). Suddenly the water level in the left canal is raised by A m as is indicated in the figure. This causes the head to change in the strip. At the right hand side the water level is unchanged. There exists an expression, which mathematically describes the effect of a sudden level rise in a strip that is unbounded on one side. We want to use this expression to computed the head in the strip. We can do this by means of mirror canals.

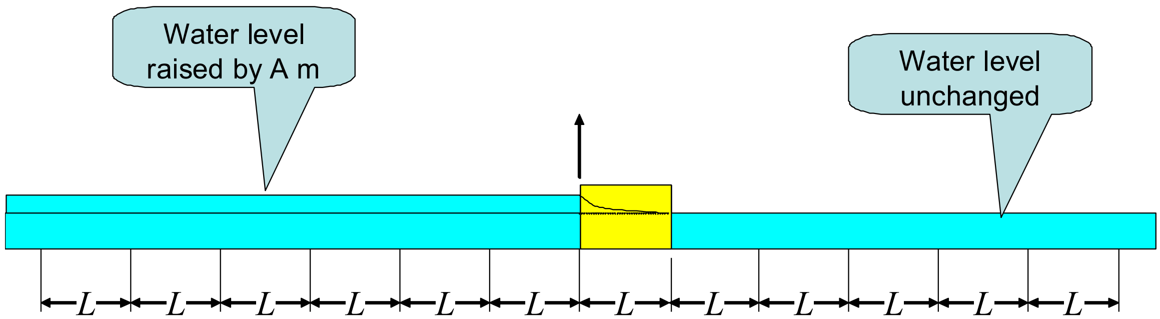
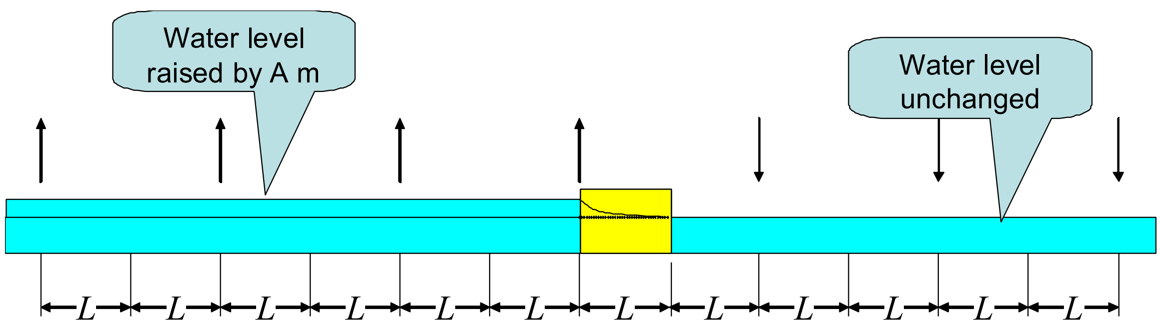


Figure 3: Strip of land of width *L* bounded by open water. The water level at the left hand side was suddenly raised by *A* m. This causes the head in the aquifer of the strip to change dynamically.

1. Irrespective of what the mathematical looks like, where would you put the mirror canals and which are positive and which are negative? Just draw an arrow respectively up or down (see figure) at the locations where you would put the mirror canal.



Question 5:

The characteristic dynamics of a groundwater systems (i.e. the time it takes for the head of a groundwater system to reach equilibrium) is related to the argument of transient groundwater flow solutions, This argument is $\sqrt{\frac{x^2 S}{4kDt}}$ in solutions for one-dimensional flow and $\frac{r^2 S}{4kDt}$ for radial flow such as in the well functions of Theis and Hantush.

1. Explain how the characteristic dynamics relate to these arguments?
2. Compare the characteristic dynamics of two systems. System two is twice as wide as system one and its transmissivity is 3 times as large and its storage coefficient 100 times as small as that of system one. How do the dynamics of these two systems relate to each other, that is: how many times faster or slower is system two compared to system one in reaching piezometric equilibrium?

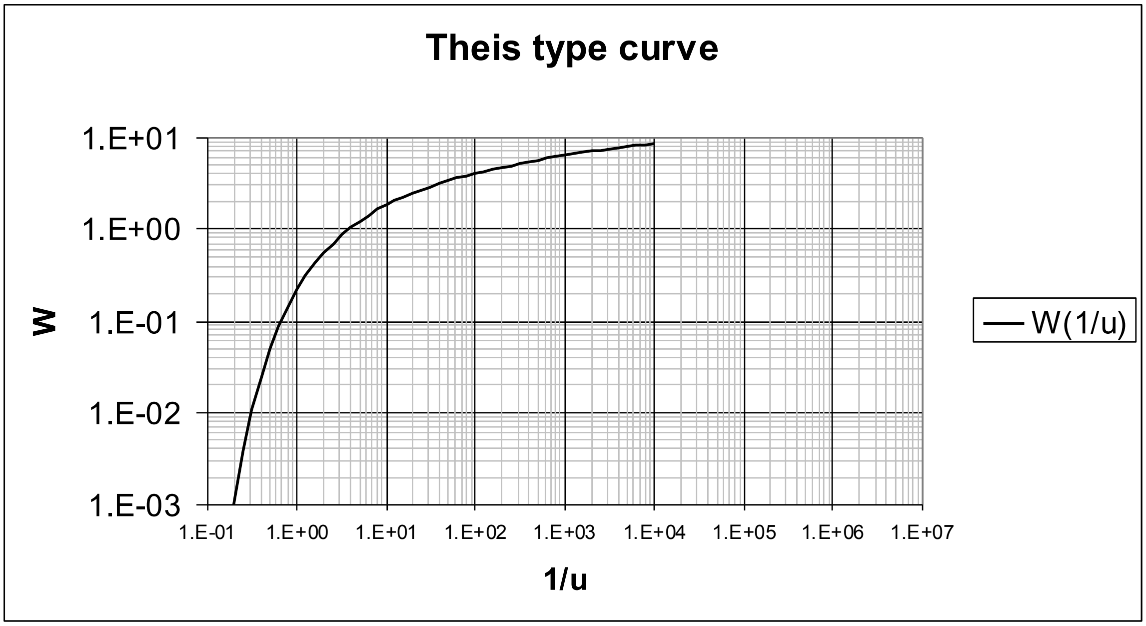
Question 6:

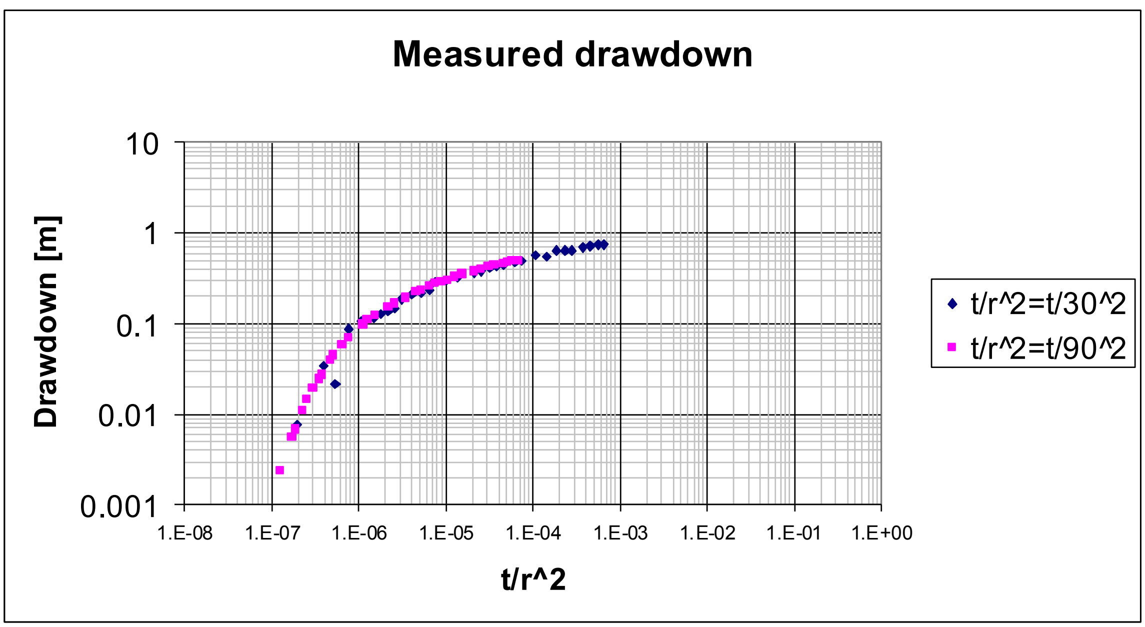
Consider a well in a semi-confined aquifer with kD=900 m^2/d, S=0.001 and c=400 d that is pumped at a discharge Q of 2400 m^3/d.

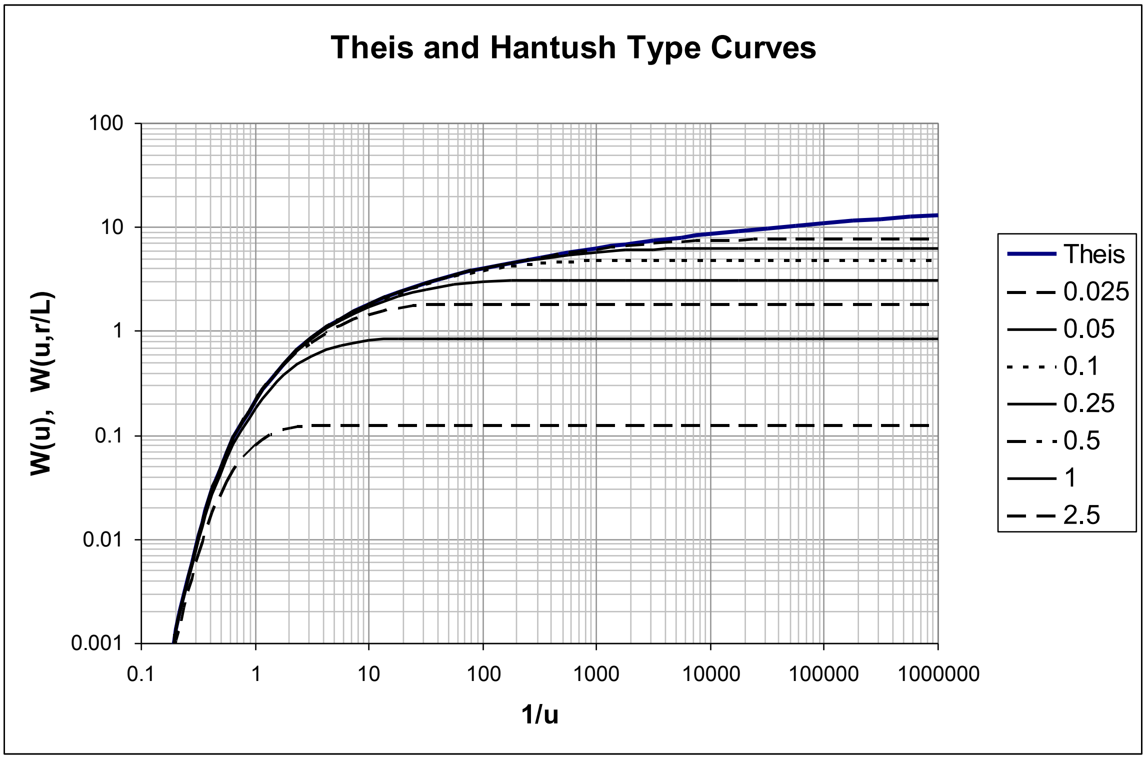
3. How long does it take before the drawdown at 60 m distance form the well becomes stationary?
4. What is the final drawdown?

Question 7:

A pumping test has been carried out in a confined aquifer. The drawdown and the Theis type curves are given in the graphs below. Theses graphs have been drawn on the same type of double logarithmic paper. The extraction of the well during the test was 1000 m^3/d. Determine the transmissivity and the storage coefficient of this groundwater system.






Theis and Hantush Type Curves
100
10
1
0.1
0.01
0.001
W(u), W(u,r/L)
0.1
1
10
100
1000
10000
100000
1000000
1/u
Theis
0.025
0.05
0.1
0.25
0.5
1
2.5

Exam: Transient Groundwater Flow, Exam Feb 2008

Question 1:

5. What is the difference between specific yield and elastic storage
6. How does the specific yield change if an already shallow water table rises further and becomes even shallower?
7. Why does this happen (make a sketch and explain)
8. Which of the two materials, gravel and fine sand, has the highest specific yield and why (assume both have the same porosity).

Question 2:

3. What do we mean by Loading Efficiency (LE) and what do we mean by Barometric Efficiency (BE)?
4. What is the difference in terms of head change if we compare a loading on land surface with an equal increase of the barometer pressure? And why?

Question 3:

Tidal flow in a confined aquifer may be described mathematically by

$s = A e^{-\alpha x} \sin\left(\omega t - \alpha x\right)$, where $\alpha = \sqrt{\frac{\omega}{2} \frac{S}{kD}}$

5. What are the different quantities in these expressions and what are their dimensions?
6. By what expression is the envelope given (the envelope describes the maximum amplitude as a function of x?)
7. How does the envelope change if the frequency of the tide would double?
8. How will the envelope change if the transmissivity would be two times less and the storage coefficient 100 times less?

Question 4:

The picture below shows a strip of land of width *L* bounded by two canals. Both the strip and the canals run perpendicular to the paper (so the picture is a cross section). Suddenly the water level in the left canal is raised by *A* m as is indicated in the figure. This causes the head to change in the strip. At the right hand side the water level is unchanged. There exists an expression which mathematically describes the effect of a sudden level rise in a strip that is unbounded on one side. We want to use this expression to computed the head in the strip. We can do this by means of mirror canals.

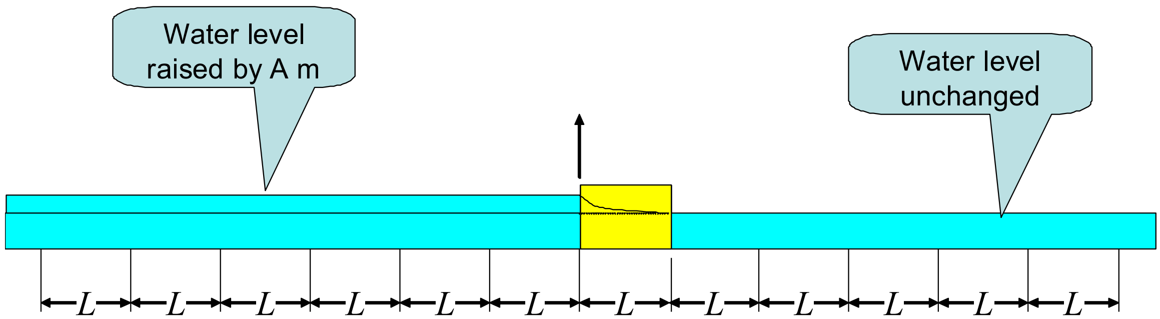
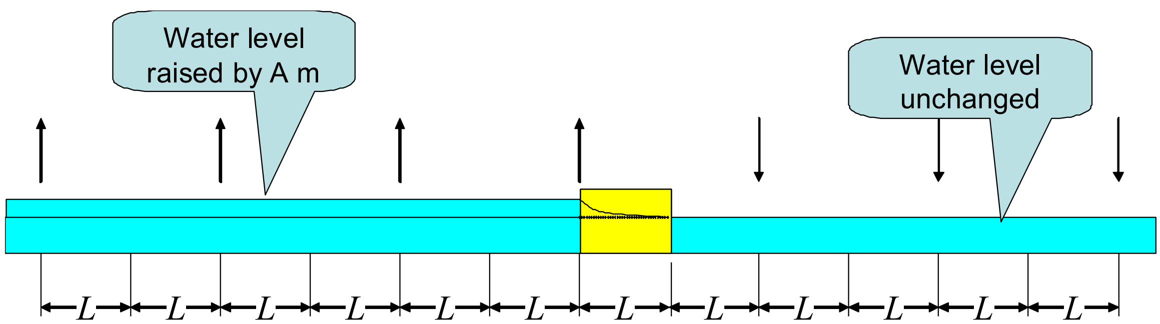


Figure 4: Strip of land of width *L* bounded by open water. The water level at the left hand side was suddenly raised by *A* m. This causes the head in the aquifer of the strip to change dynamically.

2. Irrespective of what the mathematical looks like, where would you put the mirror canals and which are positive and which are negative? Just draw an arrow respectively up or down (see figure) at the locations where you would put the mirror canal.



Question 5:

The characteristic dynamics of a groundwater systems (i.e. the time it takes for the head of a groundwater system to reach equilibrium) is related to the argument of transient groundwater flow solutions, This argument is $\sqrt{\dfrac{x^2 S}{4kDt}}$ in solutions for one-dimensional flow and $\dfrac{r^2 S}{4kDt}$ for radial flow such as in the well functions of Theis and Hantush.

5. Explain how the characteristic dynamics relate to these arguments?
6. Compare the characteristic dynamics of two systems. System two is twice as wide as system one and its transmissivity is 3 times as large and its storage coefficient 100 times as small as that of system one. How do the dynamics of these two systems relate to each other, that is: how many times faster or slower is system two compared to system one in reaching piezometric equilibrium?

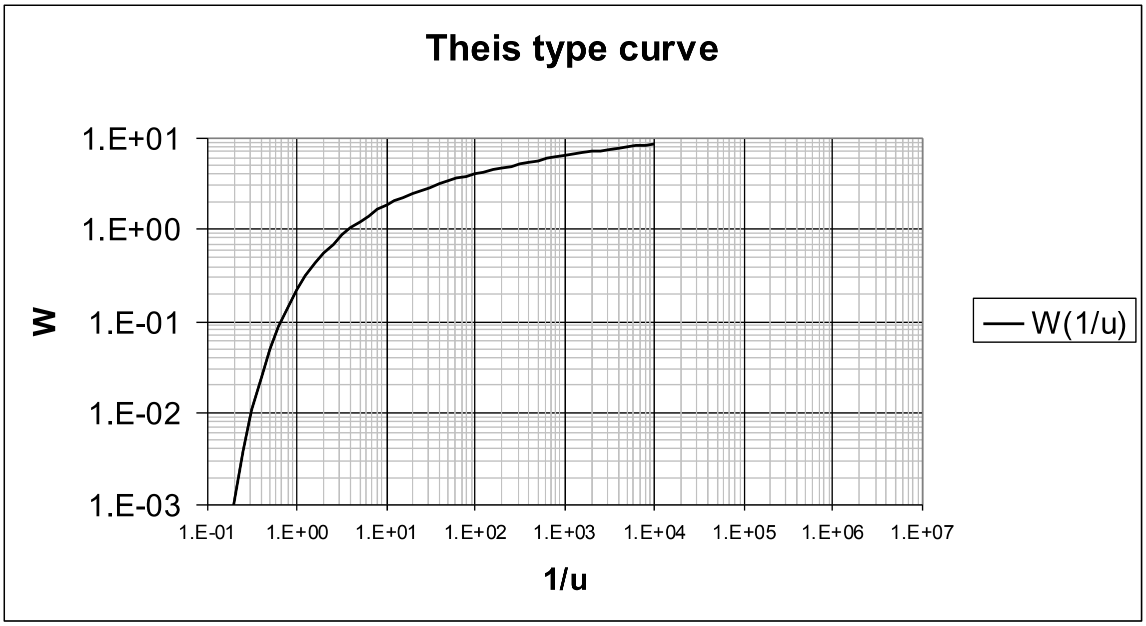
Question 6:

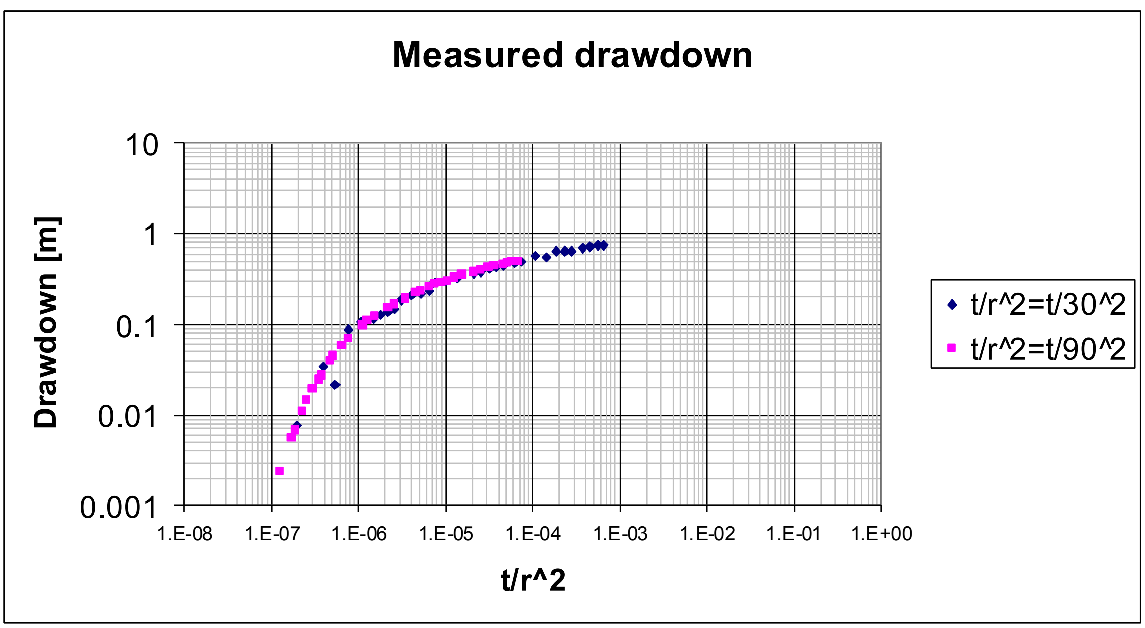
Consider a well in a semi-confined aquifer with kD=900 m^2/d, S=0.001 and c=400 d that is pumped at a discharge Q of 2400 m^3/d.

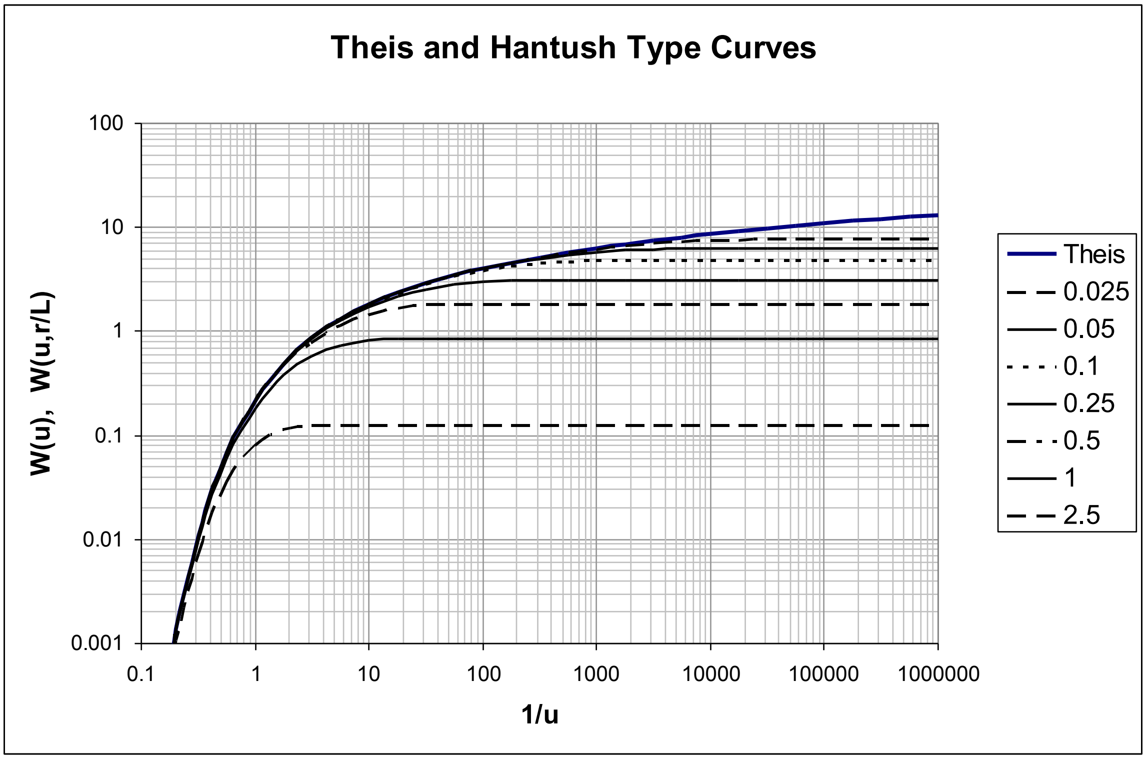
7. How long does it take before the drawdown at 60 m distance form the well becomes stationary?

Question 7:

A pumping test has been carried out in a confined aquifer. The drawdown and the Theis type curves are given in the graphs below. Theses graphs have been drawn on the same type of double logarithmic paper. The extraction of the well during the test was 1000 m^3/d. Determine the transmissivity and the storage coefficient of this groundwater system.



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0.5
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Exam February 2007

Question 1:

What is specific yield?

1. How does specific yield depend on the distance of the water table below ground level?
2. What happens to the water table in a piezometer in a confined aquifer when the barometer pressure goes up, why?

1 Diffusion equation

The diffusion equation for transient flow in one dimension is $D\frac{\partial^2 s}{\partial x^2} = \frac{\partial s}{\partial t}$

1. What is the dimension of the diffusivity D?
2. What is diffusivity D in the case of groundwater flow?
3. What is diffusivity D in the case of heat flow?

2 Fluctuation groundwater

In the case of a tidal fluctuation in a river in direct contact with an aquifer having transmissivity the fluctuation of the head may be described by

$s = s_o \exp(-\alpha x)\sin(\omega t - \alpha x)$, with $\alpha = \sqrt{\frac{\omega S}{2kD}}$

3

What is s and what does this function look like? Make a sketch of s as a function of x, and show its envelopes. (The envelope is the curve of the values between which the function fluctuates, as a function of x).

4. In the case of a double-day tide, $\omega = \frac{4\pi}{24}[h^{-1}]$. What would be the speed of the wave into the aquifer if S=0.001 and kD=500 m^2/d? (Notice the dimensions!)
5. At what distance from the river is the amplitude of the head fluctuation still only half of that in the river at x=0?
6. What happens to this distance in case the transmissivity would be 9 times a big?

4 Flow to an extraction canal

Consider an extraction canal in direct full contact with an aquifer with transmissivity kD=400 m^2/d and specific yield S_y =0.1. The water level in the canal suddenly changes by 2 m downward. The head and gradient are given by:

$s = s_o \operatorname{erfc}\left(\sqrt{\frac{x^2 S}{4kDt}}\right)$, $\frac{\partial s}{\partial x} = -s_o\sqrt{\frac{S}{\pi kDt}}\exp\left(-\frac{x^2 S}{4kDt}\right)$

3. Compute the discharge into the canal after 1d. Show the formula you use and include the dimension in your answer!
4. What is the head change *s* at 100 m from the canal after 1 and after 2 d? (Use erfc-curve further down).
5. What is the head change at 100 m from the canal after 2 days if the head in the river would change back by 2 m at *t*=1d?

5 Well in semi-confined aquifer

Consider a transient well in a semi0-confined aquifer so that Hantush's solution is valid:

$s = \frac{Q}{4\pi kD} \mathrm{W}\left(u, \frac{r}{\lambda}\right)$, $u = \frac{r^2 S}{4kDt}$, $\lambda = \sqrt{kDc}$ with kD=600 m^3/d, c=900 d, S=0.001; pumping at a rate Q=2400 m^3/d.

1. How long does it take before steady state is reached for a point at r=300 m from the well (why)? Use Hantush type curves (see graphic at the end of this exam).

6 Drawdown due to a pumping station in an unconfined aquifer

A well is situated at 100 m from an impermeable infinitely long wall. The well is pumping at a rate of 2400 m^3/d. Even though the aquifer is unconfined, the transmissivity kD may be taken as a constant equal to 600 m^2/d, while the specific yield S_y equals 0.2. The well bore has a radius of 0.25m

1. What is the drawdown at the well bore after 10 days of pumping ?
2. A well in a confined aquifer of infinite extent, with kD=1000 m^2/d and S=0.001, is pumping at a rate of 24000 m^3/d. How far would the radius of influence of this well after 100 years? The radius of influence is the radius beyond which the drawdown is considered negligible. You may exploit the logarithmic approximation of the Theis well function for large times:

$W(u) \approx \ln\left(\frac{0.5625}{u}\right)$, $u < 0.1$ with $u = \frac{r^2 S}{4kDt}$ by making it zero.

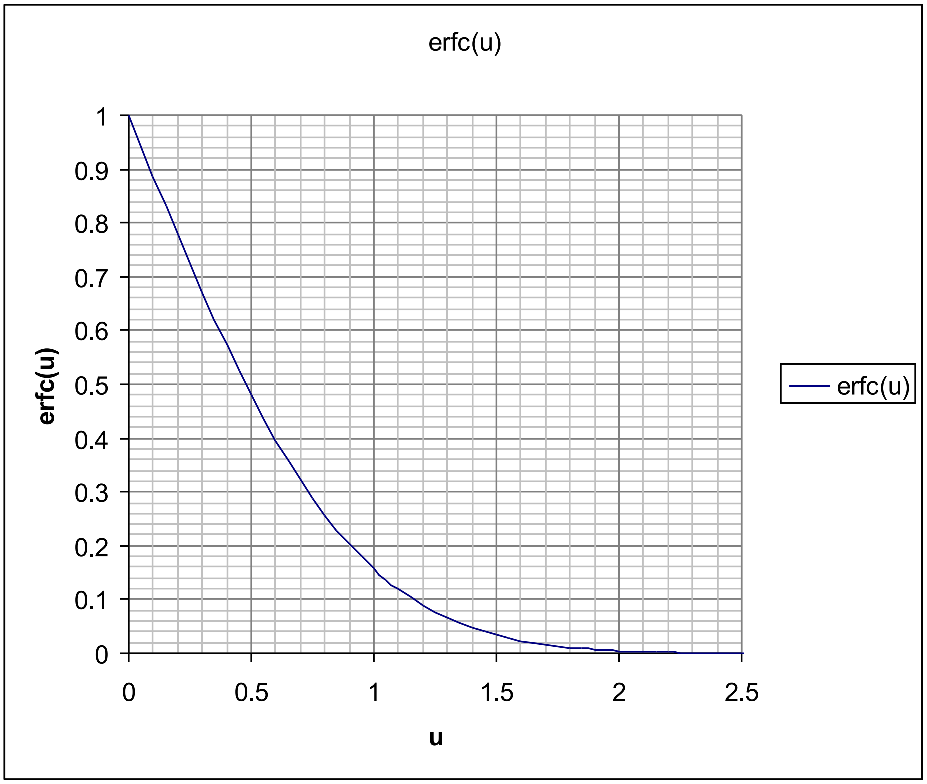


Figure 5: The function erfc(u)

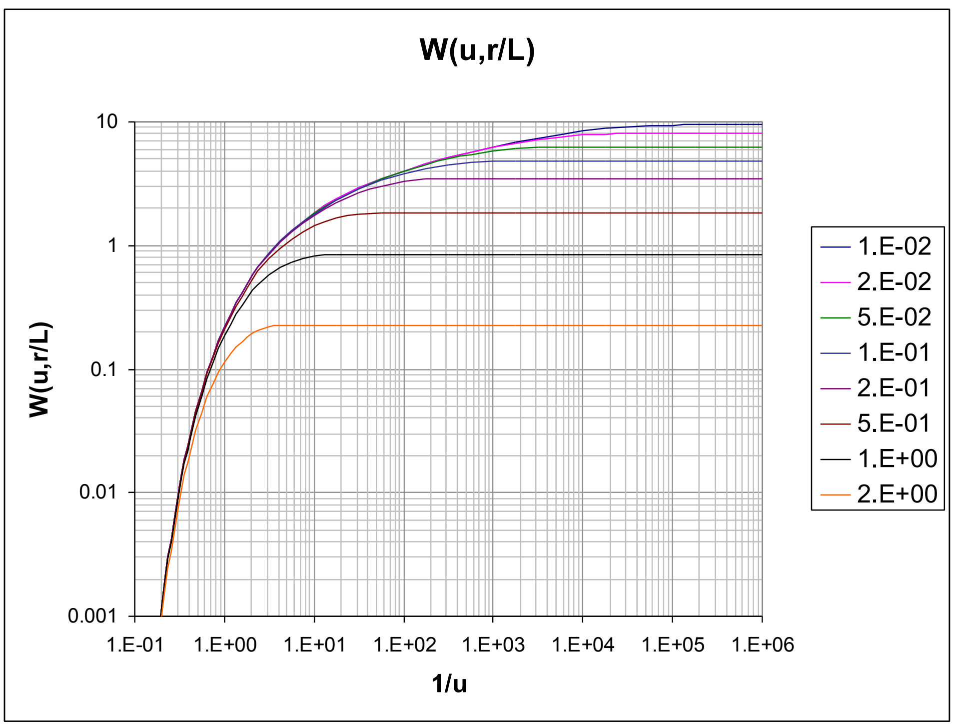


Figure 6: Theis and Hantush type curves. In case this graph is copied in black and white only, note that the lowest type curve is for the highest value of r/λ. Note that the L in the title and left axis of this figure stands for $\lambda = \sqrt{kDc}$ value

Exam module transient flow 2006

Question 1: Conceptual

a. What types of reversible storage do you know in aquifer systems, explain how it works
b. What values may you expect for the respective storage coefficients?
c. What is barometric efficiency, explain how it works.
d. When the barometric pressure increases, does the head (water table in a piezometer) in the confined aquifer rise or fall?
e. Between what values may the barometric efficiency vary?
f. What happens in a confined aquifer with the head if a load is suddenly placed on ground surface, such as a train stopping near a piezometer? What happens when it leaves? Sketch a graph showing the head versus time that you would expect in that case.

1 Characteristic time of groundwater basin

Characteristic time of groundwater basin, the partial differential equation of which reads

$$kD\frac{\partial\phi^2}{\partial x^2}=S\frac{\partial\phi}{\partial t}$$

g. What is a characteristic time of a groundwater basin that may be considered as one-dimensional of characteristic size L? (hint: Make partial differential equation dimensionless by $\xi=\frac{x}{L}, \tau=\frac{t}{T}$ and see what T is.
h. To reach equilibrium, how many times slower is a large basin compared to a small one with the same transmissivity and storage coefficient?
i. Compute the characteristic time for the following cases:
   i. Large basin: kD=500 m^2/d, system width L=100km, storage coefficient S=0.2,
   ii. Small basin: kD=100 m^2/d, system width L=100m , storage coefficient S=0.1

2 Tides in groundwater

Given: The tidal fluctuation in an aquifer in a point at distance x from the sea due to the water level fluctuation at sea with amplitude A is described by the following formula
$$s(x,t) = A\exp(-\alpha x)\sin(\omega t - \alpha x)$$
in which the damping factor is as follows $\alpha = \sqrt{\frac{\omega S}{2kD}}$, where ω is the angle velocity in radians/time or $\omega = \frac{2\pi}{T}$ where T is the time of a complete wave cycle.

Are the following expressions true or false?

j. The wave in the aquifer has a different frequency than the tide itself

k. The amplitude of the wave at a given distance from the sea becomes greater when

   i. the frequency of the tide is reduced

   ii. the storage coefficient is reduced

   iii. then the transmissivity is reduced

3 Aquifer with river

Consider an aquifer of infinite extent bounded by a fully penetrating river at x=0. At t=0 the river level suddenly changes by a height A. The change of head $s(x,t)$ in the aquifer equals in this case:

$s(x,t) = A\,erfc\left(\sqrt{\frac{x^2 S}{4kDt}}\right)$ with erfc(u) as shown in the picture below

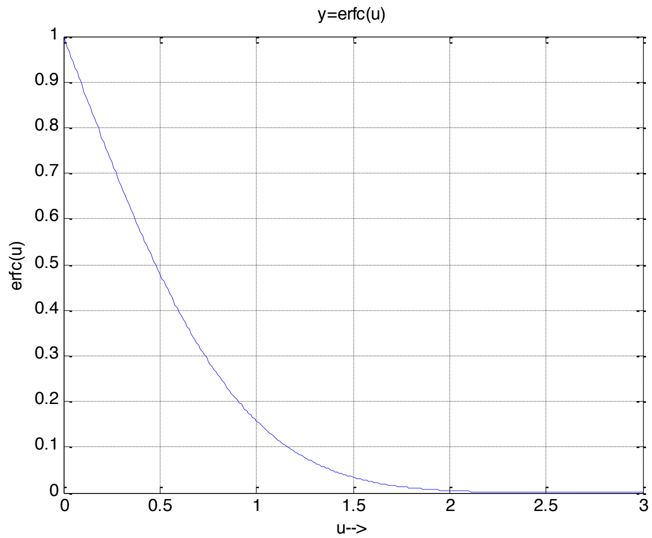


Figure: Erfc(u) versus u

l. What is the final value of the head change (the value reached after infinite time, $s(x,\infty)$?

m. What value has the argument of erfc(-), i.e. $\sqrt{\frac{x^2 S}{4kDt}}$ when the head change is half the final value?

n. If kD=400 m^2/d, S=0.1 and x=100m, after how much time is this the change of head equal to 0.5A?

o. What would be the formula if the head change occurred on time t_1 instead of time t=0?

p. How could you compute the head change at point x if the there was a sudden change of the river level of A_1 at time t=t_1 and another of A_2 at t=t_2?

4 Well in a confined aquifer

Consider a well in a confined aquifer starting an extraction of 1200 m^3/d at t=0. kD=1000 m^2/d, and S=0.001. For this case the Theis solution applies: $s = \frac{Q}{4\pi kD} W(u),\ u = \frac{r^2 S}{4kDt}$

(The type curve of Theis is given on a separate page).

q. Compute the head at r=20m after t=1 day.

r. The pump is switched off after 1 day. What is the head after 1.1 days at r=20 m?

5 Well in a leaky aquifer

Consider a transient well in a leaky aquifer. kD=400 m^2/d, c=400 d, S=0.001, so that the groundwater behaves according to Hantush's transient well formula $s = \frac{Q}{4\pi kD} W\left(u, \frac{r}{L}\right)$ with $L = \sqrt{kDc}$.

s. How long does it take until the head at r=40 m becomes steady state or virtually steady state? (Hint: look at the type curves to get u for which this is the case, note).

6 Well in an unconfined aquifer

Consider a well in an unconfined aquifer for which the Theis-solution applies (see type curve hereafter. Further given a pumping test with an extraction of 600 m^3/d during which drawdown measurements were made (see the graph with the small circles).

Interpret the test (that is: compute kD and S).

(Hint: if you can't see through the paper make the type curve thicker using a pen and hold both curves up against a light or in the direction of a window).

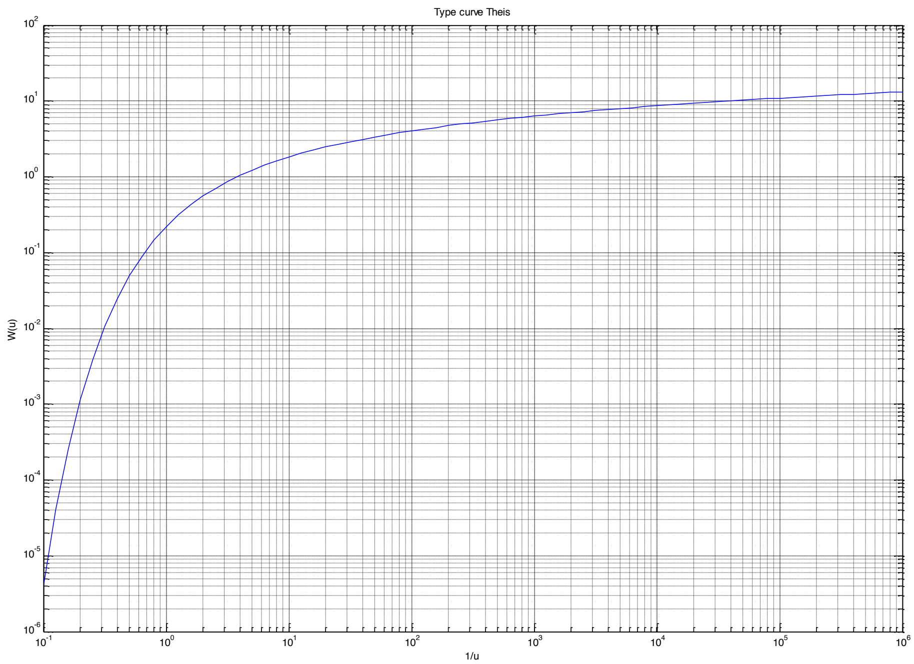


Figure: Theis type curve

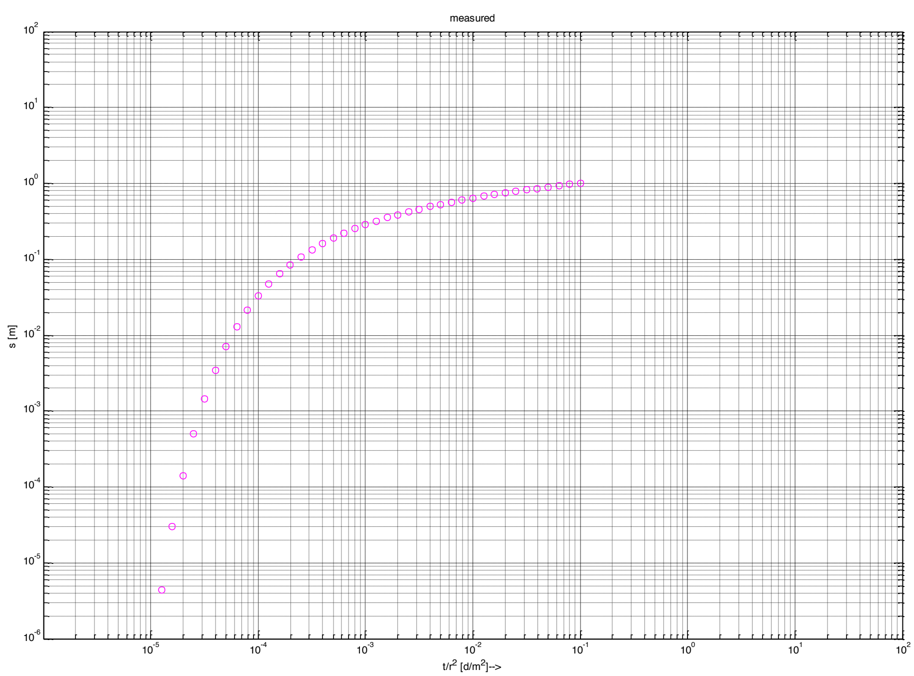
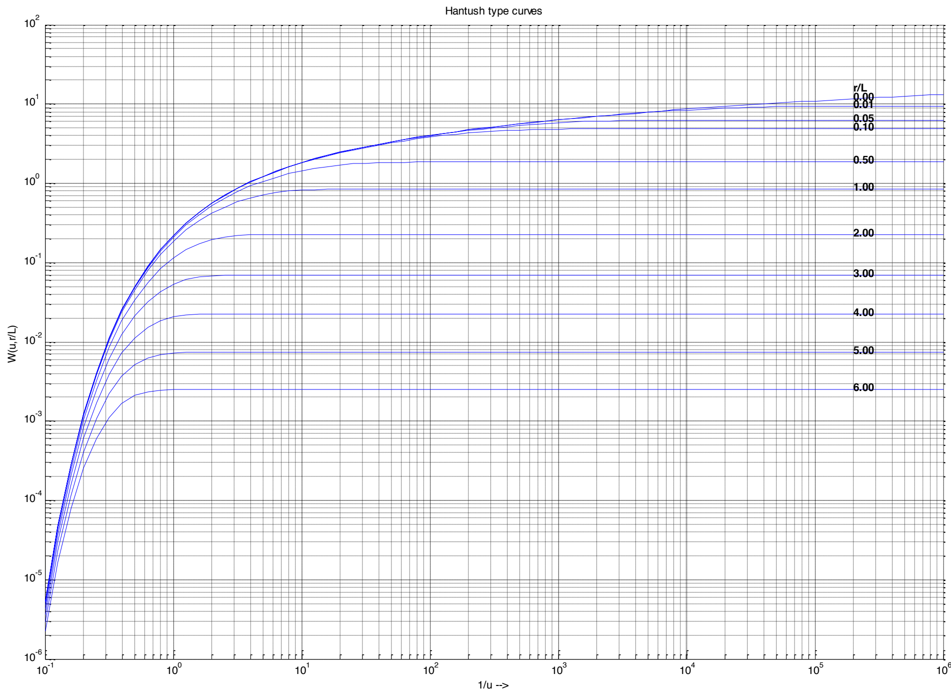


Figure: Measured data of pumping test (drawdown versus t/r^2)


Hantush type curves
W(u,r/L)
1/u -->
r/L
0.00
0.01
0.05
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