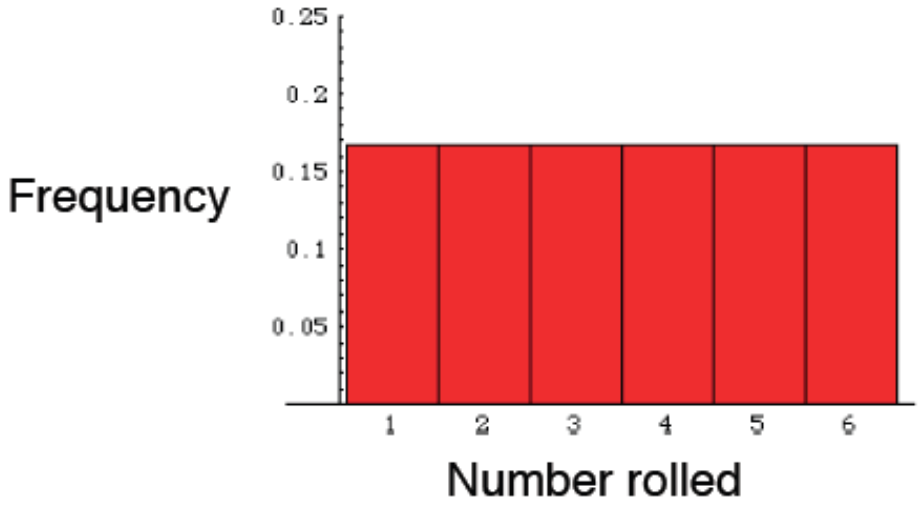
Module 2 : Probability

Frequentist and Bayesian building blocks

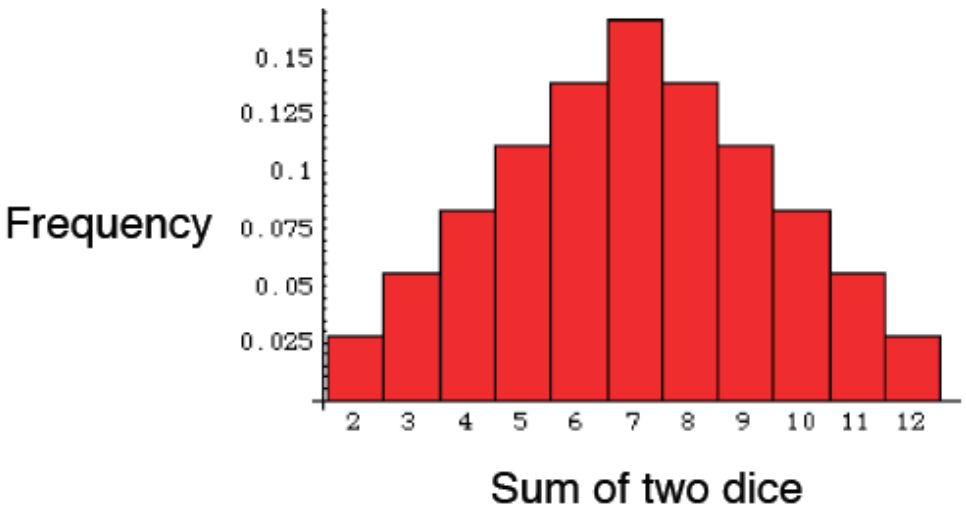
Agenda:

- Bayesian Probability
  - Structure of Bayes' Theorem: $P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{P[B|A]P[A]}{P[B]}$
  - The Monty Hall Problem: illustrating the philosophical difference with Frequentist camp - ability to update probability with new information
  - Examples:
    - Pedigree Analysis

Probability distribution for the outcome of a roll of one die:

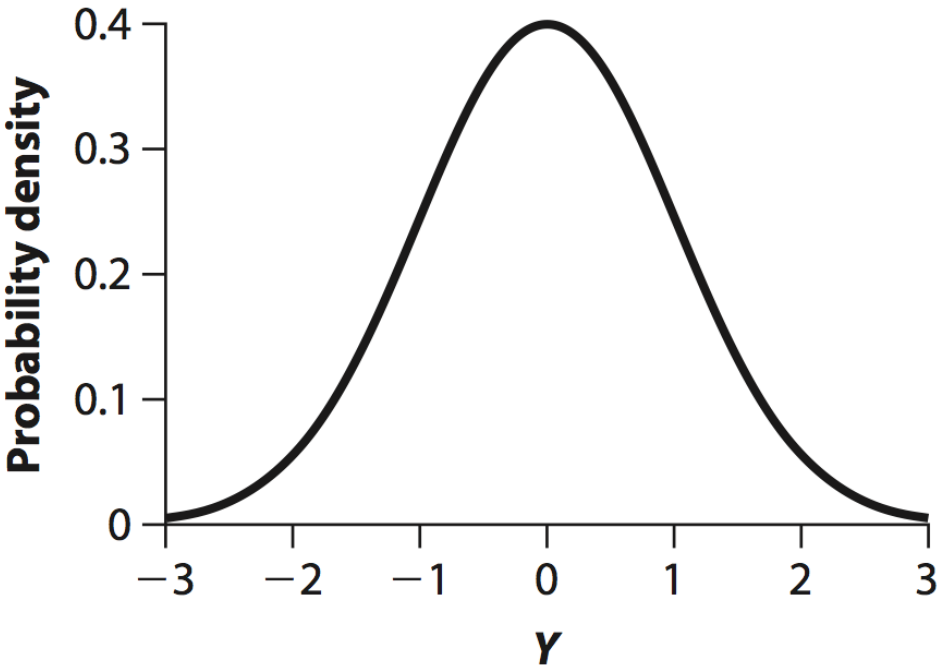


Probability distribution for the sum of a roll of two dice:

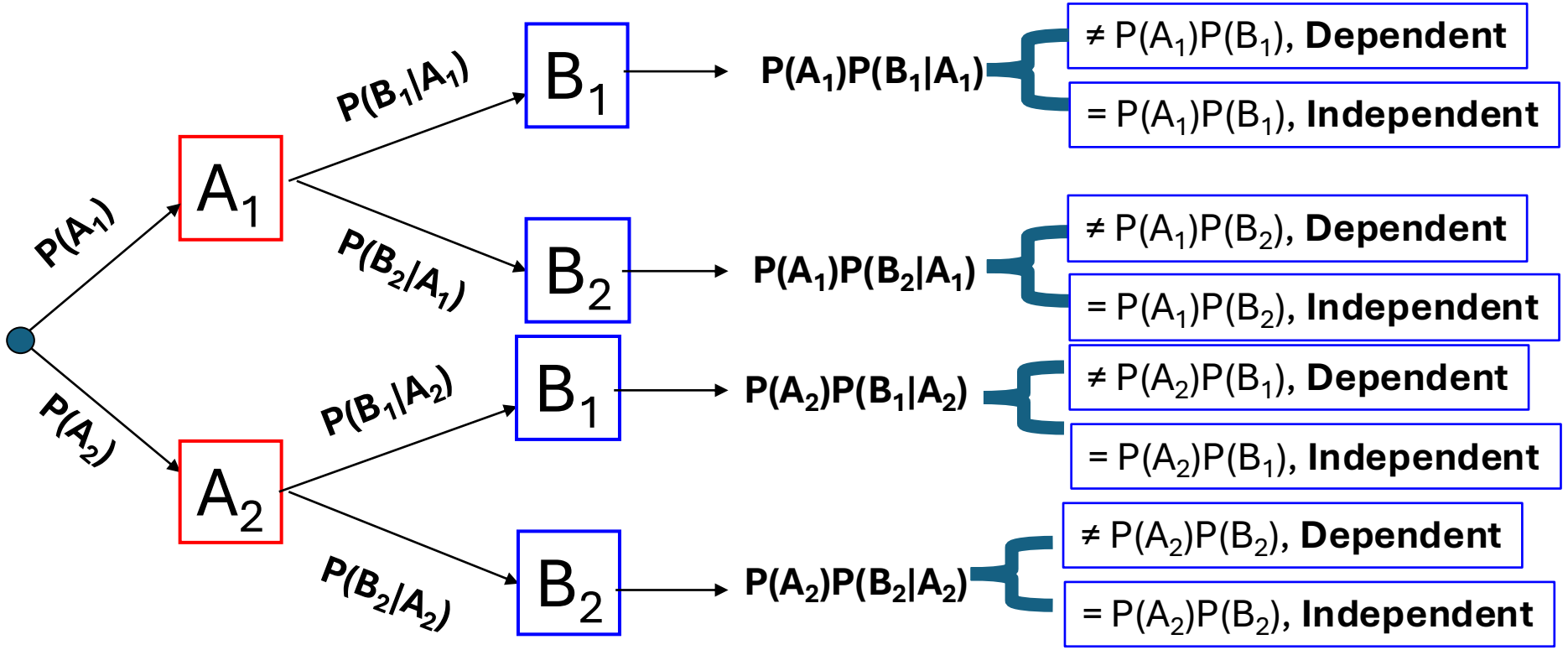


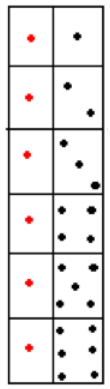
Probabilities → Frequency Distributions → Probability Curves

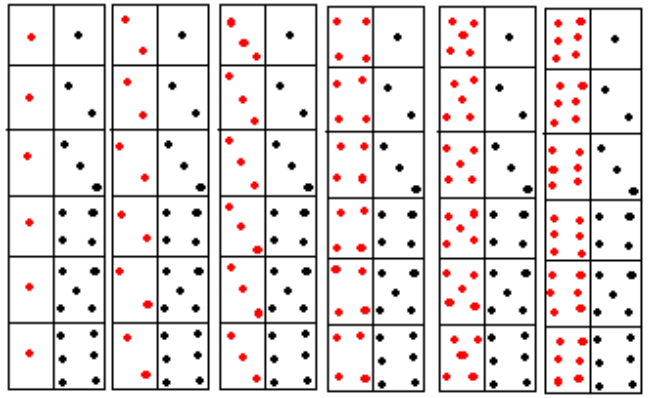
- Probabilities are the foundations for our models of how the world works
- We need distributions to conduct statistical inference



1st Variable 2nd Variable $P(A \cap B)$ **Dependent or Independent?**







$$P[A \mid B] = \frac{P[A]P[B \mid A]}{P[B]}$$

You can interpret Bayes as reducing the state space, like the following equations which use the proportion of A intersecting with B over the WHOLE universe (i.e., black die and red die both equal 1 is 1/36) and then reduce the proportion by dividing by the probability of the first event

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} \qquad P[B \mid A] = \frac{P[A \cap B]}{P[A]}$$

You might notice that both equations involve the SAME **numerator** whereas the **denominator** changes based on what event has happened <u>first</u> i.e., what we already know and what we still want to know.

Sophisticated rearrangement of the multiplication rule.....

Extra resources for understanding Bayes'

- Possibly *the best* websites to help you develop intuition about Bayes' and it gives you multiple options for how sophisticated you want your examples to be:

https://arbital.com/p/bayes_rule/?l=1zq

- Count Bayesie has also put together a guide to Bayesian statistics

which includes some useful resources if you are struggling with some of the concepts that we have covered so far in lecture:

https://www.countbayesie.com/blog/2016/5/1/a-guide-to-bayesian-statistics

- There is an interesting video about using Bayesian inference to search for sunken treasure:

http://fivethirtyeight.com/features/how-data-nerds-found-a-131-year-old-sunken-treasure/

(it has also been used to find "black boxes" of airplanes that have crashed)