Biped Patrol

Task 3.3: Think & Answer

Team Id	1452
College	IIT Bhubaneswar
Team Leader Name	Dhruval Javia
e-mail	dj15@iitbbs.ac.in
Date	January 15, 2020

Question No.	Max. Marks	Marks Scored
Q1	10	
Q2	20	
Q3	5	
Q4	5	
Q5	5	
Q6	10	
Q7	15	
Q8	8	
Q9	4	
Q10	8	
Q11	10	
Total	100	

Biped Patrol

Task 3.3: Think & Answer

Instructions:

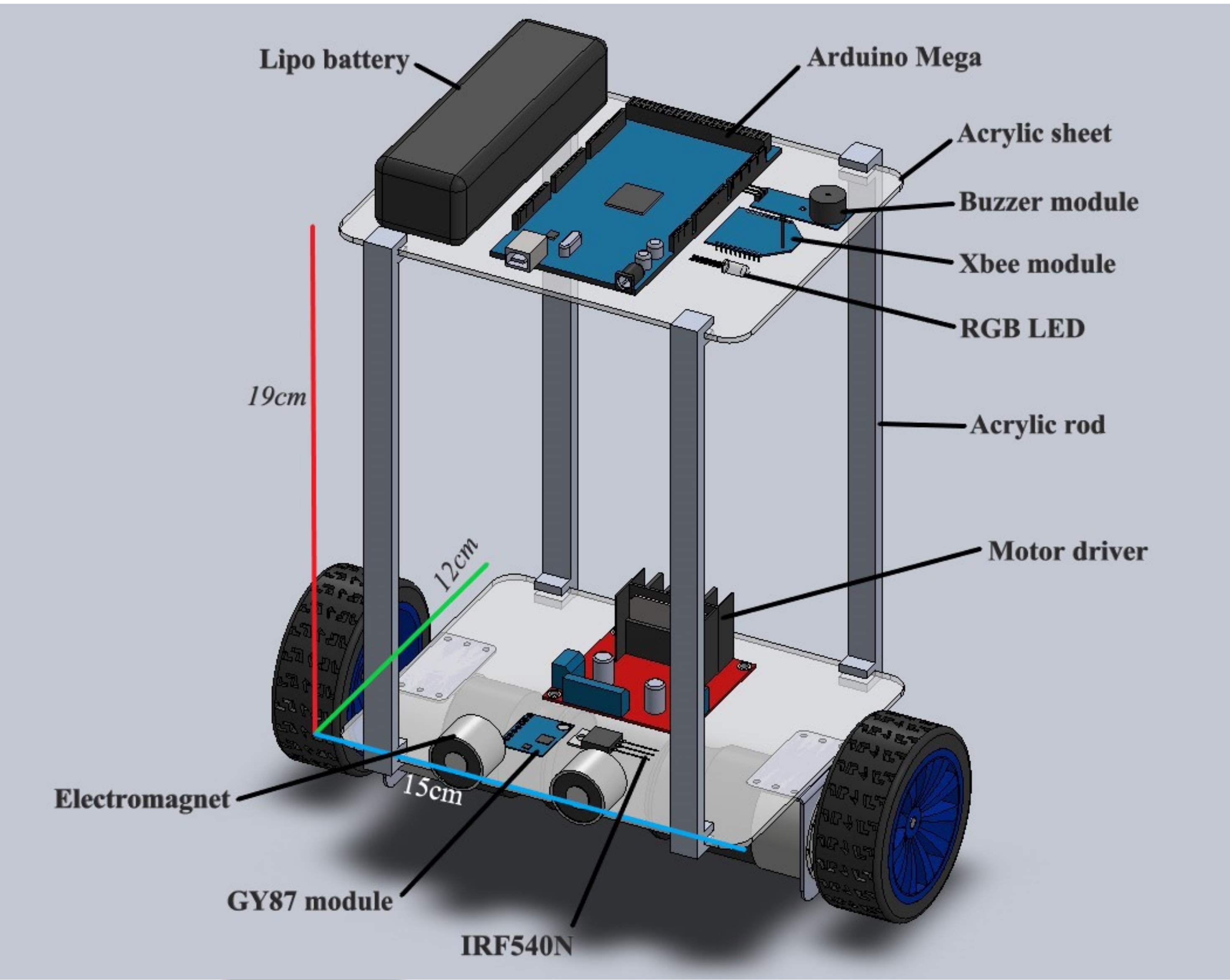
- There are no negative marks.
- Unnecessary explanation will lead to less marks even if answer is correct.
- If required, draw the image in a paper with proper explanation and add the snapshot in your corresponding answer.

Q 1. Describe hardware design for the Medbot, your team is constructing. Describe various parts with well labeled image. Give reasons for selection of design. [10]

A 1. The hardware design of the Medbot is of stacked-type design. The design consists of two rectangular platforms stacked over one another and connected by four rods placed at the four corners of the platform. The sheets for the platform and the rods are made of acrylic material. The rod and the sheet are joined together using hot-melt glue.

The lower platform consists of the motor driver, GY87 module, IRF540N, power distribution board and two electromagnet modules. The two electromagnets are placed at the edge of the platform at a considerable distance from one another. The upper platform consists of Lipo battery, Arduino Mega, Buzzer module, RGB LED and the Xbee module. The battery is placed symmetrically with respect to the wheel axis. The portion underneath the lower platform has motor, motor clamps and wheel andis attached to the lower platform with the help of nut & bolt.

The reason for selecting such a design is that it is easy to build and the components can easily be moved from one platform to another for adjusting the position of center of mass. The Lipo battery has a much greater weight amongst all the components and is thus placed on the upper platform for raising the center of mass. Also the components are placed almost symmetrical with respect to the wheel axis in order to make the center of mass lie on the vertical plane passing through the wheel axis. Two electromagnets are used in the design to simultaneously pick two boxes at a time; thus decreasing the run time.



Q 2. In Task 1.2, you were asked to model different systems such as Simple Pulley, Complex Pulley, Inverted Pendulum with and without input and stabilizing the unstable equilibrium point using Pole Placement and LQR control techniques. There you had to choose the states; Derive the equations (usually non-linear), find equilibrium points and then linearize around the equilibrium points. You were asked to find out the linear system represented in the form

$$\dot{X}(t) = AX(t) + BU(t) \tag{1}$$

Where $X(t)$ is a vector of all the state,i.e., $X(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$, and $U(t)$ is the vector of input to the system, i.e. $U(t) = [u_1(t), u_2(t), \ldots, u_m(t)]^T$. A is the State Matrix & B is the Input Matrix.

In this question, you have to choose the states for the Medbot you are going to design. Model the system by finding out the equations governing the dynamics of the system using Euler-Lagrange Mechanics. Linearize the system via Jacobians around the equilibrium points representing your physical model in the form given in equation 1.
Note: You may choose symbolic representation such as M_w for Mass of wheel, etc. [20]

A 2. Here from body we mean all the parts of robot except wheels.

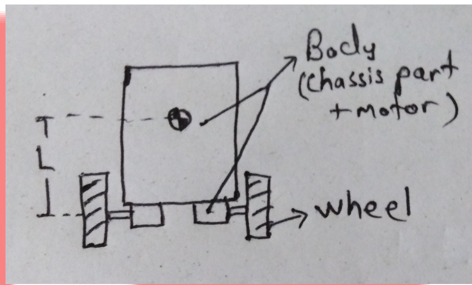


Figure 1: The body and wheel of the bot

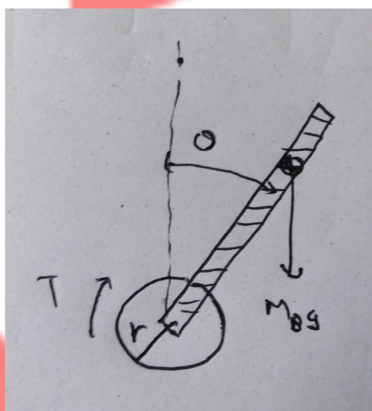


Figure 2: Major forces acting on the bot

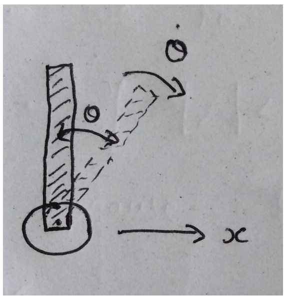


Figure 3: The states of the bot

m_B = Mass of the body
m_w = Mass of the wheel
I_B = Moment of inertia of the body about its centre of mass
I_w = Moment of inertia of the wheel
L = Height of the centre of mass of the body from the centre of the wheels
r = Radius of the wheels
T = Torque due to both the wheels

ϕ = The angular velocity of the wheel about its axis of rotation.
x = Position of centre of mass of wheel along x-axis
$\dot{x}$ = Linear velocity of centre of mass of the wheel
θ = Angle between z-axis and body of the robot
$\dot{\theta}$ = Angular velocity of the body

E_k = Kinetic Energy
E_p = Potential Energy with the plane through centre of mass of wheels as reference

Assuming the motion of wheels to be pure rolling
$x = r\phi$
$\dot{\phi} = \frac{\dot{x}}{r}$

Derivation
$E_k = (m_w + \frac{m_B}{2} + \frac{I_w}{r^2})(\dot{x})^2 + \frac{(m_B L^2 + I_B)}{2}(\dot{\theta})^2 + m_B L\dot{\theta}\dot{x}\cos(\theta)$
$E_p = m_B g L\cos(\theta)$
$\mathcal{L} = E_k - E_p = (m_w + \frac{m_B}{2} + \frac{I_w}{r^2})(\dot{x})^2 + \frac{(m_B L^2 + I_B)}{2}(\dot{\theta})^2 + m_B L\dot{\theta}\dot{x}\cos(\theta) - m_B g L\cos(\theta)$

$\frac{\partial \mathcal{L}}{\partial \theta} = m_B L\sin(\theta)(g - \dot{\theta}\dot{x})$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = (m_B L^2 + I_B)\ddot{\theta} + m_B L \ddot{x}\cos(\theta) - m_B L \dot{x}\dot{\theta}\sin(\theta)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 0$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = (m_B + 2m_w + 2\frac{I_w}{r^2})\ddot{x} + m_B L \ddot{\theta}\cos(\theta) - m_B L \dot{\theta}^2 \sin(\theta)$$

By Euler-Lagrange Equation
Q = Generalised Force
q = Generalised Co-ordinates

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = Q$$

Equations of Motion

$$\ddot{\theta} = \frac{m_B g L \sin(\theta) - m_B L \ddot{x}\cos(\theta) - T}{(m_B L^2 + I_B)}$$

$$(m_B + 2m_w + 2\frac{I_w}{r^2})\ddot{x} + m_B L \ddot{\theta}\cos(\theta) - m_B L \dot{\theta}^2 \sin(\theta) = \frac{T}{r}$$

For sake of simplicity let's take $k = \dfrac{m_B^2 L^2}{m_B L^2 + I_B}$

From Equations of Motion

$$\ddot{x} = \frac{1}{m_B + 2m_w + \frac{2I_w}{r^2} - kcos(\theta)^2}.(-k.g.cos(\theta).sin(\theta) + m_B L^2 \dot{\theta}^2 sin(\theta) + (\frac{1}{r} + \frac{k.cos(\theta)}{m_B L}).T)$$

$$\bar{x} = \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

By solving $\dot{\bar{x}} = f(\bar{x}) = 0$ the equilibrium point we get are : $\begin{bmatrix} x \\ 0 \\ 0 \\ 0 \end{bmatrix} (Unstable)$, $\begin{bmatrix} x \\ 0 \\ \pi \\ 0 \end{bmatrix} (Stable)$

Here x (first element of $\bar{x}$) is free to take any value.

Let's $\bar{x}_o = \begin{bmatrix} x \\ 0 \\ 0 \\ 0 \end{bmatrix} (Unstable\ Equilibrium)$

Linearising $\dot{\bar{x}} = f(\bar{x})$ about $\bar{x}_o$

$$\frac{\partial \ddot{x}}{\partial x}(at\,\bar{x}_o) = 0$$

$$\frac{\partial \ddot{x}}{\partial \dot{x}}(at\,\bar{x}_o) = \frac{1}{m_B + 2m_w + \frac{2I_w}{r^2} - k}.0 = 0$$

$$\frac{\partial \ddot{x}}{\partial \theta}(at\,\bar{x}_o) = \frac{1}{m_B + 2m_w + \frac{2I_w}{r^2} - k}.(-k.g)$$

$$\frac{\partial \ddot{x}}{\partial \dot{\theta}}(at\,\bar{x}_o) = \frac{1}{m_B + 2m_w + \frac{2I_w}{r^2} - k}.0 = 0$$

$$\frac{\partial \ddot{x}}{\partial T}(at\,\bar{x}_o) = \frac{\frac{1}{r} + \frac{k}{m_B L}}{m_B + 2m_w + \frac{2I_w}{r^2} - k}$$

$$\ddot{\theta} = \frac{k.g.sin(\theta)}{m_B L} - \frac{k.\ddot{x}.cos(\theta)}{m_B L} - \frac{k.T}{(m_B L)^2}$$

$$\frac{\partial \ddot{\theta}}{\partial x}(at\,\bar{x}_o) = 0$$

$$\frac{\partial \ddot{\theta}}{\partial \dot{x}}(at\,\bar{x}_o) = 0$$

$$\frac{\partial \ddot{\theta}}{\partial \theta}(at\,\bar{x}_o) = \frac{k.g}{m_B L}.\frac{m_B + 2m_w + \frac{2I_w}{r^2}}{m_B + 2m_w + \frac{2I_w}{r^2} - k}$$

$$\frac{\partial \ddot{\theta}}{\partial \dot{\theta}}(at\,\bar{x}_o) = 0$$

$$\frac{\partial \ddot{\theta}}{\partial T}(at\,\bar{x}_o) = \frac{-k}{(m_B L)^2} - \frac{k}{m_B L}.(\frac{1}{r} + \frac{k}{m_B L}).\frac{1}{m_B + 2m_w + \frac{2I_w}{r^2} - k}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \dfrac{-k.g}{m_B + 2m_w + \dfrac{2I_w}{r^2} - k} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{k.g.(m_B + 2m_w + \dfrac{2I_w}{r^2})}{m_B L.(m_B + 2m_w + \dfrac{2I_w}{r^2} - k)} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \dfrac{\dfrac{1}{r} + \dfrac{k}{m_B L}}{m_B + 2m_w + \dfrac{2I_w}{r^2} - k} \\ 0 \\ \dfrac{-k}{(m_B L)^2} - \dfrac{k}{m_B L}.(\dfrac{1}{r} + \dfrac{k}{m_B L}).(\dfrac{1}{m_B + 2m_w + \dfrac{2I_w}{r^2} - k}) \end{bmatrix}$$

Q 3. Equation 1 represents a continuous-time system. The equivalent discrete time system is represented as:

$$X(k+1) = A_d X(k) + B_d U(k) \tag{2}$$

Where $X(k)$ is a measure of the states at k_{th} sampling instant,i.e., $X(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T$, and $U(k)$ is the vector of input to the system at k_{th} sampling instant, i.e. $U(k) = [u_1(k), u_2(k), \ldots, u_m(k)]^T$. A_d is the Discrete State Matrix & B_d is the Discrete Input Matrix.

What should be the position of eigen values of A_d for system to be stable.
Hint: In frequency domain, continuous-time system is represented with Laplace transform and discrete-time system is represented with Z transform. [5]

A 3. $x[k+1] = A_d x[k] + B_d u[k]$
Taking the Z-Transform on both the sides
$z(X(z) - x[0]) = A_d X(z) + B_d U(z)$
$(zI - A_d)X(z) = B_d U(z) + zx[0]$
$X(z) = (zI - A_d)^{-1} B_d U(z) + ((zI - A_d)^{-1} x[0])z$

$y[k] = C_d x[k] + D_d u[k]$
Taking Z Transform on both the sides
$Y(z) = C_d X(z) + D_d U(z)$
$Y(z) = [C_d(zI - A_d)^{-1} B_d + D_d]U(z) + [C_d(zI - A_d)^{-1} x[0]]z$

Here, $[C_d(zI - A_d)^{-1}B_d + D_d]$ is a Transfer function

$$H(z) = C_d(zI - A_d)^{-1}B_d + D_d$$
$$(zI - A_d)^{-1} = \frac{adj(zI - A_d)}{det(zI - A_d)}$$
$$H(z) = \frac{C_d[adj(zI - A_d)]B_d + Ddet(zI - A_d)}{det(zI - A_d)}$$
$$det(zI - A_d) = 0$$

Hence z = eigenvalues of A_d = Pole of Transfer Function
Since the impulse response of the system with zero initial state is a right sided signal, the region of convergence of H(z) is |z| > p, where p = pole with the largest magnitude. Hence, system will be stable only when $|p| < 1$, as only then the unit circle will be in region of convergence.

Thus, the position of eigenvalues of A_d for the system to be stable should be such that their magnitude is less than 1.

Q 4. Will LQR control always works? If No, then why not? and if Yes, Justify your answer.
Hint: Take a look at definition of Controllable System. What is controllability? [5]

A 4. No, LQR control doesn't always work. It works when the system is stabilizable. When some of the states of the system is uncontrallable and those uncontrollable states are unstable then the cost function will become infinity, so we won't be able to optimize it, hence LQR control won't work.

Q 5. For balancing robot on two wheel i.e. as inverted pendulum, the center of mass should be made high or low? Justify your answer. [5]

A 5. Taking the torque(τ) of motor to be fixed; if the COM(Center Of Mass) is made high, the moment of inertia(I) of the bot about the wheel axis will increase as per the formula I = ml^2(where m is mass and l is height of COM) and for low COM, it will decrease. Hence, if we consider the bot to be slightly displaced from the upright position initially, the bot having a higher COM will have relatively less angular acceleration(α) and will thus, take more time to come to a specific angle as compared to the time taken for a bot with low COM to come to same angle according to the formula $\alpha = \tau/I$. Hence, the bot will be quickly balanced in case of high COM.

If the bot has a high COM; for a specific angle with the vertical, the COM will be displaced more in horizontal direction from the upright position as compared to that in case of a low COM. Hence, the threshold angle after which it is impossible to balance the bot will decrease in case of a high COM; which is a disadvantage for us.

Hence, considering the pros and cons of having high or low COM, the COM should neither be kept too high or too low but at a considerable height as per the performance requirements of the bot in a given environment.

Q 6. Why do we require filter? Do we require both the gyroscope and the accelerometer for measuring the tilt angle of the robot? Why? [10]

A 6. An electronic filter is used to remove noise or unwanted signal components from the received signal. They are usually based on the frequency aspect of a signal and attenuate either high or low frequency noise.

For measuring the tilt angle of the robot, both the gyroscope and the accelerometer are required. This is because of the nature of these two devices. A gyroscope measures the rate of change of its angle with respect to time, but not the angle itself. Hence, to obtain the angle we integrate the obtained angular speed with respect to time ; assuming some initial angle. The obtained data from the gyro will have some noise or error which will get integrated over time and will lead to an accumulation. Thus the gyro readings will drift over the period of time and give us wrong angle results. The accelerometer on the other hand measures the acceleration along the three axes of the frame in which it is kept. The orientation of the accelerometer can be obtained by noting the acceleration due to gravity along the three axes and then calculating the angles using trigonometry. But this will work only when the accelerometer is in a non-accelerating inertial frame, which hardly occurs when we deal with machines such as robots. Thus, the accelerometer will capture small jerks and vibrations and add their acceleration components to the gravity components. Hence, the angles obtained will be highly fluctuating.

Thus, it can be seen that the two sensors alone cannot be used to measure angle as the gyroscope will have low frequency drifting noise and the accelerometer will have high frequency fluctuating noise. So, to eliminate these noises and to combine the filtered data from the two sensors; low pass, high pass and complimentary filters are implemented along with these sensors.

Q 7. What is Perpendicular and Parallel axis theorem for calculation of Moment of Inertia? Do you require this theorem for modelling the Medbot?Explain Mathematically. [15]

A 7. Parallel axis theorem is used for calculating the moment of inertia of any irregularly shaped rigid body about any axis, given the moment of inertia about a parallel axis passing through body's centre of mass and the perpendicular distance between the two axes. It can be stated as,

$$I = I_{com} + ml^2$$

where I_{com} is the moment of inertia about axis passing through centre of mass of the body, m is the mass of body and l is the perpendicular distance between the two axes.

Perpendicular axis theorem is used for calculating the moment of inertia of a planar body or lamina about an axis which is perpendicular to the plane, given the moment of inertia about two axes passing through the point of intersection of the lamina and perpendicular axis and orthogonal to each other and to the perpendicular axis. It can be stated as,

$$I_z = I_x + I_y$$

where I_z is the moment of inertia about the perpendicular axis and I_x I_y are the moment of inertia about the two axes in the plane of lamina.

We will be requiring the parallel axis theorem for obtaining the moment of inertia of the whole bot about the axis passing through the centre of mass of the bot and perpendicular to the wheel axis. The individual moment of inertias of the components need to be shifted to the centre of mass of the bot for obtaining the moment of inertia of the whole bot.

Q 8. What will happen in the following situations:

(a) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit falls inside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]

(b) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit falls outside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]

(c) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit and the Medbot both fall inside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]

(d) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit and the Medbot both fall outside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]

A 8. (a) No penalty will be imposed and no points will be awarded as the First-Aid Kit has not been taken outside the store. The First-Aid Kit will not be re-positioned.

(b) No penalty will be imposed and points equal to 20 for a successful pick-up will be awarded. The First-Aid Kit will not be re-positioned.

(c) Fall penalty will be imposed but no points will be awarded. The First-Aid Kit will be re-positioned back to its initial position as the Medbot has also fallen in this case.

(d) Fall penalty will be imposed and points equal to 20 for a successful pick-up will be awarded. The First-Aid Kit will be re-positioned back to its initial position.

Q 9. What will be the points awarded if Medbot picks only one of the item from the medical store and repeatedly moves back and forth around the gravel pathway or the bridge for the entire run. [4]

A 9. For the first case when Medbot moves back and forth around gravel pathway, the points awarded will be as follows:
T = 600
$M_G = 50 \times [0.5 * 1(ERG) + 1(LRG)] = 75$
M_{PU} = 20*1(PUC) = 20
Total points awarded = (600 - T) + M_G + M_{PU} = 95

For the second case when Medbot moves back and forth around bridge, the points awarded will be as follows:
T = 600
$M_G = 50 \times [0.5 * 1(ERG) + 0(LRG)] = 25$
$M_B = 70 \times [0.5 * 0(ERB) + 1(LRB)] = 70$
M_{PU} = 20*1(PUC) = 20
Total points awarded = (600 - T) + M_G +M_B + M_{PU} = 95

Thus, in both the cases same points will be awarded.

Q 10. What are the different communication protocols you'll be using? Name the hardware interfaced related to each of the communication protocols. Explain how these communication protocols works and what are the differences between them. [8]

A 10. The communication protocols that we will be using are : I2C and UART serial communication protocol. The I2C protocol is related with the GY87 sensor and Arduino mega microcontroller and UART serial protocol is related to communication between two Xbee modules.

UART(Universal Asynchronous Reception and Transmission) is a serial communication protocol supporting bidirectional and asynchronous communication between two devices. It requires two lines for transmission and reception respectively and only two devices can communicate with each other on these line. There is no clock, hence the two devices need to be on the same baud rate to establish communication. The data is sent as a series of start bit, followed by 7/8 databits depending upon presence of parity bit and ending with a stop bit.

I2C(Inter-Integrated Circuit) is also a serial communication protocol supporting bidirectional communication but is synchronous and can handle multiple devices(masters and slaves) on two lines. The two lines are called serial data acceptance line(SDA) and serial clock line(SCL) lines respectively for data and clock signal transmission. The data is transferred as a series of start bit, 8-bit device address, 8-bit internal register address, 8-bit data and terminating with a stop bit. The master device sends the signal to every device connected with the serial bus. If the sent address bits matches with the address of a device, the device will acknowledge the master and will start receiving the data bits from it.

Q 11. Why do we require IRF540N? Provide circuit diagram for interfacing IRF540N with the microcontroller. [5+5]

A 11. IRF540N mosfet is required to amplify the current being provided to the electromagnet from Arduino mega. The current being provided by Arduino mega through output pins is 40mA which is not sufficient enough to develop considerable magnetic field strength in the electromagnet; as the field strength is directly proportional to magnitude of current. Hence IRF540N mosfet is used with electromagnet.

The circuit diagram is as follows: