Week 3 Assignment

1. Section 1.3 exercise 16

show that each conditional statement in Exercise 12 is a tautology by applying a chain of logical identities as in Example 8

a) $[\neg p \wedge (q \vee p)] \rightarrow q$ b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

c) $[p \wedge (p \rightarrow q)] \rightarrow q$ d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

a) $[\neg p \wedge (q \vee p)] \rightarrow q = \neg [\neg p \wedge (q \vee p)] \vee q$

$= [p \vee \neg (q \vee p)] \vee q$

$= [p \vee (\neg p \wedge \neg q)] \vee q$

$= [(p \vee \neg p) \wedge (p \vee \neg q)] \vee q$

$= [T \wedge (p \vee \neg q)] \vee q$

$= (T \vee q) \wedge [(p \vee \neg q) \vee q]$

$= T \wedge [p \vee (\neg q \vee q)]$

$= T \wedge [p \vee T]$

$= T \wedge T$

$= T$

b) $[(p\to q)\land(p\to r)]\to(p\to r) = [(\neg p\lor q)\land(\neg p\lor r)]\to(p\to r)$

$= [\neg p\lor(q\land r)]\to(p\to r)$

$= \neg[\neg p\lor(q\land r)]\lor(p\to r)$

$= [p\land\neg(q\land r)]\lor(\neg p\lor r)$

$= [p\land(\neg q\lor\neg r)]\lor(\neg p\lor r)$

$= [(p\land\neg q)\lor(p\land\neg r)]\lor(\neg p\lor r)$

$= (p\land\neg q)\lor(p\land\neg r)\lor(\neg p\lor r)$

$= (p\land\neg q)\lor[(p\land\neg r)\lor(\neg p\lor r)]$

$= (p\land\neg q)\lor[(p\land\neg r)\lor\neg(p\land\neg r)]$

$= (p\land\neg q)\lor T$

$= T$

c) $[p \wedge (p \to q)] \to q \equiv [p \wedge (\neg p \vee q)] \to q$

$\equiv [(p \wedge \neg p) \vee (p \wedge q)] \to q$

$\equiv [F \vee (p \wedge q)] \to q$

$\equiv (p \wedge q) \to q$

$\equiv \neg(p \wedge q) \vee q$

$\equiv \neg p \vee \neg q \vee q$

$\equiv \neg p \vee T$

$\equiv T$

d) $[(p\wedge q)\wedge(p\to r)\wedge(q\to r)]\to r = [(p\wedge q)\wedge[(p\wedge q)\to r]]\to r$

$= [(p\wedge q)\wedge(\neg(p\wedge q)\vee r)]\to r$

$= [(p\wedge q)\wedge(\neg(p\wedge q))\vee((p\wedge q)\wedge r)]\to r$

$= [F\vee[(p\wedge q)\wedge r]]\to r$

$= [(p\wedge q)\wedge r]\to r$

$= \neg[(p\wedge q)\wedge r]\vee r$

$= [\neg(p\wedge q)\vee\neg r]\vee r$

$= \neg(p\wedge q)\vee(\neg r\vee r)$

$= \neg(p\wedge q)\vee T$

$= T$

2) shows that $(p\to q)\to(p\to(p\wedge q))$

$(p\to q)\to(p\to(p\wedge q)) = (p\to q)\to(\neg p\vee(p\wedge q))$

$= \neg(p\to q)\vee(\neg p\vee(p\wedge q))$

$= \neg(p\to q)\vee((\neg p\vee p)\wedge(\neg p\vee q))$

$= \neg(p\to q)\vee(T\wedge(\neg p\vee q))$

$= \neg(\neg p\vee q)\vee(\neg p\vee q) = T$

3. Section 1.3. exercise 64

shows that the negation of an unsatistiable compound proposition is a tautology and the negation of a compound proposition that is tauology is unsatistiable

unsatistiable → Contradiction → always F

$\neg F = T =$ Tautology

Tautology → always T

$\neg T = F =$ Contradiction → unsatistiable

4. Section 1.6 exercise 4

what Rule of inference is used in each of these arguements?

a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

P: kangaroos live in Australia . q: kangroos are marsupials

$\therefore p \wedge q$ The statement is simplification

$\therefore q$

b) It is either hotter than 100 degree today or the pollution is dangerous. It is less than 100 degree today. Therefore the pollution is dangerous.

P: Today is hotter than 100 degree q: The pollution is dangerous

$\therefore p \vee q$ This statement is disjunctive syllogism

$\therefore \neg p$

$\therefore q$

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard
Therefore, Linda is a lifeguard

p: Linda is an excellent swimmer q: Linda can work as a lifeguard

$\because p$ The statement is modus ponens
$\because p \to q$
$\therefore q$

d) Steve will work at a computer company this summer. Therefore, the summer Steve will work at a computer company or he will be a beach bum.

p: Steve will work at a computer company this summer q: Steve will be a beach bum

$\because p$ The statement is addition
$\therefore p \vee q$

e) If I work all night on this homework, then I can answer all the exercises. If I answered all the exercise, then I will understand the material. Therefore, if I work all night on this homework I will understand the material.

$\because p \to q$ This statement is Hypothetical Statement
$\because q \to r$
$\therefore p \to r$

5. Is the following arguement vaild or invaild?

If the rain continues, then the river level rises. If rain continues and the river level rises, then the bridge will wash out. If the continuation of rain would cause the bridge to wash out, then a single road is not sufficient for the town. Either a single single road is sufficient for the town or the traffic engineers have made a mistake. Therefore, the traffic engineers have made a mistake.

p: The rain continues
q: river level rises
r: bridge will wash out
s: single road is not sufficient for the town
t: traffic engernieers have make a mistake

This arguement is vaild.

$$\begin{array}{l} \because p \to q \\ \because q \to r \\ \because (p \to r) \to s \\ \because s \vee t \\ \hline \therefore t \end{array} \quad \to \quad \begin{array}{l} \because p \to q \\ \quad q \to r \\ \hline \therefore p \to r \end{array} \quad \to \quad \begin{array}{l} \because (p \to r) \to s \\ \hline \therefore s \end{array} \qquad \begin{array}{l} \because s \vee t \\ \quad s \\ \hline \therefore t \end{array}$$