SCI2019 - Linear Algebra

Course coordinator: Dr. Pietro Bonizzi, Faculty of Humanities and Sciences, DKE, pietro.bonizzi@maastrichtuniversity.nl

Lecturer: Dr. Pietro Bonizzi, Faculty of Humanities and Sciences, DKE, Bouillonstraat 8-10, 6211 LH, Maastricht, The Netherlands, **room 1.006**

Email: pietro.bonizzi@maastrichtuniversity.nl
Telephone: +31 (0)43 388 3479

Course description:
Linear algebra is the branch of mathematics which is primarily concerned with problems involving linearity of one form or another. This is reflected by the three central themes of this introductory course.

The first theme is concerned with what can be recognized without doubt as the most frequently occurring mathematical problem in practical applications: how to solve a system of linear equations. For this problem a complete algebraic solution procedure is developed which provides the student with a way to deal with such problems systematically, regardless of the number of equations or the number of unknowns.

The second theme addresses linear functions and mappings, which can be studied naturally from a geometric point of view. This involves geometric ‘objects’ such as points, lines and planes, and geometric ‘actions’ such as rotation, reflection, projection and translation. One of the main tools of linear algebra is offered by matrices and vectors, for which a basic theory of matrix-vector computation is developed. This allows one to bring these two themes together in a common framework, in what turns out to be an exceptionally fruitful way. By introducing the notions of vector spaces, inner products and orthogonality, a deeper understanding of the scope of these techniques is developed, opening up a large array of rather diverse application areas.

The third theme surfaces when the point of view is shifted once more, now from the geometric point of view to the dynamic perspective, where the focus is on the effects of iteration (i.e., the repeated application of a linear mapping). This involves a basic theory of eigenvalues and eigenvectors, which have many applications in various branches of science as will be discussed. For instance, important applications in problems involving dynamics and stability, and applications to optimization problems found in operations research.

Many examples and exercises shall be provided to clarify the issues and to develop practical computational skills. They also serve to demonstrate practical applications where the results of this course can be successfully employed.

Course objectives:
In this course we provide an introduction to the main topics of linear algebra. Emphasis is on an understanding of the basic concepts and techniques. Students will:

- Develop the practical, computational skills to solve problems from a wide range of application areas.
- Obtain the insight that various seemingly different questions all boil down to the same mathematical problem of solving a system of linear equations.
- Learn to look at the same problem from different angles and learn to switch point of view (from algebraic to geometric).

Prerequisites:
Quantitative Reasoning SCI1010.

Recommendations:
It is recommended that you passed an exam in mathematics in secondary school. If your knowledge of mathematics is low or has dissipated from your memory it is recommended to prepare yourself by reading Sections 1.1 and 1.2 of the textbook.

Course contents
The following topics will be addressed during the course

- Linear equations, Vector and Matrix equations, Solution sets, Linear independence, and Matrix algebra
  Systems of linear equations lie at the heart of algebra. Moreover, they are equivalent to vector and matrix equations. They are fundamental to the understanding of concepts of spanning, linear independence, and linear transformations, which will play an essential role throughout the course. Sections of interest: 1.1-1.5, 1.7, 2.1

- Linear transformations, Matrix inverse, and Vector spaces
  Matrix algebra is fundamental in several real world applications from different fields of science, as economics, computer graphics, signal processing. The general vector space framework in particular becomes essential in dealing with discrete signals in digital control, as it will be shown through several examples.
  Sections of interest: 1.8, 2.2, 2.3, 4.1

- Column space and Null space, Basis, Coordinate systems, Dimension and Rank
  Coordinate systems pave the way to concepts as change-of-coordinates and coordinate mapping. These in turns will allow to talk about representation of investigated quantities (in terms of matrices) into new coordinate systems (new subspaces) with very special characteristics, as the one derivable from an eigenvalue decomposition of a matrix.
  Sections of interest: 4.2-4.6

- Determinants, Eigenvectors and eigenvalues
  Eigenvectors are a special set of vectors associated with a linear system of equations. The determination of the eigenvectors and eigenvalues of a system is extremely important in physics and engineering, where it is equivalent to matrix diagonalization and arises in such common applications as stability analysis or signal denoising, to name a couple. Each eigenvector is paired with a corresponding so-called eigenvalue.
  Sections of interest: 3.1, 3.2, 5.1, 5.2

- Diagonalization and Orthogonality
  In order to find an approximate solution to an inconsistent system of equations that has no actual solutions, a well-defined notion of nearness is needed. We will see how orthogonality can be used to identify the point within a subspace that is nearest to a point lying outside that subspace, and to produce approximate solutions.
  Sections of interest: 5.3, 6.1, 6.2, 6.3

- Symmetry
  Symmetric matrices are an important class of matrices which arises more often in applications, in one way or another, than other major class of matrices.
  Sections of interest: 7.1

Tentative schedule

This is merely a guideline. The pace will be adjusted as needed and topics inserted or deleted. Suitable exercises are indicated. You're not expected to make all of them but as many as you need to get a good understanding.

Section	Exercises
1.1:	3, 7, 11, 12, 15, 16, 19, 23, 24, 29, 30;
1.2:	2, 4, 7, 8, 11, 12, 16, 17, 18, 21, 22;
1.3:	3, 5, 9, 11, 12, 14, 21, 22, 26;
1.4:	7, 11, 17, 18, 21, 23, 26, 27, 29;
1.5:	5, 6, 14, 19, 20, 21, 22;
1.7:	1, 2, 7, 11, 15, 16, 21, 24, 27, 28, 37, 38;
2.1:	7, 8, 9, 10, 15, 16, 19, 20, 21, 22;
1.8:	3, 9, 19, 21, 22, 29;
2.2:	1, 4, 5, 9, 10, 11, 12, 31, 33;
2.3:	1, 3, 5, 8, 9, 10, 16, 17;
4.1:	3, 5, 6, 7, 10, 13, 14;
4.2:	2, 3, 4, 7, 8, 9, 10, 15, 23, 24;
4.3:	2, 4, 7, 8, 9, 15, 16, 21, 22, 23, 24;
4.4:	1, 7, 13, 17;
4.5:	1, 5, 7, 8, 10, 13;
4.6:	1, 2, 7, 9, 13, 18, 21;
3.1:	1, 3, 5, 14, 25, 27, 29, 39, 40;
3.2:	5, 9, 11, 12, 13, 17, 18, 21, 22, 24, 25;
5.1:	1, 2, 4, 6, 13, 14, 15, 19, 20, 21, 23, 24, 25, 26, 27, 29;
5.2:	2, 3, 7, 8, 9, 10, 18, 19, 21, 22;
5.3:	1, 3, 7, 11, 12, 17, 23, 24, 25, 26, 27;
6.1:	13, 14, 15, 16, 19, 20;
6.2:	1, 3, 5, 9, 11;

7.1: 7, 8, 12, 23;

Instructional format:
Lectures: There will be two 2-hour lectures and one 2-hour tutorial per week, consisting of:
(1) a frontal, but interactive instruction and
(2) an active training in comprehending the instructed material by spending a lot of time on solving real world problems, either individually or, as is encouraged, jointly with other participants.
During all lectures instruction and practice will alternate in line with the progress of the material in the book.

Literature:
David C. Lay, Linear Algebra and its Applications, 4th ed., Pearson Addison Wesley, 2012. Obtainable at the Academic Bookshop.

Note: There are several versions of this book on the market (hardcover, paperback, with updates, instructor's edition, etc.) but for the purpose of this course all versions are fine. The page numbering may differ between the versions.

Examination:
There will be two 2-hour written (closed-book) exams, one in the middle of the course (midterm) and one in the examination week (final exam). The midterm will cover the material covered up to that point (roughly systems of linear equations, vector spaces, dimension and rank) and the final exam will focus on the material discussed in the second half of the course (roughly determinants, eigenvectors and eigenvalues, diagonalization and symmetry). The precise material for the midterm and final exam will be announced during the lectures. Each test will consist of solving a number of open problems for testing the understanding and the computational skills.

Your final score will be the average of the 2 scores for the exams, rounded to a decimal as required by the exam regulations.
If you fail the course, then in the resit week you will have to do a 3-hour resit on the full course material. Be aware that UCM regulations apply and that you must make all efforts to pass the course in order to be eligible for a resit.

Attendance:
There is 70% attendance requirement for this course.

Calculators:
The use of graphical or ordinary calculators and cell phones is prohibited during any of the exams.