MATH2010 Advanced Calculus I, 2023-2024 Term 2

HOMEWORK 5

Due: 23:59, 1 Mar 2024

Please submit your solution to Gradescope before it is due.

Q1. Compute all the first-order and second-order partial derivatives of the function

$$u = e^{xy}\sin(4x+y).$$

Q2. Find the equation of the tangent plane to $z = x^2\cos(\pi y) - \frac{6}{xy^2}$ at $(2,-1)$.

Q3. Let

$$f(x,y) = \begin{cases} \frac{xy^5}{x^4+y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Compute $f_y(0,0)$ and $f_{yx}(0,0)$.

Q4. Let

$$f(x,y) = \begin{cases} 0, & x^2 < y < 2x^2 \\ 1, & \text{otherwise.} \end{cases}$$

Show that $f_x(0,0)$ and $f_y(0,0)$ exist, but f is not differentiable at $(0,0)$.

Q5.

$$f(x,y) = \begin{cases} \left(x^2+y^2\right)\sin\left(\frac{1}{\sqrt{x^2+y^2}}\right), & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Show that:

(i) f is continuous and differentiable at $(0,0)$.

(ii) f_x, f_y are not continuous at $(0,0)$.

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