Description of the Halfar Ice Model

This document provides a quick reference to what we refer to as the Halfar Model, which describes the diffusion of a piece of ice. Note that this model conserves the mass of ice; ice is neither created nor destroyed in this model, a known limitation that prevents extensions of the model to include melting or ice deposition. However, it is still a useful model in the context of the ASKEM evaluation.

For more information about the model than presented here, refer to Section 8.1.1 Halfar dome in the CISM documentation. Further reading can be found at Halfar 1984 and in Buehler's notes (provided in the supplementary materials). The most important equations are listed below:

- The general shallow ice approximation (SIA) ice evolution equation is

  $$\frac{\delta H}{\delta t} = \nabla \cdot (\Gamma H^{n+2} |\nabla H|^{n-1} \nabla H)$$

  (when using the Halfar Decapodes model, use the model that corresponds to this equation)
- The Glen flow law exponent, n, is usually $n = 3$. Γ is defined as: $\Gamma = \frac{2}{n+2} A(\rho g)^n$.
- For $n = 3$, the time dependent solution of the SIA using Halfar's approximation is:

$$H(t,r) = H_0 \left(\frac{t_0}{t}\right)^{\frac{1}{9}} \left[1 - \left(\left(\frac{t_0}{t}\right)^{\frac{1}{18}} \frac{r}{R_0}\right)^{\frac{4}{3}}\right]^{\frac{3}{7}}$$

Where $t_0 = \frac{1}{18\,\Gamma}\left(\frac{7}{4}\right)^3 \frac{R_0^4}{H_0^7}$, and H_0, R_0 are the central height of the dome and its radius at characteristic time $t = t_0$

The default parameter values are:

Parameter	**Value and Unit**
n (Glen's flow law exponent)	3 [unitless]
g (gravitational acceleration)	9.8101 [m/s^2]
ρ (density of ice)	910 [kg/m^3]
A (ice softness)	1E-16 [Pa^{-3}/s]