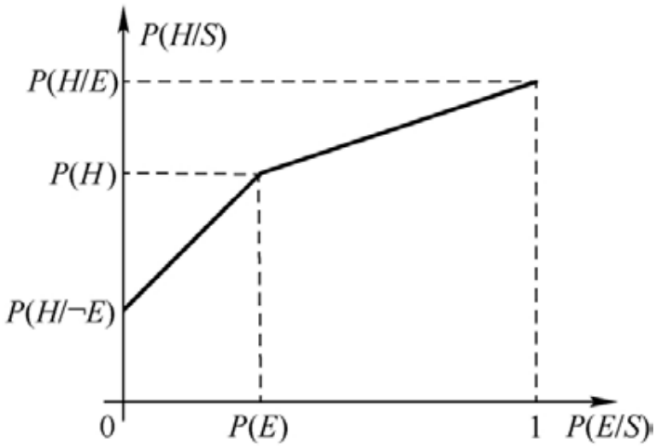
主观贝叶斯方法

1. 问题描述

在证据不确定的情况下，根据充分性度量LS、必要性度量LN、E的先验概率$P(E)$和H的先验概率$P(H)$作为前提条件，作出$P(H|S)$和$P(E|S)$的关系图，示意图如下



2. 主观贝叶斯算法描述

2.1. 表示不确定性：`IF E THEN (LS,LN) H(P(H))`

E是知识的证据，可简单可复合；H是结论

1️⃣知识不确定性表示：

1. $P(H)$：结论H的先验概率，由专家根据经验给出
2. $LS=\frac{P(E|H)}{P(E|\neg H)}\in[0,\infty)$：充分性度量，指出$E$对$H$的支持程度
3. $LN=\frac{P(\neg E|H)}{P(\neg E|\neg H)}=\frac{1-P(E|H)}{1-P(E|\neg H)}\in[0,\infty)$：必要性度量，指出$\neg E$对$H$的支持程度

➕LS,LN值，由专家给出，代表知识的静态强度

2️⃣证据不确定性的表示

1. 动态强度$P(E|S)$：对于证据E，由用户根据观测S，给出$P(E|S)$
2. 可信度$C(E|S)$：用来代替$P(E|S)$，其在PROSPECTOR(矿勘专家系统)中

$$P(E|S)=\begin{cases}\frac{C(E|S)+P(E)\times(5-C(E|S))}{5}, & 0\le C(E|S)\le 5\\ \frac{P(E)\times(5+C(E|S))}{5}, & -5<C(E|S)<0\end{cases}$$

$C(E\|S)\in\{-5,\dots,5\}$	含义
-5	观测S下证据E不存在，即$P(E\|S)=0$
5	观测S下证据E存在，$P(E\|S)=1$
0	S,E之间无关，$P(E\|S)=P(E)$

3️⃣组合证据的不确定性

$$P(E_1\wedge E_2\wedge\dots\wedge E_n|S)=min\{P(E_1|S),P(E_2|S),\dots,P(E_n|S)\}$$

$$P(E_1\vee E_2\vee\dots\vee E_n|S)=max\{P(E_1|S),P(E_2|S),\dots,P(E_n|S)\}$$

$$P(\neg E|S)=1-P(E|S)$$

2.2. 不确定的传递

2.2.1. 前置概念

1. 先验概率$P(H)$：无任何已知证据情况下结论H为真的概率
2. 后验概率$P(H|E)$：由证据E情况下结论H为真的概率
3. 几率函数$O(x)=\dfrac{P(x)}{1-P(x)}$：把位于$[0,1]$的概率映射到$[0,\infty]$，$O(x)$，$P(x)$单调性相同

2.2.2. 三种情况(推导详见课本P92)

$P(E|S)$为证据E的不确定性度量，$P(H|S)$为结论的可信度，$P(H|E)$为后验概率

1. 证据一定存在$P(E)=P(E|S)=1$时：$P(H|E)=\frac{LS\times P(H)}{(LS-1)\times P(H)+1}$
2. 证据一不存在$P(E)=P(E|S)=0$时：$P(H|\neg E)=\frac{LN\times P(H)}{(LN-1)\times P(H)+1}$
3. 证据可能存在$0<P(E|S)<1$时：

$$P(H|S)=P(H|E)\times P(E|S)+P(H|\neg E)\times P(\neg E|S)$$

1. $P(E|S)=1$时，$P(H|S)=P(H|E)=\frac{LS\times P(H)}{(LS-1)\times P(H)+1}$
2. $P(E|S)=0$时，$P(H|S)=P(H|\neg E)=\frac{LN\times P(H)}{(LN-1)\times P(H)+1}$
3. $P(E|S)=P(E)$时，$P(H|S)=P(H)$
4. 其余情况：这是EH公式，$C(E/S)$换掉$P(E/S)$便是CP公式

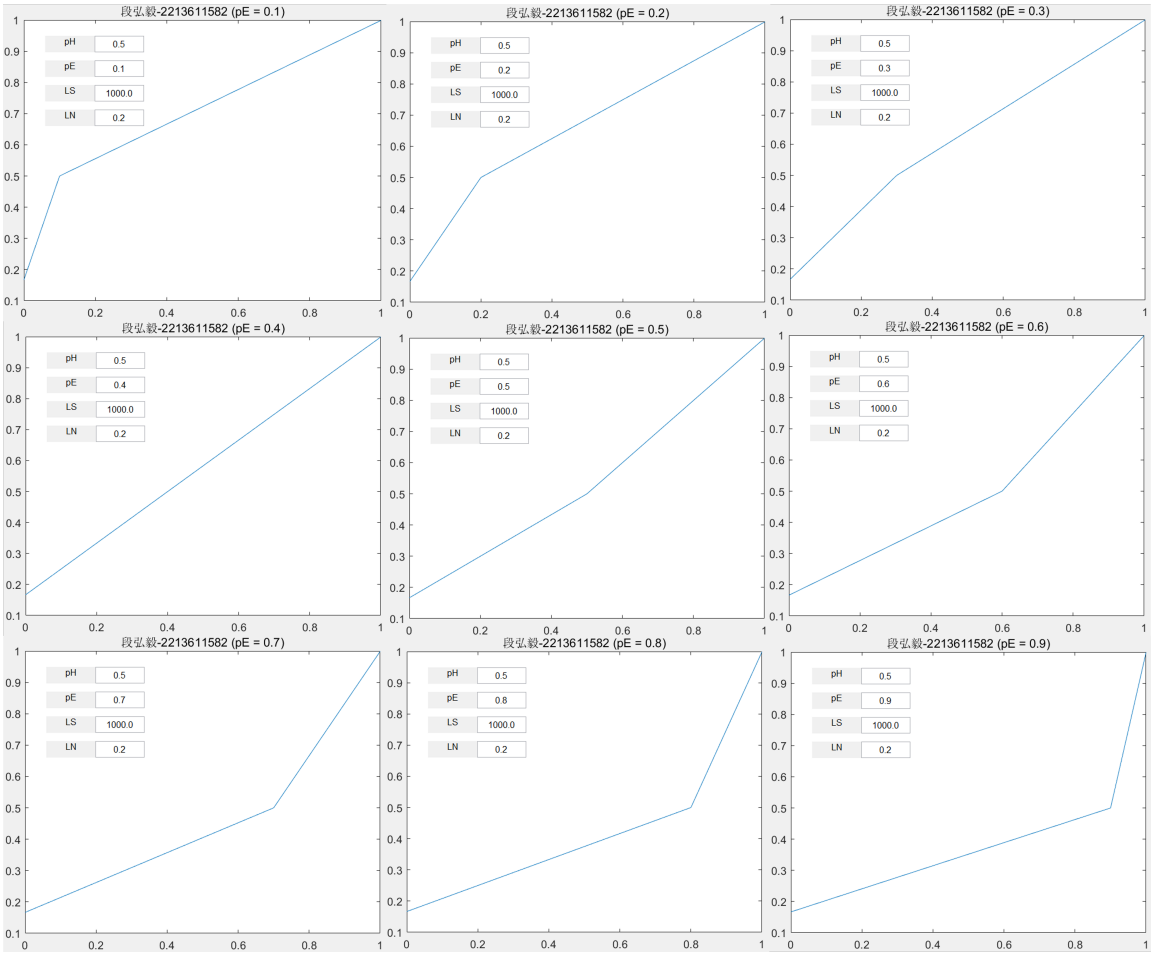
$$P(H|S)=\begin{cases}P(H|\neg E)+\frac{P(H)-P(H|E)}{P(E)}\times P(E|S), & 0\le P(E|S)<P(E)\\ P(H)+\frac{P(H|E)-P(H)}{1-P(E)}\times[P(E|S)-P(E)], & P(E)\le P(E|S)\le 1\end{cases}$$

2.2.3. LS和LN 的性质

$O(H|E)=LS*O(H)$，$O(H|\neg E)=LN*O(H)$

	LS	LN
>1	证据E对结论H有利	证据$\neg E$对结论H有利
<1	证据E对结论H不利	证据$\neg E$对结论H有利

3. 实验结果



4. 源代码Matlab版

```
function bayes_interactive_plot
 % 初始化变量
 pH = 0.5;
 pE = 0.1;
 LN = 0.2;
 LS = 1000.0;

 % 创建图形和轴
 fig = figure;
 ax = axes('Parent', fig);
 x = linspace(0, 1, 1000);
 y = EH(x, pH, pE, LN, LS);
 hPlot = plot(ax, x, y);

 % 对于每个 pE 值生成一个图形
 for n = 1:9
     pE = 0.1 * n; % 设置 pE 值

     % 创建图形和轴
     fig = figure;
     ax = axes('Parent', fig);
     y = EH(x, pH, pE, LN, LS);
     hPlot = plot(ax, x, y);
     title(ax, sprintf('段弘毅-2213611582 (pE = %.1f)', pE));

     % pH 文本框和标签
     uicontrol('Style', 'text', 'Position', [100, 350, 60, 20], 'String', 'pH');
```

```
    pHBox = uicontrol('Style', 'edit', 'Position', [160, 350, 60, 20], 'String',
'0.5', 'Callback', {@updatePlot, 'pH'});

    % pE 文本框和标签
    uicontrol('Style', 'text', 'Position', [100, 320, 60, 20], 'String', 'pE');
    pEBox = uicontrol('Style', 'edit', 'Position', [160, 320, 60, 20], 'String',
pE, 'Callback', {@updatePlot, 'pE'});

    % LS 文本框和标签
    uicontrol('Style', 'text', 'Position', [100, 290, 60, 20], 'String', 'LS');
    LSBox = uicontrol('Style', 'edit', 'Position', [160, 290, 60, 20], 'String',
'1000.0', 'Callback', {@updatePlot, 'LS'});

    % LN 文本框和标签
    uicontrol('Style', 'text', 'Position', [100, 260, 60, 20], 'String', 'LN');
    LNBox = uicontrol('Style', 'edit', 'Position', [160, 260, 60, 20], 'String',
'0.2', 'Callback', {@updatePlot, 'LN'});
 end


 function y = EH(x, pH, pE, LN, LS)
    % 计算更新后的概率值
    pH_E = (LS * pH) / ((LS - 1) * pH + 1);
    pH_negE = (LN * pH) / ((LN - 1) * pH + 1);

    y = zeros(size(x));
    for i = 1:length(x)
        pE_S = x(i);
        if pE_S <= pE
            y(i) = pH_negE + (pH - pH_negE) / pE * pE_S;
        elseif pE_S > pE
            y(i) = pH + (pH_E - pH) / (1 - pE) * (pE_S - pE);
        end
    end
 end

 function updatePlot(source, ~, paramName)
    % 根据用户输入更新图形
    newVal = str2double(source.String);
    switch paramName
        case 'pH'
            pH = newVal;
        case 'pE'
            pE = newVal;
        case 'LS'
            LS = newVal;
        case 'LN'
            LN = newVal;
    end
    y = EH(x, pH, pE, LN, LS);
    set(hPlot, 'YData', y);
 end
end
```

5. 补充：源代码C++版本

5.1. MainWindow.h

```
#ifndef MAINWINDOW_H
#define MAINWINDOW_H

#include <QMainWindow>
#include <QCustomPlot.h>

namespace Ui {
class MainWindow;
}

class MainWindow : public QMainWindow
{
 Q_OBJECT

public:
 explicit MainWindow(QWidget *parent = nullptr);
 ~MainWindow();

private slots:
 void on_pHLineEdit_textChanged(const QString &arg1);
 void on_pELineEdit_textChanged(const QString &arg1);
 void on_LSLineEdit_textChanged(const QString &arg1);
 void on_LNLineEdit_textChanged(const QString &arg1);

private:
 Ui::MainWindow *ui;
 QCustomPlot *customPlot;
 void updateGraph();
 double EH(double pE_S, double pH, double pE, double LS, double LN);
};

#endif // MAINWINDOW_H
```

5.2. MainWindow.cpp

```
#include "MainWindow.h"
#include "ui_MainWindow.h"

MainWindow::MainWindow(QWidget *parent) :
 QMainWindow(parent),
 ui(new Ui::MainWindow)
{
 ui->setupUi(this);
 customPlot = ui->widget;

 // 设置初始绘图
 updateGraph();
}

MainWindow::~MainWindow()
{
 delete ui;
}
```

```
double MainWindow::EH(double pE_S, double pH, double pE, double LS, double LN) {
 double pH_E = (LS * pH) / ((LS - 1) * pH + 1);
 double pH_negE = (LN * pH) / ((LN - 1) * pH + 1);
 if (0 <= pE_S && pE_S <= pE) {
     return pH_negE + (pH - pH_negE) / pE * pE_S;
 } else if (pE < pE_S && pE_S <= 1) {
     return pH + (pH_E - pH) / (1 - pE) * (pE_S - pE);
 }
 return 0; // 默认返回值
}

void MainWindow::updateGraph() {
 QVector<double> x(1000), y(1000);
 double pH = ui->pHLineEdit->text().toDouble();
 double pE = ui->pELineEdit->text().toDouble();
 double LS = ui->LSLineEdit->text().toDouble();
 double LN = ui->LNLineEdit->text().toDouble();

 for (int i = 0; i < 1000; ++i) {
     x[i] = i / 999.0;
     y[i] = EH(x[i], pH, pE, LS, LN);
 }

 customPlot->addGraph();
 customPlot->graph(0)->setData(x, y);
 customPlot->xAxis->setRange(0, 1);
 customPlot->yAxis->setRange(0, 1);
 customPlot->replot();
}

void MainWindow::on_pHLineEdit_textChanged(const QString &arg1) {
 updateGraph();
}

void MainWindow::on_pELineEdit_textChanged(const QString &arg1) {
 updateGraph();
}

void MainWindow::on_LSLineEdit_textChanged(const QString &arg1) {
 updateGraph();
}

void MainWindow::on_LNLineEdit_textChanged(const QString &arg1) {
 updateGraph();
}
```

5.3. `main.cpp`

```
#include "MainWindow.h"
#include <QApplication>

int main(int argc, char *argv[])
{
 QApplication a(argc, argv);
 MainWindow w;
 w.show();
 return a.exec();
}
```

6. 补充：源代码Python版本

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.widgets import TextBox


pH = 0.5
pE = 0.3
LN = 0.2
LS = 1000.0


def EH(pE_S):
 pH_E = (LS * pH) / ((LS - 1) * pH + 1)
 pH_negE = (LN * pH) / ((LN - 1) * pH + 1)
 if 0 <= pE_S <= pE:
     pH_S = pH_negE + (pH - pH_negE) / pE * pE_S
 elif pE < pE_S <= 1:
     pH_S = pH + (pH_E - pH) / (1 - pE) * (pE_S - pE)
 return pH_S


func = np.vectorize(EH)
x = np.linspace(0, 1, 1000)
y = func(x)
fig, ax = plt.subplots()
ax.set_title('Bayes')
l, = plt.plot(x, y)

plt.xlim(0, 1)
plt.ylim(0, 1)
plt.xlabel('P(E|S)', x=1)
plt.ylabel('P(H|S)', y=1.04, rotation=0)
plt.subplots_adjust(bottom=.25)


def draw():
 y = func(x)
 l.set_ydata(y)
 plt.draw()


def submit1(expression):
 global pH
 pH = float(expression)
 draw()


pH_box = fig.add_axes([.15, .1, .3, .03])
pH_text_box = TextBox(pH_box, "p(H)", textalignment="center")
pH_text_box.set_val("0.5")
pH_text_box.on_submit(submit1)


def submit2(expression):
 global pE
 pE = float(expression)
 draw()
```

```
pE_box = fig.add_axes([.55, .1, .3, .03])
pE_text_box = TextBox(pE_box, "p(E)", textalignment="center")
pE_text_box.set_val("0.3")
pE_text_box.on_submit(submit2)


def submit3(expression):
 global LS
 LS = float(expression)
 draw()


LS_box = fig.add_axes([.15, .05, .3, .03])
LS_text_box = TextBox(LS_box, "LS", textalignment="center")
LS_text_box.set_val("1000.0")
LS_text_box.on_submit(submit3)


def submit4(expression):
 global LN
 LN = float(expression)
 draw()


LN_box = fig.add_axes([.55, .05, .3, .03])
LN_text_box = TextBox(LN_box, "LN", textalignment="center")
LN_text_box.set_val("0.2")
LN_text_box.on_submit(submit4)

plt.rc("font", family='YouYuan')
plt.text(0.45, 30.25, '计算机xxx-xxx-xxxxx', fontsize=10, color='black')
plt.show()
```