MATH2010 Advanced Calculus I, 2023-2024 Term 2

HOMEWORK 7

Due: 23:59, 29 Mar 2024

Please submit your solution to Gradescope before it is due.

Q1. Let $w = x^2 e^{2y} \cos 3z$. Find the value of dw/dt at the point $(1, \ln 2, 0)$ on the curve $x = \cos t, y = \ln(t+2)$, $z = t$.

Q2. Let $F(x, y, z) = x^3 + y^3 + z^3 - 3xyz - 4$. It is given that $F(x, y, z) = 0$ defines an implicit function $z = f(x, y)$ near the point $(1, 1, 2)$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(1, 1, 2)$.

Q3. Find the maximum value of $w = xyz$ on the line of intersection of the two planes $x + y + z = 40$ and $x + y - z = 0$.

Q4. Suppose that we substitute polar coordinates $x = r \cos \theta$ and $y = r \sin \theta$ in a differentiable function $w = f(x, y)$.

(a) Show that

$$\frac{\partial w}{\partial r} = f_x \cos \theta + f_y \sin \theta$$

and

$$\frac{1}{r} \frac{\partial w}{\partial \theta} = -f_x \sin \theta + f_y \cos \theta.$$

(b) Solve the equations in part (a) to express f_x and f_y in terms of $\partial w/\partial r$ and $\partial w/\partial \theta$.

(c) Show that

$$(f_x)^2 + (f_y)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2.$$

Q5. If g_x is continuous, it is true that if

$$F(x) = \int_a^b g(t, x) dt$$

then $F'(x) = \int_a^b g_x(t, x) dt$. Using this fact and the Chain Rule, we can find the derivative of

$$F(x) = \int_a^{f(x)} g(t, x) dt$$

by letting

$$G(u, x) = \int_a^u g(t, x) dt$$

where $u = f(x)$. Find the derivatives of the following functions:

(a) $F(x) = \int_0^{x^2} \sqrt{t^4 + x^3} dt$

(b) $F(x) = \int_{x^2}^1 \sqrt{t^3 + x^2} dt$.

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