Module 4D
Supervised Machine Learning

Different flavors of REGRESSION and General Linear Models

General linear model

- Linear Model for single-factor ANOVA
- Linear Regression

$$Y = \mu + A_i \qquad\qquad A_i = \text{group mean} - \mu$$

$$Y = \alpha + \beta X$$

You are fundamentally fitting the following model in both cases

RESPONSE = CONSTANT + VARIABLE

- Analysis of covariance
- Multiple regression

General linear models:

H_0: Treatment means are same
H_A: Treatment means are not all the same

Significance of a treatment variable is tested by comparing the fit of two models, H_0 and H_A, to the data by using **F-test**

$$\text{F-test} = \frac{H_A}{H_0} = \frac{\text{Constant + Variable}}{\text{Constant}}$$

Does the additional parameter, the variable, improve the fit of the data significantly?

- ANOVA table
- P-value leads to rejection or FTR H_0
- Assumptions are same (residual plots): random sample, normal distribution, Variance of response variable is the same for all combinations of the explanatory variables

GLM: just a curated taste (there are many more)**!**

Often appropriate/useful to investigate >1 explanatory variable simultaneously

Efficiency

Interactions

Three major approaches:

Blocking

- Improve detection of treatment effects
- If nuisance variable is known and controllable

Factorial experiment

- Investigate main effects of ≥ 2 treatment variables
- Interactions

Covariates

- Confounding variables
- Nuisance variable is known but uncontrollable

Multiple factor ANOVA:

- A factor is a categorical variable
- ANOVAs can be generalized to look > 1 categorical variable at a time
  - Same principles as one-way ANOVA
    - partitioning of variance
  - Same assumptions as one-way ANOVA
    - Equal variances
    - Equal sizes
- *Not only can we ask whether each categorical variable affects a numerical variable, but also do they* ***interact*** *in affecting the numerical variable*
  - The most important aspect of multi-factor ANOVA is that we can determine whether groups differ on some dependent variable while controlling for the effects of the other independent variables
- Similar to ANCOVA but ANCOVA is more general

One-way ANOVA:

- 1 continuous dependent variable
- 1 categorical independent variable (≥ 2 groups)
- i.e., **Girls vs boys** in hours of tv watched

Multi-Factor ANOVA:

- 1 continuous dependent variable
- ≥ 2 categorical independent variables
- i.e., **Girls vs boys** in hours of tv watched in **four regions** of the United States

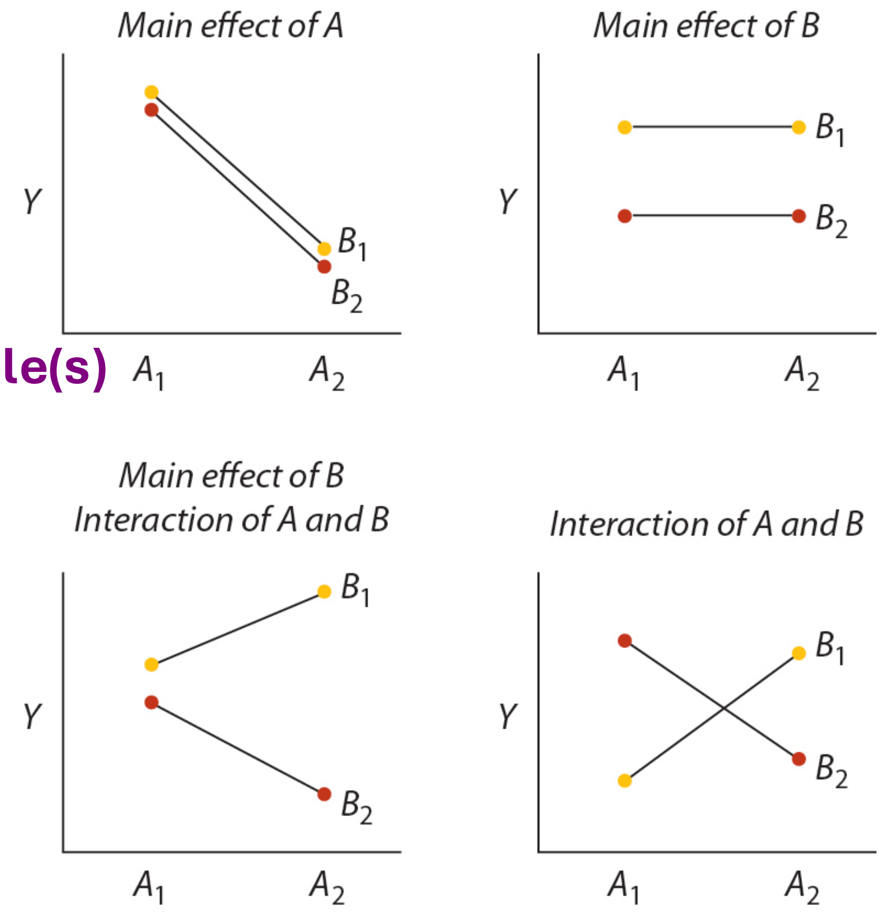
Multiple factor ANOVA:

- 1 dependent variable and ≥ 2 (independent) categorical variables
- Produces 2 interesting results:

1. **Main effects**

    - 1 F-value for each category
    - Like one-way ANOVA but with one glorious difference:

Control for (partial out the effects of) **the other independent variable(s)**



2. **Interaction effects**

Fixed Factorial Designs:

- Effects of factors (treatments) and their interactions on a response variable
  - All combinations of the two (or more) explanatory variables are investigated

- Fixed, repeatable factors

  Interaction term: if it equals 0, there is no interaction

  Main effects:
  - Factor 1 and Factor 2 since they represent the effects of that factor alone when averaged over the other factor, ie. Marginal values

  Response = Constant + Factor 1 + Factor 2 + Factor 1* Factor 2

- **F-test**
  - Contribution of each main effect and their interaction to the fit of the model to the data

Fixed Factorial Designs: Response = Constant + Factor 1 + Factor 2 + Factor 1* Factor 2

Three sets of null/alternate hypotheses to test:

1. H_0: **Main effect: Factor 1**

   $$\text{F-test} = \frac{\text{Constant + Factor 1 + Factor 2 + Factor 1*Factor 2}}{\text{Constant + Factor 2 + Factor 1*Factor 2}}$$

2. H_0: **Main effect: Factor 2**

   $$\text{F-test} = \frac{\text{Constant + Factor 1 + Factor 2 + Factor 1*Factor 2}}{\text{Constant + Factor 1 + Factor 1*Factor 2}}$$

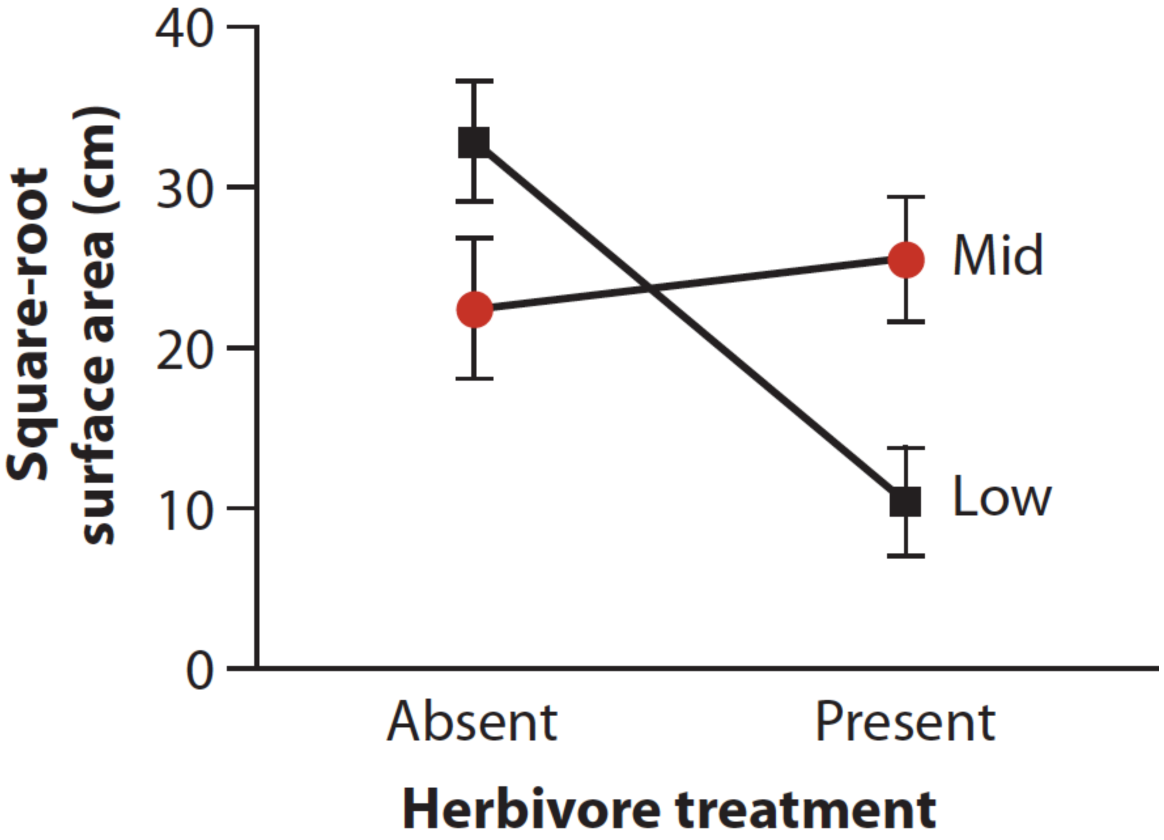
3. H_0:**Interaction effect: Factor 1*Factor 2**

   $$\text{F-test} = \frac{\text{Constant + Factor 1 + Factor 2 + Factor 1*Factor 2}}{\text{Constant + Factor 1 + Factor 2}}$$

Source of Variation	Sum of Squares	df	Mean Square	F	P
Factor 1					
Factor 2					
Interaction					
Residual					
Total					

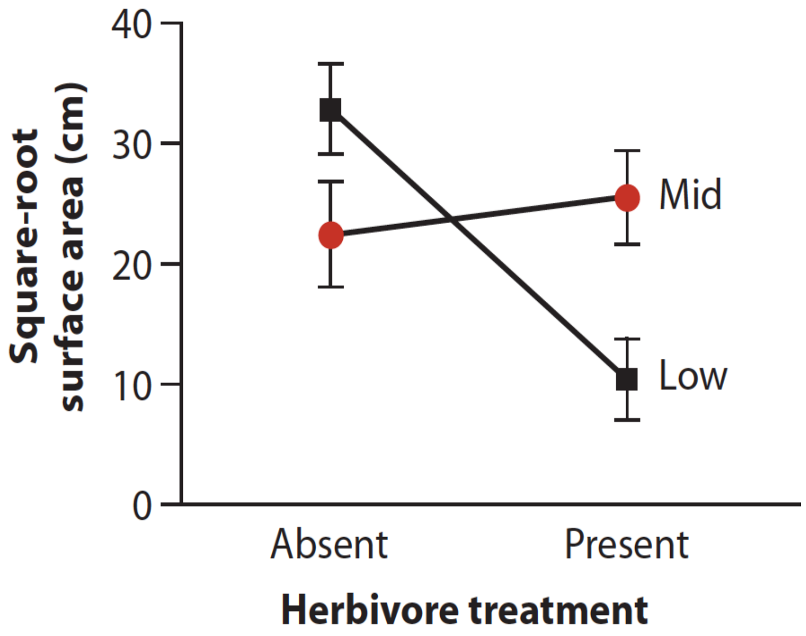
Multi- factor ANOVA

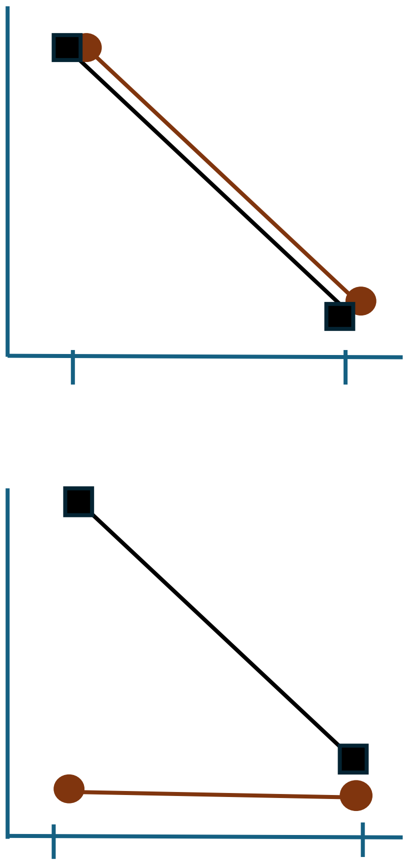
Example: Herbivores affect on red algae in an intertidal zone: exclusion and presence. Two locations variables, low tide mark and middle mark.



Multi- factor ANOVA

Example: Herbivores affect on red algae in an intertidal zone: exclusion and presence. Two locations variables, low tide mark and middle mark.





The other types of patterns that you might see on a multi-factor graph

Multi- factor ANOVA:

Testing three hypothesis pairs:

Herbivory (main effect):
H_0: **No difference between** herbivory treatments in mean algal cover
H_A: There is a difference between herbivory treatments in mean algal cover

Height (main effect):
H_0: **No difference** between height treatments in mean algal cover
H_A: There is a difference between height treatments in mean algal cover

Herbivory*Height (interaction effect):
H_0: The effect of herbivory on algal cover **does not** depend on height in the intertidal region
H_A: The effect of herbivory on algal cover **does** depend on height in the intertidal region

Source of Variation	SS	DF	MS	F	P
Herbivory	**1512.18**	1	**1512.18**	**6.36**	**0.014**
Height	**88.97**	1	**88.97**	**0.37**	**0.543**
Herbivory*Height	**2616.96**	1	**2616.96**	**11.00**	**0.002**
Residual	**14270.52**	60	**237.842**		
Total	**18488.63**	63			

Source of Variation	SS	DF	MS	F	P
herbivores	1512.18	1	1512.18	5.5227	0.02197
Residuals	16976.5	62	273.81		

Source of Variation	SS	DF	MS	F	P
height	88.973	1	88.973	0.2998	0.586
Residuals	18400	62	296.769		

Three F ratios in the table; two of them are significant.

No interaction between height and herbivory is rejected

No effect of herbivory is rejected