Module 4B
Supervised Machine Learning

Different flavors of REGRESSION and General Linear Models

Finding **a**:

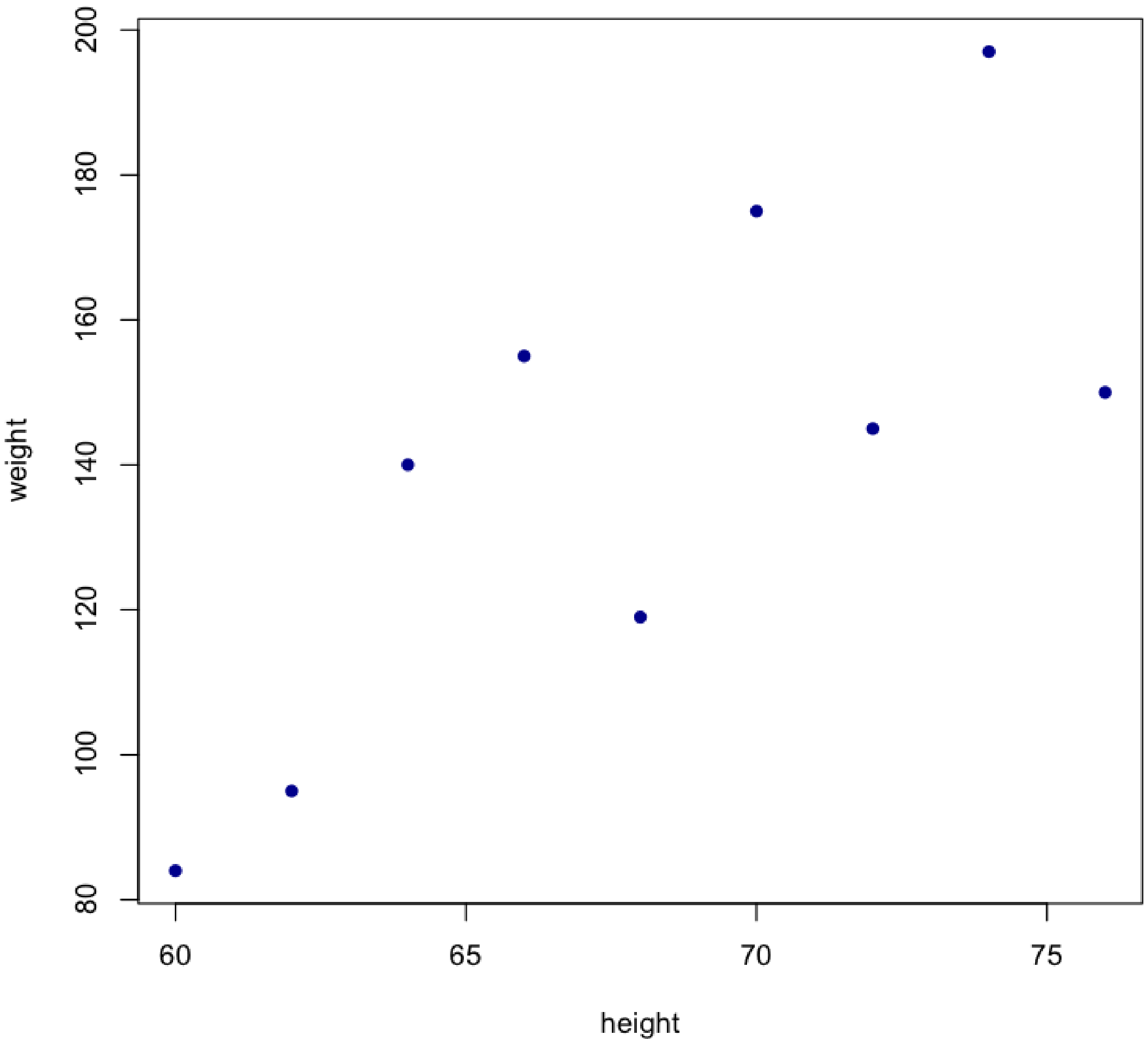
$$\boxed{\bar{Y} = a + b\bar{X}}$$

OR

$$\boxed{a = \bar{Y} - b\bar{X}}$$

Example: Predicted weight for someone who is 65 inches tall?

Height	Weight
60	84
62	95
64	140
66	155
68	119
70	175
72	145
74	197
76	150



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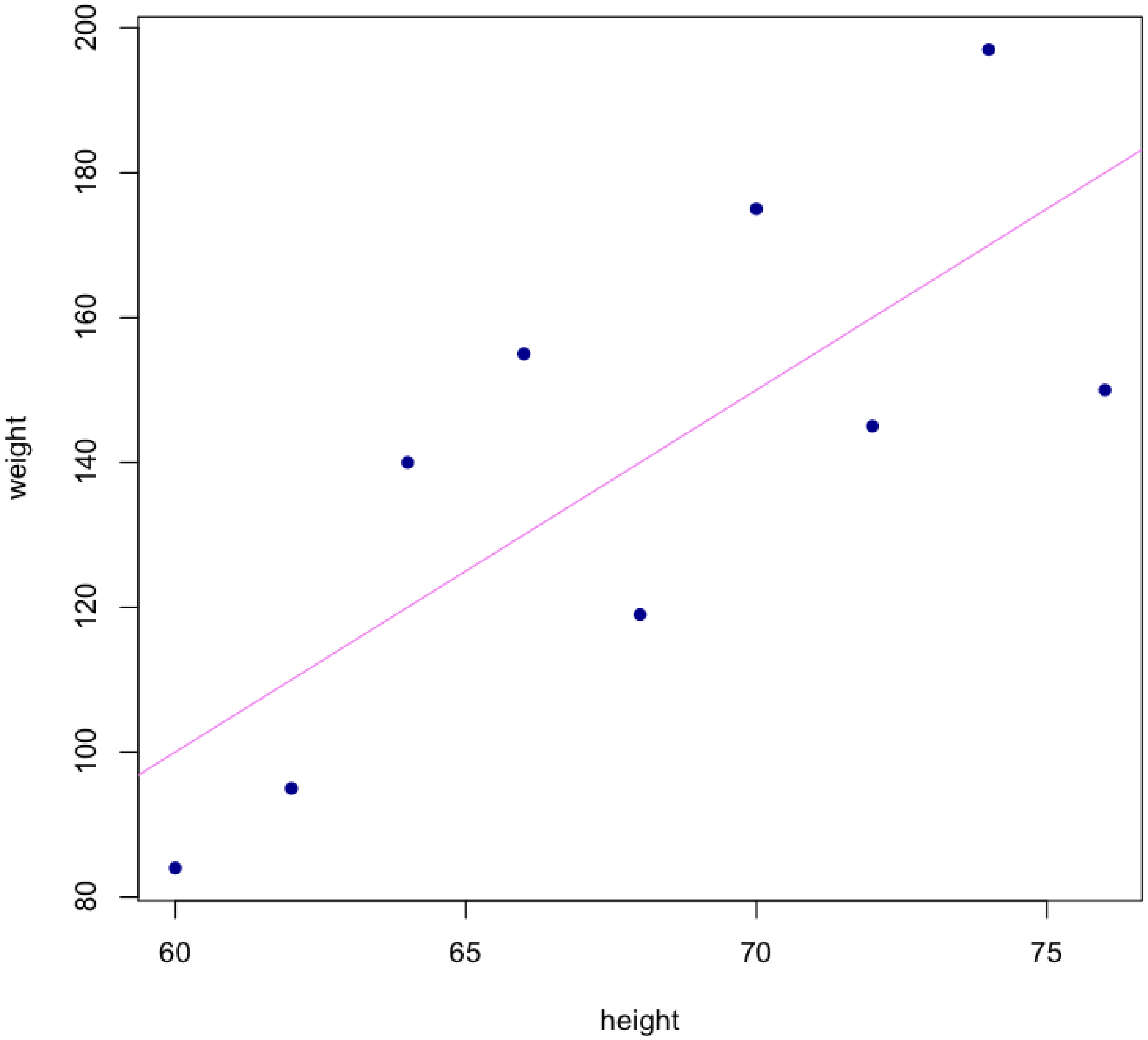


Height Weight data:

$\sum X = 612$

$\sum Y = 1260$

$n=9$

$\sum X^2 = 41856$

$\sum Y^2 = 186826$

$\sum (XY) = 86880$

$$\bar{Y} = 140$$

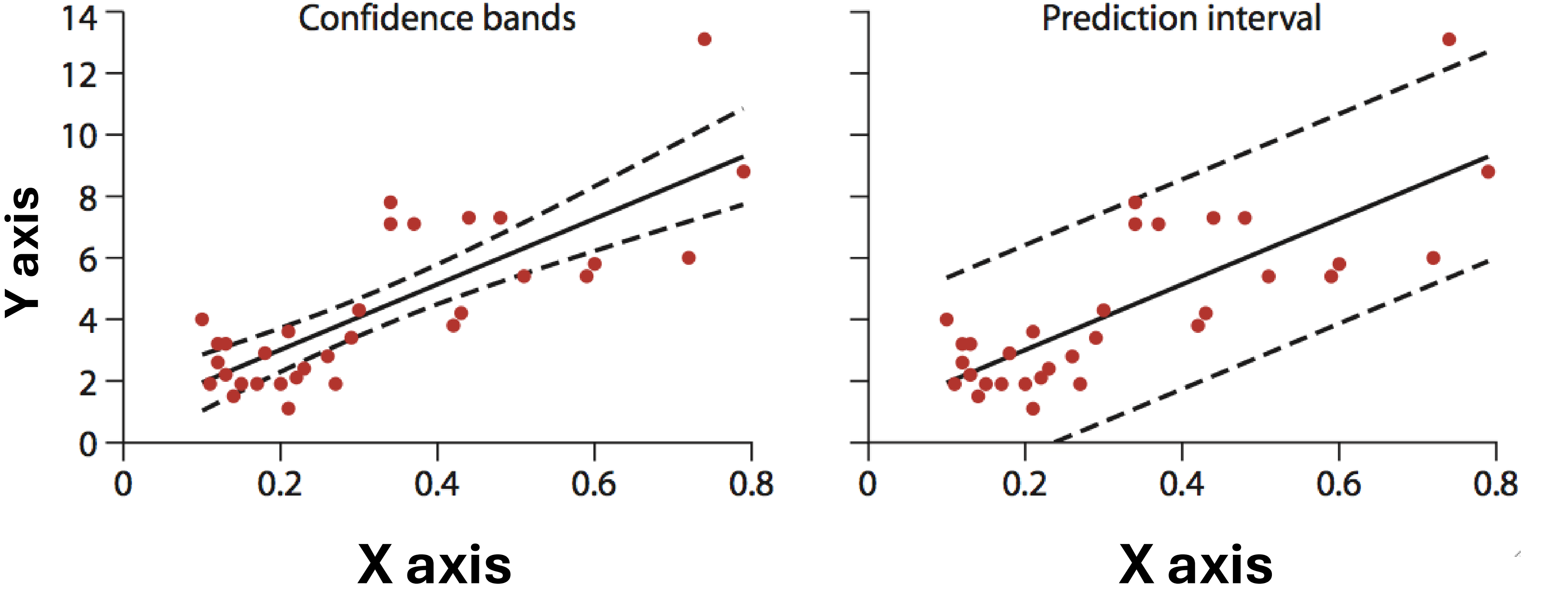
$$\bar{X} = 68$$

$b = 5$

$a = -200$

$$\boxed{\hat{Y} = -200 + 5X}$$

Prediction confidence:



Prediction confidence:

The purpose of regression is to **predict**. There are two types of prediction:

1. $\overline{Y}$ for a given X. → Confidence bands (related to Confidence Interval)

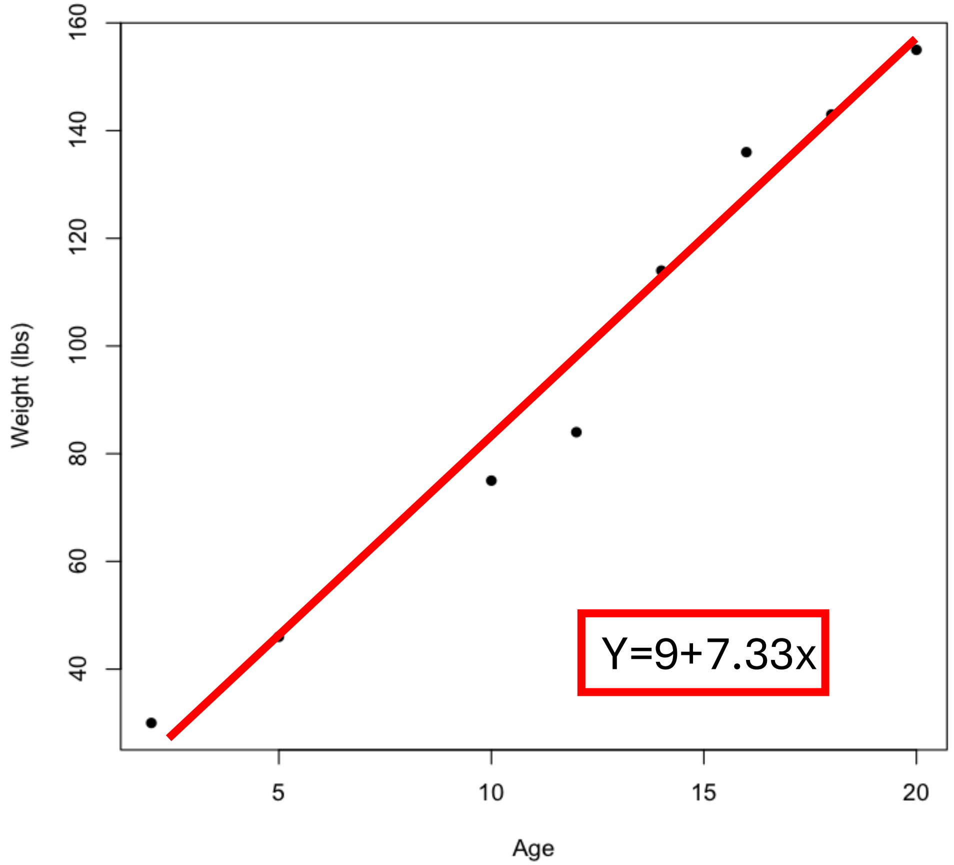
2. Single Y for a given X → Prediction bands

Both will generate $\hat{Y}$ with the same value, but the prediction of a single Y point will have a lower precision.

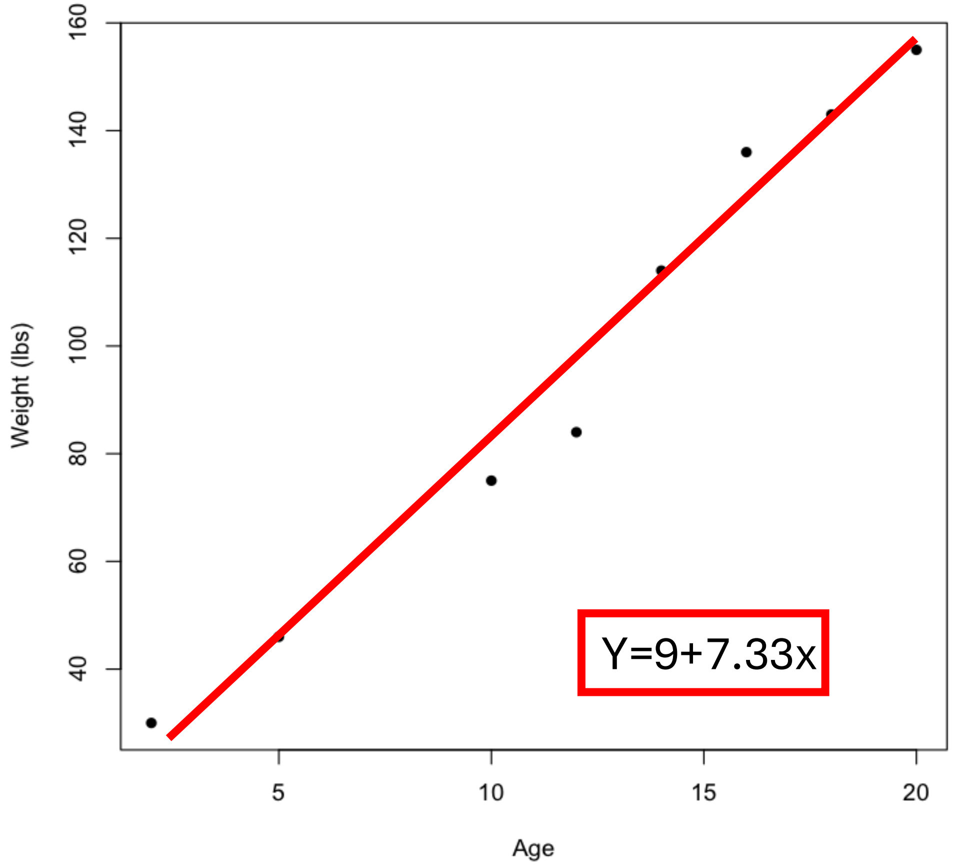
Caution! Do not extrapolate beyond the range of the data

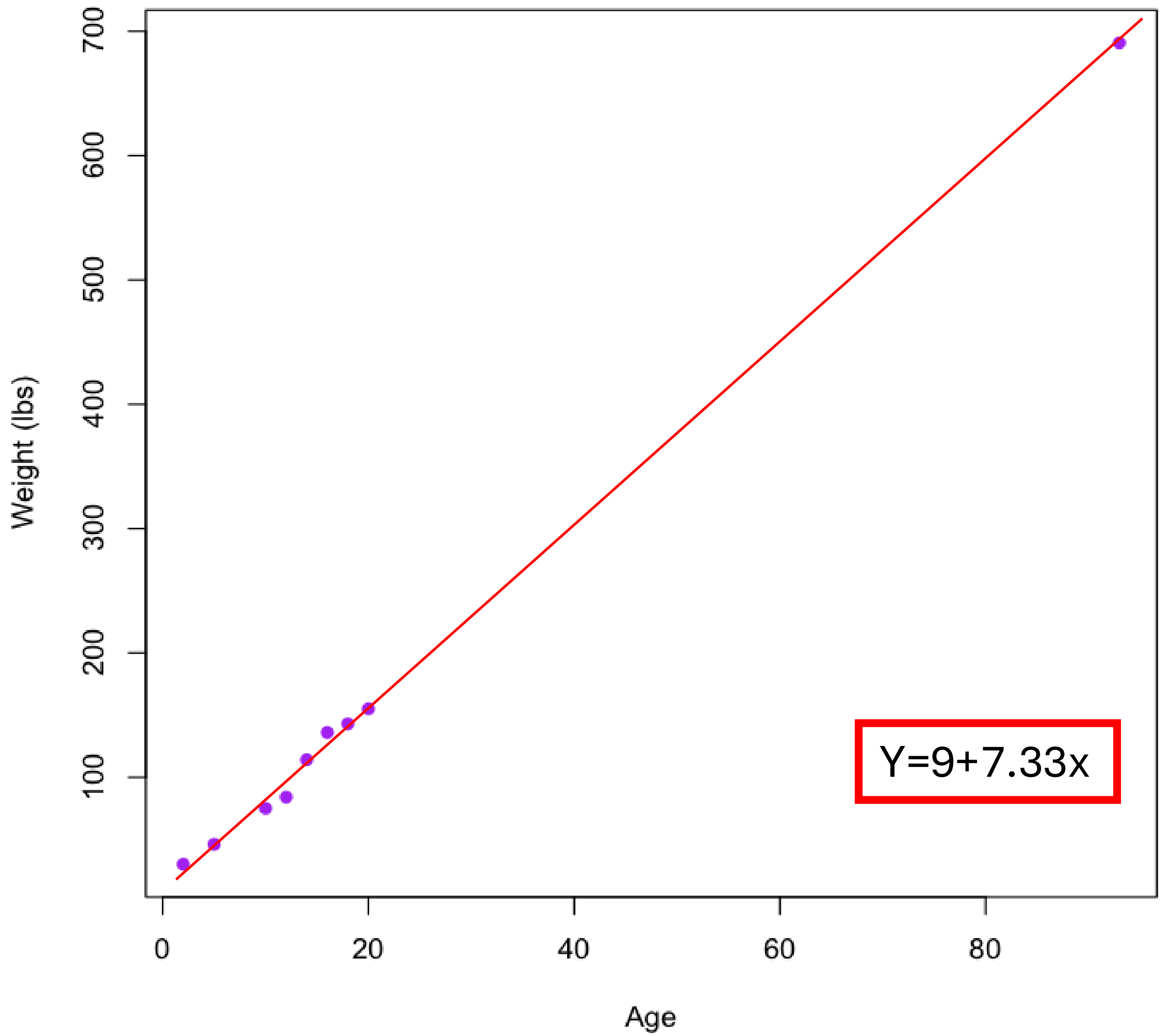
Age	Weight (lbs)	Time to run one mile	Bench Press (lbs)
2	30		
5	46		
10	75		
12	84	5:40	
14	114	5:05	
16	136	4:40	160
18	143	4:35	180
20	155	4:30	

Measurements taken over the course of an individual's life



$Y=9+7.33x$


Weight (lbs)
40
60
80
100
120
140
160
5
10
15
20
Age
Y=9+7.33x


Weight (lbs)
100
200
300
400
500
600
700
0
20
40
60
80
Age
Y=9+7.33x

Regression to the mean:

- Francis Galton invented the term to describe the observation that tall fathers had sons of average height

- He developed "regression analysis" to study this phenomenon of "regression towards mediocrity"

- results when two variables have correlation < 1
    - Individuals who are far from the mean for one of the measurements will, on average, lie closer to the mean for the other measurement

Regression fallacy:

- Tricky concept:
  - each individual has a **true** value, but the sampled value varies with time
    - the subset who scored highest on the first round included individuals who had higher values then their usual ‘true’ value
    - the second measurement captured these individuals when they happened to be closer to their own personal normal values

- failure to consider “regression towards the mean” when interpreting the results of **observational studies**

- can be a large problem when dealing with **sick** people - they are the tail of the distribution, and they might appear to improve even if the treatment applied has no real effect

Regression to the mean:

A VERY old concept:

> “You know, few sons turn out to be like their fathers;
> Most turn out worse, a few better.”
>
> (Athena speaking to Telemachus)
> - Homer, The Odyssey

Regression fallacy: Rolling a die

Student	First	Second	Second roll lower?
1	4	5	no
2	4	3	yes
3	3	-	-
4	5	5	no
5	1	-	-
6	6	5	yes
7	5	2	yes
8	6	2	yes
9	3	-	-
10	2	-	-

Remaining students have a mean value of 5 (first roll) and 3.7 (second roll)

Testing hypotheses about slope:

1. H_0: $\beta = \beta_0$ (N.B. The null hypothesis is that Y cannot be predicted from X)

   H_A: $\beta \neq \beta_0$

2. Test statistic:

$$t = \frac{b - \beta_0}{SE_b}$$

$$SE_b = \sqrt{\frac{MS_{residual}}{\Box\,(X_i - \bar{X})^2}}$$

3. significance level; df=n-2

4. Reject or FTR and:

$$\boxed{b - t_{\alpha(2),n-2} SE_b < \beta < b + t_{\alpha(2),n-2} SE_b}$$

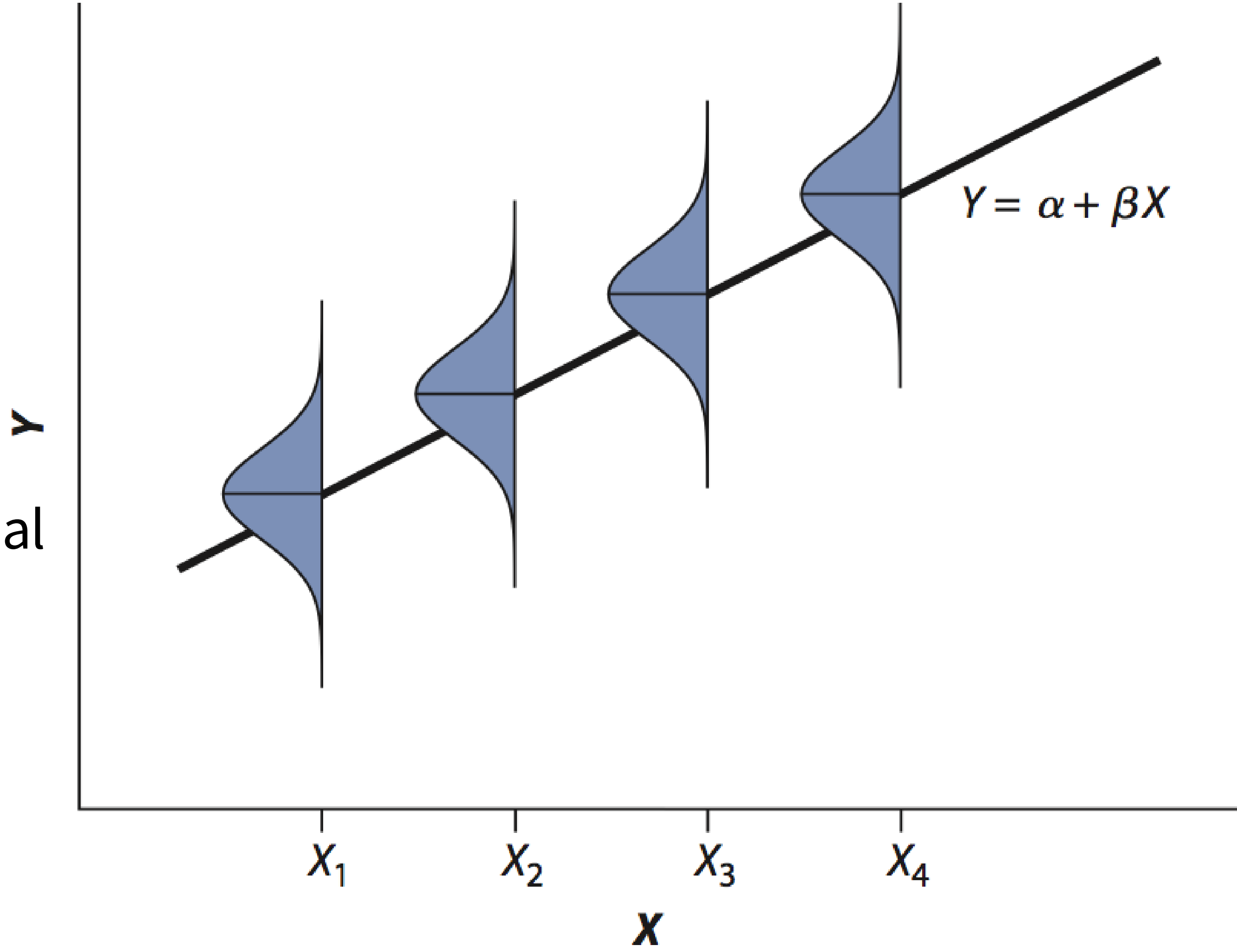
When test is two-tailed and H_0: β = 0, you can use ANOVA approach to testing regression slopes (for multiple models, too!)

- F-test versus t-test
- **If** H_0 is true, then the mean squares corresponding to the two components should be equal

Source	DF	SS	MS	F
Regression (model)	1	$\sum(\hat{Y}_i - \bar{Y}\ )^2$	$\sum(\hat{Y}_i - \bar{Y}\ )^2/1$	$MS_{regression}/MS_{residual}$
Error (residual)	N-2	$\sum(Y_i - \hat{Y}_i\ )^2$	$\sum(\hat{Y}_i - \bar{Y}\ )^2$/(n-2)	
Total	N-1	$\sum(Y_i - \bar{Y})^2$	$\sum(Y_i - \bar{Y})^2$/(n-1)	

Assumptions of Regression Analysis:

- For each X_i, there is a population of Y values whose mean lies on the ‘true’ regression line
  - For each X_i, the Y are a random sample
  - For each X_i, the Y are normally distributed
- Homoscedasticity
  - For every X_i, the variance of Y is equal
- Nothing is assumed about the distribution of X
  - It doesn’t need to be normally distributed or randomly sampled - they might be fixed by the experimenter


Y
$Y = \alpha + \beta X$
X_1
X_2
X_3
X_4
X