The Normal Distribution

What are the best values to describe a normal distribution?

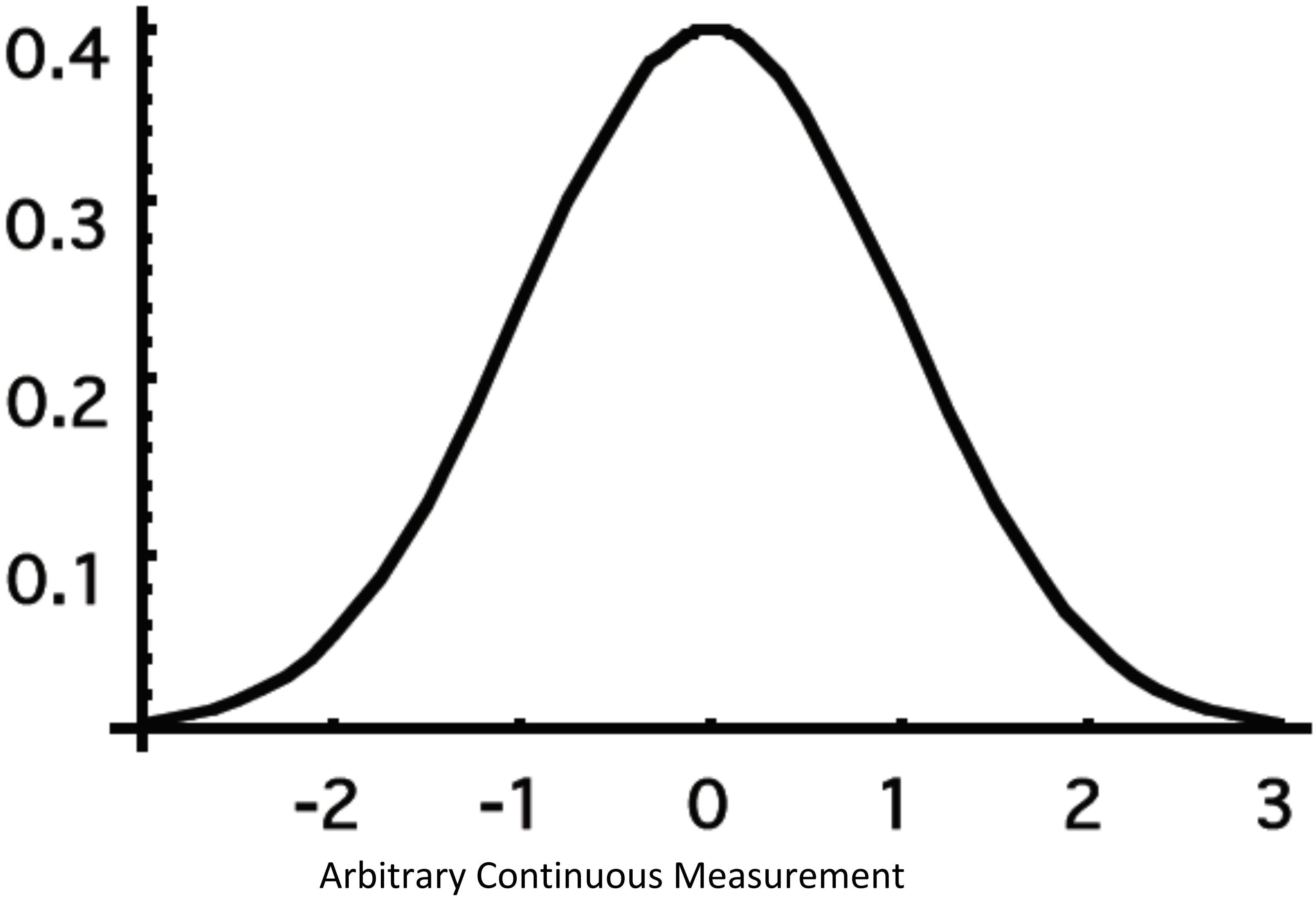
a) median and variance because they are not influenced by outliers

b) Mean and standard deviation because the data is not skewed by outliers

c) only mean, because the standard deviations are all the same with normal distributions

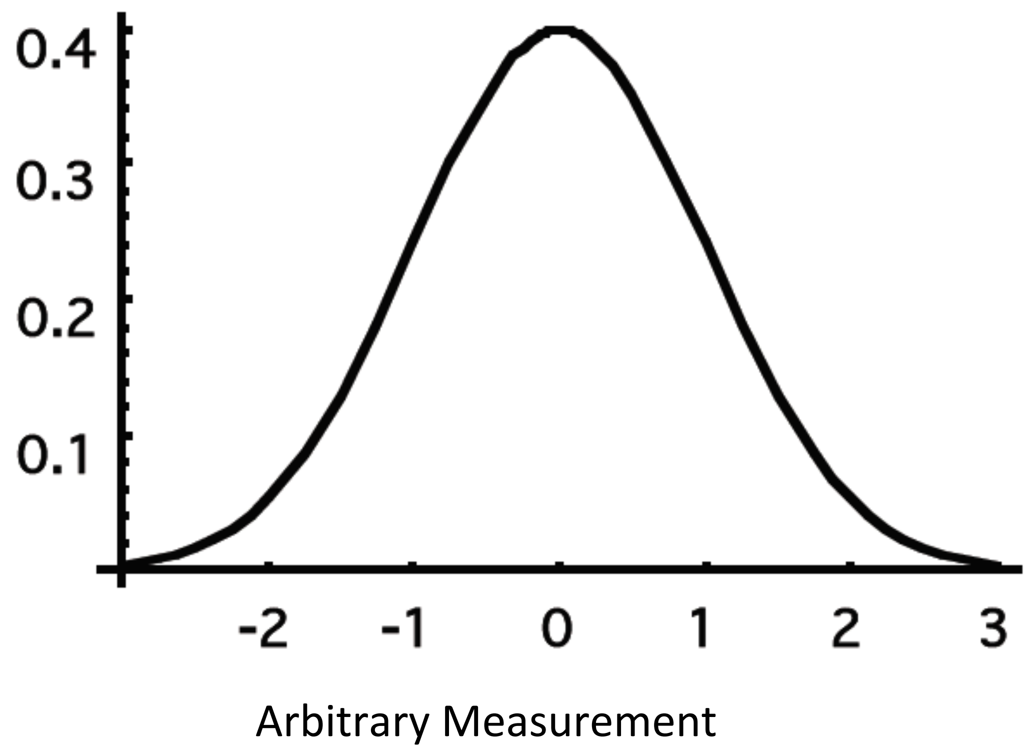
d) only mode, because that is where the densest part of the curve lies

The Normal Distribution:

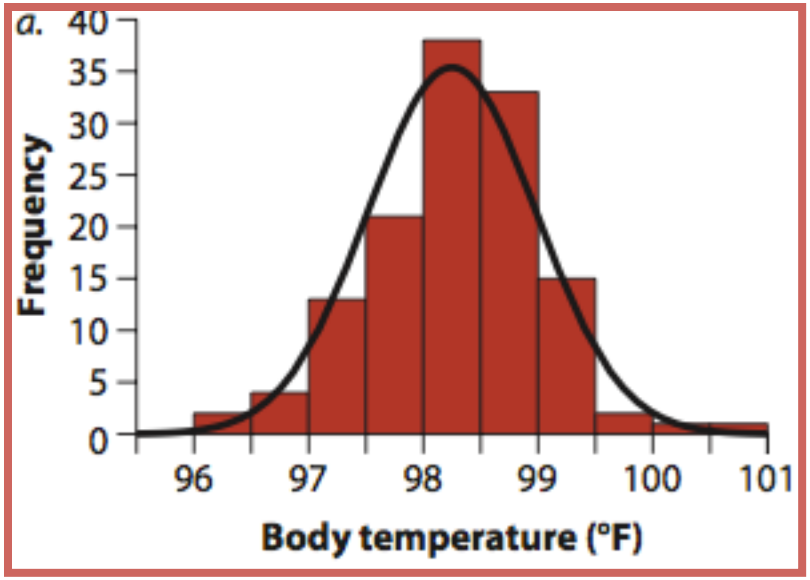


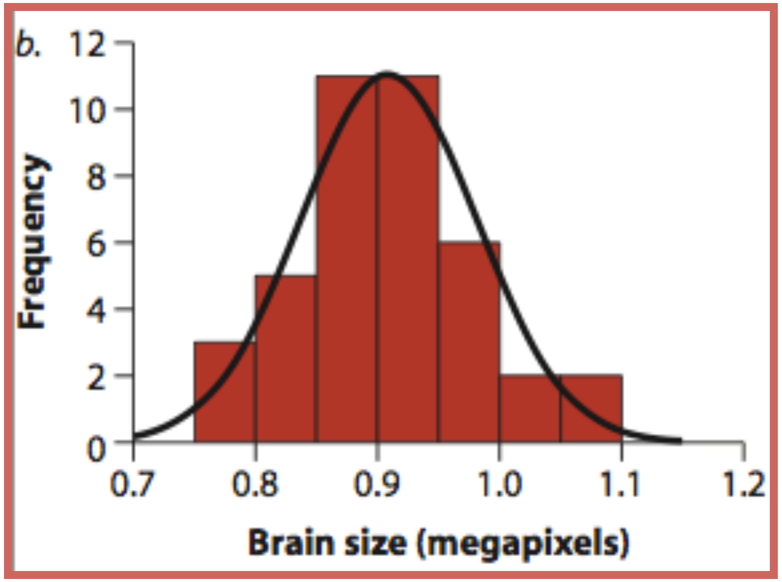
The Normal Distribution:

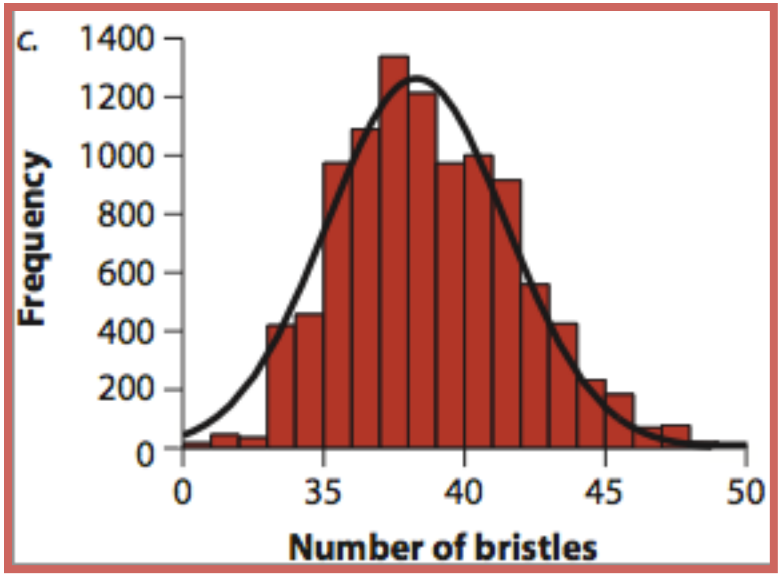
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



The Normal Distribution is common in nature:

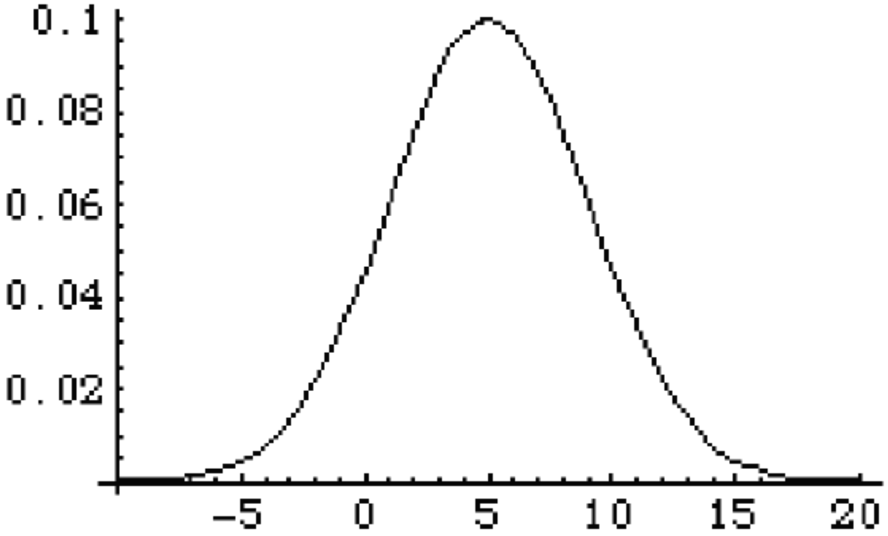




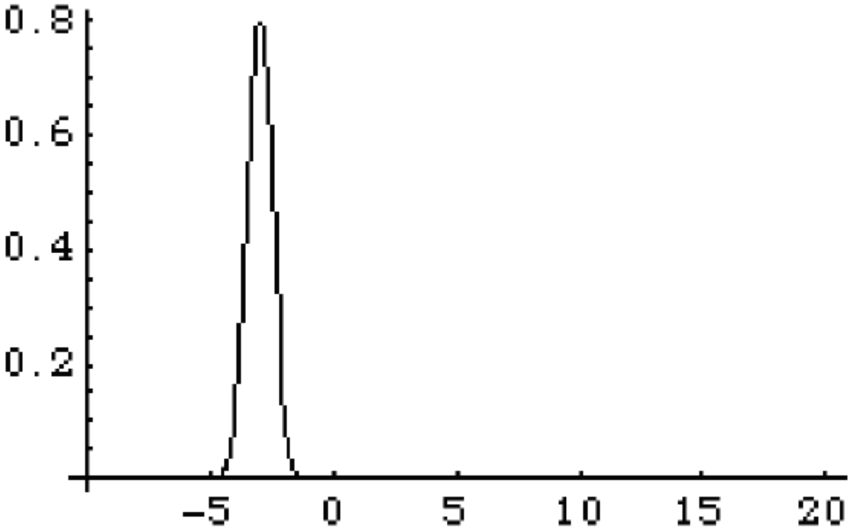


Properties of the Normal Distribution:

1. Fully described by its mean and standard deviation



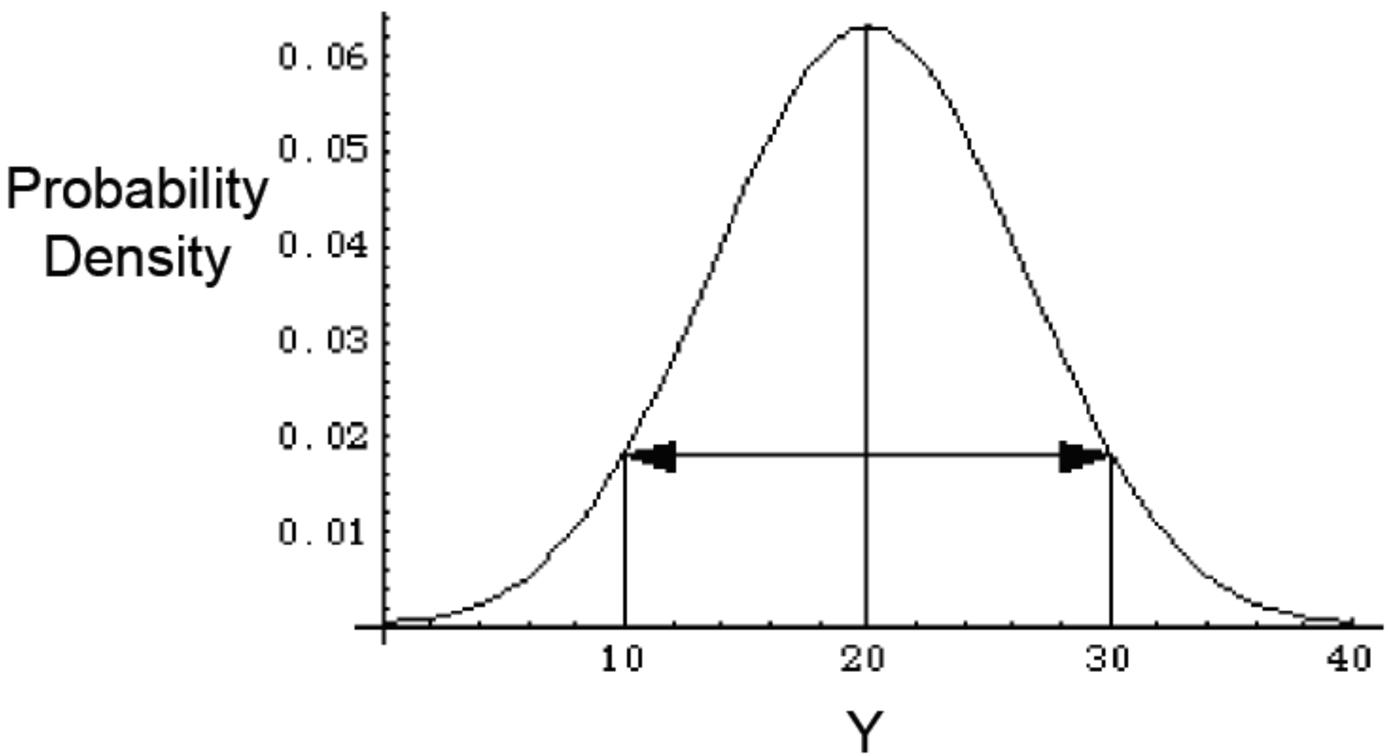
$\mu = 5; \sigma = 4$



$\mu = -3; \sigma = 1/2$

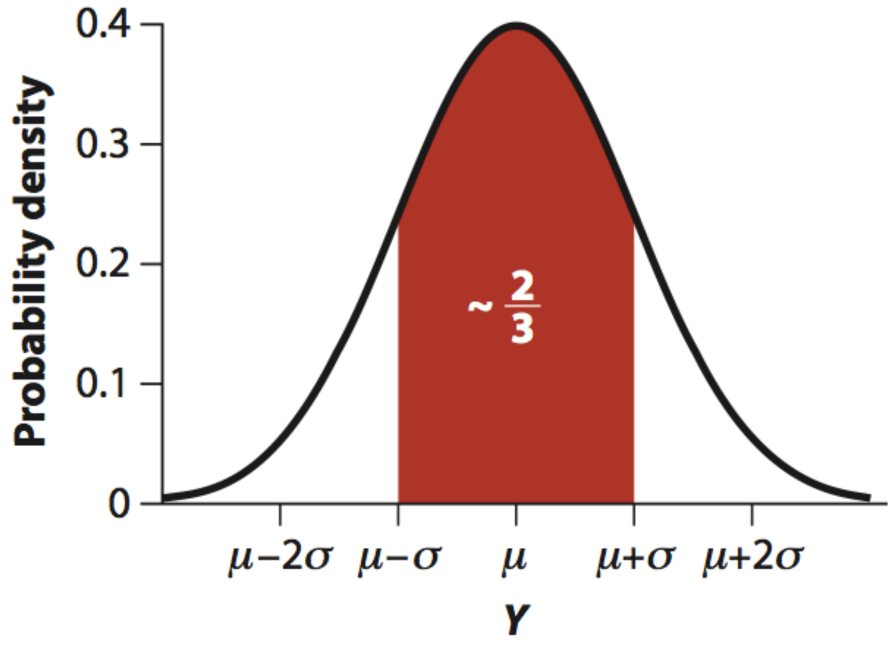
Properties of the Normal Distribution:

1. Fully described by its mean and standard deviation
2. Symmetric around its mean



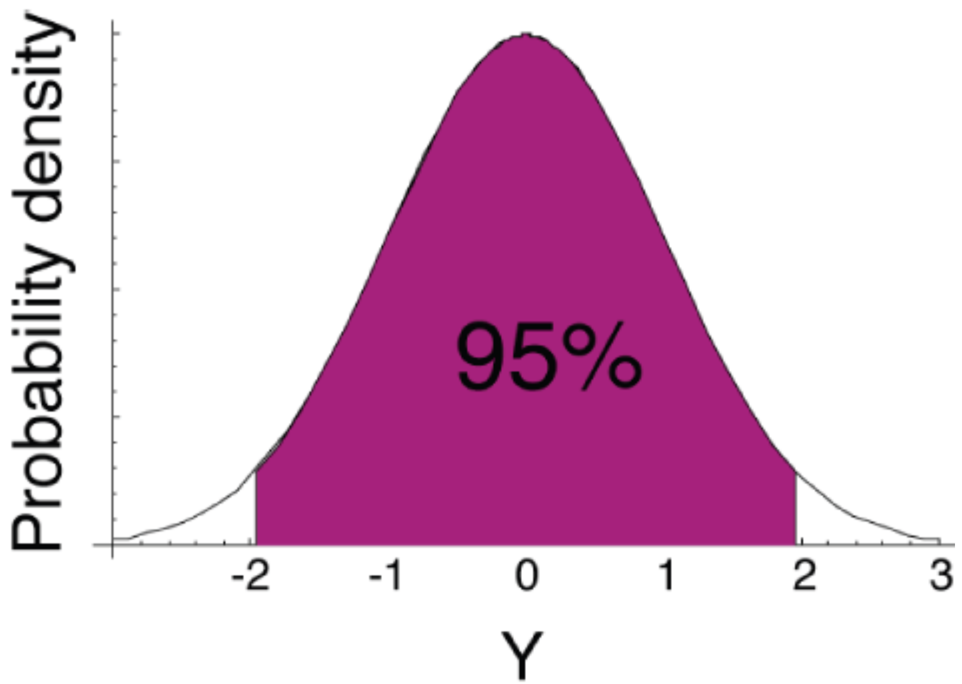
Properties of the Normal Distribution:

1. Fully described by its mean and standard deviation
2. Symmetric around its mean
3. ~ 2/3 of random draws are within one standard deviation of the mean



Properties of the Normal Distribution:

1. Fully described by its mean and standard deviation
2. Symmetric around its mean
3. ~ 2/3 of random draws are within one standard deviation of the mean
4. ~ 95% of random draws are within two standard deviations of the mean (*really, it is 1.96 SD*)



Which is the following is NOT a property of the normal distribution?

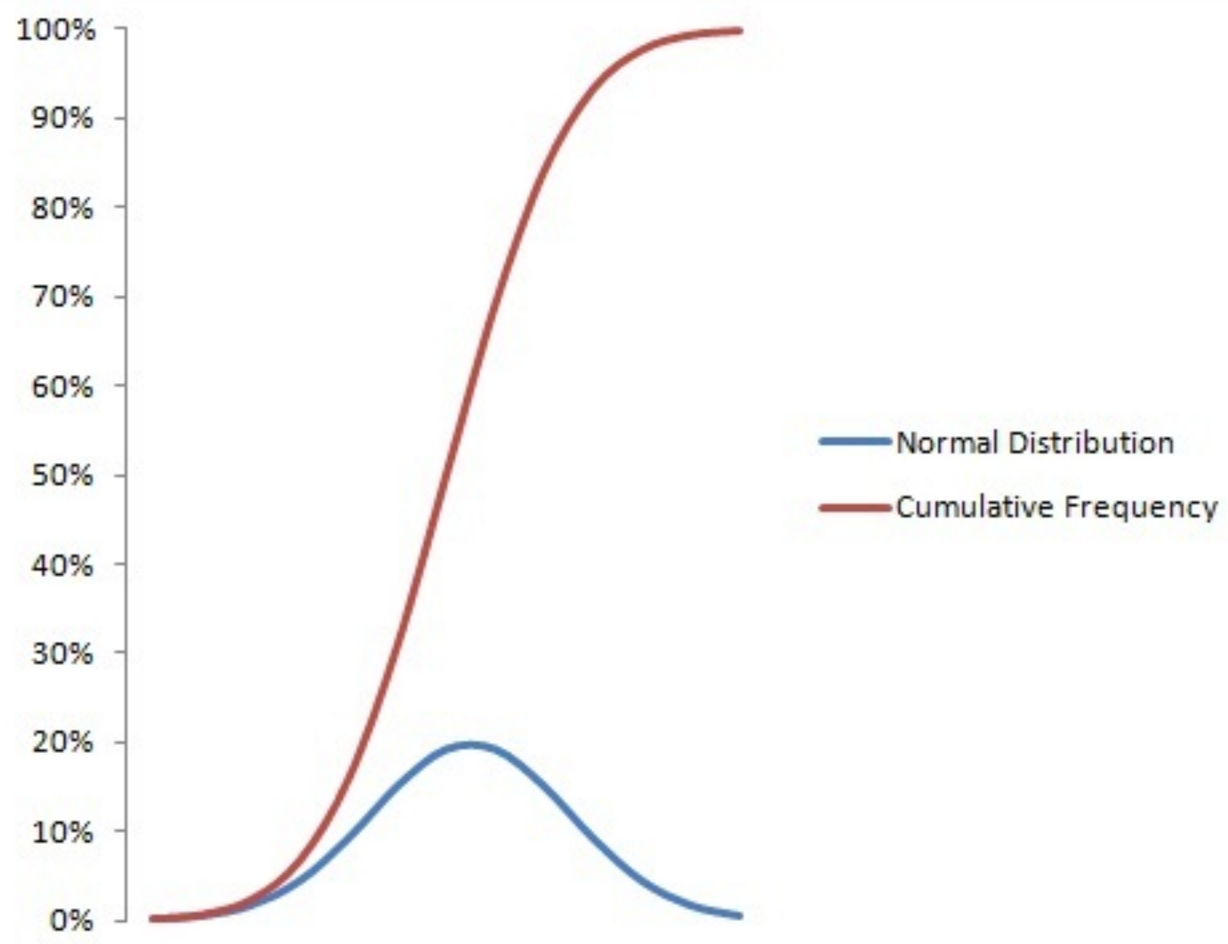
A. The probability density is highest exactly at the mean

B. The mean, mode and median are all equal

C. The normal curve is symmetrical about the mean μ

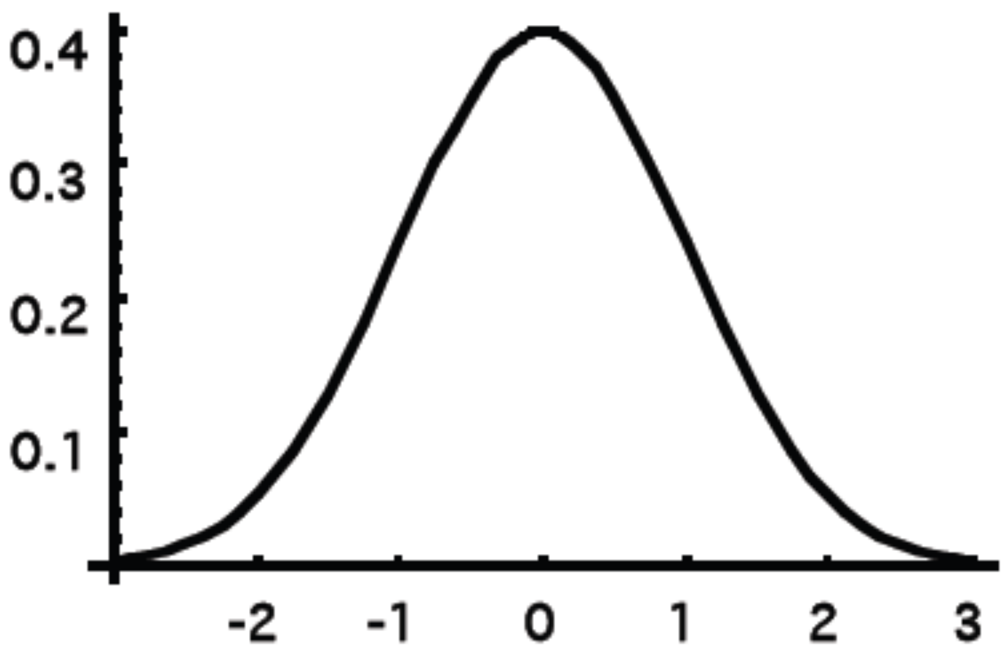
D. The probability that a random data point is within <u>two</u> standard deviation of the mean is approximately 68%

Properties of the Normal Distribution:



The Standard Normal Distribution:

- Mean is zero ($\mu = 0$)
- Standard deviation is 1 ($\sigma = 1$)



Z-scores:

- converts **raw** normally distributed scores into standard deviation units
  - useful for comparing distributions with different scales, for instance.
  - percentiles

- allows calculation of probability of variable value

- z-score indicates how far above or below the mean a value is in standard deviation units
  - how large/small is individual score **relative** to others in the distribution

The Standard Normal Distribution:

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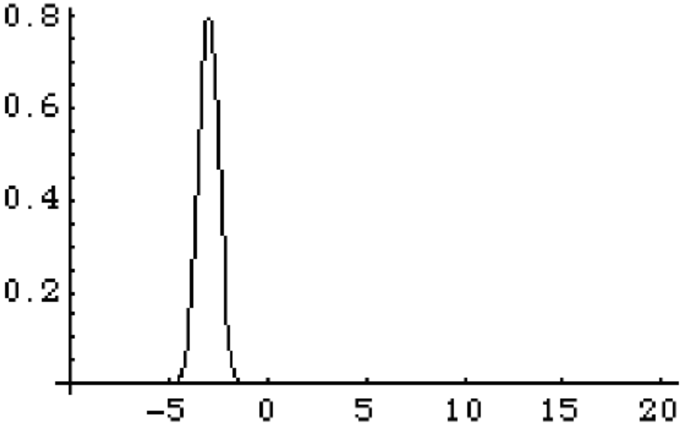
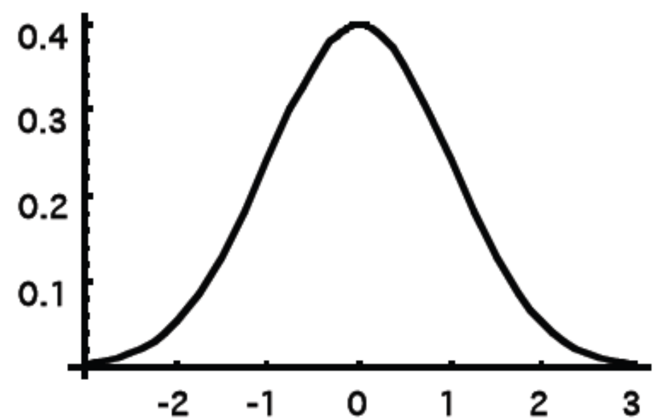


$\mu = -3$; $\sigma = 1/2$

$$\boxed{Z = \frac{X_i - \mu}{\sigma}}$$



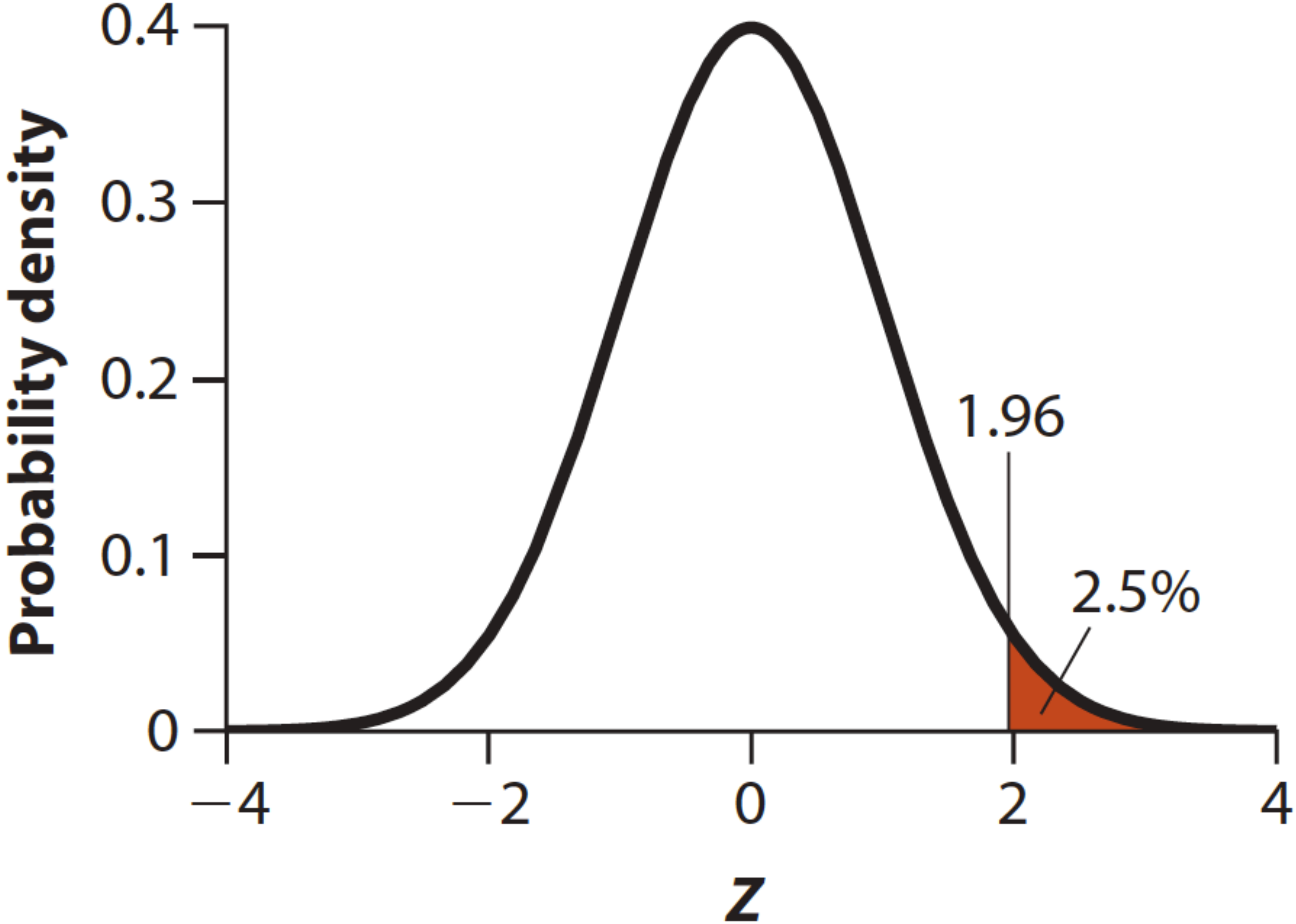
Interpret the following statements:

- Student A gets a *z-score* of -1.5 on an exam

- Student B received a *z-score* of 0.29 on the exam

- Does the *z –score* tell you sample size? What the mean score on the test was? The percentage of answers Student B got right?

The Normal Distribution



The probability of getting a random draw from a standard normal distribution greater than a given value which is the area under the curve.

Mechanics of Appendix B:

The table works for P[Z>a.bc]

First two digits of *a.bc*	Second digit after decimal (*c*) 0	1	2	3	4	5	6	7	8	9
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	$\boxed{0.02500}$	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426

For Z = 1.96 → P[Z>1.96]=0.025

Since the standard normal is symmetric:

$$P[Z > x] = P[Z < -x]$$

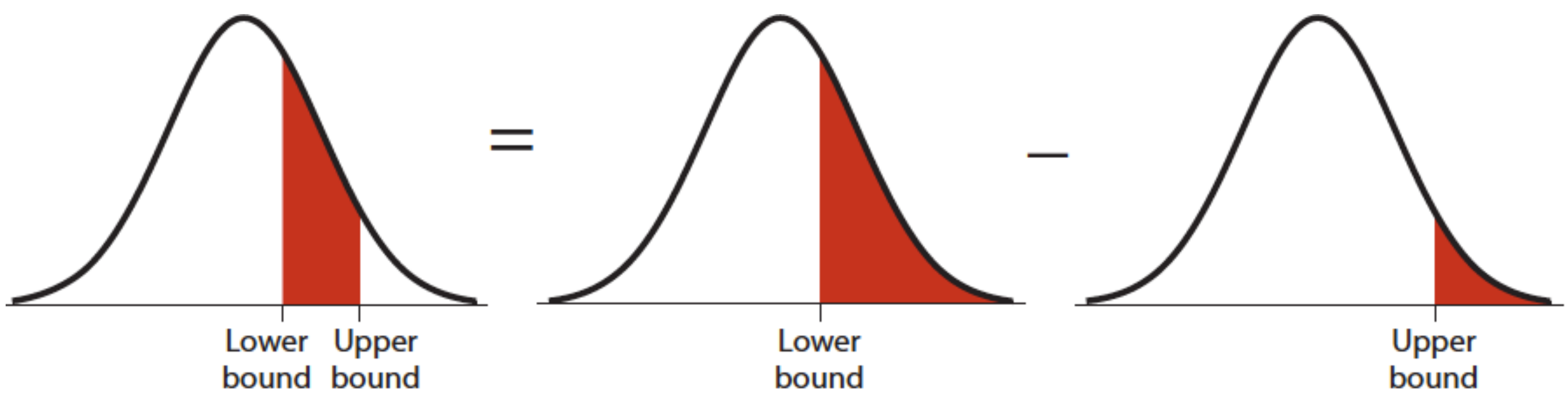
$$P[Z < x] = 1 - P[Z > x]$$

remember: instead of α, you have $\alpha/2$ at each tail

Example:

$$P[Z < -1.96] = P[Z > 1.96]$$

The Normal Distribution



$$P_{[\textit{lower bound} < Z < \textit{Upper bound}]}$$

$$= P[Z > \textit{lower bound}] - P[Z > \textit{Upper bound}]$$

Sara and Jerry took a math exam. Sara's percentile score on the exam was 35; Jerry's percentile score on the same test was 70. We know that

A. Sara scored better than 35 of her classmates.
B. Sara correctly answered half as many items as Jerry did.
C. They both scored better than average on the math exam.
D. Jerry correctly answered more items than Sara did.

- Appendix B can be defined as the probability of Z being in the interval:

$$a < Z < b$$

Which can be thought of as:

$$P(a < Z < b) = F(b) - F(a)$$

Gives rise to the general rule:

$$P(a < Z < b) = F\left(\frac{b-\mu}{\sigma}\right) - F\left(\frac{a-\mu}{\sigma}\right)$$

- Using the Z conversion, we can use the same table to find the probabilities for other normal distributions