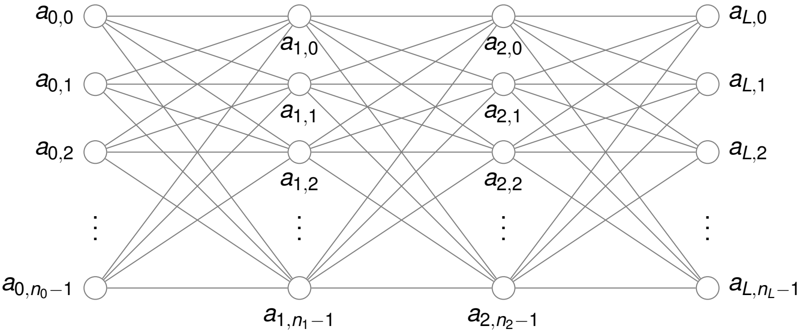
Monte Carlo Methods and Neural Networks

Noah Gamboa and Alexander Keller

Neural Networks

Fully connected layers

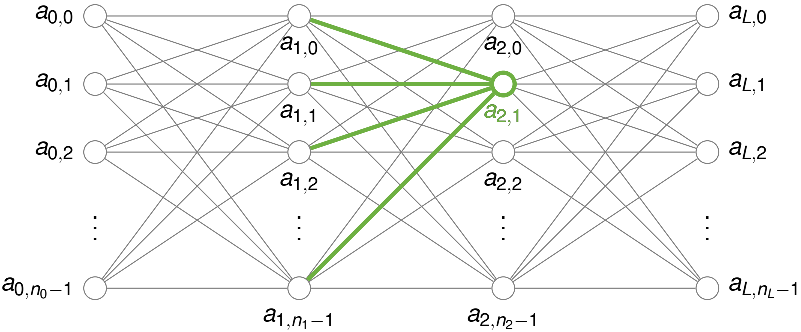
- neurons



Neural Networks

Fully connected layers

- neurons compute $\max\{0, \sum_j w_j a_{i,j}\}$



Monte Carlo Methods all over Neural Networks

Examples

- drop out
- drop connect
- stochastic binarization
- stochastic gradient descent
- fixed pseudo-random matrices for direct feedback alignment
- ...

Monte Carlo Methods all over Neural Networks

Observations

- the brain
  - about 10^{11} nerve cells with to up to 10^{4} connections to others
  - much more energy efficient than a GPU

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  - expensive to scale in depth
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- artificial neural networks
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- goal: explore algorithms linear in time and space

Partition instead of Dropout

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Guaranteeing coverage of neural units

- so far: dropout neuron if threshold $t > \xi$
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  - less random number generator calls
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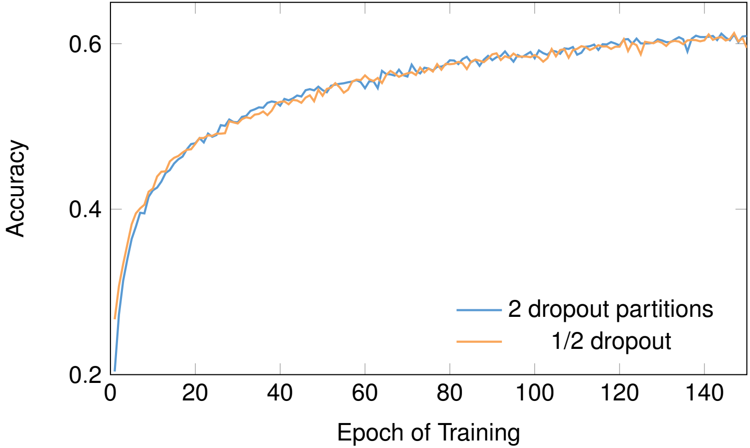
LeNet on MNIST	Average of $t = 1/2$ to $1/9$ dropout	Average of $P = 2$ to 9 partitions
Mean accuracy	0.6062	0.6057
StdDev accuracy	0.0106	0.009

Partition instead of Dropout

Training accuracy with LeNet on MNIST



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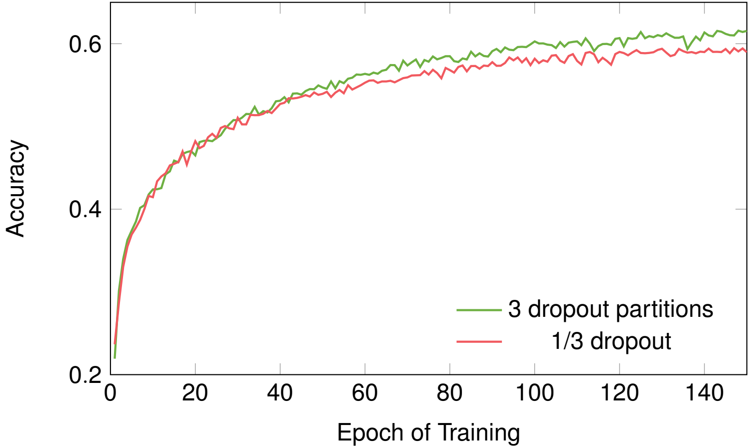

Simulating Discrete Densities

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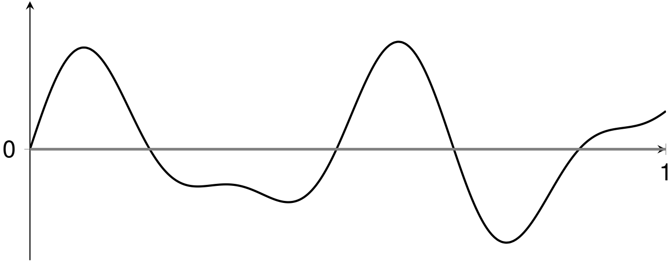
Stochastic evaluation of scalar product

- discrete density approximation of the weights



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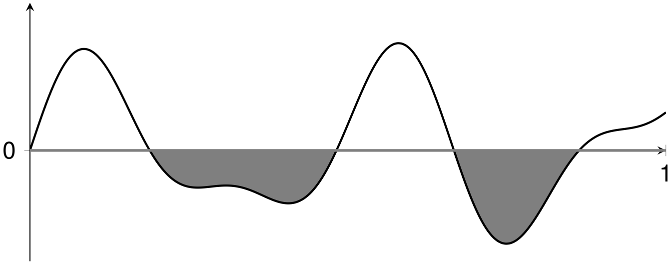

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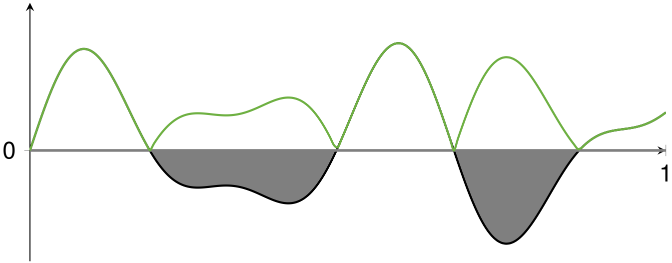

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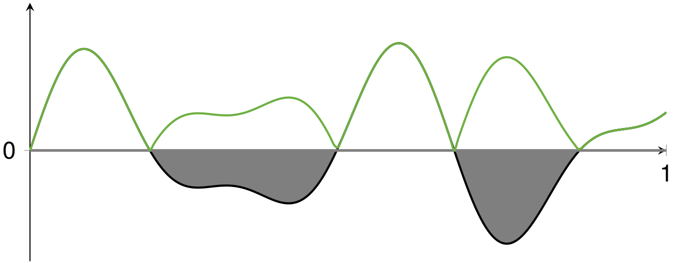

  - remember to flip sign accordingly
  - transform jittered equidistant samples using cumulative distribution function of absolute value of weights

Simulating Discrete Densities

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- partition of unit interval by sums $P_k := \sum_{j=1}^{k} |w_j|$ of normalized absolute weights

$$0 = P_0 < P_1 < \cdots < P_m = 1$$

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    select neuron $i \Leftrightarrow P_{i-1} \leq \xi < P_i$ satisfying $\text{Prob}(\{P_{i-1} \leq \xi < P_i\}) = |w_i|$

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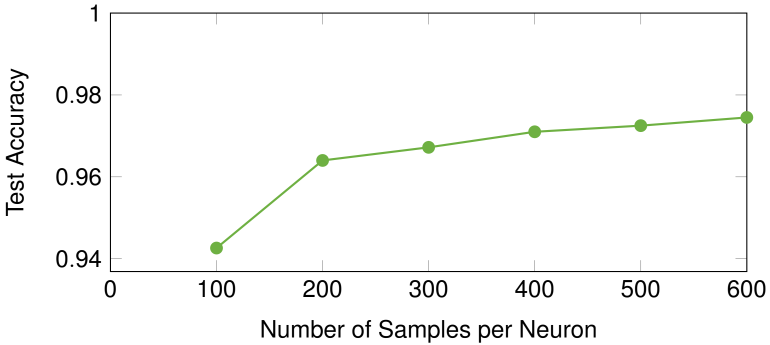
- in fact derivation of quantization to weights in $\{-1, 0, +1\}$
  - integer weights if a neuron referenced more than once
  - explains why ternary and binary did not work in some articles
  - relation to drop connect and drop out, too

Simulating Discrete Densities

Test accuracy for two layer ReLU feedforward network on MNIST

- able to achieve 97% of accuracy of model by sampling most important 12% of weights!



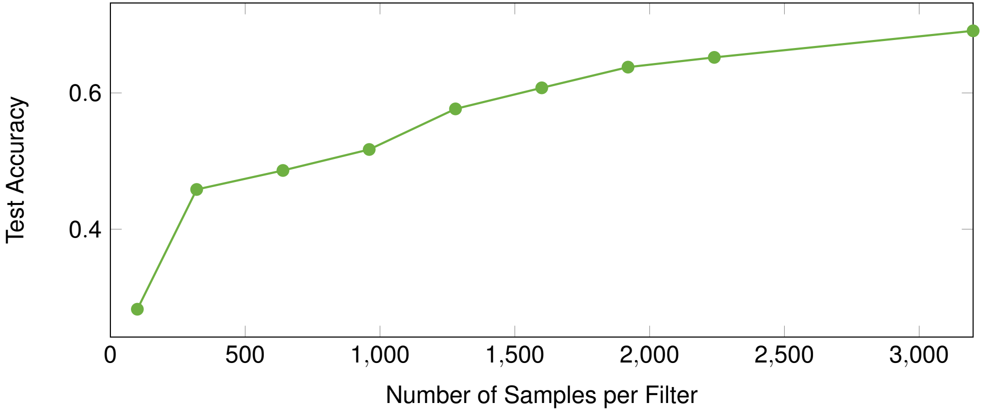
Simulating Discrete Densities

Application to convolutional layers

- sample from distribution of filter (for example, 128x5x5 = 3200)
  - less redundant than fully connected layers
- LeNet Architecture on CIFAR-10, best accuracy is 0.6912
- able to get 88% of accuracy of full model at 50% sampled

Simulating Discrete Densities

Test accuracy for LeNet on CIFAR-10



Neural Networks linear in Time and Space

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Number n of neural units

- for L fully connected layers

$$n = \sum_{l=1}^{L} n_l$$

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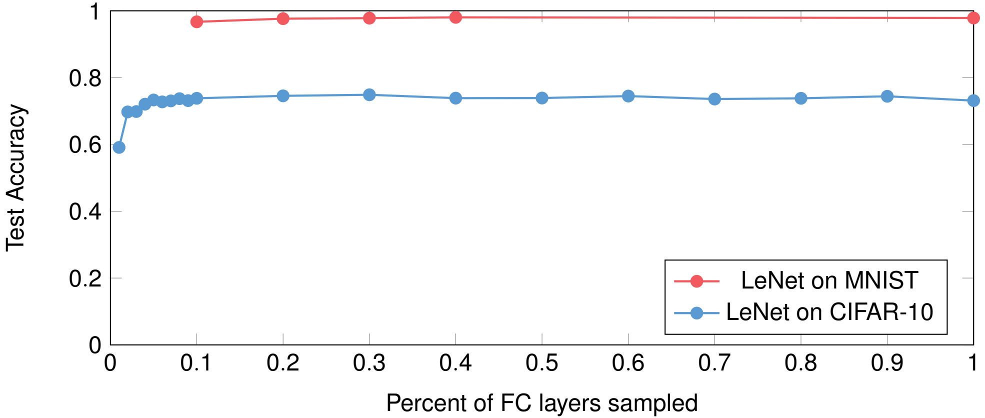
- choose number of weights per neuron such that n proportional to n_w
  - for example, constant number n_w of weights per neuron

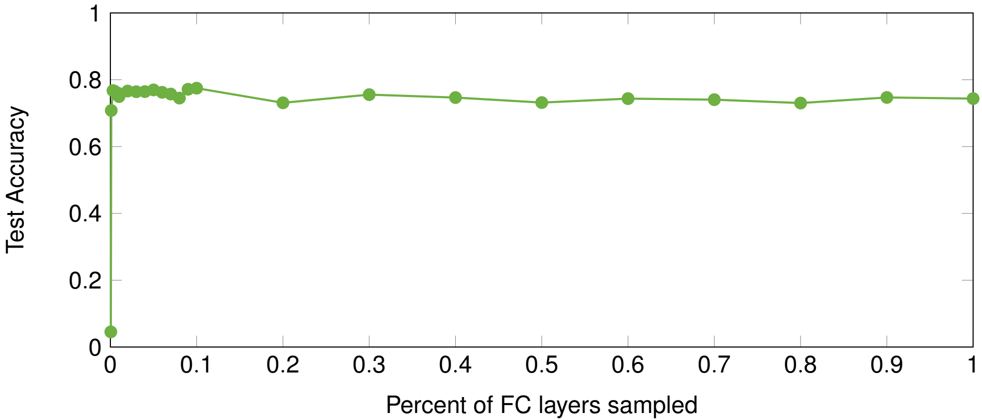
Neural Networks linear in Time and Space

Results



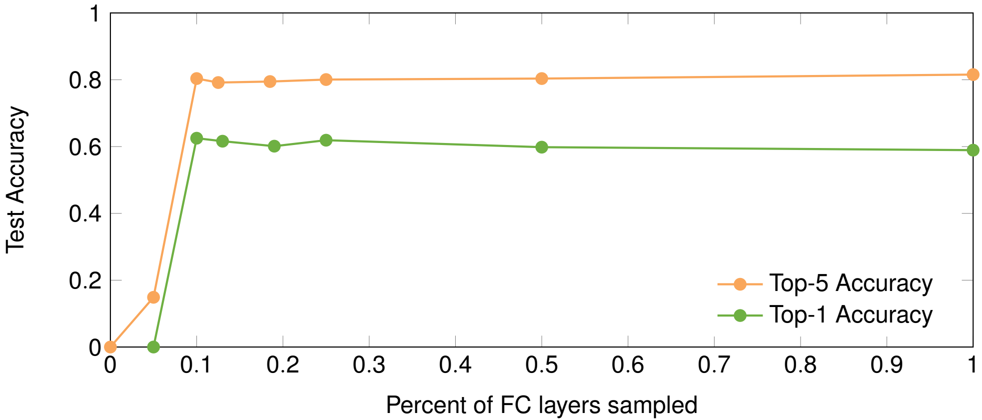
Neural Networks linear in Time and Space

Test accuracy for AlexNet on CIFAR-10



Neural Networks linear in Time and Space

Test accuracy for AlexNet on ILSVRC12



Neural Networks linear in Time and Space

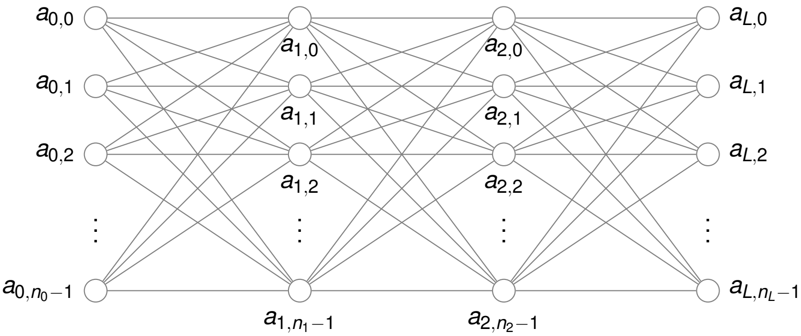
Sampling paths through networks

- complexity bounded by number of paths times depth
- strong indication of relation to Markov chains
- importance sampling by weights

Neural Networks linear in Time and Space

Sampling paths through networks

- sparse from scratch



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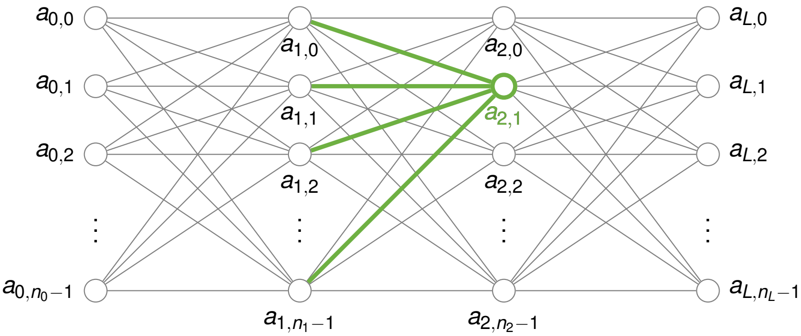

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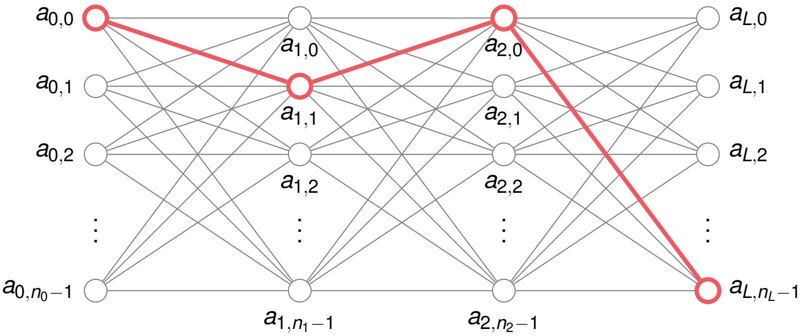

  - guaranteed connectivity

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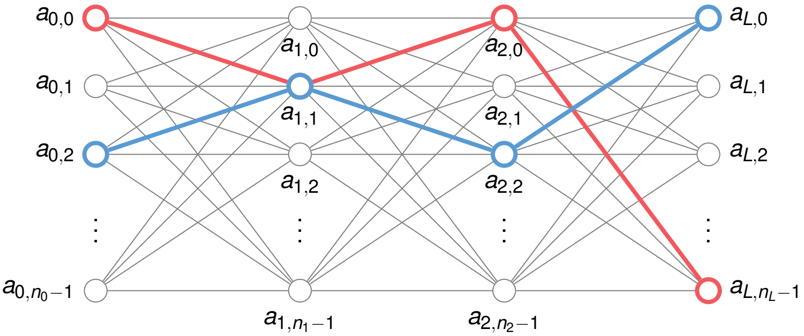

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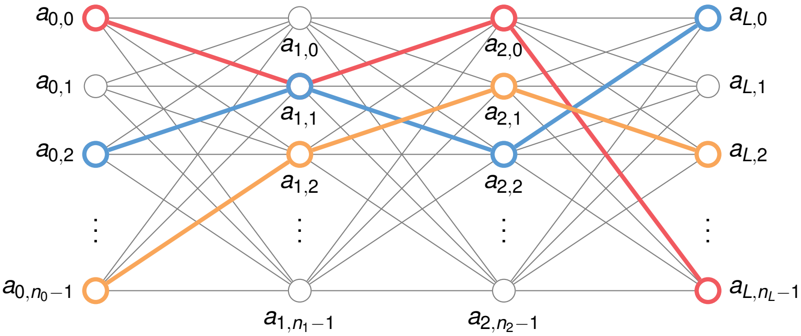

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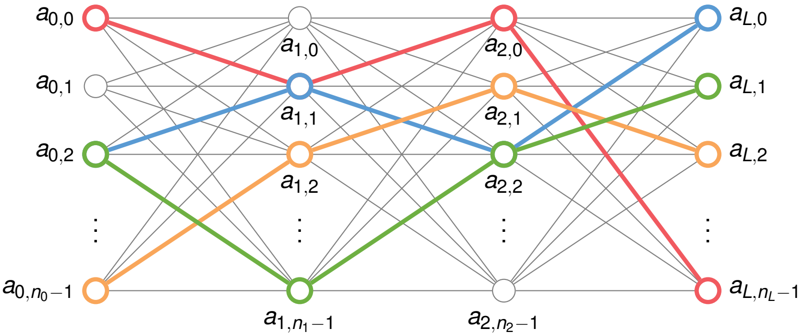

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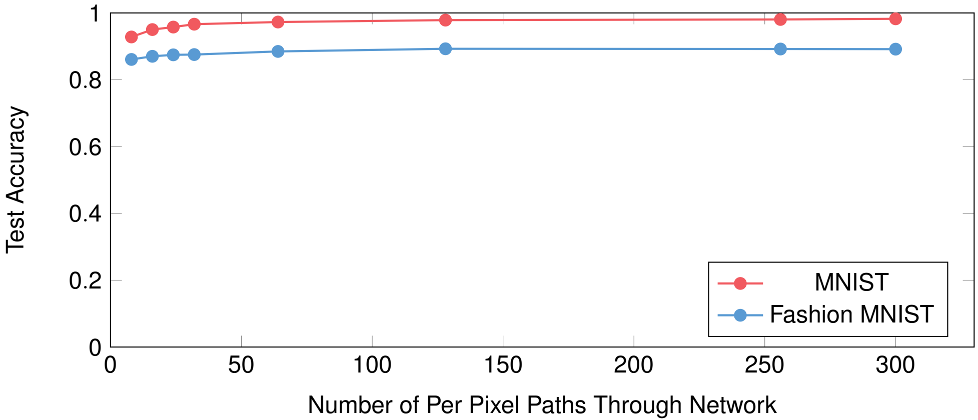
Neural Networks linear in Time and Space

Test accuracy for 4 layer feedforward network (784/300/300/10)



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- dropout partitions reduce variance
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- simulating discrete densities explains $\{-1, 0, 1\}$ and integer weights
  - compression and quantization without retraining

- neural networks with linear complexity for both inference and training
  - sparse from scratch
  - sampling paths through neural networks instead of drop connect and drop out