Week 9 Assignment

1. a) $f(n) = \pm n$ b) $f(n) = \sqrt{n^2+1}$ c) $f(n) = \frac{1}{(n^2-4)}$

a) is not a function because it may has two outputs for each input

b) is not a function from Z to R because it can not output negative number

c) is not a function from Z to R because it can not input 2 or -2

2. Find the domain of each function, and range

a) the function that assignes to each pair of positive integers the first integer of the pair.

Domain: Z^+ Range: Z^+

b) the function that assignes to each pair of positive integer its largestest decimal digital

Domain: Z^+ Range: Z^+

c) the function that assignes to a bit string the number of one minus the number of zeros in string

Domain: $Z^+, 0$ Range: Z $(1:0, 0:1) \to -1$ $(1:1, 0:0) \to 1$
$(1:0, 0:0) \to 0$

d) the function that assignes to each positive integer the largestest integer not exceeding the square root of the integer.

$(x, f(x)) = (x, \lfloor \sqrt{x} \rfloor)$ $1 \to 1$
$2 \to 1 \cdots$

Domain: Z^+ Range: Z^+

e) the function that assigns to a bit string the longest string of ones in the string

$(x, f(x)) \to (x, \text{longest } 1\cdots \text{ in } x)$ $\quad 0110111011 \to 111$

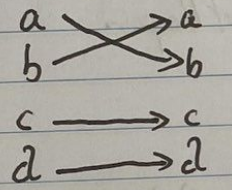
Domain: bit string Range: bit string

3 Which functions are onto? (from $\{a,b,c,d\}$ to $\{a,b,c,d\}$)

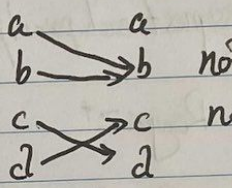
a) $f(a)=b, f(b)=a, f(c)=c, f(d)=d$

b) $f(a)=b, f(b)=b, f(c)=d, f(d)=c$

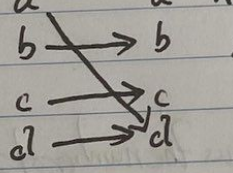
c) $f(a)=d, f(b)=b, f(c)=c, f(d)=d$

a 

one to one and onto

b 

not one to one nd onto

c 

not one to one nd onto

only function a

4 Which functions are onto? (from $\mathbb{Z}$ to $\mathbb{Z}$)

a) $f(n)=n-1$ b) $f(n)=n^2+1$ c) $f(n)=n^3$ d) $f(n)=\lceil n/2 \rceil$

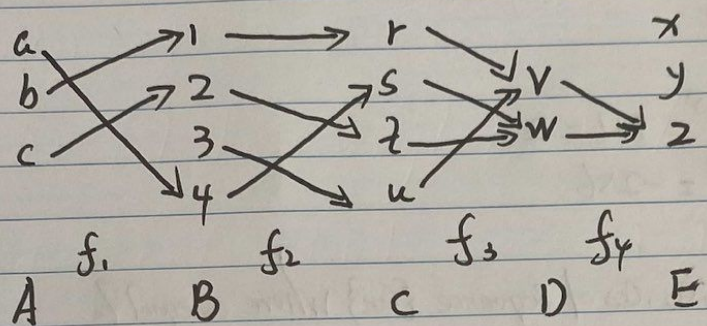
a) Domain: $\mathbb{Z}$, Co-domain: $\mathbb{Z}$, Range: $\mathbb{Z}$, codomain = Range, it is onto

b) Domain: $\mathbb{Z}$, Co-domain: $\mathbb{Z}$, Range: $\{1,2,5,10\cdots\}$, codomain ≠ Range, it is not onto

c) Domain: $\mathbb{Z}$, Co-domain: $\mathbb{Z}$, Range: $\{\cdots -9,-4,-1,0,1,4,9\cdots\}$, co-domain ≠ Range, not onto

d) Domain: $\mathbb{Z}$, Co-domain: $\mathbb{Z}$, Range: $\mathbb{Z}$, co-domain = Range, it is onto

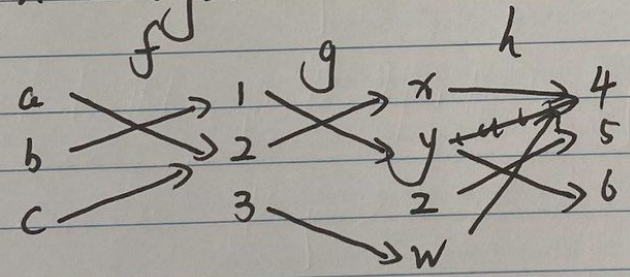
5. $f_1: A \to B$ $f_2: B \to C$ $f_3: C \to D$ $f_4: D \to E$



Are any of them invertable?

f_2 only, because if a function is invertable, it must be bijective

6 $f: A \to B$, $g: B \to C$, $h: C \to D$



find $h \circ g \circ f$

$h \circ g \circ f = h(g(f(x)))$

$h \circ g \circ f(a) = 4$ $h \circ g \circ f(b) = 6$ $h \circ g \circ f(c) = 4$

7. what is the term a_8 of the sequence $\{a_n\}$ if a_n equals

a) 2^{n-1} $a_8 = 2^7 = 128$

b) 7 $a_8 = 7$

c) $1+(-1)^n$ $a_8 = 1+(-1)^8 = 1+1 = 2$

d) $-(-2)^n$ $a_8 = -(-2)^8 = -256$

8 What are the terms of a_0, a_1, a_2, a_3 of sequence $\{a_n\}$ where a_n equal to

a) $(-2)^n$ $a_0 = (-2)^0 = 1$, $a_1 = -2$, $a_2 = 4$, $a_3 = -8$

b) 3 $a_0 = 3$, $a_1 = 3$, $a_2 = 3$, $a_3 = 3$

c) $7+4^n$ $a_0 = 7+4^0 = 7+1 = 8$, $a_1 = 11$, $a_2 = 23$, $a_3 = 71$

d) $2^n+(-2)^n$ $a_0 = 1+1 = 2$, $a_1 = 0$, $a_2 = 8$, $a_3 = 0$