UNIVERSITY OF LEICESTER

Module:

MATHEMATICAL MODELLING

by

Pranav S. Asalekar (psa8)

Module Convener:

Dr. Evegeny Mirkes

Contents

1.Introduction:

Corona Virus which is known as COVID - 19 is a disease which is caused by SARS-CoV-2 Virus. It's an Infectious disease, which had out breakage in late 2019. WHO declared COVID-19 outbreak on January 2020. COVID-19 affected globally and declared as global health emergency. This was caused by SARS-CoV-2 Virus and transmitted from human to human when came in contact. People who got this virus start experiencing respiratory issues and mild fever and sore throat symptoms. COVID-19 was more serious for the people who are in old age, heart diseases, diabetes, respiratory problems and/or cancer. Whoever get seriously ill due to virus will have less immunity and may die regardless of age barrier.

This outbreak caused pandemic and for precautionary measure this transmission required to slow down. This can be achieved with maintaining distance of more than 1 meter from others, frequently washi hand with sanitizers, avoiding gathering of people, increasing immunity, use of proper masks and getting vaccinated. If person turns out to be positive, it's required to follow the medical guidelines and isolating until recovery.

COVID Pandemic affected globally and almost all the countries required to follow certain precautionary measure as per medical and government advisory. This Virus had still not completely gone from the globe. But the situation is far better than 2020 and 2021.

To understand and study COVID outbreak, mathematical modelling comes into picture. This helps to understand the reality and to analyse or predict the future circumstances. This may also help to understand effect on the health-related subjects and to prepare or form some exploratory and/or suitable models which can help to find solutions to minimize the virus spread.

This report has been generated using the analysis done in earlier 3 Drafts. We have selected 3 countries (Spain, Italy, Portugal) for analysis. Draft 1 outlines the similarities in between the 3 countries and some approaches which are used to analyse and identifying waves of countries. Draft 2 contains SIR model of COVID which categorizes population into 3 different groups which are ‘Susceptible’(S), ‘Infected’ or ‘Infectious’(I), ‘Removed’ or ‘Recovers’(R). This identifies parameters for SIR model which will help for better prediction. Though the prediction will become more and more inaccurate as infection goes on. Draft 3 will focus on behaviour of people or population after pandemic out breakage. In initial phase of covid outbreak, people will be unaware or not serious about the infection. But as the pandemic starts growing, people will get into alarm and resistance phase and will start taking precautionary measures. As the time goes on people will become less and less alert and will get exhausted. This will again make people again ignorant and will empower the start of 2^{nd} wave of COVID. This will explain more about the people’s behaviour in accordance to pandemic but will not provide much content about evolution of COVID virus.

COVID-19 pandemics in different regions [i.e., Spain, Italy, Portugal]. We show that the model is to closely to form of the COVID-19. The graph of this model will greatly and it will be used to the regression line through the graph. More than 60,000,000 positive cases of COVID-19 infected by the most of the people has been affected in the in China, Italy, Spain, Portugal. By the year 2020 there is only 40% chance of covid-19. From the end of September, it started increasing the effected from 61% it started through the world-wide data it has 69% from over all the world. However, other three countries in the mathematical modelling, are much more data and may be of particular value when we have managing pandemic situation of the COVID-19 pandemics. To the year 2020 we have included the use of mathematical models to the progression of COVID-19.

2. About Collection of Data:

We have taken three countries which are Spain, Italy and Portugal to perform the analysis about epidemic. The choice of countries is taken in consideration as we required to take the data of countries which have population more than 1 million. The population of countries taken from web. Population of Spain, Italy and Portugal are 46796442, 60256585, 10128486 respectively. The data source is https://covid19.who.int/data . The data is public and published by WHO (World Health Organization) for informative manner. Going through data we have selected the infected people which hold daily record.

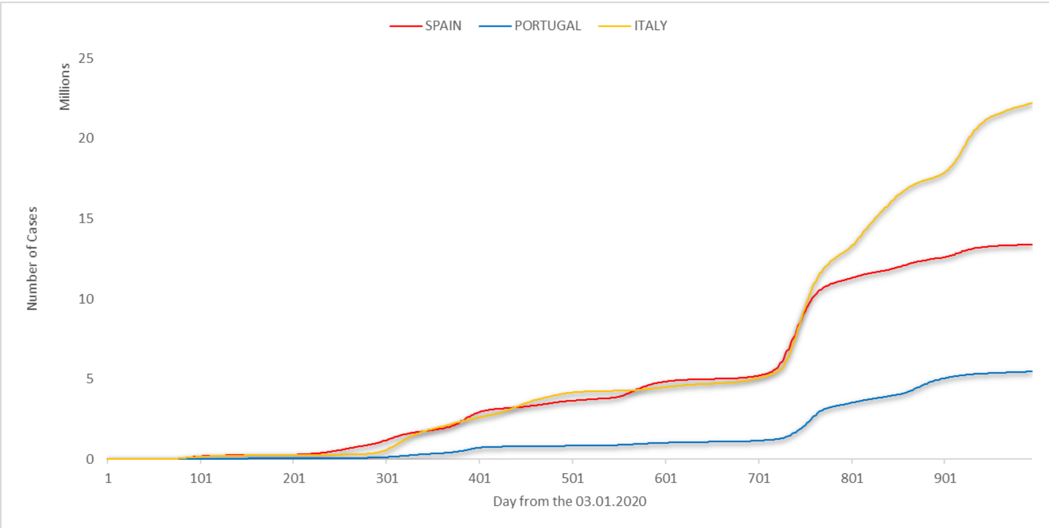

Figure 1 - Raw Data: Spain, Italy, Portugal

Figure 1 shows graph of raw data of number of cases from Italy, Spain and Portugal. The graph starts from 3rd January 2020, where the initially we can find straight horizontal line almost on the axis. Till Start of March 2020, all three countries did not have more than 10 cases per day. At the end of December 2020, we can find the outbreak of virus for all the three countries. The number of cases increase significantly after start of year 2021. This shows similarity of in the spike of covid cases. But to analyse the COVID situation further we require to dig into further.

3. Beginning of Epidemic:

In this, we will discuss about the beginning of Epidemic in Spain, Italy and Portugal. Using raw data, we can see the beginning of pandemic is bit similar for all the three countries in compare to increase in covid cases.

At the beginning of epidemic in Spain, we can find that 42 days there are cases register which can be counted by fingers. As we start going past 63 days, we can see little increase in cases. Though the bigger difference will start reflecting increase in cases from day 223. As considered the great spike in cases, day 704 will come as a changing point. The cases count ends at 13 million.

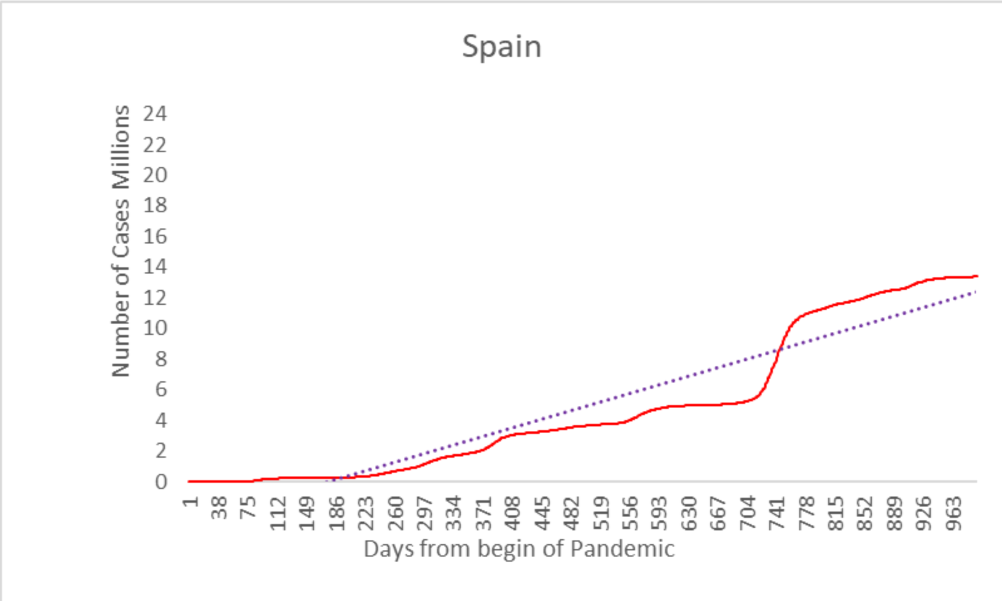

Figure 2 - Epidemic in Spain

At the start of pandemic in Italy, taking first 51 days in consideration there were no cases registered for Italy. Where the Increase in cases starts from day 52, where the case barrier started exceeding 100 on the daily basis. Though in the graph it still looks like horizontal line on the X- axis. From the day 280 and onwards we can find the rapid growth in number of covid cases. And then as the year ends there is massive increase in the cases which goes past 6 million. The gridline shows the linear growth of covid cases from 0 to 22 million.

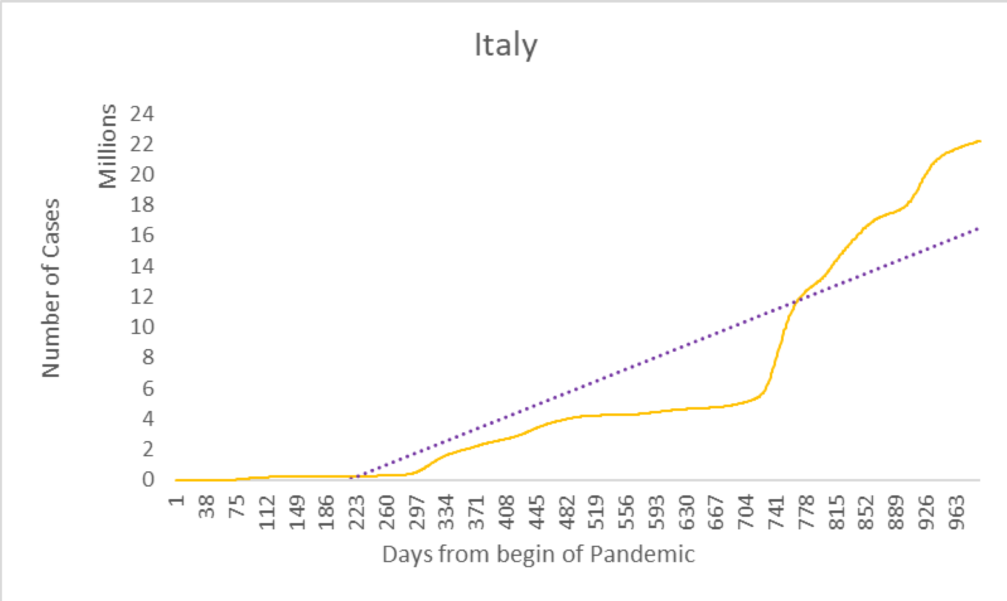

Figure 3 - Epidemic in Italy

Taking Portugal in consideration, the beginning of the pandemic almost start from day 80. First 43 days there were zero cases registered. If we compare the population-wise, Portugal have least population with 10 million. The big increase will start from day 300 and then from day 700 onwards. The linear growth ends at 5 and half million.

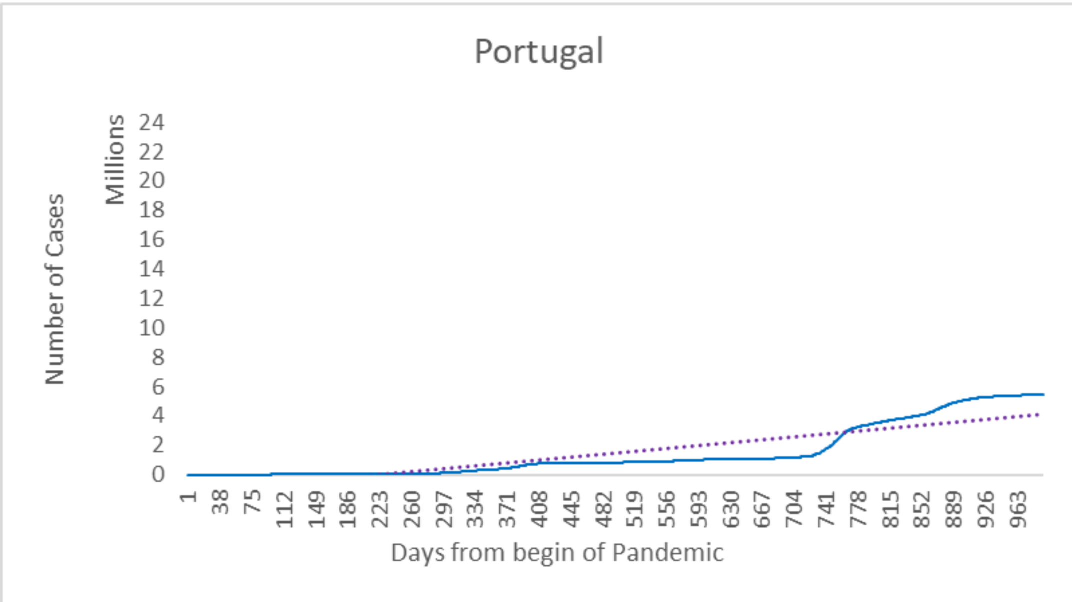

Figure 4 - Epidemic in Portugal

4. Normalization of Data:

Here, we have taken 600 days almost out of 1000 from the day the count starts exceeding 50 cases. This value is more than the half of data of the respective countries. In comparison to the population of countries, the value is minimum.

As we can find from graph, the beginning of epidemic in Italy from day 10 onwards, for Spain from day 16 onwards and for Portugal from 34. Since we have illustrated the countries in separate manner in previous section, we can easily figure out the waves of pandemic. However, comparison of different countries is not precisely clear.

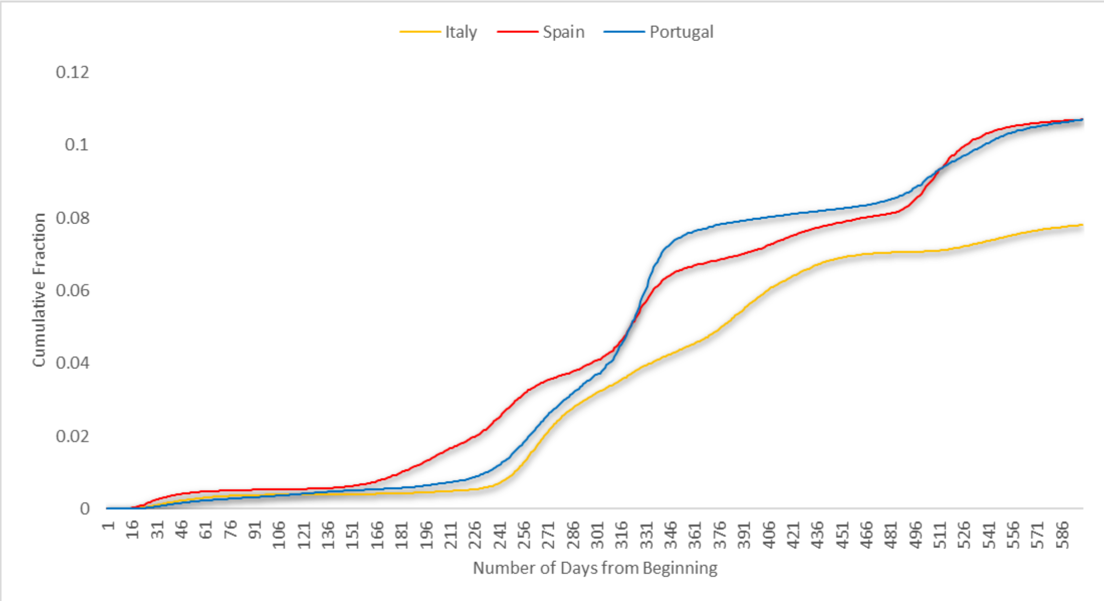

Figure 5 - Normalized Data

From help of google search engine, we have taken population as stated earlier. The population of Italy, Spain and Portugal are around 60 plus million, 47 plus million and 10 plus millions respectively.

Now we are using the population to normalize the data and find the cumulative fraction of cases where the population gets infected of COVID virus. The cases are divided by the population of specific country to get the cumulative fraction of cases.

The Figure 5, gives more idea about how the cumulative cases vs Day of epidemic graph present in best possible way. In comparative manner, Italy has less cumulative fraction than Spain and Portugal. This will help to reduce the duplicate amount of data. And align the data properly.

5. Almost Horizontal Heuristic Approach:

Now to further study about analysing model, we will split the data into the waves for each country. There are different heuristic approaches which can be used to prepare model further. Taking examples of different heuristic approaches it will have almost horizontal fragments method, finding inflection points, logarithmic approach, logistic approach.

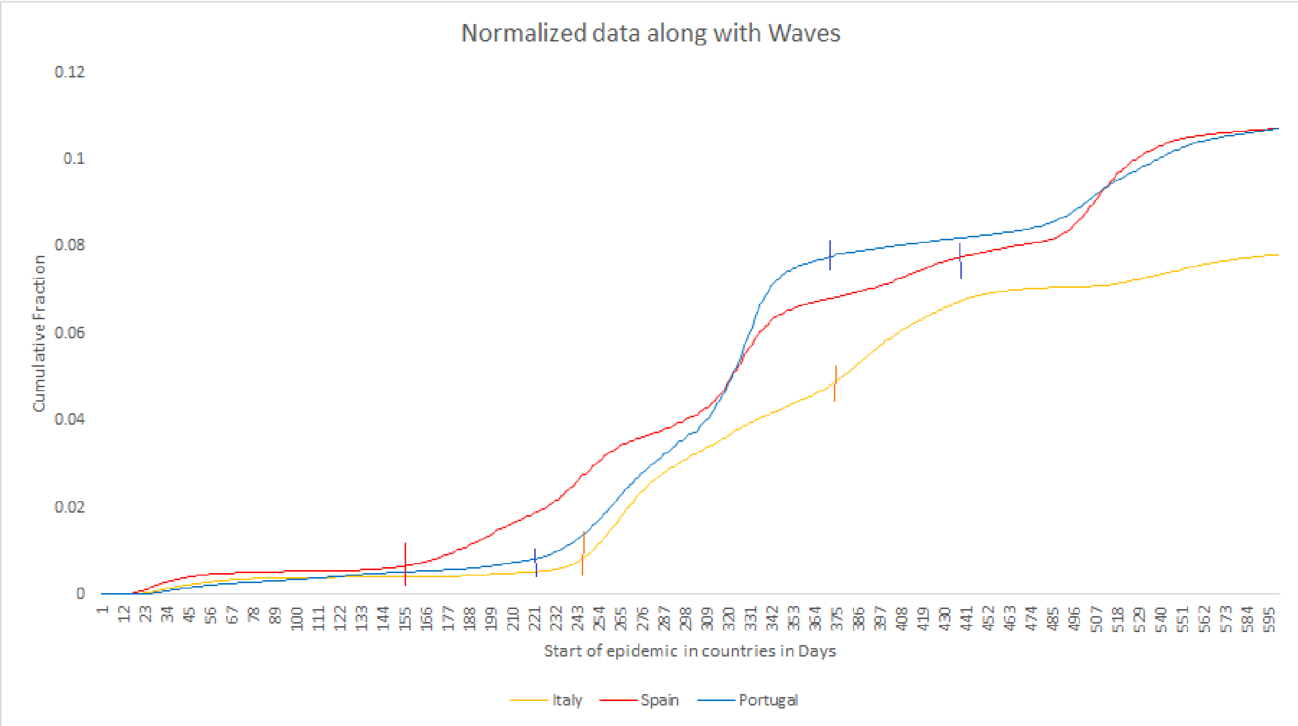

Figure 6 - Normalized Data along with Waves

In this particular analysis, we are using almost horizontal fragments heuristic approach. As earlier discussed in draft 1, The different waves for countries we have defined time series of data. The 3 countries have been divided into 3 waves each. For Italy, the first point at 256th Day, second point at 375th day. For Spain, the first point at 156th day and second point at 438th day. For Portugal, the first point at 214th day and second point at 371th day. To understand better Figure 6 illustrate the separation points which will define 3 waves for each country.

In the almost horizontal heuristic approach, we are dividing the time series graph in various waves to find the segments for good analysis by drawing lines as we find the increase in the waves.

Spain:

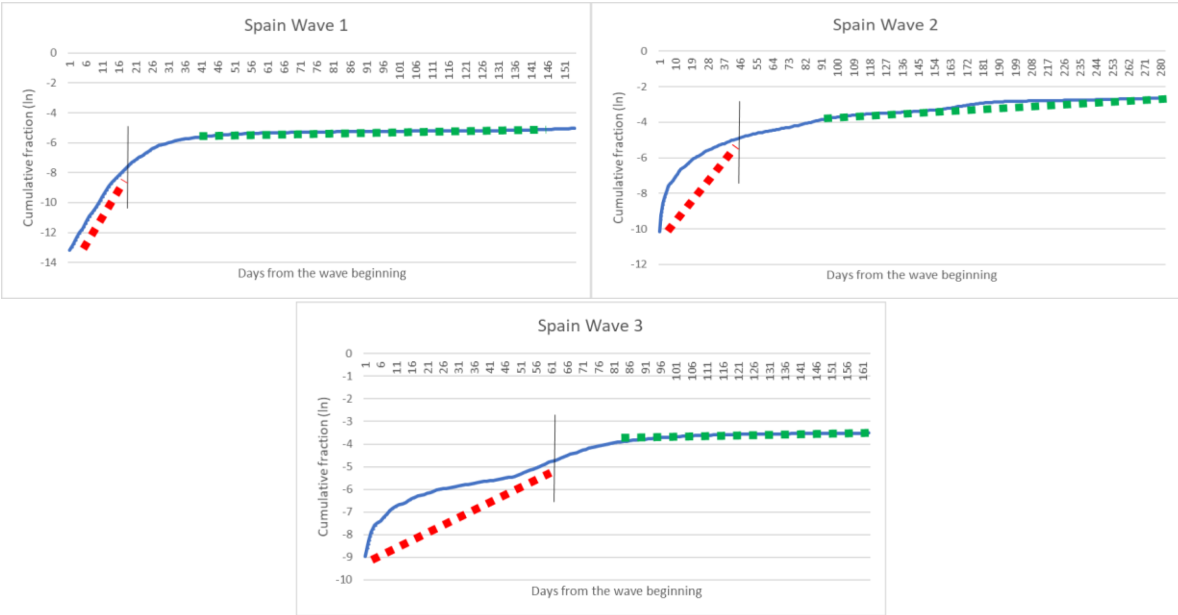

Figure 7 - Spain: Logarithm of wave - 1,2,3

In this, we can see the first horizontal interval is from 1 to 18 for wave 1. For wave 2 and 3, the horizontal intervals are from 5 to 40 and 3 to 62 respectively. Now we will estimate parameters of model using equation $\ln P(t) = a + rt$.

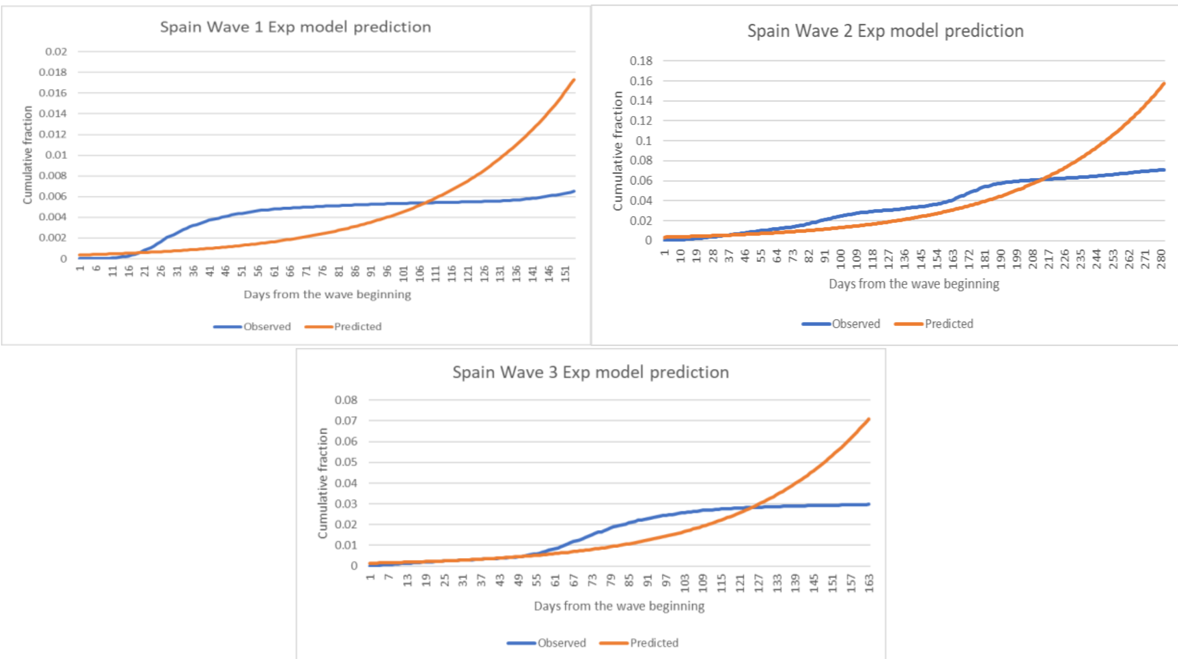

Figure 8 - Spain: Exponential Model Prediction Wave- 1,2,3

Along with the Calculation, we have found the parameters a = −7.9474 and r =0.025266 for first wave. Same has been illustrated in Figure 8. Now, we will estimate Carrying Capacity k from above calculated parameters. k is minimum at the beginning of wave and as every passing day, value of k increases. Beyond 45 days the value of k. aches maximum and then again starts decreasing. Also, in interval of day 20 to 134, value of k is less than 1, which is not useful for analysis side. At the end of first wave, the value of k is greater than 1. Considering the value of k = 0.001772 as initial estimation. This will be useful and required to estimate the parameters of logistic growth for logistic model. Similarly, for next two waves we have calculated the parameters which are stated in Table 1.

		Wave 1			Wave 2			Wave3		
Spain	Model	a	r	k	a	r	k	a	r	k
	Exponential	-7.94741	0.025266	NA	-5.71923	0.01372	NA	-6.5906	0.02421	NA
	Logistic	-1.16579	0.014972	0.00177	-3.4843	0.02808	0.0612	-3.66994	0.0323	0.046435

Table 1 - Parameters for Spain

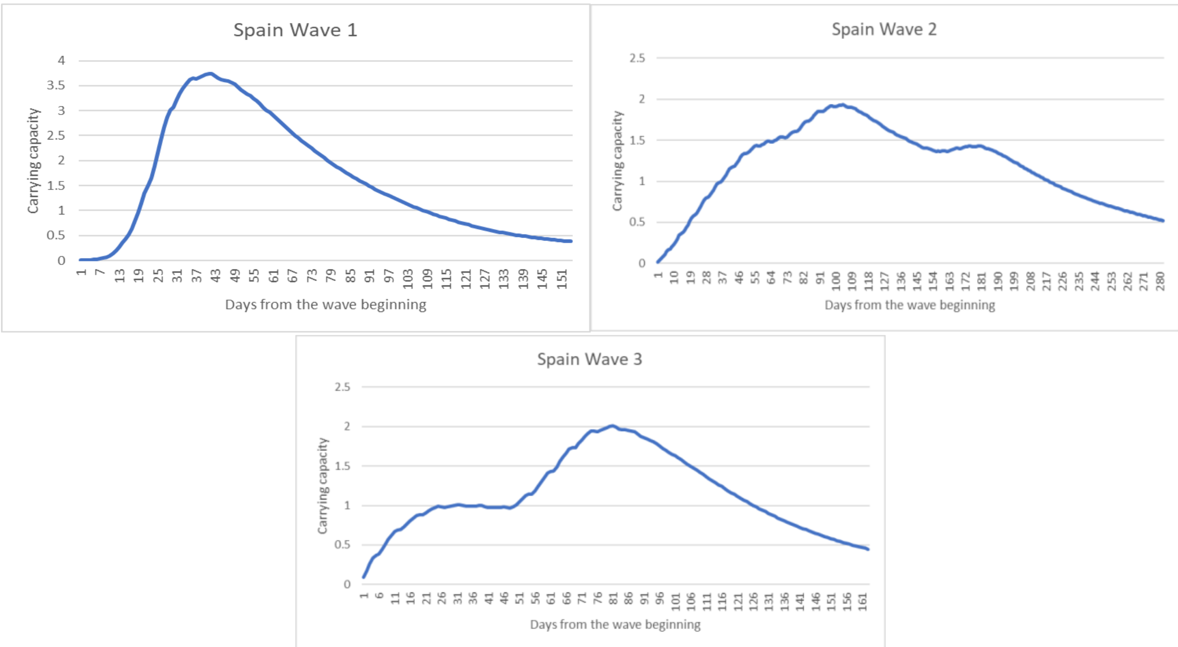

Figure 9 - Spain: Carrying capacity for Wave - 1,2,3

The Figure 9 provides information about the variation in Carrying Capacity k. To find the value of k we have used: $k(t) = \frac{P(t) + (1+\exp(a+rt))}{\exp(a+rt)}$. For wave 1, we can find the there is great spike after 10 days and it start decreasing after 50 days. For wave 2, there is instant increase since start where it again ends up decreasing once it reaches maximum. For wave 3, there is increase in value of k from the start though at certain point value goes constant and after 53 day, again instant growth in k. By the end of wave, the carrying capacity decreases gradually.

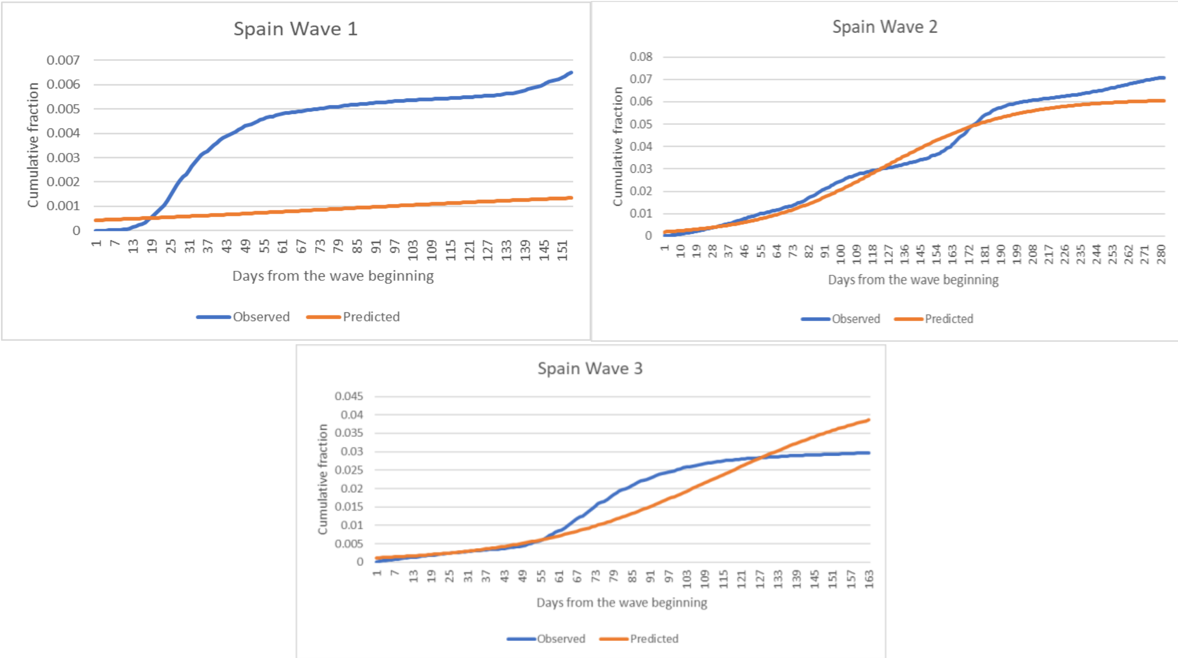

Figure 10 - Spain: Observed vs Predicted Model Wave – 1,2,3

Figure 10 illustrate the all three wave patterns which are for observed population and predicted population. For wave 1, there is a lot of difference between observed and predicted value. In comparison, wave 2 and wave 3 have good prediction of population as of observed.

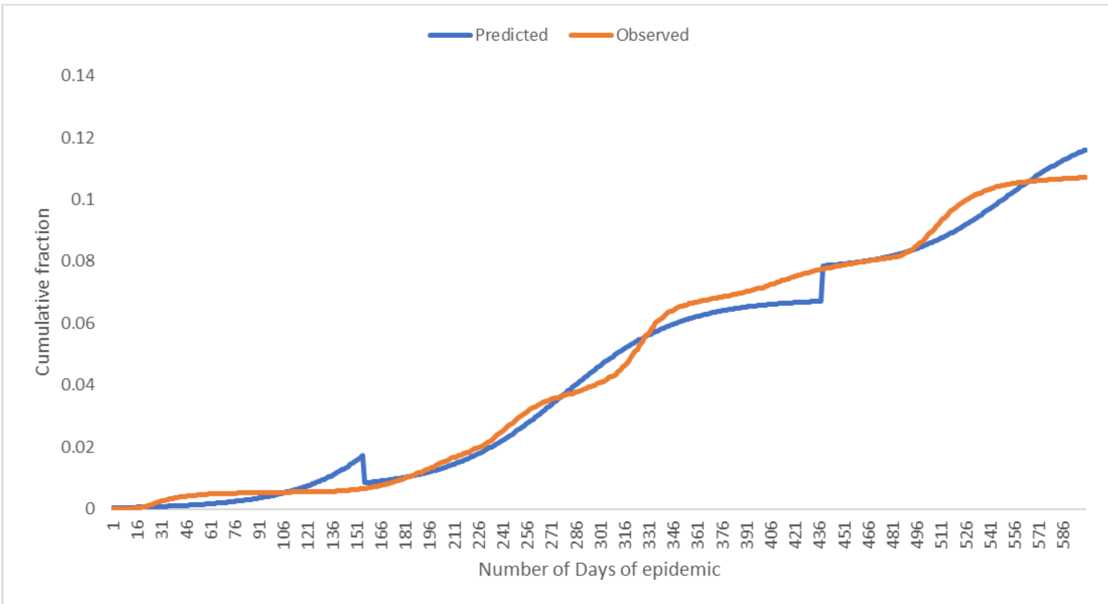

Figure 11 - Spain: Observed vs Predicted

After splitting the waves on the basis of Intervals, we have found the predicted model with help of almost horizontal method. As we can see from the Spain graph the expected value and original value are synchronous. We have taken carrying capacity (K = 0.001771603) which is less than Sum Square Error (SSE = 0.002046639) of the waves respectively.

In Figure 11, illustrating the combination of wave 1, wave 2 and wave 3 which are stated in Figure 10 earlier. The difference between end point of current wave and starting point of next wave have been joined to show the how the prediction is going on.

Italy:

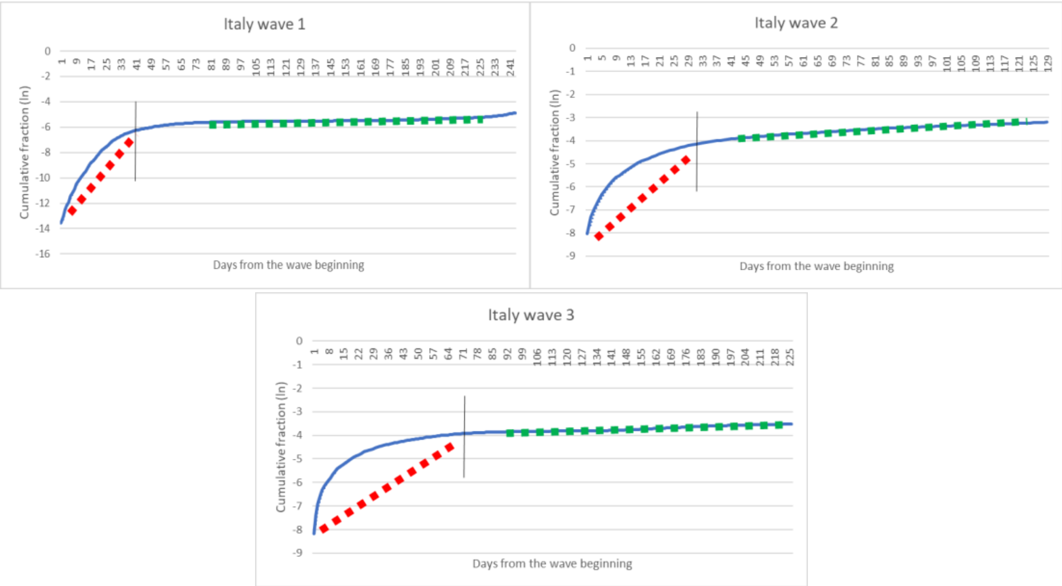

Figure 12 - Italy: Logarithm of wave -1,2,3

Here, we have taken the interval from day 2 to 46 for wave 1. For wave 2 and wave 3 the intervals are from day 2 to 130 and day 10 to 149 respectively. We have used the logarithm approach to analyse the change in wave pattern and find parameters for model.

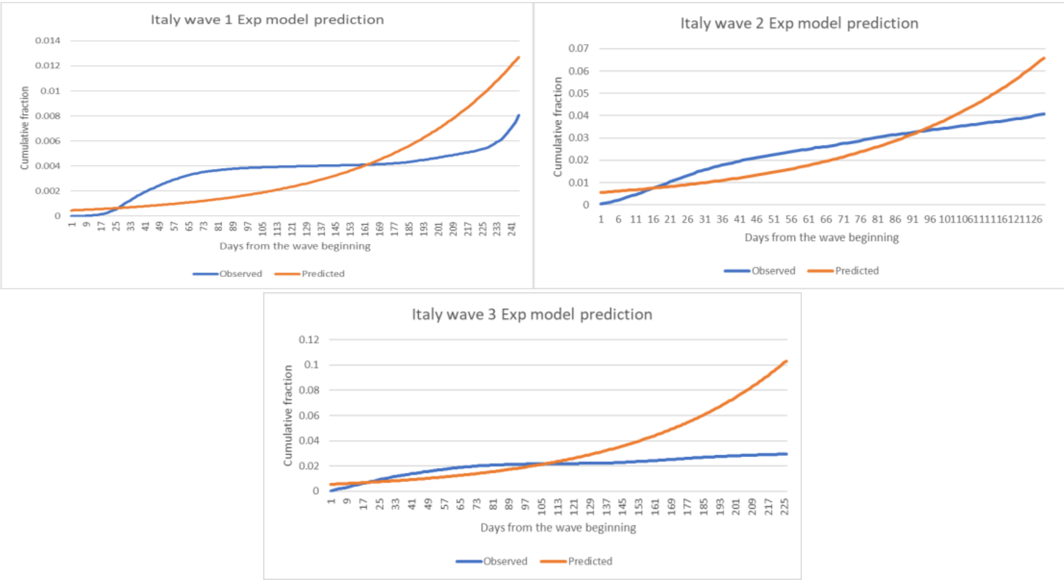

Figure 13 - Italy: Exponential Model Prediction Wave- 1,2,3

For Italy, as stated in Table 3, we have following values of parameter calculated for wave 1, wave 2 and wave 3.

		Wave 1			Wave 2			Wave3		
Italy	Model	a	r	k	a	r	k	a	r	k
	Exponential	-7.7090	0.0136	NA	-5.2064	0.0193	NA	-5.2280	0.0131	NA
	Logistic	-0.5530	0.0099	0.0029	-2.3544	0.0280	0.0587	-2.1517	0.0168	0.0524

Table 2 - Parameter for Italy

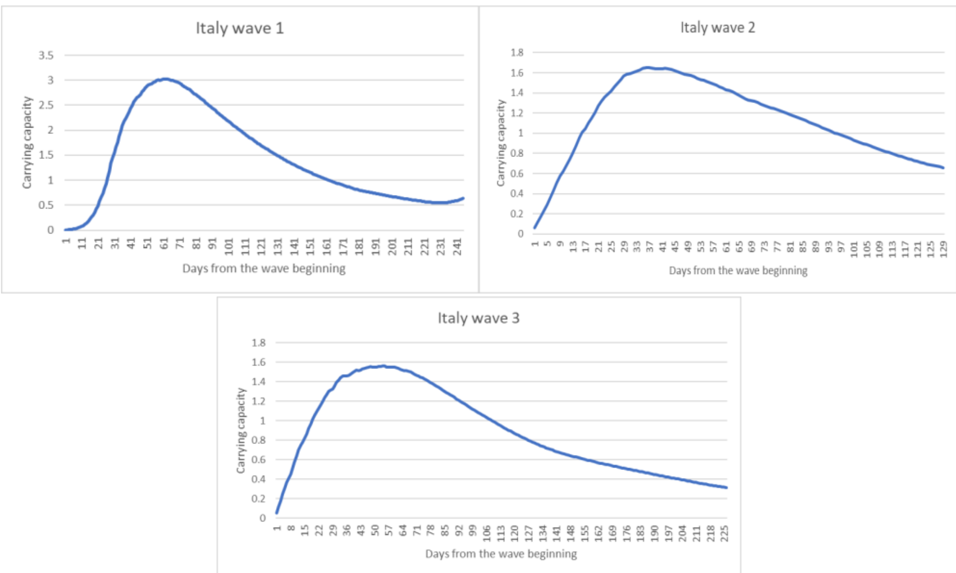

Figure 14 - Italy: Carrying capacity for Wave - 1,2,3

Figure 14 illustrate the pattern for carrying capacity k. For wave 1, we can find that value of k Is increasing after day 13. For wave 2 and wave 3, we can find somewhat similarity that value of k is increasing highly from the day 1 itself.

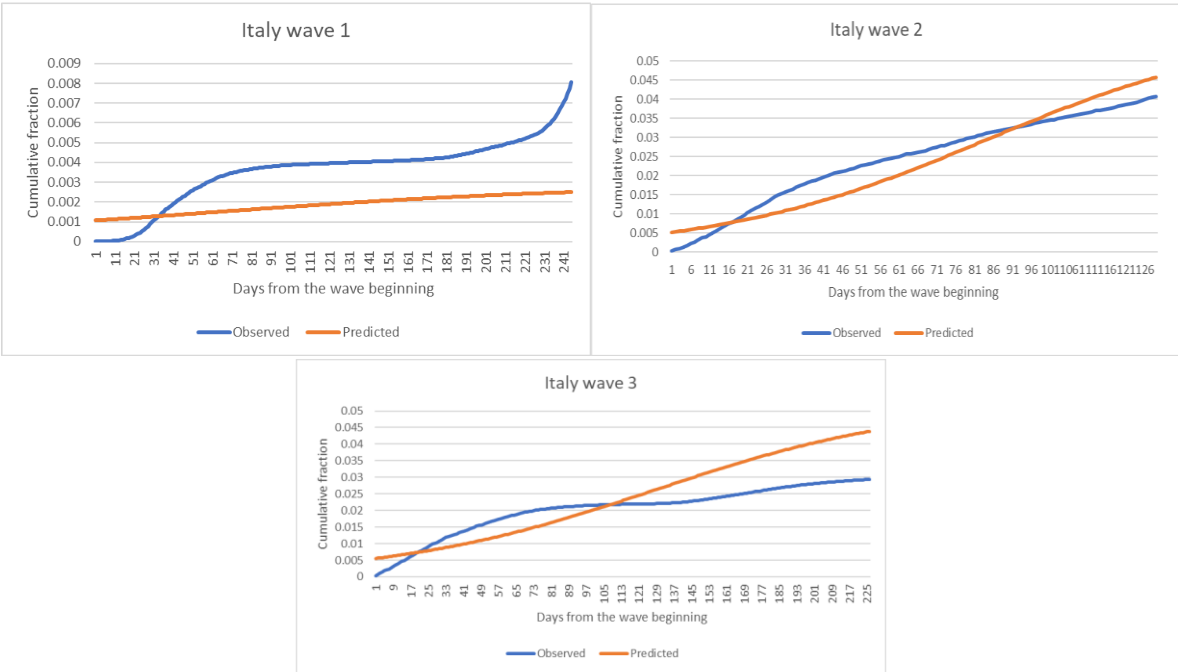

Figure 15 - Italy: Observed vs Predicted Model Wave - 1,2,3

In figure 15, We can find the prediction of all three wave as of observed values. Wave 1 have more difference in the predicted values as compare to wave 2 and wave 3.

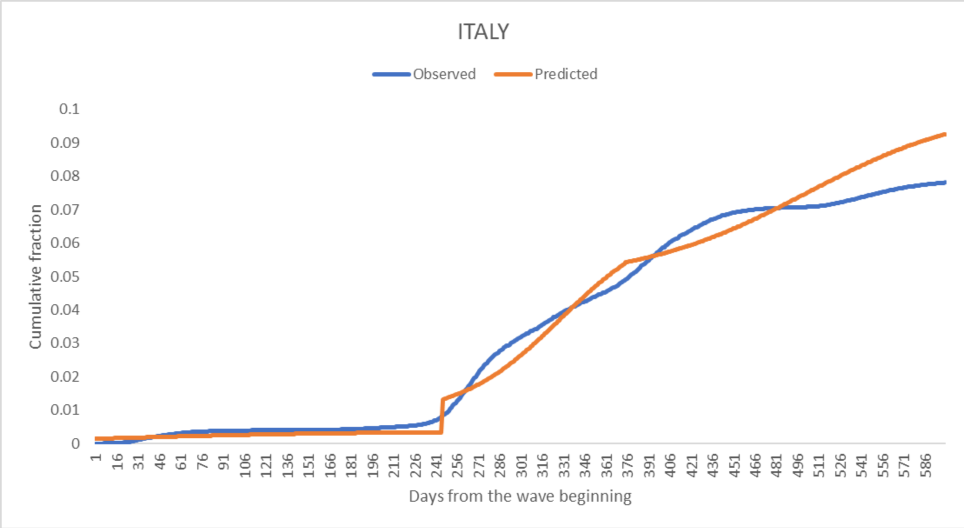

Figure 16 - Italy: Observed vs Predicted

From Italy graph we can see expected and original value are overall synchronous. Though there is some difference in final observed wave and predicted wave but which is not very huge. We have taken carrying capacity (K=0.05238831) less than Sum Square Error (SSE = 0.142577017) of the waves respectively. If we see the first wave which is predicted and observed, we don't find much difference as we have taken into consideration the minimum value of k.

The difference between the end point of first wave and start point of second wave is just slightly different. There is a slight error in the predicted value which is more than the observed value at the end of the third wave. The predicted wave and observed wave have some difference. Taking all the waves into consideration, there is not much difference between the observed and the predicted model. The Italy predicted vs observed model is not perfect but is synchronous.

Portugal:

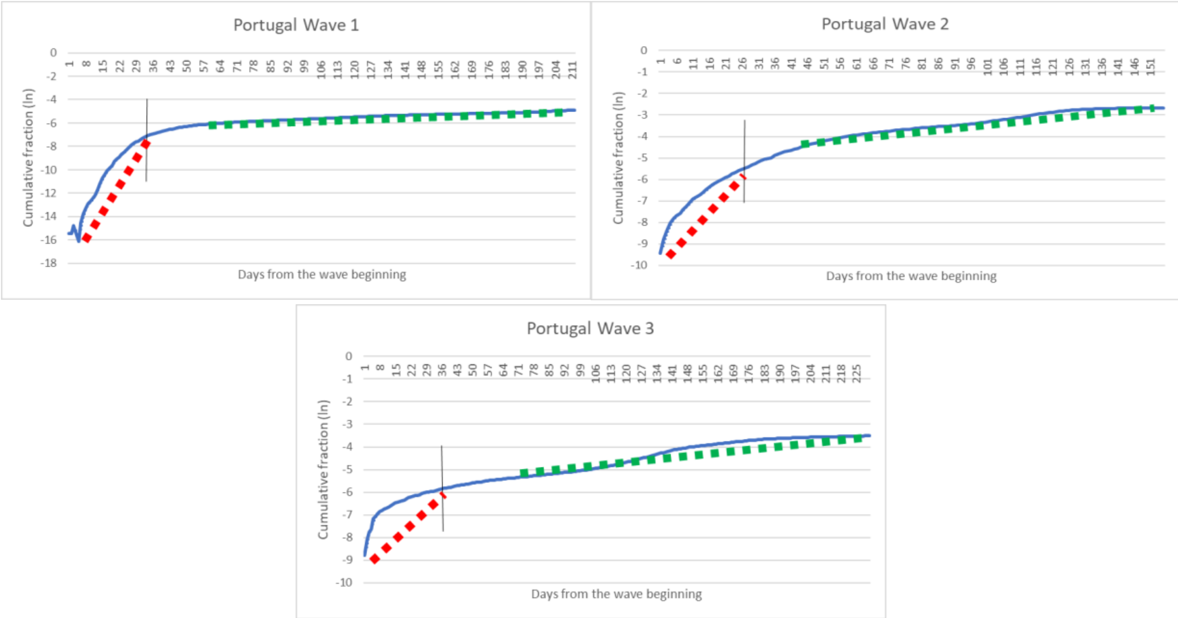

Figure 17 - Portugal: Logarithm of wave -1,2,3

As shown in Figure 17, the graphs are explaining the intervals taken into consideration for logarithmic analysis to find parameters. Wave 1 have interval from day 2 to 35, wave 2 have interval from day 6 to 27 and wave 3 have interval from day 9 to 37.

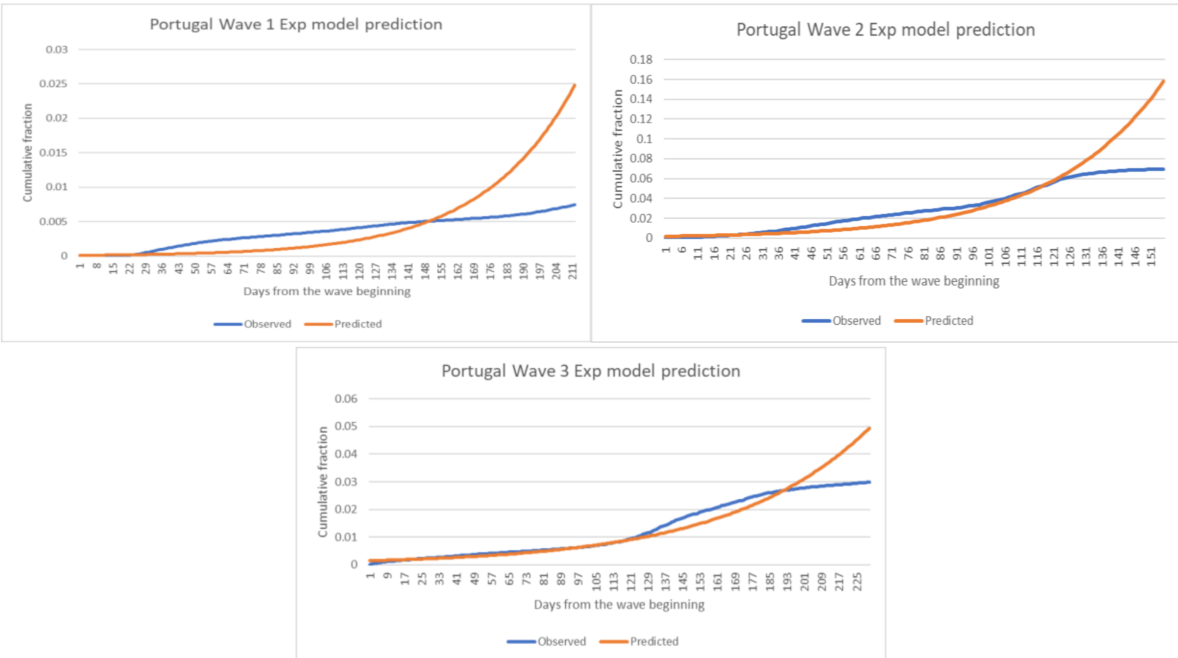

Figure 18 - Portugal: Exponential Model Prediction Wave- 1,2,3

For Portugal, in Table 3, we have following values of parameter calculated for wave 1, wave 2 and wave 3.

		Wave 1			Wave 2			Wave3		
	Model	a	r	k	a	r	k	a	r	k
Portugal	Exponential	-9.1169	0.0256	NA	-6.4065	0.0294	NA	-6.5682	0.0154	NA
	Logistic	-3.2928	0.0332	0.0045	-3.2677	0.0392	0.0453	-0.0846	0.0048	0.0069

Table 3 - Parameters for Portugal

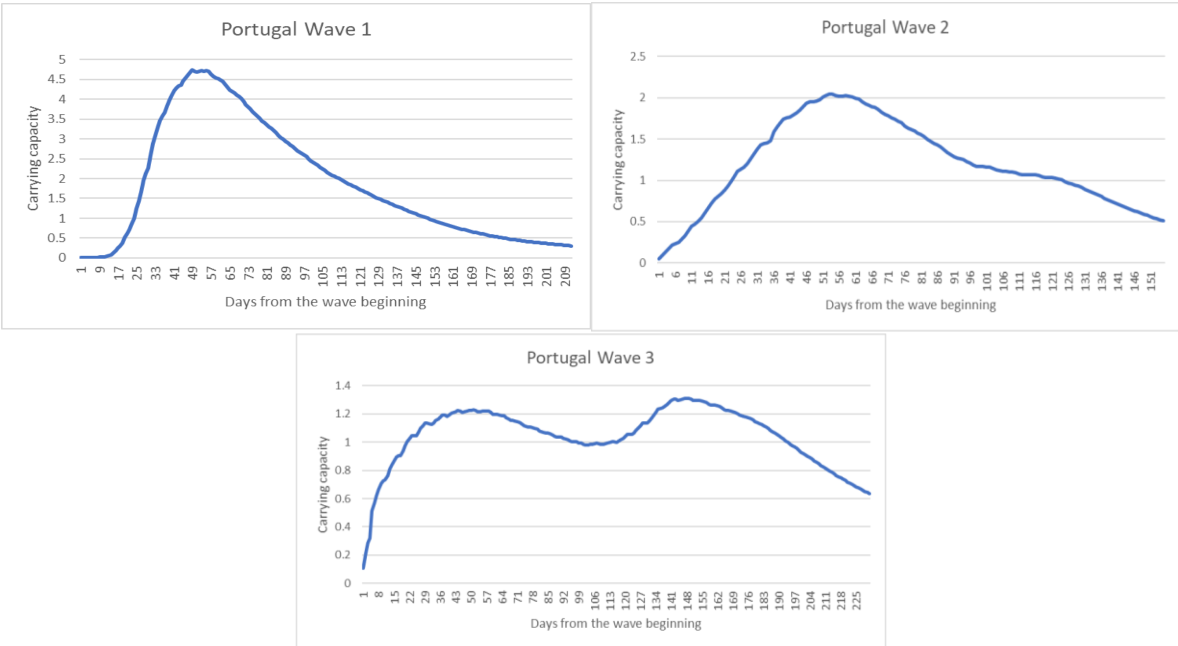

Figure 19 - Portugal: Carrying capacity for Wave - 1,2,3

The graphs explained about the Carrying capacity for all the three wave of Portugal in Figure 19. We can find the linear growth of k in wave 2. For wave 1, the horizontal growth starts growing rapidly from day 18. For wave 3, the growth in the value of k is started from day 1 itself.

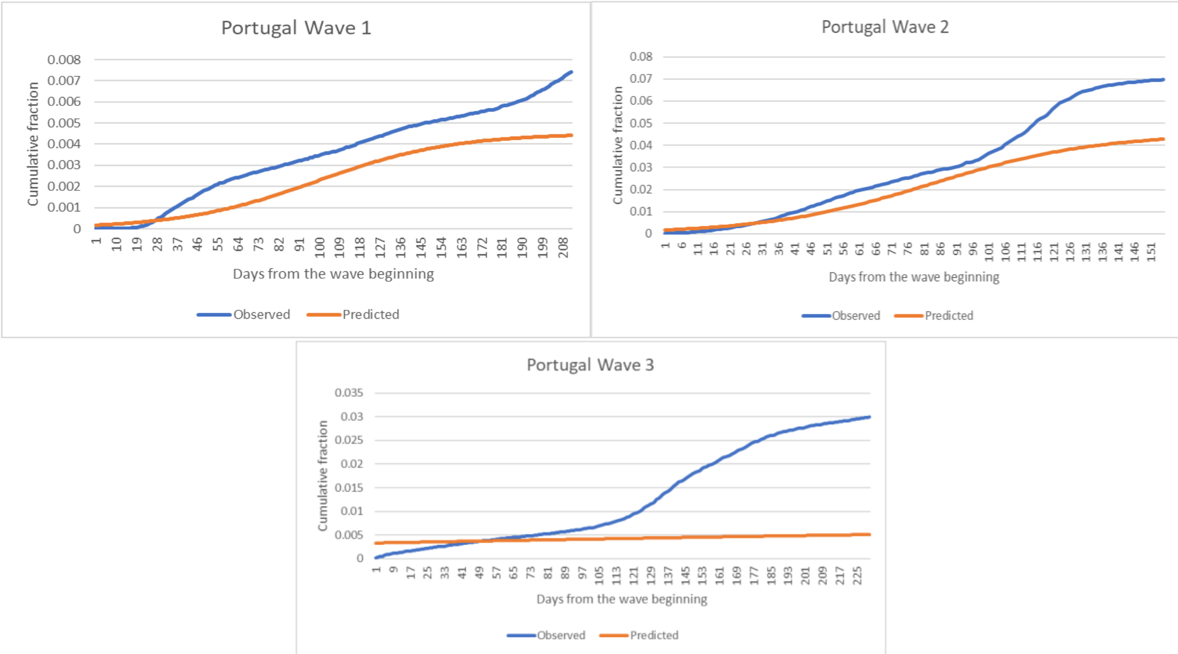

Figure 20 - Portugal: Observed vs Predicted Model Wave - 1,2,3

Figure 20, we can find the analysis of predicted vs observed values for the three waves. Wave 1 and wave 2 have not big difference in the predicted values as of observed ones. Though in wave 3, we can find high difference in ending of wave.

As we can see from the graph of Portugal, expected value and original value are less synchronous. There are predicted waves which shows asynchronous behaviour with observed waves. After combining all three waves, we can state that the predicted vs observed model is asynchronous as the final point of previous wave and first point of next wave having difference which is not minimum. Here also we have taken carrying capacity K which is less than Sum Square Error of the waves respectively. We have taken into consideration the minimum value of k but there is some error between the predicted model and the observed model. The difference between the end point of first wave and start point of second wave is slightly different which is also the case with the second and third wave. Taking all the waves into consideration, there is some difference between the observed and the predicted model in this case.

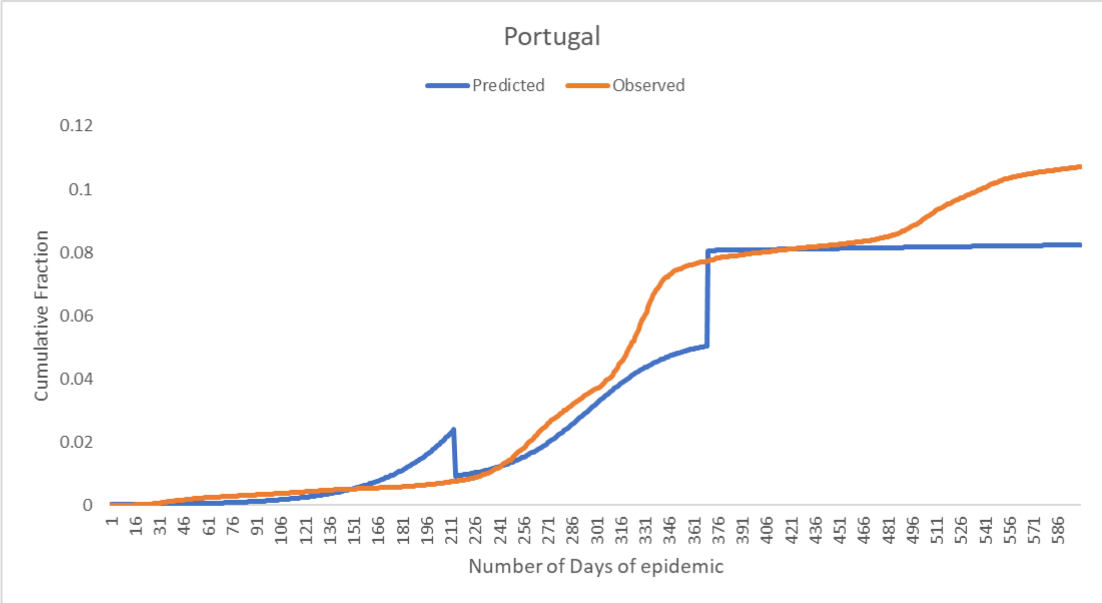

Figure 21 - Portugal: Observed vs Predicted

All the 3 selected countries have similar number of waves as observed from the graphs. By comparing all three countries (Italy, Spain, Portugal), we can find the synchronous and asynchronous model. All the three countries have similar number of waves which are three. By using almost horizontal method, we have calculated predicted vs observed model for these three countries.

In analysis of each of individual wave of three countries. The countries have similarities at the start of pandemic and increase in the cases. But the predicted values have huge difference as of observed values for country Portugal.

We can find that the Italy and Spain model is synchronous, as it is predicting somewhat similar plot as of observed model. Though the waves have minimum difference in the intervals. Taking Portugal, in consideration we can see the predicted model and observed model have difference, which is not minimum. Hence, the model is Asynchronous.

6. SIR MODEL:

SIR model is an epidemic-logical model which computes the people infected with infectious illness over the time. The name of this model is derived from the equation of Susceptible, Infected and Recovered People. $S(t)$ is the number of susceptible people, $I(t)$ stand for number of infected people and $R(t)$ stands for number of recovered people. Kermack-McKendrick Model is simplest SIR model.

In this we have the excel data of the modelling approach we have used the mathematical modelling on the covid-19 situation.

In this discussion we have used model susceptible-infected-removed (SIR) model that provides a data to investigate within a covid time.

The SIR model indicates that susceptible-infected-removed with the total population that has been defined constant. The number of susceptible individuals does not show here in such time. In that particular we used to calculate the time of the different countries. In this we used to calculate

The different country is significant the time for the spreading of the disease in a various cities and communities like Spain, Italy, Portugal.

We used to calculate the waves by using the python file by the covid -19 cases from the effected data from the excel sheet we find to compare the SIR model to improve the mean percentage error. In the SIR Model is determined by two parts one is infection rate α and recovery rate β.

The SIR model is governed by three equations, where at time t, is the size of the susceptible population $S_{t,}$ I_t is the size of the infected population, R_t is the size of the recovered population, and N is the total population.

$$N = S(t) + I(t) + R(t)$$

We assume in our training data we have for daily information on, $I(t)$ the number of new cases, and $R(t)$ the number of recovered cases on

each day t. Given these data, we calculate the starting condition of the wave, or a given wave n, the parameters are only dependent on the initial conditions of the wave (number of susceptible, infected, and recovered at the start of the wave.

As per the SIR model, when the S is closer to 1, we will get exponent $r = a - b$.

Using the given value of b = 0.1, we will be taking the first 10 days to measure the $R(0)$ i.e., $\tau = \frac{1}{10}$. Now, we will get the $R(0)$ using the first 10 days' cases. This helps us to find the people infected at the start of the pandemic.

Next, we will calculate $a = r + b$. This will mathematically provide the SIR parameters.

$$\frac{dS}{dT} = -aSI$$

$$\frac{dI}{dT} = aSI - bI$$

$$\frac{dR}{dT} = bI$$

	r	b	a
Spain	0.036976	0.1	0.136976
Portugal	0.048408	0.1	0.140408
Italy	0.020584	0.1	0.120584

Table 4 - Coefficient a, b

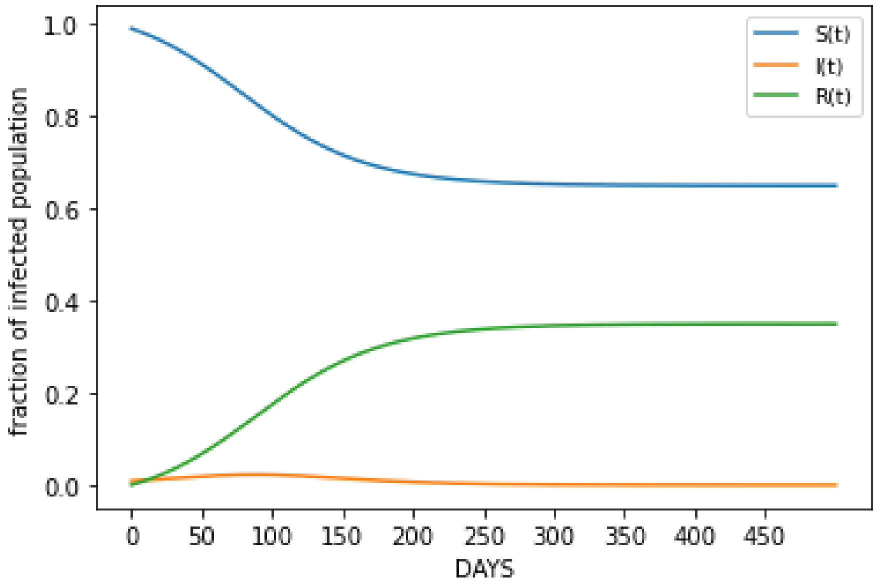

Figure 22 - SIR MODEL - Italy

Now taking values of S, I, R in consideration we illustrated the graph for SIR model for Italy. This graph shows the rate of infection was not high at the time of start so as the rate of susceptible and the death rate. After 89 days, we can notice the change in Infected rate and rate of death increasing and rate of Susceptible decreasing. This Shows The decrease in the waves after day 223 for infection rate. Rate of susceptible and Recovered/Death started normalizing.

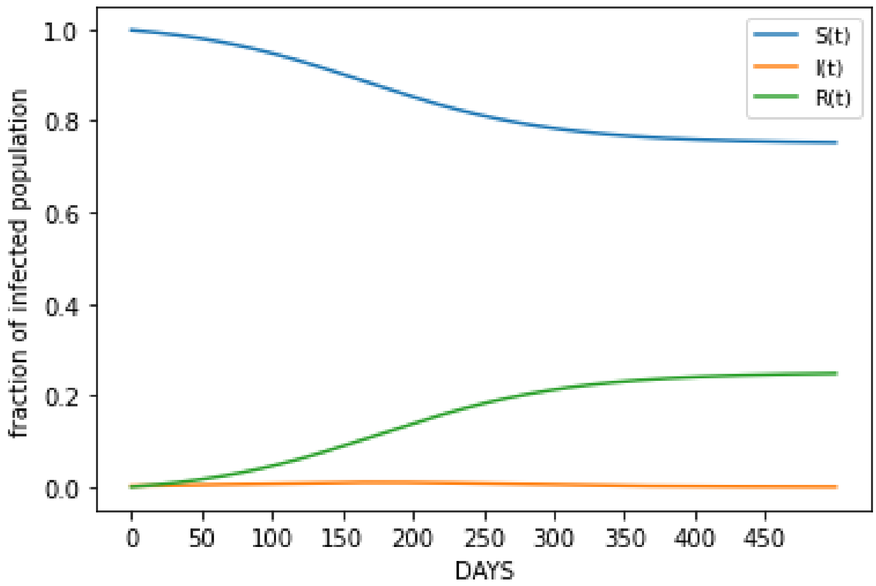

Figure 23 - SIR MODEL - Spain

For Country Spain, we can find that the recovery rate is increasing after 84 Days, till that the rate of recovery was not drastic. We can see the change in infected rate along with susceptible rate. This is due to the second wave of covid in Spain after 160 days. At the end of second wave, we can get the stabilized or have less change in S, I, R.

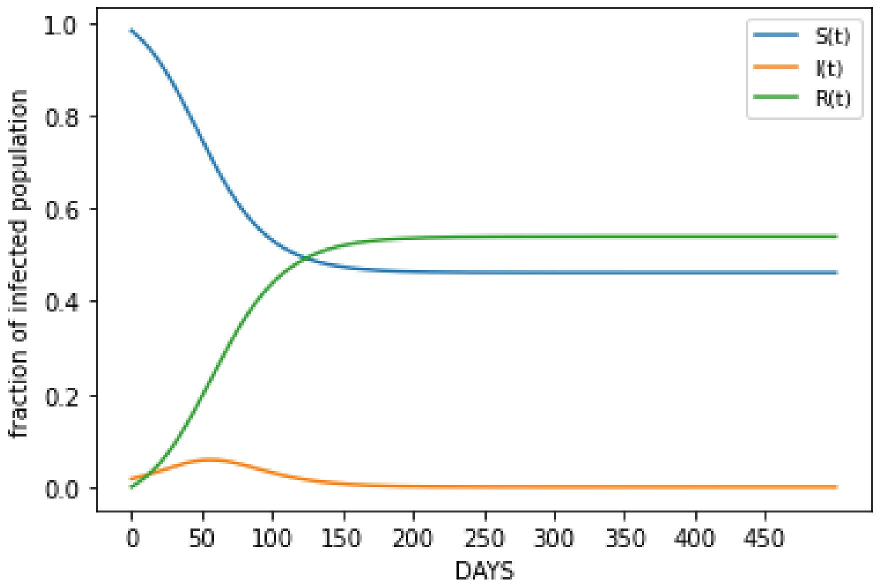

Figure 24 - SIR MODEL - Portugal

Now for Portugal, we can see the rate of change in Infected at day 106 where we can find a sudden spike in rate of recovery/death and decrease in rate of susceptible. This is due to the increase in number of cases at the time of wave 1 end. At the time, start of second wave, the infection rate is going down as the line starts normalizing the at after day 250.

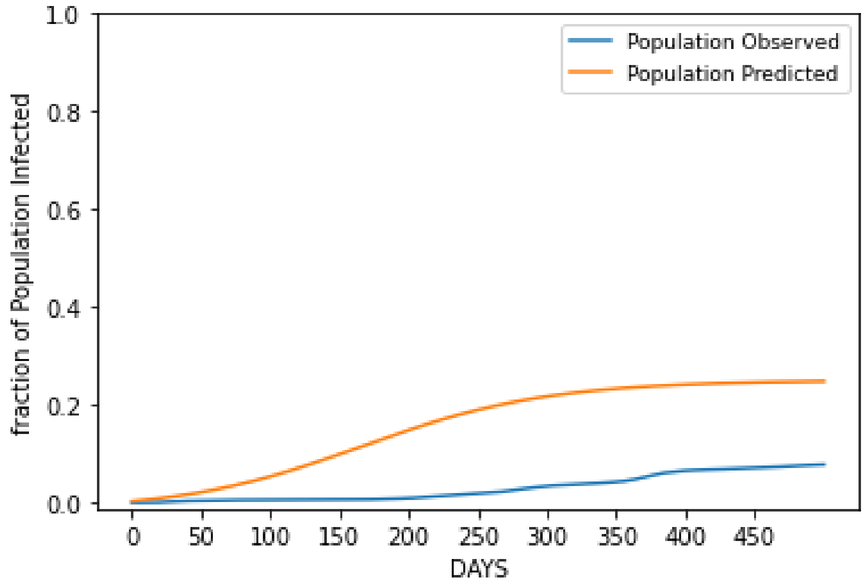

Figure 25 - Spain - Observed vs Predicted

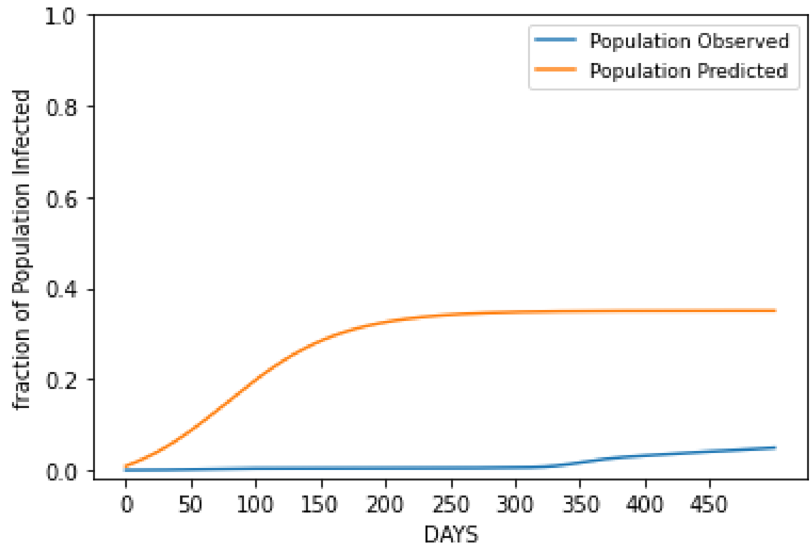

Figure 26 - Italy - Observed vs Predicted

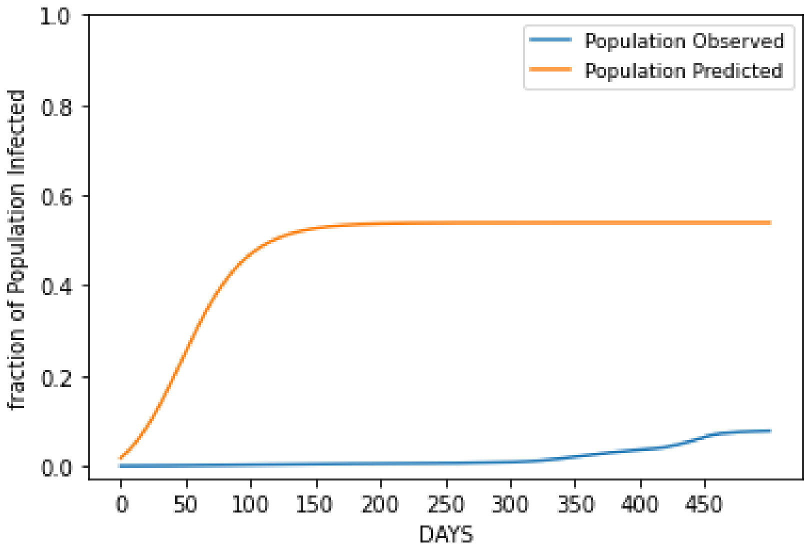

Figure 27 - Portugal - Observed vs Predicted

Here we have taken different values of I (0). Let us consider the five different values of I (0) as follows:

$$I01 = \frac{I0}{4}, I02 = \frac{I0}{2}, I03 = I0 * 2, I04 = I0 * 2, I05 = I0 * 2$$

We can find the change in different waves of Infections on this basis as below. This shows that the more the rate of infection will have less rate of susceptible and more rate of recovery when the cure if found. In the below graph we can see if I0 is quarter then there is less rate of Infection in compare to the population observed. And so, on I05 have more rate of infection. The following graphs Figure illustrates the model of different Initial values of I0 for Spain, Portugal, Italy respectively.

If we compare the three countries, we can state that the rate of change of infection affects more in Italy than Spain and Portugal. As we know, Italy had very bad phase of covid in between the wave 1 and wave 2. Where the number of recoveries were very low while infection rate was high.

If we take summerize the graphs we can find that if the rate of infection is half or quarter times then it is less than the observed population. But as the rate of infection starts multiplying, we can say the observed value of population is less.

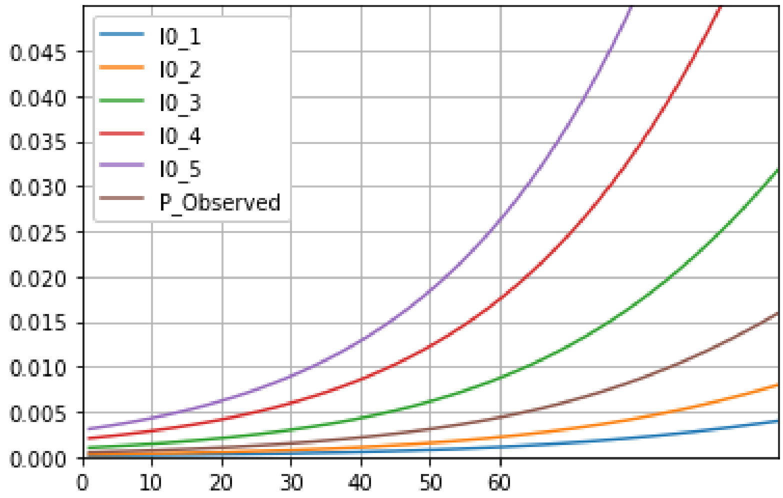

Figure 28 - Different Values of I(0) - Spain

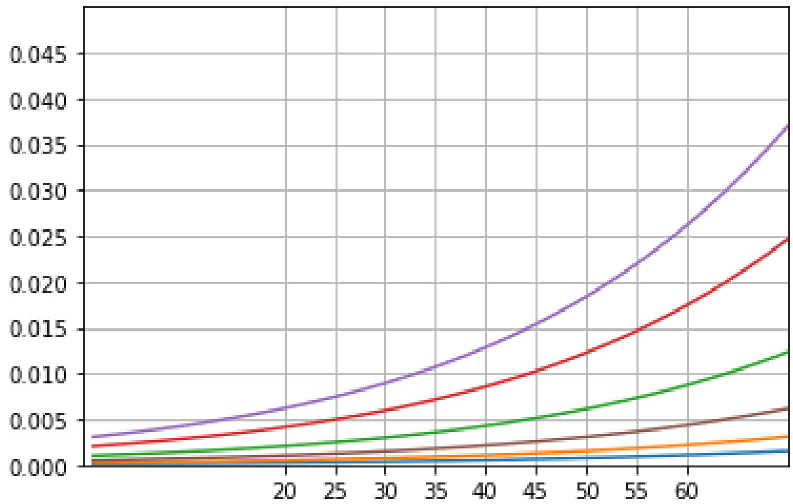

Figure 29 - Different Values of I(0) - Portugal

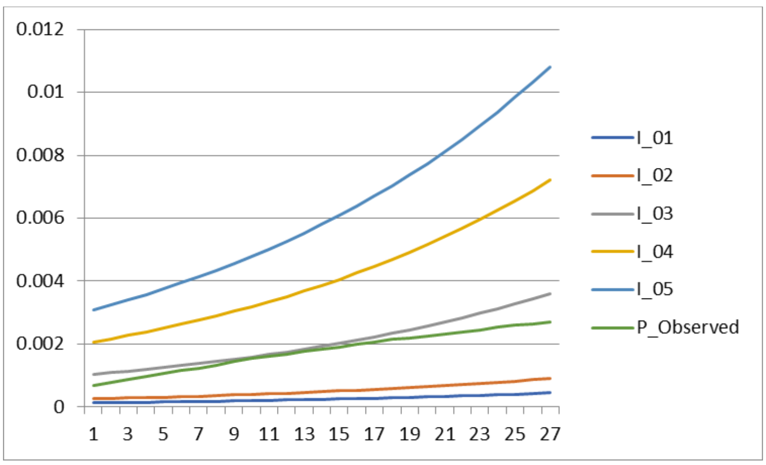

Figure 30 - Different Values of I(0) - Italy

Country	a	b	MSE
Spain	0.1369	0.1	0.1041
Portugal	0.1031	0.1	0.1305
Italy	0.1561	0.1	0.1134

Table 5 - Optimal Parameters and MSE

Country	MSE1	MSE2	MSE3	MSE4	MSE5
Portugal	0.000104	0.000425	0.006869	0.005814	0.008502
Spain	0.008555	0.034219	0.547514	2.190056	4.927627
Italy	0.003524	0.012421	0.089514	0.468951	0.778231

Table 6 - Different MSE calculated for all three countries

For this research, we have considered the whole data r and a. Which has included confirmed, deaths and cases infected more than once per day. We have used the data of the individuals of each person affected by covid. All data are taken in excel and python data The number of confirmed cases and deaths has been graphically illustrated in the Portugal graph. It shows that there is a begins between 23 to 73 and 214 to 371 are recovered. we can say that the Portugal graph has started and includes the reported cases of people who have been tested. When it knows the actual number of individuals with got COVID-19 some people recovered. All of the people in Portugal have been tested. We can see the graph through the Portugal data set. We have b= 0.1, a= 0.140583868. The country has not the most infected patients from Portugal. We see that Portugal has started from wave 1 it started from 1 to 151, and there are not too many people who got affected` coming to wave 2 it has started 1 to 225. Coming to wave 3 is from 18 to 181.

For Italy, we have taken interval from 27 to around 80 where we can see the change in exponential wave. If we compare this on basis of interval taken, the value of slope is higher than the one used in first draft. This can be optimized more.

Here, the factor b is constant i.e., 0.1, if we consider the changing the b factor, we can improve the SIR model. Another thing is there are some number of people who were susceptible and then got infected and recovered and got infected again. So, this type of people will also make the difference.

SIR model is, used to analyse the spread of COVID-19 during the covid disease weather the person got or not. Susceptible–Infected–Removed (SIR) epidemic model and proposes a Generalized SIR (GSIR) model that is an integrative model encompassing multiple waves of daily reported cases. Existing growth function models of epidemic have been shown as the special cases. Dynamic modelling of the parameters reflects the impact of policy decisions, social awareness, and the availability of medication during the pandemic. There are many mathematical approaches for the modelling and analysis of the spread of infectious, contagious, or both types of diseases in the human population. The mathematical models provide important information such as the basic reproduction number.

The effects on second waves are due to the vaccine supply and availability, deaths caused due to mutation of the COVID. Possibility of people who had covid and have other diseases including diabetes, tuberculosis, asthma and people who are of older age. This causes the spike in the cases for Spain, Portugal and Italy. It started to normal after the finding cure for the mutation and double vaccination, booster vaccine dosage. Also, as the availability and supply of covid vaccines such as Sputnik, COVAXIN, COVISHIELD, etc.

These factors also played an important role in second covid wave as well as third covid wave which are not considering currently. Also, on the basis of the above-mentioned factors we can improve the model. So, we can achieve near to correct analysis of SIR model.

The SIR Model Overview:

A model in which they considered a fixed population with only three compartments: susceptible

$S(t)$ is used to conclude that each person who is infected with the disease at time t, or those susceptible to the disease of the population.

$I(t)$ denote the individuals of each person who have been infected with the disease and are capable of the disease to those in the population

$R(t)$ is the compartment used for the individuals of each person who has been got covid and then who did not get a disease, either due to immunity or due to death. Those in this category are not able to be infected again or to transmit the infection from one person to another person

There are many modifications of the SIR model, including those that include births and deaths coming to recovery there is no immunity (SIS model) where immunity will be at a short period of time (SIRS)

There are two types of epidemic models stochastic and deterministic we finally concluded that is a disease for estimating the disease for the probability distributions of covid disease by allowing for a random person to person and it depends on the chance variations in the risk of exposure, disease, and other illness dynamics. It is a level of disease dissemination in small or large populations that can be used from the stochastic methods.

We are using the population which is approximately correct where we are considering the equation $S + I + R = N$. In these particular, natural births and natural deaths are not taken into consideration. Here, even though the covid can be fatal, the recovery rate cannot distinguish in between recovered individuals or deaths.

Another point which is not taken into the consideration that β and γ are constant. This means, it is unaffected by the age of patients, virus mutations, natural deaths or any other death or additional diseases.

7. General Adaption Syndrome & Crowd Effect in SIR MODEL:

Now as we will be using classical theory of stress and GAS (General adaption Syndrome). In GAS, it describes psychological changes that organism goes through in three stage process. The three phases of stress are Alarm, Resistance and Exhaustion. This Theory was proposed by a medical doctor and researcher, Dr. Hans Selye.

We will assume that there are four types of human behaviour which are divided into four sub-populations in S:

$S_{ing} - these\ are\ the\ people\ who\ do\ not\ worry\ about\ epidemic$
$S_{al} - these\ are\ the\ people\ who\ are\ in\ Alarm\ Phase$
$S_{res} - these\ are\ the\ people\ who\ are\ very\ rational\ and\ safe\ behaviour\ in\ resistance\ state$
$S_{exh} - these\ are\ the\ people\ who\ are\ tire\ of\ epidemic, behaves\ unsafe\ and\ does\ not\ react\ on\ alarm\ situation\ and\ are\ in\ Exhaystion\ state$

So now considering the proposed sub-population, we have equation as below:

$$S = S_{ing} + S_{al} + S_{res} + S_{exh}$$

Initial stage, $S(0) = S_{ing}(0)$ and similarly all remaining components have population zero i.e., $S_{al}(0) = S_{res}(0) = S_{exh}(0) = 0$. We have now equation as:

$$S = S_{ign} + S_{res} + S_{exh}$$

Also, we can state that, the transition phase of SIR model is as:

$$S_{ign} \rightarrow S_{res} \rightarrow S_{exh} \rightarrow S_{ign}$$

Now with using Mass Action Law formalism, we have SIR:

$$S_{ign} + I \rightarrow 2I$$
$$I \rightarrow R$$

And Stress reaction as:

$$S_{ign} + I \rightarrow S_{res} + I$$
$$S_{res} \rightarrow S_{exh}$$
$$S_{exh} \rightarrow S_{ign}$$
$$S_{exh} + I \rightarrow 2I$$

Now we will modify this into kinetic equation:

Reaction Rate	Stoichiometric Vector
$\gamma_1 = aS_{ign}I$	$(-1,0,0,1,0)$
$\gamma_2 = k_1S_{ign}I$	$(-1,1,0,0,0)$
$\gamma_3 = k_2S_{res}$	$(0,-1,1,0,0)$
$\gamma_4 = aS_{exh}I$	$(0,0,-1,1,0)$
$\gamma_5 = bI$	$(0,0,0,-1,1)$
$\gamma_6 = k_3S_{exh}$	$(1,0,-1,0,0)$

Table 7 - Reaction Rate and Stoichiometric Vector

Now we have modified the differential equations as well, which will help to analyse the transition phases further:

$$\frac{dS_{ign}}{dt} = -aS_{ign} - k_1S_{ign}I + k_3S_{exh}$$
$$\frac{dS_{res}}{dt} = k_1S_{ign}I - k_2S_{res}$$
$$\frac{dS_{exh}}{dt} = k_2S_{res} - aS_{exh}I + k_3S_{exh}$$
$$\frac{dI}{dt} = aS_{ign}I + aS_{exh}I + bI$$
$$\frac{dR}{dt} = bI$$

From these equations, we have unknown constants $k_1,\ k_2 and\ k_3$ and those are as below:

$k_1 = 1$; This is reaction rate constant for $S_{ign} + I \to S_{res} + I\ which\ is\ 1$. (Transition rate is $kS_{sign}I$. In this we re assuming that the I is close to 1, after that the ignorant people will change the behaviour to resistant with time 1 day.

$k_2 = \frac{1}{50}$; This is reaction rate constant for $S_{res} \to S_{exh}\ which\ is\ \frac{1}{50}$. (This is where people will be in tired or Resistance State after 50 days)

$k_3 = \frac{1}{100}$; This is reaction rate constant for $S_{exh} \to S_{ign}\ which\ is\ \frac{1}{100}$. (This is where people will be in Exhaustion state and will return to Ignorant i.e., Initial phase)

Now further we will discuss model in detail for Spain, Italy & Portugal.

Spain:

Now for this model we are using old parameters from SIR Model worked earlier. We had earlier used differential equation and then integrated to predict the model. Now we are using the same tactic to predict the new adapted model.

To improve the prediction for earlier model we have assumed the values for k_1, k_2, k_3 as $1, \frac{1}{50}, \frac{1}{100}$ respectively. Here, in Figure 31 we can see that predicted values have been changed as we have used value of k_2. So, if any person who is in ignorant phase comes in contact of infected person, then they shift to resistance phase in a day. Below graphs in Figure 31 and Figure 32 shows the similar interpretation of values $k_2 and\ k_3$.

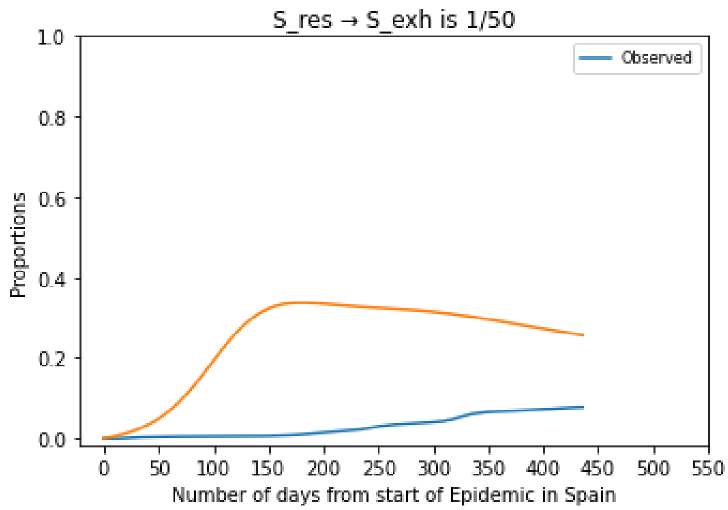

Figure 31 - For k2 = 1/50 Spain

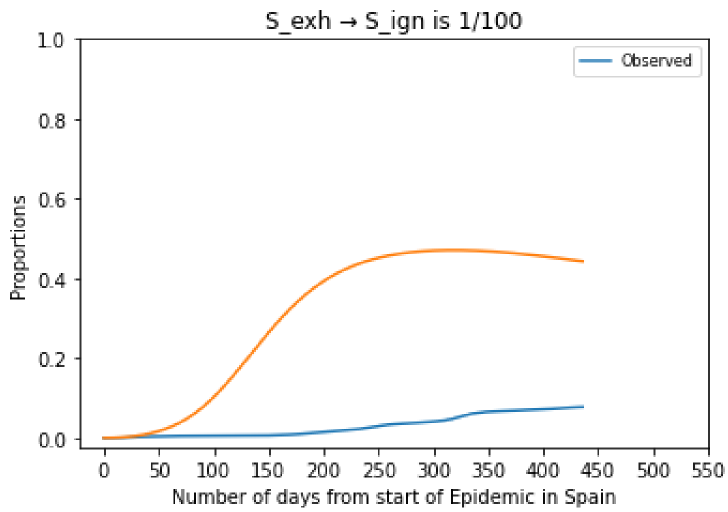

Figure 32 - For k3 = 1/100 Spain

Now, in Figure 33 we can find that combination of $\mathrm{k}_2$and k_3 which is more accurate. Below is the SIR model which is predicted after the calculating the derivatives and then integration of values. This model will define how behaviour of epidemic in the Spain. We can predict how the parameters are affecting for wave 2. The recovery rate is getting higher for the wave 2, where the people are becoming less ignorant as the wave progress. The MSE here is 0.005506399.

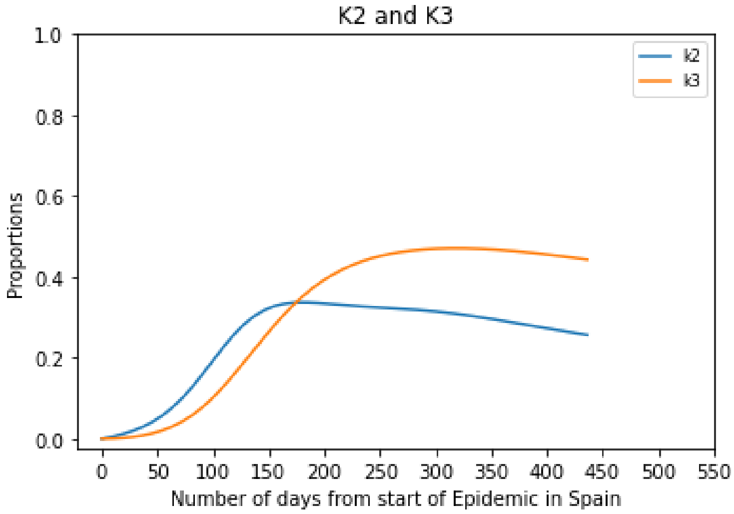

Figure 33 - For $k_2$and k_3 Spain

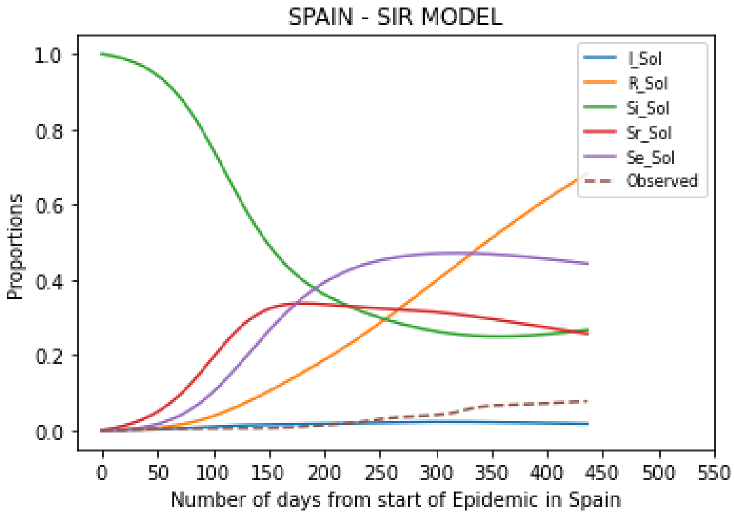

Figure 34 - GAS-SIR MODEL for Spain

Italy:

As discussed for Spain, we have predicted for Italy as well. We can find from figure 35, the resistance to exhausted rate changes as per the observed data. In Figure 36, we can check the change in the exhausted to ignorance state for Italy. The parameters are calculated using linear regression of first wave only. We can find the change in prediction as the values of k changes.

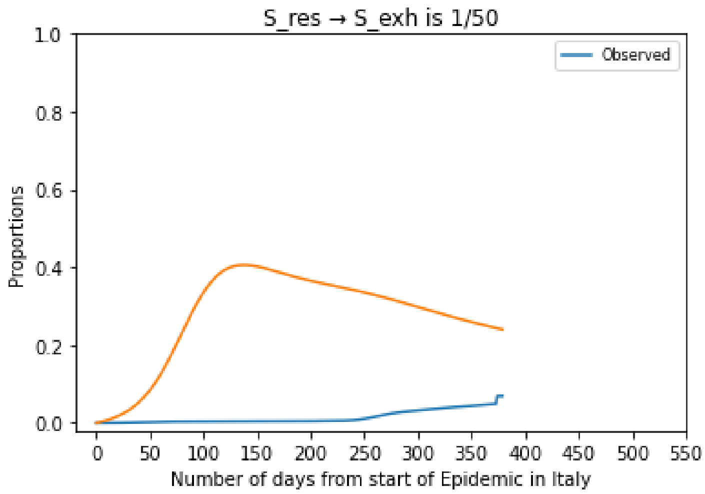

Figure 35 - For $k_2 = \frac{1}{50}$ *Italy*

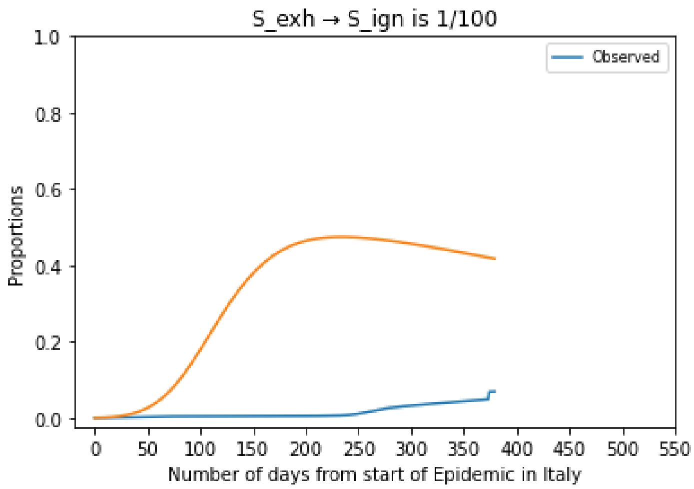

Figure 36 - For $k_3 = \frac{1}{100}$ *Italy*

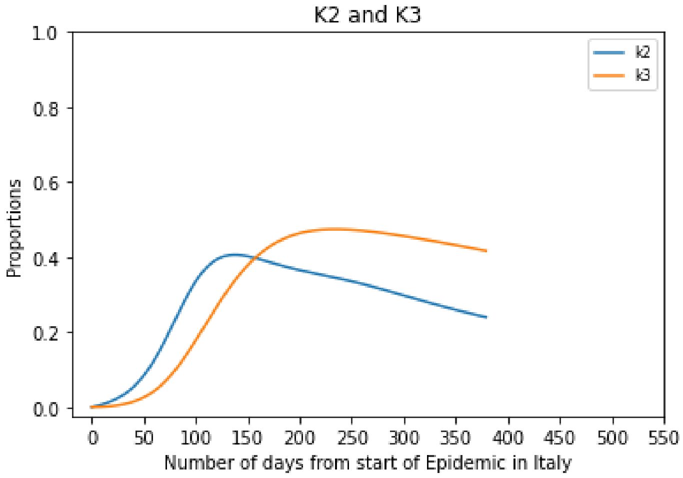

Figure 37 - For $k_2 and\ k_3$ Italy

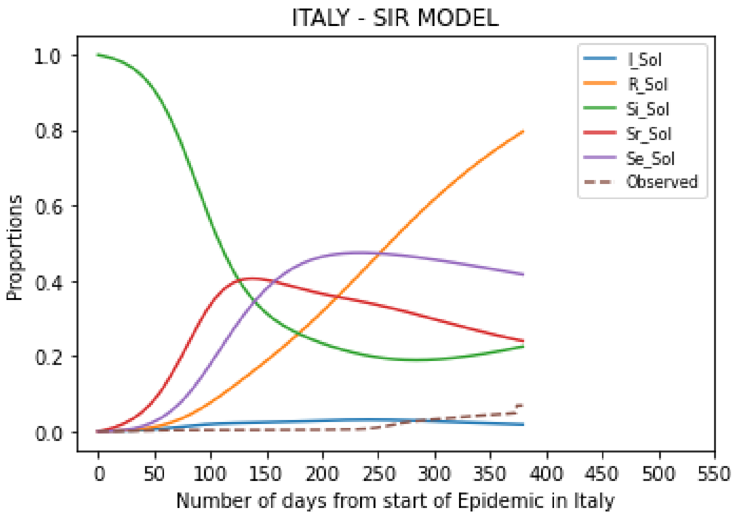

Figure 38 – GAS-SIR MODEL for Italy

In Figure 38, we have predicted the SIR model for Italy where we can illustrate the effect on wave 2 as Susceptible states changes. The rate of recovery in wave 2 goes higher. The rate of Ignorance is going down as in wave 2 in compare to wave 1. MSE for Italy is 0.003260215

Portugal:

For Portugal, we have done the same analysis as Italy and Spain. In Figure 9,10, we have illustrated the model for $k_2 and\ k_3$ along with observed data. In Figure 11, we can find the $k_2 and\ k_3$ graph which shows rate of change in susceptible states. We have predicted SIR model for Portugal where the MSE is 0.140583868

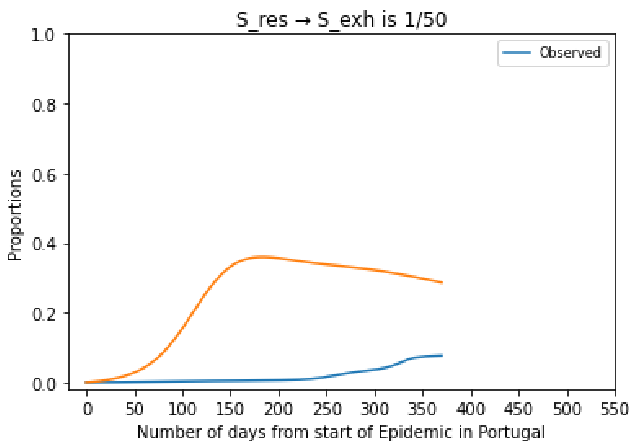

Figure 39 - for $k_2 = \frac{1}{50}$ Portugal

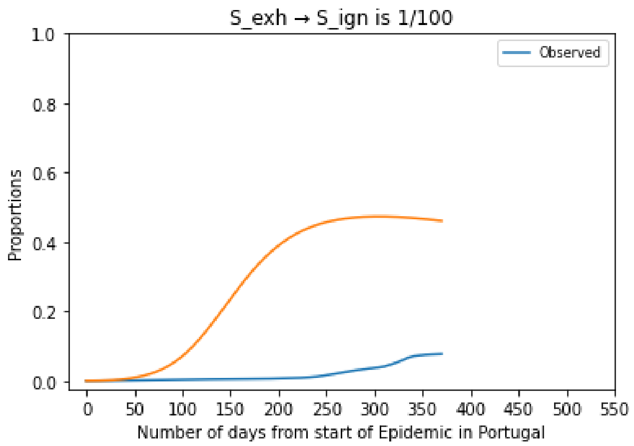

Figure 40 - For $k_3 = \frac{1}{100}$ Portugal

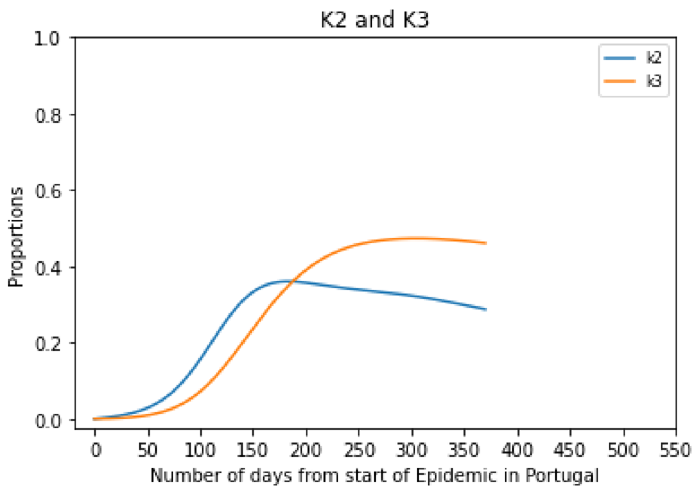

Figure 41 - For $k_2$and k_3 Portugal

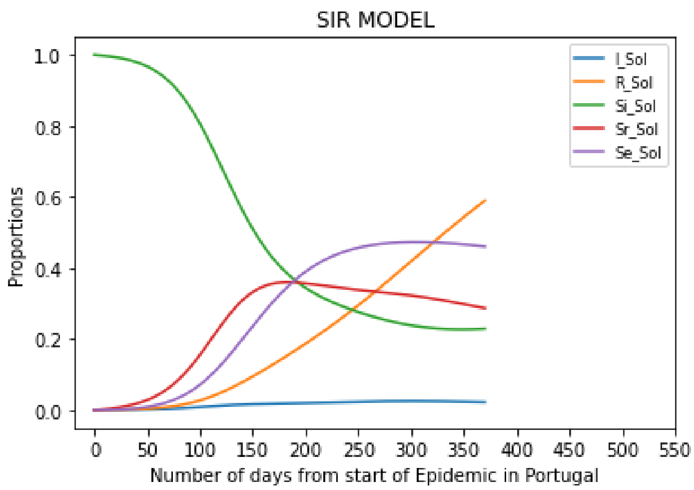

Figure 42 - SIR MODEL for Portugal

Figure 16, 17, 18 for Italy and Figure 19, 20, 21 for Portugal. This Models describe about the optimal co-efficient which will predict the SIR Model for Spain, Italy and Portugal respectively. The values a and b , we have found the model. We have predicted the model with different values of a and b. The Table 2, 3, 4 provides the optimal value of a, b and MSE calculated for the three predicted models for Spain, Italy and Portugal Respectively.

SPAIN	a	b	MSE
Observed	0.136976529	0.1	0.005506399508725173
Optimized 1	0.086976529	0.05	0.07668246142030434
Optimized 2	0.236976529	0.2	0.006428103999584364
Optimized 3	0.536976529	0.5	0.05439521522536944

Figure 43 - Optimal Values of a, b and MSE for Spain

Here, we can find that the optimal model for Spain is Model 2, where the MSE is 0.006428. The wave 2, we can find that rate of recovery is very high as the rate of Ignorance and Resistance goes down.

ITALY	a	b	MSE
Observed	0.148408252	0.1	0.0032602152350464523
Optimized 1	0.098408252	0.05	0.030875756445652807
Optimized 2	0.248408252	0.2	0.021603166563544537
Optimized 3	0.548408252	0.5	0.2070333051458477

Figure 44 - Optimal Values of a, b and MSE for Italy

In this, we can describe observed model is better than optimal models as the MSE is 0.00326. The wave 2 we can see a linear growth in Recovery rate. The rate of Ignorance and rate of exhaustion is almost constant for wave 2

PORTUGAL	a	b	MSE
Observed	0.140583868	0.1	0.05439521522536944
Optimal 1	0.090583868	0.05	0.06498252982811707
Optimal 2	0.240583868	0.2	0.006239538934586664
Optimal 3	0.540583868	0.5	0.06185253516740087

Figure 45 - Optimal Values of a, b and MSE for Portugal

Here, we can find that the second model is optimal as the MSE is 0.006234. Wave 2 we have high recovery rate and the rate of ignorance had decreased.

We can predict that the recovery rate is getting higher as the wave 1 ends and it ends at high note for wave 2 as well. Where the fixed number of contacts which are sufficient to spread the covid and infected group who will recover per day are playing pivotal role for all three countries. Where the optimal values and optimal model differs. This might be due to panic outbreak of covid is managed as the restriction are place on the people. The people who were ignorant and came in contact of infected became resistant to covid and started taking precautionary measure. Also, supply of vaccines and population density important in this aspect.

In this particular, we are using crowd effect where the alarm state increasing linearly. So, assuming this state the people who comes in contact of infected person, goes to resistant phase from ignorant phase rapidly.

$$S_{ign} + 2I \rightarrow S_{res} + 2I$$
$$S_{ign} + I \rightarrow S_{res} + I \; r = k_1 S_{ign} I$$
$$S_{ign} + 2I \rightarrow S_{res} + 2I \; r' = q \; S_{ign} I^2$$

This will give us to find the crowd effect, where we can find value of q as below:

$$q = \frac{k1}{I_p}$$

Where I_p is percent of population is infected.

Below graphs Figure 46, 47, 48 illustrates the SIR model for Crowd Effect on Spain, Italy and Portugal respectively.

In this, the value of $I_p = 0.2$ i.e., 2 % of Population. And $k_1 = 1$ which is unchanged.

The MSEs 0.0014831034, 0.0003073181, 0.0008371483 are for Spain, Italy and Portugal respectively

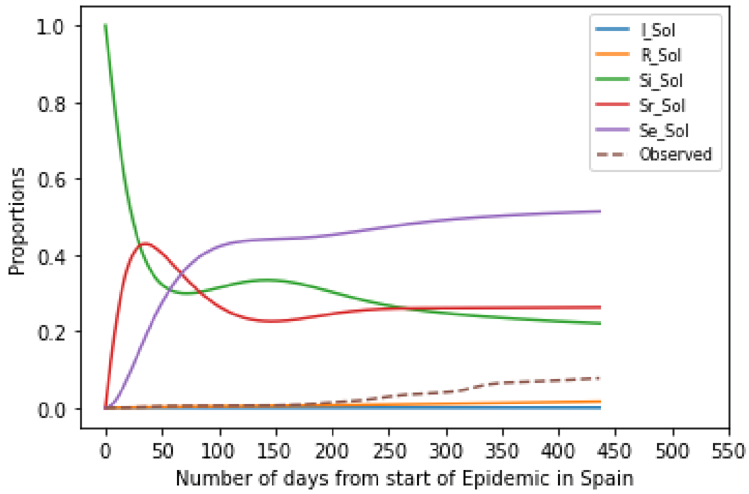

Figure 46 - Crowd Effect - Spain

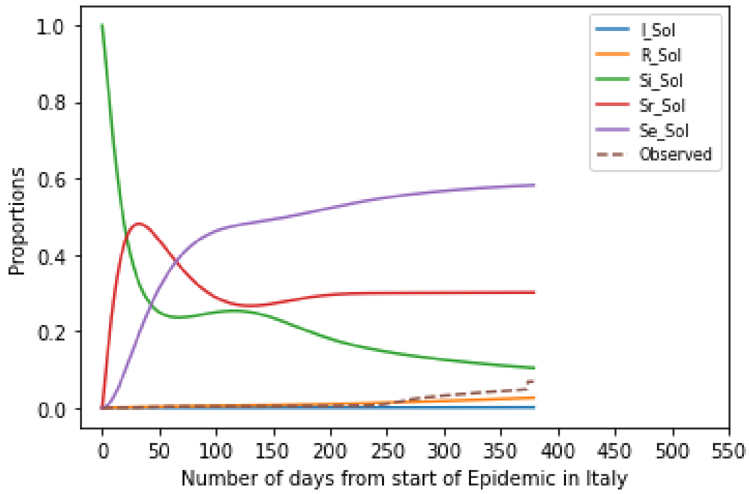

Figure 47 - Crowd Effect - Italy

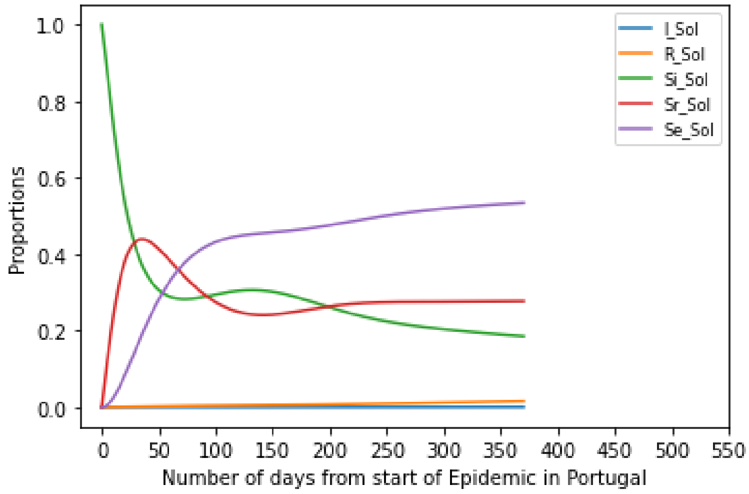

Figure 48 - Crowd Effect – Portugal

Now we are using different values of I_p to calculate the predicted model and MSE for all the three countries.

We are assuming and changing the values of $k_1, k_2, k_3 and\ I_p$ as given in the Table 5. Also, we can find the values of MSE for the predicted models as well. This does help to analyse the effect of crowd and optimize the SIR model.

Spain:

For Spain, when the $I_p = 10\%$, we can find that MSE is least compare to other models. We are assuming the values of $k_1 = \frac{1}{5}$, $k_2 = \frac{1}{45}$ $and\ k_3 = \frac{1}{110}$. This shows the rate of recovery is growing linearly where the rate of exhaustion state is higher than the rate of recovery.

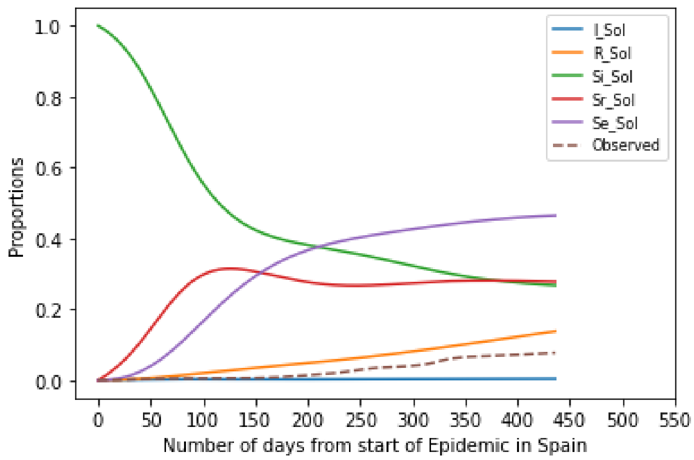

Figure 49 - Ip=3% - Crowd Effect - ESP

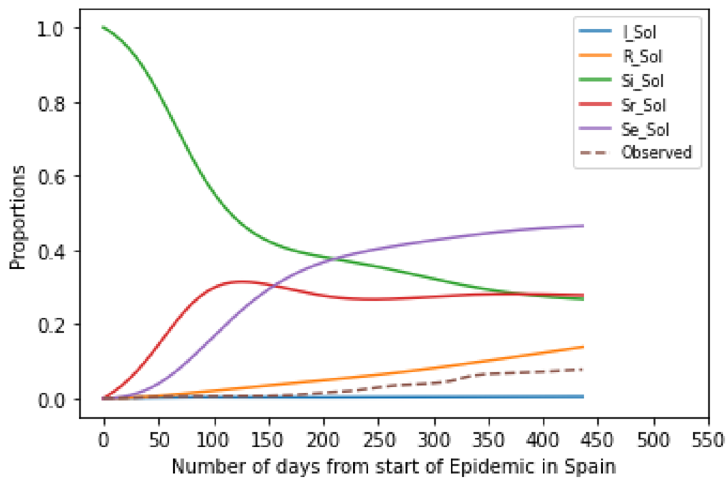

Figure 50 - Ip=5% - Crowd Effect – ESP

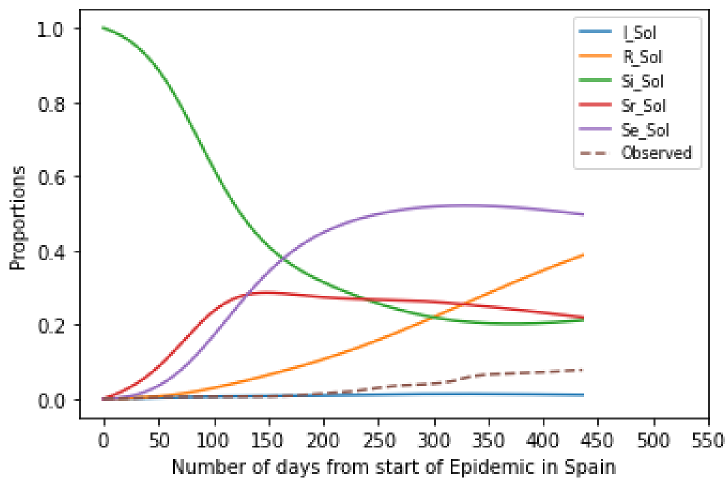

Figure 51 - Ip=10% - Crowd Effect – ESP

***<u>Italy</u>*:**

For Italy, we can find the $I_p = 3\%$ is better model as compared to remaining models where we are taking assumption of $k_1 = \frac{1}{10}$, $k_2 = \frac{1}{70}$ and $k_3 = \frac{1}{130}$. Recovery rate is not higher but ignorance and exhaustion state rate is increasing as the wave 2 arrives. The MSE Is better as compared to the remaining models.

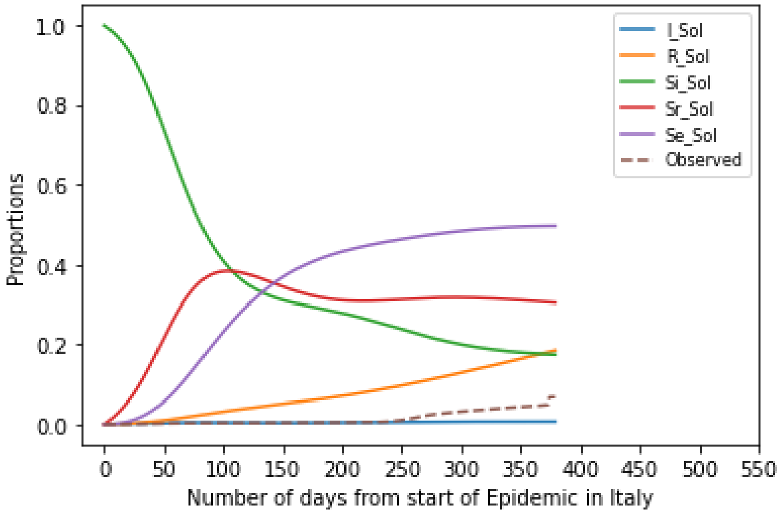

Figure 52 - Ip=3% - Crowd Effect - ITA

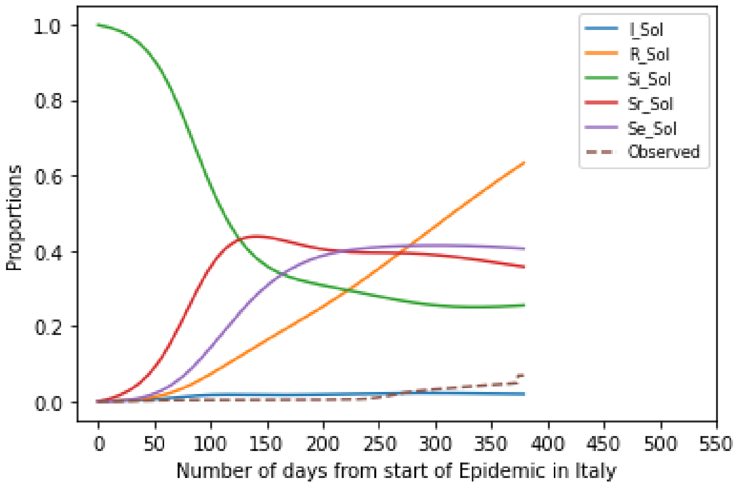

Figure 53 - Ip=5% - Crowd Effect - ITA

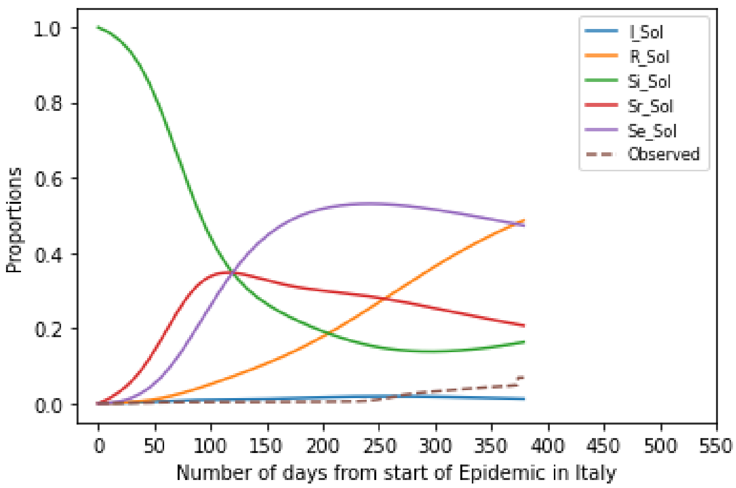

Figure 54 - Ip=10% - Crowd Effect - ITA

Portugal:

For Portugal, we can state that in term of recovery rate the second model is better where as if we compare the MSE we can state the third model is better where the $I_p = 10\%$. The rate of recovery is lesser. The recovery rate is better for second model where $I_p = 5\%$.

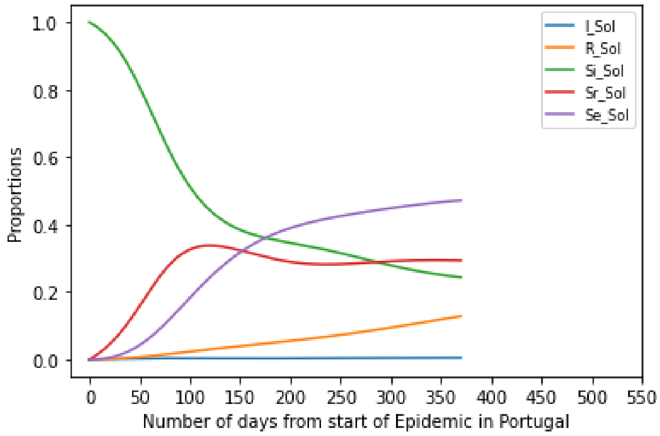

Figure 55 - Ip=3% - Crowd Effect - POR

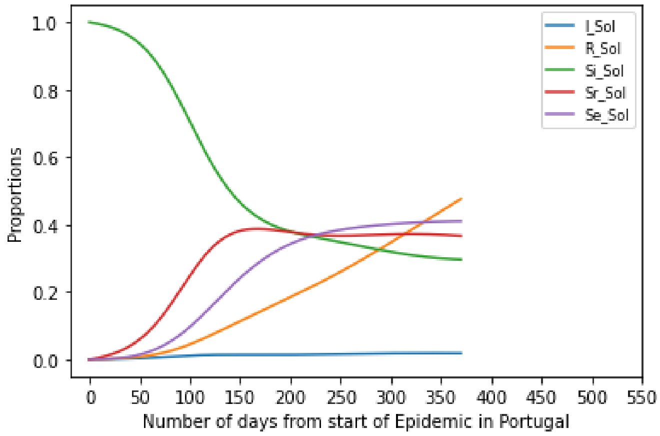

Figure 56 - Ip=5% - Crowd Effect - POR

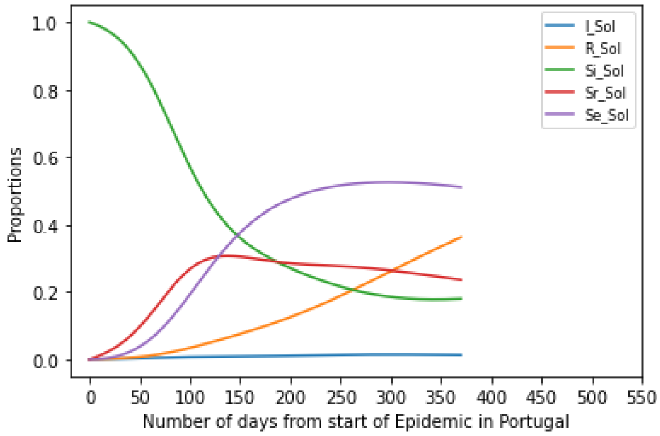

Figure 57 - Ip=10% - Crowd Effect - POR

These above graph from Figure 49 to Figure 57, provides the predicted model of crowd effect where the $I_p = 3\%$, 5% and 10% for all three countries. As provided in table, we have analysed the Mean Square Error for the values of I_p. For Spain, when $I_p = 10\%$ it has minimum MSE. Similarly for Italy and Portugal the minimum MSE is at $I_p = 10\%$

I_p	k_1	k_2	k_3	MSE_Spain	MSE_Italy	MSE_Portugal
3%	$\frac{1}{10}$	$\frac{1}{70}$	$\frac{1}{130}$	0.0029396278	0.0007568264	0.0018717482
5%	$\frac{1}{20}$	$\frac{1}{65}$	$\frac{1}{90}$	0.0129215081	0.0068226598	0.0099789152
10%	$\frac{1}{5}$	$\frac{1}{45}$	$\frac{1}{110}$	0.0013387396	0.0024013196	0.0008064990

Table 8 - Crowd Effect Iterations

Currently, there is a situation of COVID-19. We used the exponential model to assess its prevalence, it is necessary to have models modelling of the impact of various quarantine measures by the state. The SIR model, which is implemented using a modelling based on data which took on the random data was improved. Methods of comparing the parameters of the SIR model with real geographical, social and medical indicators have been developed. That allows the modelling of the spatial distribution of COVID-19 as a interact with each other. The developed model also allows us to assess the impact of quarantine b/w the models from certain interval to interval at a time.

In compare to the aspects taken into the consideration in all the covid model, this model is better. Where this model is not perfect example to do the analysis but this will provide a lot overview about the effect of covid in the countries. We can predict from what certain point the increase of cases has started and predict how we can cure the next stage. This also require more factors such as lockdown effect, population density, vaccinations and vaccination supply. More importantly the mutations of virus which can be spread in less time or more time and the severity of virus. This also needs to be taken in consideration. If the alarm state where $I_p$is greater will help to boost the resistance rate and increase the recovery rate and may help to decrease the infected rate. As we already found in Spain and Portugal where $I_p = 10$ %.

8. Understanding, Conclusion & Future work:

In the mathematical modelling, we have introduced to many concepts and some of we have used in analysing and preparing this report. The concepts like simple growth model, exponential growth model, discrete time model, logistic growth, continuous time logistic model, Malthus growth model stoichiometry, Kinetics, SIR model, SIS model, and many more.

Understanding mathematical importance in the aspect of analysis data, we have used some of the concepts in the prediction of COVID data for the three countries. The concepts which can be highlighted are SIR Model, Almost Horizontal method, Logistic growth, exponential model, Stoichiometry, Continuous time logistic model, Crowd effect.

The main objective of SIR model study is to evaluate the models for healthcare purposes which are able to predict the pandemics similar to COVID-19. This model study about the virus transmission and specifically Susceptible, Infected and Recovered People. The model is applied to perform prediction on the available data.

This will mostly analyse the growth of virus and how it is affecting as the time goes on. The prediction about how the rate of infection and recovery will come in picture. From this we can predict how Recovery will happen if the values of Infection changed. This will help to analyse the module and prepare the heath care related theories or applications. Infectious diseases are major health related issue in the world. And mathematical modelling helps to understand the infectious disease spread across the population. Stochastic model is used for better accuracy.

The SIR model helps to predict the infection rate, death rate, recovery rate and to predict the spread of infection over the population. Also, we can analyse why the rate of infection or death has been increased or decreased in specific time period and what precautionary measure can play important role to build better model. This will eventually help in health care areas.

Now, how can the model be better or what can make the model better. If we consider the SIR model, we have used GAS theory and Crowd effect in the final draft, which help to analyse the change in Susceptible growth in time period and how it affected further. The more things might come in focus if we go deep into other issues. Briefing about the area such as the lockdown effect how would have affected on the specific element. The birth rate at the time of pandemic and natural deaths occurred at the time of pandemic which are not considered in the data. The S, I, R effect in village or city or metropolitan city areas. The rate of people infected for several times. People who had covid after vaccination. How vaccination worked in the preliminary stages and further stages. These and many more aspects which are not fully taken into consideration can be answered further. The accuracy plays the important part, if the model is not accurate then it would predict wrong data and analysis will trouble or have no use.

In conclusion the SIR model is good for the prediction on the basis of first wave. Though in the second wave the model becomes more inaccurate as the spread of virus grows. This is possibly due to reasons like mutations of covid virus, or the government restriction as per the country, or the vaccination awareness and distribution or supply or the immunity of people who have high risk diseases. Imposing the restriction but not following proper guidelines and protocols can be one of the reasons which affection 2^{nd} wave model being more inaccurate. Focusing on the country specific, Italy had worse epidemic effect as the recovery rate found to be less than the rate of infection. The SIR model is not totally reliable one but this can help to build and analyse the plans for countries. There are more cases which are unknown and the data which can help to understand model better is missing. The data and some aspects which are not considered such as immunity may have considered in the model preparation. The new covid restriction or rules and regulation which are imposed by the governments of each country can as well help to make the model better. The age group of population which have affected by covid can help to make model better as there will be more impact on old and people who have weak immunity or have diseases. All the mentioned aspects required to be considered to predict as much better model which will be highly accurate and can be used for further research and applications.

9. References:

1. https://www.who.int/health-topics/coronavirus#tab=tab_1
2. Weinstein A., Kermack-McKendrick Model, https://mathworld.wolfram.com/Kermack-McKendrickModel.html
3. Saad Awadh Alanazi, M. M. Kamruzzaman, Madallah Alruwaili, Nasser Alshammari, Salman Ali Alqahtani, and Ali Karime, Measuring and Preventing COVID-19 Using the SIR Model and Machine Learning in Smart Health Care
4. Hamdi Friji, Raby Hamadi, Hakim Ghazzai, Hichem Besbes, Yehia Massoud, "A Generalized Mechanistic Model for Assessing and Forecasting the Spread of the COVID-19 Pandemic", IEEE Access, vol.9, pp.13266-13285, 2021
5. Godio A., Pace F., Vergnano A. "SEIR Modeling of the Italian Epidemic of SARS-CoV-2 Using Computational Swarm Intelligence." Int. J. Environ. Res. Public Health. 2020;17(10):3535. doi: 10.3390/ijerph17103535
6. R. Jayatilaka, R. Patel, M. Brar, Y. Tang, N.M. Jisrawi, F. Chishtie, J. Drozd, and S.R. Valluria,, "A mathematical model of COVID-19 transmission", doi: 10.3390/ijerph17103535 , Published online 2021 Dec 4. doi: 10.1016/j.matpr.2021.11.480
7. Christina Pagel, Christian A Yates,"Role of mathematical modelling in future pandemic response policy", BMJ 2022; 378 doi: https://doi.org/10.1136/bmj-2022-070615 (Published 15 September 2022)
8. Georgia Perakis, Divya Singhvi, Omar Skali Lami, Leann Thayaparan, "COVID-19: A multiwave SIR-based model for learning waves", First published: 29 January 2022 https://doi.org/10.1111/poms.13681
9. Charlotte Alger, Kaitlin Todd, "The SIR Model of Disease Spread", December 2015
10. Wikipedia, COVID-19 pandemic in Italy, https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Italy
11. Wikipedia, COVID-19 pandemic in Spain, https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Spain
12. Wikipedia, COVID-19 pandemic in Portugal, https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Portugal