SCI3006 Mathematical Modelling

Course coordinator

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Course lecturers and tutors

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Semester	Period	ECTS	Concentration
Spring	4	5	Sciences

Prerequisites

SCI2019 Linear Algebra and SCI2018 Calculus.

Objectives

- To have the ability to interpret dynamical phenomena as mathematical systems and to cast them into such form.
- To understand the basic concepts of linear systems theory.
- To be familiar with analysis techniques for linear systems, to understand their behavior and interaction.
- To become familiar with some application areas of mathematical systems and models.

Description of the course

To describe natural phenomena and processes, mathematical models are widely used. The focus in this course shall be on dynamical models (i.e., where time plays a role) in particular those that have interaction with the environment through inputs and outputs. Mathematical systems theory provides the framework to deal with such models in a systematic and useful way.
First we consider some general aspects of mathematical modeling. Then we briefly address dynamical systems without inputs and outputs - but which may show nonlinear behavior. We study basic properties such as equilibrium points, linearization, and stability.
We then switch to linear dynamical models with inputs and outputs. They are used in many different areas of the natural sciences and in engineering disciplines. We discuss the following topics and concepts. Linear difference and differential equations, Laplace transforms, transfer functions of linear systems; controllability, observability, minimality; system representations with an emphasis on state-space representations and canonical forms; stability; the interconnection of linear systems including feedback; frequency domain analysis and the relationship with filter theory, Fourier analysis, and time series analysis.
To demonstrate the applicability of the techniques and concepts, many examples from science and engineering are mentioned and briefly discussed.

Literature

- Lecture notes, electronically provided

Recommended background literature (not compulsory):

- R.J. Vaccaro, Digital Control. *A State-Space Approach*, McGraw-Hill International Editions, 1995. ISBN: 0-07-066781-0.
- D.W. Jordan and P. Smith, *Nonlinear Ordinary Differential Equations*, 2nd ed., (Oxford Applied Mathematics and Computing Science Series), Clarendon Press, 1987.

Instructional format

Lectures and exercises in a mixed and interactive way.

Attendance

A minimum attendance of 85% of the tutorial meetings is required. When the attendance requirement is not met, an additional assignment is given, consisting of a selected set of exercises.

Examination
Form: a written midterm and a written final exam with open questions.
Weighting: The final grade consists of 50% of the midterm result plus 50% of the final exam result.
Optionally, one may take a final exam on all of the course material, in which case the final grade consists of 20% of the midterm exam result and 80% of the final exam result.
No participation in the midterm exam implies a midterm grade of zero, still used in combining grades to arrive at a final grade as indicated above.
The resit exam is on all of the course material. No weighting with previous partial scores is performed: the new grade replaces all previous grades.

Tentative course schedule

Here's a brief overview of the planned contents of the lectures and tutorial sessions for each week. Tutorial sessions will be supplied with exercise sets. Before the midterm and final exams, old exams will be practiced and discussed.

Week 1: Modelling and Dynamics
An introduction to mathematical models and the model cycle. Examples of classical models are given, such as the predator-prey model. Dynamics: we discuss the element of time. Then we focus on nonlinear dynamical models, equilibria, linearization, phase diagrams and stability.
Week 2: Input-Output Systems
Input-output systems and high-order differential or difference equations are introduced. The Laplace transform and the z-transform are explained and applied. The transfer function of a system is introduced and the relationship Y = H U is derived. Partial fraction expansion is exercised and the impulse response is discussed. Stability of input-output systems is explained.
Week 3: State-Space Systems
We arrive at the concept of the state of a system. We formulate a state-space system (A,B,C,D) as a system of first-order equations. The transfer function and the relationship to input-output systems are discussed. State-space transformations, parameter redundancy and canonical forms are studied and exercised. Interconnection of systems (cascade, parallel and feedback) is exercised.

Midterm exam, taking place in week 4

Week 4: Discrete versus Continuous
Sampling a continuous-time system under the ZOH-principle gives a discrete-time system. Matrix exponentials are involved, they are defined and explained. The pole-mapping formula relates continuous-time and discrete-time systems. Stability is preserved under sampling.
Week 5: Sinusoidal Fidelity
The response of stable systems to harmonic inputs is studied. Interpretation of the transfer function in terms of frequency response follows: this leads us to the Bode diagrams. We discuss the relationship with Fourier transformation and Fourier analysis. An extra session on examples and a demonstration of the matlab "control system toolbox" is given.
Week 6: State Feedback
Controllability and observability, minimality and system order are discussed, as is the controllable canonical form. Pole-placement by state-feedback is explained and exercised. This allows one to build a simple but powerful state-feedback regulator if controllability holds.

Final exam, taking place in week 7