

Probability

Probability:

Two major splits in how probability is defined:

Frequency Interpretation:

Frequency of a particular outcome (an event)
across many random trials

Subjective Interpretation:

Subjective belief or opinion of the chance that a
particular outcome (an event) will be realized

Random Experiment:

The process of observing the outcome of a chance event

ex - one roll of a die is an experiment

Sample Space:

A list of all possible elementary outcomes of an experiment

ex - $\{1, 2, 3, 4, 5, 6\}$

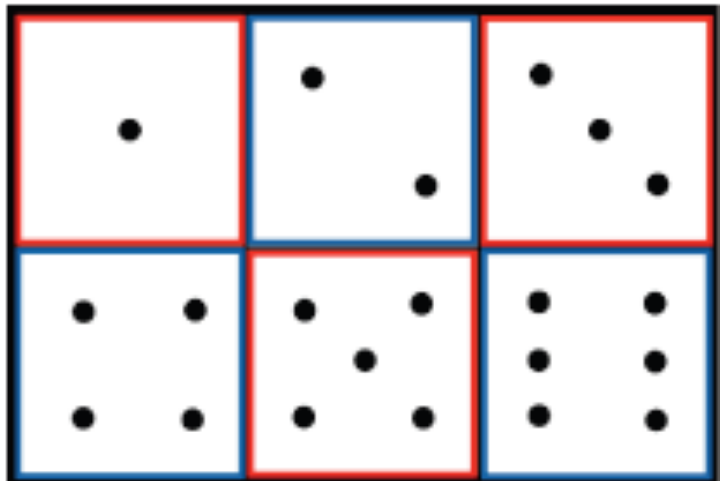
Event:

A set made up of elements from the sample space

ex - $\{1, 2, 3, 4, 5, 6\}$ or “an even number”

Examples of Sample Space:

One Die Toss



$\{1, 2, 3, 4, 5, 6\}$

One Coin Toss



$\{\text{heads}, \text{tails}\}$

<https://xkcd.com/2090/>

FEATHERED DINOSAUR VENN DIAGRAM

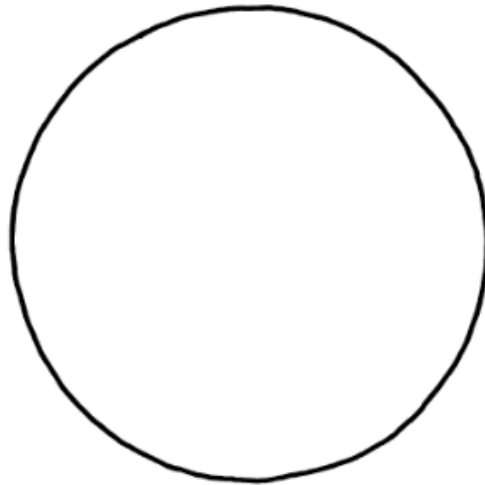
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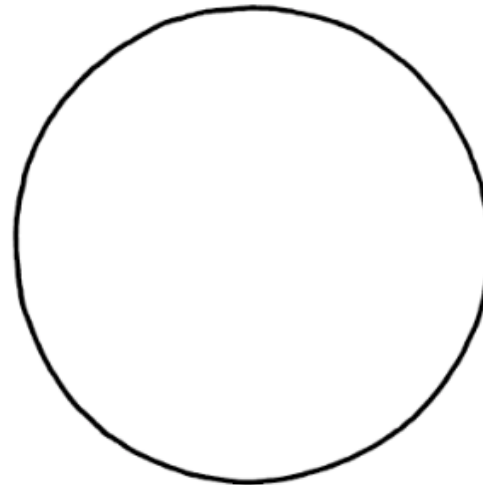
RANDOM

NEXT >

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PEOPLE WHO DON'T THINK
FEATHERED DINOSAURS
SOUND SCARY

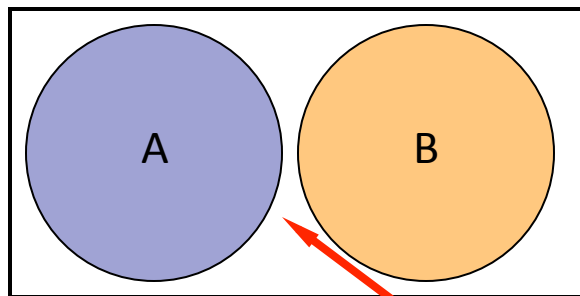


PEOPLE WHO
HAVE TRIED TO
FIGHT AN OSTRICH

Mutually Exclusive:

Two events that cannot be simultaneously true.

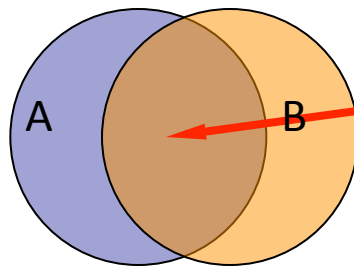
Illustrated with a Venn Diagram:



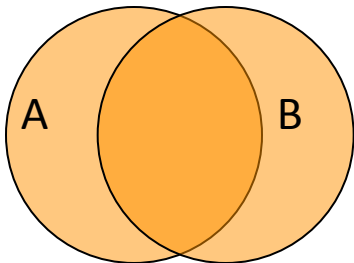
Area of 'square' is 1

$$P(A \cap B) = 0$$

Venn Diagram:

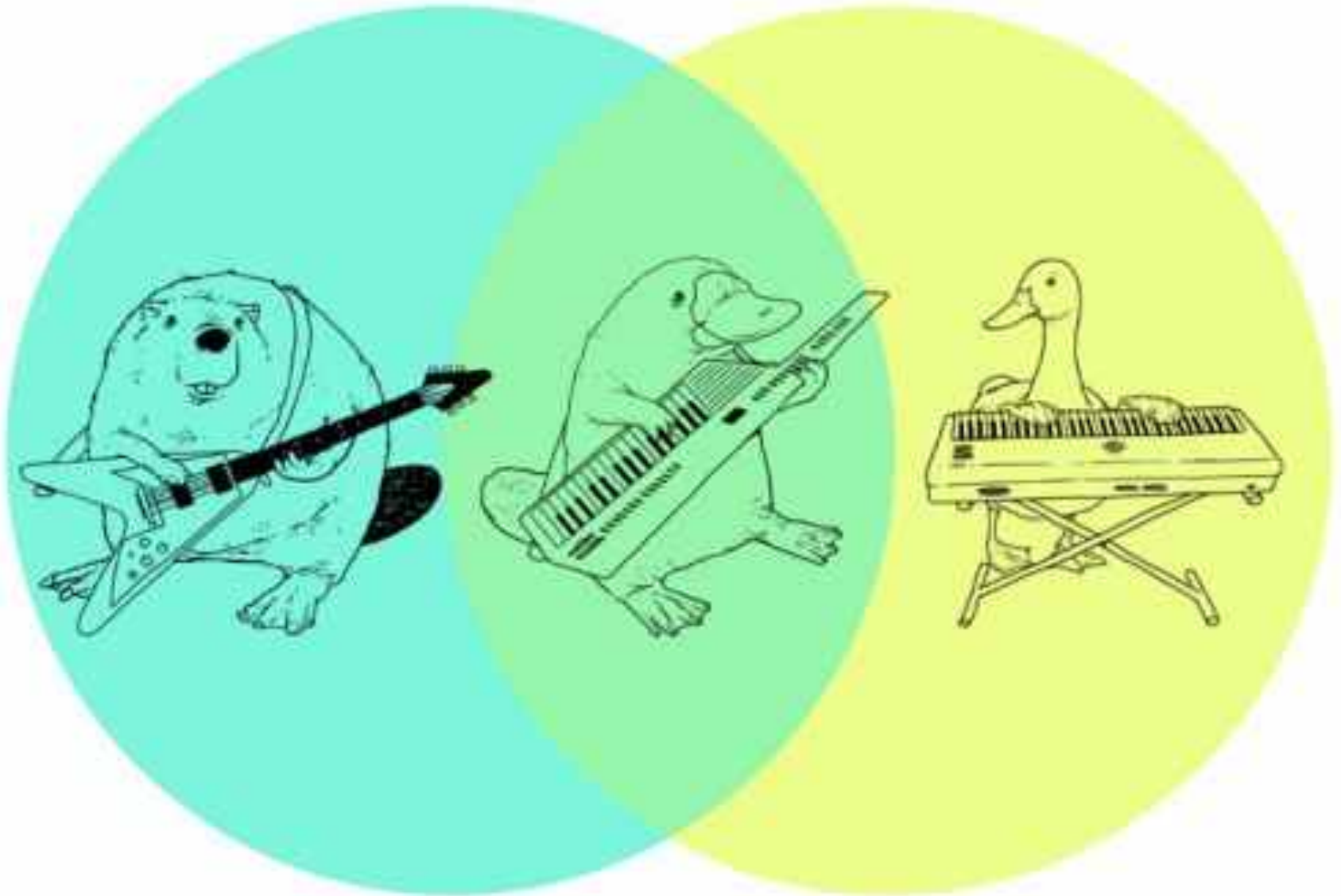


$$P(A \cap B) \neq 0$$



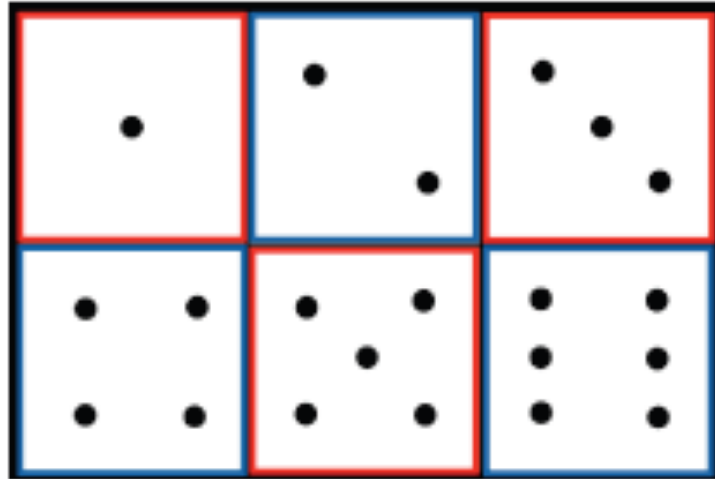
$$P(A \cup B) \neq 0$$

The BEST Venn diagram



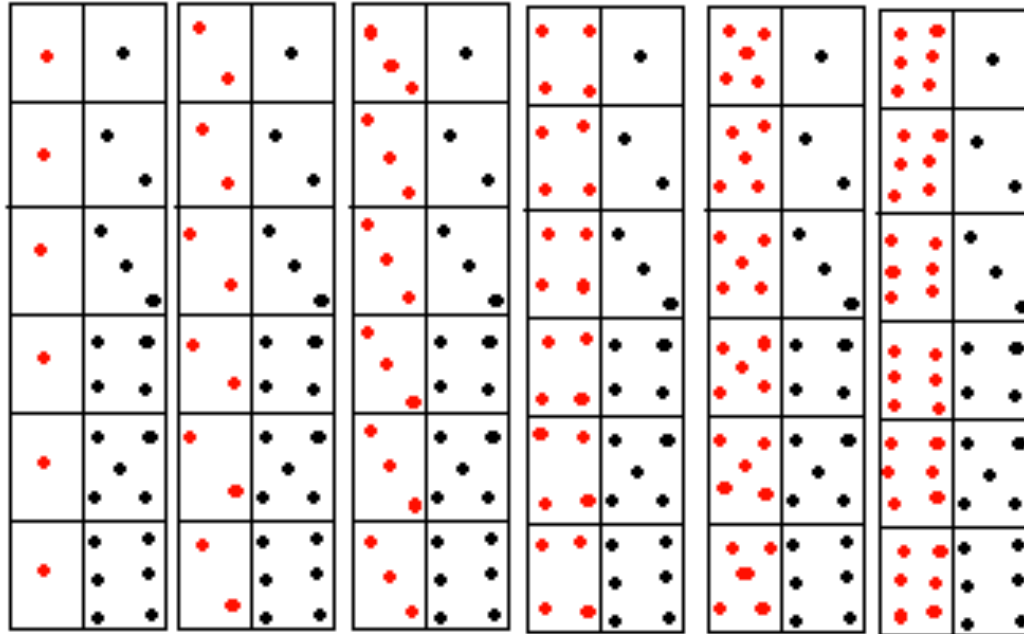
<http://i.imgur.com/CsaCFLc.jpg>

Probability



$$P(\text{rolling } 1) = P(\text{rolling } 2) = \dots = P(\text{rolling } 5) = P(\text{rolling } 6) = 1/6$$

Probability



A little review:

A sample space is made up of elementary outcomes

ex. One roll of a die has a sample space = $\{1,2,3,4,5,6\}$

An event is a subset of the elementary outcomes

ex. Two dice are rolled. The event of interest is the two dice faces add up to 3. What is the probability of this event?

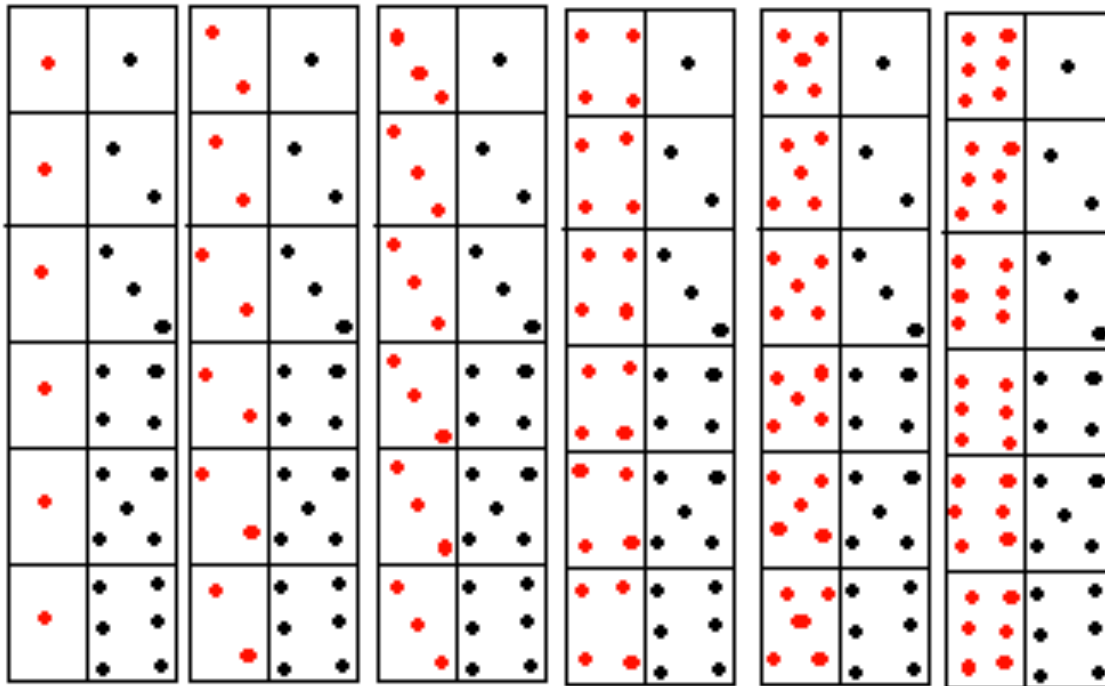
The probability of this event is the sum of elementary outcomes in this subset.

Probability

ex. Two dice are rolled. The event of interest is the two dice faces add up to 3. What is the probability of this event?

The probability of this event is the sum of the probabilities of the elementary outcomes in this subset.

The full set of elementary outcomes of rolling two dice



Which of the following best describes the reason for your answer in the preceding question?

- A. The fourth ball should not be red because too many red ones have already been picked.
- B. The picks are independent, so every color has an equally likely chance of being picked.
- C. This color is just as likely as any other color.

Discussion:

What if a coin is flipped five times and comes up heads each time. Is a tail "due" and therefore more likely than not to occur on the next flip?

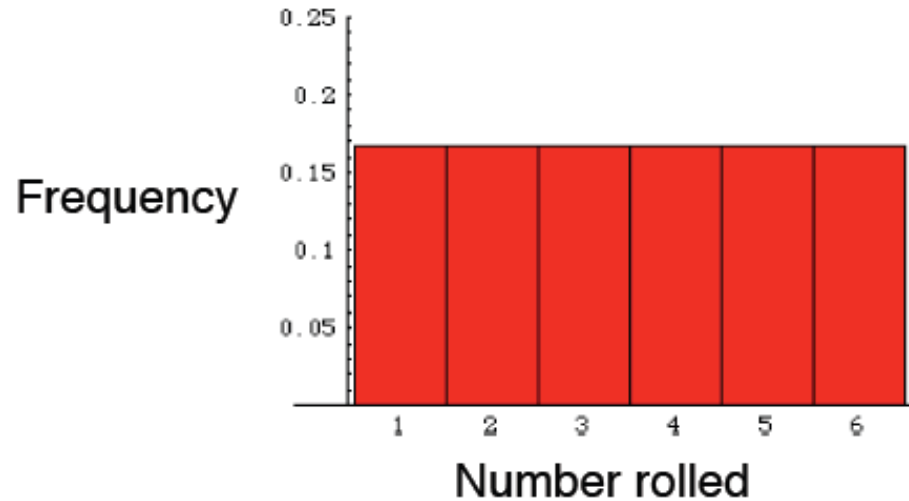
<http://onlinestatbook.com/2/probability/gambler.html>



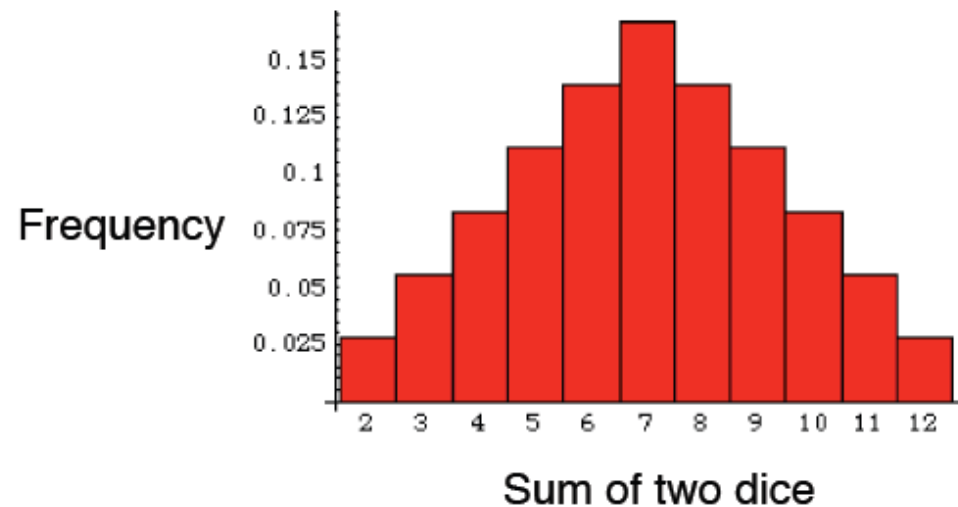
<https://www.youtube.com/watch?v=7hx4gdlfamo>

Probability

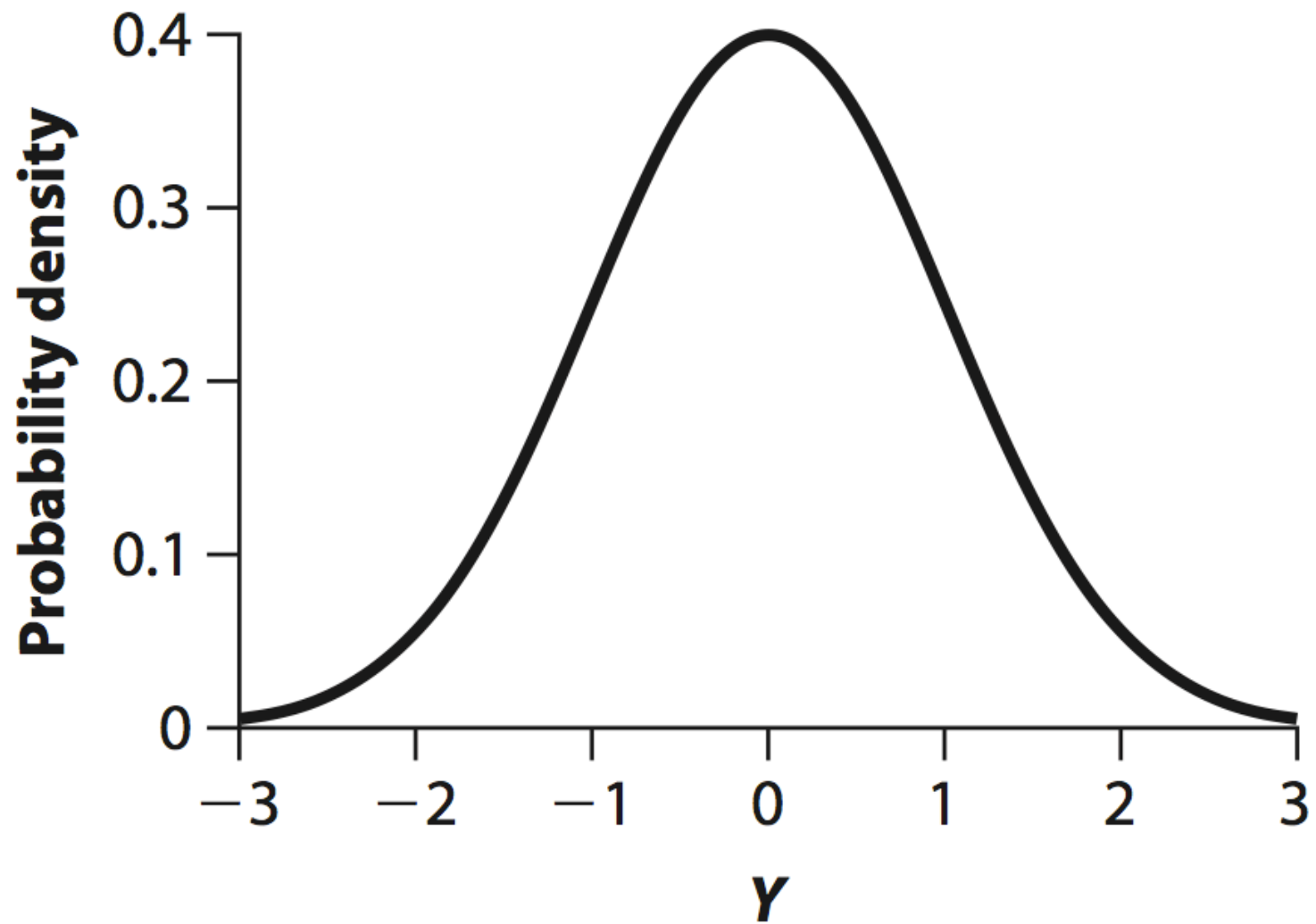
Probability distribution for the outcome of a roll of one die:



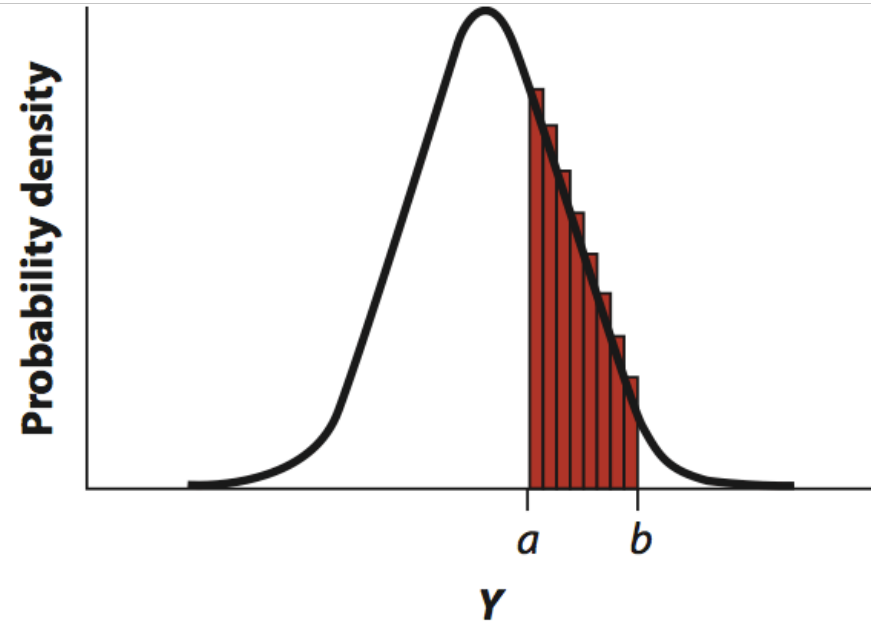
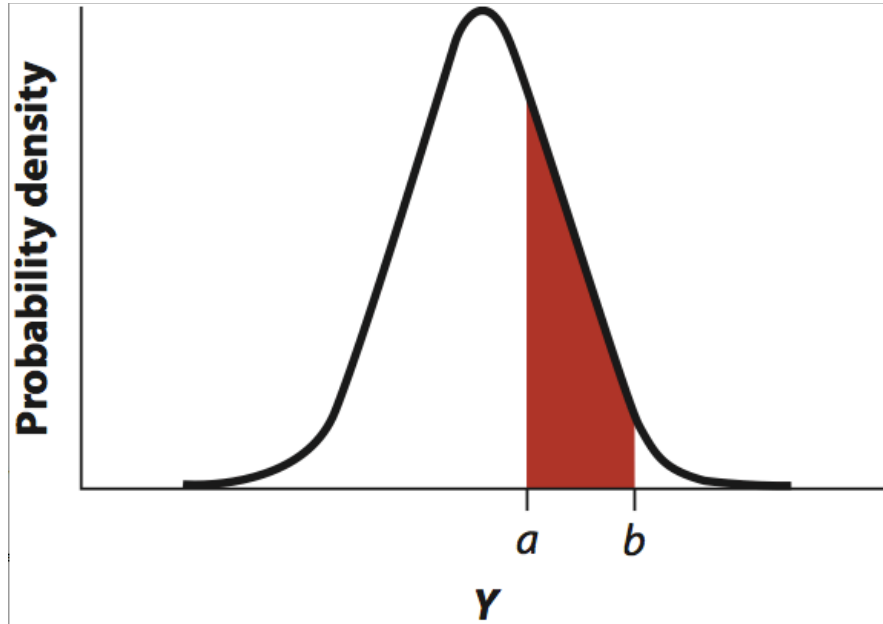
Probability distribution for the sum of a roll of two dice:



Probability



Probability



THREE LOGIC operations are done on probabilities:

AND, OR, NOT

E AND F: The event E and the event F both occur

E OR F: The event E or the event F occur (or they both do)

NOT E: The event E does not occur

Probability

Ex. Event A = Black die is 1, Event B = Red die is 1

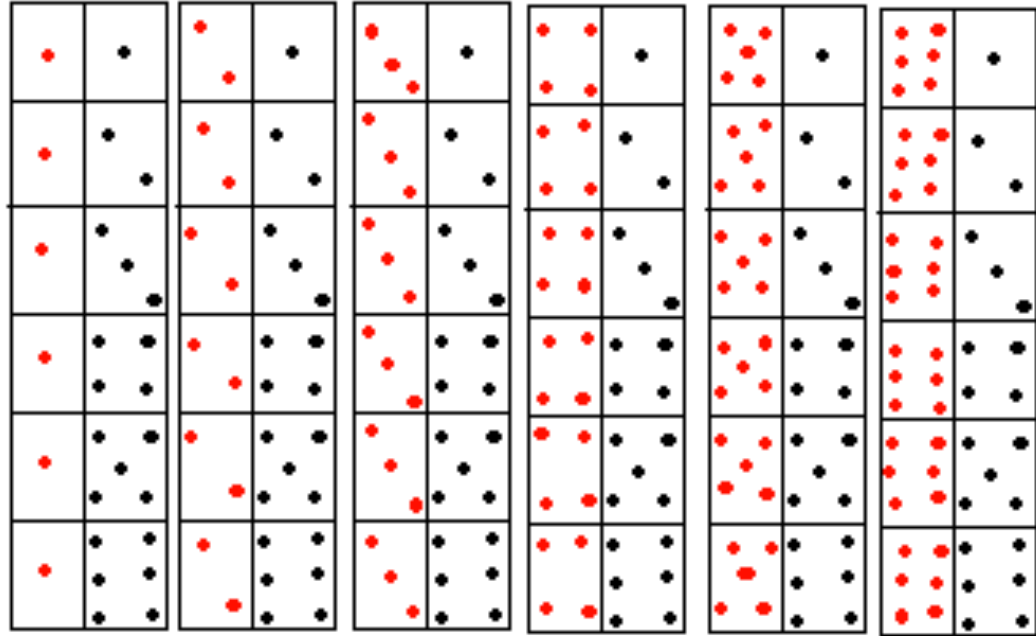
$P(A \text{ OR } B) = ?$

a. $6/36$

b. $12/36$

c. $11/36$

d. $1/36$



Probability

Ex. Event A = Black die is 1, Event B = Red die is 1

$P(A \text{ AND } B) = ?$

a. $6/36$

b. $12/36$

c. $11/36$

d. $1/36$

