Goodness of Fit

Goodness of Fit

We have already seen a specific category of X^2 Goodness of fit test that gives us EXACT probabilities: The **Binomial Test**. This is a GoF test that is limited to categorical variables with two outcomes only!

χ^2 Goodness of fit test:

Compares observed counts to those predicted by a discrete probability distribution

Fitting Discrete Models:

• A goodness-of-fit test compares observed counts to a discrete probability distribution

Example: Days of the week when babies born

• Discrete distribution is a probability distribution which describes discrete numerical random variables

Example:

Number of heads (10 flips of a coin)
Number of flowers in a square meter
Number of disease outbreaks in a year

Hypotheses for the χ^2 test:

H_0: *The data come from a particular discrete probability distribution*

H_A: *The data do not come from that distribution*

Test statistic for the χ^2 test:

$$\chi^2_{df} = \sum_i \frac{(Observed_i - Expected_i)^2}{Expected_i}$$

Degrees of Freedom:

d.o.f. = # categories - 1 - # est. parameters

Each of the following accurately represents characteristics of the Chi-Square distribution **except** for:

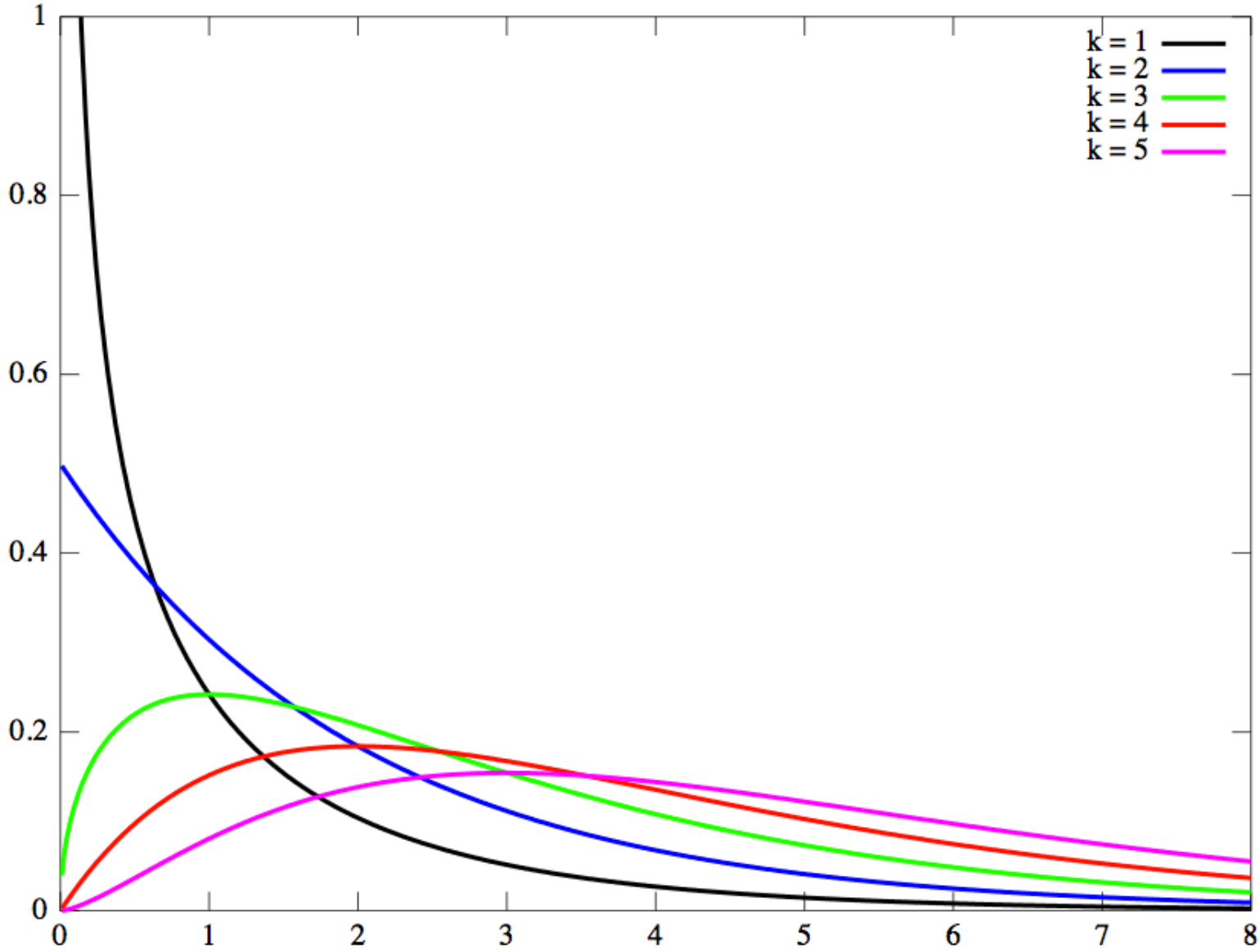
A. As the degrees of freedom increase, the critical value of the Chi-Square distribution becomes larger

B. The region of rejection is always in the left-tail of the Chi-Square distribution

C. It is a positively skewed distribution

D. Its shape depends on the number of degrees of freedom

χ^2 Distribution:



Finding the P-value of χ^2 distr^n:

- P-value of χ^2 test uses only right hand tail of distribution

- χ^2 distribution is continuous so probability is measured by area under curve ---> either use a computer statistical package or get really really good at calculus

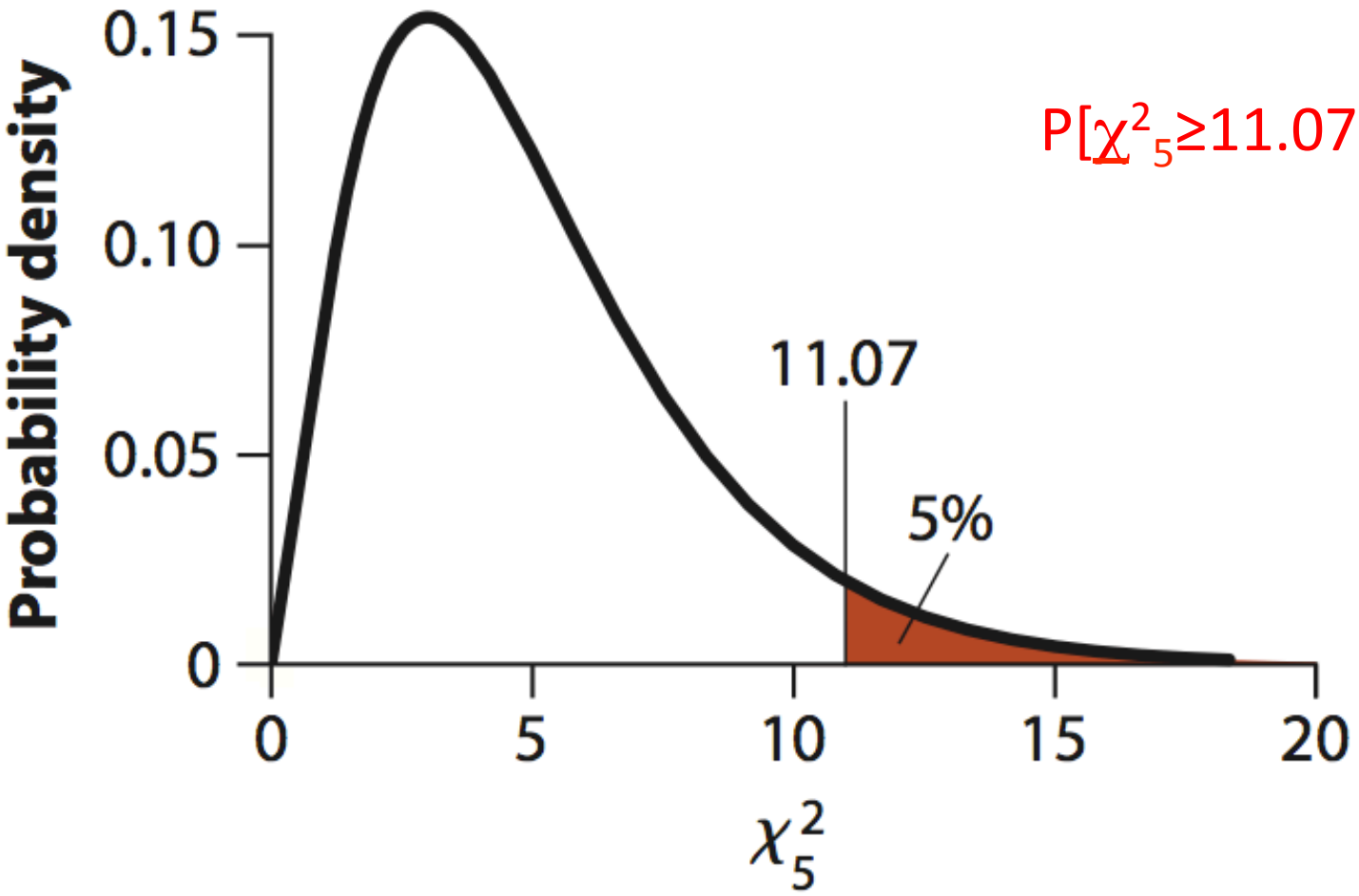
Finding **critical values for** χ^2 distr^n:

- **Critical value**: values of the test statistic that marks the boundary of a specified area of the sampling distribution

- Statistical Table A (pg 703)

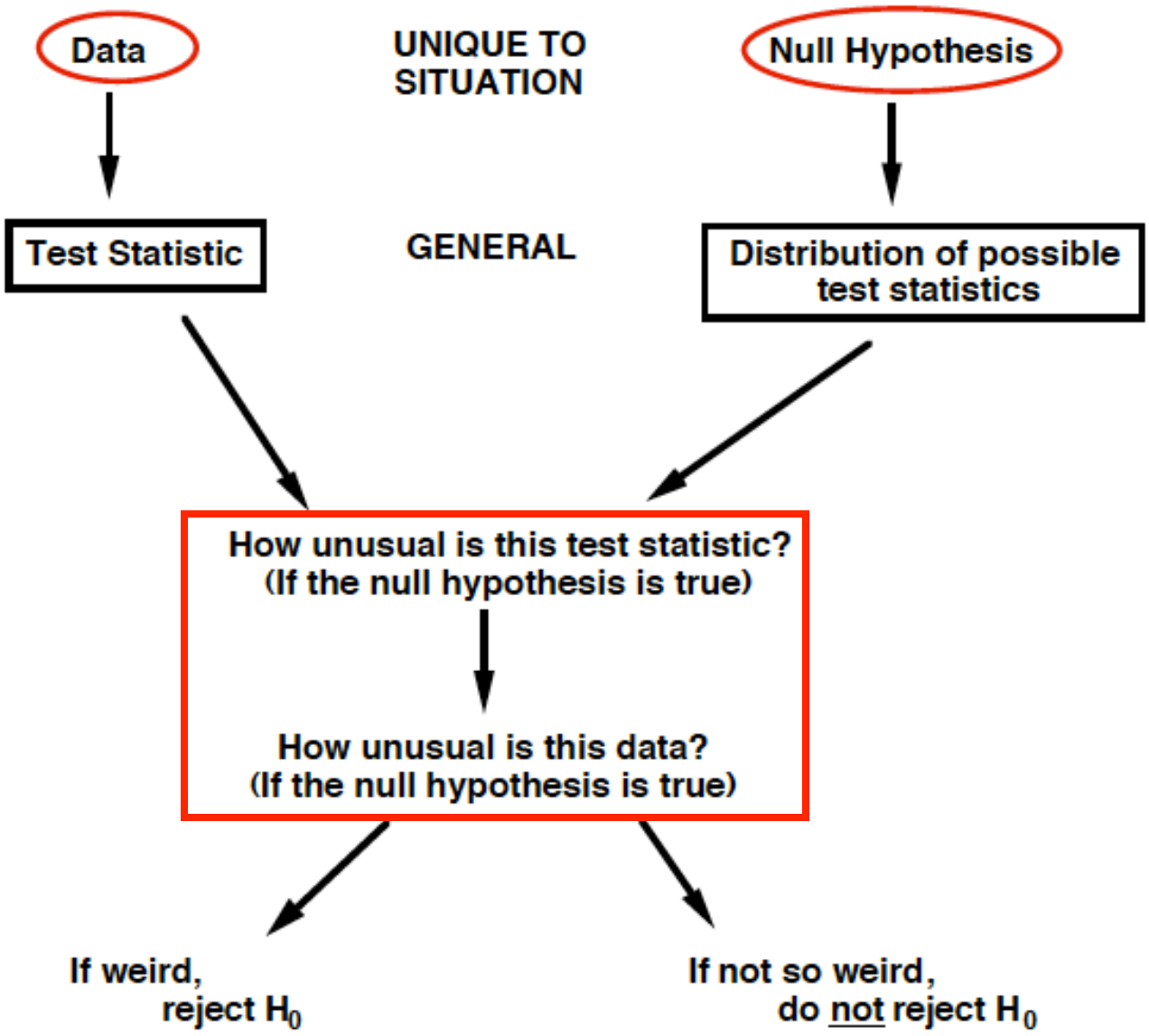
df	0.999	….	0.95	0.05
1	0.0000016		0.00393	3.84
…				
5	0.21		1.15	**11.07**
6	0.38		1.64	12.59

Finding **critical values for** χ^2 distr^n:



$P[\chi^2_5 \geq 11.07] = 0.05$

Test Statistics and Hypothesis Testing



More of the χ^2 test:

Assumptions:

- No more than 20% of categories have ***expected*** frequencies < 5

- No category with ***expected*** frequencies < 1

  – You can sometimes work around these assumptions by chopping up your categories in a different manner

χ^2 Goodness of Fit test

"A model is a mathematical tool that mimics how we *think* a natural process works.." (p.187)

Life is interesting when a model doesn't fit the data because it suggests that at least one of the major assumption about how we think about the process is wrong

All models are wrong, but some are useful

- George E.P. Box