Two-sample Design:

– Assumptions:

- **Random Sample** is **normally distributed** in both populations --> sampling distribution for difference between sample means is also normal

- **Standard deviation** is the same in both populations --> if this is not true, use Welch's approximate t-test instead*

- two-sample t-test is fairly robust to violations of assumptions if *n* is similar between the two groups.

Two-sample Design:

Unlike in a paired t-test, there are two variables from two entirely different populations. Instead of one variable describing the difference, $\bar{d}$, you have two: $\bar{Y}_1$ - $\bar{Y}_2$

Two-sample Design:

Standard Error of $\bar{Y}_1 - \bar{Y}_2$:

$$SE_{\bar{Y}_1 - \bar{Y}_2} = \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Two-sample Design:

Pooled sample variance:

- Weighted average; the average of the variances of the samples weighted by their degrees of freedom

$$s_p^2 = \frac{df_1 s_1^2 + df_2 s_2^2}{df_1 + df_2}$$

BEHRENS-FISHER Problem

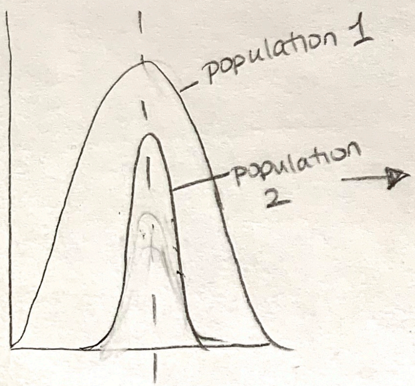
- when variances of two populations are not equal
- we can illustrate the problem with two extreme situations

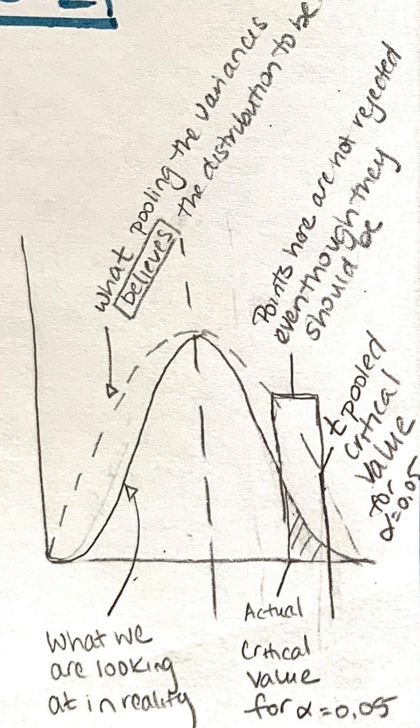
- Two sample t-test is not robust when $s_1^2 \neq s_2^2$ and $n_1 \neq n_2$

Situation 1

larger sample has larger variance:

$\boxed{n_1 >> n_2 \,;\, \sigma_1 > 3\sigma_2}$



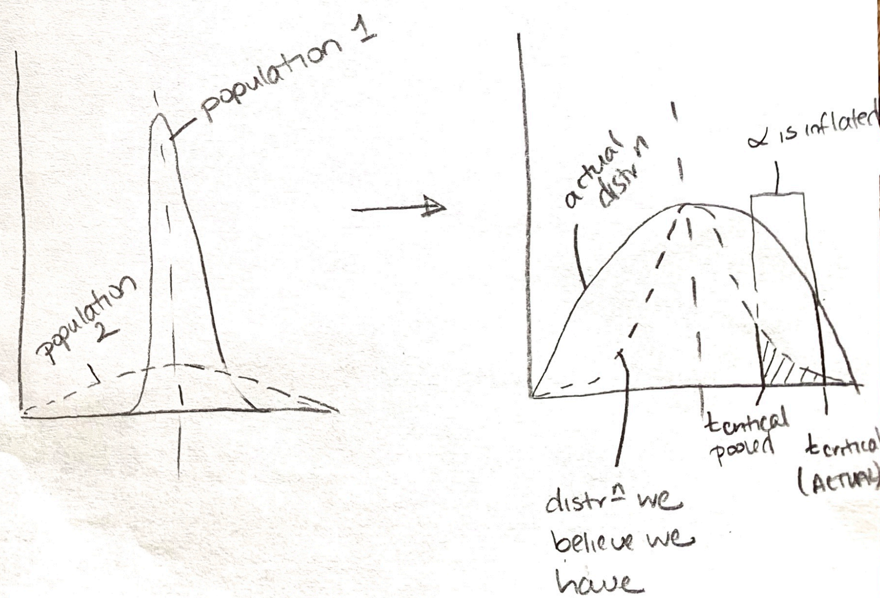


- pooled variance allows the larger sample, with its much larger variance, to contribute more

Situation 2

larger sample has smaller variance

$\boxed{n_1 >> n_2 \,;\, \sigma_2 > 3\sigma_1}$



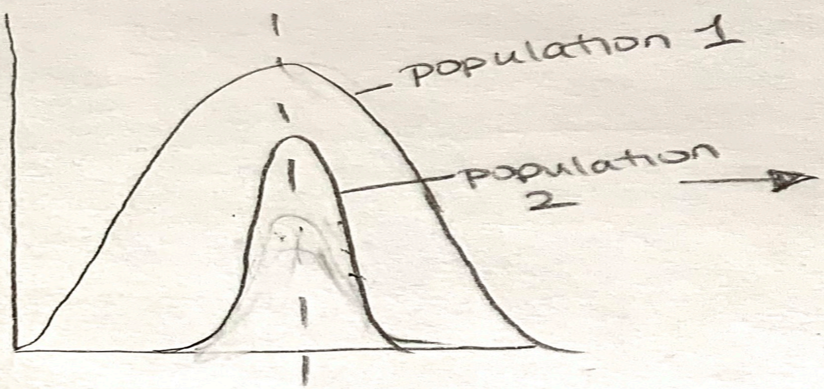
BEHRENS-
Proble

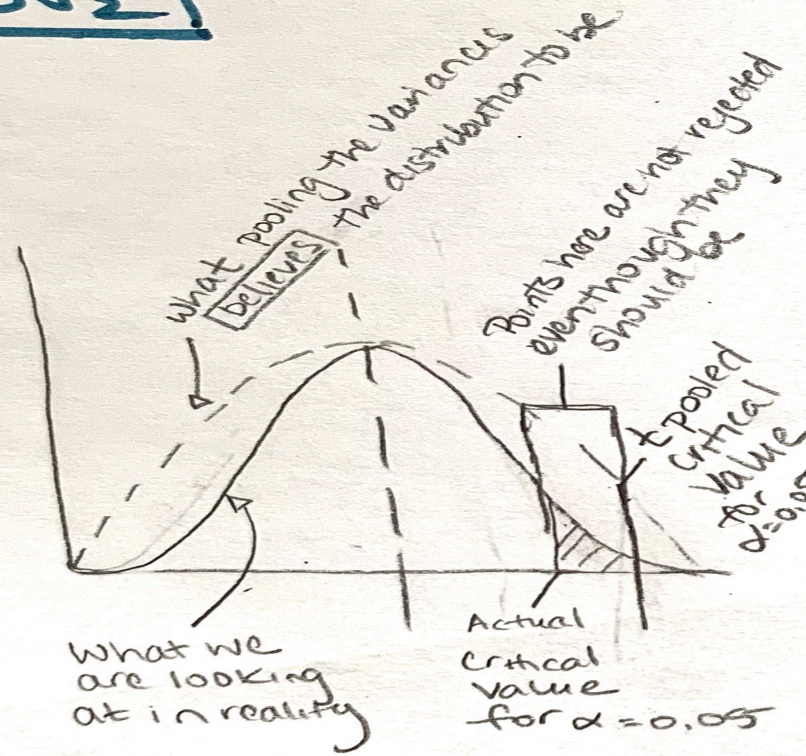
- **Two sample t-test is n**

Situation 1

larger sample has larger variance:

$\boxed{n_1 >> n_2 \; ; \; \sigma_1 > 3\sigma_2}$





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-FISHER

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Situation 2

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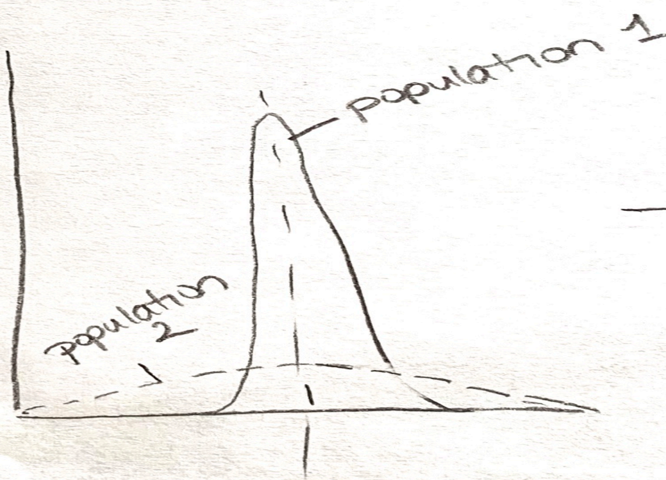

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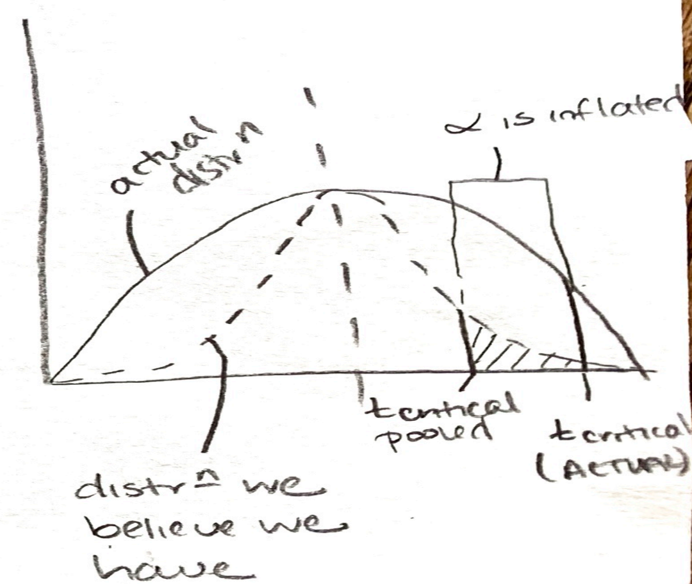

Two-sample Design:

Student's t-distribution of two-sample design:

- Compares the means of a numerical variable between two populations

$$t = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{SE_{\bar{Y}_1 - \bar{Y}_2}}$$

Total degrees of freedom:

$df = df_1 + df_2 = n_1 + n_2 - 2$

– Two means are estimated, so subtract 2

Two-sample Design:

Example: 2 genotypes of lettuce: *susceptible* and *resistant*. Do these genotypes differ in fitness in the absence of aphids.

The proxy for fitness that is measured are number of buds.

Two-sample Design:

Example: 2 genotypes of lettuce: *susceptible* and *resistant*. Do these genotypes differ in fitness in the absence of aphids.

	Susceptible	Resistant
Mean number of buds	**720**	**582**
SD of number of buds	**223.6**	**277.3**
Sample size	**15**	**16**

Both of the distributions are normally distributed

Two-sample Design:

Example: 2 genotypes of lettuce: *susceptible* and *resistant*. Do these genotypes differ in fitness in the absence of aphids.

	Susceptible	**Resistant**
Mean number of buds	**720**	**582**
SD of number of buds	**223.6**	**277.3**
Sample size	**15**	**16**

H_0: There is no difference between the number of buds in susceptible and resistant plants ($\mu_1 = \mu_2$)

H_A: There is a difference between the number of buds in susceptible and resistant plants ($\mu_1 \neq \mu_2$)

Two-sample Design:

Example: 2 genotypes of lettuce: *susceptible* and *resistant*. Do these genotypes differ in fitness in the absence of aphids.

	Susceptible	**Resistant**
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t-test:

df = 15+16 - 2 = 29

$\alpha = 0.05$

$$s_p^2 = \frac{df_1 s_1^2 + df_2 s_2^2}{df_1 + df_2} = \frac{14(223.6)^2 + 15(277.3)^2}{14+15} = 63909.9$$

$$SE_{\bar{Y}_1 - \bar{Y}_2} = \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{63909.9\left(\frac{1}{15} + \frac{1}{16}\right)} = \sqrt{8255.02} = 90.86$$

Two-sample Design:

Example: 2 genotypes of lettuce: *susceptible* and *resistant*. Do these genotypes differ in fitness in the absence of aphids.

	Susceptible	Resistant
Mean number of buds	**720**	**582**
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H_0: There is no difference between the number of buds in susceptible and resistant plants ($\mu_1 = \mu_2$)
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Two sample t-test:
Assumptions have been met

$$t = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{SE_{\bar{Y}_1 - \bar{Y}_2}} = \frac{(720 - 582)}{90.86} = 1.52$$

$\alpha = 0.05$

Critical value: $t_{0.05(2),29} = 2.05$: t < 2.05 so Fail to reject. These data are not sufficient to say there is resistance

Confidence Interval: Two-sample Design:

$$\boxed{(\bar{Y}_1 - \bar{Y}_2) - t_{\alpha(2),df} SE_{\bar{Y}_1 - \bar{Y}_2} < \mu_1 - \mu_2 < (\bar{Y}_1 - \bar{Y}_2) + t_{\alpha(2),df} SE_{\bar{Y}_1 - \bar{Y}_2}}$$

$138 - 2.05(90.86) < \mu_1 - \mu_2 < 138 + 2.05(90.86)$

$-48.21 < \mu_1 - \mu_2 < 324.26$

Note: this interval includes 0 which supports our conclusion (FTR)

What is the total degree of freedom and the standard error of difference between $mean_1 = 32$ and $mean_2 = 30$, given that $n_1 = 10$, $n_2 = 15$; $s_1 = 2$ and $s_2 = 3$. (The variable in both population is normally distributed)

A. df = 23; SE = 1.13

B. df = 24; SE = 1.13

C. df = 23; SE = 1.08

D. df = 24; SE = 1.08

E. df = 23; SE = 0.66