

$$I = \int_a^b f(x) dx \approx \int_a^b p_1(x) dx$$

$$\int_a^b \frac{x-b}{a-b} f(a) dx + \int_a^b \frac{x-a}{b-a} f(b) dx$$

$$\frac{f(a)}{a-b} \int_a^b x-b dx + \frac{f(b)}{b-a} \int_a^b x-a dx$$

$$\frac{f(a)}{a-b} \left(\frac{x^2}{2} - bx \right) \Big|_a^b + \frac{f(b)}{b-a} \left(\frac{x^2}{2} - ax \right) \Big|_a^b$$

$$\frac{f(a)}{a-b} \left(\left(\frac{b^2}{2} - b^2 \right) - \left(\frac{a^2}{2} - ab \right) \right) + \frac{f(b)}{b-a} \left(\left(\frac{b^2}{2} - ab \right) - \left(\frac{a^2}{2} - a^2 \right) \right)$$

$$\frac{f(a)}{a-b} \left(\frac{-b^2 - a^2 + 2ab}{2} \right) + \frac{f(b)}{b-a} \left(\frac{b^2 - 2ab + a^2}{2} \right)$$

$$\frac{f(a)}{a-b} \left(\frac{-(a-b)^2}{2} \right) + \frac{f(b)}{b-a} \left(\frac{(b-a)^2}{2} \right)$$

$$-\frac{f(a)(a-b)}{2} + \frac{f(b)(b-a)}{2}$$

$$-\frac{f(a)a + f(a)b + f(b)b - f(b)a}{2}$$

$$= \boxed{\frac{b-a}{2} (f(a) + f(b))}$$