COMP9334 Assignment1
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Q1
(a)According to service law:
Service demand D(j)=U(j)/X(0)
X(0)=1231/(30*60)=0.6839
U(Disk1)
Service demand at each device:

device	D(j)	utilization
Disk1	754ms	0.516
Disk2	826ms	0.565
Disk3	1028ms	0.703
CPU	895ms	0.612

(b)
Using bottleneck analysis to determine the asymptomic bound on thr system:
When there are 40 active terminals

Disk2 has highest service demand 1030ms.
The maximum system throughput is 1/1030=0.97 jobs/s
According to the throughput bound:
$$X(0) <= min[\frac{1}{max(D_i)}, \frac{N}{\Sigma_{i=1}^{k} D_i + thinktime}]$$

Assume t0 is the crosspoint of the two boundary lines.
1/(0.76+0.838+1.03+0.897+27)t0=0.97
1/30.525*t0=0.97
t0=29.6
Since N=40>29.6
Therefore the asymptotic bound on the system throughput is 0.97 jobs/s

(c)

When the number of terminal is 40
response time+thinking time=number of terminals /system throughoutput
Therefore response time=40/0.97-27=14.23s

Q2
(a)

According to queuing theory:
The possibility that an incoming arrival is blocked=Probability that there are m customers in the system.
In this case, m=4
Service rate=1/mean service time=1/10min
Therefore,
$\lambda = 20/60 = 1/3(transactions/min)$
$\mu = 1/10(transactions/min)$

$\rho = \frac{\lambda}{\mu} = 10/3$

$P_m = \frac{\frac{\rho^m}{m!}}{\sum_{k=0}^{m} \frac{\rho^k}{k!}}$

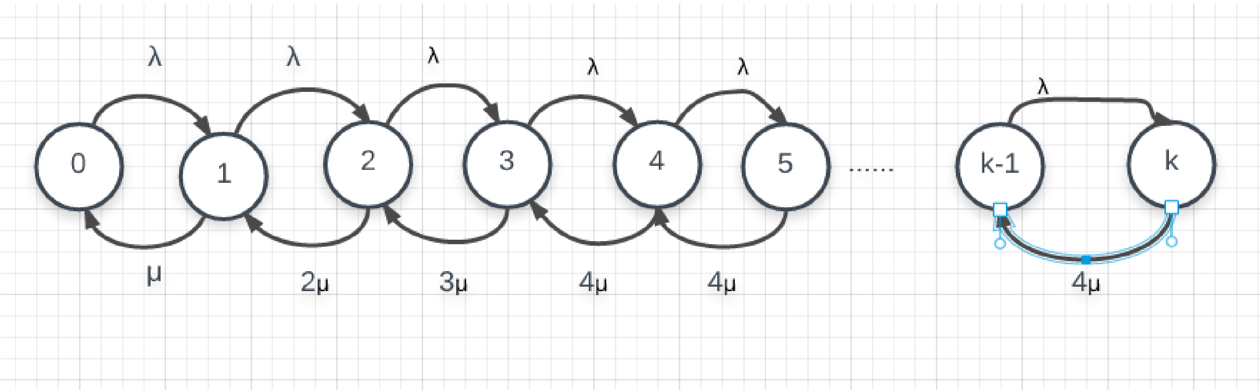
Possibility that an incoming call is rejected $P_m = 0.2425$

(b)

(i)

Decrease the loss rate less than 50% means the loss rate is less than 0.2425*0.5=0.12125

Formularte Markov chain for 4 operators and M holding slots as follows:



Definition of state:

State=0 :all operators are ilde

State=1 only one operators is busy

State=2 only 2 operators are busy

State=3 only 3 operators are busy

State=4 All operators are busy

State=5 All operators are busy,1 task in queue

…

State=k All operators are busy, k-4 tasks in queue

(ii)

balance equation:

$\lambda P_0 = \mu P_1$

$\lambda P_0 + 2\mu P_2 = (\mu + \lambda) P_1$

$\lambda P_1 + 3\mu P_3 = (2\mu + \lambda) P_2$

$\lambda P_2 + 4\mu P_4 = (3\mu + \lambda) P_3$

$\lambda P_3 + 4\mu P_5 = (4\mu + \lambda) P_4$

$\lambda P_4 + 4\mu P_6 = (4\mu + \lambda) P_5$

…

$4\mu P_k = \lambda P_{k-1}$

(iii)

k=M+4

$P_1 = \rho P_0$
$P_2 = \rho^2/2P_0$
$P_3 = \rho^3/6P_0$
$P_4 = \rho^4/24P_0$
$P_5 = \rho^4/24 * \rho/4P_0$
$P_6 = \rho^4/24 * (\rho/4)^2 P_0$
…
$P_k = \rho^4/24 * (\rho/4)^{k-4} P_0$

$(1 + \rho + \rho^2/2 + \rho^3/6 + \rho^4/24 + (\rho^4/24)\Sigma_{i=5}^{k}((\rho/4)^{k-4}))P_0 = 1$
$P_0 = \frac{1}{1+\rho+\rho^2/2+\rho^3/6+\rho^4/24+(\rho^4/24)\Sigma_{i=5}^{k}((\rho/4)^{k-4})}$

(iv)
Steady state probabilities

P0= 0.0312
P1= 0.104
P2= 0.173
P3= 0.193
P4= 0.161
P5= 0.134
P6= 0.111
P7= 0.0929

Smallest value of M is 3
Using python code to calculate.

(v)
The time an accepted call will be wait :
Navg=3.659
throughoutput=20/h
Using little's law:
$responsetime = 3.659/20(1 - P_7) = 0.20h$
mean service time=1/6 h
wating time=response time-service time=0.035h

Q3

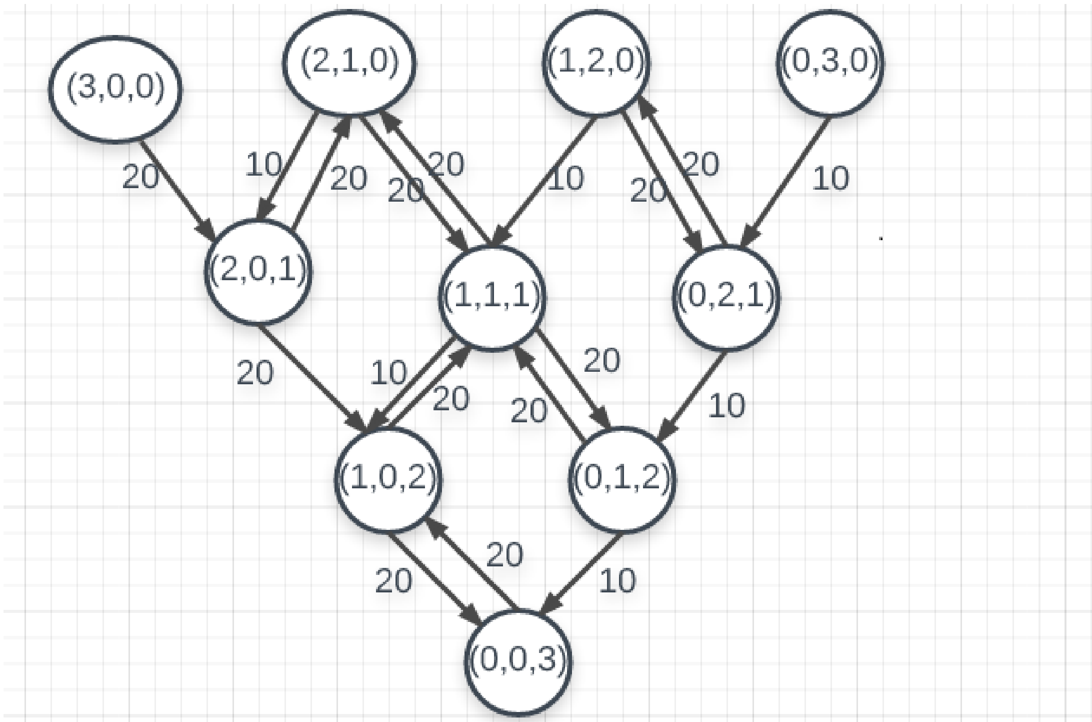
(a)

A list of 3-tuple states:

(#CPU1,#CPU2,#Disk)

(3,0,0)
(2,1,0)
(1,2,0)
(0,3,0)
(2,0,1)
(1,1,1)
(0,2,1)
(1,0,2)
(0,1,2)
(0,0,3)

The transition rate between states:



(b)

A=

$$\begin{pmatrix} 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & -20 & -20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 30 & 0 & 0 & 0 & -20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ -20 & -10 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 \\ 0 & -20 & -10 & 0 & 0 & 50 & 0 & -20 & -20 & 0 \\ 0 & 0 & -20 & -10 & 0 & 0 & 30 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20 & -10 & 0 & 40 & 0 & -10 \\ 0 & 0 & 0 & 0 & 0 & -20 & -10 & 0 & 30 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{1}$$

x=

$$\begin{pmatrix} P(3,0,0) \\ P(2,1,0) \\ P(1,2,0) \\ P(0,3,0) \\ P(2,0,1) \\ P(1,1,1) \\ P(0,2,1) \\ P(1,0,2) \\ P(0,1,2) \\ P(0,0,3) \end{pmatrix} \tag{2}$$

B=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{3}$$

Balance equation:

$$A * x = B$$

(c)

P(3,0,0)=0
P(2,1,0)=0.1579
P(1,2,0)=0
P(0,3,0)=0
P(2,0,1)=0.0395
P(1,1,1)=0.1974
P(0,2,1)=0
P(1,0,2)=0.2039
P(0,1,2)=0.1316
P(0,0,3)=0.2697

(d)

Throughput=Utilisation*Service rate
Disk utilisation=P(2,0,1)+P(1,1,1)+P(0,2,1)+P(1,0,2)+P(0,1,2)+P(0,0,3)=0.8421
Throughput=0.8421*20=16.842 transactions/s

(e)
Use Little's Law
CPU1 utilization=0.1579 +0.0395 +0.1974+0.2039 =0.5987
CPU1 throughout=utilization*service rate=0.5987*20 =11.974 transactions/s
N=(0.1579+0.0395)*2+(0.1974+0.2039)*1=0.7961

Response time of CPU1:
R=N/x with N=0.7961
=0.7961/11.974=66ms

(f)
Using little's law
mean jobs in Disk=(0.0395+ 0.1974)*1+(0.2039+0.1316)*2+0.2697*3=1.717
The time user has to wait:

R=N/x=1.717/16.842=102ms
waiting time=response time -service time=102-50=52ms