Module 3A : Thinking in Distributions

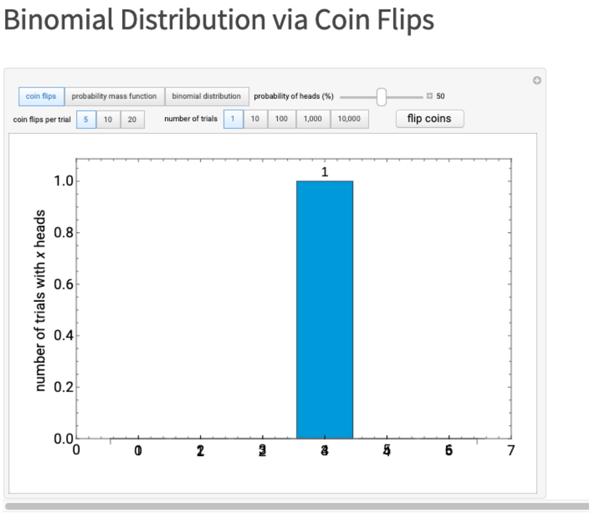
Building block for Hypothesis Testing

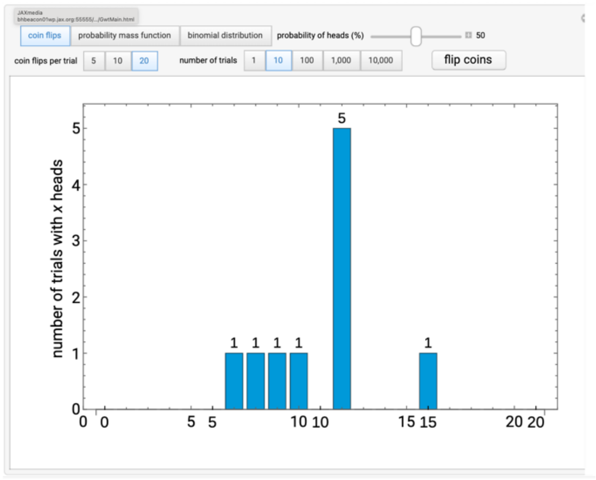
Agenda:

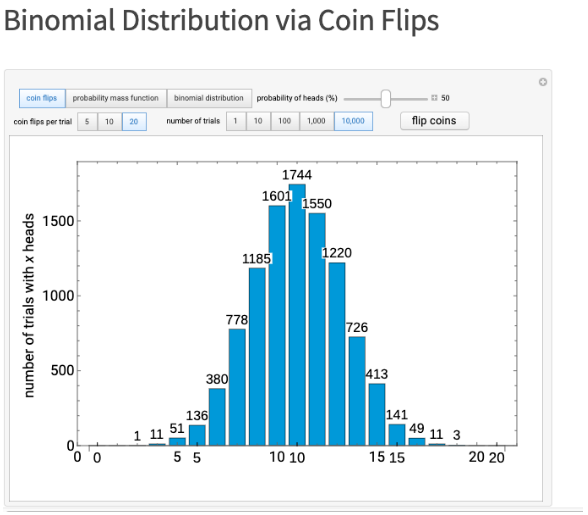
- Major distributions:
  - **Discrete Distributions**
    - Bernoulli
    - Binomial
    - Poisson
    - Hypergeometric
  - **Continuous Distributions**
    - Normal
    - Uniform
    - Exponential
    - Gamma
- Interactive simulations
- **Central Limit Theorem**
  - **Sampling Distribution of the mean**

Probabilities build up to distributions

Gives information about location and spread of the data





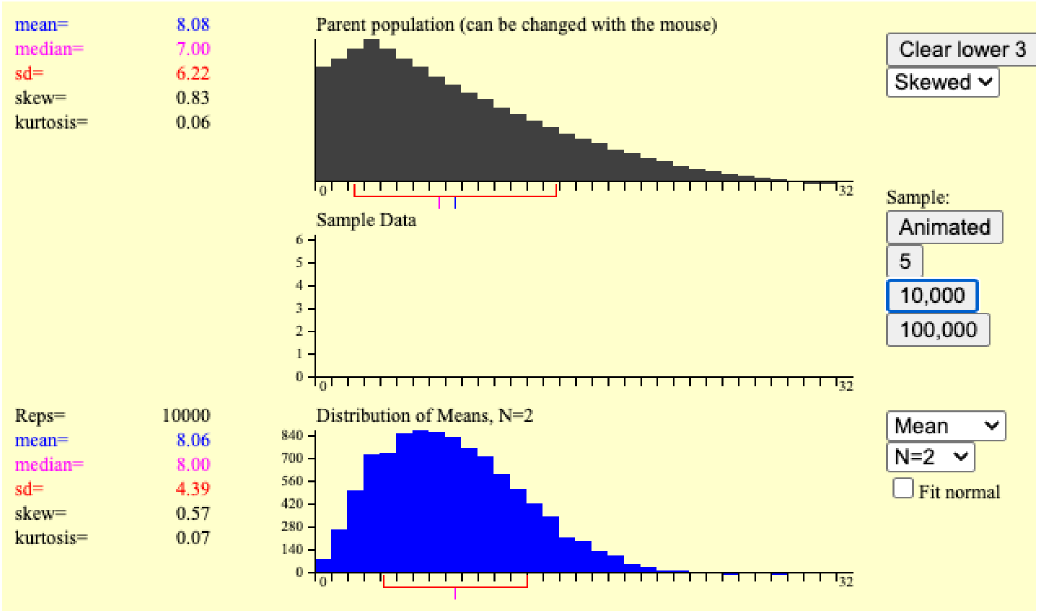


From Probability to Distributions: how randomness becomes predictable

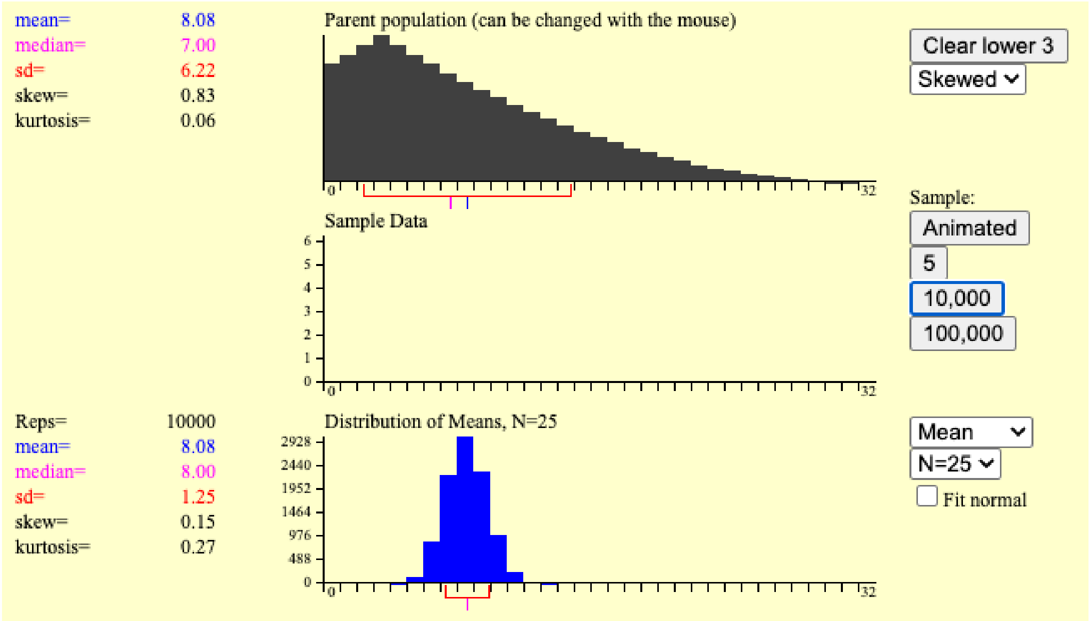
https://demonstrations.wolfram.com/BinomialDistributionViaCoinFlips/

Central Limit Theorem

- CLT allows us to assume that any sampling distribution of the mean is normally distributed…. **Even if the random variables are from a highly skewed distribution** (you will need to increase n if you are sampling from a highly non-normal distribution)
  - https://onlinestatbook.com/stat_sim/sampling_dist/



- sampling n=2, 10000 times



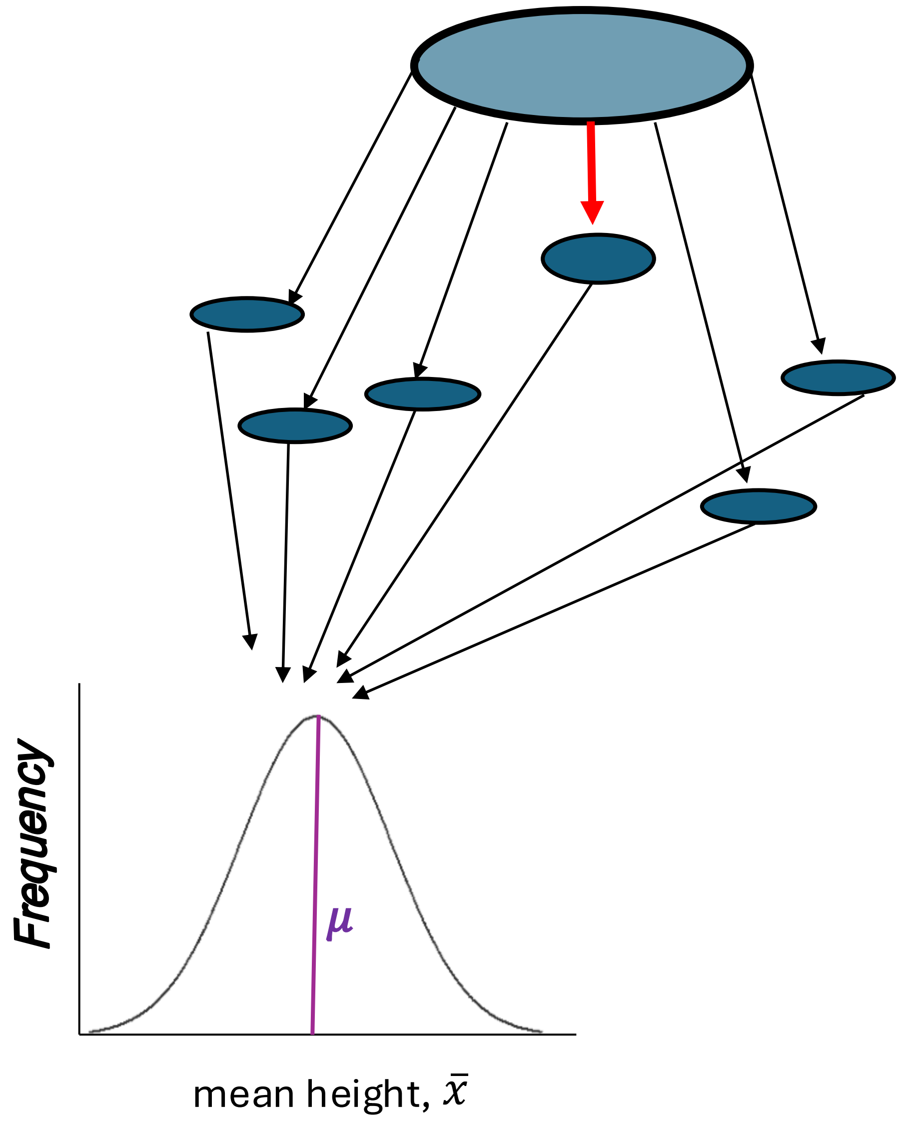
- sampling n=25, 10,000 times

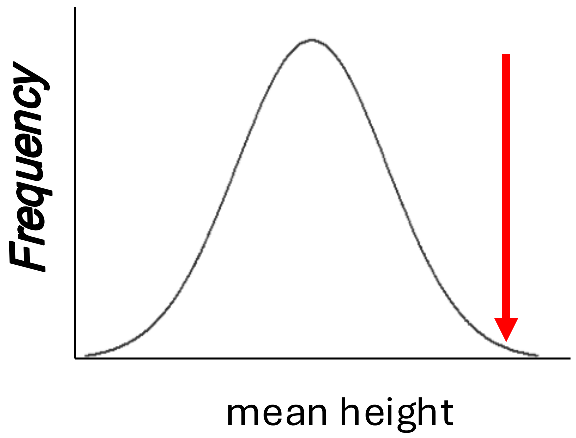
both from Skewed distributed Variable, but have different $SE_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

- We want to know something about this population
- We can't measure everyone, so we take a sample

How 'good' is the sample?

We imagine taking an infinite number of samples from the ***null distribution***





Module 3 : Hypothesis Testing

Applied Epistemology: A Framework for how we know things scientifically

<u>Agenda:</u>

1. H_O/H_A: Our model of the test universe (the distribution of the variable)
2. Test & assumptions: are the assumptions met? Is the test valid?
3. Quantitative evidence: **p-value**, or critical value.
   - False positive =Type I (α), False Negative = Type II (β), Type III errors
   - Sensitivity, Specificity, Power → confusion matrix, ROC/AUC curve
   - Positive Predictive Power, Negative Predictive Power
   - Confusion Matrix
   - **ROC/AUC curve**
4. Conclusion & uncertainty/estimation

Your pipeline for hypothesis testing in statistics

Step 1

Formulate your null **hypothesis**

- Null hypothesis is ***only*** *hypothesis that is tested*
- Falsification: *want* to reject your null

Step 2

Identify appropriate **test statistic**

- Assumptions of your test

Step 3

Quantify the results of your test

- **P value** or comparison to **critical values**
- How *unusual* is your data?

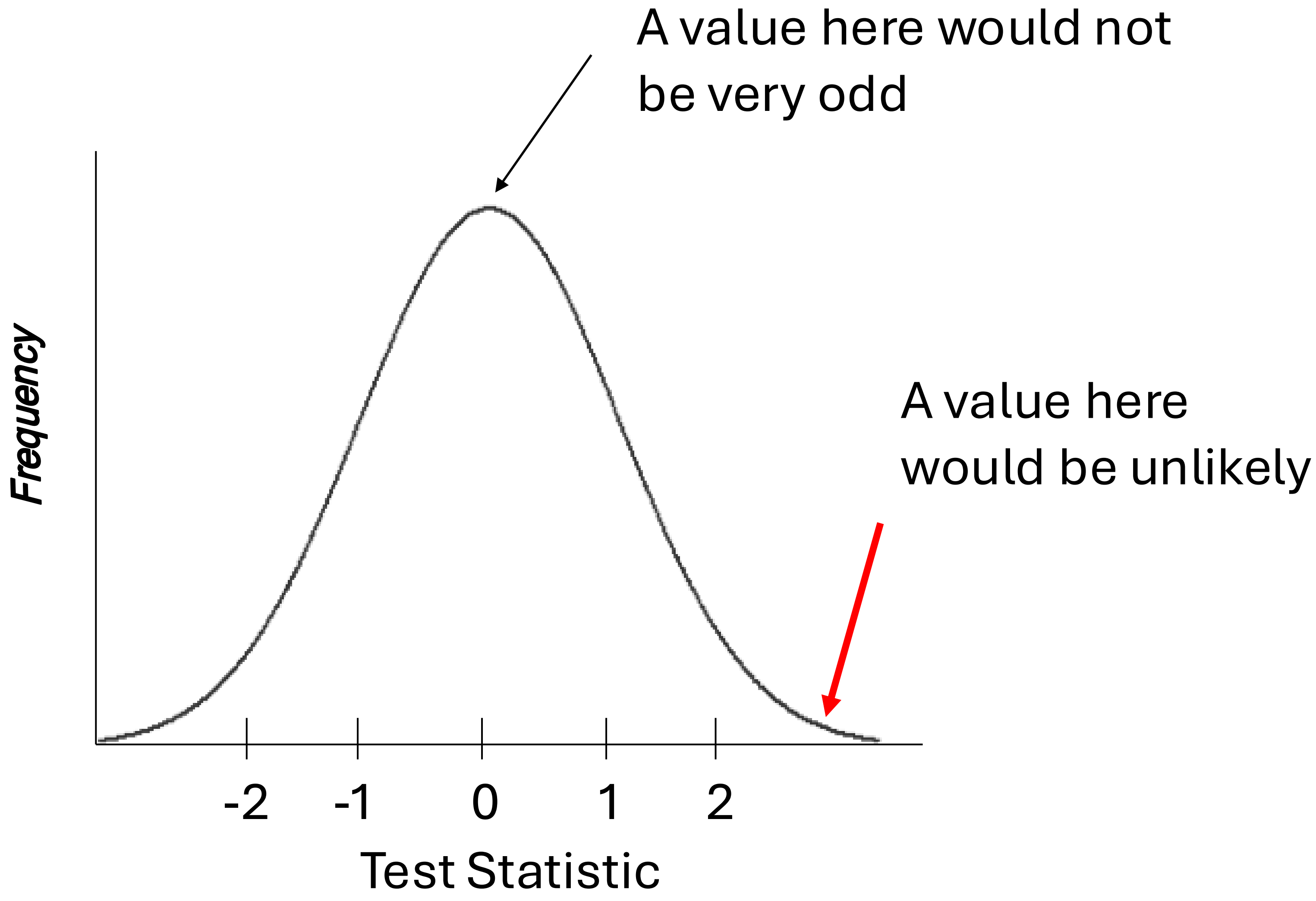
Step 4

Conclude: reject or fail to reject

- based on alpha value
- if appropriate, confidence interval of the parameter

P-Value:

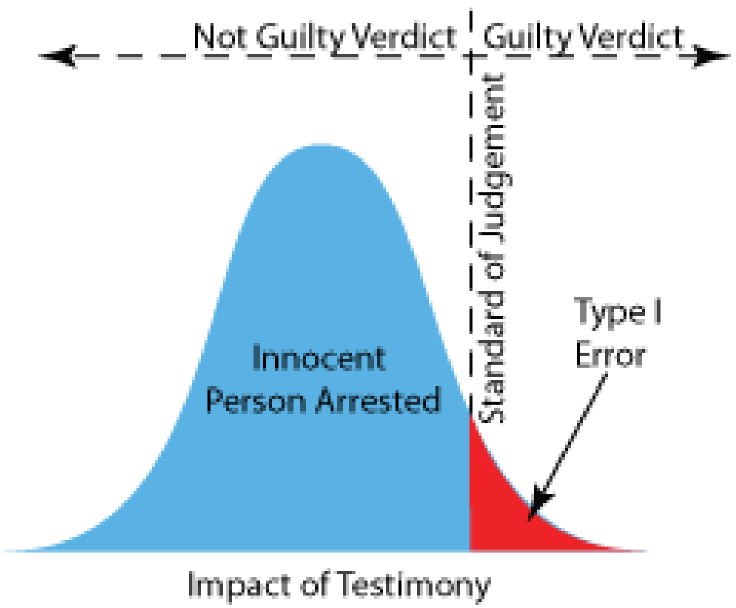
Probability of obtaining data that are <u>equal to or even more extreme</u> than the value assuming the null hypothesis is true



Type I (α) error:

Rejecting a true null hypothesis

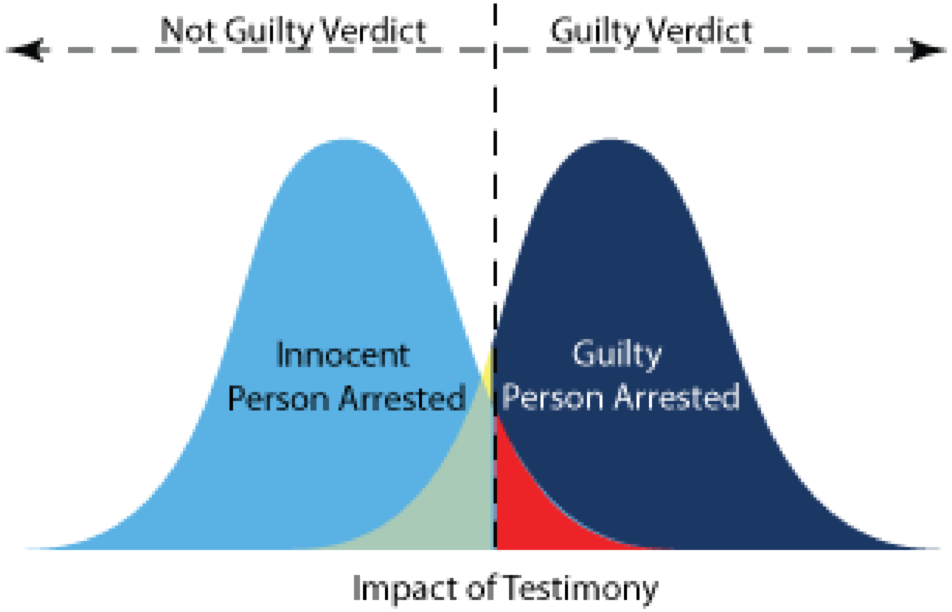
$$P(\text{reject } H_0|H_0 = \text{true}) = \alpha$$



Type II (β) error:

Not rejecting a false null hypothesis

$$P(\text{Fail to reject } H_0|H_0 \text{ is not true}) = \beta$$



http://www.intuitor.com/statistics/T1T2Errors.html

	No Disease (H_0 true)	Disease (H_A true)
Fail To Reject H_0	No Error (specificity*)	Type II
Reject H_0	Type I	No Error (power, sensitivity*)

Definitions:

* (This is a rate) **Specificity** = P[FTR|Ho is True]/P[Total Ho is true] = True Negative Rate

α=type I= P[Reject|Ho is True] = False Positive

β=type II error = P[FTR|Ho is **not** True] = False Negative

Power= P[Reject|Ho is **not** True] = True Positive

* (This is a rate) **Sensitivity** = P[Reject|Ho is **not** True]/P[Total Ho is NOT True] = True Positive Rate

Power can be increased by increasing the sample size (n)