CS 2305: Discrete Mathematics for Computing I

Lecture 01

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What is Discrete Mathematics?

- A fairly broad branch of mathematics

Mathematics

Continuous

- **Focus on processes/phenomena that change in a continuous fashion**
- **Infinite in scope: one entity/state blends smoothly into the next**
- **No "holes" between successive values**
  - **Example: real numbers**

Discrete

- **Focus on processes/phenomena that consist of a sequence of individual steps**
- **Finite in scope: abrupt changes in values**
- **"Holes" between successive values**
  - **Example: integers**

Why Study Discrete Mathematics?

- Provides the mathematical background needed for more advanced courses in computer science
  - Concepts, terminologies, methods etc.
- Concepts learned here will be useful in many other disciplines like advanced mathematics, philosophy, economics, linguistics etc.

Kinds of Problems Solved Using Discrete Mathematics[1]

- How many ways can a password be chosen following specific rules?
- How many valid Internet addresses are there?
- What is the probability of winning a particular lottery?
- Is there a link between two computers in a network?
- How can I identify spam email messages?
- How can I encrypt a message so that no unintended recipient can read it?
- How can we build a circuit that adds two integers?

Kinds of Problems Solved Using Discrete Mathematics 2

- What is the shortest path between two cities using a transportation system?
- Find the shortest tour that visits each of a group of cities only once and then ends in the starting city.
- How can we represent English sentences so that a computer can reason with them?
- How can we prove that there are infinitely many prime numbers?
- How can a list of integers be sorted so that the integers are in increasing order?
- How many steps are required to do such a sorting?
- How can it be proved that a sorting algorithm always correctly sorts a list?

What You Will Learn from this Course

- Basic concepts of mathematical logic
- Different formal proof strategies
- Introduction to various *discrete structures* which are used to represent discrete mathematical objects and study their behavior
  - sets, functions, relations, graphs, trees etc.
- Introduction to number theory, with focus on concepts with important computer science applications
- Introduction to algorithms and computational complexity of algorithms

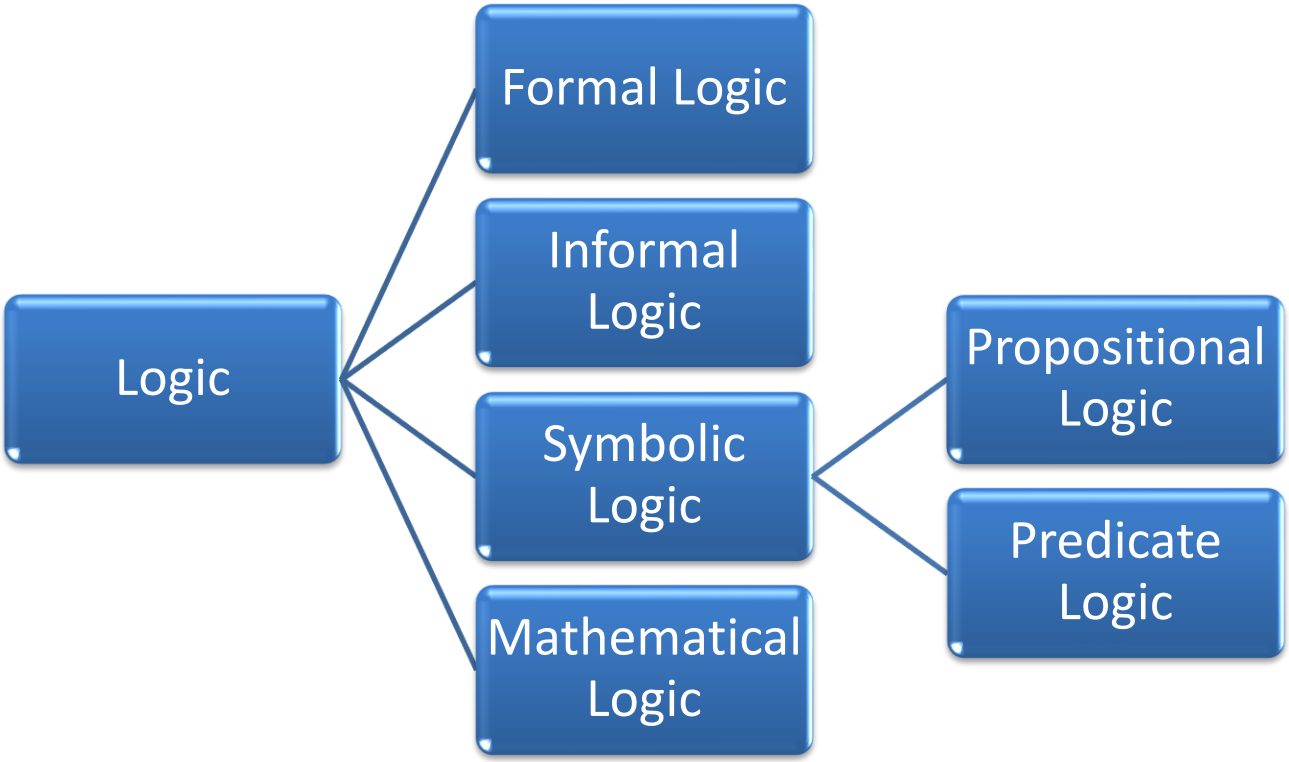
Text Sections Expected to be Covered

- Chapter 1
- Chapter 2 (Sec 2.1 – 2.3)
- Chapter 3 (Sec 3.1 – 3.2)
- Chapter 4 (Sec 4.1. – 4.2)
- Chapter 5 (Sec 5.1 – 5.2, 5.4.1)
- Chapter 6 (Sec 6.1 – 6.3)
- Chapter 9 (Sec 9.1)
- Chapter 10 (Sec 10.1 – 10.5) (as much as time permits)
- Chapter 11 (Sec 11.1) [time permitting]

What is Logic?

- Logic is the study of formal reasoning
  - Major focus is on determining what may be correctly inferred from a given collection of facts
- Statements in a spoken languages are often ambiguous in their meaning but statements in logic have a well defined meaning
- Logic is useful in any field in which it is important to analyze precise statements

Main Branches of Logic



Source: https://www.philosophybasics.com/branch_logic.html

Propositional Logic
(Propositional Calculus)

Propositions

- A *proposition* is a **declarative** sentence that is either true or false, but not both
  - The truth or falsity of a proposition is called its **truth value**
- Examples of propositions:

a) The Moon is made of green cheese.
b) Trenton is the capital of New Jersey.
c) Toronto is the capital of Canada.
d) 1 + 0 = 1
e) 0 + 0 = 2

- Examples that are not propositions.

a) Shut the door.
   - Imperative (command)

b) What time is it?
   - Interrogative (question)

c) What a funny story he told us!
   - Exclamative

d) $x + 1 = 2$
   - May be true or false, depending upon the value of x

Classroom Exercise

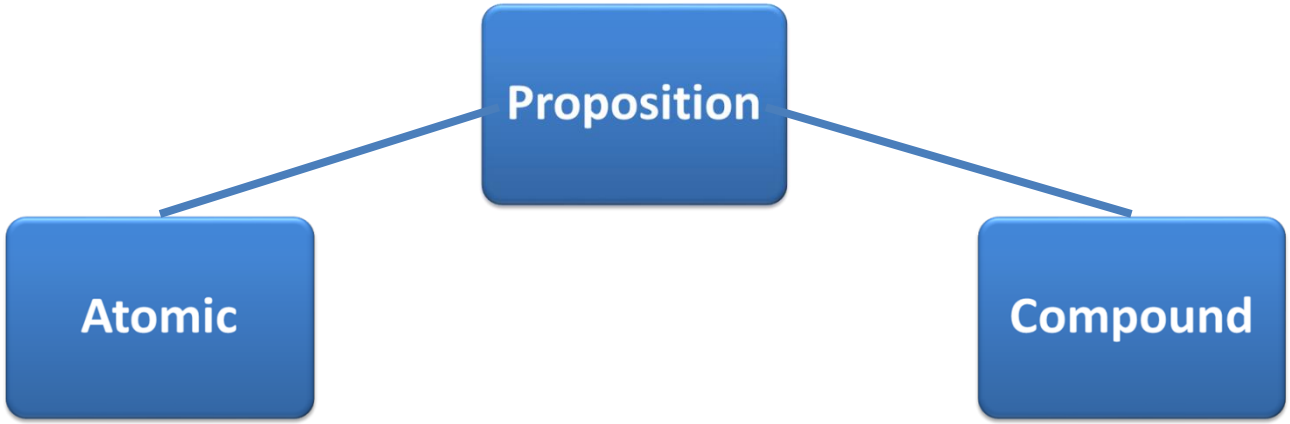
- Which of these sentences are propositions? What are the truth values of those that are propositions?
  - Boston is the capital of Massachusetts.
  - 5 + 7 = 10.
  - x + 2 = 11.
  - Is it raining outside?
  - Wood floats on water.
  - Bill Gates is very generous.

Formal Representation of Propositions

- In mathematical logic propositions are represented as **propositional variables** (or **sentential variables**) and denoted by letters p, q, r, s, …
- Each propositional variable has a truth value of true (denoted as **T**) or false (denoted as **F**)

The area of logic that deals with propositions is called propositional calculus or propositional logic.

Types of Propositions



- **A simple declarative statement that cannot be expressed in terms of simpler propositions**

- **A proposition that is formed from simple(r) propositions, using <u>logical operators</u>**

Logical Operators

- Negation ¬
- Conjunction ∧
- Disjunction ∨
- XOR ⊕
- Implication →
- Biconditional ↔

Connectives

Truth Table

- A table that lists the output truth value of a compound proposition for each combination of the truth values of its constituent propositions

p	q	r	Output
F	F	F	T
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	T
T	T	F	T
T	T	T	F

Logical Operator: Negation

The *negation* of a proposition *p* is denoted by $\neg p$ (and sometimes as $\bar{p}$ or ~p or p' or !p) and has this truth table:

p	$\neg p$
T	F
F	T

- The truth value of the negation of p ($\neg$p) is the opposite
- of the truth value of p
- **Example**: If *p* denotes "The earth is round.", then $\neg p$ denotes "It is not the case that the earth is round," or more simply "The earth is not round."

Classroom Exercise

- Find the negation of these propositions
  - Michael's PC runs Linux
    - It is not the case that Michael's PC runs Linux
    - Michael's PC does not run Linux.
  - Iron floats in water
    - It is not the case that iron floats in water
    - Iron does not float in water
    - Iron sinks in water
  - 100 is an even number
    - It is not the case that 100 is an even number
    - 100 is not an even number
    - 100 is an odd number

Logical Operator: Conjunction

- The *conjunction* of propositions *p* and *q* is denoted by $p \wedge q$ and has this truth table:

p	*q*	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- The conjunction p ∧ q is true when both p and q are true and is false otherwise.
- **Example**: If *p* denotes "I am at home." and *q* denotes "It is raining." then $p \wedge q$ denotes "I am at home and it is raining."

Classroom Exercise

Create a conjunction from the following simple propositions:

- The sun is shining
- It is raining
  - The sun is shining **and** it is raining
  - The sun is shining **but** it is raining

Create a conjunction from the following simple propositions:

- Rebecca's PC has more than 16 GB free hard disk space
- The processor in Rebecca's PC runs faster than 1 GHz
  - Rebecca's PC has more than 16 GB free hard disk space, **and** the processor in Rebecca's PC runs faster than 1 GHz
  - Rebecca's PC has more than 16 GB free hard disk space, and its processor runs faster than 1 GHz