

$$\vec{U}_1 = (3, 1, 0, 1)$$

$$\vec{U}_2 = (1, 2, 1, 1)$$

$$\vec{U}_3 = (-1, 0, 2, -1)$$

$$\boxed{\vec{V}_1 = \frac{1}{\sqrt{11}} (3, 1, 0, 1)}$$

$$\text{Proj}_{V_1} U_2 = \frac{U_2 \cdot V_1}{V_1 \cdot V_1} V_1 = \frac{6}{\sqrt{11}} \cdot \frac{1}{\sqrt{11}} (3, 1, 0, 1) = \left(\frac{18}{11}, \frac{6}{11}, 0, \frac{6}{11} \right)$$

$$\vec{V}_2 = (1, 2, 1, 1) - \left(\frac{18}{11}, \frac{6}{11}, 0, \frac{6}{11} \right)$$

$$\vec{V}_2 = \left(-\frac{7}{11}, \frac{16}{11}, 1, \frac{5}{11} \right)$$

$$\boxed{\vec{V}_2 = \frac{1}{\sqrt{451}} (-7, 16, 11, 5)}$$

$$S_2 = \text{gen} \left\{ \frac{1}{\sqrt{11}} (3, 1, 0, 1), \frac{1}{\sqrt{451}} (-7, 16, 11, 5) \right\}$$

$$\begin{aligned} \text{Proj}_{S_2} U_3 &= (U_3 \cdot V_1) V_1 + (U_3 \cdot V_2) V_2 \\ &= \frac{-4}{\sqrt{11}} \frac{1}{\sqrt{11}} (3, 1, 0, 1) + \frac{24}{\sqrt{451}} \frac{1}{\sqrt{451}} (-7, 16, 11, 5) \\ &= \frac{-4}{11} (3, 1, 0, 1) + \frac{24}{451} (-7, 16, 11, 5) \\ &= \left(-\frac{12}{11}, -\frac{4}{11}, 0, -\frac{4}{11} \right) + \left(\frac{-168}{451}, \frac{384}{451}, \frac{264}{451}, \frac{120}{451} \right) \\ &= \left(-\frac{60}{41}, \frac{20}{41}, \frac{24}{41}, -\frac{4}{41} \right) \end{aligned}$$

$$\vec{V}_3 = (-1, 0, 2, -1) - \left(-\frac{60}{41}, \frac{20}{41}, \frac{24}{41}, -\frac{4}{41} \right)$$

$$\vec{V}_3 = \left(\frac{19}{41}, -\frac{20}{41}, \frac{58}{41}, -\frac{37}{41} \right)$$

$$\vec{V}_3 = \frac{1}{41} (19, -20, 58, -37)$$

$$\boxed{\vec{V}_3 = \frac{1}{\sqrt{5494}} (19, -20, 58, -37)}$$

$$P_W(b) = C_1 V_1 + C_2 V_2 + C_3 V_3$$

$$C_1 = (-3, -3, 8, 9) \cdot (3, 1, 0, 1) \frac{1}{\sqrt{11}}$$

$$-9 - 3 + 9$$

$$C_1 = \frac{-3}{\sqrt{11}}$$

$$C_2 = (-3, -3, 8, 9) \cdot (-7, 16, 11, 5) \frac{1}{\sqrt{451}}$$

$$C_2 = \frac{106}{\sqrt{451}}$$

$$C_3 = (-3, -3, 8, 9) \cdot (19, -20, 58, -37) \frac{1}{\sqrt{5494}}$$

$$C_3 = \frac{134}{\sqrt{5494}}$$

$$P_W(b) = \frac{-3}{\sqrt{11}} \frac{1}{\sqrt{11}} (3, 1, 0, 1) + \frac{106}{\sqrt{451}} \frac{1}{\sqrt{451}} (-7, 16, 11, 5) + \frac{134}{\sqrt{5494}} \frac{1}{\sqrt{5494}}$$

$$(19, -20, 58, -37)$$

$$= \frac{-3}{11} (3, 1, 0, 1) + \frac{106}{451} (-7, 16, 11, 5) + \frac{1}{41} (19, -20, 58, -37)$$

$$= \left(-\frac{9}{11}, -\frac{3}{11}, 0, \frac{3}{11} \right) + \left(-\frac{742}{451}, \frac{1696}{451}, \frac{1166}{451}, \frac{530}{451} \right) + \left(\frac{19}{41}, -\frac{20}{41}, \frac{58}{41}, -\frac{37}{41} \right)$$

$$\left(-\frac{101}{41}, \frac{143}{41}, \frac{106}{41}, \frac{37}{41} \right) + \left(\frac{19}{41}, -\frac{20}{41}, \frac{58}{41}, -\frac{37}{41} \right)$$

$$\boxed{\text{Proj}_W(b) = (-2, 3, 4, 0)}$$