MATH2010 Advanced Calculus I, 2023-2024 Term 2

HOMEWORK 3

Due: 23:59, 2 Feb 2024

Please submit your solution to Gradescope before it is due.

Q1. Find a parametric equation for the line tangent to the helix

$$\mathbf{r}(t) = (\sqrt{2}\cos t, \sqrt{2}\sin t, t)$$

at the point where $t = \pi/4$.

Q2. Let $\mathbf{u}$ be a unit vector in $\mathbb{R}^3$. Find the arc length of the curve $\mathbf{r}(t) = P_0 + t^2\mathbf{u}$, $0 \le t \le 1$.

Q3. Show that the equations $x = r\cos\theta$, $y = r\sin\theta$ transform the polar equation

$$r = \frac{k}{1 + e\cos\theta}$$

into the Cartesian equation

$$\left(1 - e^2\right)x^2 + y^2 + 2kex - k^2 = 0.$$

Q4. Convert the equation written in spherical coordinates into an equation in Cartesian coordinates.

(i) $\rho\cos\varphi\sin\varphi\sin\theta = 3$;

(ii) $\rho - \cos\varphi = 2 + \cos^2\varphi$.

Q5. Determine whether the following subsets of $\mathbb{R}^3$ are open, closed, bounded and connected. No justification is needed.

(i) $\{(x, y, z) \in \mathbb{R}^3 : |x| + |y| + |z| = 1\}$;

(ii) $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \in \mathbb{Z}\}$;

(iii) $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 - z^2 > 1\}$.

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