MATH2010 Advanced Calculus I, 2023-2024 Term 2

HOMEWORK 4

Due: 23:59, 23 Feb 2024

Please submit your solution to Gradescope before it is due.

Q1. Find the limit $\lim\limits_{(x,y)\to(0,0)} \frac{e^y \sin x}{x}$.

Q2. Find the limit of $f(x,y) = \frac{x^3 - xy^2}{x^2+y^2}$ as $(x,y) \to (0,0)$ or show that the limit does not exist.

Q3. Does knowing that $|\sin(1/x)| \leq 1$ tell you anything about

$$\lim_{(x,y)\to(0,0)} y \sin \frac{1}{x}?$$

Give reasons for your answer.

Q4. Let $f(x,y) = \begin{cases} 1, & y \geq x^4 \\ 1, & y \leq 0 \\ 0, & \text{otherwise.} \end{cases}$

Find each of the following limits, or explain that the limit does not exist.

(a) $\lim\limits_{(x,y)\to(0,1)} f(x,y)$

(b) $\lim\limits_{(x,y)\to(2,3)} f(x,y)$

(c) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$

Q5. Evaluate the following limits or show they do not exist.

(a) $\lim\limits_{(x,y)\to(2,-4)} \frac{y+4}{x^2y - xy + 4x^2 - 4x}$

(b) $\lim\limits_{(x,y)\to(0,0)} \left(2x^2 + y^2\right) \sin \frac{1}{\sqrt{x^2+4y^2}}$.

(c) $\lim\limits_{(x,y)\to(0,0)} \frac{x^5 y^2}{x^{10} - y^4}$.

(d) $\lim\limits_{(x,y)\to(1,-1)} \frac{xy+1}{x^2 - y^2}$

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