RK$_{22}$

Por definición del método de RK

$$y_{i0} = y_i \tag{1}$$

$$y_{i1} = y_i + hA_{10}f[x_{i0}, y_{i0}] \tag{2}$$

$$y_{i2} = y_i + hA_{20}f[x_{i0}, y_{i0}] + hA_{21}f[x_{i1}, y_{i1}]. \tag{3}$$

Por definición de P.V.I y por (2)

$$y_{i1} = y_i + hA_{10}y_i' \tag{4}$$

haciendo serie de Tarylor

$$\begin{aligned} y(x_{i1}) &= y(x_i + \theta_1 h) \\ &= y(x_i) + \theta_1 h y'(x_i) \end{aligned} \tag{5}$$

Igualando (4) con (5) tenemos que $A_{10} = \theta_1$.

Por otra parte considerando (3)

$$\begin{aligned} y_{i2} &= y_i + hA_{20}y'(x_i) + hA_{21}y'(x_i+\theta_1 h) \\ &= y_i + hA_{20}y'(x_i) + hA_{21}[y'(x_i) + \theta_1 h y''(x_i) + \tfrac{\theta_1^2 h^2}{2} y'''(x_i) + \cdots] \\ &= y_i + h[A_{20}+A_{21}]y'(x_i) + h^2 A_{21}\theta_1 y''(x_i) + \tfrac{h^3}{2} A_{21}\theta_1^2 y'''(x_i) + \cdots \end{aligned} \tag{6}$$

aplicando serie de Taylor

$$y(x_{i+1}) = y(x_i+h) = y(x_i) + hy'(x_i) + \frac{h^2}{2}y''(x_i) + \cdots \tag{7}$$

Igualando (6) con (7) tenemos que $A_{20} + A_{21} = 1$ y $A_{21}\theta_1 = \frac{1}{2}$.