Problem 4.8.6 -1

$$J = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\lambda^2 - (a+d)\lambda + (ad - bc) = 0$$

(i)

$$A + X \xrightarrow{k_1} 2X \qquad \frac{dx}{dt} = k_1 x(t) A - k_2 x^2(t)$$

$$X + X \xrightarrow{k_2} Y$$

$$Y \xrightarrow{k_3} B \qquad \frac{dy}{dt} = k_2 x^2(t) - k_3 y(t)$$

(ii)

$$A k_1 x^{ss} - k_2 (x^{ss})^2 = 0 \;, \quad x^{ss}(A k_1 - k_2 x^{ss}) = 0$$

$$k_2 (x^{ss})^2 - k_3 y^{ss} = 0 \;, \quad y^{ss} = \frac{k_2}{k_3}(x^{ss})^2$$

$$\Rightarrow \quad x^{ss} = 0 ,\; y^{ss} = 0$$

$$x^{ss} = \frac{Ak_1}{k_2} ,\; y^{ss} = \frac{Ak_1}{k_3}$$

(iii)

$$J = \begin{bmatrix} Ak_1 - 2k_2 X & 0 \\ 2k_2 X & -k_3 \end{bmatrix}$$

if b or $c = 0$

$\Rightarrow \lambda_1 = a , \lambda_2 = d$

positive，發散

$$x^{ss} = 0, \; y^{ss} = 0 \quad \begin{cases} \lambda_1 = Ak_1 \\ \lambda_2 = -k_3 \end{cases}$$

unstable *

$$x^{ss} = \frac{Ak_1}{k_2}, \; y^{ss} = \frac{Ak_1}{k_3} \quad \begin{cases} \lambda_1 = -Ak_1 \\ \lambda_2 = -k_3 \end{cases}$$

stable *

(iv)

$\cancel{A + X \xrightarrow{k_1} 2X}$ $[X] = 0$

$\cancel{X + X \xrightarrow{k_2} Y}$ $[X] = 0$

$Y \xrightarrow{k_3} B$

exponential decay *

Problem 4.8.6 - 2

(i)

$$A + X \xrightarrow{k_1} 2X$$

$$X + Y \xrightarrow{k_2} 2Y$$

$$Y \xrightarrow{k_3} B$$

$$\frac{d}{dt} x(t) = Ak_1 x(t) - k_2 x(t) y(t)$$

$$\frac{d}{dt} y(t) = k_2 x(t) y(t) - k_3 y(t)$$

(ii)

$$Ak_1 x - k_2 xy = 0 \quad , \quad (Ak_1 - k_2 y) x^{ss} = 0 \quad , \quad x^{ss} = 0 \; , \; y^{ss} = 0$$

$$k_2 xy - k_3 y = 0 \quad , \quad (k_2 x - k_3)\, y^{ss} = \quad , \quad x^{ss} = \frac{k_3}{k_2} \; , \; y^{ss} = \frac{Ak_1}{k_2}$$

(iii)

$$J = \begin{bmatrix} Ak_1 - k_2 y & -k_2 x \\ k_2 y & k_2 x - k_3 \end{bmatrix} \qquad J = \begin{bmatrix} Ak_1 & 0 \\ 0 & -k_3 \end{bmatrix} \qquad \lambda_1 = Ak_1 \quad \lambda_2 = -k_3$$

$$J = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad J = \begin{bmatrix} 0 & -k_3 \\ Ak_1 & 0 \end{bmatrix} \qquad \lambda_1 = \sqrt{Ak_1 k_3}\, i \quad \lambda_2 = -\sqrt{Ak_1 k_3}\, i$$

$$\lambda^2 - (a+d)\lambda + (ad - bc) = 0 \qquad \lambda^2 + Ak_1 k_3 = 0$$

unstable

(iv)

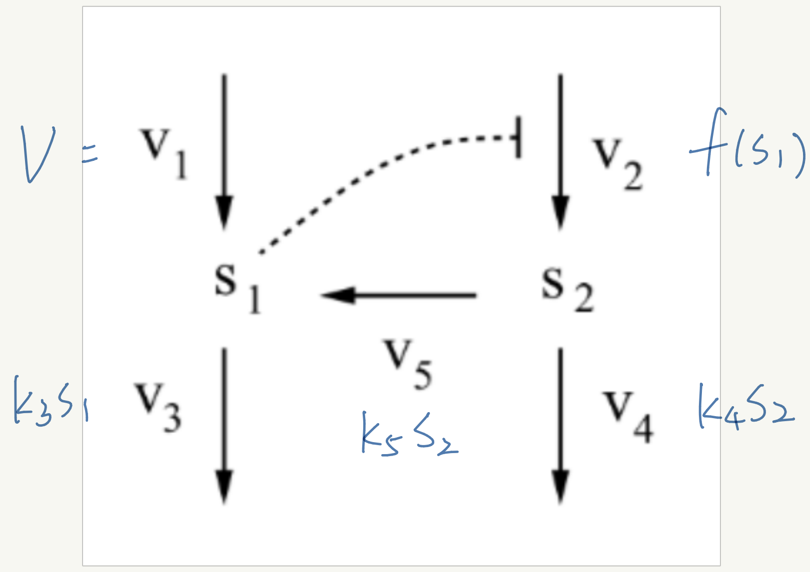
$$\cancel{A + X \xrightarrow{k_1} 2X} \quad [x] = 0$$

$$\cancel{X + Y \xrightarrow{k_2} 2Y} \quad [x] = 0$$

$$Y \xrightarrow{k_3} B$$

exponential decay

Problem 4.8.7



$V \cdot k_3 \cdot k_4 \cdot k_5$: + constant

$f(s_1) \Rightarrow$ take positive value,

is a decreasing function. ($s_1 \uparrow$ $f(s_1) \downarrow$)

$$\frac{d}{dt} s_1(t) = V - k_3 s_1(t) + k_5 s_2(t)$$

$$\frac{d}{dt} s_2(t) = \frac{k_2}{1+(s_1(t)/K)^n} - k_4 s_2(t) - k_5 s_2(t)$$

s_1 - nullcline is defined by

$$V - k_3 s_1(t) + k_5 s_2(t) = 0$$

$$s_1(t) = \frac{V + k_5 s_2(t)}{k_3} \quad *$$

s_2 - nullcline is defined by

$$0 = \frac{k_2}{1+(s_1(t)/K)^n} - k_4 s_2(t) - k_5 s_2(t)$$

$$s_2(t) = \frac{k_2}{[1+(s_1(t)/K)^n](k_4+k_5)} \quad *$$