

Maximum likelihood part 2

Log-likelihood tests

Log-Likelihood ratio test:

- Compare how well two models fit the data
- One of the null hypothesis has particular parameters; the other is the alternative hypothesis which has free parameter values. This is a similar situation to general linear models (chapter 18).

Log-Likelihood ratio =

$$\ln \left[\frac{\text{likelihood}[\text{Maximum_likelihood_hypothesis}]}{\text{likelihood}[\text{Null_hypothesis}]} \right]$$

$G = 2 * \text{Log-Likelihood ratio}$

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- Compare how well two models fit the data

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- **Test statistic:**

$$\chi^2 = 2 \log \text{likelihood_ratio}$$

* If H_0 is true, G follows χ^2 distribution

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Example: 3 out of 8 individuals are male

H₀: 50% are male

H_A: not 50% are male

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$$L[p = 3/8 | 3 \text{ males}, 5 \text{ females}] = \binom{8}{3} (3/8)^3 (1 - 3/8)^5 = 0.2816$$

Remember: $L[p = 3/8 | 3 \text{ males}, 5 \text{ females}] = P[3 \text{ males}, 5 \text{ females} | p = 3/8]$

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Likelihood of Null Hypothesis:

$$L[p = 0.5] = \binom{8}{3} (0.5)^3 (1 - 0.5)^5 = 0.2188$$

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$$\ln \left[\frac{L[p = 3 / 8]}{L[p = 0.5]} \right] = \ln \left[\frac{0.2816}{0.2188} \right] = 0.2526$$

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$$\chi^2 = 2(0.2526) = 0.5051$$

$df = 1 \leftarrow$ fixed one variable in the null hypothesis (p)

$$\chi^2_{0.05,1} = 3.84, \text{ FTR } H_0$$