# Handbook of Blind Source Separation

Independent Component Analysis and Applications

Edited by P. Comon and C. Jutten



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# About the editors

The two editors are pioneering contributors of ICA. They wrote together the first journal paper on ICA, which appeared in *Signal Processing*, published by Elsevier in 1991, and received a best paper award in 1992, together with J. Hérault.

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# Preface

In signal processing, a generic problem consists in separating a useful signal from noise and interferences. Classical approaches of the twentieth century are based on *a priori* hypotheses, leading to parameterized probabilistic models. Blind Source Separation (BSS) attempts to reduce these assumptions to the weakest possible.

As shown in this handbook, there are various approaches to the BSS problem, depending on the weak *a priori* hypotheses one assumes. The latter include either statistical independence of source signals or their sparsity, among others.

In order to prepare this book, among the best worldwide specialists have been contacted to contribute (cf. page xxiii). One of them, Serge Degerine, has passed away unexpectedly during the writing of Chapter 7. We would like to dedicate this book to his memory.

This handbook is an extension of another book which appeared in 2007 in French, and published by Hermes. The present version contains more chapters and many additions, provided by contributors with international recognition. It is organized into 19 chapters, covering all the current theoretical approaches, especially Independent Component Analysis, and applications. Although these chapters can be read almost independently, they share the same notations and the same subject index. Moreover, numerous cross-references link the chapters to each other.

Pierre Comon and Christian Jutten

vector of components  $x_p$ ,  $1 \le p \le P$ 

# Glossary

 $\mathbf{X}$ 

```
sources, observations, separator outputs
s, x, y
N
                              number of sources
P
                              number of sensors
T
                              number of observed samples
                              convolution
                              matrix with components A_{ij}
A
A, B
                              mixing and separation matrices
G, W, Q
                              global, whitening, and separating unitary matrices
ģ
                              Fourier transform of g
\hat{\mathbf{s}}
                              estimate of quantity s
                              probability density of x
p_{\mathbf{x}}
                              joint score function
ψ
                              marginal score function of source s_i
\varphi_i
Φ
                              first characteristic function
Ψ
                              second characteristic function
\mathbb{E}\mathbf{x}, \mathbb{E}\{\mathbf{x}\}
                              mathematical expectation of x
I\{y\} or I(p_y)
                              mutual information of y
K\{\mathbf{x};\mathbf{y}\} or K(p_{\mathbf{x}};p_{\mathbf{y}})
                              Kullback divergence between p_x and p_y
H\{\mathbf{x}\} or H(p_{\mathbf{x}})
                              Shannon entropy x
\mathscr{L}
                              likelihood
\mathcal{A}, \mathcal{B}
                              mixing, and separating (nonlinear) operators
cum\{x_1,\ldots,x_p\}
                              joint cumulant of variables \{x_1, ..., x_p\}
\operatorname{cum}_{R}\{y\}
                              marginal cumulant of order R of variable y
```

#### 822 Glossary

 $\mathbf{Q}^{\mathrm{T}}$  transposition

Q<sup>H</sup> conjugate transposition

**Q**\* complex conjugation

Q<sup>†</sup> pseudo-inverse

Υ contrast function

 $\mathbb{R}$  real field

 $\mathbb{C}$  complex field

**Â** estimator of mixing matrix

diag A vector whose components are the diagonal of matrix A

Diag a diagonal matrix whose entries are those of vector a

trace A trace of matrix A

det A determinant of matrix A

mean a arithmetic average of component of vector a

 $\ddot{s}(v)$  Fourier transform of process s(t)

⊗ Kronecker product between matrices

• i contraction over index j

krank{A} Kruskal's k-rank of matrix A

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