a)

$x_{n+1} = 4x_n - x_n^2$

$x_n = 4x_{n-1} - x_{n-1}^2 \longrightarrow x_{n+1} = 4(4x_{n-1} - x_{n-1}^2) - (4x_{n-1} - x_{n-1}^2)^2 = 16x_{n-1} - 4x_{n-1}^2 - (16x_{n-1}^2 - 8x_{n-1}^3 + x_{n-1}^4)$

$x_1 = 16 \operatorname{sen}^2\theta - 16 \operatorname{sen}^4\theta \longrightarrow 16\operatorname{sen}^2\theta(1 - \operatorname{sen}^2\theta) = 16\operatorname{sen}^2\theta\cos^2\theta = 16(\operatorname{sen}\theta\cos\theta)^2$

$= 16\left(\frac{\operatorname{sen}(2\theta)}{2}\right)^2 = 4\operatorname{sen}^2(2\theta)$

$x_2 = 16\operatorname{sen}^2(2\theta) - 16\operatorname{sen}^4(2\theta) = 16\operatorname{sen}^2(2\theta)\left(1 - \operatorname{sen}^2(2\theta)\right) = 16(\operatorname{sen}(2\theta)\cos(2\theta))^2 = 16\left(\frac{\operatorname{sen}(4\theta)}{2}\right)^2$

$= 4\operatorname{sen}^2(4\theta)$

Entonces, se deduce que x_n siempre tiene la forma de: $x_n = 16(\operatorname{sen}(\alpha_{n-1}\theta)\cos(\alpha_{n-1}\theta))^2$ donde α_n es $2(\alpha_{n-1})$ y $\alpha_0 = 1$ por lo que $\alpha_n = 2^n$. Entonces:

$x_n = 16(\operatorname{sen}(2^{n-1}\theta)\cos(2^{n-1}\theta))^2 = 4\operatorname{sen}^2(2^n\theta)$

b) $x_{n+1} = 4x_n - 4x_n^2 \qquad x_0 = \operatorname{sen}^2\theta$

$x_1 = 4\operatorname{sen}^2\theta - 4\operatorname{sen}^4\theta = 4\operatorname{sen}^2\theta(1 - \operatorname{sen}^2\theta) = 4(\operatorname{sen}\theta\cos\theta)^2 = \operatorname{sen}^2 2\theta$

$x_2 = 4\operatorname{sen}^2(2\theta) - 4\operatorname{sen}^4(2\theta) = 4\operatorname{sen}^2(2\theta)\cos^2(2\theta) = \operatorname{sen}^2(4\theta)$

Entonces, se deduce que x_n siempre tiene la forma de: $x_n = 4\operatorname{sen}^2(\alpha_{n-1}\theta)\cos^2(\alpha_{n-1}\theta)$ donde α_n es $2(\alpha_{n-1})$ y $\alpha_0 = 1$ por lo que $\alpha_n = 2^n$. Entonces:

$x_n = 4\operatorname{sen}^2(2^{n-1}\theta)\cos^2(2^{n-1}\theta) = \operatorname{sen}^2(2^n\theta)$