Module 5E: A Parametric Test

Z-scores & RNAseq analysis

<u>Agenda:</u>

1. Z-scores
2. RNASeq

Z-scores:

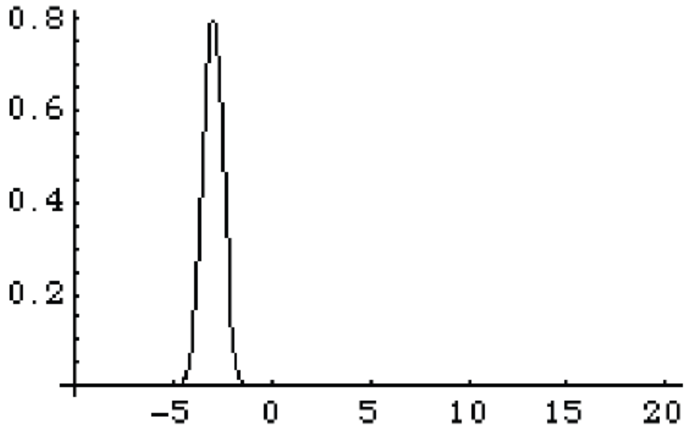
- converts **raw** normally distributed scores into standard deviation units
  - useful for comparing distributions with different scales, for instance.
  - percentiles

- allows calculation of probability of variable value

- z-score indicates **how far above or below the mean a value** is in standard deviation units
  - how large/small is individual score **relative** to others in the distribution

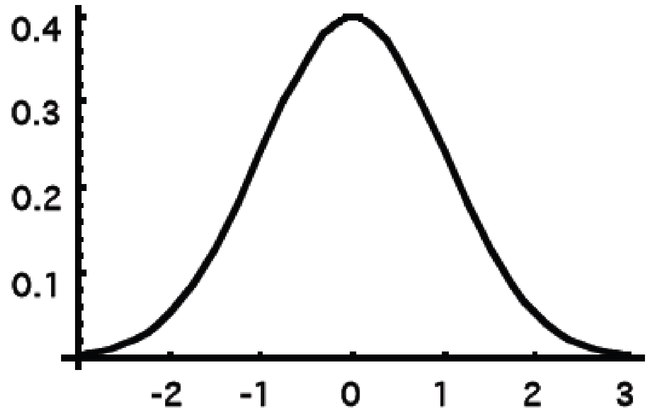
The Standard Normal Distribution:

- Mean is zero ($\mu = 0$)
- Standard deviation is 1 ($\sigma = 1$)



$\mu = -3$; $\sigma = 1/2$

$$\boxed{Z = \frac{X - \mu}{\sigma}}$$



3 major motivations:

- Z score tells us how many standard deviations our normally distributed variable is from the mean

$$Z = \frac{\text{Raw score - Mean}}{\text{Standard deviation}}$$

- Confidence interval: $(a,b) = \bar{x} \pm z_{\alpha/2}(\sigma / \sqrt{2})$

- Determine proportion of scores that fall between two raw scores

- Allows us to use standard normal table

Example: British Spies. MI5 says that a man must be shorter than 180.3 cm tall to be a spy. The mean height of British men is 177.0 cm, with standard deviation 7.1 cm, and with a normal distribution.

What proportion of British men are excluded from a career as a spy by this height criteria?

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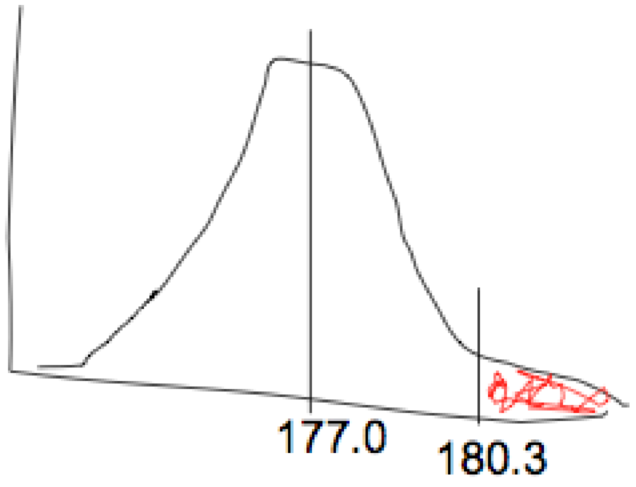

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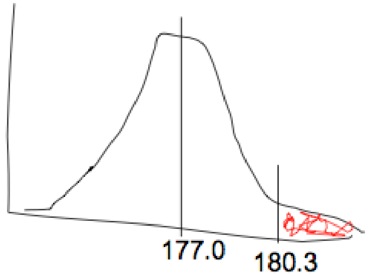

Step 2: Transform into Standard Normal

$$\boxed{\begin{aligned} &\mu = 177.0cm \\ &\sigma = 7.1cm \\ &X = 180.3cm \\ &P[height > 180.3] \end{aligned}}$$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{180.3 - 177.0}{7.1}$$

$$Z = 0.46$$

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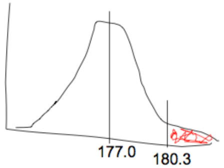

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$$Z = 0.46$$

Step 3: Look up probability online

https://www.z-table.com/

	x.x0	x.x1	x.x2	.x3	x.x4	x.x5	x.x6	x.x7	x.x8	x.x9
0.0	0.5	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476

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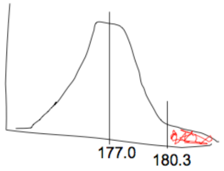

$$\boxed{\begin{aligned}&\mu = 177.0cm\\&\sigma = 7.1cm\\&X = 180.3cm\\&P[height > 180.3]\end{aligned}}$$

$$Z = \frac{X-\mu}{\sigma}$$

$$Z = \frac{180.3-177.0}{7.1}$$

$$Z = 0.46$$

Step 3: Look up probability in Z table

P[Z > 0.46] = 0.32276

So, P[height > 180.3] = 0.32276

The fraction of British males who are too tall to be spies is approx. 1/3.

Example: For a particular year, the average SAT-math scores was 517 (out of 800) with a standard deviation of 100. What score marks the **90%** percentile?

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0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.1335	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330

<u>Example:</u> For a particular year, the average SAT-math scores was 517 (out of 800) with a standard deviation of 100. What score marks the **90%** percentile?

$$Z = 1.285 = \frac{X_i - 517}{100}$$

$$128.5 + 517 = X_i$$

$$X_i = 645.5$$

Example: What if your friend casually mentioned to you that they had scored a 750 while you had scored 425 on the math section of the SAT? What percentage of scores is between you?

Step 1: Convert raw score into *z-score*:

$$\boxed{Z = \frac{X_i - \mu}{\sigma}}$$

$$\boxed{Z = \frac{425 - 517}{100} = -0.92}$$

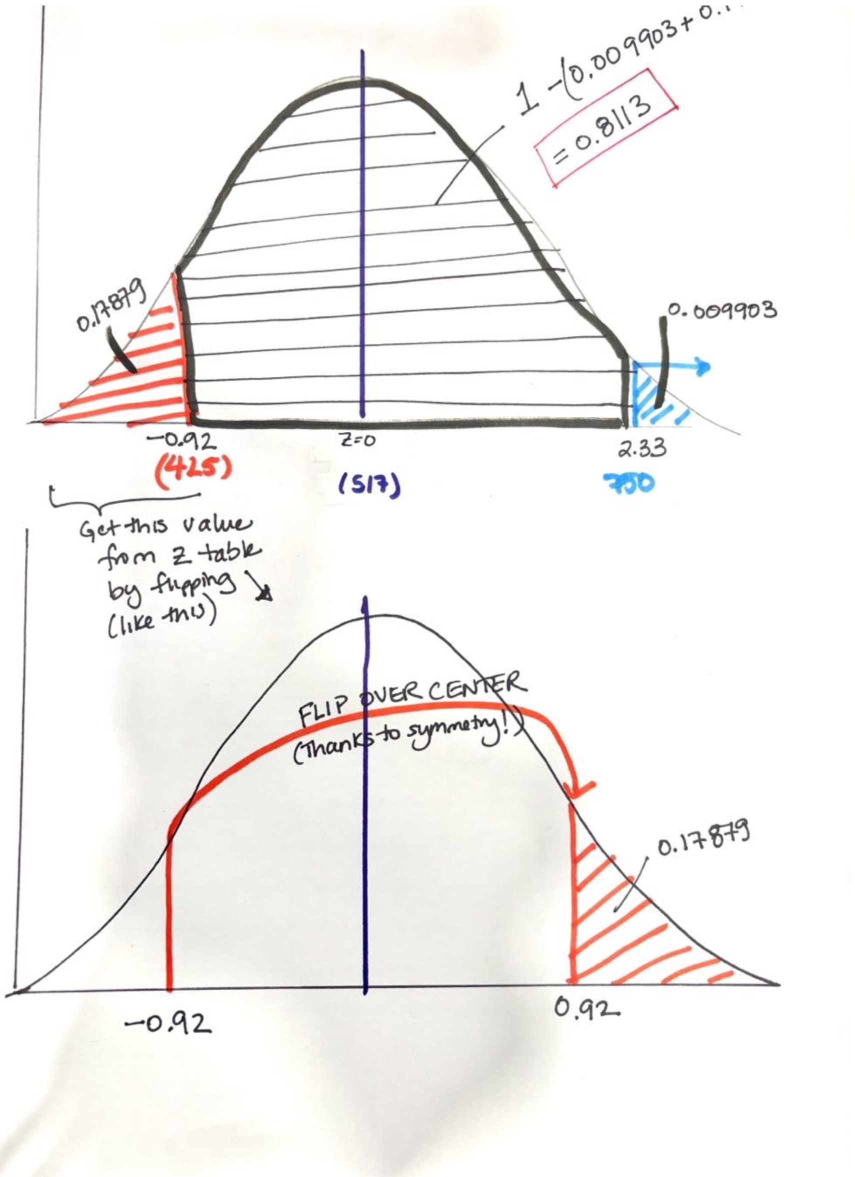
$$\boxed{Z = \frac{750 - 517}{100} = 2.33}$$

Step 2: Find the proportion of the normal distribution that falls below a score of:

1. -0.92
2. 2.33

***** I have added few slides that explain the logic and solution of this question that I did not cover on the original video, but is now on LP_BI_Module5E_Appendix***

Example: What if your friend casually mentioned to you that they had scored a 750 while you had scored 425 on the math section of the SAT? What percentage of scores is between you? ***Hint: Draw the question out**



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Step 1: Convert raw score into *z-score*:

Step 2: Find the proportion of the normal distribution that falls below a score of:

1. -0.92
   - Notice that we are dealing with a negative number
   - The Table only reports positive scores.
   - symmetry – the proportion of the distribution that falls above the score is identical whether it is positive or negative
   - **Draw it out** – it is then easy to see that you are looking for (0.5-0.17879) = 0.3212
   - This score, btw, corresponds to a percentile of 17.88%
2. 2.33
   - According to Normal Table, this score corresponds to 1-0.009903 or the 99.1 percentile.

Example: What if your friend casually mentioned to you that they had scored a 750 while you had scored 425 on the math section of the SAT? What percentage of scores is between you?

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1. -0.92
    - (0.5-0.17879) = 0.3212
    - This score, btw, corresponds to a percentile of 17.88%
2. 2.33
    - According to Appendix B, this score corresponds to 1-0.00990 or the 99.01 percentile.

The difference between the two scores is: **0.3212+0.4901 = 0.8113**

Sample means are normally distributed:

If a variable itself is normally distributed then the distribution of sample means, $\overline{Y}$, is also normally distributed

Sampling distribution for $\overline{Y}$:

The range of different values for $\overline{Y}$ that could have been obtained by sampling, and their associated probabilities, constitute the sampling distribution for $\overline{Y}$.

Sample means are normally distributed:

If a variable itself is normally distributed then the distribution of sample means, $\overline{Y}$, is also normally distributed

- The mean of the sample means is μ
- The standard deviation of the sampling distribution for $\overline{Y}$ is called the *Standard error:*

$$\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{n}} \xrightarrow{\text{approx}} SE_{\overline{Y}} = \frac{s}{\sqrt{n}}$$

Standard error **is the spread/variation statistic for a "collection" of means (or proportions)**.

It can be thought of as the theoretical variation present when you repeat a study many times.

This contrasts with standard deviation which is used to describe the natural variation of something that you can measure

Standard Normal applied to sampling means:

• Sample means distributed normally with mean equal to μ and standard error then:

$$Z = \frac{\overline{Y} - \mu}{\sigma_{\overline{Y}}}$$

• The means of samples taken from a normal distribution are themselves normally distributed but….

•….the sampling distribution of sample means is *approximately normal* **even when the distribution of Y is not normal** *if the sample size is large enough (depends on the shape of the data)*

Central Limit Theorem:

The sum (or mean) of a large number of measurements randomly sampled from any population is approximately normally distributed.

Normal Approximation to the Binomial Distribution*:

- Remember the binomial distribution?
    - discrete
    - number of successes in n independent trials n
- Number of successes is a sum
- mean = np
- stand dev = $\sqrt{np(1-p)}$

Standard Normal Approximation to the Binomial Distribution:

1. State H_0 and H_A
2. Test Statistic
3. P-Value or Critical value/Compare to critical value (remember to double it!)

$$\boxed{P[NumSuccesses \square X] = P[Z > \frac{X - np}{\sqrt{np(1-p)}}]}$$

4. State a conclusion