

# ANOVA example

Example: Body temperature of squirrels in low, medium and hot environments



## Squirrel body temperature data (Celsius)

- **Cold:** 30.4, 31.0, 31.2, 31.0, 31.5, 30.4, 30.6, 31.1, 31.3, 31.9, 31.4, 31.6, 31.5, 31.4, 30.3, 30.5, 30.3, 30.0, 30.8, 31.0
- **Warm:** 36.3, 37.5, 36.9, 37.2, 37.5, 37.7, 37.5, 37.7, 38.0, 38.0, 37.6, 37.4, 37.9, 37.2, 36.3, 36.2, 36.4, 36.7, 36.8, 37.0, 37.7
- **Hot:** 40.7, 40.6, 40.9, 41.1, 41.5, 40.8, 40.5, 41.0, 41.3, 41.5, 41.3, 41.2, 40.7, 40.3, 40.2, 41.3, 40.7, 41.6, 41.5, 40.5

$H_0$ : Mean body temperature is the same for all three groups of squirrels

$H_A$ : At least one of the three is different from the others

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<b>GROUP</b>	$\bar{X}$	<b>s</b>	<b>n</b>
<b>Cold</b>	31.0	0.551	20
<b>Warm</b>	37.2	0.582	21
<b>Hot</b>	41.0	0.430	20

$$N = \sum n = 20 + 21 + 20 = 61$$

## Squirrel Mean square error:

$$\begin{aligned}SS_{error} &= \sum df_i s_i^2 \\&= 19(0.551)^2 + 20(0.582)^2 + 19(0.430)^2 \\&= 16.1\end{aligned}$$

$$df_{error} = 19 + 20 + 19 = 58$$

$$MS_{error} = 16.1/58 = 0.277$$

## Squirrel mean squares groups:

$$\bar{X}_T = \frac{20(31.0) + 21(37.2) + 20(41.0)}{20 + 21 + 20} = 36.4$$

$$SS_{groups} = \sum n_i (\bar{X}_i - \bar{X})^2$$

$$\begin{aligned} SS_{groups} &= 20(31.0 - 36.4)^2 + 21(37.2 - 36.4)^2 + 20(41.0 - 36.4)^2 \\ &= 1015.7 \end{aligned}$$

# Squirrels mean squares group

$$df_{\text{groups}} = k - 1 = 3 - 1 = 2$$

$$MS_{\text{groups}} = SS_{\text{groups}} / df_{\text{groups}} = 1015.7 / 2 = 507.9$$



The test statistic for ANOVA is F:

$$\begin{aligned} F &= MS_{\text{groups}} / MS_{\text{error}} \\ &= 507.9 / 0.277 \\ &= 1834.7 \end{aligned}$$

$$F_{0.05(1),2,58} = 3.15$$

Since  $1835 \gg 3.15$ , we know that  $P < 0.05$  and we can reject  $H_0$ .

The variance in the sample group means is bigger than expected given the variance within sample groups so at least one of the groups has a population mean different from another group

Results are presented in ANOVA Table:

Source of variation	Sum of Squares	df	Mean Squares	F-ratio	P
Groups (treatment)	1015.7	2	507.9	1834.7	<0.0001
Error	16.1	58	0.277		
Total	1031.8	60			

$$R^2 = SS_{\text{groups}} / SS_{\text{error}} = 0.984$$