**So far, we have learned a great many things about probability:**

1. Sample space is made up of elementary outcomes
2. Events can be elementary outcomes or groupings of elementary outcomes
3. Logic operators on probabilities: AND, NOT, OR
4. General Addition rule: $P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$
5. IF events A and B are mutually exclusive, then general addition rule collapses into special addition rule: $P(A \bigcup B) = P(A) + P(B)$
6. General Multiplication rule: P[A and B]=P[A|B]xP[B]
7. If events A and B are independent, general multiplication rule collapses into special multiplication rule
   - allows one to test whether or not two events are independent

8. What about if they are not independent?

# Manipulating Probabilities

<u>Example:</u> *Nasonia vitripennis*, a parasitoid wasp, lays eggs in fly pupae; larval wasps then hatch inside, feed on host, and emerge as adults; the males and females then mate on the spot.

***Nasonia* females manipulate sex of their offspring depending on if host fly pupa previously parasitized**.

- If host not yet parasitized, then *Nasonia* lays mainly female eggs and produces only a few males (one male can fertilize multiple females).
- If host already parasitized, then *Nasonia* lays mostly male eggs.

The state of the host encountered by a female and the sex of an egg laid are **<u>dependent variables</u>** (Werren, 1980)

# Manipulating Probabilities

# Manipulating Probabilities

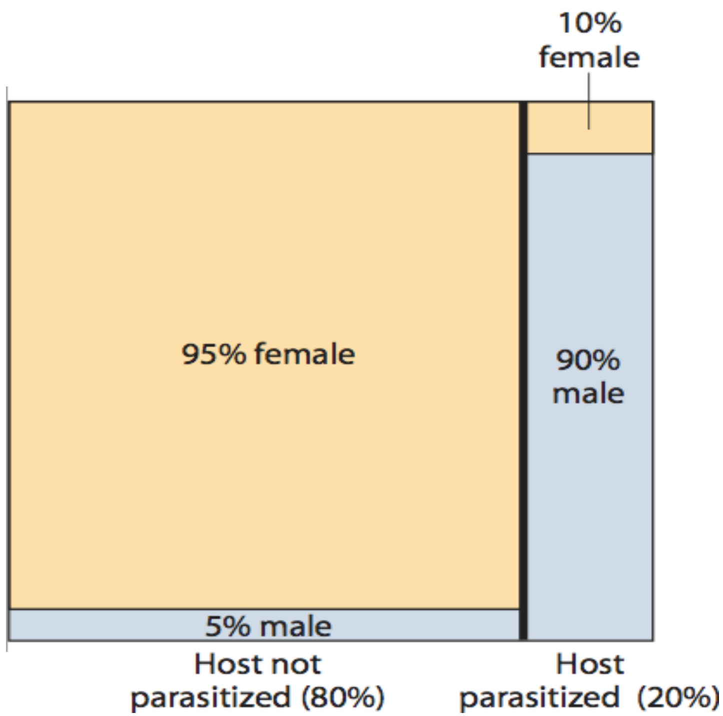
* If host not yet parasitized, then *Nasonia* lays mainly female eggs and produces only a few males (one male can fertilize multiple females).

* If host already parasitized, then *Nasonia* lays mostly male eggs.

State of host (parasitized, not parasitized)

**Possibly Dependent variable based on mosaic plot (chapter 2):**

Sex of egg (male, female)



# Manipulating Probabilities

Example: Offspring of two carriers (Nn x Nn):

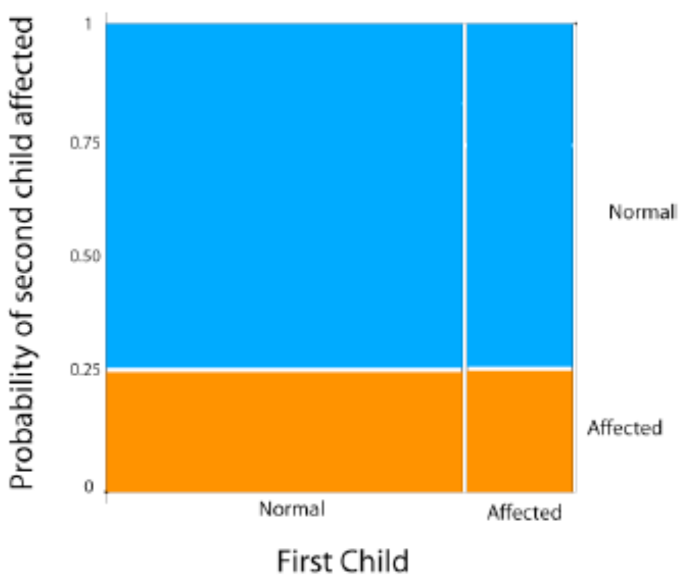
P[night blindness]=0.25

| | **N** | **n** |
|---|---|---|
| **N** | NN | Nn |
| **n** | nN | nn |

*What is the probability that two kids from this family both have night blindness?*

*P[(1st child night blindness) AND (2nd child night blindness)]*
*= 0.25 x 0.25 = 0.0625*

**Possibly Independent variable based on mosaic plot:**



## Manipulating Probabilities

Probability trees provide a straightforward method to determine independence or dependence between variables

- Map out probabilities of all mutually exclusive outcomes of variables

Additional Benefits:

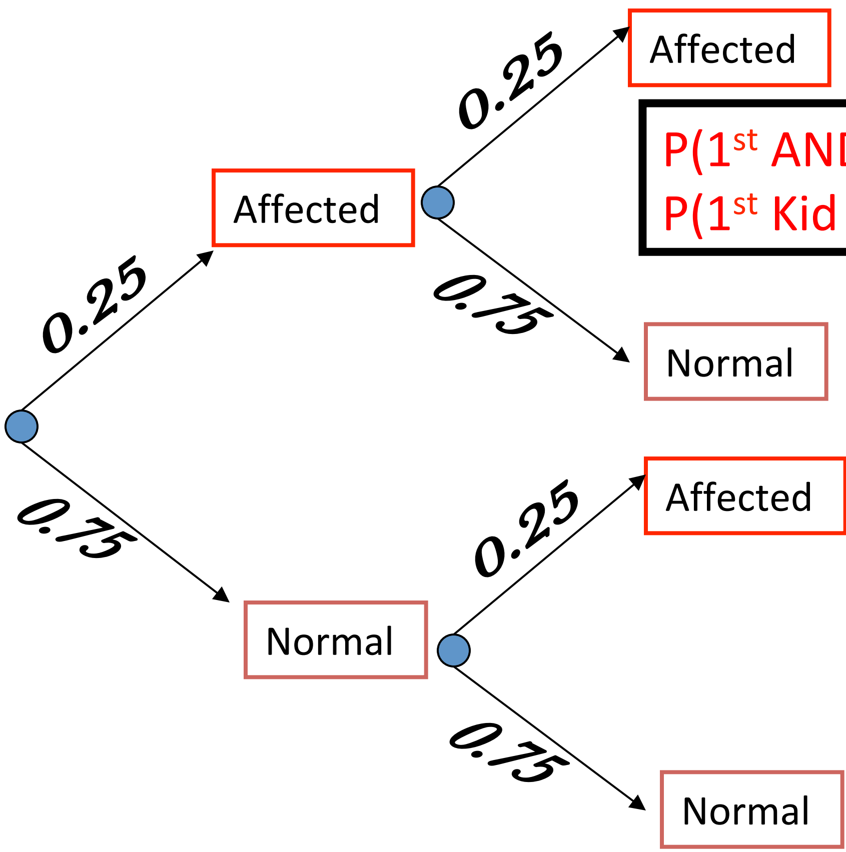
- easy to calculate the probability of any possible outcome sequence for the variables under consideration

- easy to double check that all possibilities have been enumerated

# Manipulating Probabilities

## Probability Trees:

Phenotype of 1st Child — Phenotype of 2nd Child — Probability:




$0.25 \times 0.25 = 0.0625$

$P(1^{st} \text{ AND } 2^{nd} \text{ kids affected}) = P(2^{nd}|1^{st})P(1^{st}) =$
$P(1^{st} \text{ Kid affected}) \times P(2^{nd} \text{ Kid affected})$

$0.25 \times 0.75 = 0.1875$

$0.75 \times 0.25 = 0.1875$

$0.75 \times 0.75 = 0.5625$

# Manipulating Probabilities

**So far, we have learned a great many things about probability:**

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8. What about if they are not independent?

A large population of giant pandas has five alleles at one gene labeled: $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$, $\mathbf{A_4}$, $\mathbf{A_5}$. They have corresponding frequencies in the population of: 0.1, 0.15,0.6, 0.05, 0.1. In this randomly mating population, the two alleles present in any individual are independently sampled from the population as a whole.

a. What is the probability that a single allele chosen at random from this population either $\mathbf{A_1}$ or $\mathbf{A_4}$?
b. What is the probability that the individual has **two** $\mathbf{A_1}$ alleles?
c. What is the probability that an individual is ***not*** $\mathbf{A_1A_1}$?
d. What is the probability, if you drew two individuals at random from this population that neither of them would have an $\mathbf{A_1A_1}$ genotype?
e. What is the probability, if you drew two individuals at random from this population that at least one of them would have an $\mathbf{A_1A_1}$ genotype?
f. What is the probability that three randomly chosen individuals would have ***no*** $\mathbf{A_2}$ or $\mathbf{A_3}$ alleles?

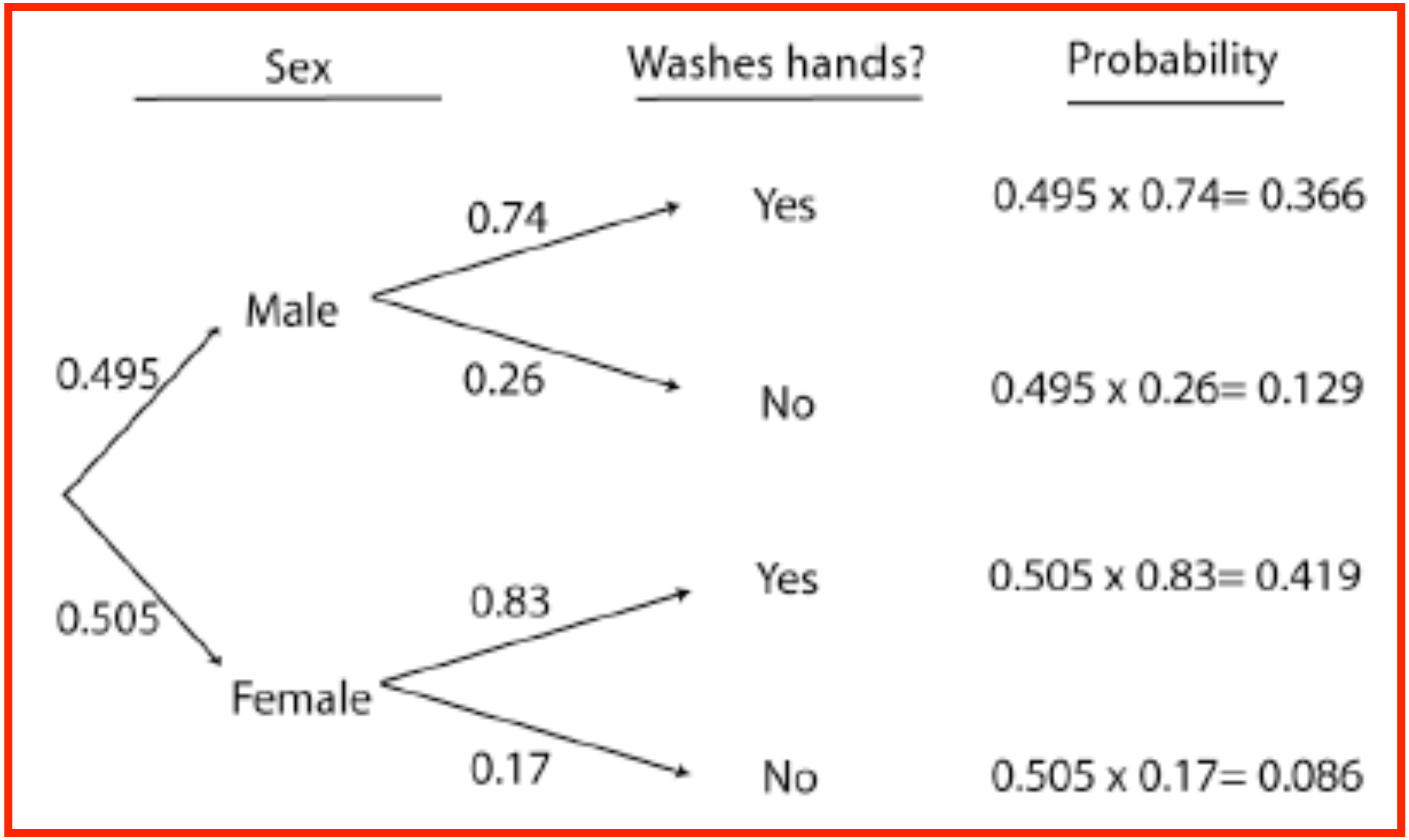
# Manipulating Probabilities

Example: Is washing your hands after using the washroom dependent on gender?

P[male] = 0.495

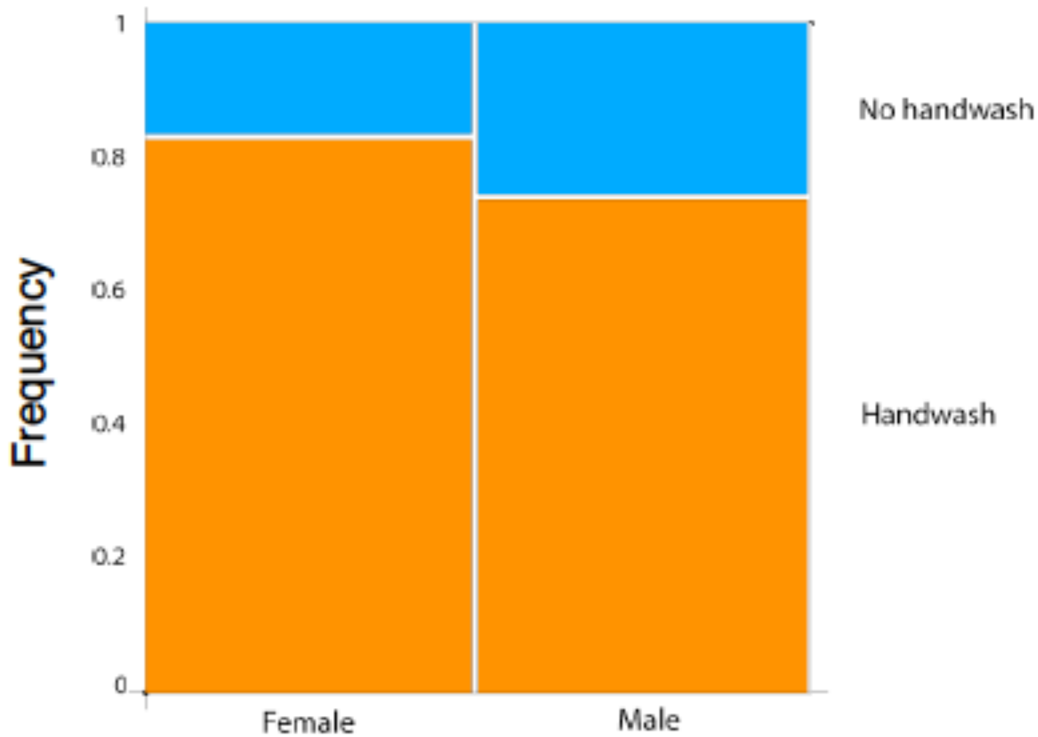
P[male washes his hands]=0.74

P[female washes her hands]=0.83



# Manipulating Probabilities

Are gender and hand washing independent variables?



## Conditional Probability:

*The probability that an event occurs given that a condition is met*

**P[X|Y] = P[X and Y]/P[Y]**

This is read as "the probability of X given Y"

It means: the probability of X if Y is true

Fancier way of writing the total probability of an event:

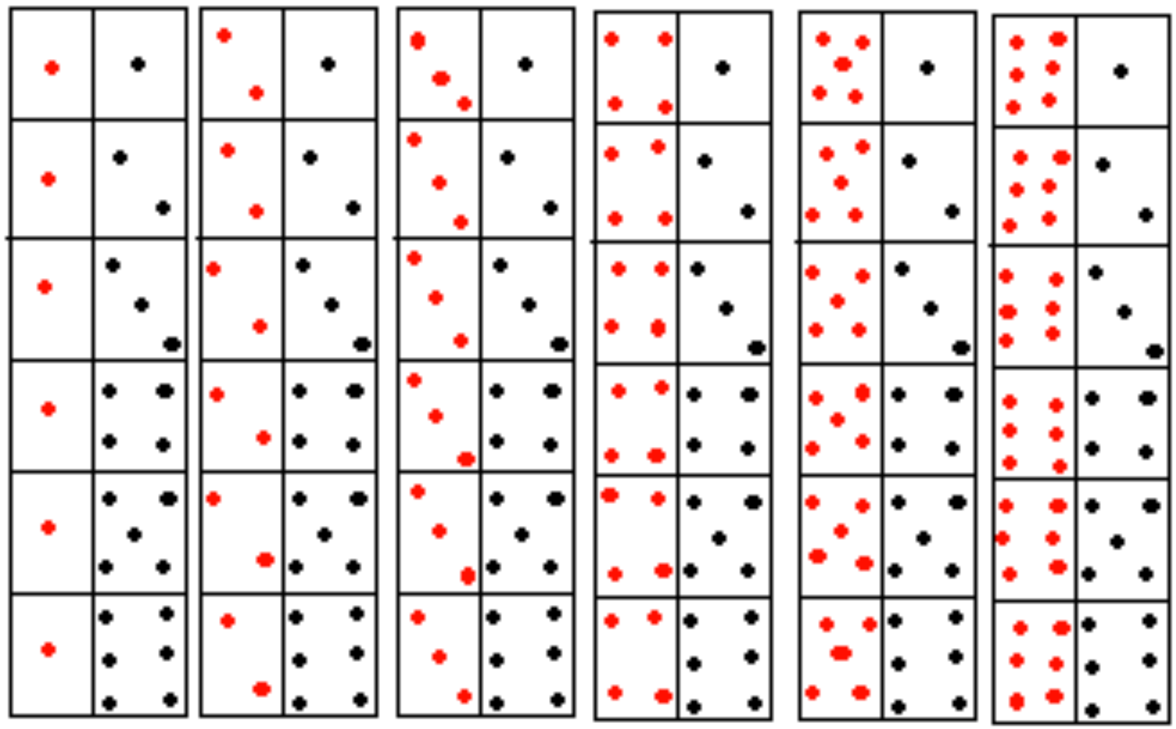
$$\boxed{P[X] = \sum_{Y} P[X \mid Y] P[Y]}$$

# Manipulating Probabilities

Example: Conditional Probability: **P[X|Y] = P[X and Y]/P[Y]**

Earlier: *What is the probability that two dice will sum to three?*

*-this is really asking **P[X and Y]** where **X=** red die 1 or 2 and **Y** = 1 or 2*



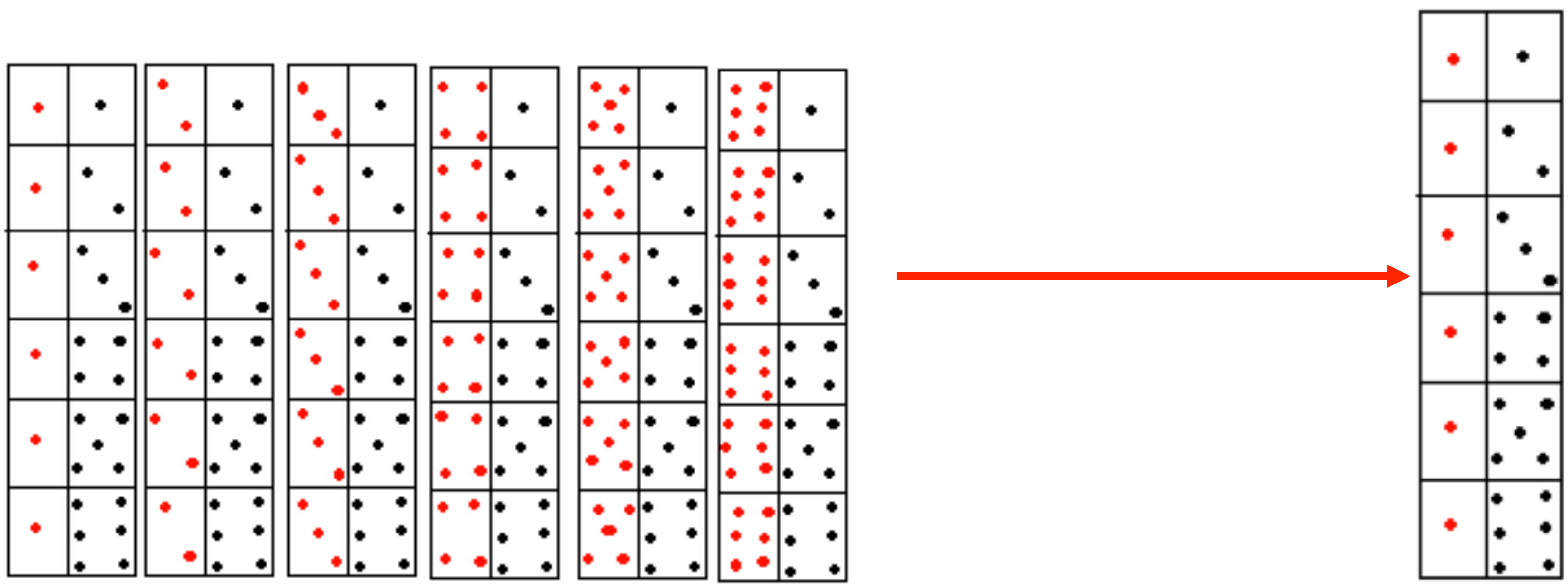
# Manipulating Probabilities

Example:

Earlier: *What is the probability that two dice will sum to three?*

Now: **what if we already have rolled the first die and know That we have a one? Event X=1**

Reduced state space, from 36 to 6:



*P[Sum to three] =2/36*

P[Sum to three]=1/6

# Manipulating Probabilities

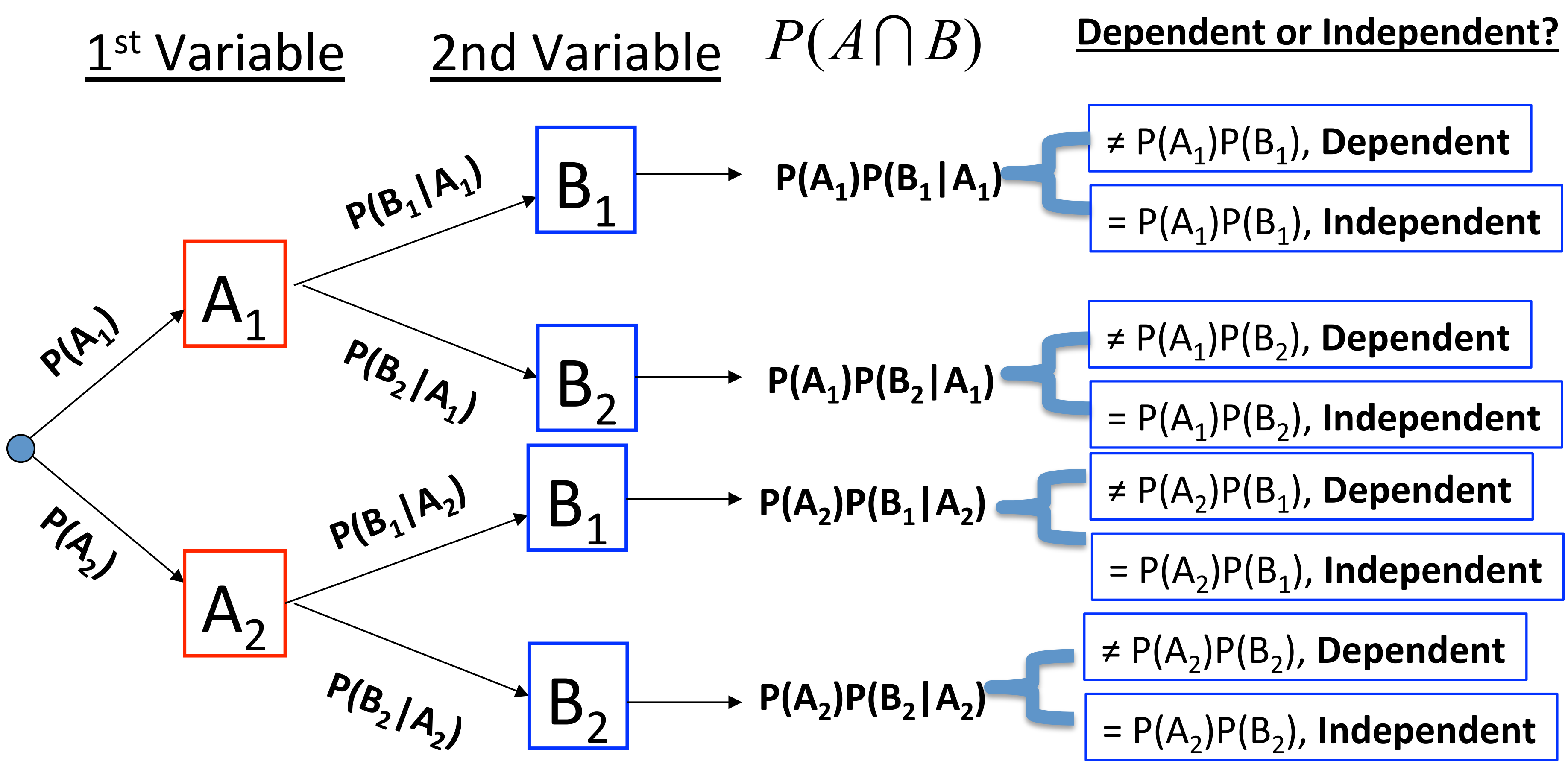
- **Very** important to understand conditional probability before we tackle Bayes'

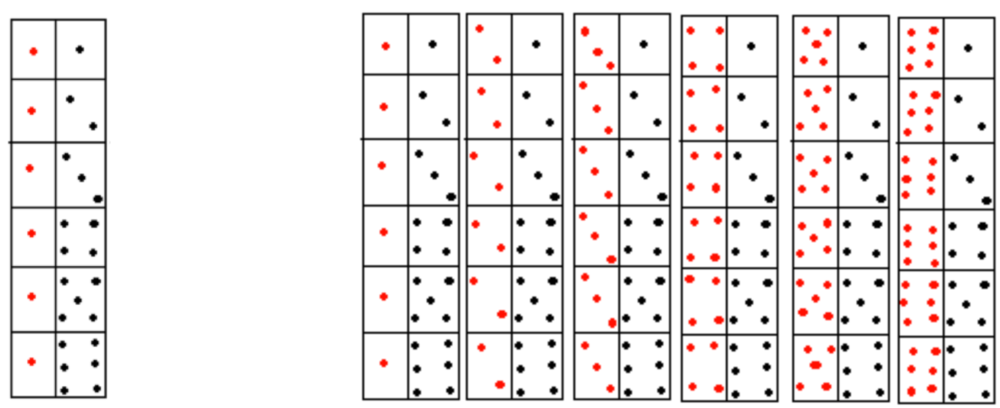
- Conditional probability can be a little confusing; sometimes using a Venn diagram with **3** events instead of 2 makes it clearer.

- Some students have struggled with the difference between $P(A \cap B)$ and P(A|B)

- **Remember:**

$$\boxed{P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)}$$



In this example, you will need to use the law of total probability to get **$P(B_1)$** and **$P(B_2)$**