

The space of states is defined inductively as follows:

$$\sigma ::= \sigma \otimes \sigma \mid x \rightarrow \nu$$

That is, a state is either a product of two states, or is a mapping from variables to values.

An assertion ( $P$ ,  $Q$ , etc) is a mapping from states to propositions, as in traditional Hoare Logic:

$$P : \sigma \rightarrow \text{Prop}$$

$$\begin{array}{c}
\text{PART} \frac{c_0 \text{ part } c_1 \cdot c_2 \quad \Gamma \vdash \{P\} \ c_1 \ \{R\}}{\Gamma \vdash \{\text{pairwise } P \ R\} \ c_1 \times c_2 \ \{\text{pairwise } R \ Q\}} \\
\Gamma \vdash \{P\} \ c_0 \ \{Q\} \\
\text{SPLIT} \frac{\Gamma \vdash \{P_0\} \ c_0 \ \{Q_0\} \quad \forall st_0, st_1. P(st_0 \otimes st_1) \Rightarrow P_0 st_0 \wedge P_1 st_1}{\Gamma \vdash \{P\} \ c_0 \times c_1 \ \{Q\}} \\
\text{COMM} \frac{\Gamma \vdash \{\text{commute } P\} \ c_1 \times c_0 \ \{\text{commute } Q\}}{\Gamma \vdash \{P\} \ c_0 \times c_1 \ \{Q\}} \quad \text{ASSOC} \frac{\Gamma \vdash \{\text{associate } P\} \ (c_0 \times c_1) \times c_2 \ \{\text{associate } Q\}}{\Gamma \vdash \{P\} \ c_0 \times (c_1 \times c_2) \ \{Q\}} \\
\text{IF} \frac{\Gamma \vdash \{P \wedge e\} \ c_0 \times c_2 \ \{Q\} \quad \Gamma \vdash \{P \wedge \neg e\} \ c_1 \times c_2 \ \{Q\}}{\Gamma \vdash \{P\} \ \text{if } e \text{ then } c_0 \text{ else } c_1 \text{ fi } \times c_2 \ \{Q\}} \\
\text{WHILE} \frac{\Gamma \cup \{P\} \ \text{while } e \text{ do } c_0 \text{ done } \times c_1 \ \{Q \wedge \neg e\} \vdash \{P\} \ \text{if } e \text{ then } c_0 \text{ else skip fi} ; \ \text{while } e \text{ do } c_0 \text{ done } \times c_1 \ \{Q\}}{\Gamma \vdash \{P\} \ \text{while } e \text{ do } c_0 \text{ done } \times c_1 \ \{Q \wedge \neg e\}}
\end{array}$$

$$\begin{aligned}
&\text{pairwise } P \ Q \ (\sigma_0 \otimes \sigma_1) := P \ \sigma_0 \wedge Q \ \sigma_1 \\
&\text{commute } P \ (\sigma_0 \otimes \sigma_1) := P \ (\sigma_1 \otimes \sigma_0) \\
&\text{associate } P \ ((\sigma_0 \otimes \sigma_1) \otimes \sigma_2) := P \ (\sigma_0 \otimes (\sigma_1 \otimes \sigma_2))
\end{aligned}$$

$$\text{E-Prod} \frac{(c_0, st_0) \Downarrow (st'_0) \quad (c_1, st_1) \Downarrow (st'_1)}{(c_0 \times c_1, st_0 \otimes st_1) \Downarrow (st'_0 \otimes st'_1)}$$

A more typical sequencing rule can be derived from PART and SPLIT:

$$\text{SEQ} \frac{c_0 \text{ part } c_1 \cdot c_2 \quad \Gamma \vdash \{P\} \ c_1 \ \{R\} \quad \Gamma \vdash \{R\} \ c_2 \ \{Q\}}{\Gamma \vdash \{P\} \ c_0 \ \{Q\}}$$