UNIVERSIDAD DE CONCEPCION
FACULTAD DE CIENCIAS
FISICAS Y MATEMATICAS
DEPARTAMENTO DE INGENIERIA MATEMATICA

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Solución Listado 9 (Números Complejos)

3. **a)** $\dfrac{1}{2+3i} = \dfrac{2}{13} - \dfrac{3}{13}i$

b) $-4(1+\dfrac{i}{12}) + 4(1-\dfrac{i}{12}) = -\dfrac{2}{3}i$

c) $i + \dfrac{1}{i^{11}} = 2i$

d) $\dfrac{1+2i}{(1-2i)(-1-i)} = -\dfrac{1}{10} - \dfrac{7}{10}i$

7. **a)** $z_1 = \sqrt{1/2} + \sqrt{1/2}i$ y $z_2 = -\sqrt{1/2} - \sqrt{1/2}i$.

b) $z = 2$

c) $z = -\dfrac{2}{3} - \dfrac{i}{3}$

d) $z = 12$

e) $z = 1/3$

f) $z = \dfrac{3}{2} - 2i$

8. **a)** $S = \{(x,y) \in \mathbb{C} \colon x^2 + y^2 \leq 4\}$

b) $S = \{(0,5)\}$

c) $S = \{(x,y) \in \mathbb{C} \colon (x+1)^2 + (y-2)^2 > 9\}$

d) $S = \{(x,y) \in \mathbb{C} : y \leq 1\}$

e) $S = \{(x,y) \in \mathbb{C} \colon (x-1)^2 + y^2 \geq 1\}$

f) $S = \{(x,y) \in \mathbb{C} : x < y\}$

9. **a)** $(1+i)^{40} = 2^{20}$

b) $(1-i)^{21} = 2^{10}(i-1)$

c) $(\dfrac{1+\sqrt{3}i}{1-i})^{16} = -2^7(1+\sqrt{3}i)$

10. **a)** $(-2+2i)^5 = \begin{cases} 2^7(1-i) \\ 2^{\frac{15}{2}} cis(-\frac{\pi}{4}) \end{cases}$

b) $(3cis(-\dfrac{\pi}{4}))^4 = \begin{cases} -3^4 \\ 3^4 cis(-\pi) \end{cases}$

c) $(1+i)^{-\frac{1}{4}} = \begin{cases} 2^{-\frac{1}{8}}[\cos(\frac{\pi}{16}) - \operatorname{sen}(\frac{\pi}{16})i] \\ 2^{-\frac{1}{8}} cis(-\frac{\pi}{16}) \end{cases}$

e) $[2cis(-\frac{\pi}{3})]^{-4} = \begin{cases} 2^{-\frac{9}{2}}(1+i) \\ 2^{-4}e^{\frac{\pi}{4}i} \end{cases}$

12. **c)** $\arctan(a) + \arctan(b) + \arctan(c) = \arctan(\frac{a+b+c-abc}{1-ab-ac-bc})$

13. **a)** $z \in \{\sqrt{2}e^{-\frac{\pi}{12}i}, \sqrt{2}e^{\frac{11\pi}{12}i}\}$

b) $z = 2^{\frac{1}{16}}e^{\frac{\pi}{12}i}, 2^{\frac{1}{16}}e^{-\frac{\pi}{12}i}, 2^{\frac{1}{16}}e^{\frac{3\pi}{4}i}, 2^{\frac{1}{16}}e^{\frac{7\pi}{12}i}, 2^{\frac{1}{16}}e^{\frac{17\pi}{12}i}, 2^{\frac{1}{16}}e^{\frac{5\pi}{4}i}$

c) $z = 2^{\frac{1}{8}}e^{-\frac{\pi}{16}i}, 2^{\frac{1}{8}}e^{\frac{7\pi}{16}i}, 2^{\frac{1}{8}}e^{\frac{15\pi}{16}i}, 2^{\frac{1}{8}}e^{\frac{23\pi}{16}i}$

e) $z = e^{\frac{\pi}{8}i}, e^{\frac{9\pi}{8}i}$

14. **a)** $\sqrt[4]{1+i} = 2^{\frac{1}{8}}e^{-\frac{\pi}{16}i}, 2^{\frac{1}{8}}e^{\frac{7\pi}{16}i}, 2^{\frac{1}{8}}e^{\frac{15\pi}{16}i}, 2^{\frac{1}{8}}e^{\frac{23\pi}{16}i}$

c) $\sqrt[3]{8} = 2, -1+\sqrt{3}i, -1-\sqrt{3}i$

d) $\sqrt[5]{-i} = e^{\frac{3\pi}{10}i}, e^{\frac{7\pi}{10}i}, e^{\frac{11\pi}{10}i}, e^{\frac{3\pi}{2}i}, e^{\frac{19\pi}{10}i}$

16. **a)** 2^5

b) $2^{\frac{7}{20}}$

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semestre otoño 2006.