Alcohol relapse — Study A

A.1

The first study in this series of examples is an investigation into the causes of relapse among alcoholics, conducted at the Addiction Research Unit, London, by Dr G. K. Litman and Dr A. N. Oppenheim, and our thanks are extended to them and their co-workers for allowing us to use their data. We begin the analysis with a simple univariate analysis of variance, in order to illustrate and help fix the ideas introduced in Chapter 2. Having introduced the principal questions under investigation in this more familiar setting of simple univariate analysis, in Section A.2 we proceed to a multivariate analysis of variance, using a parallel set of data taken from the same study.

The study consists of a sample survey of people presenting themselves for treatment of alcoholism and heavy drinking problems at several hospitals and other related agencies. The data were collected over a one-year period, with a view to a long-term follow-up of all those participating. The data in the present analysis consist of information taken at the point of the patient's admission into treatment. A central interest in the study was to inquire how vulnerable people with drinking problems are to relapse, and in what way this vulnerability is perceived. The measures with which we are here concerned were made with a 'Relapse Precipitants Inventory' (RPI) which all of the participants in the study were asked to fill out. This questionnaire was developed in a previous survey and listed a variety of situations which were likely cues for drinking. Respondents scored these items according to how dangerous they found them in precipitating a relapse into heavy drinking. The sum of these item scores provided a total score for the entire inventory and it is this total which we now analyse.

The sample was drawn from a year's total consecutive admissions to treatment for alcoholism in the hospitals and agencies referred to above. These consecutive admissions were subjected to various exclusion criteria for a variety of administrative and ethical reasons. This left over the one-year period 256 cases, of which 5 failed to give full information on the Relapse Precipitants Inventory. The study data thus comprise the remaining 251 cases, of which roughly 25% are female and 75% are male. The main interest of the study concentrated on future relapse or non-relapse, but it was also important to assess the present position of the subjects in the study. In particular, information was collected on their personal histories and previous troubles with heavy drinking. This enabled the researchers to characterize the subjects by the number of previous relapses they had experienced, prior to their current heavy drinking period and consequent intake into the study. It is therefore possible to inspect the relationship between previous experience of relapse and present perceived vulnerability to relapse.

Although it would be possible to look for a linear relationship between these two measures by means of a regression or correlation analysis, it was thought that the spread of experience of previous relapses would not make for a simple linear relationship throughout the range. Furthermore, this particular item of information is not easy to define accurately and consequently there are fairly arbitrary elements introduced into its measurement. It was therefore decided to categorize the people in the sample into three crude groups according to the rough count of previous relapses. These groups were defined as

- those never having previously experienced relapse after trying to give up heavy drinking;
- those who claimed to have relapsed, but no more than two or three times before;
- those who had a longer history of relapse of four or more times.

These three groups are referred to as 'new', 'recent' and 'long-standing'. The numbers in each group are shown in Table A.1, along with the mean score for the group on the RPI. The score is a summation of the scores on 25 individual items, with a maximum score of 75.

Our first analysis is therefore a univariate analysis of variance with one factor at three levels representing the experience of the subject. The intention is to inspect the influence of the 'experience' factor on the response measure, vulnerability. As described in Section 2.1, in a simple analysis of variance we usually define group effects, that is, the effects of the factor at its three levels, as the difference between the individual group mean and the overall mean of the three groups. Such effects are represented in terms of a set of contrasts by the following:

$$\begin{array}{rrr} 1-1/3 & -1/3 & -1/3 \\ -1/3 & 1-1/3 & -1/3 \\ -1/3 & -1/3 & 1-1/3 \end{array}$$

Table A.1 Numbers in cells and means on the RPI score for the three levels of the factor 'experience'.

Factor level	*No. in cell*	*Observed mean*
Group 1 (long-standing)	125	40.00
Group 2 (recent)	90	34.31
Group 3 (new)	36	31.78
Overall	251	36.78

Table A.2 Possible alternative sets of contrasts constituting the complete comparison of the three groups shown in Table A.1 and the estimates of the effects they define.

Group 1	*Group 2*	*Group 3*	
Obs. mean scores:			
40.00	34.31	31.78	
Grand mean:			Corresponding est'd effect:
1/3	1/3	1/3	35.363
Standard contrasts:			Corresponding est'd effects:
1 − 1/3	− 1/3	− 1/3	4.637
− 1/3	1 − 1/3	− 1/3	−1.052
− 1/3	− 1/3	1 − 1/3	−3.585
Pairwise differences contrasts:			Corresponding est'd effects:
1	−1	0	5.687
0	1	−1	2.533
1	0	−1	8.222
Polynomial contrasts:			Corresponding est'd effects:
−1	0	1	−8.222
−1	2	−1	−3.156
Helmert contrasts:			Corresponding est'd effects:
1	−1/2	−1/2	6.956
0	1	−1	2.533

These three contrasts (or indeed, any two of them), it can easily be shown, constitute a complete comparison for the factor, as described in Section 2.2. The estimated values of the effects are given by the values of these three contrasts when applied to the sample means and these are shown in Table A.2. Note that these effects, which compare the group mean with the overall mean, are simply multiples of the effects which compare each mean with the average of the other two groups. This is easy to see from the following three contrasts, which are simply 1½ times the previous set:

1	− 1/2	− 1/2
− 1/2	1	−1/2
− 1/2	− 1/2	1

Again as was indicated in Section 2.2, this implies that the effects are equivalent, in that each measures the same set of relationships.

Alternatively, we can define the effects as they were first described in Section 2.2, as all possible pairwise differences between the group means. In this particular case, of three groups only, we have three pairwise comparisons and these are also given Table A.2, along with the estimated values of the effects defined in this manner. These estimated values in the complete comparison are again, of course, just the value of the contrasts applied to the observed group means. The two sets are alternative representations for the complete comparison of the levels of the factor 'experience'.

In this example the levels of the factor have an order to them, in that they progress from many relapses in the 'long-standing' group to none in the 'new' group. Under such circumstances it is often desirable to look for trends across the three groups and in particular the polynomial trends, linear and quadratic. Effects defined this way, as trends, are also shown in Table A.2. It can be readily verified that these two contrasts also provide a complete comparison for the factor. They constitute one particular summary of all the pairwise differences described previously (see Section 2.2).

Although a case can be made for using the two polynomial contrasts to represent the ordered effects in this study, it was felt on balance that they were not an interesting way of describing the differences between the levels of the factor. Prior considerations led the researchers to the view that the first group, that of long-standing relapsing heavy drinkers, would be considerably different from the two other groups which had only new or recent histories of problematic drinking. These two groups, it was felt, would show no great differences from each other, whereas their joint differences from the group which contained a large number of very long-standing problem drinkers would be quite marked. It was therefore decided that the effects of primary interest for this factor are the two defined by the two contrasts shown in the last part of Table A.2. Contrast one represents the marked difference between the first group and the other two, and contrast two represents the difference between these last two groups, 'new' and 'recent'. The table also gives the value of these contrasts calculated on the observed group means. Note that these two contrasts (which, incidentally, are the Helmert contrasts referred to in Section 2.11) provide yet another alternative representation of the complete comparison for the three groups, a fact which is again easily verified. It is with this last set of contrasts that the analysis shall concern itself. It is important to appreciate that these contrasts were decided upon simply from a priori considerations and interests, without reference to the observed data. As described in Section 2.2, they are conceived as being relationships between the population means of the three groups; when applied to the sample means they produce the estimates of these population effects which are given alongside them in Table A.2.

The estimates for these two contrasts were produced with a standard computer program, although a program is scarcely needed for such simple computations. However, computing the sums of squares for the related tests of significance is a different matter. Table A.3 is the standard analysis of variance table, although some readers may be used to one which omits the sum of squares due to the grand mean. It shows the familiar sums of squares attributable to the factor calculated with such a program. Note that this overall sum of squares for

Table A.3 Univariate analysis of variance of the RPI score for the data shown in Table A.1.

(i) Analysis of variance for the complete comparison.

Source	*d.f.*	*SS*	F-*statistic*	*(sig-level)*
Within groups	248	75 345.5		
Grand mean	1	339 561.0		
Between groups	2	2 745.4	4.518	(.012)

(ii) Analysis of variance for the partitioned between-groups degrees of freedom, showing the 'unique' contribution of each to the complete comparison.

Source	*d.f.*	*SS*	F-*statistic*	*(sig-level)*
Within groups	248	75 345.5		
Grand mean	1	339 561.0		
1st Helmert (1 vs. 2, 3)	1	2 729.9	8.985	(.003)
2nd Helmert (2 vs. 3)	1	165.0	0.543	(.462)

the factor – that is, for the complete comparison of all levels – would be the same value no matter which of the sets of contrasts were used to represent the complete comparison. The within-group sums of squares are also shown in the table. The usual F-test would compare the mean square associated with the sums of squares due to the comparison with the mean square for error in the familiar way. This F-statistic would be used to test the overall null hypothesis that there are no differences between any groups (see Section 4.2). Such a test of hypothesis would be the appropriate one when searching for any, or general, differences between the three groups. In this particular experiment, however, the overall null hypothesis of no differences is not the one to test, for the researchers had in mind two particular hypotheses which they desired to test separately. These two hypotheses are the ones represented by the contrasts themselves, namely that group 3 is equal to group 2 and that group 1 is different from this average. There is no particular reason to construct a simultaneous test of these two distinct hypotheses.

Thus the second half of Table A.3 shows the tests of the hypotheses in which we are actually interested. The two separate contrasts are shown as partitioned sources of variation, each with one degree of freedom. The sum of squares opposite each entry is the sum of squares attributable to the contrast when allowing for everything else in the complete comparison – in this case, that means just the other contrast. These are referred to as the uniquely attributable sums of squares for the contrasts, or more simply as the 'unique' sums of squares. They measure the need to include the particular contrast in the comparison, that is, they are each used to test the null hypothesis that the corresponding population contrast value is zero. We note in passing that the two sums of squares do not add up to the overall sum of squares for the complete comparison (in spite of the two contrasts 'appearing' orthogonal) because of the imbalance in the design (see Section 2.6).

The two relevant F-statistics are quoted alongside the corresponding unique sums of squares in Table A.3 and are, as usual, the ratio of the mean square for the hypothesis to the mean square for error. Each hypothesis is tested separately at the 1% level, this high level being chosen simply because a lot of data are available (251 observations in all), allowing stronger conclusions to be drawn. The first test of hypothesis – that the first Helmert contrast (comparing the 'long-standing' group with other two) is zero – is rejected, showing that the 'recent' and 'new' group average is significantly different from the 'long-standing' group. We can see from the bottom line of the table that the null hypothesis that groups 2 ('recent') and 3 ('new') are the same is not rejected. Thus there is no evidence for this effect being anything other than zero. The contrast can be omitted from the comparison and the remaining partial comparison – that is, the contrast representing the two groups' joint difference from the 'long-standing' group – is adequate to explain the observed data.

Under these circumstances of groups 2 and 3 being equal, we might wish to proceed to estimate the incomplete comparison which comprises the adequate model, that is, the single contrast representing this joint difference between the 'long-standing' group and the other two. Recall from Section 2.3 that this is equivalent to pooling groups 2 and 3 for the purposes of estimating their average difference from the 'long-standing' group. The value for the contrast estimated as an incomplete comparison, with group 2 restricted equal to group 3 by the omission of the corresponding contrast from the comparison, is shown in Table A.4(i) as 6.413. Note that the contrast coefficients which are shown as

$$1 \qquad -1/2 \qquad -1/2$$

are the 'conceptual' weights on the population group means referred to in Section 2.3; they are not the efficiently reweighted coefficients actually applied to the observed group means to produce the estimated value of the effect.

Most computer programs provide the facility for reconstructing, or 'predicting' as it is sometimes termed, the observed group means from the effects in a diminished model of this sort. Thus the reconstructed means shown in Table A.4(i) are formed from the single significant group effect and the overall mean. These are the estimates of the group means made under the assumption that the 'recent' and 'new' groups have equal values.

We can also test in this diminished model whether the size of the effect is non-zero. The null hypothesis is that the population value of the first Helmert contrast is zero, and the effect therefore superfluous, even when it is the only one in the design. Thus we need to use the sum of squares for the partial comparison

Table A.4 (i) Estimation of effects under the assumption that 2nd Helmert (1 vs. 3) has population value of zero, along with estimated group means under this assumption.

	Conceptual coefficients			*Est'd effect*
Grand mean:	1/3	1/3	1/3	35.725
1st Helmert:	1	−1/2	−1/2	6.413
Est'd group means:	Grp 1	Grp 2	Grp 3	
	40.00	33.587	33.587	

Table A.4 (ii) Analysis of variance for the minimal adequate model (i.e. 2nd Helmert contrast assumed zero).

Source	*d.f.*	*SS*	F-*statistic*	*(sig-level)*
Within groups	248	75 345.5		
Grand mean	1	339 561.0		
1st Helmert (1 vs. 2, 3)	1	2 580.4	8.493	(.004)

(i.e. for just the remaining contrast) in an *F*-test. The result, shown in Table A.4(ii), is a highly significant one and we reject the null hypothesis: there is such an effect, and so it constitutes a minimal adequate explanation of the data.

Before turning to the multivariate analysis, we shall briefly recapitulate on the procedures used here. It was decided *a priori* to represent the complete comparison between groups by the two Helmert contrasts. Each of these was to be the subject of a separate test of hypothesis that the associated relationship between group means was zero. Only one of these null hypotheses was rejected, that which declared a significant difference between the first group and the other two. This contrast, or incomplete comparison, thus constituted an adequate explanation of the data; and the estimated value of this incomplete comparison was recomputed, by using the information that the mean of group 2 is equal to the mean of group 3.

A.2

We turn now to a multivariate analysis of variance on a set of variables from this same study. It will be recalled that the RPI was developed from work on a previous survey. This work had identified three areas of vulnerability to relapse, which were incorporated in the 'Relapse Precipitants' questionnaire. These three areas gave rise to three separate scores, one for each, as follows:

(UM) unpleasant mood states; for example, depression;
(ES) euphoric states and related situations; for example, celebrations and parties; and
(LV) an area designated as lessened vigilance; for instance, a temptation to believe that one or two drinks would cause no problem.

We shall now parallel the analysis of the preceding section, but using these three separate scores to characterize the individuals in the three groups, 'long-standing', 'recent' and 'new'. In fact, this analysis was the more important analysis in the research, the univariate analysis of the whole RPI being presented here rather for the purpose of giving an initial description of the questions under study. Accordingly we shall, in analysing these data, ignore the fact that we have already univariate results for a related problem. (The raw data for this study are given in Table A.11.)

We thus have for each subject three response variables representing three vulnerability scores; between subjects the design consists, as before, of one factor ('experience') at three levels. The questions in this analysis concerning these three levels of experience are the same as those in the preceding analysis; that is, the two separate questions of firstly, whether 'recent' (group 2) and 'new' (group 3) are different; and secondly, whether 'long-standing' (group 1) is different from their average. In this analysis, however, we are concerned with the groups being equal or different across the profile of scores representing the three areas of vulnerability. We are therefore interested in whether the group mean vectors (not just univariate group means) differ. There are no particular relationships postulated between these three variables so the questions are general ones, asking simply whether the variables as a set can be used to identify the between-groups relationships described above (see Section 3.2). Table A.5 shows the mean vectors for the three groups under study.

As in the univariate example above, we begin the analysis by estimating the effects in the complete comparison, represented by the two Helmert contrasts

Table A.5 Numbers in cells and mean vectors on the three RPI scales for the three levels of the factor 'experience'.

Factor level	*No. in cell*	*Observed mean vector*		
		(UM)	*(ES)*	*(LV)*
Group 1 (long-standing)	125	21.60	11.98	6.42
Group 2 (recent)	90	18.24	10.68	5.39
Group 3 (new)	36	17.33	9.94	4.50
Overall	251	19.78	11.22	5.77

previously described. These are shown in Table A.6 and we note that for each effect we now produce a vector of values, one for each variable. Along with the effects relating to the Helmert contrasts is shown the grand mean vector (the overall level for each variable). This differs from the simple sample mean shown at the foot of Tàble A.5, because it is fitted in conjunction with the contrasts and therefore uses the conceptual coefficients shown directly. We now proceed to test the hypotheses that the Helmert effects are zero in the population. As in our univariate case, the research arguments lead us to test the two hypotheses separately.

Table A.6 The Helmert contrasts for the complete comparison of the three groups shown in Table A.5 and the estimates of the effects they define.

Grp 1	*Grp 2*	*Grp 3*	*(UM)*	*(ES)*	*(LV)*
Grand mean:			Corresponding effect vector:		
1/3	1/3	1/3	[19.06	10.87	5.43]
Helmert contrasts:			Corresponding effect vector:		
1	−1/2	−1/2	[3.81	1.47	1.47]
0	1	−1	[0.91	0.73	0.89]

Table A.7 is a parallel to Table A.3, but shows matrices of sums of products whereas Table A.3 shows only sums of squares. For each response variable in the multivariate analysis there is a column and a row in the sums of products matrix, and the diagonal entry in the matrix corresponds to the sums of squares that would be used for that variable in a straightforward univariate analysis, as was described in Section 3.1. Here, however, we are interested in all three variables simultaneously and in the relationships between them. In other words, we compare the whole matrix of sums of products for the hypothesis with the whole matrix of sums of products for the error term.

The topmost entry is the within-groups sums of products matrix **E** that we shall use for the error term in the significance tests. The second entry is the hypothesis matrix for the grand mean, which we shall ignore since we do not wish to test whether this is zero in the present example. The next entry, with two degrees of freedom, is the hypothesis matrix relevant to the complete comparison between groups. It is included here only to make the parallel with Table A.3 exact and we shall ignore it since, as we argued in the previous analysis, we do not wish to test the compound hypothesis that there are no group differences. We wish only to perform tests on the two individual hypothesis matrices shown for the two separate hypotheses relating to the first Helmert contrast and the second Helmert contrast. The sums of products matrices for these partitioned sources are given in the lower part of the table. To test the null hypothesis that group 1 equals the average of groups 2 and 3 in their mean vectors, we compute the

Table A.7 Multivariate analysis of variance of the three RPI scales and the factor 'experience' shown in Table A.5.

Source	*d.f.*	*SP matrix*			*Pillai statistic*
		(UM)	(ES)	(LV)	
Within groups	248	30 637	—	—	
(**E** matrix)		10 152	10 440	—	—
		3 878	2 251	1 707	
Grand mean	1	98 251	—	—	
(**H** matrix)		55 733	31 615	—	—
		28 688	12 262	8 364	
Between groups	2	841.76	—	—	.071
(**H** matrix)		361.09	342.07	—	(sig .006)
		311.49	138.61	123.30	
1st Helmert	1	819.57	—	—	.068
(**H** matrix)		359.75	157.91	—	(sig .001)
		316.45	138.91	122.19	
2nd Helmert	1	21.35	—	—	.013
(**H** matrix)		17.18	13.83	—	(sig .347)
		20.83	16.76	20.32	

matrix ratio $\mathbf{H}\cdot\mathbf{E}^{-1}$, using the hypothesis matrix **H** relating to the first Helmert contrast. The three eigenvalues of this matrix ratio (see Section 4.3) are

0.0734 0.0 0.0

The test statistic we shall use is the Pillai–Bartlett statistic (see Section 4.3) and this is shown in Table A.7 opposite the appropriate entry. This statistic is significant and we reject the null hypothesis, concluding that there is a difference between the mean vector of group 1 and the mean vector of groups 2 and 3. The second hypothesis is tested in an identical way, using the hypothesis sums of products matrix relevant to the second contrast. The eigenvalues of this matrix ratio are

0.0133 0.0 0.0

and the derived Pillai–Bartlett criterion is shown in the table. This does not yield a significant value. As in the univariate analysis above, we are testing to see whether the contrast representing the specified relationship needs to be included in the overall comparison in order to give an adequate description of the data. Our conclusion is that the first contrast alone is adequate.

Having decided that the first Helmert contrast is an adequate model for the data by itself, we can re-estimate its value on the three response scales using the

Table A.8 (i) Estimation of effects under the assumption that the 2nd Helmert contrast (2 vs. 3) has population value of zero, along with estimated group means under this assumption.

Conceptual coefficients			*Estimated effects*		
Grp 1	Grp 2	Grp 3	(UM)	(ES)	(LV)
Grand mean:			Corresponding effect vector:		
1/3	1/3	1/3	[19.19	10.97	5.56]
1st Helmert:			Corresponding effect vector:		
1	−1/2	−1/2	[3.62	1.52	1.28]
Estimated group mean vectors					
			(UM)	(ES)	(LV)
Grp 1 'long-standing'			[21.60	11.98	6.42]
Grp 2 'recent'			[17.98	10.47	5.14]
Grp 3 'new'			[17.98	10.47	5.14]

Table A.8 (ii) Multivariate analysis of variance for the minimal adequate model (i.e. 2nd Helmert contrast assumed zero).

Source	*d.f.*	*SP matrix*			*Pillai statistic*
		(UM)	(ES)	(LV)	
Within groups	248	30 637	—	—	
(**E** matrix)		10 152	10 440	—	—
		3 878	2 251	1 707	
Grand mean	1	98 251	—	—	
(**H** matrix)		55 733	31 615	—	—
		28 688	12 262	8 364	
1st Helmert	1	820.41	—	—	.059
(**H** matrix)		343.91	144.16	—	(sig .002)
		290.67	121.85	102.98	

information that groups 2 and 3 have equal mean vectors. The results are shown in Table A.8(i), with the reconstructed values of the group means estimated under this restriction. We can see that in this particular example the estimate of the effect is only slightly changed. However, in general such estimates are more efficient, given that the restriction is appropriate.

This procedure is sometimes referred to as 'trimming the analysis' of its superfluous effects and we can proceed to look at the relationships of the variables under these restrictions. Having shown that the three variables UM, ES and LV as a set distinguish the 'long-standing' group from the others, we may use the canonical representation of the set to highlight the structure of this discriminant ability. There is only one canonical variable for this comparison, since it comprises only one degree of freedom (see Section 3.10). Calculations using a computer program give the optimum combination of the three responses as being obtained by using the respective weights

0.247 − 0.172 0.939

on the standardized variables – that is, on each response after dividing it by its (within-group) standard deviation. This standardization allows us to compare the relative importance of the responses in contributing to the canonical variable, even though they may be measured on very different scales. It can be seen that the primary weighting is on the third response, LV, and we conclude that this measure alone will give almost as good a discrimination.

A.3

We turn now to a second way of analysing these multivariate data. We seek to answer a different question in this analysis, one which was not part of the initial research aims but which is included here to demonstrate the relationship between the two multivariate analyses. The question we will ask is 'has anything been gained in analysing the differences between the groups by using three separate scales for vulnerability, instead of just a simple overall level?'

The analysis of Section A.2 looked at the profiles of the groups on the three variables without making any specific hypotheses about the sort of relationships that might occur between these measures. It is an example of what in Sections 3.2 and 3.8 is referred to as an unstructured question about the variables. We have now effectively structured the question by asking about specific, predetermined combinations of the variables, namely their overall average (the general level of response) and the difference of each of them from this overall average. We therefore consider them explicitly as three separate levels of a single between-variables factor which we shall call 'type'. Thus we are in a position to use the same structures and transformations described for the between-subjects design in Section 2.15, separating an overall mean level of response from a complete comparison between the variables.

Section 3.4 explained the need to have some sort of commensuration across the variables if we are going to use specified linear combinations of them and be able to interpret the results. In the preceding profile analysis it was not necessary, since no such transformations were specified. In this present context we note that each variable is a sum of several items in the RPI (each item being scored in an identical way), so an overall total of the variables is interpreted as the extent to which a subject is measured as vulnerable to relapse by the inventory. Differences between the scales are similarly differences in these extents. We will expect to find large differences between the scales, since the number of items in each is not the same, and such a finding will not contribute to our understanding of the data. None the less, whether these differences are the same for each group or larger in some than others is a useful question to ask.

We shall define our between-variable factor effects as the following set of combinations:

V0:	1	1	1
V1:	1	−1	0
V2:	1	0	−1

The first (V0) defines the overall level and the second two (V1, V2) are contrasts forming a complete comparison for the 'type' factor. Although we have chosen a version of a particular comparison in Section 2.15, contrasting scale 1 with each of scales 2 and 3 in turn, we shall see that for the purposes of this particular analysis it does not matter which way we choose to summarize the 'type' comparison. It is important only that we separate the overall level from the factor effects. Seen in this perspective, we may consider the previous analysis in Section A.2 of the groups' profiles across the three variables to be an analysis of the mean response level *within* each level of the 'type' factor. In the present analysis we are considering the differences *between* levels of the 'type' factor. To highlight the relationship of this analysis to that of Section A.2 we shall ask of the between-subjects design the same questions that we considered there, namely the tests of hypotheses about the Helmert contrasts between groups. We shall demonstrate the analysis in two stages, beginning with a simple multivariate analysis of one between-variables factor ('type') and one between-subjects factor ('experience'), although this first analysis does not precisely answer our primary question.

Table A.9 shows this analysis of variance. The between-variables factor produces two separate sets of new variables transformed from the original three scales. As stated above, these are

(a) the single variable representing the overall level of the 'type' factor (V0); and
(b) the pair of variables representing the factor effects – the differences between 'types' (V1, V2).

Note that in the between-subjects factor the degrees of freedom have been partitioned to provide two separate hypotheses, but in the between-variables factor there is no requirement, in this example, to partition the degrees of freedom. The effects of this factor are tested simultaneously as a complete comparison with two degrees of freedom.

Each set is analysed separately against the between-subjects design, exactly as in Section A.2. The first part of the table shows the tests for the first 'set', the single variable V0. This of course is therefore a simple univariate analysis, the sums of products matrix reducing to the familar sum of squares, and the analysis is performed in the usual way. The *F*-statistic of univariate analysis is equivalent

Table A.9 Multivariate analysis of variance of the between-variables factor 'type' and the between-subjects factor 'experience' for the data shown in Table A.5 for partitioned between-groups degrees of freedom (showing the 'unique' contribution of each to the complete comparison).

(i) Analysis for the variable representing overall level of response (V0).

Source	*d.f.*	*SS* *(V0)*	F-*statistic*
Within groups	248	75 345.5	
Grand mean	1	339 561.0	
Between groups	2	2 745.1	4.518 (sig .012)
1st Helmert (1 vs. 2, 3)	1	2 729.9	8.985 (sig .003)
2nd Helmert (2 vs. 3)	1	165.0	0.543 (sig .462)

(ii) Analysis of the two variables representing the effects of the factor 'type' (V1, V2).

Source	*d.f.*	*SP matrix*		*Pillai statistic*
		(V1)	*(V2)*	
Within groups	248	20 722	—	
(**E** matrix)		18 857	24 586	
Grand mean	1	18 399	—	.687
(**H** matrix)		30 112	49 280	—
Between groups	2	277.57	—	.015
(**H** matrix)		307.78	342.07	(sig .451)
1st Helmert	1	257.98	—	.013
(**H** matrix)		282.27	308.85	(sig .188)
2nd Helmert	1	0.81	—	.0001
(**H** matrix)		0.10	0.01	(sig .987)

(see Section 4.3) to the Pillai–Bartlett statistic (and in fact to all of the four multivariate statistics) when our multivariate 'set' consists of just one variable. Moreover, our definition of the original scales implies that the overall level V0, which is the total of the three scales, is identical to the RPI total score analysed in Section A.1. The reader may compare the analyses of Tables A.9(i) and A.3 to vindicate this statement, and we shall not comment further on these tests except to restate the conclusion that the higher overall level of perceived vulnerability distinguishes the first group from the others.

The second set of variables provides a bivariate analysis for the effects of the 'type' factor and this is shown in Table A.9(ii). It parallels exactly the format of the previous multivariate analysis in Section A.2 (Table A.7) except that the sums of products matrices now have only two rows and two columns, one for V1 and one for V2. Note that because the analysis deals with them as a bivariate pair, and not individually, the ensuing tests would produce the same results regardless of which particular contrasts were used to summarize the comparison of 'type' levels (Section 4.2 discusses this invariance of multivariate tests). The first entry after the within-groups error matrix is the hypothesis matrix for testing whether the grand mean across the whole sample is zero for both V1 and V2. This would simply be testing whether there are any effects on average due to 'type' – that is, whether there are differences between the three variables which constitute the levels of the factor. These are precisely the differences anticipated in the discussion above which will arise because the numbers of items in the original scales differ. We are not unduly concerned with testing this hypothesis, but note simply from the size of the test statistic that our expectations are met.

The next entry, which we will again ignore as we did in the previous analyses, relates to the test of the compound null hypothesis that no between-group differences exist (see Section 2.7) and we turn instead to the tests of each of the two separate hypotheses concerning the Helmert contrasts. Taking the ratio of each hypothesis matrix in turn to the error matrix produces the two test statistics shown. If we continue to use an individual test level of .01 type I error, we see that neither of these hypotheses is rejected. Thus we conclude, for each effect, that it is not required in addition to the presence of the other in order to explain the data. It is still possible that either effect, by itself, might be significant for we have only shown that the complete comparison is not required. However, we shall not proceed to these tests but instead shall pause to consider why it was stated above that this first-stage analysis does not answer our main question of interest, 'have we gained from using the three scales UM, ES and LV instead of one overall score?'

By testing in the manner above, for instance, the null hypothesis that the second Helmert contrast is zero on the responses, we are asking whether the mean vector for V1 and V2 in group 2 is equal to that in group 3. We are asking therefore whether the response variables V1 and V2, which themselves represent differences between the original scales, carry any information which will distinguish group 2 from group 3. Now the question in which we are interested clearly asks whether the differences between the original scales carry any *additional* information about the groups – information additional to that carried by the overall total score. We therefore do not want simply to test the 'type' effect, but need to test the 'type' effect when controlling for the overall level of response. (Sections 3.4 and 3.8 discuss this point.) It may seem strange to test for 'type' carrying additional information about an hypothesis when the previous analysis showed that it was anyway unable to provide enough

Table A.10 Analysis of the two variables representing the effects of the factor 'type' (V1, V2) controlling for the variable representing the overall level of response (V0).

Source	*d.f.*	*SP matrix*		*Pillai statistic*
		(V1)	*(V2)*	
Within groups	247	14 450	—	
(**E** matrix)		8 189	6 583	
Regression	1	6 321	—	.819
(**H** matrix)		10 668	18 004	(sig .000)
Grand mean	1	119.49	—	.163
(**H** matrix)		378.32	717.46	—
Between groups	2	11.60	—	.037
(**H** matrix)		5.52	76.12	—
1st Helmert	1	2 729.89	—	.035
(**H** matrix)		918.22	308.85	(sig .013)
2nd Helmert	1	165.03	—	.011
(**H** matrix)		1.45	0.01	(sig .252)

information to reject that hypothesis. However, such a situation is possible, since effects can be masked by the 'noise' of large experimental error and by other effects. We now proceed to a second analysis in which the 'type' main effect is tested controlling for the overall level by using it as a covariate, whereas the preceding analysis tested the main effect and the overall level completely separately.

Table A.10 shows this analysis and it is in the same form as the analysis of the 'type' effects in Table A.9(ii), except that an extra source of variation is included, that due to regression of the responses on the covariate. Thus we have a bivariate analysis of V1 and V2 ('type' differences) after allowing for the single covariate, V0 (overall response level). The entry for the regression relationship, instead of being the sum of squares familiar in univariate regression, is a sums of products matrix which can be used as a hypothesis matrix in testing for a null regression.

A glance at this table shows from the error matrix that the within-group variation has been considerably reduced from that shown in Table A.9(ii). This reduction is due to the effect of the responses' regression on the covariate. This regression effect is tested in the usual multivariate manner, by using the matrix ratio $\mathbf{H} \cdot \mathbf{E}^{-1}$ and the Pillai–Bartlett test statistic shows a very high level of significance. Passing to the tests for the individual Helmert contrasts, we see in each case that although the variation attributed to these sources has been reduced, the reduction in error variation has more than compensated. The test of the hypothesis that group 2 is equal to group 3 is not rejected, but the test that group 1 is different from their average is only fractionally short of achieving significance. We may conclude that even in a test at this high level of significance, there is evidence that the differences between the three types of response scales in the RPI provide extra information for distinguishing the first group of people (the 'long-standing' group) from the others.

The nature of this extra information is not hard to see. The profile analysis in Section A.2 showed the third scale, LV, to be the most important in discriminating between the 'long-standing' group and the others. It has, however, a very small range of scores compared with the other scales, so that a raw total of the three swamps its effect. Its differences from the other two scales are lost in their greater scale of error variation and these differences fail to discriminate between the groups. It is only when controlling for the highly variable level of overall endorsements that we can see the effects due to this minor scale. Thus it is important to separate the three types of perceived vulnerability in order to demonstrate the ability of the LV scale to distinguish the 'long-standing' relapsers from the rest.

Table A.11 Raw data for the variables used in the RPI.

	Grp	*UM*	*ES*	*LV*		*Grp*	*UM*	*ES*	*LV*
1	2	27	0	3	23	2	13	9	7
2	1	37	19	6	24	1	15	12	3
3	2	2	2	6	25	1	25	18	6
4	3	24	15	3	26	1	23	11	4
5	1	34	13	4	27	1	26	6	9
6	1	37	10	7	28	1	18	0	2
7	1	34	23	9	29	1	21	18	6
8	3	7	9	7	30	1	0	4	3
9	3	10	0	0	31	1	14	7	2
10	3	6	7	6	32	2	14	13	0
11	2	30	16	9	33	1	10	13	7
12	3	24	21	8	34	1	13	18	9
13	1	22	6	8	35	3	41	16	9
14	1	24	7	7	36	1	24	15	3
15	2	12	14	4	37	2	1	6	3
16	1	31	13	9	38	2	15	16	7
17	2	2	3	3	39	2	6	2	0
18	3	3	4	0	40	1	34	16	6
19	1	26	9	9	41	1	37	19	9
20	1	33	14	9	42	2	2	4	0
21	1	18	7	6	43	1	28	6	9
22	1	16	8	9	44	2	21	15	8

Table A.11 (continued)

	Grp	UM	ES	LV		Grp	UM	ES	LV
45	3	13	3	3	93	1	24	9	8
46	1	32	9	8	94	1	37	19	9
47	2	25	2	9	95	2	3	2	0
48	3	0	0	0	96	1	16	12	8
49	2	27	13	4	97	2	14	5	7
50	1	12	10	4	98	1	35	16	6
51	2	11	11	7	99	1	35	15	7
52	1	30	13	6	100	1	34	21	9
53	2	32	19	5	101	1	34	23	7
54	1	9	7	1	102	2	31	7	9
55	1	32	24	9	103	1	35	10	5
56	2	31	18	5	104	1	24	7	6
57	3	26	16	3	105	1	29	16	9
58	3	0	2	1	106	1	17	3	9
59	1	5	3	4	107	1	11	12	6
60	1	15	5	3	108	2	39	23	9
61	1	29	18	4	109	1	21	10	9
62	1	28	21	9	110	1	11	3	4
63	2	16	14	9	111	2	33	17	9
64	3	15	5	8	112	2	0	0	0
65	1	27	15	7	113	2	31	14	7
66	2	21	12	5	114	1	18	14	2
67	1	42	23	9	115	3	41	12	9
68	2	21	12	9	116	1	5	10	9
69	1	16	0	3	117	3	30	12	6
70	1	26	0	9	118	2	3	9	6
71	1	20	2	5	119	2	9	8	2
72	1	1	1	1	120	1	17	20	7
73	3	15	23	9	121	3	0	10	3
74	1	16	13	3	122	2	22	10	5
75	2	6	6	4	123	3	20	6	1
76	1	19	13	6	124	3	14	3	4
77	1	34	23	9	125	1	7	7	3
78	1	26	21	7	126	3	14	12	5
79	2	28	22	7	127	2	14	1	5
80	2	36	21	9	128	2	40	5	5
81	1	19	16	9	129	1	24	9	7
82	1	32	18	8	130	2	15	5	5
83	2	3	5	4	131	1	8	17	6
84	1	9	16	9	132	2	5	4	3
85	2	0	6	7	133	1	25	1	9
86	1	8	4	5	134	2	17	2	9
87	1	42	22	9	135	1	17	18	8
88	1	41	21	8	136	1	26	18	8
89	2	16	10	2	137	1	29	10	6
90	2	7	12	4	138	1	26	15	9
91	2	36	11	7	139	1	999	999	999
92	3	18	10	9	140	2	20	14	9

	Grp	UM	ES	LV		Grp	UM	ES	LV
141	1	10	17	9	189	1	35	15	9
142	3	31	20	6	190	2	17	5	5
143	1	18	7	3	191	2	29	16	7
144	1	22	13	4	192	2	32	20	8
145	2	21	9	6	193	1	13	5	3
146	2	3	7	4	194	2	37	22	9
147	2	16	1	0	195	1	26	13	6
148	1	15	12	7	196	1	17	9	4
149	3	7	5	1	197	1	28	10	5
150	2	18	12	4	198	3	28	8	6
151	1	27	11	9	199	3	5	3	0
152	1	18	9	8	200	2	25	19	8
153	2	0	0	0	201	3	19	23	3
154	1	8	14	6	202	2	33	14	5
155	2	7	7	6	203	2	7	17	4
156	1	7	4	5	204	1	19	13	6
157	2	19	21	8	205	1	32	13	9
158	2	30	16	8	206	3	0	3	0
159	3	6	16	8	207	1	35	18	999
160	1	28	13	9	208	2	16	18	2
161	1	17	8	3	209	1	39	18	9
162	2	17	13	6	210	1	28	20	3
163	1	4	4	3	211	2	40	4	3
164	2	12	16	6	212	1	18	9	8
165	1	2	1	3	213	2	11	11	6
166	2	15	3	6	214	1	11	21	9
167	1	28	21	6	215	2	38	999	8
168	1	14	10	2	216	2	23	12	7
169	2	13	14	7	217	1	32	20	9
170	2	0	0	0	218	2	14	3	3
171	2	0	6	2	219	2	10	6	4
172	1	6	1	2	220	3	32	7	4
173	1	35	16	9	221	1	999	999	999
174	1	23	3	8	222	2	22	19	5
175	3	10	5	6	223	1	0	0	0
176	1	29	20	9	224	2	29	16	6
177	1	23	12	6	225	1	0	10	8
178	2	4	0	2	226	1	23	9	7
179	1	11	13	8	227	1	37	22	9
180	2	34	16	7	228	2	25	17	6
181	3	0	0	0	229	1	17	10	3
182	1	26	5	8	230	2	18	3	2
183	3	15	15	5	231	2	19	22	8
184	1	3	5	2	232	1	16	4	5
185	3	34	11	7	233	2	0	2	2
186	2	6	19	6	234	1	24	16	8
187	1	18	12	5	235	2	24	14	6
188	1	3	5	7	236	2	19	5	7

Table A.11 (continued)

	Grp	UM	ES	LV		Grp	UM	ES	LV
237	3	33	19	3	247	2	22	20	7
238	1	15	10	7	248	1	21	15	6
239	1	24	16	7	249	2	17	9	6
240	2	33	14	7	250	1	28	23	9
241	1	26	3	5	251	1	15	17	9
242	2	26	12	8	252	3	11	5	6
243	1	26	16	6	253	3	39	15	6
244	2	40	20	9	254	2	30	17	9
245	3	33	17	7	255	2	22	11	3
246	2	30	13	8					