Mann-Whitney U-Test

- Compares the central tendencies of two groups using ranks
    - uses ranks of measurements to test whether or not frequency distribution of two groups are the same
- Equivalent to Kruskall-Wallis test but for less than 2 categories
    - Small samples lead to little power
    - All group samples are random samples
    - Distribution of the variable has the same shape in every population
- Nonparametric version of two-sample t-test

Mann-Whitney U-Test

Method:

1. Declare hypotheses
2. Rank all individuals from both groups together in order (smallest to largest)
3. Sum ranks for all individuals in each group
4. Calculate test statistic, U

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$$\boxed{U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1}$$

$$\boxed{U_2 = n_1 n_2 - U_1}$$

U_1 is the number of times an individual from pop 1 has a lower rank than an individual from pop 2 out of all pairwise comparisons

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4. Calculate test statistic, U
5. U is the larger of U_1 or U_2
6. Determine the P-value by comparing the observed U with the critical value of the null distribution for U (table E)
7. If both samples have n > 10, use Z transformation

$$Z = \frac{2U - n_1 n_2}{\sqrt{n_1 n_2 (n_1 + n_2 + 1)/3}}$$

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How to deal with tied ranks:

- Determine the ranks that the values would have been assigned if they were slightly different
- Average these ranks, and assign **that** average to each tied individual
- Count all those individuals when deciding the rank for the next largest individual

Intuitive way to understanding MWU test

- View the MWU test as the 'extremeness' of the distributions of the two populations

Example: two populations each have three data points:

1 1 1 0 0 0

There are twenty orderings of rank for this example:

{111 000, 110 100, 110 010, 110 001, 101 100, 101 010, 101 001, 100 110, 100 101 etc.}

Note: you can calculate this using (6 choose 3) = $\frac{6*5*4*3*2*1}{3*2*1(3*2*1)}$ = 20

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Assumptions:

- Both samples are random samples
- Both populations have the same shape of distribution
    - **They can both be skewed but it must be in the same direction**
    - **Same variance**

Mann-Whitney U-Test

PP: When intruding lions take over a pride of females, they often kill most or all of the infants in the group in order to reduce the time until females are again sexually receptive. A long term study measured the time to reproduction of female lions after losing cubs to infanticide and compared this to the time to reproduction of females who had lost their cubs to accidents. The data are not normally distributed within the two groups and we have been unable to find a transformation that makes them normal.

Accidental: 110, 117, 133, 135, 140, 168, 171, 238, 255

Infanticide: 211, 232, 246, 251, 275

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1. H_0: $\mu_{accidental} = \mu_{infanticide}$

   H_A: $\mu_{accidental} \neq \mu_{infanticide}$

In words: the mean time to reproduction of female lions who have lost cubs due to accident or infanticide is the same.

Mann-Whitney U-Test

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2. Rank all individuals from both groups together

110, 117, 133, 135, 140, 168, 171, 211, 232, 238, 246, 251, 255, 275

3. Sum ranks for all individuals in each group

Accidental (**R_1**): 1+2+3+4+5+6+7+10+13 = 51

Infanticide (**R_2**) : 8+9+11+12+14 = 54

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4. Calculate test statistic, U (NB: U is larger of U_1 or U_2):

$$U_1 = n_1 n_2 + n_1(n_1+1)/2 - R_1 = (9)(5) + (9)(10)/2 - 51$$

$$= 45+45-51 = 39$$

$$U_2 = n_1 n_2 - U_1 = 45 - 39 = 6$$

$$\boxed{U = 39}$$

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4. Calculate test statistic, U (NB: U is larger of U_1 or U_2): **U = 39**

5. Determine critical value from Table E

Critical Value is 38 for n_1= 9, n_2 = 5 for alpha =0.05

Our value of **U** is > Critical value so we can reject the null hypothesis that females have equal times to reproduction regardless of how their previous cub died.