Assignment week 06

1. Section 1.7, exercise 06

Use direct proof to show that the sum of two even integer is even

Any even number can be represented as: $2k$

Assume there are two even integers m and n, where $m=2x$, $n=2y$

Then, $m+n=2x+2y=2(x+y)$

Because $2(x+y)$ is in form of $2k$, so $2(x+y)$ is even, so $m+n$ is even

Therefore, for any two even integers, the sum of them must be also even

2. Section 1.7, exercise 8

Prove that if n is a perfect square, then $n+2$ is not a perfect square

Assume $n=m^2$, $n+2=m^2-(-2)=m^2-(\sqrt{2})^2=m^2-(\sqrt{2}i)^2$

$m^2-(\sqrt{2}i)^2=(m+\sqrt{2}i)(m-\sqrt{2}i)$

$m+\sqrt{2}i$ and $m-\sqrt{2}i$ will never equal for any value of m

So $(m+\sqrt{2}i)\neq(m-\sqrt{2}i)$, so m^2+2 will never be a perfect square

Therefore if n is a perfect square, $n+2$ will never be a perfect square

3 section 1.7, Exercise 10

Use a direct proof to show that the production of two rational number is rational

Assume we have two rational number m, n, where $m = \frac{a}{b}$, $n = \frac{c}{d}$

then $m \times n = \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ which is a rational number

therefore, the product of two rational number is also rational

4. If an integer n is divisible by 3, then show that n^2+3n is divisible by 9

Assume $n = 3m$, then $n^2+3n = (3m)^2 + 3(3m) = 9m^2 + 9m = 9(m^2+m)$

therefore if n is divisible by 3, n^2+3n is divisible by 9

5. Prove that if r^2 is irrational, then r is irrational

Assume r^2 is irrational and r is rational, where $r = \frac{a}{b}$

then $r^2 = r \times r = \frac{a}{b} \times \frac{a}{b} = \frac{a^2}{b^2}$ which is rational, and conflict with r^2 is irrational,

therefore, when r^2 is irrational, r must be also irrational

6. Section 1.7, exercise 20

Prove that if n is an integer and $3n+2$ is even, then n is even use

1) proof by contraposition 2) proof by contradiction

1) $A \to B \equiv \neg B \to \neg A$

if n is odd, then $n=2k+1$

$3n+2 = 3(2k+1)+2 = 6k+5$ which is odd

Since we have shown $\neg B \to \neg A$, So $A \to B$ which is if $3n+2$ is even, then n is even

2) prove opposite is false

Assume $3n+2$ is even and n is odd

then we let $n=2k+1$, So we have $3n+2 = 3(2k+1)+2$

$3(2k+1)+2 = 6k+5$ which is odd and conflict with $3n+2$ is even.

Therefore, if $3n+2$ is even, then n must be even

7. Show that there is no smallest integer

Assume n is the smallest integer.

then we have $n-1 \geq n$ because n is the smallest $n-1$ can not be smaller than n

then we have $n-1-n \geq n$, which equal to $-1 \geq 0$

$-1 \geq 0$ is false, then $n-1 \geq n$ is also false $(p \leftrightarrow q, \neg q \leftrightarrow \neg p)$

So $n-1$ not smaller than n is false in any condition, which means $n-1$ is smaller than n

Because $n-1 < n$, So n can never be the smallest integer

8 show that if the sum of two integers is less than 100, then at least one of the number is less than 50

Assume integer m and n, $m+n < 100$, and both of them are not smaller than 50

then we have $m+n < 100$, $m = 50+a$, $n = 50+b$, where a and b are non negative number

then $m+n = 50+a+50+b = 100+(a+b)$, because a and b are non-negative

So $a+b \geq 0$, hencely, $100+(a+b) \geq 100$, which indicate $m+n \geq 100$

$m+n \geq 100$ contradict with $m+n < 100$, So m and n can not be not smaller than 50 same time

therefore at least one of them will be smaller than 50

9 prove that the difference between any irrational number and any rational number is irrational

Assume m is irrational, n is rational and $n = \frac{a}{b}$, and sum of them is rational as $\frac{c}{d}$

$m - n = m - \frac{a}{b} = \frac{c}{d}$

So $m = \frac{c}{d} + \frac{a}{b} \quad \frac{cb}{db} + \frac{ad}{db} = \frac{cb+ad}{db}$

because $\frac{cb+ad}{db}$ is rational which conflicted with m is irrational,

So it is impossible to have m-n being rational number

therefore, the difference between an irration number and an rational number is irrational