Course Examination
1st Term 2020-21

ENGG2760A-D

ENGG 2760/ESTR 2018: Probability for Engineers
The Chinese University of Hong Kong, Fall 2020
Duration: 2 hours

Thursday 17 December
Final Examination

You must work alone and cite any external references that you use as stipulated in the CUHK academic honesty guidelines. Please submit your solution by 8.45pm on Thursday 17 December electronically at `https://course.cse.cuhk.edu.hk/~engg2760`.

Each question is worth 10 points. Explain your answers clearly.

1. Let A and B be arbitrary events. Which of the following is true? If you answer yes, prove it using the axioms of probability. If you answer no, provide a counterexample.
   (a) $\mathrm{P}(A|B) + \mathrm{P}(A|B^c) = 1$.
   (b) $\mathrm{P}(A \cap B | A \cup B) \leq \mathrm{P}(A|B)$.

2. n independent random numbers are sampled uniformly from the interval $[0, 1]$.
   (a) If $n = 10$, what is the probability that exactly 4 of of them are greater than 0.7?
   (b) If $n = 50$, use the Central Limit Theorem to estimate the probability that their sum is between 20 and 25 (inclusive).

3. Companies A and B produce lightbulbs. Their lifetimes are exponential random variables with mean 2 years for company A and 1 year for company B.
   (a) A shop sources 3/4 of its lightbulbs from company A and the remaining 1/4 from company B. If a random lifebulb from the shop survived for 2 years, how likely is it to have been produced by company B?
   (b) What is the probability that a lightbulb produced by company B outlasts one produced by company A? Assume their lifetimes are independent.

4. Alice takes T hours to travel to Bob's house, where T is a random variable with PDF

$$f_T(t) = \begin{cases} 1/t^2, & \text{when } t \geq 1 \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the CDF (cumulative distribution function) $F_T(t) = \mathrm{P}(T \leq t)$.
   (b) The distance between Alice's and Bob's house is one mile so that Alice travels at a speed $V = 1/T$ miles per hour. What is Alice's expected speed $\mathrm{E}[V]$?

5. A group of 10 boys and 10 girls is randomly divided into 5 teams A, B, C, D, E of 4 children each.
   (a) What is the probability that all children in team A are of the same gender?
   (b) Is the probability that all teams are of mixed gender more than 50% or not? Justify your answer.

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