請勿攜去
Not to be taken away

香 港 中 文 大 學
The Chinese University of Hong Kong

二 0 二一至二二年度上學期科目考試
Course Examination 1st Term, 2021-22

科目編號及名稱
Course Code & Title : ENGG2760A/ESTR2018: Probability for Engineers

時間 小時 分鐘
Time allowed : 2 hours minutes

學號 座號
Student I.D. No. : Seat No. :

Instructions:

- This is a closed-book exam. You may use an approved calculator from the University's list.
- Answer **ALL** questions. The total mark is **100**.
- Answers must be given in English. Answers without sufficient intermediate steps may lose some marks.
- Unless otherwise specified, numerical answers should be either **exact** or corrected to **3 significant figures**.
- The exam paper and scratch paper must be returned along with the answer book at the end of the exam.

1. (8 marks) The length X of certain items produced in a particular workshop are normally distributed with mean μ cm and standard deviation σ cm. It is known that 10.56% of the items have a length less than 45 cm and 22.66% have a length greater than 49 cm. Find the values of μ and σ. (You may use the standard normal distribution table at the end of the question paper.)

2. (10 marks) Let X and Y be two independent random variables where X follows an exponential distribution with mean 1 and Y follows a uniform distribution over [0, 2]. Calculate $P(X > Y)$.

3. (12 marks) Assume that the lifetime T (hours) of a certain electronic component follows exponential distribution with the mean value of 400 hours. Answer the following questions.
   (a) (6 marks) What is the probability that the lifetime of the electronic component exceeds 200 hours?
   (b) (6 marks) Assuming that an electronic component has been used for 200 hours, what is the probability that it can continue to be used for at least another 200 hours?

4. (10 marks) Let X and Y be two continuous random variables with the joint probability density function:

$$f_{XY}(x,y) = \begin{cases} 2, & \text{for } y + x \leq 1, x > 0, y > 0. \\ 0, & \text{otherwise.} \end{cases}$$

   Calculate $Cov(X,Y)$ and the correlation coefficient $\rho(X,Y)$.

5. (10 marks) Suppose you have a bag with the same number of red, green, orange and blue balls. You repeatedly draw a ball at random from the bag, observe its color, and

place it back in the bag. Let Y be the number of red balls you get out of $n = 400$ drawings. Derive an upper bound on $P(Y \geq 200)$ using Markov's inequality and Chebyshev's inequality respectively.

6. (16 marks) Let X and Y be two independent Uniform (0, 1) random variables. Find the PDF of the random variables for

   (a) (5 marks) $Z_1 = e^X$

   (b) (5 marks) $Z_2 = -3\log X$

   (c) (6 marks) $Z_3 = X + Y$

7. (16 marks) Let X be a Uniform (0,1) random variable. Suppose that we know $Y \mid X = x$ follows a Geometric (x) distribution.

   (a) (5 marks) Find the PMF $P_{Y|X}(2 \mid x)$.

   (b) (5 marks) Find $P_Y(2)$.

   (c) (6 marks) Find the posterior density of X given $Y = 2$, i.e., $f_{X|Y}(x \mid 2)$.

8. (18 marks) 500 balls are drawn without replacement altogether from a bin with 600 black balls and 400 white balls.

   (a) (4 marks) What is the expected number of black balls drawn among the 500 balls?

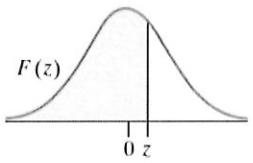
   (b) (7 marks) What is the variance of the number of black balls drawn?

   (c) (7 marks) Use Chebyshev's inequality to show that the probability you drew fewer than 200 black balls is no more than 1%. Justify your answer.

The following is the standard normal distribution table.

Table 3

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5973	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998									
4.0	0.99997									
5.0	0.9999997									

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