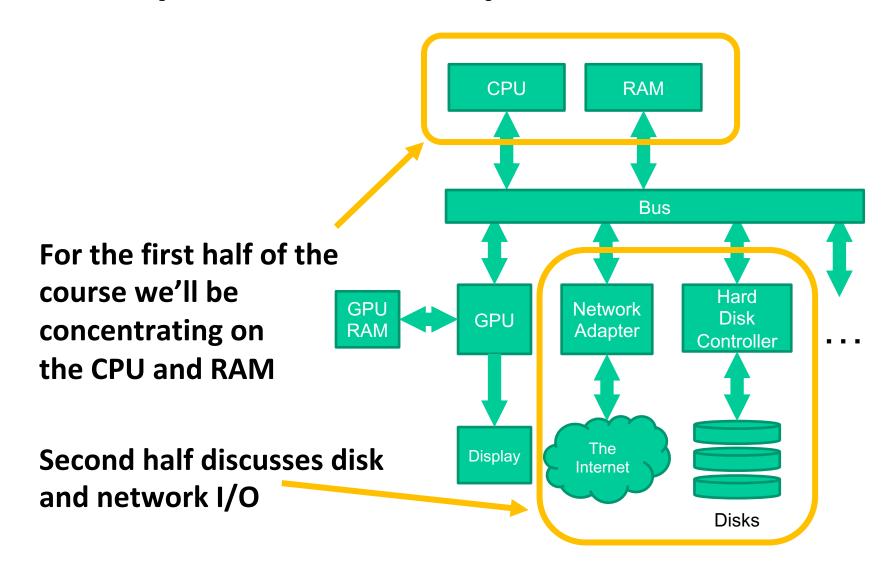
## Bits, Bytes and Integers

Introduction to Computer Systems

Danfeng Shan Xi'an Jiaotong University

## Roadmap – Inside a Computer



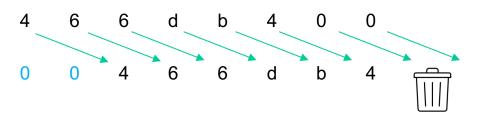
#### **Course Teaser (Things You May Know Already)**

What value do you get if you arithmetic right-shift the hexadecimal number 0x466db400 by eight bits?

Hexadecimal is just like decimal ... if you have sixteen fingers.

"Eight bits" is two hex digits.

"Arithmetic right shift" is division by a power of two.



## Today: Bits, Bytes, and Integers

#### Representing information as bits

**Bit-level manipulations** 

#### Integers

Representation: unsigned and signed

Conversion, casting

Expanding, truncating

Addition, negation, multiplication, shifting

Summary

Representations in memory, pointers, strings

## **Everything is bits**

#### Each bit is 0 or 1

#### By encoding/interpreting sets of bits in various ways

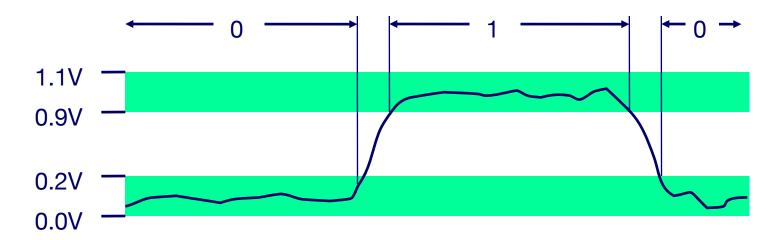
Computers determine what to do (instructions)

... and represent and manipulate numbers, sets, strings, etc...

#### Why bits? Electronic Implementation

Easy to store with bistable elements (双稳态器件)

Reliably transmitted on noisy and inaccurate wires



## For example, can count in binary

#### **Base 2 Number Representation**

Represent 15213<sub>10</sub> as 11101101101<sub>2</sub>

Represent 1.20<sub>10</sub> as 1.001100110011[0011]...<sub>2</sub>

Represent  $1.5213 \times 10^4$  as  $1.1101101101101_2 \times 2^{13}$ 

## **Encoding Byte Values**

#### Byte = 8 bits

Binary 00000000<sub>2</sub> to 11111111<sub>2</sub>

Decimal: 0<sub>10</sub> to 255<sub>10</sub>

Hexadecimal 00<sub>16</sub> to FF<sub>16</sub>

Base 16 number representation

Use characters '0' to '9' and 'A' to 'F'

Write FA1D37B<sub>16</sub> in C as

- 0xFA1D37B
- 0xfa1d37b

He	Pe	Bind
0	0	0000
1	1	0001
2	2	0010
3	0 1 2 3	0011
4	4	0100
0 1 2 3 4 5 6 7 8	4 5 6 7 8	0101
6	6	0110
7	7	0111
8	8	1000
	9	1001
A	10	1010
B C	11	1011
U	12	1100
D	13	1101
E	14	1110
F	15	1111

15213:	0011	1011	0110	1101
	3	В	6	D

## **Example Data Representations**

C Data Type	Typical 32-bit	Typical 64-bit
char	1	1
short	2	2
int	4	4
long	4	8
float	4	4
double	8	8
pointer	4	8

## **Example Data Representations**

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## **Example Data Representations**

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char	1	1
short	2	2
int	4	4
long	4	8
float	4	4
double	8	8
pointer	4	8
	"ILP32"	"LP64"

## **Today: Bits, Bytes, and Integers**

#### Representing information as bits

#### **Bit-level manipulations**

#### Integers

Representation: unsigned and signed

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## **Boolean Algebra**

#### **Developed by George Boole in 19th Century**

Algebraic representation of logic

Encode "True" as 1 and "False" as 0

And

Or

A&B = 1 when both A=1 and B=1

A|B = 1 when either A=1 or B=1 or both

Not

Exclusive-Or (Xor)

 $^{\sim}$ A = 1 when A=0

 $A^B = 1$  when A=1 or B=1, but not both

## **General Boolean Algebras**

#### **Operate on Bit Vectors**

Operations applied bitwise

	01101001	01101001		01101001		
<u>&amp;</u>	01010101	<u>  01010101</u>	^_	01010101	~	01010101
	01000001	01111101		00111100		10101010

## **Bit-Level Operations in C**

#### Operations &, |, ~, ^ Available in C

Apply to any "integral" data type
long, int, short, char, unsigned
View arguments as bit vectors
Arguments applied bit-wise

# Het Decime Binary 0 0 0000 1 1 0001 2 2 0010

_	
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
	2 3 4 5 6 7 8 9 10 11 12 13 14

## **Contrast: Logic Operations in C**

#### **Contrast to Bit-Level Operators**

Logic Operations: &&, ||,!

View 0 as "False"

Anything nonzero as "True"

Always return 0 or 1

Early termination

#### **Examples (char data type)**

 $!0x41 \rightarrow 0x00$ 

 $!0x00 \rightarrow 0x01$ 

 $!!0x41 \rightarrow 0x01$ 

 $0x69 \&\& 0x55 \rightarrow 0x01$ 

 $0x69 \mid \mid 0x55 \rightarrow 0x01$ 

p && \*p (avoids null pointer access)

Watch out for && vs. & (and || vs. |)... Super common C programming pitfall!

## **Shift Operations**

#### Left Shift: x << y

Shift bit-vector **x** left **y** positions

Throw away extra bits on left
 Fill with 0's on right

#### Right Shift: $x \gg y$

Shift bit-vector **x** right **y** positions

Throw away extra bits on right

Logical shift

Fill with 0's on left

Arithmetic shift

Replicate most significant bit on left

#### **Undefined Behavior**

Shift amount < 0 or ≥ word size

Argument x	<mark>0</mark> 1100010
<< 3	00010 <i>000</i>
Log. >> 2	00011000
Arith. >> 2	00011000

Argument x	10100010
<< 3	00010 <i>000</i>
Log. >> 2	00101000
Arith. >> 2	11101000

## **Today: Bits, Bytes, and Integers**

Representing information as bits Bit-level manipulations

#### **Integers**

Representation: unsigned and signed, negation

Conversion, casting

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## **Question?**

```
int foo = -1;
unsigned bar = 1;
(foo < bar) == true ?</pre>
```

## **Encoding "Integers"**

#### **Unsigned**

Given a bit w bits long...

Given a bit vector 
$$x$$
,  $B2U(x) = \sum_{i=0}^{w-1} x_i \cdot 2^i$   $w$  bits long...

#### Signed (twos complement)

B2T(x) = 
$$-x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$
 Sign Bit

#### Examples (w = 5)

$$0 + 8 + 0 + 2 + 0 = 10$$

$$16 + 8 + 0 + 2 + 0 = 26$$

$$-16 + 8 + 0 + 2 + 0 = -10$$

## **Negation: Complement & Increment**

#### Negate through complement and increase

Why?

$$-x + x = 0 \text{ (by definition)}$$
 $-x + x = 1111...111 == -1$ 
 $-x + x + 1 == 0$ 
 $(-x+1) + x == 0$ 
 $-x + 1 == -x$ 
 $-x + x + 1 == 0$ 
 $-x + 1 == -x$ 

#### **Example:** x = 15213

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
~x	-15214	C4 92	11000100 10010010
~x+1	-15213	C4 93	11000100 10010011
У	-15213	C4 93	11000100 10010011

## **Complement & Increment Examples**

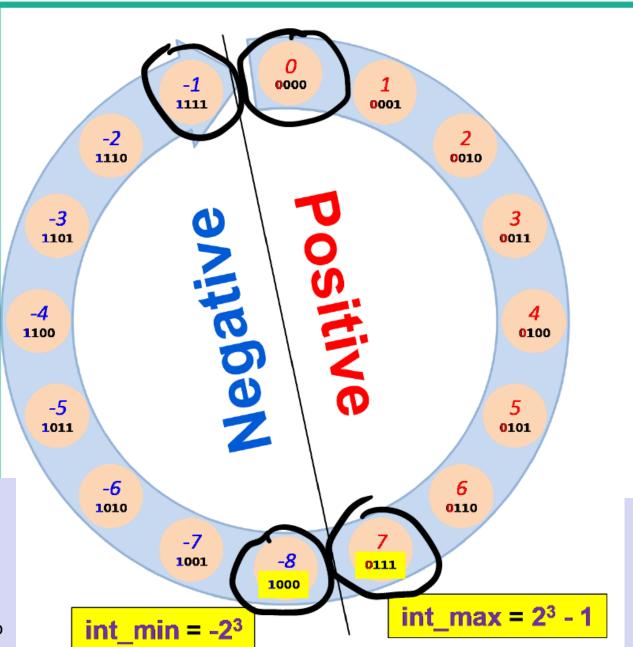
$$x = 0$$

	Decimal	Hex	Binary
0	0	00 00	00000000 00000000
~0	-1	FF FF	11111111 11111111
~0+1	0	00 00	00000000 00000000

$$x = T_{\min}$$

	Decimal	Hex	Binary
x	-32768	80 00	10000000 00000000
~x	32767	7F FF	01111111 11111111
~x+1	-32768	80 00	10000000 00000000





values: -1, -2, ..., -8

Eight negative

Mathematicians would prefer it if a 4-bit signed number could represent values -8...8, but that's  $2^4 + 1$  values, so they won't all fit.

Eight *non*-negative values: 0, 1, ..., 7

What if we made a 4-bit signed number only represent values -7...7? Then we wouldn't be using bit pattern 1000...

## Today: Bits, Bytes, and Integers

Representing information as bits
Bit-level manipulations
Integers

Representation: unsigned and signed, negation

Conversion, casting

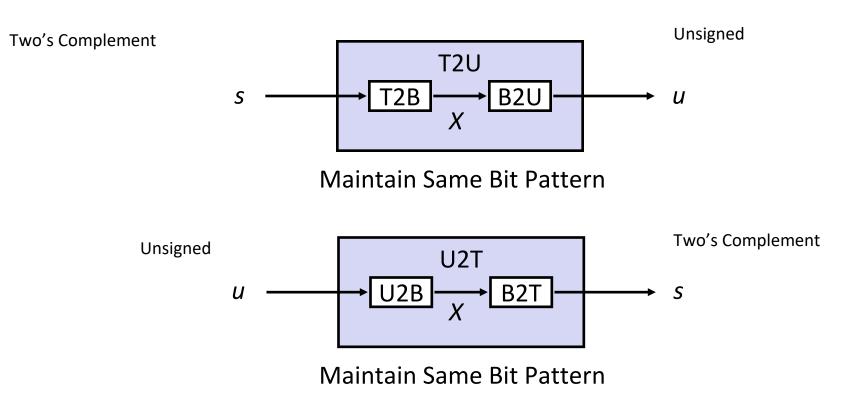
Expanding, truncating

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Summary

Representations in memory, pointers, strings

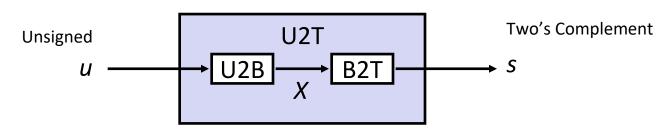
## **Mapping Between Signed & Unsigned**



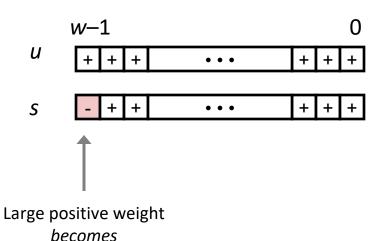
Mappings between unsigned and two's complement numbers:

**Keep bit representations and reinterpret** 

## **Relation between Signed & Unsigned**



Maintain Same Bit Pattern

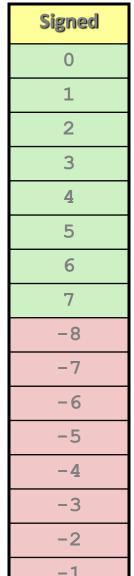


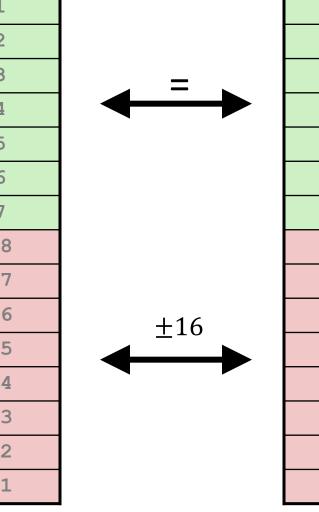
Large negative weight

25

## Mapping Signed ↔ Unsigned

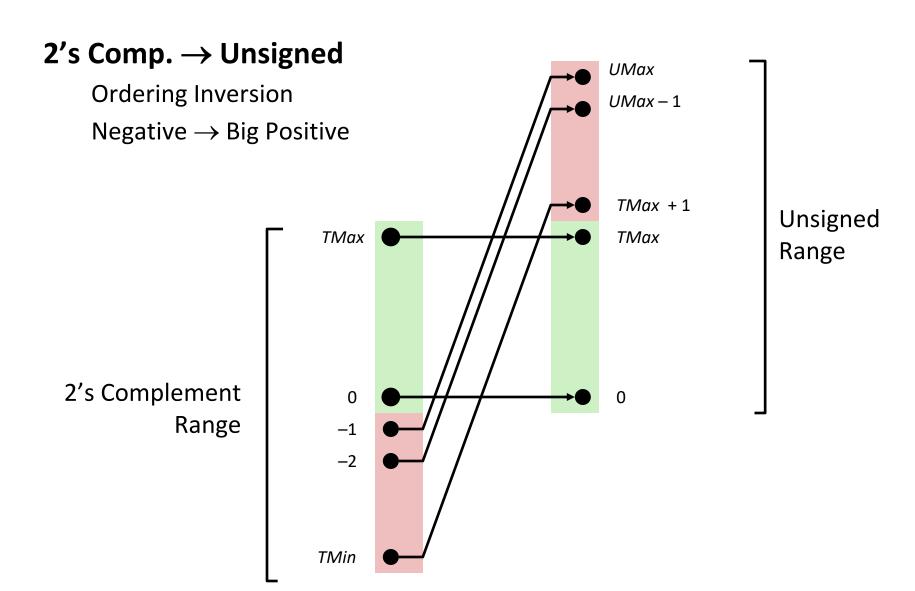
Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111





Unsigned	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

#### **Conversion Visualized**



## Signed vs. Unsigned in C

#### **Constants**

```
By default are considered to be signed integers Unsigned if have "U" as suffix OU, 4294967259U
```

#### Casting

```
Explicit casting between signed & unsigned same as U2T and T2U
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

Implicit casting also occurs via assignments and procedure calls

## **Casting Surprises**

#### **Expression Evaluation**

If there is a mix of unsigned and signed in single expression, signed values implicitly cast to unsigned

Including comparison operations <, >, ==, <=, >=

Examples:

Constant 1	Constant 2	Relation	Evaluation
0	0 <b>u</b>	==	Unsigned
-1	0	<	Signed
-1	0υ	>	Unsigned
INT_MAX	INT_MIN	>	Signed
(unsigned) INT_MAX	INT_MIN	<	Unsigned
-1	-2	>	Signed
(unsigned)-1	-2	>	Unsigned
INT_MAX	((unsigned)INT_MAX) + 1	<	Unsigned
INT_MAX	(int)(((unsigned)INT_MAX) + 1)	>	Signed

## **Question?**

example02.c

```
int foo = -1;
unsigned bar = 1;
foo < bar == true ?</pre>
```

## Summary Casting Signed ↔ Unsigned: Basic Rules

Bit pattern is maintained

**But reinterpreted** 

Can have unexpected effects: adding or subtracting 2<sup>w</sup>

**Expression containing signed and unsigned int** 

int is cast to unsigned!!

## Today: Bits, Bytes, and Integers

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Bit-level manipulations
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Summary

Representations in memory, pointers, strings

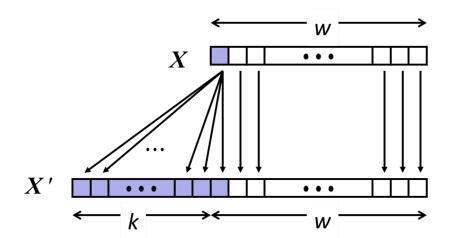
## **Question?**

example03.c

```
int x = 0x8000;
short sx = (short) x;
int y = sx;
```

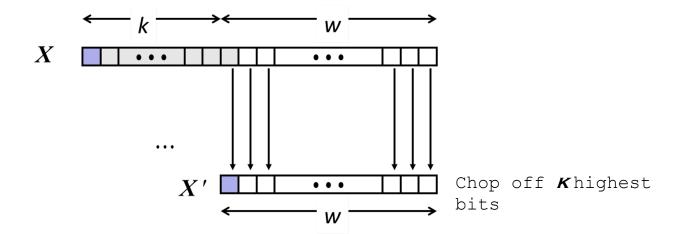
## **Sign Extension and Truncation**

#### **Sign Extension**

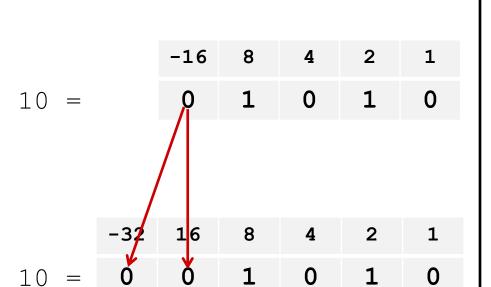


Make K copies of sign bit

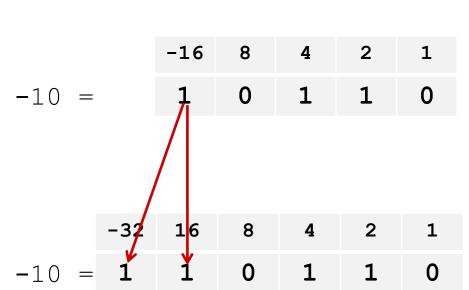
#### **Truncation**



## **Sign Extension: Simple Example**



Positive number



Negative number

## **Truncation: Simple Example**

No sign change

$$-16$$
 8 4 2 1  $-6$  = 1 1 0 1 0

$$-8$$
 4 2 1  $-6$  = 1 0 1 0

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

$$-8$$
 4 2 1  $-6$  = 1 0 1 0

$$-16$$
 8 4 2 1  $-10$  = 1 0 1 1 0

## **Question?**

example03.c

```
int x = 0x8000;
short sx = (short) x;
int y = sx;
```

## **Today: Bits, Bytes, and Integers**

Representing information as bits
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Representation: unsigned and signed

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Representations in memory, pointers, strings

## **Unsigned Addition**

Operands: w bits

True Sum: w+1 bits

Discard Carry: w bits

и	• • •	
+ <i>V</i>	• • •	
u + v	• • •	

 $UAdd_w(u, v)$ 

## 

# 

7	7	010
5	5	0101
6	6	0110

7	7	011
8	8	100

#### 

#### 

#### D

#### 

#### **Standard Addition Function**

Ignores carry output

unsigned char	+	1110 1101	 E9 + D5	233 + 213
	_		 	

## **Unsigned Addition**

Operands: w bits

True Sum: w+1 bits

Discard Carry: w bits

и	• • •	
+ V	• • •	
u + v	• • •	

#### **Standard Addition Function**

Ignores carry output

unsigned cha		1110 1101		E9 + D5	233 + 213
	1	1011	1110	1BE	446
		1011	1110	BE	190

 $UAdd_w(u, v)$ 

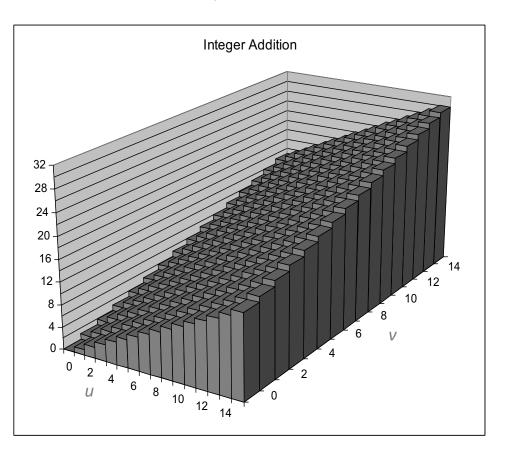
## Hex Decimal Binary

1	•	•
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
В	11	1011
С	12	1100
D	13	1101
E	14	1110
F	15	1111

## Visualizing (Mathematical) Integer Addition

#### **Integer Addition**

4-bit integers u, vCompute true sum Add<sub>4</sub>(u, v) Values increase linearly with u and vForms planar surface  $Add_4(u, v)$ 

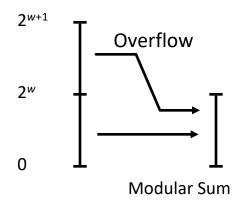


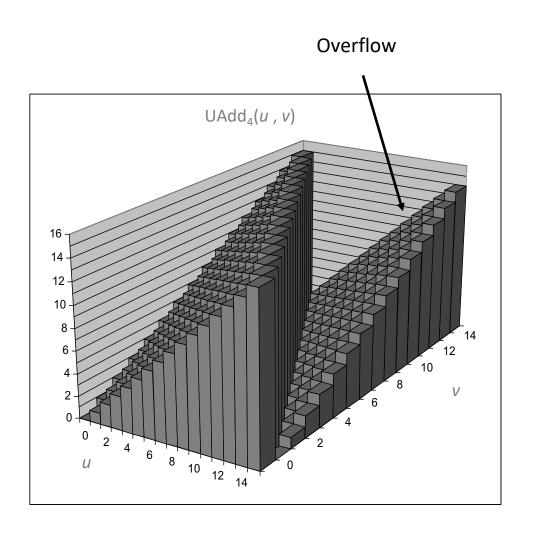
## **Visualizing Unsigned Addition**

#### **Wraps Around**

If true sum  $\geq 2^w$ At most once

#### True Sum





## **Two's Complement Addition**

#### TAdd and UAdd have Identical Bit-Level Behavior

Signed vs. unsigned addition in C: int s, t, u, v; s = (int) ((unsigned) u + (unsigned) v); t = u + vWill give s == t 1110 1001 **E9** -231101 0101 -43 + D5 1011 1110 -66 1BE -66 BE

## Visualizing 2's Complement Addition

#### **Values**

4-bit two's comp. Range from -8 to +7

#### **Wraps Around**

If sum  $\geq 2^{w-1}$ 

**Becomes** 

negative

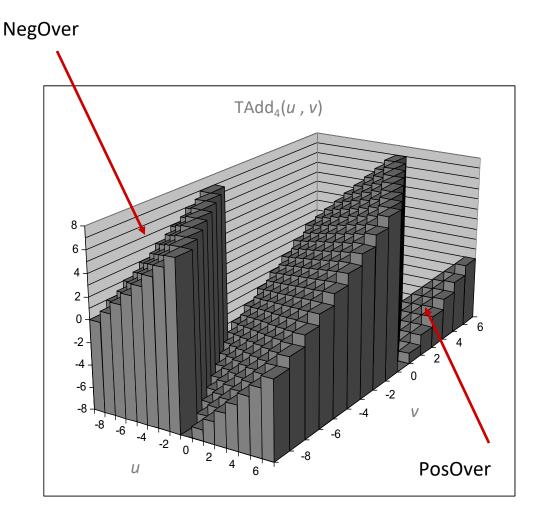
At most once

If sum  $< -2^{w-1}$ 

**Becomes** 

positive

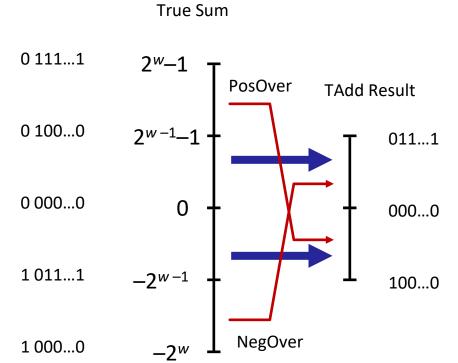
At most once



### **TAdd Overflow**

#### **Functionality**

True sum requires w+1 bits
Drop off MSB
Treat remaining bits as 2's comp. integer



## Today: Bits, Bytes, and Integers

Representing information as bits
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## **Shifting**

#### Left Shift: x << y

Shift bit-vector  $\mathbf{x}$  left  $\mathbf{y}$  positions

Throw away extra bits on left

Fill with 0's on right

Equivalent to multiplying by  $2^{y}$ 

#### Right Shift: $x \gg y$

Shift bit-vector x right y positions

Throw away extra bits on right

Two kinds:

"Logical": Fill with 0's on left "Arithmetic": Replicate most significant bit on left

Almost equivalent to dividing by  $2^y$ 

#### **Undefined Behavior (in C)**

Shift amount < 0 or ≥ word size

Argument x	01100010
<< 3	00010000
Logical >> 2	00 <mark>011000</mark>
Arithmetic >> 2	00 <mark>011000</mark>

Argument x	10100010
<< 3	00010000
Logical >> 2	00 <mark>101000</mark>
Arithmetic >> 2	11101000

## Multiplication

#### Goal: Computing Product of w-bit numbers x, y

Either signed or unsigned

#### But, exact results can be bigger than w bits

Unsigned: up to 2w bits

Result range:  $0 \le x * y \le (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$ 

Two's complement min (negative): Up to 2w-1 bits

Result range:  $x * y \ge (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$ 

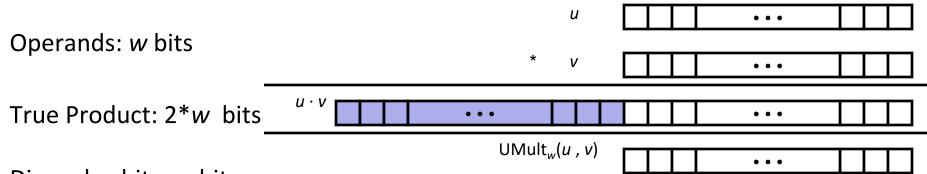
Two's complement max (positive): Up to 2w bits, but only for  $(TMin_w)^2$ 

Result range:  $x * y \le (-2^{w-1})^2 = 2^{2w-2}$ 

#### So, maintaining exact results...

would need to keep expanding word size with each product computed is done in software, if needed

## **Unsigned Multiplication in C**



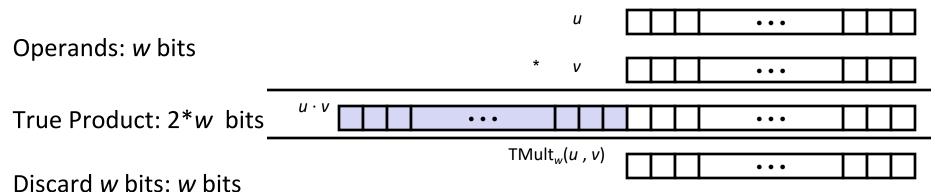
Discard w bits: w bits

#### **Standard Multiplication Function**

Ignores high order w bits

		1110	1001		E9		233
*		1101	0101	*	D5	*	213
1100	0001	1101	1101	C	1DD		49629
		1101	1101		DD		221

## Signed Multiplication in C



## **Standard Multiplication Function**

Ignores high order w bits

Some of which are different for signed

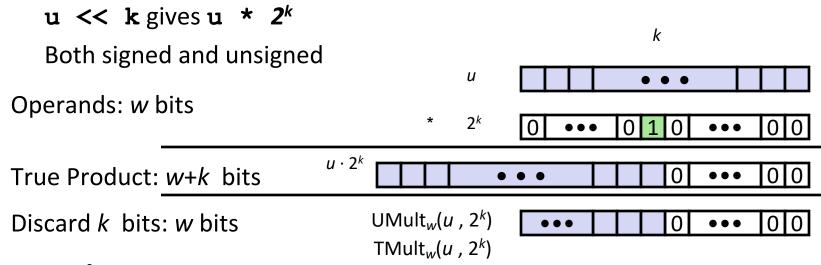
vs. unsigned multiplication

Lower bits are the same

		1110	1001		<b>E9</b>		-23
*		1101	0101	*	D5	*	-43
0000	0011	1101	1101	(	)3DD		989
		1101	1101		DD		-35

## **Power-of-2 Multiply with Shift**

#### **Operation**



#### **Examples**

$$u \ll 3 == u * 8$$
  
 $(u \ll 5) - (u \ll 3) == u * 24$ 

Most machines shift and add faster than multiply Compiler generates this code automatically

## **Today: Bits, Bytes, and Integers**

Representing information as bits
Bit-level manipulations
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Representation: unsigned and signed

Conversion, casting

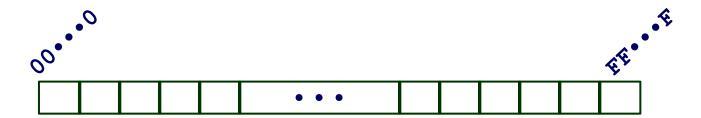
Expanding, truncating

Addition, negation, multiplication, shifting

Summary

Byte order in memory, pointers, strings

## **Byte-Oriented Memory Organization**



#### Programs refer to data by address

Imagine all of RAM as an enormous array of bytes

An address is an index into that array

A pointer variable stores an address

### System provides a private address space to each "process"

A process is an instance of a program, being executed

An address space is one of those enormous arrays of bytes

Each program can see only its own code and data within its enormous array

We'll come back to this later ("virtual memory" classes)

### **Machine Words**

#### Any given computer has a "Word Size"

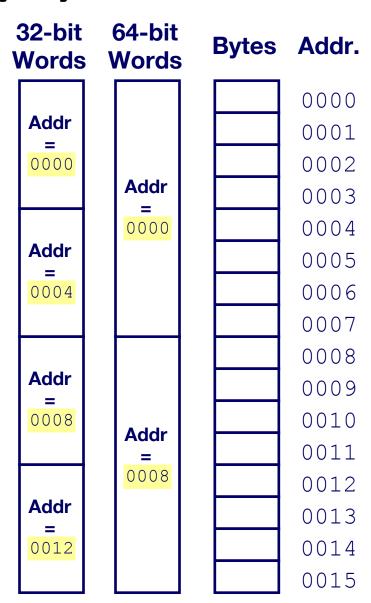
Nominal size of integer-valued data and of addresses

Historically, most machines used 32 bits (4 bytes) as word size Limits addresses to 4GB (2<sup>32</sup> bytes)

Machines still support multiple data formats
Fractions or multiples of word size
Always integral number of bytes

## Addresses Always Specify Byte Locations

Address of a word is address of the first byte in the word Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



## **Example Data Representations**

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
char	1	1	1
short	2	2	2
int	4	4	4
long	4	8	8
float	4	4	4
double	8	8	8
pointer	4	8	8

## **Question?**

example04.c

```
struct foo {
   char mem1[3]; // 3 bytes
   int mem2; // 4 bytes
   char mem3; // 1 byte
};
sizeof(struct foo) = ?
```

## **Byte Ordering**

# So, how are the bytes within a multi-byte word ordered in memory?

#### **Conventions**

Big Endian: Sun, PPC Mac, network packet headers

Least significant byte has highest address

Little Endian: x86, ARM processors running Android, iOS, and

Windows

Least significant byte has lowest address

## **Byte Ordering Example**

#### **Example**

Variable x has 4-byte value of 0x01234567 Address given by &x is 0x100

Big Endian		0x100	0x101	0x102	0x103			
			01	23	45	67		
Little Endian		0x100	0x101	0x102	0x103			
			67	45	23	01	-	

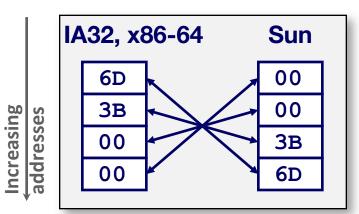
## **Representing Integers**

**Decimal: 15213** 

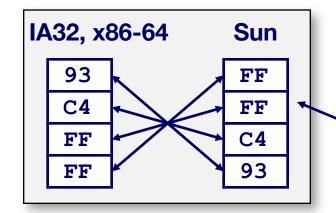
**Binary:** 0011 1011 0110 1101

**Hex:** 3 B 6 D

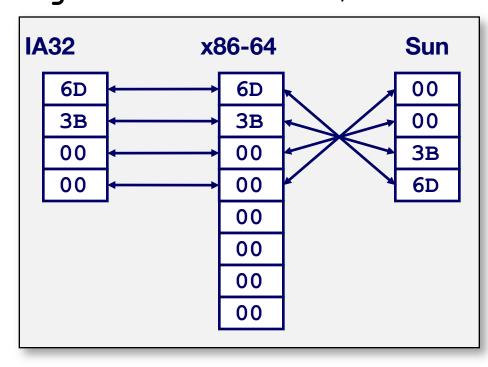




int B = -15213;



long int C = 15213;



Two's complement representation

## **Examining Data Representations**

#### **Code to Print Byte Representation of Data**

Casting pointer to unsigned char \* allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len){
    size_t i;
    for (i = 0; i < len; i++)
        printf("%p\t0x%.2x\n",start+i, start[i]);
    printf("\n");
}</pre>
```

#### **Printf directives:**

%p: Print pointer

%x: Print Hexadecimal

## show\_bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

### Result (Linux x86-64):

```
int a = 15213;

0x7fffb7f71dbc 6d

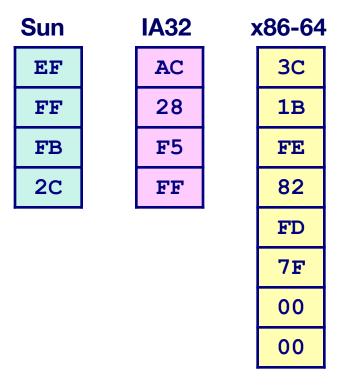
0x7fffb7f71dbd 3b

0x7fffb7f71dbe 00

0x7fffb7f71dbf 00
```

## **Representing Pointers**

```
int B = -15213;
int *P = &B;
```



Different compilers & machines assign different locations to objects

Even get different results each time run program

## Representing Strings

char S[6] = "18213";

#### **Strings in C**

Represented by array of characters

Each character encoded in ASCII format

Standard 7-bit encoding of character set

Character "0" has code 0x30

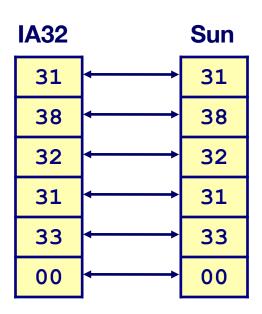
Digit i has code 0x30+i

String should be null-terminated

Final character = 0

#### **Compatibility**

Byte ordering not an issue



## Representing x86 machine code

#### x86 machine code is a sequence of *bytes*

Grouped into variable-length instructions, which look like strings...

But they contain embedded little-endian numbers...

#### **Example Fragment**

Address	Instruction Code	Assembly Rendition		
8048365:	5b	pop %ebx		
8048366:	81 c3 ab 12 00 00	add \$0x12ab, %ebx		
804836c:	83 bb 28 00 00 00 00	cmpl \$0x0,0x28(%ebx)		

### **Deciphering Numbers**

Value:

Pad to 32 bits:

Split into bytes:

Reverse:

0x12ab

0x000012ab

00 00 12 ab

ab 12 00 00