# Module 2:
# **Inference for a Normal Population**

Different flavours of t tests

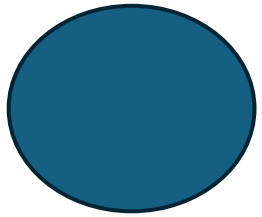
# Hypothesis testing for means using t tests Agenda

1. **Why do we use Student t-tests instead of Z scores?**
2. **What are the three types of t-tests**

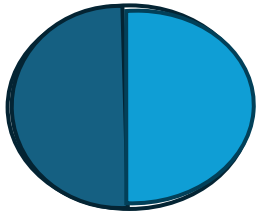
- **One sample t tests**
  - ❑ Assumptions
  - ❑ When assumptions not met, use median and rank → **Signed test**
- **Paired t test**
  - ❑ Assumptions
- **Two sample t test**
  - ❑ Assumptions
  - ❑ When variances aren't equal → **Welch's approximate t test**
  - ❑ Other assumptions not met: median and rank → **Mann Whitney U test**

# Part 2: **What** t tests? We will look at the following t-tests:

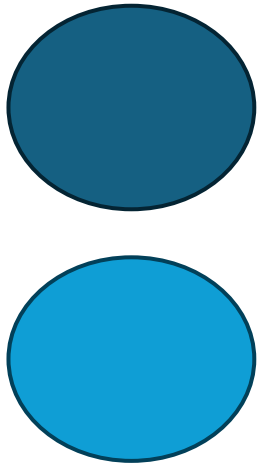
1. Comparing one mean:
   a. **One-sample t-test**
2. Comparing two means:
   **a. Paired t-test**
   **b. Two-sample t-test**



one sample



paired



two sample

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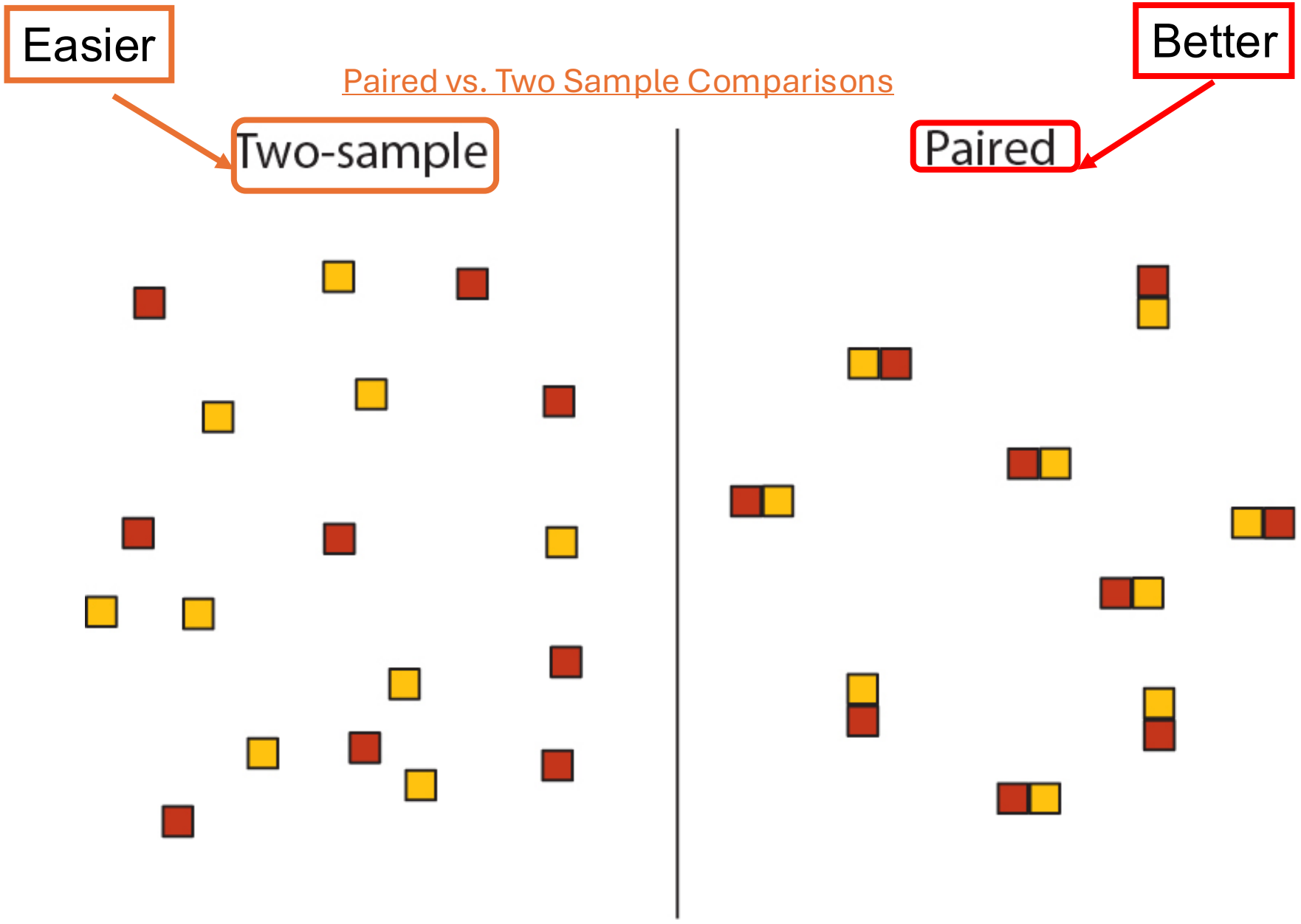
*Each of the above tests have **slightly different assumptions** which allow our conclusions to be supported. We will investigate what happens when these assumptions are violated and how robust our various t-tests are to violations.*

## Paired vs. Two Sample Comparisons



# Applications of one sample t-test

Researchers are studying the body weight of mice to understand the impact of a high-fat diet on genetically modified (GM) mice. They can collect the following data: body weight.

**One-Sample t-test:** Does the **body weight of the GM mice** differ significantly from a **known population mean weight** of non-GM mice?

**Two-Sample t-test:** Does the body weight of GM mice on a **high-fat diet** differ from the body weight of a GM mice on a **standard diet**?

**Paired t-test:** They measure the body weights of a group of GM mice **before** and **after** they are switched from a normal diet to a high fat diet to see if there's a significant change in weight within the same group.

Four general ways to address violations:

- Ignore
  - sometimes you can use a method even if assumptions are violated
  - especially true if sample sizes are large an violations are not extreme
- Transform
  - attempt to  force normality and other assumptions onto data
  - We will investigate various tools
  - work that does not always pay off
- Use Non-parameter method
  - classes of methods that do not require assumption of normality
  - not cost free! Often lose power etc.
- **Computationally Intensive Methods**
  - Simulation
  - Bootstrap
  - randomization/permutation test

## Welch's approximate t-test:

- it is used when comparing means of two populations that are normally distributed but have unequal variances

$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{4.8 - 0.51}{\sqrt{\frac{3.26^2}{10} + \frac{0.89^2}{10}}} = 4.01$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{\left(\frac{3.26^2}{10} + \frac{0.89^2}{10}\right)^2}{\frac{\left(\frac{3.26^2}{10}\right)^2}{9} + \frac{\left(\frac{0.89^2}{10}\right)^2}{9}} = 10.33$$

$$(\bar{Y}_1 - \bar{Y}_2) - t_{\alpha(2),df} SE_{\bar{Y}_1 - \bar{Y}_2} < \mu_1 - \mu_2 < (\bar{Y}_1 - \bar{Y}_2) + t_{\alpha(2),df} SE_{\bar{Y}_1 - \bar{Y}_2}$$

(you might notice it looks an awful lot like the other t statistic confidence intervals)

# Mann-Whitney U-Test

- Compares the central tendencies of two groups using ranks
  - uses ranks of measurements to test whether frequency distribution of two groups are the same
  - Small samples lead to little power
  - All group samples are random samples
  - Distribution of the variable has the same shape in every population

- Nonparametric version of two-sample t-test

# Continue to add to your flow chart!