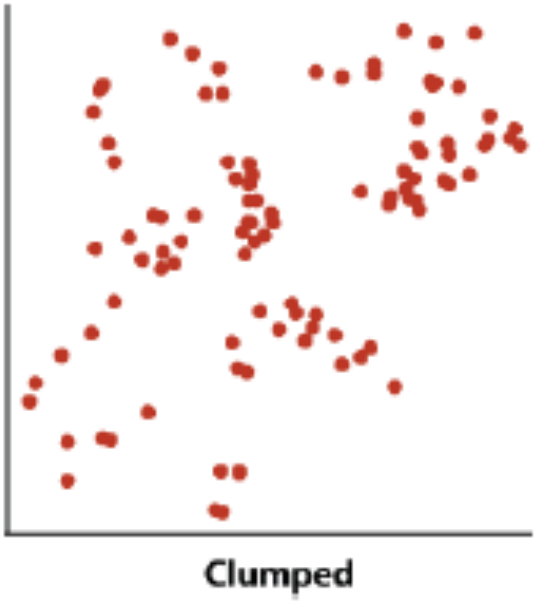
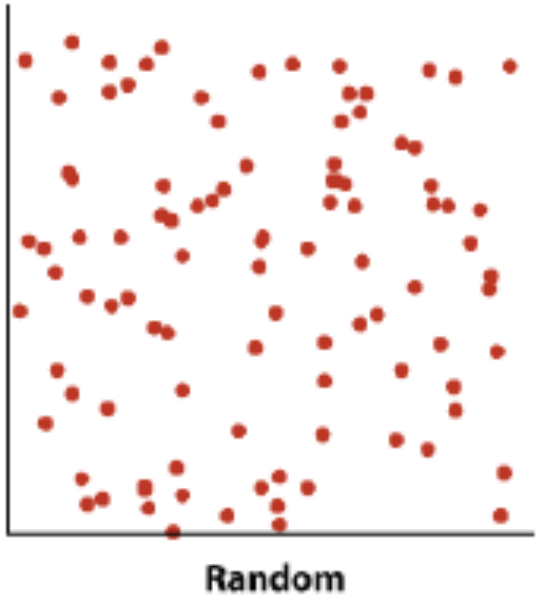
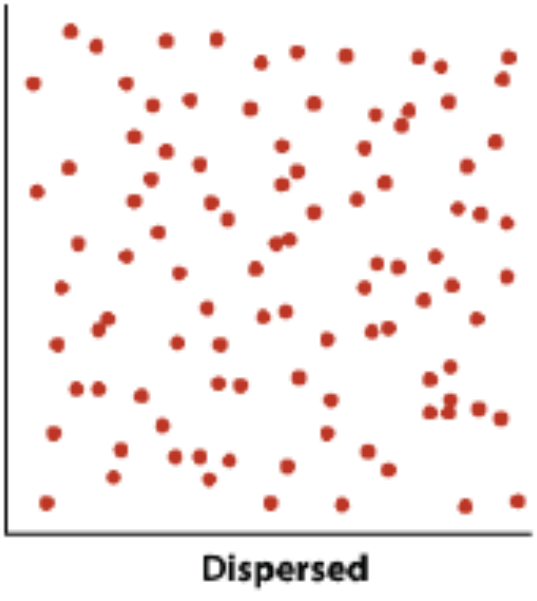
Fitting the Poisson Distribution:

The Poisson Distribution describes the probability of getting X successes in a block of time or space when the successes happen independently of each other and occur with equal probability at every point in time or space.

χ^2 Goodness of Fit test







χ^2 Goodness of Fit test

Poisson Distribution:

$$P[X] = \frac{e^{-\mu}\mu^{X}}{X!}$$

χ^2 Goodness of Fit test

Example: Mass extinctions (example 8.6): random or concentrated in periods of time? Fossil Marine invertebrates families extinctions in 76 blocks of time of similar duration (Raup Sepkoski, 1982).

If extinction is random, then the number of extinctions per block of time will be Poisson.

If not, then they could be either clumped or dispersed.

χ^2 Goodness of Fit test

Num Extinctions (X)	Frequency
0	0
1	13
2	15
3	16
4	7
5	10
6	4
7	2
8	1
9	2
10	1
11	1

Num Extinctions (X)	Frequency
12	**0**
13	0
14	1
15	0
16	2
17	0
18	0
19	0
20	1
>20	0
Total	76

χ^2 Goodness of Fit test

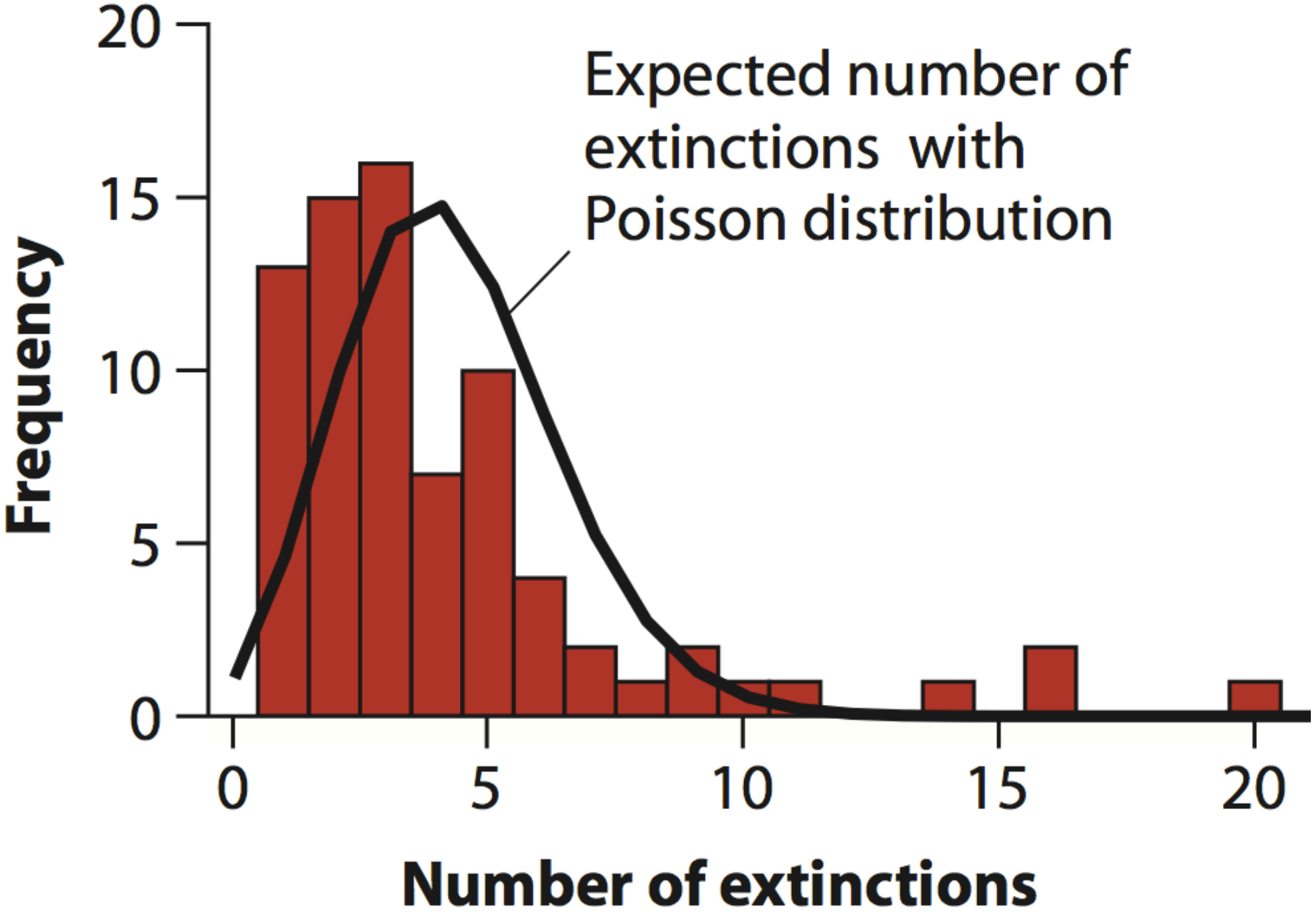
H_0: The number of extinctions per unit of time has a Poisson distribution

H_A: The number of extinctions per unit of time does *NOT* have a Poisson distribution

Estimate μ:

$$\overline{X} = \frac{(0X0) + (13X1) + (15X2) + \ldots}{76} = 4.21$$

χ^2 Goodness of Fit test



χ^2 Goodness of Fit test

Num Extinctions(X)	Observed Frequency	Expected Frequency
0	0	1.13
1	13	4.75
2	15	10.00
3	16	14.03
4	7	14.77
5	10	12.44
6	4	8.72
7	2	5.24
8	1	2.76
9	2	1.29
≥10	6	0.86
Total	76	76

χ^2 Goodness of Fit test

Num Extinctions(X)	Observed Frequency	Expected Frequency
0 or 1	13	5.88
2	15	10.00
3	16	14.03
4	7	14.77
5	10	12.44
6	4	8.72
7	2	5.24
>8	9	4.91
Total	76	76

χ^2 Goodness of Fit test

$$\chi^2 = \frac{(13-5.88)^2}{5.88} + \frac{(15-10.00)^2}{10.00} + \ldots = 23.93$$

Critical value for χ^2 is given in statistical table A as 12.59.
In fact, P-value < 0.001. Therefore, we can reject the null hypothesis and conclude that the extinction record for these fossils do not fit a Poisson distribution

I failed to mention this in the video but the degrees of freedom for this example:

DoF = 8 categories − 1 − 1 estimate = 6 degrees of freedom

Warning:

*** When you 're-bin' your data to ensure that the assumptions of the χ^2 gof test are met, you might need to update your degrees of freedom since they are based on the number of categories!**

χ^2 Goodness of Fit test

Variance = Mean:

If Variance > Mean, then CLUMPED

- visual hint: histogram is 'u-shaped'

If Variance < Mean, then DISPERSED

- points are spread uniformly in space or time

- This may bit a bit confusing if you are a BMG major, because in molecular genetics, we refer to the "overdispersed molecular clock" which is really saying that variance > mean number of substitutions.

χ^2 Goodness of Fit test

$$\chi^2 = \frac{(13-5.88)^2}{5.88} + \frac{(15-10.00)^2}{10.00} + \ldots = 23.93$$

Critical value for χ^2 is given in statistical table A as 12.59.
In fact, P -value < 0.001. Therefore, we can reject the null hypothesis and conclude that the extinction record for these fossils do not fit a Poisson distribution.

Since the sample variance is 13.72, we can also say that not only do we reject the null hypothesis that extinction patterns follow the Poisson distribution, we can also say that extinction events are clumped

Rejecting a null hypothesis of a Poisson distribution of successes implies that

A- Success are not independent

B- The probability of a success occurring is constant over time or space.

C-The probability of a success occurring is NOT constant over time or space.

D- A and B

E- A and C