

12-Month ASKEM Climate Modeling Scenarios

Nirupama Bhattacharya, Christina Gilligan, Howell Yee, Laurence Audenaerd

The views, opinions, and/or findings contained in this report are those of The MITRE Corporation and should not be construed as an official Government position, policy, or decision, unless designated by other documentation. This technical data deliverable was developed using contract funds under Basic Contract No. W56KGU-18-D-0004.

1. **(Low Complexity)** This scenario focuses on Zero-Dimensional (0-D) and One-Dimensional (1-D) Energy Balance Models (EBMs), gradually adding complexity. (*Derived in part from the [Climate Laboratory](#)*)

- a. Assume Earth behaves like a blackbody radiator governed by $OLR = \sigma T_s^4$ where:
 - The outgoing longwave radiation (OLR) emitted by Earth has a measured global annual mean value of 238.5 W/m^2 .
 - T_s is the global average surface temperature.
 - $\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}$ is the Stefan-Boltzmann constant

What is T_s under the blackbody radiation assumption, and how does it compare to measurements?

- b. Now, correct OLR to account for the fact that only a fraction of energy emitted by Earth at the surface makes it out to space, due to the greenhouse effect, $OLR = \tau \sigma T_s^4$, where $\tau = 0.612$ is the transmissivity, or fraction of emissions from the surface of the Earth, that make it to space. What is T_s under this corrected expression, and how does it compare to measurements?
- c. Now we consider a simple 0-dimensional EBM:

$$C \frac{dT_s}{dt} = \underbrace{(1 - \alpha)Q}_{ASR} - \underbrace{\tau \sigma T_s^4}_{OLR}$$

where:

- ASR is the ‘absorbed shortwave radiation’, which is part of the incoming energy from the sun that is not reflected back, and is absorbed into the Earth system.
- OLR is the ‘outgoing longwave radiation’ emitted by Earth (incorporating the greenhouse effect).
- T_s is the global average surface temperature.
- C is the heat capacity of the Earth system, which is approximated by the amount of energy required to heat 100 meters of water. It is governed by the equation, $C = f \rho c_w H$, where f is the fraction of the area of interest covered by water (as a whole, the fraction of Earth covered by water is 0.7), ρ is the density of water ($1025 \frac{\text{kg}}{\text{m}^3}$), c_w is the specific heat of water ($4186 \frac{\text{J}}{\text{kg}^\circ\text{C}}$), and H is the depth of the

Distribution Statement A. Approved for public release: distribution is unlimited

MITRE Public Release Case Number 24-2089

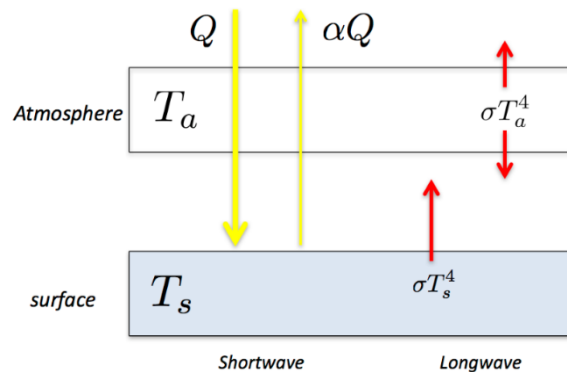
© 2025 The MITRE Corporation.

water that is heated or cooled. For this problem, you can assume $f = 1$, and $H = 100m$.

- $\tau = 0.612$ is the transmissivity, or fraction of emissions from the surface of the Earth, that make it to space.
- $\sigma = 5.67E - 8 \frac{W}{m^2 K^4}$ is the Stefan-Boltzmann constant.
- $Q = 341.3 \frac{W}{m^2}$ is the annual global average insolation, or the energy received from incoming solar radiation per unit time per unit surface.
- $\alpha = 0.299$, is the planetary albedo, which is the fraction of Q that is reflected back into space.

Determine the equilibrium temperature? Set $T_s(0) = 288 K$. How long does it take to approach the equilibrium temperature (within some reasonable margin)? Plot T_s over this period.

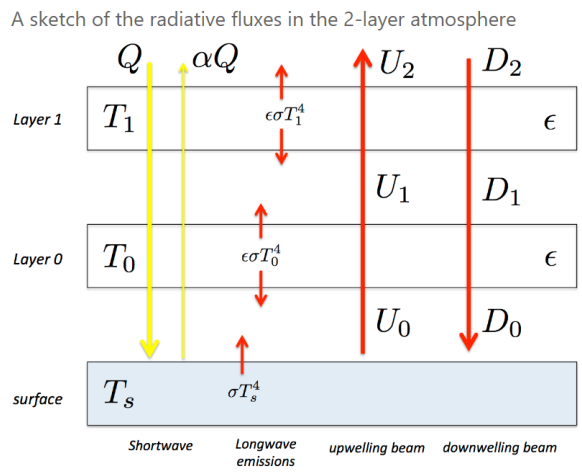
- Let's assume that global warming affects the atmospheric composition and cloudiness, such that α is increased to 0.31, and τ is decreased to 0.58. Set $T_s(0)$ to the equilibrium temperature in question 1c. How long does it take to approach the new equilibrium temperature due to the perturbation? Does the change (and direction of change) in equilibrium temperature make sense given the nature of the perturbation?
- Now let's represent a simple greenhouse model by adding an atmosphere layer that absorbs and emits longwave radiation. This model makes the following simplifying assumptions (shown in this diagram):



- The atmosphere can be represented by a single layer with temperature T_a
- The atmosphere is transparent to shortwave solar radiation (shown in the diagram above with the yellow arrows), and completely opaque to infrared radiation (shown in the diagram above the red arrows).
- The surface absorbs $(1 - \alpha)Q$ amount of shortwave radiation.
- Both surface and atmosphere emit blackbody radiation.
- There are no other heat transfer mechanisms.

What are the temperatures for the surface and atmosphere, at radiative equilibrium? To answer this, you will need to balance energy for each layer. How does the surface temperature compare to observed data? Why do you think there is a difference?

- f. To add more realism into our model, we add yet another layer to our atmosphere, so that our model is now represented by the diagram below. This is known as the two-layer grey gas model (named this because emission and absorption processes have no wavelength/spectral dependence).



This model makes the following assumptions:

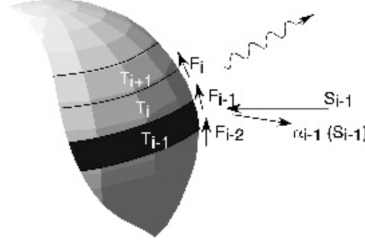
- The atmosphere is divided into two layers of equal mass; the dividing line is the 500 hPa pressure level.
- Each atmosphere layer absorbs a fraction epsilon, ϵ , of the longwave radiation incident upon it and is the same for both layers. Therefore, $(1 - \epsilon)$ of the longwave radiation is transmitted through each layer.
- Each atmosphere layer is still fully transparent to shortwave radiation (shown in the diagram by the yellow arrows)
- For the atmospheric layers, absorptivity = emissivity (Kirchoff's Law).
- According to this model, we find that

$$OLR = (1 - \epsilon)^2 \sigma T_s^4 + \epsilon(1 - \epsilon) \sigma T_0^4 + \epsilon \sigma T_1^4.$$

- What is the effect of varying ϵ from $[0,1]$? What value of ϵ best matches observed values of $OLR = 238.5 \frac{W}{m^2}$; $T_s = 288 K$; $T_0 = 275 K$; $T_1 = 230 K$?
- What is the radiative equilibrium solution, and how does it compare to observed temperatures? If there is a difference, why do you think that is the case?
- Automate the extension of this model to include an arbitrary number of atmospheric layers

- g. One-Dimensional Energy Balance Model (EBM): One of the issues with simpler models such as the ones described in the above sections, is that they lack important processes like heat transport. In moving from 0-D to 1-D models, we now think about regional climates and energy transfer between latitude zones.

One-dimensional EBM

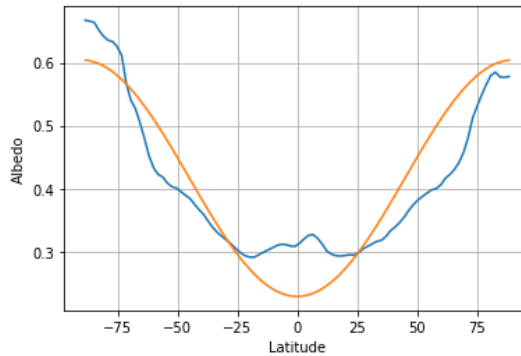


Consider the following 1-D EBM, also known as the Budyko-Sellers model:

$$C(\varphi) \frac{\partial T_s(\varphi, t)}{\partial t} = \underbrace{(1 - \alpha(\varphi))Q(\varphi, t)}_{ASR} - \underbrace{A + BT_s(\varphi, t)}_{OLR} + \underbrace{\frac{D}{\cos \varphi} \frac{\partial}{\partial \varphi} \left(\cos \varphi \frac{\partial T_s(\varphi, t)}{\partial \varphi} \right)}_{\text{heat transport}}$$

where,

- Surface temperature T_s is a function of both time t and latitude φ .
- Here we choose a simple parameterization of OLR (although this could be made more complex) with $A = 210 \frac{W}{m^2}$, representing longwave emissions at 0°C , and $B = 2 \frac{W}{m^2C}$, representing the increase in emissions per degree. These values were chosen based on tuning with reanalysis data from the National Oceanic and Atmospheric Administration (NOAA) - National Centers for Environmental Prediction (NCEP).
- We estimate albedo $\alpha(\varphi) \approx \alpha_0 + \alpha_2 P_2(\sin \varphi)$ where $P_2(x) = \frac{1}{2}(3x^2 - 1)$ is the 2nd Legendre polynomial, $\alpha_0 = 0.354$ (global planetary albedo), $\alpha_2 = 0.25$. This function (orange line) was chosen as a relatively good fit to observed data (blue line, from NCEP reanalysis) where we get the correct global mean and representation of equator-to-pole gradient in the albedo, to reflect increased albedo towards the poles due to greater surface snow and ice.



- $C(\varphi)$ is effective heat capacity of the atmosphere at latitude φ , in units $\frac{J}{m^2 \circ C}$. We describe this as a function of latitude so this model can be generalized different land-ocean fractions at different latitudes (although that is not the focus of this problem). For this scenario we can assume this is a constant value given by $C = f\rho c_w H$, as described in 1c, but here assume more realistic values with $f = 0.70$, and the depth of the ocean mixed layer $H = 70m$.
- Insolation values $Q(\varphi, t)$ can be obtained and averaged to get annual mean values, as described here: <https://brian-rose.github.io/ClimateLaboratoryBook/courseware/insolation.html#the-daily-average-zenith-angle>
- Assume the horizontal diffusivity of the climate system $D = 0.6 \frac{W}{m^2 \circ C}$

Assume annual mean values (in other words, assume a timestep of 1 year), and solve the full time-varying 1D EBM as defined in this problem, assuming an initial temperature profile that is warm at the equator and cold at the poles: $T_s(\varphi, 0) = 12 - 40P_2(\sin \varphi)$, where P_2 is the 2nd Legendre polynomial as defined earlier. Assume no-flux boundary conditions at the poles. What is the steady state equilibrium temperature profile as a function of latitude, $T_{s,eq}(\varphi)$?

- Now add a seasonal insolation cycle to 1g, such that $Q(\varphi, t) = Q^* \sin(\omega t) + Q_0(\varphi)$, where $\omega = \frac{2\pi}{year}$, $Q_0(\varphi)$ is the annual mean at a given latitude, and Q^* is the amplitude of seasonal variations.
- We will now add an interactive snow and ice line to our 1D EBM, to reflect the idea that as the climate warms, snow and ice will retreat poleward, and planetary albedo will decrease. Keeping all else the same as 1g, let

$$\alpha(\varphi, T(\varphi)) = \begin{cases} \alpha_0 + \alpha_2 P_2(\sin \varphi) & T(\varphi) > T_f \text{ (no ice)} \\ \alpha_i & T(\varphi) \leq T_f \text{ (ice covered)} \end{cases}$$

where $P_2(x) = \frac{1}{2}(3x^2 - 1)$ is the 2nd Legendre polynomial (as defined earlier), $\alpha_0 = 0.354$ (global planetary albedo), $\alpha_2 = 0.25$, $\alpha_i = 0.62$, threshold temperature $T_f = -10^\circ C$, and diffusivity to $D = 0.55 \frac{W}{m^2 \circ C}$.

- Solve for the 1D EBM with this step function for the latitude and temperature-dependent albedo, until an equilibrium state is reached. How does the result compare with 1g?
- Beginning from the equilibrium state in i.I, what is the impact of a small radiative forcing (let $A_{new} = A - 4.5 \frac{W}{m^2}$)? Is the impact equal across all latitudes?
- Beginning from the equilibrium state in i.I, what is the impact of changing energy output from the Sun, by lowering insolation? Treat this as a climate forcing process - lower the insolation in small increments, and let the system evolve to a new equilibrium state after each increment. Is there a point at which there is a major change in the equilibrium temperature profile?

IV. Now continue from the result of i.III by reversing the forcing process and increasing insolation back to its original value, at the same rate that you decreased it in i.III, waiting for the system to reach equilibrium after each increment. Do you arrive at the same equilibrium state that you were at the beginning of i.III? Is the result what you expected, and if not, can you explain what might be the reason?

2. (Low-Medium Complexity) Solving the 2D and 3D Navier-Stokes Systems of Equations

Background Review:

I. Which Conservation Laws matter for climate?

1. Mass
2. Energy
3. Momentum
4. Angular Momentum
5. Charge

II. What is the *continuity equation* and why is it important?

a. **2D Navier Stokes:** Solve the 2D Navier-Stokes equation, as defined in the following:

https://nbviewer.org/github/barbagroup/CFDPython/blob/master/lessons/14_Step_11.ipynb

$$\begin{aligned}\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} &= -\rho \left(\frac{\partial u}{\partial x} \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} \right)\end{aligned}$$

Where ν is viscosity, ρ is density, p is pressure, and u is fluid velocity.

Use the following parameters:

Parameter	Value
nx (number of grid points, x-axis)	41
ny (number of grid points, y-axis)	41
nt (number of timesteps to calculate)	100
nit (pseudo-time variable supports Poisson pressure calculation to ensure divergence free field or ease the initial spike)	50
c(wave speed)	1
dt(amount of time each timestep covers)	0.001
ρ : "rho" (density)	1
ν : "nu" (viscosity)	0.1

Distribution Statement A. Approved for public release: distribution is unlimited

MITRE Public Release Case Number 24-2089

© 2025 The MITRE Corporation.

Set the following Initial Conditions and Boundary Conditions (Pressure Poisson; Cavity Flow):

$$u = 1 \text{ at } y = 2 \text{ (the "lid");}$$

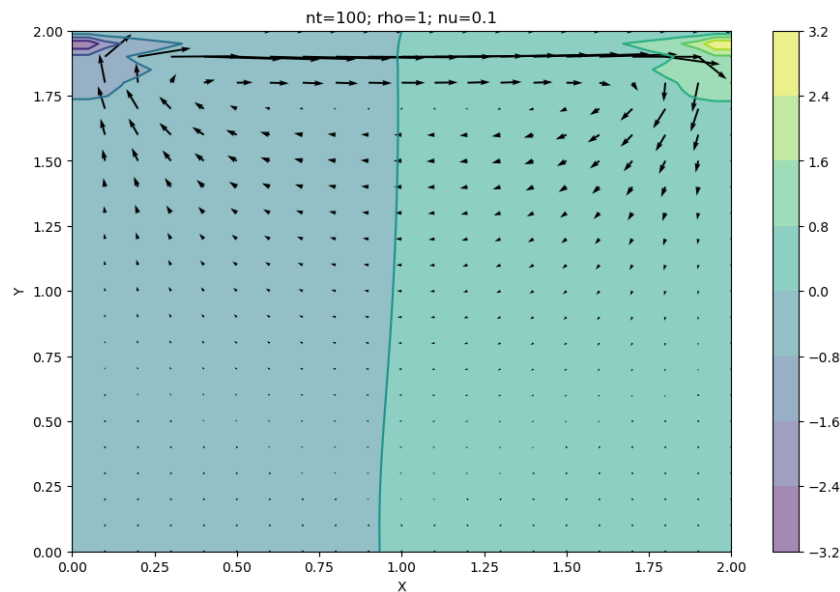
$$u, v = 0 \text{ on the other boundaries;}$$

$$\frac{\partial p}{\partial y} = 0 \text{ at } y = 0;$$

$$p = 0 \text{ at } y = 2$$

$$\frac{\partial p}{\partial x} = 0 \text{ at } x = 0, 2$$

Plot the Navier Stokes velocity field and recreate the following figure:



- b. Redo 2a but now vary the number of timesteps (nt) to find at what point the field stabilizes (reaches an equilibrium state). Visual inspection will suffice (no need to calculate and/or minimize differences).
 - i. For $nt = 500$, are there changes in the pressure zones compared to the simulation output at $nt = 100$?
 - ii. For $nt = 700$, are there changes in the pressure zones compared to the simulation output at $nt = 500$?
 - iii. For $nt = 1000$, are there changes in the pressure zones compared to the simulation output at $nt = 700$?
 - iv. For $nt = 1200$, are there changes in the pressure zones compared to the simulation output at $nt = 1000$?

- c. Explore the influence of viscosity, n , on velocity field: Alter the viscosity variable, n , keep the other variable declarations and initial conditions the same.
 - i. Vary nt . Note the changes in the pressure zones as a function of viscosity. How does viscosity appear to influence the pressure zones?
 - ii. Do the observed changes in the field make intuitive sense?
 - iii. Would lower viscosity cause an increase or decrease in the loss of kinetic energy?
 - iv. Would adding salt to water increase or decrease the viscosity of water? Why?
 - v. How would the increase or decrease in temperature influence the viscosity? Why?
- d. **3D Navier-Stokes:** Implement the 3D Navier Stokes as described in the following publications and code. Observe the influence of viscosity, n , by *decreasing and increasing* its value from the initial value in the code. Do the observed changes in the field make intuitive sense?
 - i. Martin Bazant, Henry Moffatt, *Exact solutions of the Navier-Stokes equations having steady vortex structures*, Journal of Fluid Mechanics, Volume 541, pages 55-64, 2005.
https://math.mit.edu/~bazant/papers/pdf/Bazant_2005_J_Fluid_Mech_MM3.pdf
 - ii. Johannes Burgers, *A mathematical model illustrating the theory of turbulence*, Advances in Applied Mechanics, Volume 1, pages 171-199, 1948.
 - iii. C Ross Ethier, David Steinman, *Exact fully 3D Navier-Stokes solutions for benchmarking*, International Journal for Numerical Methods in Fluids, Volume 19, Number 5, March 1994, pages 369-375.
<https://www.ljll.math.upmc.fr/~frey/papers/Navier-Stokes/Ethier%20C.R.,%20Steinman%20D.A.,%20Exact%20fully%203d%20Navier-Stokes%20solutions%20for%20benchmarking.pdf>
 - iv. Python code:
https://people.sc.fsu.edu/~jburkardt/py_src/navier_stokes_3d_exact/navier_stokes_3d_exact.py

Navier_stokes_3d_exact.py	Reference
def uvwp_burgers (nu, n, x, y, z, t):	Burgers paper
def resid_burgers (nu, n, x, y, z, t):	Burgers paper
def resid_ethier (a, d, n, x, y, z, t):	Ethier and Steinman: Eqn 15a; 15b; 15c; 15d
def uvwp_ethier (a, d, n, x, y, z, t):	Ethier and Steinman: Eqn 15a; 15b; 15c; 15d

3. (Low Complexity) Halfar Dome Unit Test

- a. Refer to Section 8.1.1 Halfar dome in the [CISM documentation](#). Extract the relevant equations to model a dome of ice analytically. Further reading can be found at [Halfar 1984](#) and in Buehler's notes (provided as a file in the starter kit). Parts b and c are based off of the CISM documentation.

- The general shallow ice approximation (SIA) ice evolution equation is $\frac{\delta H}{\delta t} = \nabla \cdot (\Gamma H^{n+2} |\nabla H|^{n-1} \nabla H)$.
- The Glen flow law exponent, n , is usually $n = 3$. G is defined as: $\Gamma = \frac{2}{n+2} A (\rho g)^n$.
- Set acceleration $g = 9.8 \text{ m/s}^2$, $A = 1 \times 10^{-16} \text{ Pa}^{-3}/\text{s}$, and density $\rho = 910 \text{ kg/m}^3$, which are the values used by CISM. We can define $t_0 = \frac{1}{18 \Gamma} \left(\frac{7}{4}\right)^3 \frac{R_0^4}{H_0^7}$.
- Using these definitions, the time dependent solution of the SIA using Halfar's

$$\text{approximation is: } H(t, r) = H_0 \left(\frac{t_0}{t}\right)^{\frac{1}{9}} \left[1 - \left(\left(\frac{t_0}{t}\right)^{\frac{1}{18}} \frac{r}{R_0}\right)^{\frac{4}{3}}\right]^{\frac{3}{7}}$$

- b. Using these equations, model a glacier with a range of initial radii and thicknesses at different time steps. What happens to thin glaciers as opposed to thick glaciers over time? How does the initial radius affect glacial evolution? The Halfar approximation is one approach to solving this, and reference code can be found in [runHalfar.py](#), in the CISM repository.
- c. What other values of r and A are used in the literature? Redo 3b with these literature values – what is the impact on the model output?
- d. Now redo 3b, but numerically solve SIA with finite difference equations. You can use [this notebook](#) as a reference for one way to do this. As an extension, model a sloped bed instead of a flat bed ice sheet. What is the impact of using a sloped bed versus a flat bed?
- e. **Challenge:** Extract the entire [CISM libglimmer library](#) (~40 Fortran files), redo 3.b and compare with the Halfar approximation (like the example in the CISM documentation shows).

4. (Medium Complexity) Oceananigans Scenario. This example is from the [Oceananigans's wind mixing and convection tutorial](#). We simulate the wind and convection mixing at the ocean surface boundary layer in 3 dimensions. Model the ocean wind mixing and convection, outputting the changes in vertical velocity, temperature, salinity, and eddy viscosity.

- The general advection equation is $u \cdot \nabla = u_x \frac{\partial}{\partial x} + u_y \frac{\partial}{\partial y} + u_z \frac{\partial}{\partial z}$.
- Start with a $32(x \text{ direction}) \times 32(y \text{ direction}) \times 24(z \text{ direction})$ grid with 2-meter grid spacing horizontally while vertically varying the spacing so that there is higher resolution near the ocean surface.

- We define the seawater buoyancy using a linear equation of state (EOS) as $b = g(\alpha T - \beta S)$ where $g = 9.8 \frac{m}{s^2}$, $\alpha = 2 \cdot 10^{-4} K^{-1}$ (expansion coefficient), and $\beta = 8 \cdot 10^{-4} ppt^{-1}$ (Haline contraction coefficient).
 - The boundary conditions are: $200 W m^{-2}$ surface cooling, density of the surface of the ocean of $1026 kg/m^3$ and the heat capacity of sea water $3991 J/K/kg$.
 - The temperature gradient is set at $0.01 K/m$.
 - The velocity field is blowing over the ocean's surface at $10 m/s$. The drag coefficient is $c_b = 2.5 \cdot 10^{-3}$. The density of the air at the ocean's surface is $\rho_{air} = 1.225 \frac{kg}{m^3}$.
 - The evaporation rate of the ocean into the atmosphere is $evap = \frac{10^{-3}}{hour} m/s$ (1 mm per hour). However, the true evaporation rate depends on the salinity of the water and is therefore $evap = \frac{10^{-3}}{hour} \cdot S$ where S is the salinity.
 - The initial conditions are a linearly stratified fluid. Include noise for an extra challenge.
 - a. Solve the advection equation
 - b. Repeat 4a but now include the Coriolis force
 - c. What is the impact of using a different bottom boundary layer, as described in https://clima.github.io/OceananigansDocumentation/stable/generated/tilted_bottom_boundary_layer/ ?
5. (Medium Complexity) Coupling oceanic models for biogeochemical transport-kinetics.

Chlorofluorocarbons 11 and 12 (CFC-11 and CFC-12), and sulfur hexafluoride (SF6), are synthetic chemicals manufactured for industrial and domestic applications from the early 20th century onwards. CFC-11 (CCl3F) is a volatile liquid at room temperature and was widely used in refrigeration. CFC-12 (CCl2F2) is a gas at room temperature, and, like CFC-11, was widely used as a refrigerant, and additionally as an aerosol propellant. SF6 (SF6) is also a gas at room temperature, with a range of applications based around its property as an excellent electrical insulator (often replacing more toxic alternatives). All three are relatively inert chemicals that are both non-toxic and non-flammable, and their wide use has led to their accumulation within the Earth's atmosphere. Large-scale production of CFC-11 and CFC-12 began in the 1930s, while production of SF6 began in the early 1950s.

- a. Using NEMO, generate the atmospheric concentration time-histories for each of the three gases CFC-11, CFC-12, and SF6 in both the Northern Hemisphere (NH) and the Southern Hemisphere (SH) from 1955 to 2015 (Note: SF6 production began in the 1950s).
- b. Thought experiment: Based on the solubility (chemical and physical properties) of CFC-11, CFC-12 and SF6, where would these inert gases be most soluble in the ocean water? Start from the Arctic (North Pole) and move to the Antarctica (South Pole). Going East to West, consider the Atlantic Ocean, Indian Ocean, and Pacific Ocean.

- c. Using Eqn 2.12 and supporting Table 2.1, plot out the solubility distribution. Does the plotted output make sense with the above *thought experiment* (5b)?
- d. Given CFCs and SF6 are not naturally occurring, what ocean body (and what part) should contain the highest concentration?
- e. Based on 5d, where would you propose placing probes or monitoring stations to alert you that *some larger scale* phenomena may be underway?
- f. Describe at least 2 mechanisms (or processes, non-anthropogenic) by which the ocean will become a source of CFCs and SF6.
- g. Based on 5a above, what would be the apparent exchange rate between the Northern and Southern hemisphere for the CFCs and SF6.
- h. **Reference materials:**
 - I. NEMO Consortium. <https://www.nemo-ocean.eu/>
 - II. NEMO Ocean Engine. Reference Guide. <https://zenodo.org/record/6334656>
 - III. NEMO TOP PISCES. Reference Guide. <https://zenodo.org/record/1471700>
 - IV. NEMO User Guide. <https://sites.nemo-ocean.io/user-guide/>
 - V. NEMO Tracer CFC Code (Repo: <https://forge.nemo-ocean.eu/nemo/nemo>)
 1. /nemo/src/TOP/CFC/*.F90
 - a. Example: trcsms_cfc.F90. Schmidt number coefficients are set at subroutine cfc_init().
 - VI. Data: From NEMO download (repository)
 1. CFCs_CDIAC.dat (/nemo/cfg/ORCA2_OFF_TRC/EXPREF)
 - VII. Data: US Geotrace Program
 1. <https://www.bco-dmo.org/dataset/820232>
 2. <https://www.bco-dmo.org/dataset/820232/data>
 - VIII. Data: CFC concentration
 1. <https://www.nature.com/articles/s41586-021-03260-5>

Transport-reaction equation (Eqn 2.1 from NEMO TOP)

$$\frac{\partial C}{\partial t} = S(C) - \frac{1}{b_i} \left[\frac{\partial e_{2u} e_{3u} u C}{\partial i} + \frac{\partial e_{1v} e_{3v} uv, C}{\partial i} \right] + \frac{1}{e_{3t}} \frac{\partial w C}{\partial k} + D^l C + D^v C \quad (2.1)$$

Air-Sea flux (Eqn 2.8 from NEMO TOP)

$$F_{cfc} = K_w \cdot (C_{sat} - C_{surf}) \cdot (1 - f_i) \quad (2.8)$$

F_{cfc} is air-sea flux

K_w is the piston velocity

C_{sat} is the saturation concentration (in mol m⁻³)

C_{surf} is the local surface concentration (in mol m⁻³)

f_i is the fractional sea-ice cover of the local ocean (range: 0.0 for ice free ocean .. 1.0 for ice covered ocean)

Distribution Statement A. Approved for public release: distribution is unlimited

MITRE Public Release Case Number 24-2089

© 2025 The MITRE Corporation.

Saturation concentration (Eqn 2.9 from NEMO TOP)

$$C_{sat} = Sol \cdot P_{cfc} \quad (2.9)$$

Sol is the gas solubility (in mol m⁻³ pptv⁻¹)

P_{cfc} is the atmosphere concentration of CFC (pptv, parts per trillion by volume)

Piston velocity (Eqn 2.10 from NEMO TOP) relates to the wind “driving” CFC gas into the sea surface at a particular temperature.

$$K_w = X_{conv} \cdot a \cdot u^2 \cdot \sqrt{\frac{Sc(T)}{660}} \quad (2.10)$$

K_w is the piston velocity, function of a 10 m wind speed (in m s⁻¹)

$X_{conv} = 0.01 / 3600$, a conversion factor that changes the piston velocity from cm h⁻¹ to m s⁻¹

a is a constant = 0.251 (in cm h⁻¹ / (m s⁻¹)²)

u is the 10 m wind speed in m s⁻¹

Sc is the Schmidt number

Schmidt number (Eqn 2.11 from NEMO TOP) dimensional less number that relates momentum diffusivity (kinematic viscosity) and mass diffusivity.

$$Sc = a0 + (a1 \cdot T) + (a2 \cdot T^2) + (a3 \cdot T^3) + (a4 \cdot T^4) \quad (2.11)$$

Sc is the Schmidt number

a_x coefficients in Table 2.2

Solubility (Eqn 2.12 from NEMO TOP)

$$\ln(Sol) = a_1 + \frac{a_2}{T_X} + a_3 \cdot \ln T_X + a_4 \cdot T_X^2 + S \cdot (b_1 + b_2 \cdot T_X + b_3 \cdot T_X^2) \quad (2.12)$$

T_X is (T + 273.16) / 100, a function of temperature

a_x and b_x coefficients are specific for each gas (Table 2.1)

Table 2.1.: Coefficients for fit of the CFCs solubility (Source: NEMO TOP v 4.2.0. Coefficients for Eqn 2.12)

Gas	a1	a2	a3	a4	b1	b2	b3
CFC-11	-218.0971	298.9702	113.8049	-1.39165	-0.143566	0.091015	-0.0153924
CFC-12	-229.9261	319.6552	119.4471	-1.39165	-0.142382	0.091459	-0.0157274
SF6	-80.0343	117.232	29.5817	0.0	0.0335183	-0.0373942	0.00774862

Table 2.2.: Coefficients for fit of the CFCs Schmidt number (Source: NEMO TOP v 4.2.0. Coefficients for Eqn 2.11)

Gas	a0	a1	a2	a3	a4
CFC-11	3579.2	-222.63	7.5749	-0.14595	0.0011874
CFC-12	3828.1	-249.86	8.7603	-0.1716	0.001408
SF6	3177.5	-200.57	6.8865	-0.13335	0.0010877

Distribution Statement A. Approved for public release: distribution is unlimited

MITRE Public Release Case Number 24-2089

© 2025 The MITRE Corporation.

6. (Hard Complexity, Challenge) The [Community Earth Systems Model \(CESM\)](#) is a coupled climate/Earth systems modeling framework, which is in many ways analogous to the Space Weather Modeling Framework (SWMF). It allows for a variety of component models (representing different climate subsystems) to be coupled in different configurations. Extract the POP2 OGCM (ocean global circulation model), and the CICE5 sea ice model, and run a simulation with these models coupled together.

