Assumptions of Parametric Tests

*** There is a typo in the textbook for the confidence interval of Welsh's approximate t-test!**

* The confidence interval looks like all the other t-test confidence intervals (not the way it looks in the textbook). Here is the proper format:

$$(\bar{Y}_1 - \bar{Y}_2) - t_{\alpha(2),df} SE_{\bar{Y}_1 - \bar{Y}_2} < \mu_1 - \mu_2 < (\bar{Y}_1 - \bar{Y}_2) + t_{\alpha(2),df} SE_{\bar{Y}_1 - \bar{Y}_2}$$

<u>Assumptions of parametric tests:</u>

- Random Samples
- Populations are normally distributed
- <u>for two sample t-test</u>: Populations have equal(ish) variances
  - Welsh's approx t-test
  - How do we tell when populations don't have <u>equal</u> variances?

Comparing Variances:

Hypotheses:

H_0: $\sigma^2_1 = \sigma^2_2$

H_A: $\sigma^2_1 \neq \sigma^2_2$

Methods:

1. **The F-Test of equal variances**

2. **Levene's test for homogeneity of variances**

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Methods:

1. **The F-Test of equal variances STATISTICAL TABLE D**

- If variances are equal, this should be 1

$$\boxed{F = \frac{s_1^2}{s_2^2}}$$

- Put larger sample variance on top
  - forces it to be a one tailed test
- two different degrees of freedom:
  - **df = n_i − 1**
- very sensitive to assumption that both populations are normally distributed

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Example: Variation in insect genitalia (the above picture is from damsel fly) between polygamous species and monogamous species of insects: Lock and key or sexual selection? Goran Arnqvist expresses this data as a 'morphometric dimension'

	Polygamous	Monogamous
Mean	-19.3	10.25
Sample Variance	243.9	2.27
Sample Size	7	9

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Example: Variation in insect genitalia between polygamous species and monogamous species.

$$\boxed{\begin{aligned} s_1^2 &= 243.9 \\ s_2^2 &= 2.27 \end{aligned}}$$

$$\boxed{F = \frac{s_1^2}{s_2^2} = \frac{243.9}{2.27} = 107.4}$$

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$$\boxed{F = \frac{s_1^2}{s_2^2} = \frac{243.9}{2.27} = 107.4}$$

- $df_1 = 7-1 = 6$
- $df_2 = 9 - 1 = 8$

$$F_{0.025,6,8} = 4.7$$

Comparing Variances:

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Example: Variation in insect genitalia between polygamous species and monogamous species.

Why is the critical value $\alpha/2$?

- by putting the larger variance in the numerator, we are forcing F to be greater than 1
- By the null hypothesis there is a 50:50 chance of either s^2 being greater so we want the higher tail to include just $\alpha/2$

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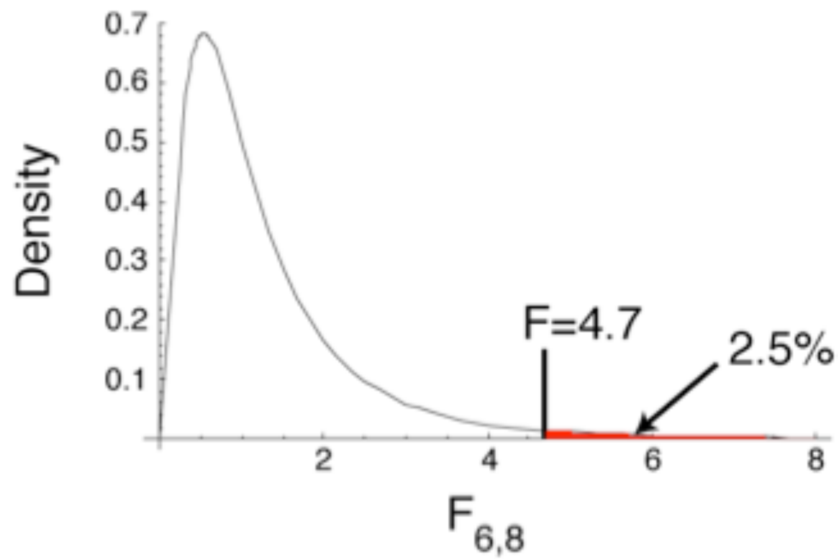

Comparing Variances:

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Example: Variation in insect genitalia between polygamous species and monogamous species.

Conclusion:

The F = 107.4 from the data is greater than the critical value for F with 6 and 8 dof so we can reject the null hypothesis that the variances of the two groups are equal.

The variance in insect genitalia is much greater for polygamous species than monogamous species.

You could confirm this by determining the Confidence Interval of each of the two variances and seeing if they overlap…

$$\frac{6*243.9}{X^2_{0.025,6}} < \sigma^2_1 < \frac{6*243.9}{X^2_{0.975,6}} \quad ; \quad \frac{8*2.27}{X^2_{0.025,8}} < \sigma^2_2 < \frac{8*2.27}{X^2_{0.975,8}}$$

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$$101.27 < \sigma^2_1 < 1180.16; \quad 1.04 < \sigma^2_2 < 8.33$$

Comparing Variances:

Hypotheses:

H_0: $\sigma^2_1 = \sigma^2_2$
H_A: $\sigma^2_1 \neq \sigma^2_2$

Methods:

1. **The F-Test of equal variances**

2. Levene's test for homogeneity of variances

- more robust than F-test
- calculations are complex so you should know that it exists and why you would use it but you do not need to know how to do it for this class.

When assumption of equal(ish) variances is violated?

- Two sample t test is robust to violations of equal variances until $variance_1 > 3variances_2$
- After that point, your conclusions can't be trusted so you need to use a different test:

  - **Welsh's approximate t test**

Welsh's approximate t-test:

- it is used when comparing means of two populations that are normally distributed but have unequal variances

The real world: Welsh’s approximate t test

- Often used as the ‘default’ method by programs such as R since it accounts for the Behrens-Fisher problem when variances are very unequal and still gives correct type I error rate when variances are equal (ish).

- However, for this class I will still expect you to test to see if variances are within assumptions of student’s t test and to use the two sample t-test if appropriate

<u>Welsh's approximate t-test:</u>

- it is used when comparing means of two populations that are normally distributed but have <u>unequal variances</u>

- <u>Experimental Design:</u>
  - 20 randomly selected burrowing owl nests
  - Randomly divided into two groups of 10 nests each
  - One group given extra dung; the other not
  - Counted number of dung beetles in owls' diets

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- Summary of beetles consumed:

	Dung added	**No Dung added**
Mean	4.8	0.51
s	3.26	0.89

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- Summary of beetles consumed:

	Dung added	No Dung added
Mean	4.8	0.51
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- Hypotheses:
  - H_0: Owls catch the same number of dung beetles with or without extra dung ($\mu_1 = \mu_2$)
  - H_A: Owls do not catch the same number of dung beetles with or without extra dung ($\mu_1 \neq \mu_2$)

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- Complicated formula found at the end of the chapter

$$\boxed{t = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$

$$\boxed{df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}\right)}}$$

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	Dung added	No Dung added
Mean	4.8	0.51
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$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{4.8 - 0.51}{\sqrt{\frac{3.26^2}{10} + \frac{0.89^2}{10}}} = 4.01$$

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There is a typo in the textbook for the formula for the confidence interval for welsh's approximate t-test. Here is the actual formula:

(you might notice it looks an awful lot like the other t statistic confidence intervals)

$$(\bar{Y}_1 - \bar{Y}_2) - t_{\alpha(2),df} SE_{\bar{Y}_1 - \bar{Y}_2} < \mu_1 - \mu_2 < (\bar{Y}_1 - \bar{Y}_2) + t_{\alpha(2),df} SE_{\bar{Y}_1 - \bar{Y}_2}$$

More Comparing Two Means

Welsh's approximate t-test:

- it is used when comparing means of two populations that are normally distributed but have <u>unequal variances</u>

- **<u>Conclusion:</u>**

$t_{0.05(2),10} = 2.23$

Since $t = 4.01 > 2.23$

We can reject the null hypothesis with $P < 0.05$.

$$(\overline{Y}_1 - \overline{Y}_2) - t_{\alpha(2),df} SE_{\overline{Y}_1 - \overline{Y}_2} < \mu_1 - \mu_2 < (\overline{Y}_1 - \overline{Y}_2) + t_{\alpha(2),df} SE_{\overline{Y}_1 - \overline{Y}_2}$$

$$(\overline{Y}_1 - \overline{Y}_2) - t_{\alpha(2),df} SE_{\overline{Y}_1 - \overline{Y}_2} < \mu_1 - \mu_2 < (\overline{Y}_1 - \overline{Y}_2) + t_{\alpha(2),df} SE_{\overline{Y}_1 - \overline{Y}_2}$$

$$4.29 - 2.23 * (1.069) < \mu_1 - \mu_2 < 4.29 + 2.23(1.069)$$

$$1.91 < \mu_1 - \mu_2 < 6.67$$

The above range does not contain "0" so it supports our rejection of the null hypothesis AND it is greater than "0" so extra dung near burrowing owls nests increases the number of dung beetles eaten.