Module 2:

Inference for a Normal Population

Different flavours of t tests

Hypothesis testing for means using t tests Agenda

1. **Why do we use Student t-tests instead of Z scores?**

2. **What are the three types of t-tests**

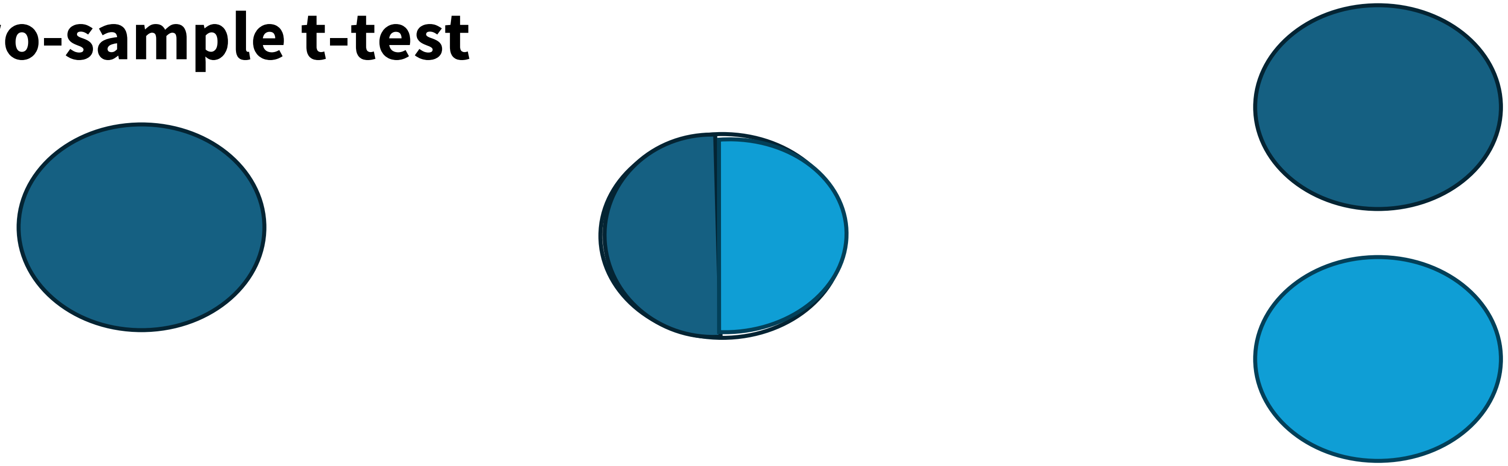
- **One sample t tests**
  - Assumptions
  - When assumptions not met, use median and rank → **Signed test**
- **Paired t test**
  - Assumptions
- **Two sample t test**
  - Assumptions
  - When variances aren't equal → **Welch's approximate t test**
  - Other assumptions not met: median and rank → **Mann Whitney U test**

We won't have time to cover everything in detail – nor every example I give - so here is another reference that outlines the different t tests:

https://statisticsbyjim.com/hypothesis-testing/t-tests-1-sample-2-sample-paired-t-tests/

Part 2: **What** t tests? We will look at the following t-tests:

1. Comparing one mean:
   a. **One-sample t-test**
2. Comparing two means:
   a. Paired t-test
   b. Two-sample t-test



one sample paired two sample

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*Each of the above tests have **slightly different assumptions** which allow our conclusions to be supported. We will investigate what happens when these assumptions are violated and how robust our various t-tests are to violations.*

Researchers are studying the body weight of mice to understand the impact of a high-fat diet on genetically modified (GM) mice. They can collect the following data: body weight.

One-Sample t-test: Does the **body weight of the GM mice** differ significantly from a **known population mean weight** of non-GM mice?

Two-Sample t-test: Does the body weight of GM mice on a **high-fat diet** differ from the body weight of a GM mice on a **standard diet**?

Paired t-test: They measure the body weights of a group of GM mice **before** and **after** they are switched from a normal diet to a high fat diet to see if there's a significant change in weight within the same group.

Example: What is the **95% confidence interval** for the mean whisker twitching rate of sedated mice. The rate of twitching (hz) from 8 mice:

0.9, 1.4, 1.2, 1.2, 1.3, 2.0, 1.4, 1.6

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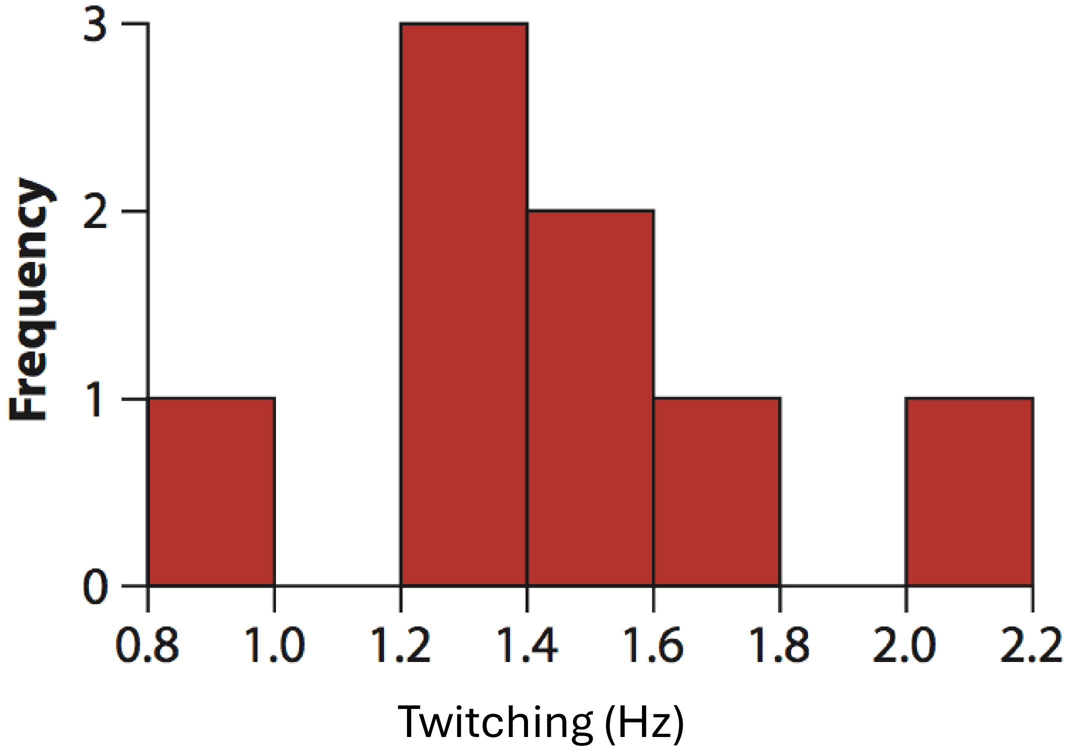


$$\bar{Y} = 1.375 \qquad dof = n - 1 = 7$$

$$s = 0.324$$

$$n = 8$$

$$t_{\alpha(2),df} = t_{0.05(2),7} = 2.36$$

Inference for a Normal Population

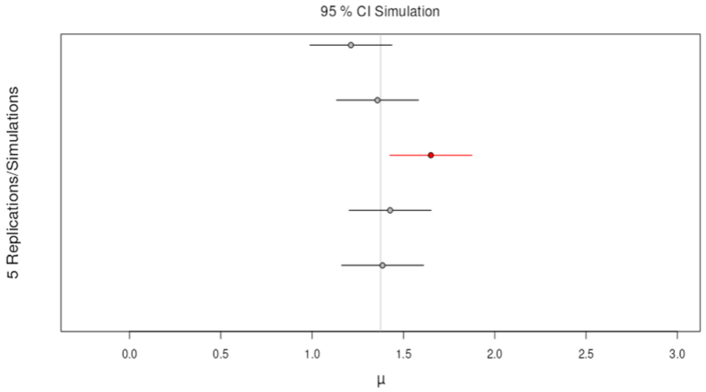
Example: What is the **95% confidence interval** for the mean whisker twitching rate of sedated mice. The rate of twitching (hz) from 8 mice:

0.9,1.4,1.2,1.2,1.3,2.0,1.4,1.6

Answer:

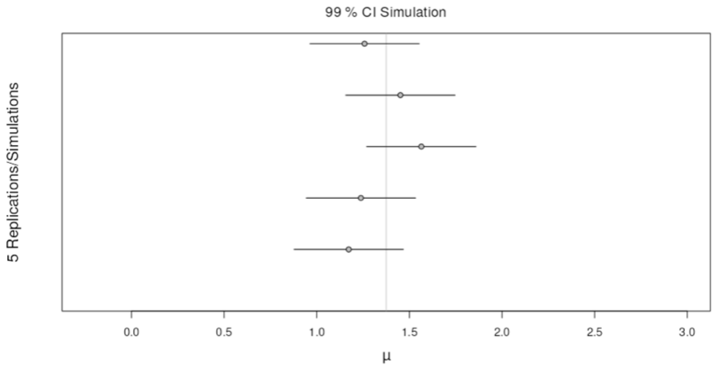
$$\bar{Y} \pm t_{0.05(2),7} SE_{\bar{Y}} = 1.375 \pm 0.115(2.36)$$

$$\boxed{1.10 < \mu < 1.65 \text{ (95\% Confidence Interval)}}$$



$$\bar{Y} \pm t_{0.01(2),7} SE_{\bar{Y}} = 1.375 \pm 0.115(3.50)$$

$$\boxed{0.97 < \mu < 1.78 \text{ (99\% Confidence Interval)}}$$



https://shiny.rit.albany.edu/stat/confidence/

The one-sample t test:

Compares the mean of a random sample from a normal population with the population mean proposed in a null hypothesis

H_0: True mean equals μ_0
H_A: True mean *does not* equal μ_0

Assumptions:

- The variable is normally distributed
- The sample is a random sample

One Sample t-Test

H_0: True mean equals μ_0
H_A: True mean *does not* equal μ_0

Test Statistic:

$$\boxed{t = \frac{\bar{Y} - \mu_0}{SE_{\bar{Y}}} = \frac{\bar{Y} - \mu_0}{s / \sqrt{n}}}$$

One Sample t Test

Example: What is the **95% confidence interval** for the mean whisker twitching rate of sedated mice. The rate of twitching (hz) from 8 mice: **0.9,1.4,1.2,1.2,1.3,2.0,1.4,1.6**

Moderate sedation (using Isoflurane) cause C57BL mice to have 0.5 Hz whisker twitching. Are these mice moderately sedated?

Step 1:

H_0: These mice are moderately sedated, $\mu = 0.5$

H_A: These mice are not moderately sedated, $\mu \neq 0.5$

One Sample t Test

Example: What is the **95% confidence interval** for the mean whisker twitching rate of sedated mice. The rate of twitching (hz) from 8 mice: **0.9,1.4,1.2,1.2,1.3,2.0,1.4,1.6.** Moderate anesthesia (using Isoflurane) cause C57BL mice to have 0.5 Hz whisker twitching. Are these mice moderately sedated?

H_0: These mice are deeply sedated, $\mu_0 = 0.5$

H_A: These mice are not deeply sedated, $\mu_0 \neq 0.5$

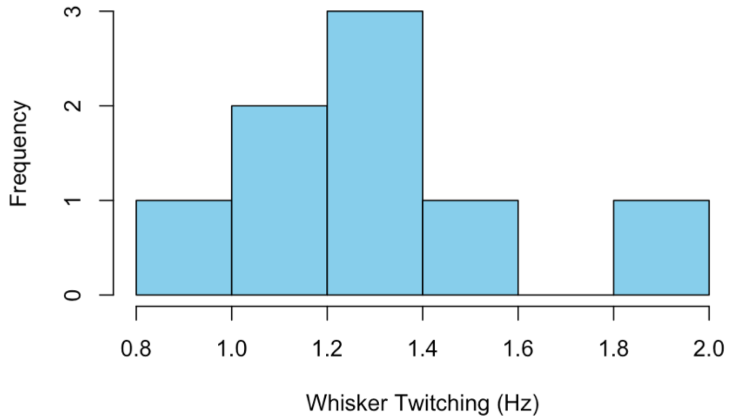
Step 2: conduct the test (and investigate assumptions)

$\bar{X} = 1.375$

s = 0.324

n= 8

df = 7



- Small sample size
- A slight outlier at 2
- Still normal enough for the one sample t test

$$t_{0.05,7} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1.375 - 0.5}{0.324/\sqrt{8}} = 7.63$$

One Sample t Test

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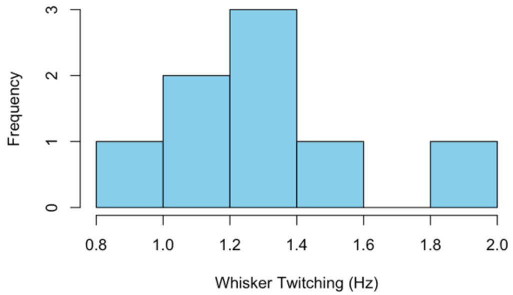


$$t_{0.05,7} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1.375 - 0.5}{0.324/\sqrt{8}} = 7.63$$

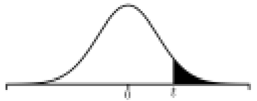
Step 3: quantify evidence (use critical value

$t_{0.05,7} = 2.365$

$t_{0.05,7} < t_{0.05,7}$ so this is farther in the tails

https://www.stat.purdue.edu/~lfindsen/stat503/t-Dist.pdf

Critical Values for Student's t-Distribution.



	Upper Tail Probability: $Pr(T > t)$									
df	0.2	0.1	0.05	0.04	0.03	0.025	0.02	0.01	0.005	0.0005
1	1.376	3.078	6.314	7.916	10.579	12.706	15.895	31.821	63.657	636.619
2	1.061	1.886	2.920	3.320	3.896	4.303	4.849	6.965	9.925	31.599
3	0.978	1.638	2.353	2.605	2.951	3.182	3.482	4.541	5.841	12.924
4	0.941	1.533	2.132	2.333	2.601	2.776	2.999	3.747	4.604	8.610
5	0.920	1.476	2.015	2.191	2.422	2.571	2.757	3.365	4.032	6.869
6	0.906	1.440	1.943	2.104	2.313	2.447	2.612	3.143	3.707	5.959
7	0.896	1.415	1.895	2.046	2.241	2.365	2.517	2.998	3.499	5.408
8	0.889	1.397	1.860	2.004	2.189	2.306	2.449	2.896	3.355	5.041
9	0.883	1.383	1.833	1.973	2.150	2.262	2.398	2.821	3.250	4.781
10	0.879	1.372	1.812	1.948	2.120	2.228	2.359	2.764	3.169	4.587

One Sample t Test

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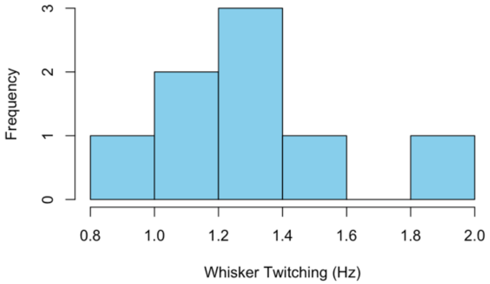


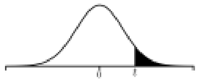
$$t_{0.05,7} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1.375 - 0.5}{0.324/\sqrt{8}} = 7.63$$

Step 3: quantify evidence (use critical values in table or run in R/Python)

$t_{0.05,7} = 2.365$

$t_{0.05,7} < t_{0.05,7}$ so this is farther in the tails

Critical Values for Student's *t*-Distribution.



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	Upper Tail Probability: $Pr(T > t)$									
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Step 4: Conclude

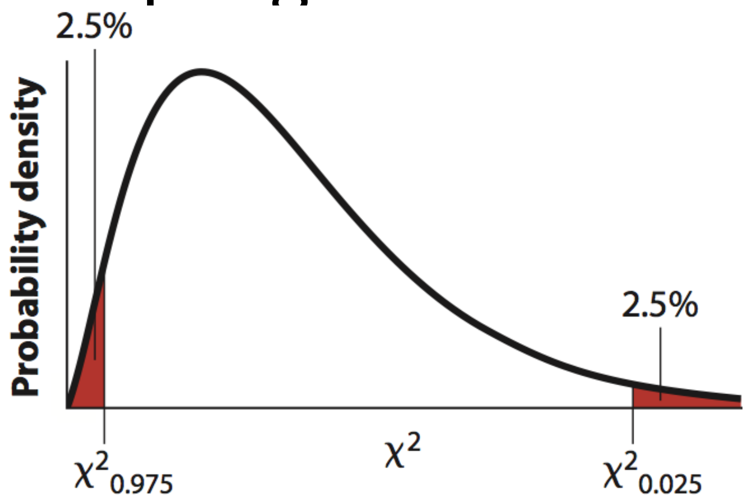
Reject the null hypothesis

Estimating the Standard Deviation and Variance of a Normal Population:

- Y is a normally distributed variable then the sampling distribution of:

$$(n-1)\frac{s^2}{\sigma^2} \sim \chi^2_{n-1}$$

is the χ^2 distribution with dof = n-1



- This can be rearranged to give CI

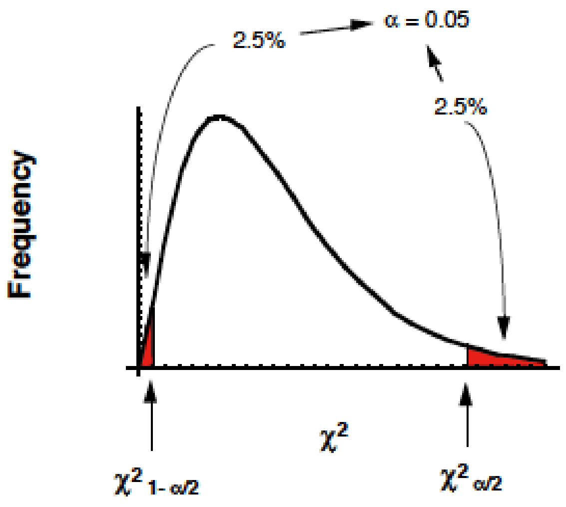
$$\boxed{\frac{df * s^2}{\chi^2_{\frac{\alpha}{2},df}} < \sigma^2 < \frac{df * s^2}{\chi^2_{1-\frac{\alpha}{2},df}}} \longrightarrow \sqrt{\frac{df * s^2}{\chi^2_{\frac{\alpha}{2},df}}} < \sigma < \sqrt{\frac{df * s^2}{\chi^2_{1-\frac{\alpha}{2},df}}}$$

Inference for a Normal Population

<u>Example</u>: What is the **95% confidence interval** for the mean whisker twitching rate of sedated mice. The rate of twitching (hz) from 8

<u>Answer:</u> $\bar{Y} \pm t_{0.05(2),7} SE_{\bar{Y}} = 1.375 \pm 0.115(2.36)$

$1.10 < \mu < 1.65$ (95% CI)



<u>We can also add CI for variance!</u>

df=n-1=7

$s^2 = (0.324)^2 = 0.105$

$$\boxed{\frac{df * s^2}{\chi^2_{\frac{\alpha}{2},df}} < \sigma^2 < \frac{df * s^2}{\chi^2_{1-\frac{\alpha}{2},df}}}$$

$$\frac{df * s^2}{\chi^2_{\frac{\alpha}{2},df}} < \sigma^2 < \frac{df * s^2}{\chi^2_{1-\frac{\alpha}{2},df}}$$

$$\frac{7(0.324)^2}{16.01} < \sigma^2 < \frac{7(0.324)^2}{1.69}$$

$$0.0459 < \sigma^2 < 0.435$$

df / X	0.999	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.001
1	1.6 E-6	3.9E-5	0.00016	0.00098	0.00393	3.84	5.02	6.63	7.88	10.83
2	0	0.01	0.02	0.05	0.1	5.99	7.38	9.21	10.6	13.82
3	0.02	0.07	0.11	0.22	0.35	7.81	9.35	11.34	12.84	16.27
4	0.09	0.21	0.3	0.48	0.71	9.49	11.14	13.28	14.86	18.47
5	0.21	0.41	0.55	0.83	1.15	11.07	12.83	15.09	16.75	20.52
6	0.38	0.68	0.87	1.24	1.64	12.59	14.45	16.81	18.55	22.46
7	0.6	0.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28	24.32
8	0.86	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.95	26.12

$$\boxed{\begin{aligned} \chi^2_{\frac{\alpha}{2},df} &= \chi^2_{0.025,7} = 16.01 \\ \chi^2_{1-\frac{\alpha}{2},df} &= \chi^2_{0.975,7} = 1.69 \end{aligned}}$$

Example A professor wants to test if her introductory class has a good grasp of basic concepts. 6 students are randomly chosen and given a proficiency test. The professor wants the class to be able to score above 70 on the test.

62, 92, 75, 68, 83, 95

Can the professor be at least **90%** certain that the mean score for the class on the test would be 70%?

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Can the professor be at least 90% certain that the mean score for the class on the test would be 70%?

Step 1: H_o: $\mu \leq 70$; H_A: $\mu > 70$

Step 2: Use a one sample t test. Calculate the relevant values from our data: sample mean = 79.17

sample stand. dev. = 13.17

t-value: $$t = \frac{79.17-70}{13.17/\sqrt{6}} = \frac{9.17}{5.38} = 1.71$$

Step 3: compare to the critical value of **t** that has **5 df** and alpha = 0.10 (one-tailed so you don't divide alpha by 2); $t_{0.10,5}=1.476$

Step 4: Since 1.71 > 1.476, we reject Ho and with 90% confidence state that the true class mean on the math test would be at least 70%. The 90% Confidence Interval:

$79.17 - 1.476*5.38 < \mu < 79.17 + 1.476*5.38$

$71.23 < < \mu < 87.11$ (note that 70 is not included)