

# Hypothesis Testing

# Your pipeline for hypothesis testing in statistics

Step 1

Formulate your **null hypothesis**

- How *unusual* is your data?



Step 2

Identify appropriate **test statistic**

- Assumptions of your test



Step 3

**Quantify** the results of your test

- **P value** or comparison to **critical values**



Step 4

**Conclude: reject or fail to reject** based on alpha value

- if appropriate, confidence interval of the parameter

## Hypothesis testing automates binary decision making:

- If  $p\text{-value} < \alpha$  (also called significance level)
  - Reject null hypothesis
- If  $p\text{-value} > \alpha$ 
  - Fail to reject null hypothesis
- We can outline steps that help us make decisions
- **Remember: What is statistically significant is somewhat arbitrary:**  
p-value of 0.04999 is not so different from 0.050001

# Does wearing a red shirt help win during a wrestling match?



16 out of 20 rounds had more red-shirted than blue-shirted winners in the 2004 Olympics in wrestling, taekwondo and boxing.

# Hypothesis Testing

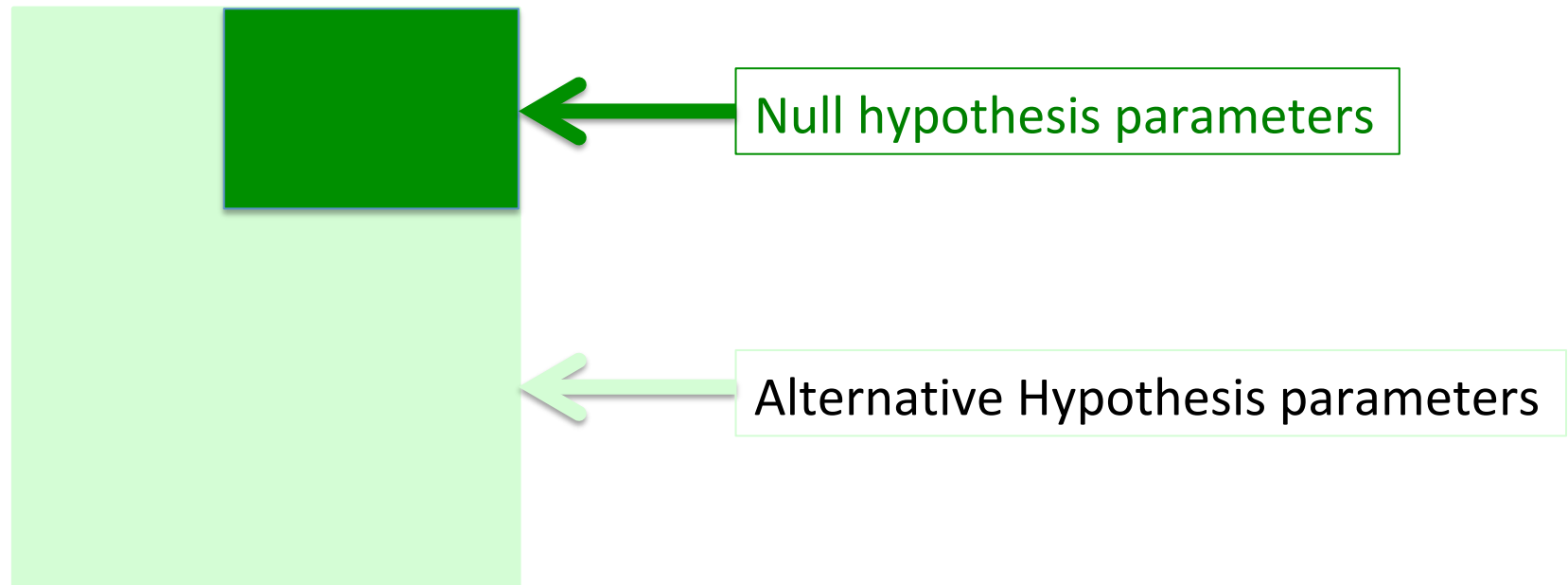
Does wearing a red shirt help win in combat sports?

## **Step 1: Formulate Hypothesis**

# Four steps in hypothesis testing:

## 1. Formulate Hypothesis

- o Most of the mental effort
- o Quantifies how unusual data is *if you assume that the null hypothesis is true*
- o  $H_0$  and  $H_A$  - mutually exclusive

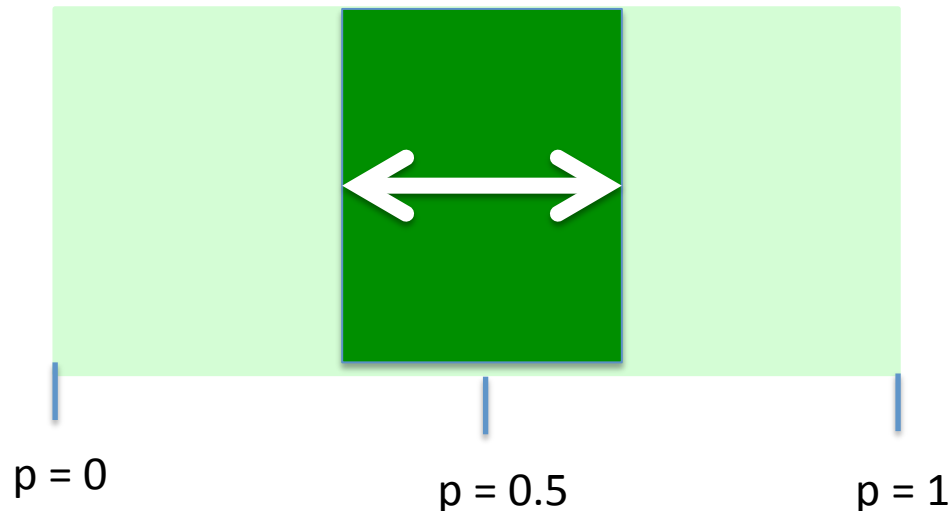


Does wearing a red shirt help win in combat sports?

## Step 1: Formulate Hypothesis

$H_0$ : Red and blue shirted athletes are equally likely to win (proportion = 0.5)

$H_A$ : Red and blue shirted athletes are not equally likely to win (proportion  $\neq 0.5$ )



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#### **Step 2: Identify test statistic**



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16 out of 20 red shirted winners

--> proportion = **0.8**

This is a discrepancy of **0.3** from  $H_0$ . Can it be due to chance alone?

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**Step 3: Calculate the P-Value/Compare to critical values or fixed Significance**

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*If  $H_0$  is true, what is the chance of observing a test statistic with a value at least as extreme as the one we have observed?*

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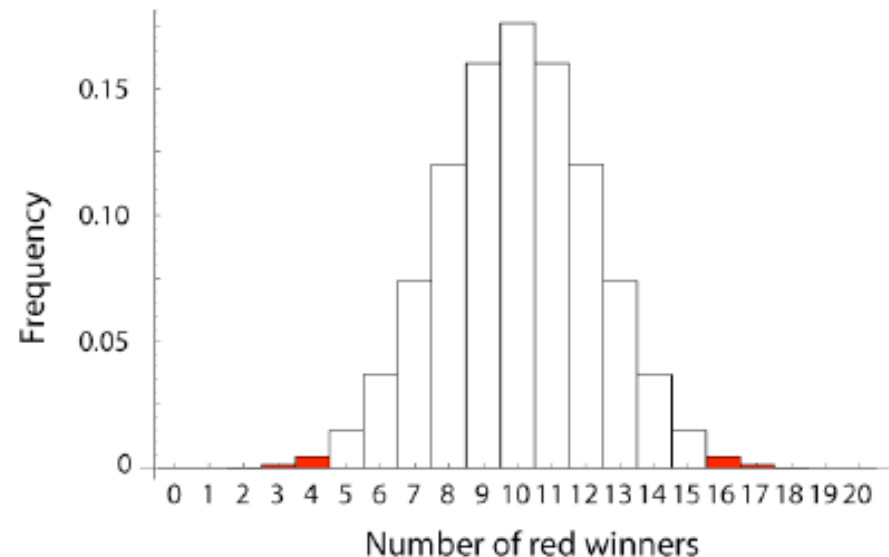
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The Binomial Distribution Null Distribution of the sample proportion

explains this type of proportion data (explained in the next chapter)

If  $H_0$  is true, what is the chance of observing a test statistic value *at least as extreme* as the one we have observed?



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The P-value from the null distribution of the proportion is calculated as:

$$\begin{aligned} P &= 2 \times [P[16] + P[17] + P[18] + P[19] + P[20]] \\ &= 0.012 \end{aligned}$$

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What is alpha?

$$\alpha = 0.05 \quad \text{and} \quad P\text{-value} = 0.012$$

$$P < \alpha \quad \text{so we can reject } H_0$$

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### Step 3(a): Calculate the P-Value

$$P = 2 \times [P[16] + P[17] + P[18] + P[19] + P[20]] = 0.012$$

### Step 3(b): Compare to a fixed significance

$\alpha = 0.05$  and P-value = 0.012

$P < \alpha$  so we can reject  $H_0$

## Step 4: ALWAYS CONCLUDE

Athletes in red and blue shirts are not equally likely to win



Someone claims they make 90% of the shots they make on goal in soccer. If this is tested, what would be the null hypothesis?

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a) You do not have enough information to make a null hypothesis because you don't know how they will do the test

b)  $\bar{x} = 0.9$

c) The person does not have a mean of making 90% of their soccer shots

d) The person does have a mean of making 90% of their soccer shots