01-C2

绪论

渐近复杂度：多项式

Computational problems can be feasibly computed on some computational device only if they can be computed in polynomial time.

\- A. Cobham & J. Edmonds

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$\mathcal{O}(1)$: constant

❖ 常数：$2 = 2024 = 2024 \times 2024 = \mathcal{O}(1)$ //含RAM的所有基本操作，甚至

$2024^{2024} = \mathcal{O}(1)$

❖ **从渐近的角度来看，再大的常数，也要小于递增的变数** //尽管实际并非如此

❖ [General Twin Prime Conjecture, de Polignac 1849]

For every natural number k, there are infinitely many prime pairs p and q such that $p - q = 2k$

❖ [Yitang Zhang, April 2013] $k \le 35{,}000{,}000$

[Terence Tao, May 2013] $k \le 6{,}500{,}000$

[Polymath Project, April 2014] $k \le 123$

...

$\mathcal{O}(1)$: constant

❖ **这类算法的效率最高** //总不能奢望不劳而获吧

❖ **什么样的代码段对应于常数执行时间?** //应具体分析...

❖ **一定不含循环?**

```
for ( i = 0; i < n; i += n/2024 + 1 );    //常数
for ( i = 1; i < n; i = 1 << i );    //log*n，几乎常数
```

❖ **一定不含分支转向?**

```
if ( (n + m) * (n + m) < 4 * n * m ) goto UNREACHABLE;    //不考虑溢出
```

❖ **一定不能有（递归）调用?**

```
if ( 2 == (n * n) % 5 ) O1op(n);    //𝒪(1)-time Operation
...
```

$\mathcal{O}(\log^c n)$: poly-log

❖ 对数 $\mathcal{O}(\log n)$： $\ln n$ $\lg n$ $\log_{100} n$ $\log_{2024} n$ //为何不注明底数?

❖ 常底数无所谓： $\forall\ a, b > 1,\ \ \log_a n \ = \ \boxed{\log_a b} \cdot \log_b n \ = \ \Theta(\log_b n)$

❖ 常数次幂无所谓： $\forall\ c > 0,\ \ \log n^c \ = \ c \cdot \log n \ = \ \Theta(\log n)$

❖ 对数多项式： $123 \cdot \log^{321} n + \log^{205}(7 \cdot n^2 - 15 \cdot n + 31) \ = \ \Theta(\log^{321} n)$

❖ 这类算法非常有效，复杂度无限接近于常数： $\forall\ c > 0,\ \ \log n \ = \ \mathcal{O}(n^c)$

$\mathcal{O}(n^c)$: polynomial

❖ **多项式：** $a_k \cdot n^k + a_{k-1} \cdot n^{k-1} + \cdots + a_2 \cdot n^2 + a_1 \cdot n + a_0 \;=\; \mathcal{O}(n^k),\;\; a_k > 0$

$$100 \cdot n + 2024 \;=\; \mathcal{O}(n) \qquad \sqrt{23 \cdot n - 472} \times \sqrt{101 \cdot n + 2024} \;=\; \mathcal{O}(n)$$

$$(100 \cdot n - 532) \cdot (20 \cdot n^2 - 445 \cdot n + 2024) \;=\; \mathcal{O}(n^3) \qquad (2024 \cdot n^2 - 129)/(1911 \cdot n - 37) \;=\; \mathcal{O}(n)$$

$$\sqrt[3]{2 \cdot n^3 - \sqrt[3]{3 \cdot n^4 - \sqrt{4 \cdot n^5 + \sqrt{5 \cdot n^6 + \sqrt{6 \cdot n^7 + \sqrt{7 \cdot n^8 + \sqrt{8 \cdot n^9 + n^{2024}/\sqrt{n^{16} - 5 \cdot n^9 + 2024}}}}}}} \;=\; \mathcal{O}(n^7)$$

❖ **线性 (linear function) ：所有$\mathcal{O}(n)$类函数**

❖ **从$\mathcal{O}(n)$到$\mathcal{O}(n^2)$：本课程编程习题主要覆盖的范围**

❖ **这类算法的效率通常认为已可令人满意，然而...这个标准是否太低了?** //P难度!