Module 2 : Probability

Frequentist and Bayesian building blocks

Agenda:

- Bayesian Probability
  - Structure of Bayes' Theorem: $P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{P[B|A]P[A]}{P[B]}$
  - The Monty Hall Problem: illustrating the philosophical difference with Frequentist camp - ability to update probability with new information
  - Examples:
    - Pedigree Analysis

Bayes probability definition:

“Probability is a measure of belief associated with the occurrence of an event “

- Probability is subjective and can be **updated** when new information is available. Start with a **PRIOR** probability (or belief) and update it with a **LIKELIHOOD** and end with a **POSTERIOR** probability.

Ex. Will it rain tomorrow?

- **Frequentist Answer**: probability of rain in a place is sum of days of rain over time.
- **Bayesian Answer**: it will depend on whether it is Summer or Winter.

Probability:

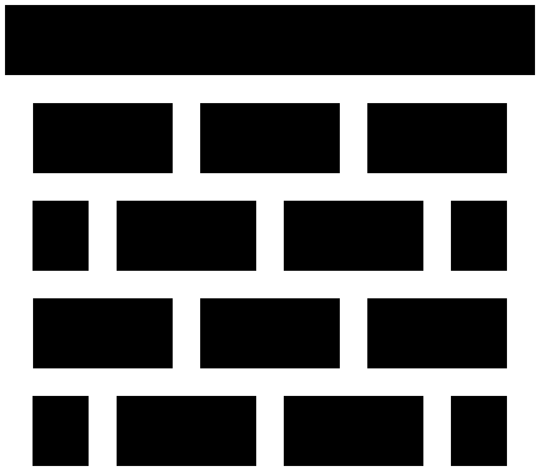
Two major splits in how probability is defined:

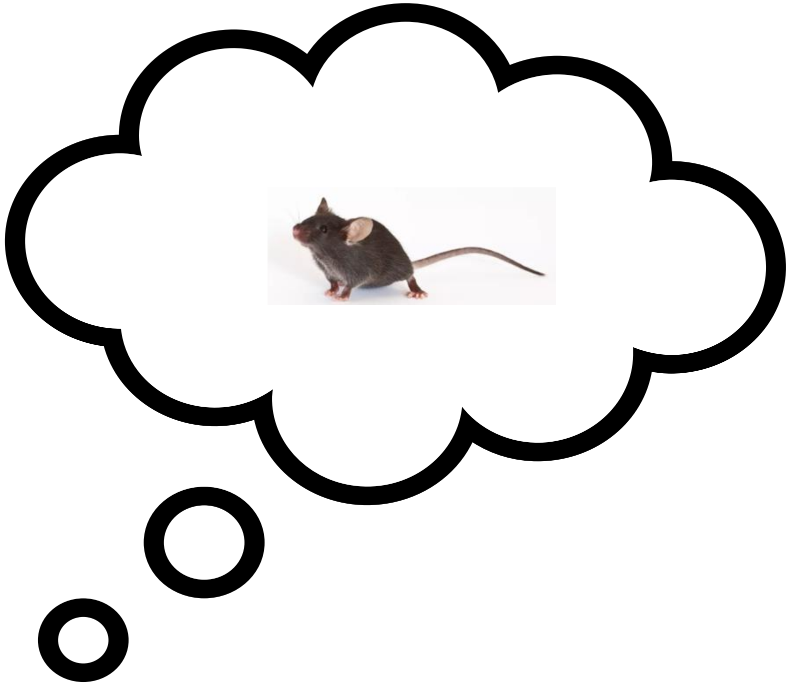
Frequency Interpretation:

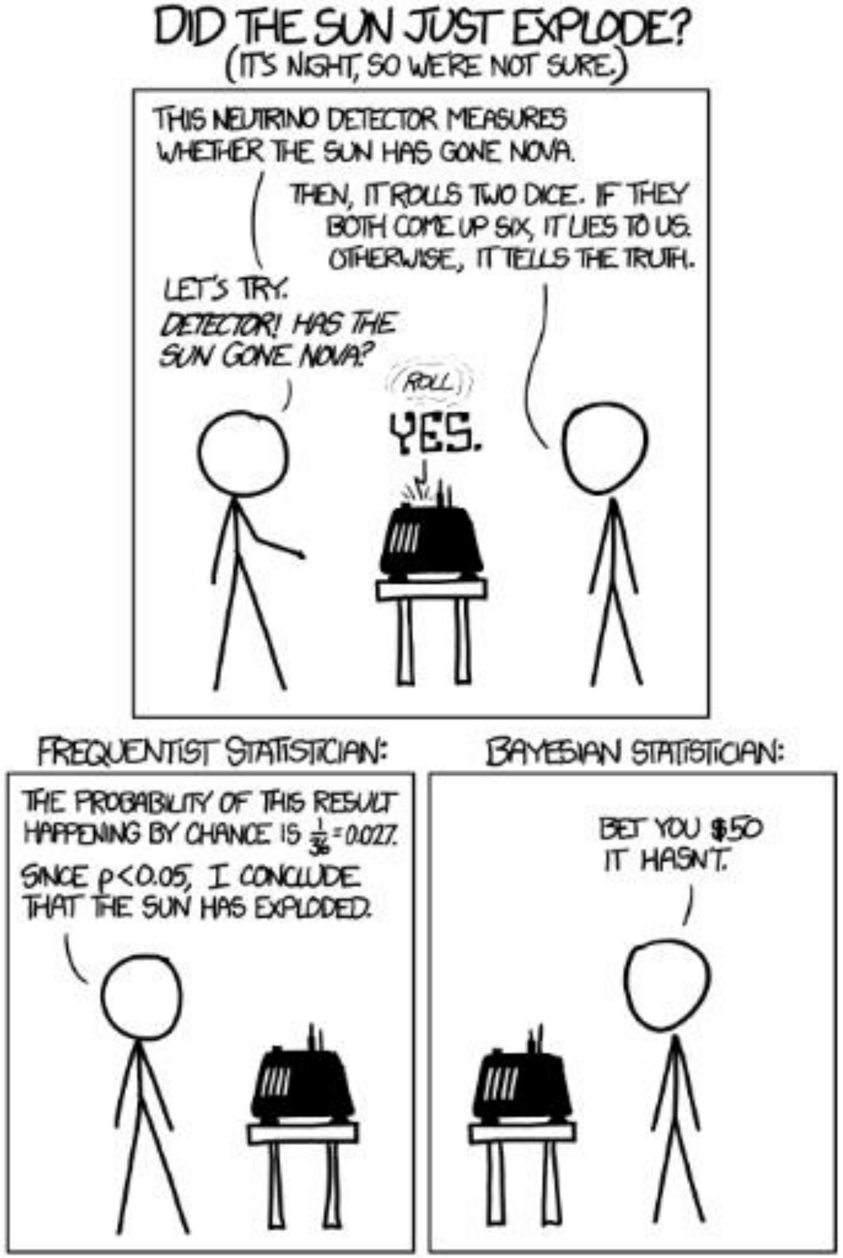
Frequency of a particular outcome <u>(an event)</u> across many random trials

Subjective (Bayesian) Interpretation:

Subjective belief or opinion of the chance that a particular outcome <u>(an event)</u> will be realized

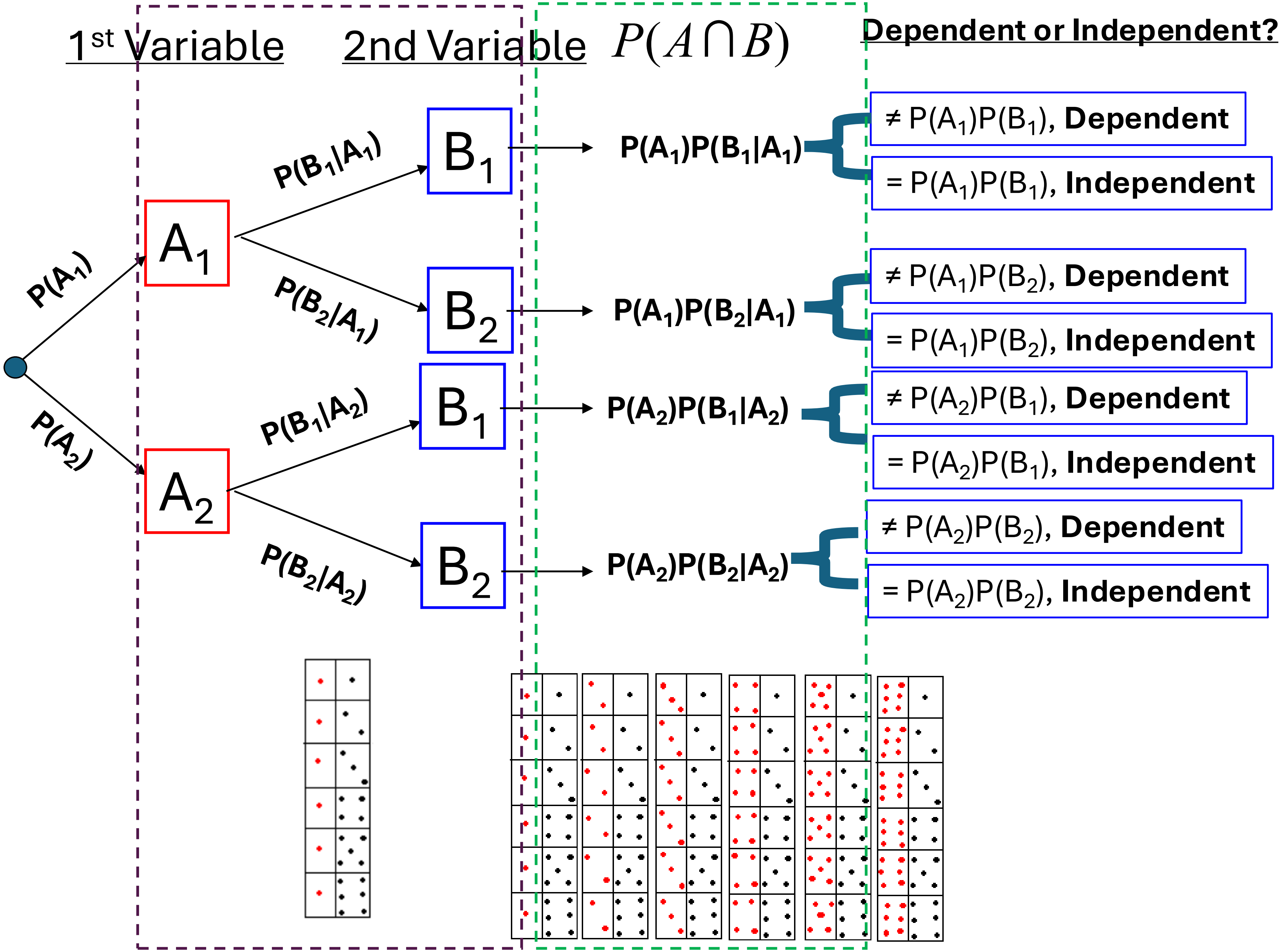






http://xkcd.com/1132/

- **Bayes inference is different from what we normally do**
- Probability is a way of quantifying uncertainty and assumes that with random sampling we can infer meaningfully from repeated experiments (**Frequentist)**
  - With enough information, and properly designed sampling, we can accurately estimate the **unknown but constant parameter value with increasing precision**
- Some phenomenon make sense under frequentist definition….
  - If we toss a fair coin, what is the probability of 10 heads in a row?
  - If we assign treatments randomly to subjects, what is the probability that a sample mean difference between treatments will be greater than 20%?
  - What is the probability that, given the null hypothesis is true, of obtaining data that is at least as extreme as that observed?
- Some phenomenon don't….
  - What is the probability that polar bears will be extinct in 30 years?
  - What is the probability that hippos are sister group to whales?
- There is no random trials inherent in certain meaningful questions (Hippos either ARE or ARE NOT sister groups; polar bears will either be EXTINCT or NOT in 30 years)
  - **The unknown parameter value is not a constant truth; it has inherent uncertainty** and no matter how much information you get, it will not have a constant value.
  - Probability is subjective and can be updated when new information is available. You start with a prior probability and update it with a likelihood and end with a posterior probability.


1st Variable
2nd Variable
$P(A \cap B)$
Dependent or Independent?
$P(A_1)$
$P(A_2)$
A_1
A_2
$P(B_1|A_1)$
$P(B_2|A_1)$
$P(B_1|A_2)$
$P(B_2|A_2)$
B_1
B_2
B_1
B_2
$P(A_1)P(B_1|A_1)$
$P(A_1)P(B_2|A_1)$
$P(A_2)P(B_1|A_2)$
$P(A_2)P(B_2|A_2)$
$\neq P(A_1)P(B_1)$, **Dependent**
$= P(A_1)P(B_1)$, **Independent**
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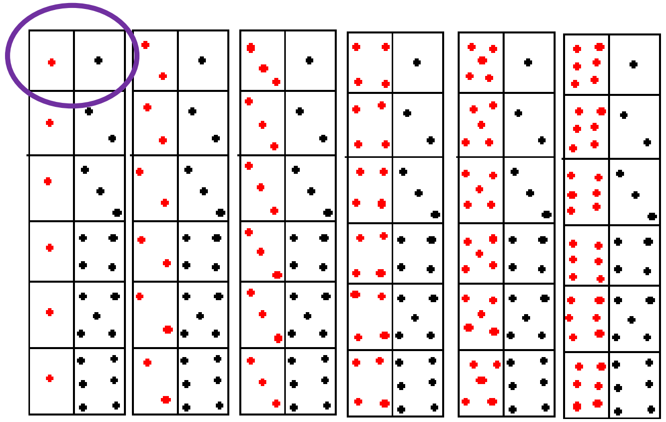
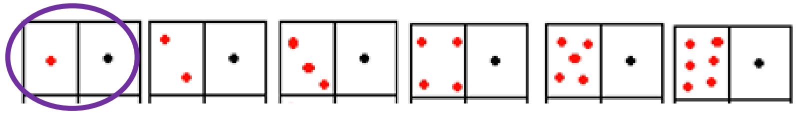
$= P(A_2)P(B_2)$, **Independent**

P(A∩B) Versus P(A|B)

$P(red=1 \cap black=1) = 1/36$



$P(red=1|black=1) = 1/6$



Let's warm up our brains with a bit of a riddle (it uses a similar 'updated knowledge' logic that the Monty Hall problem uses):

Inside of a dark closet are five hats: three blue and two yellow. Knowing this, three students go into the closet, and each selects a hat in the dark and places it unseen upon their head.

Once outside the closet, no one can see their own hat. The first student looks at the other two, thinks, and says, "I cannot tell what color my hat is." The second hears this, looks at the other two, and says, "I cannot tell what color my hat is either." The third student has an eye infection and temporarily can't see. The temporarily blind student says, "Well, I know what color my hat is." What color is their hat?

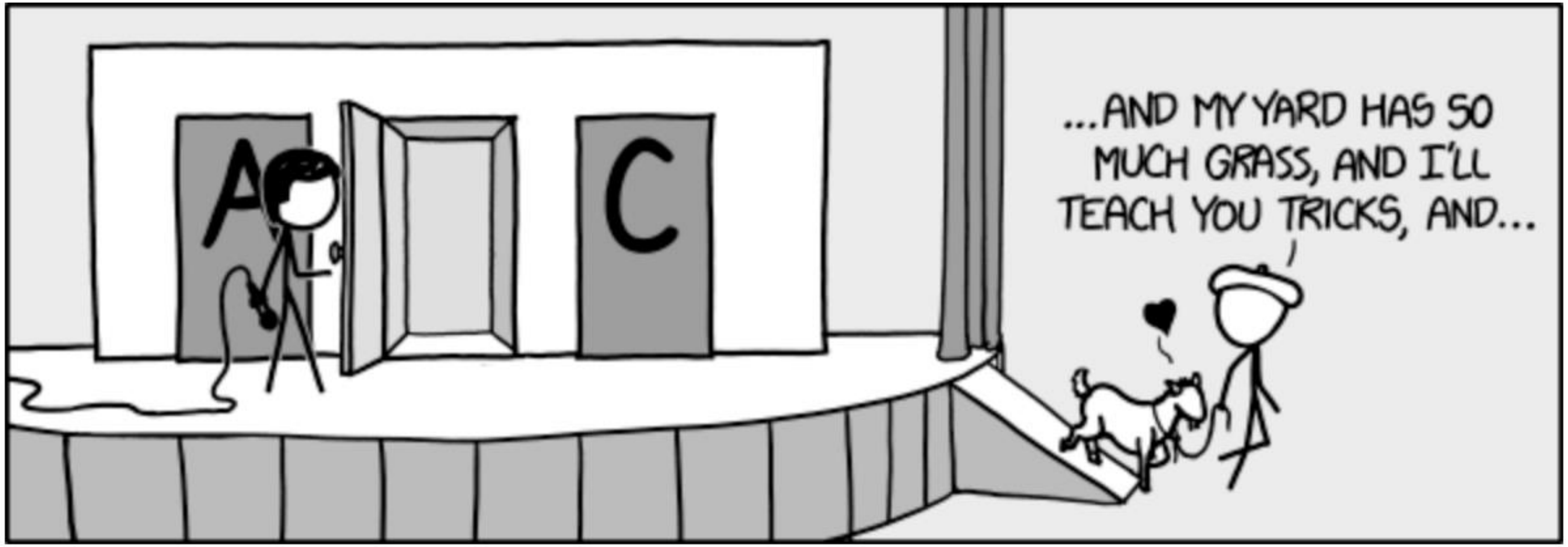
Monty Hall Problem

https://montyhall.io/

- **1,000,000. P(picking the car) = 1/1,000,000.**
- **Pick a door – two doors have goats, and one door has a car**
- **Monty Hall then opens one of the remaining doors and ask….**

“Do you want to switch your door or stay with the door your have chosen?”

Should you switch or stay?



https://xkcd.com/1282/

Good overview of Monty Hall problem:

- https://www.statisticshowto.com/probability-and-statistics/monty-hall-problem/

The TV show "MythBusters" have a short segment (first 15 minutes or so) on Monty Hall problem:

https://www.youtube.com/watch?v=oWWNZ_eciGI

You might also be interested in Bertrand's box paradox which is similar...