The Normal Distribution Applied

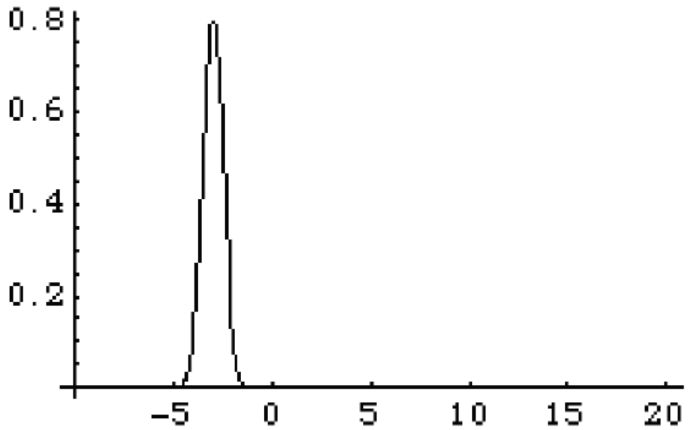
Review:

- Transform any normal distribution into the standard normal distribution by

$$\boxed{Z = \frac{X - \mu}{\sigma}}$$

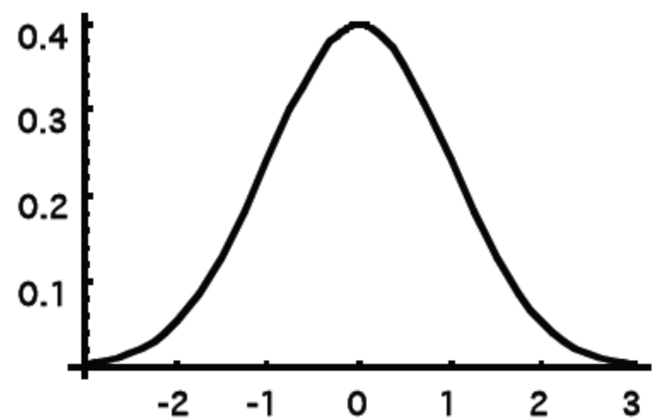
The Standard Normal Distribution:

- Mean is zero ($\mu = 0$)
- Standard deviation is 1 ($\sigma = 1$)



$\mu = -3$; $\sigma = 1/2$

$$\boxed{Z = \frac{X - \mu}{\sigma}}$$



3 major motivations:

- Z score tells us how many standard deviations our normally distributed variable is from the mean

Z = Raw score - Mean / Standard deviation

- Confidence interval: $(a,b) = \bar{x} \pm z_{\alpha/2}(\sigma / \sqrt{2})$

- Determine proportion of scores that fall between two raw scores
- Allows us to use standard normal table

Example: British Spies. MI5 says that a man has to be shorter than 180.3 cm tall to be a spy. The mean height of British men is 177.0 cm, with standard deviation 7.1 cm, and with a normal distribution.

What proportion of British men are excluded from a career as a spy by this height criteria?

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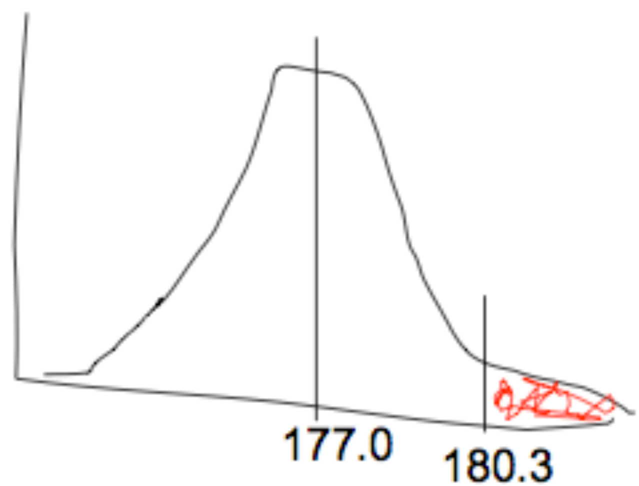

Applications of the Normal Dist'n

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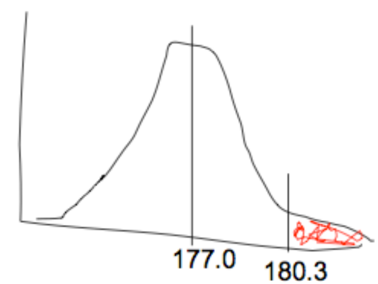

Step 2: Transform into Standard Normal

$$\boxed{\begin{aligned} \mu &= 177.0cm \\ \sigma &= 7.1cm \\ X &= 180.3cm \\ P[height &> 180.3] \end{aligned}}$$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{180.3 - 177.0}{7.1}$$

$$Z = 0.46$$

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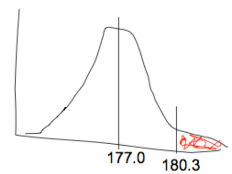

$$\boxed{\begin{aligned} \mu &= 177.0cm \\ \sigma &= 7.1cm \\ X &= 180.3cm \\ P&[height > 180.3] \end{aligned}}$$

$$Z = \frac{X-\mu}{\sigma}$$

$$Z = \frac{180.3-177.0}{7.1}$$

$$Z = 0.46$$

Step 2: Transform into Standard Normal

Step 3: Look up probability in Appendix B

	x.x0	x.x1	x.x2	.x3	x.x4	x.x5	x.x6	x.x7	x.x8	x.x9
0.0	0.5	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476

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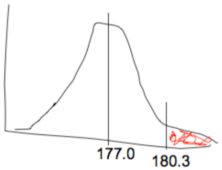

$$\boxed{\begin{aligned} \mu &= 177.0cm \\ \sigma &= 7.1cm \\ X &= 180.3cm \\ &P[height > 180.3] \end{aligned}}$$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{180.3 - 177.0}{7.1}$$

$$Z = 0.46$$

P[Z > 0.46] = 0.32276

So, P[height > 180.3] = 0.32276

The fraction of British males who are too tall to be spies is approx 1/3.

Example: For a particular year, the average SAT-math scores was 517 (out of 800) with a standard deviation of 100. What score marks the **90%** percentile?

a. 681.5

b. 645.5

c. 527

d. 568.7

The Normal Distribution

	x.x0	x.x1	x.x2	.x3	x.x4	x.x5	x.x6	x.x7	x.x8	x.x9
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0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.1335	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330

<u>Example:</u> For a particular year, the average SAT-math scores for was 517 (out of 800) with a standard deviation of 100. What score marks the 90% percentile?

Step 1: Determine *z-score* which marks 90% percentile

- Using table B, we find the probability of 0.10 (which is between values 0.10027 and 0.09853) which gives a *z-score* of **1.285**

Step 2: Convert *Z-score* back into original unit of measurement:

$$\boxed{X_i = \mu + z * \sigma}$$

$$X_i = 517 + 1.285(100)$$

$$X_i = 645.5$$

Example: What is your friend casually mentioned to you that they had scored a 750 while you had scored 425 on the math section of the SAT? What percentage of scores is between you?

Hint: Draw the question out; work with at least one friend!

a. 32.5%

b. 16.98%

c. 81.13%

d. 17.878%

Example: What is your friend casually mentioned to you that they had scored a 750 while you had scored 425 on the math section of the SAT? What percentage of scores is between you?

Step 1: Convert raw score into *z-score*:

$$\boxed{Z = \frac{X_i - \mu}{\sigma}}$$

$$Z = \frac{425 - 517}{100} = -0.92$$

$$Z = \frac{750 - 517}{100} = 2.33$$

Step 2: Find the proportion of the normal distribution that falls below a score of:

1. -0.92
2. 2.33.

<u>Example:</u> What is your friend casually mentioned to you that they had scored a 750 while you had scored 425 on the math section of the SAT? What percentage of scores is between you?

Step 1: Convert raw score into *z-score*:

Step 2: Find the proportion of the normal distribution that falls below a score of:

1. -0.92
   - Notice that we are dealing with a negative number
   - Appendix B only reports positive scores.
   - symmetry – the proportion of the distribution that falls above the score is identical whether it is positive or negative
   - **Draw it out** – it is then easy to see that you are looking for (0.5-0.17879) = 0.3212
   - This score, btw, corresponds to a percentile of 17.88%

2. 2.33
   - According to Appendix B, this score corresponds to 1-0.00990 or the 99.1 percentile.

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1. -0.92
   - (0.5-0.17879) = 0.3212
   - This score, btw, corresponds to a percentile of 17.88%
2. 2.33
   - According to Appendix B, this score corresponds to 1-0.00990 or the 99.01 percentile.

The difference between the two scores is: 0.3212+0.4901 = 0.8113