

MATH2010 Advanced Calculus I, 2023-2024 Term 2

HOMEWORK 1

Due: 23:59, 19 Jan 2024

Please submit your solution to Gradescope before it is due.

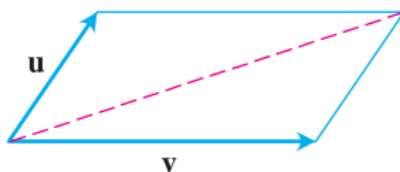
Q1. Show that for any $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$,

$$-\|\mathbf{a} - \mathbf{b}\| \leq \|\mathbf{a}\| - \|\mathbf{b}\| \leq \|\mathbf{a} - \mathbf{b}\|.$$

Q2. Find the following vectors:

- (i) The vector \overrightarrow{OP} where O is the origin and P is the midpoint of segment RS , where $R = (2, -1)$ and $S = (-4, 3)$
- (ii) The sum of \overrightarrow{AB} and \overrightarrow{CD} , where $A = (1, -1)$, $B = (2, 0)$, $C = (-1, 3)$, and $D = (-2, 2)$
- (iii) The unit vectors making an angle $\theta = 2\pi/3$ with the positive x -axis

Q3. Let two vectors \mathbf{u} and \mathbf{v} determine a parallelogram. Use dot product to show that if $\|\mathbf{u}\| = \|\mathbf{v}\|$, then the indicated diagonal makes equal angles with \mathbf{u} and \mathbf{v} .



Q4. Let \mathbf{u}, \mathbf{v} be two vectors in \mathbb{R}^3 . We denote by

$$\text{Proj}_{\mathbf{v}} \mathbf{u} := \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \mathbf{v}$$

the projection of \mathbf{u} onto \mathbf{v} .

1. Show that $\|\text{Proj}_{\mathbf{v}} \mathbf{u}\| \leq \|\mathbf{u}\|$.
 2. Under what circumstance(s) does the equality hold in Part (1)?
- Q5.** Using the formula of projection in **Q4**, show by direct calculation that $(\mathbf{u} - \text{Proj}_{\mathbf{v}} \mathbf{u}) \cdot \text{Proj}_{\mathbf{v}} \mathbf{u} = 0$.
- Q6.** Prove that a parallelogram is a rectangle if and only if its diagonals are equal in length.