MATH2010 Advanced Calculus I, 2023-2024 Term 2

HOMEWORK 6

Due: 23:59, 8 Mar 2024

Please submit your solution to Gradescope before it is due.

Q1. Can the function $f(x,y) = \frac{\sin x \sin^3 y}{1-\cos(x^2+y^2)}$ be defined at $(0,0)$ in such a way that it becomes continuous there?

Q2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function and let $(a,b) \in \mathbb{R}^2$. We define two single variable functions $g(x) = f(x,b)$ and $h(y) = f(a,y)$.

(a) If $g(x)$ is continuous at $x = a$ and $h(y)$ is continuous at $y = b$, does it follow that f is continuous at (a,b) ? Why?

(b) If $f(x,y)$ is continuous at (a,b), does it follow that $g(x)$ is continuous at $x = a$ and $h(y)$ is continuous at $y = b$? Why?

Q3. Compute the gradient for the following functions.

(a) $f(x,y,z) = \sec(x+y) + \frac{1}{y+z}$.

(b) $f(x,y) = \frac{x+y}{x-y}$.

Q4. Let $f(x,y,z) = xy+yz+zx$. Using the limit definition, find the directional derivative of f at the point $\mathbf{a} = (1,-1,1)$ along the direction of $\mathbf{v} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

Q5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function. Fix a point (x_0, y_0). Consider a linear approximation of $f(x,y)$ near (x_0,y_0), defined by

$$l(x,y) = f(x_0,y_0) + r(x-x_0) + s(y-y_0),$$

where $r, s \in \mathbb{R}$ are constants. Suppose the error

$$\epsilon(x,y) = f(x,y) - f(x_0,y_0) - r(x-x_0) - s(y-y_0)$$

satisfies

$$\lim_{(x,y)\to(x_0,y_0)} \frac{\epsilon(x,y)}{\sqrt{(x-x_0)^2+(y-y_0)^2}} = 0.$$

Show that the partial derivatives $f_x(x_0,y_0)$ and $f_y(x_0,y_0)$ exist with $r = f_x(x_0,y_0)$ and $s = f_y(x_0,y_0)$.

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