Module 2 : Probability

Frequentist and Bayesian building blocks

<u>Agenda:</u>

- Bayesian Probability
  - Structure of Bayes' Theorem: $P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{P[B|A]P[A]}{P[B]}$
  - The Monty Hall Problem: illustrating the philosophical difference with Frequentist camp - ability to <u>update</u> probability with new information
  - Examples:
    - Pedigree Analysis

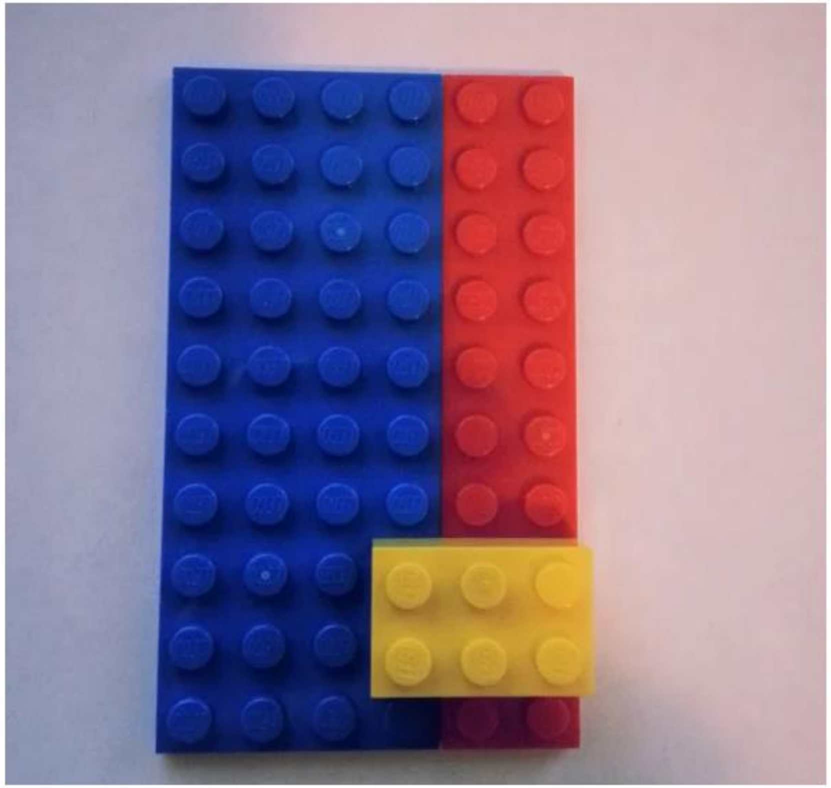
https://www.countbayesie.com/blog/2015/2/18/bayes-theorem-with-lego

Introducing Bayes' Theorem

Bayes' Theorem states:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

As far as formulae go this one isn't too scary, it doesn't even have a Σ! But what is actually happeni
here? Let's pull out some Lego bricks and put some concrete questions to our equation.



Lego Brick Probability Space

$$\boxed{P[A \mid B] = \frac{P[A]P[B \mid A]}{P[B]}}$$

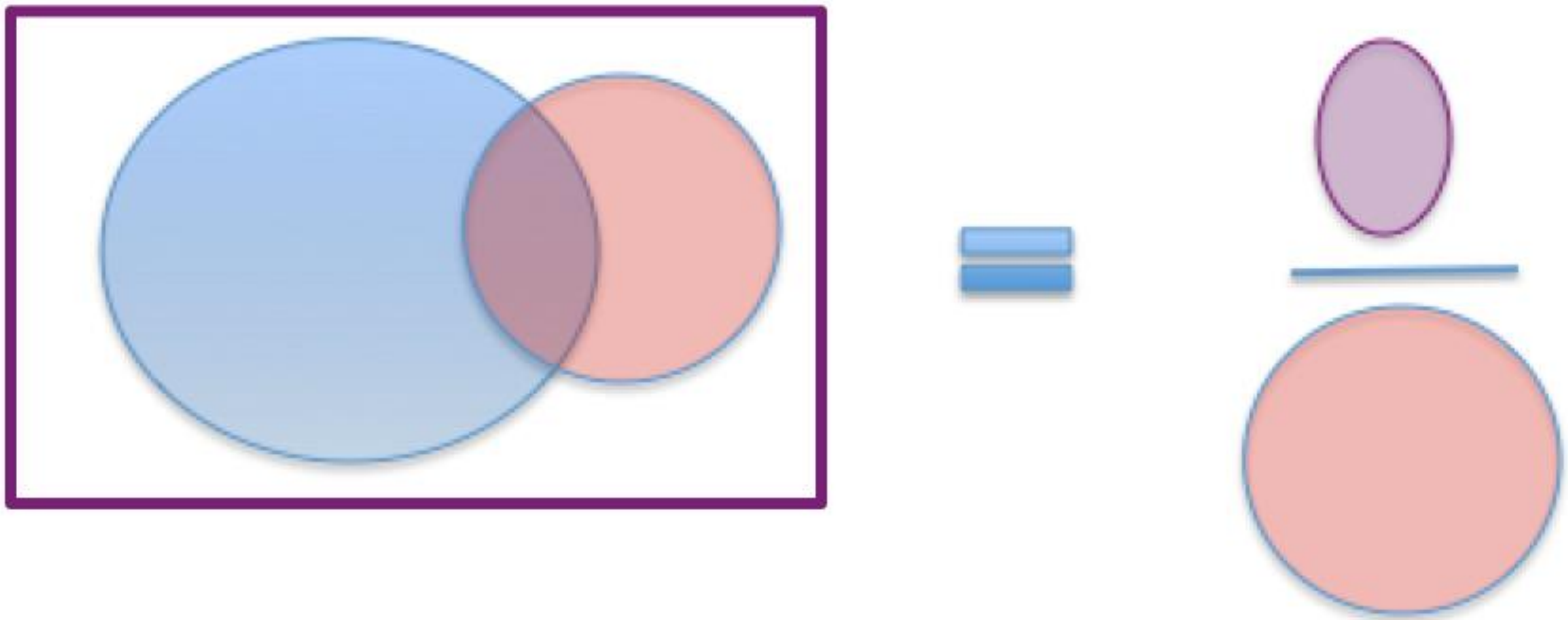
You can interpret Bayes as reducing the state space, like the following equations which use the proportion of A intersecting with B over the WHOLE universe (i.e., black die and red die both equal 1 is 1/36) and then reduce the proportion by dividing by the probability of the first event

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} \qquad P[B \mid A] = \frac{P[A \cap B]}{P[A]}$$

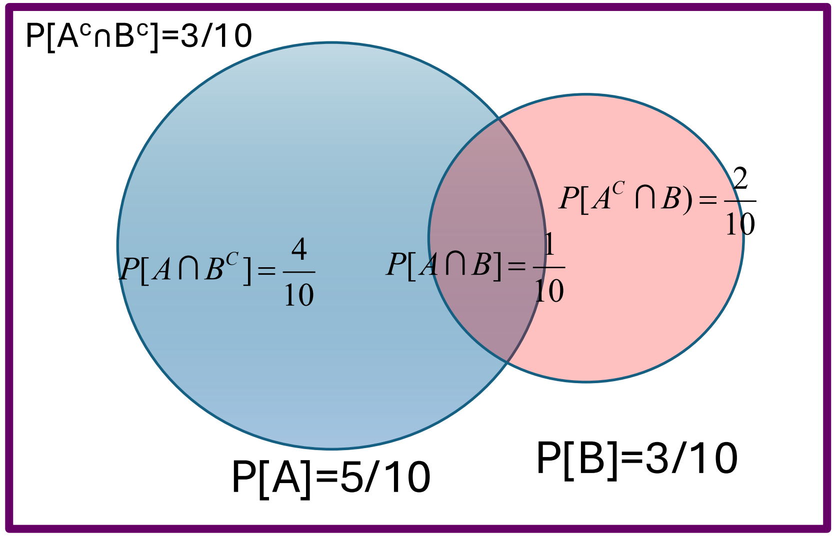
You might notice that both equations involve the SAME **numerator** whereas the **denominator** changes based on what event has happened <u>first</u> i.e., what we already know and what we still want to know.

This is a sophisticated rearrangement of the multiplication rule.

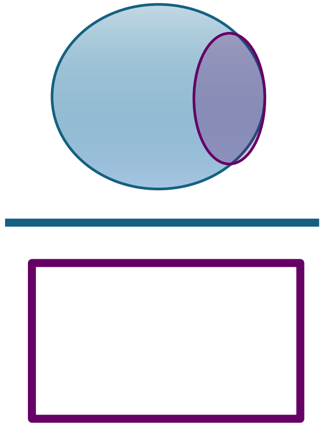
$$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{P[B|A]P[A]}{P[B]}$$



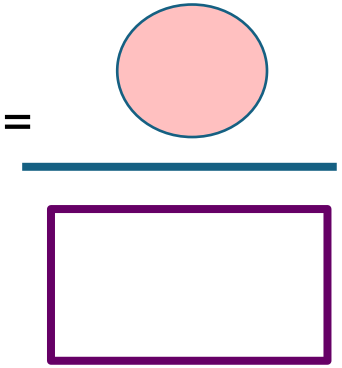
$$\boxed{P[A \mid B] = \frac{P[A]P[B \mid A]}{P[B]}}$$



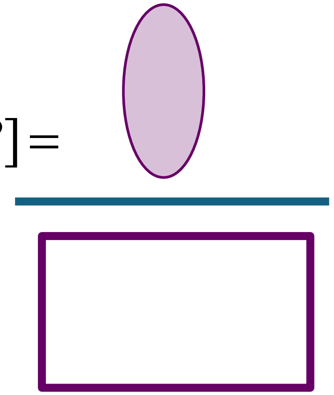
P[A] =



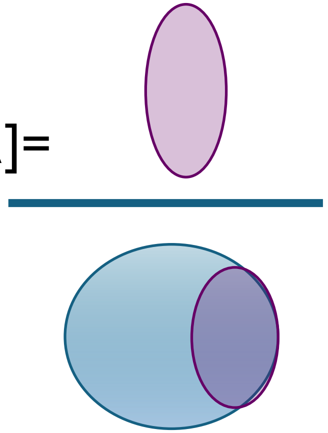
P[B] =

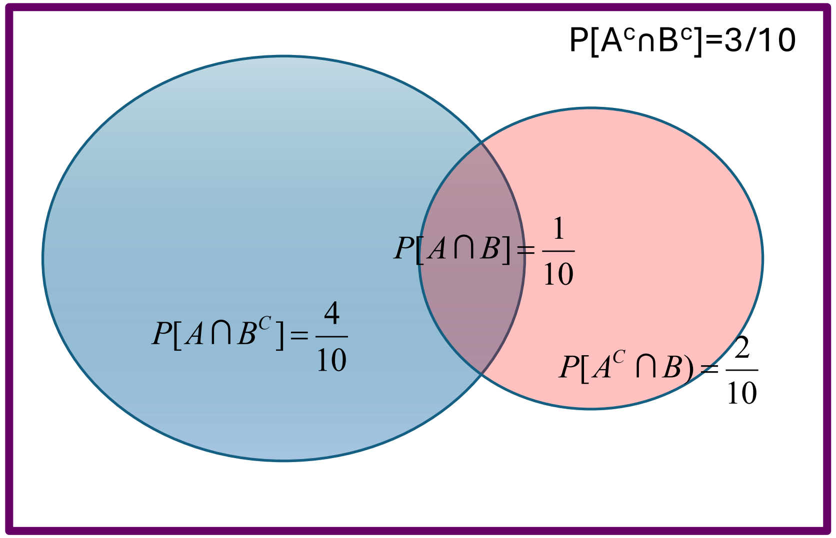


$P[A \cap B] =$



P[B|A]=





$$\boxed{P[A \mid B]=\frac{P[A]P[B \mid A]}{P[B]}}$$

P[A]=5/10

P[B] = 3/10

$$P[A \cap B]=\frac{1}{10}$$

$$\mathbf{P[A|B]=\frac{1/10}{3/10}=\frac{1}{3}}$$

$$\mathbf{P[B|A]=\frac{1/10}{5/10}=\frac{1}{5}}$$

<u>**Bayes' theorem in words:**</u> The conditional probability of **A (the hypothesis) given B (the data)** is the conditional probability of **B (data)** given **A (hypothesis)** scaled by the relative probability of **A compared to B**

$$\mathbf{P[Hypothesis|DATA] = \frac{P[Data|Hypothesis]P[Hypothesis]}{P[Data]}}$$

The **PRIOR** hypothesis:
The original probability of the hypothesis without any additional information

The **LIKELIHOOD** interpreted as:
P(observation GIVEN the hypothesis)

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} = \frac{P[A]P[B \mid A]}{P[B]}$$

the **POSTERIOR probability** interpreted as the P(hypothesis GIVEN the observation)

The **observation/data/ Evidence** that has been observed