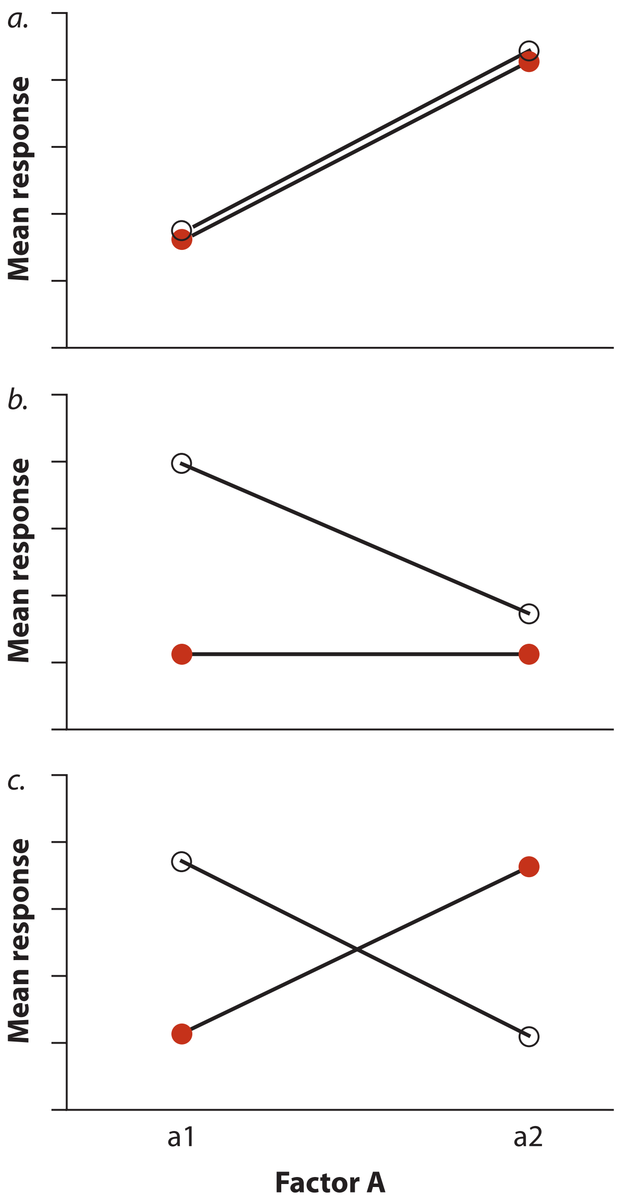
Multi-Factor ANOVA

Which graph belongs to which description?



1. No main effect of A or B but an interaction
2. A main effect of A and B and an interaction
3. A main effect of A, no main effect of B and no interaction

* It may be obvious but the two different levels of B are indicated by the red dot (for one level of B) and the open circle (second level of B).

A. (a,1), (b,2),(c,3)
B. (a,3),(b,1), (c,2)
C. (a,3),(b,2), (c,1)
D. (a,2),(b,1), (c,3)

Multiple factor ANOVA:

- A factor is a categorical variable
- ANOVAs can be generalized to look a more than one categorical variable at a time
  - Same principles as one-way ANOVA
    - partitioning of variance
  - Same assumptions as one-way ANOVA
    - Equal variances
    - Equal sizes
- *Not only can we ask whether each categorical variable affects a numerical variable, but also do they* ***interact*** *in affecting the numerical variable*

One-way ANOVA:

- 1 continuous dependent variable
- 1 categorical independent variable (≥ 2 groups)
- ie. Girls vs boys in hours of tv watched

Multi-Factor ANOVA:

- 1 continuous dependent variable
- ≥ 2 categorical independent variables
- ie. Girls vs boys in hours of tv watched in four regions of the United States

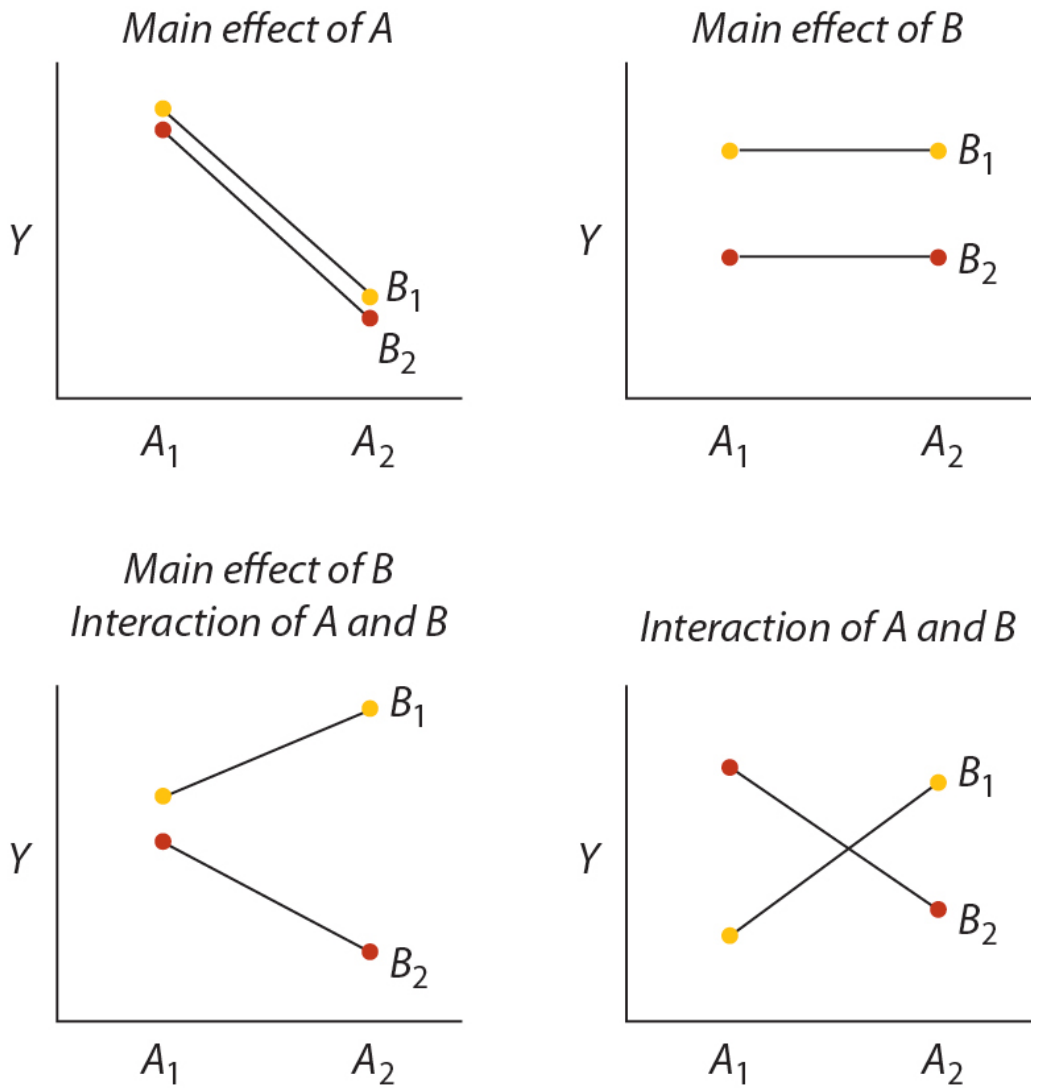
Multiple factor ANOVA:

- 1 dependent variable and ≥ 2 (independent) categorical variables
- Produces 2 interesting results:
  - **Main effects**
  - **Interaction effects**

Multiple factor ANOVA:

- One dependent variable and ≥ 2 categorical variables
- Produces 2 interesting results:
  - **Main effects**
    - 1 F-value for each category
    - Similar to the results if used one-way ANOVA but with one glorious difference:
      - Control for (partial out the effects of) **the other independent variable(s)**
  - **Interaction effects**

Multi- factor ANOVA:



Multi- factor ANOVA:

The most important aspect of multi-factor ANOVA is that we can determine whether or not groups differ on some dependent variable while controlling for the effects of the other independent variables

* Similar to ANCOVA but ANCOVA is more general

Fixed Factorial Designs:

- Effects of factors (treatments) and their interactions on a response variable
  - All combinations of the two (or more) explanatory variables are investigated

- Fixed, repeatable factors
  - **Interaction term**: if it equals 0, there is no interaction
  - **Main effects:**
    - Factor 1 and Factor 2 since they represent the effects of that factor alone when averaged over the other factor, ie. Marginal values

Response = Constant + Factor 1 + Factor 2 + Factor 1* Factor 2

- **F-test**

Fixed Factorial Designs:

Response = Constant + Factor 1 + Factor 2 + Factor 1* Factor 2

– **F-test:**

- contribution of each main effect and their interaction to the fit of the model to the data

– **Three sets of null/alternate hypotheses to test:**

1. H_0: Factor 1

$$\text{F-test} = \frac{\text{Constant + Factor 1 + Factor 2 + Factor 1*Factor 2}}{\text{Constant + Factor 2 + Factor 1*Factor 2}}$$

2. H_0: Factor 2

$$\text{F-test} = \frac{\text{Constant + Factor 1 + Factor 2 + Factor 1*Factor 2}}{\text{Constant + Factor 1 + Factor 1*Factor 2}}$$

3. H_0: Factor 1*Factor 2

$$\text{F-test} = \frac{\text{Constant + Factor 1 + Factor 2 + Factor 1*Factor 2}}{\text{Constant + Factor 1 + Factor 2}}$$

Fixed Factorial Designs:

Response = Constant + Factor 1 + Factor 2 + Factor 1* Factor 2

– **Three sets of null/alternate hypotheses to test:**

1. H_0: **Main effect: Factor 1**

$$\text{F-test} = \frac{\text{Constant + Factor 1 + Factor 2 + Factor 1*Factor 2}}{\text{Constant + Factor 2 + Factor 1*Factor 2}}$$

2. H_0: **Main effect: Factor 2**

$$\text{F-test} = \frac{\text{Constant + Factor 1 + Factor 2 + Factor 1*Factor 2}}{\text{Constant + Factor 1 + Factor 1*Factor 2}}$$

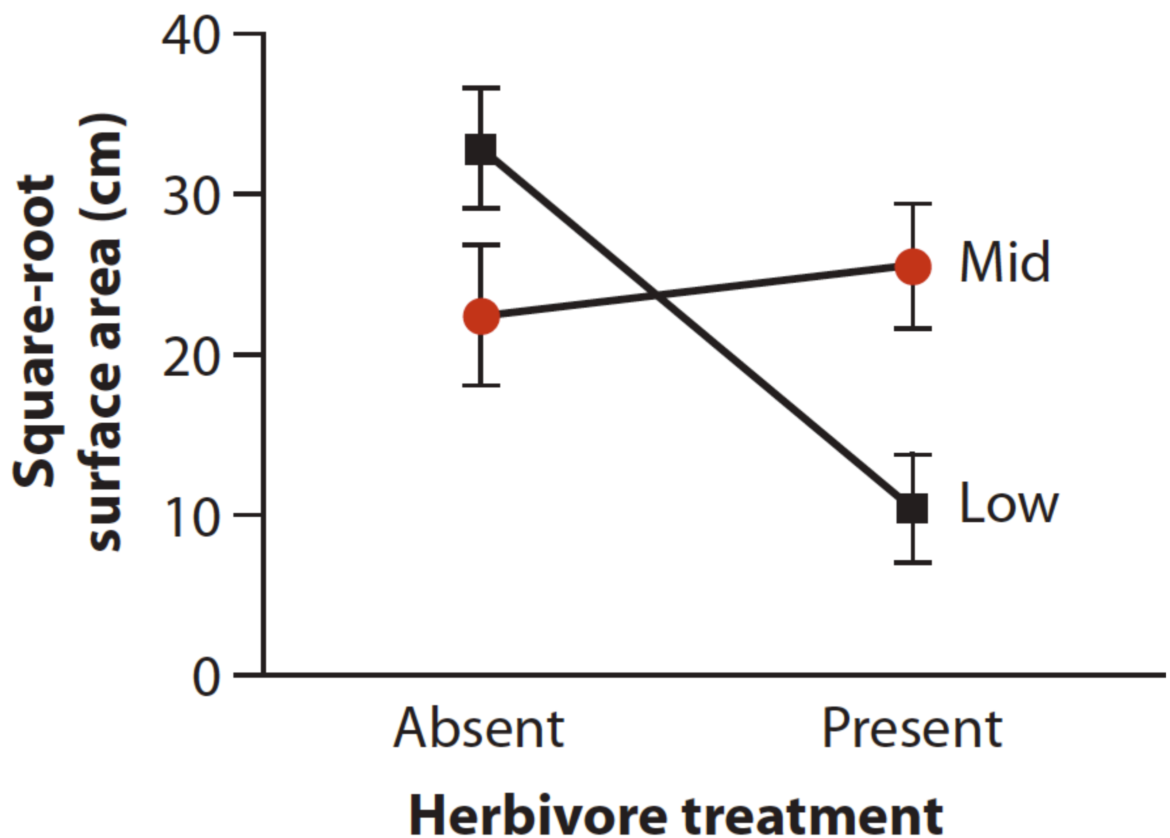
3. H_0:**Interaction effect: Factor 1*Factor 2**

$$\text{F-test} = \frac{\text{Constant + Factor 1 + Factor 2 + Factor 1*Factor 2}}{\text{Constant + Factor 1 + Factor 2}}$$

Source of Variation	Sum of Squares	df	Mean Square	F	P
Factor 1					
Factor 2					
Interaction					
Residual					
Total					

Multi- factor ANOVA:

Example: Herbivores affect on red algae in an intertidal zone: exclusion and presence. Two locations variables, low tide mark and middle mark.



Multi- factor ANOVA:

Testing three hypothesis pairs:

Herbivory (main effect):

H_0: **No difference between** herbivory treatments in mean algal cover

H_A:There is a difference between herbivory treatments in mean algal cover

Height (main effect):

H_0: **No difference** between height treatments in mean algal cover

H_A:There is a difference between height treatments in mean algal cover

Herbivory*Height (interaction effect):

H_0: The effect of herbivory on algal cover **does not** depend on height in the intertidal region

H_A: The effect of herbivory on algal cover **does** depend on height in the intertidal region

Multi- factor ANOVA:

Sources of variation	Sum of Squares	df	Mean Squares	F	P
Herbivory	1512.18	1	1512.18	6.36	0.014
Height	88.97	1	88.97	0.37	0.543
Herbivory*Height	2616.96	1	2616.96	11.00	0.002
Residual	14270.52	60	237.842		
Total	18488.63	63			

Multi- factor ANOVA:

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	Df	Sum Sq	Mean Sq	F value	Pr(>F)
herbivores	1	1512.2	1512.18	5.5227	0.02197
Residuals	62	16976.5	273.81		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
height	1	89	88.973	0.2998	0.586
Residuals	62	18400	296.769		

Multi- factor ANOVA:

- Three F ratios in the table; two of them are significant.
- No interaction between height and herbivory is rejected
- No effect of herbivory is rejected