Assignment week 5

8. Let $Q(x,y)$ be the statement "student $x$ has been a contestant on quiz show $y$". Express each of these sentences in terms of $Q(x,y)$, quantifiers, and logical connectives, where the domain of $x$ is all students at your school, for $y$ consist all quiz shows on TV.

a) There is a student your school who has been a contestant of a TV quiz show.

$\exists x \exists y (Q(x,y))$

c) There is a student at your school who has been a contestant on Jeopardy and Wheel of Fortune.

$\exists x (Q(x, \text{Jeopardy}) \land Q(x, \text{wheel of fortune}))$

e) At least two students from your school have been contestants of Jeopardy.

$\exists x \exists z ((x \neq z) \land (Q(x, \text{Jeopardy}) \land Q(z, \text{Jeopardy})))$

two unique students

10. Let $F(x,y)$ be the statement "$x$ can fool $y$", where the domain of main consists of all people in the world. Use quantifiers to express each of these statements

a) Everybody can fool Fred

$\forall x (F(x, \text{Fred}))$

c) Everybody can fool somebody

$\forall x \exists y (F(x,y))$

e) Everyone can be fooled by someone

$\forall x(\exists y(F(y,x)))$

for every $x$ exist any

g) Nancy can fool exactly two people

$\exists x \exists y \forall z\,((x \neq y) \wedge ((x=z) \vee (y=z)) \wedge (F(Nancy,x)) \wedge (F(Nancy,y)))$

two unique exactly two

i) No one can fool himself or herself

$\forall x(\neg(F(x,x)))$

14. For each of these arguements, explain which rules of inference are used each step.

b) "Each of five roomates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathmatic can take a course in algorithms. Therefore, all five roomates can take a course in algorithms next year.

$P(x)$: $x$ is a roomate

$q(x)$: $x$ has taken a course in discrete math

$r(x)$: $x$ can take a course in algorithms

premises: $\forall x(P(x) \rightarrow q(x))$, $\forall x(q(x) \rightarrow r(x))$

using Universal instantiation: $\dfrac{\forall x(P(x) \rightarrow q(x))}{\therefore P(c) \rightarrow q(c)}$ $\dfrac{\forall(q(x) \rightarrow r(x))}{q(c) \rightarrow r(c)}$

using Hypothetical syllogism:

$P(c) \rightarrow q(c)$

$q(c) \rightarrow r(c)$

$\therefore P(c) \rightarrow r(c)$

using Universal generalization: $\dfrac{P(c) \rightarrow r(c) \text{ for all } c}{\therefore \forall x(P(x) \rightarrow r(x))}$

d) "There is someone in this class who has been to France. Everyone who goes to France visits the Louvres. Therefore, some one in this class has visited the Louvres.

$P(x)$ : $x$ in this class room  premises: $\exists x(P(x) \land q(x))$, $\forall x(q(x) \to r(x))$
$q(x)$ : $x$ has been to France
$r(x)$ : $x$ has visited Louvres

using existential instantiation: $\dfrac{\exists x(P(x) \land q(x))}{\therefore P(c) \land q(c) \text{ for some element } c}$

using universal instantiation: $\dfrac{\forall x(q(x) \to r(x))}{\therefore q(c) \to r(c) \text{ for all element } c}$

using simplification, modus ponens: $\dfrac{P(c) \land q(c)}{\therefore q(c)}$  $\dfrac{P(c) \land q(c)}{\therefore P(c)}$  $\dfrac{\begin{array}{c} q(c) \\ q(c) \to r(c) \end{array}}{\therefore r(c)}$

using conjunction: $\dfrac{\begin{array}{c} P(c) \\ r(c) \end{array}}{\therefore P(c) \land r(c)}$

using existential generalization: $\dfrac{\text{there exist } P(c) \land r(c) \text{ for some element } c}{\therefore \exists x(P(x) \land r(c))}$

16. For each of these arguements determine whether the arguement is correct or incorrect and explain why.

b) A convertible car is fun to drive. Issaac's car is not a convertible, therefore, Isaac's car is not fun to drive.

$P(x)$: $x$ is a convertible car    premesis: $\forall x(P(x) \rightarrow q(x)), \neg P(\text{Isacc's car})$

$q(x)$: $x$ is fun to drive

using Universal instantiation: $\dfrac{\forall x(P(x) \rightarrow q(x))}{\therefore P(c) \rightarrow q(c)}$

then, we have:

$$\begin{array}{l} P(c) \rightarrow q(c) \\ \underline{\neg P(\text{Isacc's car})} \\ \therefore \neg q(\text{Isacc's car}) \end{array}$$

then, replace c with Isacc's car:

$$\begin{array}{l} P(\text{Isacc's car}) \rightarrow q(\text{Isacc's car}) \\ \underline{\neg P(\text{Isacc's car})} \\ \therefore \neg q(\text{Isacc's car}) \end{array}$$

This is an invalid arguement as it is fallacy of denying the hypothesis

d) All lobstermen set at least a dozen traps, Hamilton is a lobsterman, Therefore, Hamilton sets at least a dozen of traps:

$P(x)$: $x$ is a lobsterman

$q(x)$: $x$ has set a dozens of traps.

premises: $\forall x(P(x) \rightarrow q(x))$

$P(\text{Hamilton})$

using universal instantiation: $$\frac{\forall x(P(x) \rightarrow q(x))}{\therefore P(c) \rightarrow q(c)}$$

using Modus pones, replace c with Hamilton: $$\frac{\begin{array}{c}P(\text{Hamilton}) \rightarrow q(\text{Hamilton}) \\ P(\text{Hamilton})\end{array}}{\therefore q(\text{Hamilton})}$$

Arguement is vaild