Theory Construction and Modelling Techniques

Code: COR1005

2017/2018
Period 4

Contents

General Information

Introduction

Ever wondered what the weather will be like tomorrow? How birds organize themselves in formation patterns? Who will be elected as president? These questions are difficult to answer, since they all relate to natural systems whose behaviour arises from complex interactions. For example, the individual preference of a voter for one candidate over the other is influenced by many factors such as mass media, the preference of other voters, or recent (political) events. Because of the excess of factors and the complexity of the interactions, it is quite difficult to make reliable predictions about who will become the next president.

Scientists employ models in an attempt to gain a deeper understanding of natural phenomena. Such models reveal important properties of the real phenomenon or system by focusing on factors that are believed to be important and ignoring others that are assumed to be less important. For instance, a simplified model of voting behaviour may take into account the factors “individual preference” and “preference of other voters” while ignoring other factors. Surely, the model will not be accurate in predicting the future president when prior to the elections a major terrorist attack or other political event occurs. On the other hand, the model may be very accurate in predicting preference formation within a group of voters.

Models allow us to approach complex questions in systematic ways. Such questions are present everywhere and it is through modelling that we can try to find some answers. Those questions may be broad at the start and described in fuzzy terms. However, as part of the modelling process those questions become more specific and structured.

Modelling helps us to break down what we are studying into variables, understand relations or correlations between them and even predict the future. The very purpose of that is all about describing, explaining and predicting the phenomena we encounter in the world around us.

An important part of modelling is experimental verification. The predictions of the model are compared to actual observations. A successful model yields predictions that agree with the observations and can be used to predict future events and to gain a deeper understanding of the underlying principles.

Aim and approach

This course provides an introduction to theorizing and modelling. It is relevant for a wide range of other courses that are offered at UCM.

The aim of the course is to familiarize students with model systems within the different disciplines of Sciences, Social Sciences and Humanities. Its aims are:

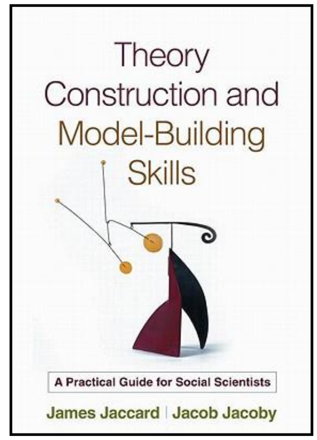
- To offer a broad overview of scientific models and modelling techniques in different disciplines;
- To teach students how to work with models in different academic fields;
- To teach students how to model a specific situation by using general models and modelling techniques.

The course starts with a short introduction into models, followed by several case studies that illustrate their usefulness in various contexts. Exposing students to models used both in academia and every-day thinking, the course fosters a deeper understanding of natural and social phenomena.

Throughout the course, students are encouraged to link models to specific situations and examples from daily-life. The final report allows students to use the knowledge gained in the course to analyse a phenomenon/situation of their own interest. This can be done either by conducting thought experiments, applying and redefining existing models or designing one’s own model.

The interactive lectures help students to gain a broad understanding of different kinds of modelling techniques. A special workshop is offered in order to trigger interests, thoughts and ideas and find ways of translating them into an individual and structured academic report.

Readings and resources



The main book used in this course is *Theory Construction and Model-Building Skills* by Jaccard and Jacoby (2010). In addition to the book, for several tasks a list of PDF-files is offered via the Student Portal. Some of these readings provide a theoretical framework for the models discussed in the different case studies; the other readings are a selection of examples of the application of the models in science. Browse smartly through the example-readings offered and concentrate on those that might be of interest for you.

As a lot of the resources consist of articles or chapters from books, you may sometimes have the feeling that you lack context, or that names crop up that you have not seen before. Go to any good introduction or handbook on the topic and look these up, if it impedes understanding of the argument. As a last resort you can always try and ask the members of your tutorial group.

Please refer to Appendix A for an overview of required and recommended resources per task.

Attendance requirement

The attendance requirement for the ten tutorial meetings is 85%. You are allowed to miss two regular tutorials without further consequences, providing that you have a valid reason. If you miss more meetings, you will have to apply for an additional assignment (request forms are available at the Office of Student Affairs). In order to qualify for an additional assignment you have to have valid reasons for all missed sessions. If you don't meet the attendance requirement, you are not eligible for a resit. You automatically fail the course if you miss over 30% of the scheduled meetings.

The practical session in the sixth week of the course is mandatory and is not considered one of the regular tutorial meetings to which the 85% attendance requirement applies.

Assessment

This course contains two moments of assessment:

1. a written final exam consisting of five open questions (counting towards 60% of the overall course grade). There will be two questions about the first four tasks of the course and four questions about the four case studies – for more information on the setup of the questions, see below;
2. a written assignment. You will receive one combined grade (counting towards 40% of the overall course grade) for:
    a. a 2500-3000 word report (excluding references) on a modelled phenomenon – for more information, see below;
    b. bonus and minus points for:
        i. the draft version of your report (insufficient-fail = -0.5, sufficient-good = 0)
        ii. two peer reviews (insufficient-fail = -0.5, sufficient-good = 0, excellent/distinguished = +0.5)

More details on the assessment criteria will be provided during the lectures in this course. Slides from the lectures will be posted via the Student Portal.

Final exam

The exam in this course consists of open questions that each includes three parts:

a. Conceptual level: *Define, identify, enumerate, classify, describe, list;* student shows concrete, reductive understanding of the topic; **2 points**
b. Relational level: *Compare, contrast, explain, integrate, relate;* student shows understanding of several components which are integrated conceptually showing understanding of how the parts contribute to the whole; **3 points**
c. Extended abstract level: *analyse, apply, theorize, generalize, predict, create, hypothesize, reflect, generate, evaluate;* student is able to conceptualize at a higher level of abstraction and apply; **5 points**

The topics discussed in this course will be divided over a total of six questions on the exam. Questions 1 and 2 focus on the first four tasks. Students have to answer both (full) questions. Questions 3 to 6 focus on the four case studies. Students have to answer three out of four (full) questions, adding up to a total of five full questions. For each question students can obtain ten points. The grade for the exam is calculated by means of: (obtained points/total possible points) x 10.

Although the attendance for lectures is not mandatory, the content adds to the pre- and post-discussions in the tutorial group meetings. Therefore, the content of the lectures is considered part of the examination.

During the lecture timeslot in the third week there will be time reserved to practice a few illustrative exam questions.

Report and peer review

The purpose of the written assignment in this course is teaching students modelling skills and letting them express that in a report. By doing that, we hope to achieve a transfer of both the improved modelling and writing skills to for example other courses, skills trainings and projects at UCM. In addition, students will be able to apply in a report what they have learned from the course book and from the case studies which was offered on a more theoretical level in this course.

Topic of the report

The writing assignment in this course gives you an opportunity to develop and elaborate on a modelled phenomenon. It allows you to search for a model or modelling technique from either the course or outside of the course and use this to model the phenomenon of interest.

Modelling is all about describing, explaining and predicting. This report is an exercise in describing a phenomenon or event, explaining it by means of a model and predicting by using the model. Be aware that the course is at an introductory level. The writing assignment does not set any higher claims. For example, you are not required to gather data for empirical verification.

Note: You are not allowed to use a paper/report from either this course or another course which you failed in the past and improve that and hand that in. Your choice of topic and approach need to be new in that sense. However, they may obviously fit your academic interests as expressed in other courses at UCM and your curriculum.

Format of the report

The report should be between 2500 to 3000 words, excluding references. If the word count is between 2250-2500 or 3000-3250, 0.5 points are subtracted from the final grade for the report. If the word count is below 2250 or above 3250, the report will not be graded and considered a fail. As it is a rather short report, do not make too many subdivisions. Use the structure that will be presented in the workshop in this course.

When you use information from a scientific article, university textbook or internet source, make sure you include the reference. Not doing so, is a form of plagiary. Do not take this lightly. Copying and pasting pieces of text from the internet is as much a form of plagiary as is copying from a book!

In addition to scientific articles or university textbooks, there is nothing against using the internet as a source. However, if you use an internet site, always put a reference to it in your report; omitting to do so and being caught in doing it, results in an annulment of the whole report.

Reports are to be handed in before the deadline (see overview below). This deadline cannot be extended. Failure to comply with it loses you all credits on the writing assignment.

Format of the draft

Although it is hard to set a strict guideline for the number of words for your draft, you can take 1800 as a minimum. Keep in mind that a draft is more than an outline.

It is very difficult, not to say impossible, to give constructive feedback on a list of bullet points. In addition to that, the more elaborate your draft is, the more feedback you can expect and the less work you need to do in order to finish your final report. In essence, a draft is a nearly complete and finished report, with the exception that it can still be changed.

Feel free to add a list of questions you still have with regard to the content of your own report (facts, theories, sources, interpretations).

The quality of your draft report counts towards the final grade for your written assignment.

Format of the review

You have to write a review for two drafts. In addition, two of your fellow students are requested to read your draft and provide feedback. The peer reviews will be assessed. The quality of your peer reviews counts towards the final grade for your written assignment.

The reviews should be constructive; indicating that something is well written and easy to read, may sound very nice, but doesn't help the author to improve anything. Guidelines on writing the peer reviews will be provided during the workshop on modelling and report writing in this course.

The peer reviews will be discussed in your tutorial group. This meeting is considered a practical session rather than a regular tutorial meeting. Therefore the attendance is mandatory and 100%.

Remember to **bring two copies of each review with you to the tutorial meeting** (one for the tutor and one for the fellow student whose draft version you reviewed). In addition, upload your peer reviews in Assignments in the Student Portal. **Do not forget to merge them into one document since you can upload only one file.**

Below you find guidelines on what each review should include:

- A brief summary of the report (approximately 200 words): This allows both the reviewer and the author to check whether the article is understood.
- Major comments and criticisms of the report (minimum 500 words): Starts with the larger issues and ends with details. Use criteria for the report as a guideline. Explain in full sentences why certain areas are a positive aspect of the report or a matter of concern and provide suggestions on how to improve. This part of the peer review focuses mainly on the content part of the assessment criteria and hence the steps and phases of modelling the phenomenon at hand.
- Minor comments and criticisms of the article (approximately 100 words): Once the major concerns have been laid out, it is welcome to point out minor comments and criticism (e.g. mislabelling of figures or tables, spelling or grammar mistakes, etc.). This element of the peer review focuses on the delivery part of the assessment criteria.

Workshop

A workshop is offered which is used as a starting point for picking an interesting phenomenon and applying and/or adapting an existing model to that specific phenomenon. It will help you to select a topic and consider possible explanations and models for your selected topic.

Besides providing you with detailed information on the requirements of the written assignment for this course, the workshop serves the purpose of triggering your interest, thoughts, ideas and creativity. It will do so by means of providing you with a number of examples of modelled situations that students in previous editions came up with. This will support you in working towards the report in which you will explain a phenomenon or situation that caught your interest.

Resit

Students who initially fail the course, but who have complied with the compulsory attendance requirement and took part in all of the assessment during the course, are eligible for one resit.

The overall grade for this course consists of a grade for the written assignment (40%) and a grade for the exam (60%). The resit serves the purpose of lifting your overall grade to sufficient/above 5.5. It does not replace the overall grade. Hence, the resit can be either on the final exam OR on the final report version of the written assignment. In case of a resit, you will have to redo that part of the assessment for which you received the lowest grade. In case of redoing the report, you can either rewrite the original report, incorporating the feedback you received, or choose a new topic and write a totally new report. In addition, bonus and minus points as described above, will be annulled and the grade for the report replaces the grade for this assignment.

In order to receive a grade for the exam you will have to do a serious attempt at passing the exam. If it is not deemed a serious attempt you will not receive a grade and you will not qualify for a resit. The same thing holds for the written assignment. If it is not deemed a serious attempt you will not receive a grade and you will not qualify for a resit.

Course coordinators

Questions or issues regarding the course content and e.g. assessment should first be discussed with the tutorial group and tutor. Those questions can also be discussed with the coordinators, but preferably during or directly after the lectures. The coordinators will be present during all of the lectures.

Please note that the coordinators will not reply to individual e-mails that could have been dealt with during tutorial group meetings or during lectures.

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Overview of meetings, lectures and deadlines

week	meeting	Post-discussion	Pre-discussion	deadlines
1	Tutorial group meeting 1		Task 1	
	Introductory Lecture – L. Bevers and W. van Dellen			
	Tutorial group meeting 2	Task 1	Task 2	
2	Tutorial group meeting 3	Task 2	Task 3	
	Workshop Modelling – L. Bevers and W. van Dellen			
	Tutorial group meeting 4	Task 3	Task 4	
3	Tutorial group meeting 5	Task 4	Case study 1 + Reflection report	
	Lecture Models of Growth and Decay – L. Bevers and W. van Dellen			
	Tutorial group meeting 6	Case study 1	Case study 2 + Reflection report	
4	Tutorial group meeting 7	Case study 2	Case study 3	
	Lecture Network Systems – L. Bevers and W. van Dellen			
	Tutorial group meeting 8	Case study 3	Case study 4	
5	Tutorial group meeting 9	Case study 4	Reflection Case studies and/or Report	
	Lecture Desert – T. Dekker			
	Tutorial group meeting 10	Reflection Case studies and/or Report	Reflection Case studies and/or Report	
6	*No meeting, write peer review*	*No meeting, write peer review*	*No meeting, write peer review*	**Monday March 19th 2018 09.00AM:** Deadline draft version report – upload via the Student Portal > Assignments and send to peer reviewers.
	No lecture			
	Mandatory practical session	Discuss draft version report and peer review	Discuss draft version report and peer review	**Thursday March 22nd 2018 09.00AM:** Deadline peer reviews – upload via the Student Portal > Assignments and bring to class.
7	Final exam			
				Thursday March 29th 2018 11.00PM/23:00h: Deadline final report – upload via the Student Portal > Assignments and drop a hardcopy in OSA mailbox.

Please check the schedule provided by the Office of Student Affairs and announcements on the Student Portal as well.

Tasks & Case studies

Task 1. Naïve Theories?

Children have theories about the world they live in. The following example is based on an experiment by Lautrey and Mazens*: in an experimental set-up children were asked to think about the properties of heat.
The experimenter asked the children if it was possible to feel heat through walls, through a metal box or through a cardboard box and to explain why or why not.

Rémy: “No because heat can’t go through walls, concrete, because it [heat] is small and it is like dust.”
Daniel: “No we can’t but when there is a door, we can because there are holes.”
Lionel: “It passes like that, through here, because there is a very small space.”
Jessica: “No, because it’s made of metal and it’s a lot harder than cardboard, so the smoke can’t leave.”
Marlies: “Yes, maybe the heat is stronger than the box. It can sometimes pass.”
Michelle: “Yes because cardboard is softer. The smoke can go outside here because the box is soft and everything can go out.”
Lorena: “Yes, because it is smoke and the smoke can pass between something, like a ghost.”
Nassir: “Because it is transparent, the smoke, so it can go up, up.”
Julie: “Since it’s hot, the little bottle will make steam on the cardboard box and the cardboard box will get hot.”

So, as is evident from the above, children are little scientists. They try to formulate explanations for why the world behaves as it does. Grown up science isn’t all that different from what children do. Some things are actually the same. Rémy for example has the idea that heat is like dust and, that because heat is like dust, it can behave the way it does, i.e. go through walls. Julie has the idea that a hot bottle will make steam and the steam in turn will heat the card board. Without even realizing it, these kids make sense out of the world by conceptualizing. They assign different properties or “values” to the different materials they are confronted with. Although you might not agree with the models the children propose, you must admit that they at least contain some of the criteria a good theory should meet!

* Lautrey, J. & Mazens, K. (2004). Is children’s naive knowledge consistent? A comparison of the concepts of sound and heat. *Learning and Instruction, 14*, 399–423

Task 2. Reflecting on UCM – The Dean's Dilemma (1)

At the 10-year anniversary party of UCM, the former deans, 'Big Dean' Louis Boon and 'Small Dean' Harm Hospers, are having a discussion about UCM's academic calendar. The Big Dean claims that in his times, students had to work much harder as compared to the current batch of students. Obviously the Small Dean protests this claim vigorously, standing in for his beloved students.

"Your shift from periods of 8 weeks to 7 weeks turns them into lazy, spoiled brats," the Big Dean says.

"That is most certainly not the case! The reflection week allows them to get some rest and reflect on their program. It clearly shows that it works, the overall performance has increased since the introduction of the reflection week!" the Small Dean opposes.

It so happens to be that the two coordinators of Theory Construction and Modelling Techniques overhear the Deans arguing. Always ready to teach people a thing or two, they step in.

"Before shouting at each other, shouldn't you first do some instantiation? Define what you mean with 'performance' and 'lazy', Small Dean." Wilfred suggests.

"And while you're at it, why don't you perform a thought experiment?!" Lonneke adds.

Ignoring the two nuisances for the moment, The Big Dean and the Small Dean decide to forget their argument and enjoy another drink.
However, over the weekend, the Small Dean keeps thinking about the discussion. On Monday he turns to Wilfred and Lonneke.

"So, what did you say I need to do? Constipation?" he asks.

"No no no, you're confusing it with parsimony. Instantiation!" Wilfred replies promptly, "Define 'performance'. And then, using logic and reasoning, construct a contingency table and derive relevant theoretical propositions from this experiment. You know what? Don't do anything at all! We'll ask our students to do the thought experiment for you. I'm sure they will come up with some hypotheses that you could test empirically!"

The Small Dean wanders off, mumbling: "I never thought I would turn to TCMT to save face…"

Task 3. Cause and Effect – The Dean's Dilemma (2)

Dean Mathieu is having one of his regular meetings with the UCM Management Team. At the end of the meeting, as usual, there is room for miscellaneous remarks and questions.

Mathieu: "Let's hear it. Any last issues to discuss?"

Anouk: "Yes, I've got one. I have received an e-mail from both our student representatives and the UM's Green Office on whether we could take more care of wastepaper management. It seems our students hardly consider recycling their e-readers, printouts etc. What are we going to do about that?"

Tessa: "And I would like to add some. I have been approached by the UM Board on promoting healthy behaviour among students. The minister of Health and the minister of Education have enforced the university and other educational programs to help solve current issues regarding the practicing of safe sex, smoking ánd drinking behaviour among students. It seems that the use of condoms has dropped, they start smoking more again and drinking is off the scale. Both high schools and universities need to address the issues more and we've basically been asked to come up with solutions. Got any ideas?"

Mathieu sighs and frowns: "These are a lot of issues, people. I am not sure whether we can solve them soon, let alone all at once since they are so entirely different. I will look into it. End of meeting…"

The group leaves and Mathieu contemplates some more on how to solve all these issues. Then he wonders whether they actually are that different. If the underlying behaviour, the causes and effects, the very variables are quite similar then he may be able to hit several birds with one stone.

And he starts drawing…

Task 4. The More, the Merrier? – UCM's Bounded Growth

University College Maastricht, the year is 2005. Ever since the start of the program an increasing number of students has applied for the college and attended it. Whereas in 2002 a total of 100 students applied and 80 got accepted to the program, in three years a steady application of 300 has been reached and 150 are selected.

Excited, but unsure about what the future might bring for the college, Stefan Hatchhock and Steffi Count - two recruitment officers - have a discussion on the development of the student population and discuss how they can best predict the years to come.

Stefan: "The number of students is increasing exponentially! If we go on like this we will not fit our building any longer, haha!"

Steffi: "Although I'd love to share your enthusiasm, I do not think that our student population will increase THAT fast. I think what we are looking at is a power function at most."

Stefan: "Humm, maybe you are right, but aren't we confusing number of applications with number of students that get through the application procedure? True, maybe the number of applications is increasing exponentially and will continue to do so, but over time we will see that we will have to stick to a steady number of new students each year."

Steffi: "Of course we will. Now that I come to think of it, we will not move to another building and I estimate that the maximum number of students we can have in this building is 650. Once that is reached, we cannot have an ever-increasing influx of new students anymore but have to consider a constant number. That is assuming of course that we will have a consistent efflux consisting of a 10% drop out of first semester students each year, and a constant number of students graduating each year."

Stefan: "But when will we know we have reached that point…? Can we even make a rough prediction on that…?"

Case study 1. Koalas, Cats, and Kangaroos, Regimes and Protesters

Australia has one of the most diverse selections of animals on earth: about 7% of the world's species. Most of those animals are unique, in that they live only in the island continent. Researchers, however, say that 20% of the country's surviving mammals are threatened with extinction, along with 12% of its birds.

At a nature reserve near Canberra, people are now trying to preserve wild-life by restoring former farmlands to its original state before the settlers arrived. Not only did these colonialists chop down trees, thereby destroying the habitats of native animals, they also brought with them invasive plants and animals like cats. There are now thought to be more than 15 million feral cats running wild, eating the native animals and competing for their food.[†]

Europe's past has seen several regimes that remained in power for decades, such as those in the German Democratic Republic, Czechoslovakia and Romania. The use of force to strike down opponents seemed an effective strategy for those regimes to stay in power. However, as we all know, at one point in history the regimes were overthrown and replaced.

More recently, in other countries such as Egypt, Libya, and Syria, we see similar patterns in which coercion and protests resulted in the rebellions taking over and new governments being formed. However, in Egypt the protests returned and the rebellions felt that the revolution did not bring the desired democracy yet.

The developments in Syria are yet another story. Excessive violence has been used and the regime assumed that this would lead to an end of the protests and attempts to overhaul. So far, however, it has triggered a state of civil war.

There are politicians and analysts who claim that the future will be that sooner or later all regimes will be replaced by democratically installed governments. These claims may be based on historically similar situations in other countries. However, others argue that this may not necessarily be the case, or that the protests will start again after they seemingly faded out. Then the call for change returns.

No one really knows what the future might bring.

[†] http://www.aljazeera.com/news/2015/07/australia-battle-save-threatened-species-150705105649032.html

Case study 2. Game Theory

Strategies and Equilibriums

Topics
- Games, best responses and Nash equilibrium
- Dominant and dominated strategies
- Pareto dominated Nash equilibrium
- Nash equilibrium in dominated strategies

Game theory and Nash equilibrium

John von Neumann
1903-1957

Game theory arguably started its existence as a separate discipline of economics in 1944, with the publication of "Theory of games and economic behaviour" by John von Neumann and Oscar Morgenstern. Oscar Morgenstern was an economist; John von Neumann was more of a "homo universalis". Of Hungarian origin, he was recognized as a mathematical prodigy at a very early age. He worked in Germany until WW II forced him to flee to the US, where he became one of the founders of the Mathematical Institute at Princeton. He was, among other things, the inventor of software, a quantum physicist and one of the contributors to the Manhattan project which led to the development of the nuclear bomb during WW II.

Game theory aims to give insight in strategic behaviour by modelling competitive environments (e.g. competition between firms) as games and trying to deduce optimal behaviour within the resulting model. We will do the same in this case. Let us start by defining a game.

Definition. A game consists of the following ingredients: first of all, there is a set $\boldsymbol{N = \{1,\ldots,n\}}$ of players. Each player $\boldsymbol{i}$ has a set $\boldsymbol{S_i}$ of strategies at his disposal. When the game is played, each player simply selects one strategy $\boldsymbol{s_i}$ out of his set of strategies $\boldsymbol{S_i}$. This way we get a vector – $\boldsymbol{(s_1,\ldots,s_n)}$ – representing the strategies chosen by all the players. Such a vector is called a strategy profile. Once this strategy profile has been chosen, each player $\boldsymbol{i}$ receives a payoff $\boldsymbol{u_i}$ which depends on the strategy profile, i.e. on the strategies chosen by all the players: $\boldsymbol{u_i(s_1,\ldots,s_n)}$. Then the game ends.

Game: Any set of circumstances that has a result dependent on the actions of two of more players
Player: A strategic decision maker within the context of the game

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This is a very simple, even simplistic, description of a game. In fact, this description is called the *one-shot representation* (or, more fashionably, the *strategic form*) of a game. The simplicity is due to the interpretation of a strategy in our game definition.

In our definition, a strategy is viewed as an entire *plan of action* that a player might make before the onset of the game. In this plan of action, a player is supposed to document all possible decisions he wishes to make in *every conceivable* situation in the game that might possibly occur.

Example: Chess as a one-shot game
We all know what a move is in a game of chess. However, a *move* in chess is not the same as a *strategy*! In the one-shot representation of chess, a *strategy* is a plan of action that specifies *for each conceivable configuration of the chess pieces on the board* which move one would take! Suppose that we would require a player to record such a strategy in a book. Given that the number of possible configurations of chess pieces on the chess board is something in the order of 10^{46}, this would have to be quite a voluminous book.

Nevertheless, our simplistic game description is quite useful, because the resulting model, the one-shot game, is relatively easy to analyse, by means of the well-known *Nash equilibrium*. The 1951 paper of John F. Nash, "*Theory of games and economic behaviour*", which introduced the Nash equilibrium concept, is a second milestone in the history of game theory and its effect on economic theory. The term "Nash equilibrium" was actually coined later; Nash himself modestly referred to Nash equilibriums as "equilibrium points".

Like von Neumann, John Nash was a mathematician. Nash's two major contributions to economic theory are the invention of axiomatic bargaining and the Nash equilibrium. Axiomatic bargaining is a discipline in economics that is based on Nash's master thesis, written when he was 20. The Nash equilibrium concept is based on his PhD thesis, finished at the age of 22. After that, the story rapidly became much sadder: Nash suffered from paranoid schizophrenia for more than 30 years. In 1994 he received the Nobel Prize, together with Reinhard Selten and John Harsanyi. The movie "A beautiful mind", starring Russell Crowe, is based on his life, after the book with the same name by Sylvia Nasar.

John Forbes Nash Jr.
1928-2015

Example: How to spend the evening

Anne and Bob are two UCM students who spend a lot of time together. Tonight, they have to decide how they want to spend their evening. Their options are either to go to the Open Mic Night (strategy *O*), or to attend the weekly Meditation meeting (strategy *M*). Bob would like to attend the meditation meeting; it gives him a utility of 1. He is not terribly fond of open mic nights, which scores zero on his utility scale. Anne on the other hand feels differently about these issues. Attending an open mic night is valued at utility level 1, while she couldn't care less about meditation: zero utility. Further, Bob and Anne do like each other very much and being together gives both of them an additional 2 units of utility on top of their scores mentioned so far.

Since Bob and Anne are currently both at their work without the option to discuss matters, they have to decide separately where to go to. We can model this decision problem as a *bimatrix game*.

$$\begin{array}{ccc} & & \text{Bob} \\ & & \begin{array}{cc} O & M \end{array} \\ \text{Anne} & \begin{array}{c} O \\ M \end{array} & \begin{bmatrix} 3,2 & 1,1 \\ 0,0 & 2,3 \end{bmatrix} \end{array}$$

Anne is called the *row player*, because her choice determines in which row we have to look to determine the payoffs. Bob is the *column player*. So, if both Anne and Bob choose *M*, then we end up in the lower row and the right-hand column, where we can see that the payoff for Anne (the first number in the corresponding cell of the matrix) equals 2: she is happy to be with Bob, but doesn't like to meditate. The second number, 3, is the payoff for Bob: 1 unit of utility for meditating, plus 2 units of utility for being together with a good friend.

Now let us analyse this bimatrix game along the path that Nash has set out for us. First put yourself in the shoes of Anne.

$$\begin{array}{ccc} & & \text{Bob} \\ & & \begin{array}{cc} O & M \end{array} \\ \underline{\text{Anne}} & \begin{array}{c} O \\ M \end{array} & \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \end{array}$$

Since you have not talked to Bob today, you do not know what he is going to do. But you can make the following simple observation: *if* Bob is going to go to the open mic night, then you have a choice between going to the open mic night as well with a payoff of 3, or going to the meditation meeting with a payoff of zero. So, in that case your *best response* to Bob's choice is to go to the open mic night. In the same way, going to the meditation meeting is your best response if Bob decides to go to the meditation meeting. These choices are indicated below by an asterisk (*).

$$\begin{array}{cc} & \text{Bob} \\ & \begin{array}{cc} O & M \end{array} \\ \underline{\text{Anne}} \quad \begin{array}{c} O \\ M \end{array} & \begin{bmatrix} 3^* & 1 \\ 0 & 2^* \end{bmatrix} \end{array}$$

Of course, Bob can perform a similar thought experiment. In the matrix below, his best responses are also indicated with an asterisk:

$$\begin{array}{cc} & \underline{\text{Bob}} \\ & \begin{array}{cc} O & M \end{array} \\ \text{Anne} \quad \begin{array}{c} O \\ M \end{array} & \begin{bmatrix} 2^* & 1 \\ 0 & 3^* \end{bmatrix} \end{array}$$

Altogether, we end up with the following best response choices in our bimatrix game:

$$\begin{array}{cc} & \text{Bob} \\ & \begin{array}{cc} O & M \end{array} \\ \text{Anne} \quad \begin{array}{c} O \\ M \end{array} & \begin{bmatrix} 3^*, 2^* & 1,1 \\ 0,0 & 2^*, 3^* \end{bmatrix} \end{array}$$

We see that the cell in the matrix that corresponds to the pair of strategies (*O,O*) has two asterisks, one from Anne and one from Bob. This means that, when Bob chooses *O*, it is a best response for Anne to choose *O* as well; and vice versa, if Anne chooses *O*, it is a best response for Bob to do so as well. We express this phenomenon of being mutual best responses by saying that the strategy pair (*O,O*) is a *Nash equilibrium*.

Apparently the above "How to spend the evening" game has two Nash equilibriums: (*O,O*) and (*M,M*).

Definitions. A strategy s_i of player i is a best response against a profile $(s_1,\dots,s_{i-1},s_{i+1},\dots,s_n)$ of strategies of the other players when $u_i(s_1,\dots,s_{i-1},s_i,s_{i+1},\dots,s_n) \geq u_i(s_1,\dots s_{i-1},t_i,s_{i+1},\dots,s_n)$ for every possible other strategy t_i that player i could use.

In words, strategy s_i is a best response for player i against the profile $(s_1,\dots,s_{i-1},s_{i+1},\dots,s_n)$ when the payoff to player i when using strategy s_i is at least as high as any other payoff he could realize by playing another strategy t_i (that is, given that the other players stick to playing the strategies in $(s_1,\dots,s_{i-1},s_{i+1},\dots,s_n)$!).

A strategy profile $(s_1,\dots,s_n)$ is a Nash equilibrium when for each player i, the strategy s_i is a best response against the profile $(s_1,\dots,s_{i-1},s_{i+1},\dots,s_n)$ of strategies used by the other players.

Strategy: A complete plan of action that documents all possible decisions a player will take given the set of circumstances that might arise within the game
Strategy profile: A set of strategies for all players which fully specifies all actions in a game. A strategy profile must include one and only one strategy for every player.
Best response: The strategy (or strategies) which produces the most favourable outcome for a player, taking other players' strategies as given.

Bimatrix: **bold**: payoffs for row player; underlined: payoffs for column player

			column player ← → possible strategies of column player		
			X	Y	Z
row player	↑ possible strategies of row player ↓	A	**10** , 5	**5** , 11	**17**, 22
		B	**0** , 4	**3** , 13	**11** , 2
		C	**16** , 4	**4** , 16	**0** , 0

The notion of Nash equilibrium can be applied in many other different environments. The best way to get a feeling for a Nash equilibrium, is to apply it in all sorts of different models. Always remember: when you want to test whether you are in a Nash equilibrium, you have to ask yourself the question: do I play a best response *given* the behaviour of the other players (no matter how ludicrous the other players' behaviour may seem to you!).

Exercise 1)
Consider the following 3x3 bimatrix game:

$$\begin{array}{ccc} & & \text{Column player} \\ & & \begin{array}{ccc} L & C & R \end{array} \\ & T & \\ \text{Row player} & M & \begin{bmatrix} 3,5 & 0,5 & 4,3 \\ 4,2 & 4,6 & 4,-2 \\ 12,3 & 1,1 & 4,3 \end{bmatrix} \\ & B & \end{array}$$

a. Compute the best responses for both the row player and the column player.
b. Determine the Nash equilibriums of this bimatrix game.

There is a certain peculiar feeling to the Nash equilibrium. After these examples, you may wonder: if I were in that situation, having to make a decision, how could I be certain that the others will make their choices accordingly? How do I know in advance what the others will choose? E.g., consider Anne, if she knows that Bob plans to go to the meditation meeting, then her best response is to do the same. But she doesn't know this when making her choice; when Bob goes to the open mic night, her best response would be to choose her other option, i.e. to go to the open mic night too!

The question "how do we manage to coordinate on this or that specific Nash equilibrium" was never posed by Nash. Instead, the question he asked was: *if* we end up in a certain situation (no matter how or when we got there), then under what conditions do we want to *stay* there? And his answer was: *if* we end up in a certain situation and we are all inclined to stay there, then this situation should be a Nash equilibrium. After all, if the situation is *not* a Nash equilibrium, then at least one of us has the inclination to move somewhere else.

Another way of thinking about this is the *no regret* thought experiment. Since we play a one-shot game with simultaneous moves (by our definition of a strategy), we do not know what other players will choose at the moment we decide which strategy to choose ourselves. But *if* we would have known the others' choices, would we have regretted our choice? Put differently, in retrospect, having seen what the other players did and having seen what happened in the game, do we regret having done what we did? If so, we did not play a Nash equilibrium. If no one has any regrets, we were playing a Nash equilibrium.

The approach of Nash only says that we will end up in a Nash equilibrium. His line of reasoning does not answer the question in *which* Nash equilibrium we will end up. That is a matter of initial conditions and coordination among the players: an entirely different problem altogether.

Peculiar Nash equilibriums

In many situations, the Nash equilibrium is a decent predictor of behaviour. Nevertheless, it is not the final authoritative answer. In this section you will see a few examples of Nash equilibriums that are not quite convincing as a solution.

Definition. A payoff scheme $\boldsymbol{x = (x_1,\dots,x_n)}$, where $\boldsymbol{x_i}$ is the payoff to individual $\boldsymbol{i}$, *Pareto dominates* a payoff scheme $\boldsymbol{y = (y_1,\dots,y_n)}$ if $\boldsymbol{x_i \geq y_i}$ for each individual $\boldsymbol{i}$ and $\boldsymbol{x_k > y_k}$ for at least one individual $\boldsymbol{k}$.

In words, the payoff scheme $\boldsymbol{x}$ is better according to the Pareto criterion than payoff scheme $\boldsymbol{y}$ if under $\boldsymbol{x}$ every individual gets a payoff that is at least as high as the payoff under $\boldsymbol{y}$, while *at least* one individual is even *strictly* better off under $\boldsymbol{x}$ than under $\boldsymbol{y}$.

In this case we also say that the payoff scheme $\boldsymbol{y}$ is *Pareto dominated*.

Nash equilibrium: Situation where both players play a best response (no player has an inclination to change; 'no regret rule'), given the response of the other player.

Pareto dominant outcome: Given an initial outcome (e.g. a Nash equilibrium), there is another outcome in the game in which at least one player is better off without making any other player worse off. NB. Pareto looks at the entire game and takes into account the outcome for all players.

Exercise 2) Pareto dominated Nash equilibrium.
Consider the following game between the university colleges Maastricht (UCM) and Utrecht (UCU): Both colleges have the opportunity to promote (*P*) or to refrain from promotion (*NP*). The pool of prospective students contains 600 students. When both colleges choose the same strategy (either to promote, or to refrain from promotion) they split the number of prospective students equally. When one college promotes and the other doesn't, the one that promotes gets all the prospective students. However, promotion may scare away prospective students, which will result in students deciding to attend University College Venlo (UCV) and thus a reduction of 100 on the number of prospective students for one's college, irrespective of the choice of the opponent.

a. Model this game as a 2x2 bimatrix game.
b. Compute the best responses for both colleges (UCM and UCU) and determine the Nash equilibrium.
c. Explain why the outcome in the Nash equilibrium is Pareto dominated.

Despite being Pareto dominated, the Nash equilibrium in exercise 2 is in another sense nevertheless quite a convincing one. It is called a *Nash equilibrium in dominant strategies*.

Definition. A strategy $\boldsymbol{s_i}$ of a player $\boldsymbol{i}$ is *dominant* when the payoff of that strategy for player $\boldsymbol{i}$ is at least as high as the payoff of any other strategy, *irrespective* of the strategy choices of the other players.

A strategy $\boldsymbol{s_i}$ of a player $\boldsymbol{i}$ is *dominated* when player $\boldsymbol{i}$ has another strategy $\boldsymbol{t_i}$ such that, given any profile of strategies of the other players, the payoff of $\boldsymbol{t_i}$ is at least as high as the payoff of $\boldsymbol{s_i}$, while the payoff of $\boldsymbol{t_i}$ is sometimes even strictly higher than the payoff of $\boldsymbol{s_i}$.
Dominant strategy: when the possible payoff in one strategy is higher than the possible payoff in any other strategy. This occurs when one strategy is better than any other strategy for one player, no matter how that player's opponents may play. Opposite: dominated strategy.

- *Strategy B dominates strategy A: choosing B always gives a better outcome than choosing A.*
- *Strategy B is dominated by strategy A: choosing B never gives a better outcome than choosing A, no matter what the other player(s) do.*

Exercise 3)
Consider the following game between the two housing companies StudentflatMaastricht and Jules. Both companies have two options, either to accommodate (*A*) or to start a price war (*P*). The market on which the two companies compete can generate a profit of at the most 20 units. When both companies choose to accommodate, they share this profit equally. When one company starts a price war and the other accommodates, the company that chose the price war captures the entire profit. When both companies choose price war, profits dissipate almost entirely and both companies enjoy a payoff of only 5 units.

a. Model this game as a 2x2 bimatrix game.
b. Compute the best responses for both companies; determine the dominant strategies and determine the Nash equilibrium.
c. Explain why the outcome in the Nash equilibrium is Pareto dominated.

The housing company game addressed in exercise 3, as well as the promotion game of exercise 2, are (from a strategic perspective) equivalent to the well-known Prisoner's dilemma.

Auction and Voting

<u>Topics</u>

- Economics and equilibrium
- Auction design
- Voting theory, a troubled discipline

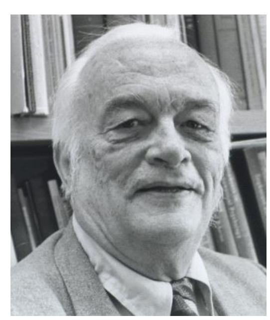
In this case we apply the theory from the previous case to different settings. The first application, **auction design**, is a text book case where game theory and Nash equilibrium are used to actually *solve* problems in economics. The second application, **voting theory**, is an example of a field in social choice theory where game theory does not provide any clear-cut solutions and answers.

Auction design

One of the more recent success stories in applications for game theory is the area of auction design. The first notable event in this particular success story is related in the following example.

Example: The Vickrey auction
Suppose an auctioneer wants to sell a single indivisible item to two interested bidders. Bidder I has a valuation v_1 between zero and one for the item and bidder II has a valuation v_2 between zero and one for the item. These valuations are private information. That is, both players know their own valuation for the item, but not the valuation of the other bidder.

The auction is executed as follows. Both bidders simultaneously hand in a sealed bid: b_1 and b_2. The bidder with the highest bid gets the item – in


William Vickrey
1914-1996

case of a draw, we allocate the item randomly. However, contrary to the traditional auction, the highest bidder does not pay his own bid, but *the bid of the other bidder*.

This at first sight somewhat peculiar auction format was invented by William Vickrey in 1961. For the invention of this simple but effective auction mechanism he was awarded the Nobel Prize in Economics in 1996, together with James Mirrlees. Sadly enough he died three days after the public announcement of his Nobel laureate, so he was never able to actually collect the prize.

Exercise 4)

Imagine you are bidder II. Bidder I has made a sealed bid of b_1.

a. Suppose that $b_1 \leq v_2$. Argue that bidding $b_2 = v_2$ is a best response.
b. Suppose that $b_1 > v_2$. Argue that also in this case bidding $b_2 = v_2$ is a best response.
c. Argue that truthful bidding is a dominant strategy.

As you can see, the Vickrey auction is a very elegant sales mechanism. In this auction, a bidder can follow a very simple and straightforward strategy, namely to bid his valuation truthfully. This is always a best response, no matter what the opponent bids. And if bidders behave according to this simple recipe, the resulting allocation is efficient. William Vickrey was indeed rightfully awarded a Nobel laureate.

Example: multiple items and the VCG mechanism

Auction design as an applied field really took off in the 90s of the previous century under the influence of the UMTS license auctions for mobile telephone networks. A typical UMTS license gives a mobile network provider the right to provide mobile telephone services in a particular country or region for a period of about 10 to 15 years. In a typical UMTS auction about 5 such nationwide licenses were up for sale. Depending on the specifics, revenues on a single license ranged from hundred million euros (€100.000.000,-) in Switzerland up to more than 8 *billion* (€8.000.000.000,-) euros in Germany. You can easily imagine there must have been some sense of urgency to get the design of these auctions just right.

We will not go into the details of all these auctions. There is one detail however that is important to mention. In several of these auctions a mobile network provider could not just bid for a nationwide network license, such a national license had to be built up by collecting a sufficient amount of *regional* licenses. This particular aspect of some of the UMTS auctions was the source of a lot of added complexity in the bidding process. Again, we will not go into details, but we will highlight one facet of this extra complication: the need for mobile telephone providers *to buy more than one license* in order to create value for themselves. Needless to add that most providers were aiming to build a national network rather than being content with one or two regional licenses.

Suppose you are a national authority in charge of selling six licenses for regional mobile telephone services. There are three providers, Frank, Ben, and Aida, who have shown an interest in these licenses. You have conducted a bit of research and you estimated the *marginal valuations* of each of the three providers for additional licenses. Your findings are reported in the following table:

# licenses	F	B	A
1	200	220	180
2	200	200	160
3	90	170	140
4	60	60	80
5	0	40	60
6	0	0	20

The amounts in the table are expressed in € mln. So, for example the valuation of Ben for receiving 3 licenses is 220+200+170=590 million euros. In other words, the 170 for 3 licenses in the third column indicates that, when Ben already owns 2 licenses, receiving a third license increases his valuation from 220+200=420 to 420+170=590 million euro's.

Notice that this means that licenses are *homogeneous*, meaning that providers do not care exactly *which* of the 6 licenses they obtain, they only care about *how many* licenses they get. However, valuations are not *anonymous*; different providers may and do, have different valuations for licenses.

Your aim is to assign the licenses in such a way that the *total valuation* is maximized. So, when the above numbers are correct, this means that you should assign 2 licenses to Frank, 3 to Ben, and 1 to Aida, for a total valuation of 200+200+220+200+170+180=1170 million euro's. Notice that this aim is, at least in principle, different from revenue maximization: you as a governmental institution are not primarily concerned with the amount of money you manage to reap in the sales process, but you simply want to assign licenses to providers that are able to make good use of them[‡]!

This brings us to the proverbial million dollar question: how do you get to know these valuations? Of course, like we assumed so far, you can try to estimate them. But it is a fact that the providers themselves have far better knowledge regarding these valuations than you will ever manage to obtain. The best thing to do is of course to get the providers to actually tell you their valuation for licenses. But strategic considerations may bring them to misreport their valuations.

So, to rephrase the previous question: *how do you persuade the providers to report their valuations truthfully to you, the central authority?* This is one of the central issues in present-day auction design. How do we organize/design the auction in such a way that bidders in the auction are inclined to report their valuations truthfully? The objective underlying this explicit wish is not to squeeze as much cash out of the bidders as we possibly can, but to assign licenses – or, more generally, whatever we are auctioning – in a *Pareto optimal* way, that is in such a way that it maximizes total valuation.

<u>The VCG mechanism</u>

For the sales of UMTS licenses there is in fact an auction design that establishes exactly this objective. It is known as the Vickrey-Clarke-Groves mechanism[§] after three economists who, more or less independently of each other, discovered this mechanism in different settings.

The VCG mechanism operates in three phases. We will discuss its principles in the context of our UMTS example.

Phase 1. *The bidding phase*

In this phase each of the three providers Frank, Ben, and Aida are asked to report their valuations. So, a bid is simply a list of marginal valuations, one reported marginal valuation for each additional license. Of course there is no way to check whether providers are reporting truthfully, so they can misreport if they wish to do so.

Phase 2. *The allocation phase.*

In this phase the licenses are assigned to providers. This is done in the most naïve and straightforward way you can possibly think of. We act *as if* the bids from phase 1 are truthful and we assign licenses in such a way that given these bids total valuation is maximized.

Phase 3. *The payment scheme.*

This is the clever part. So far nothing forced the providers to report truthfully. This however is done by the way payments are calculated. In fact, each provider is asked to pay what is called his own *externality.* It is calculated as follows:

[‡] In this context there is a famous quote from former vice president Al Gore, who opened the December 1994 Broadband PCS spectrum auction (the US equivalent of the UMTS auction) proclaiming "Now we're using the auctions to put licenses in the hands of those who value them the most."

[§] In auction design lingo, the somewhat formal phrase "mechanism" is often used as a substitute for "auction".

Suppose that the numbers in the above tables were the actual bids by the providers. In that case Frank is awarded 2 licenses. How much should Frank pay for his licenses? In order to compute that, we are going to perform a thought experiment: Suppose that Frank had not participated in the auction and only Ben and Aida would have bid. What would the allocation of licenses have looked like?

# licenses	B	A
1	220	180
2	200	160
3	170	140
4	60	80
5	40	60
6	0	20

Given these bids and given the way we allocate licenses, both Ben *and* Aida would have gotten 3 licenses, with a total (reported) valuation of 220+200+170+180+160+140=1070. And now that Frank is present, Ben and Aida only have a (reported) valuation of 220+200+170+180=770, because Aida receives two licenses less. The *difference* 1070-770=300 between these two amounts is what Frank has to pay for his two licenses. This amount can be seen as the damage that Frank inflicts upon the other providers by his presence in the auction. This is exactly what economists call an externality.

Note that a quicker way of approaching these calculations is by simply adding up the “damage” one inflicts because of participating in the game. In the example above: Because Frank is present, Ben and Aida get two licenses less to distribute. If Frank had not been there, Aida would have received two more licenses and he valuated those for 160+140=300. This is the exact amount that Frank has to pay. Since the licenses are worth 200+200=400 to him, the net utility (= valuation – payment) for Frank is 400-300=100.

Exercise 5)

a. Compute the VCG payments for Ben and Aida.
b. Compute the net utility (= valuation for licenses won - payment) for both Ben and Aida.
c. Observe that net utilities are automatically positive, i.e. you never pay more than the value of what you get. Can you explain why this happens?

In all these calculations we automatically assumed that the valuations reported by the providers were the true valuations. It is beyond this short introduction to explain why truthful reporting is indeed a sensible strategy under the VCG mechanism – in fact, it can be seen that truthful reporting is a *dominant* strategy.

Voting theory, a troubled discipline

After the success stories of the use of game theory in auction design we shortly discuss voting theory as a third application, this time to exemplify the limitations of game theory, or better perhaps the limitations of voting procedures in general.

In the general setting of voting theory there is a group of individuals, usually referred to as voters. Each individual is assumed to have his or her own personal opinion on one or several issues under discussion. The type of issue or issues can be anything, ranging from presidential elections and admission of new members to the European Union to convictions in a court of law. Voting theory designs and studies procedures to aggregate all those individual opinions into one single opinion on which a collective decision can be based.

Example: strategic voting

Suppose that three individuals, Abbie, Bo and Chris, who are members of a political party, have to vote for a representative in parliament for their party. There are three candidates, namely Xavier, Yomanda and Zinedine.

The preferences of Abbie, Bo and Chris over the three candidates are given by a *ranking.* These rankings are shown in the following table.

ranking	**A**	**B**	**C**
1	Z	Y	X
2	Y	Z	Y
3	X	X	Z

So, for example Abbie prefers to have Zinedine as a representative in parliament. Second in her ranking is Yomanda, and Xavier is her least preferred candidate.

Now, in order to select among the three candidates, Abbie, Bo and Chris agree to use the following *voting procedure* to appoint a representative: All three members (Abbie, Bo and Chris) write down the name of the candidate they vote for (Xavier, Yomanda or Zinedine) on a piece of paper. After the votes are cast, the candidate with the highest number of votes in favour of him or her will be appointed as the representative. In case all three candidates receive exactly one vote, the procedure is considered to be in deadlock, no representative is appointed and this is considered to be the worst alternative by all three voters[**].

We can model this situation as a *game in strategic form.* The players of the game are Abbie, Bo and Chris. Each player has three strategies, namely to vote for either Xavier or Yomanda or Zinedine. The payoffs are as follows: If an individual's most preferred candidate is appointed as representative, the payoff to that individual is 3. If the second-best candidate is appointed, the payoff is 2. The payoff is 1 for the worst candidate and it is zero when the votes end up in deadlock and no representative is appointed. Consider the strategy profile (*Y,Y,X*), indicating that Abbie and Bo (the first two coordinates in this vector) voted for Yomanda and Chris voted for Xavier. Under this strategy profile Yomanda is appointed as representative according to the above voting procedure. So, Chris and Abbie get a payoff of 2, while Bo sees her most preferred candidate elected for a payoff of 3.

Exercise 6)

a. Explain why the strategy profile (*Y,Y,X*) constitutes a Nash equilibrium in the game we defined just now.
b. Explain why, given the votes of Bo and Chris in the above strategy profile, strategic voting (that is, wilfully misreporting your preferences) is a profitable activity for Abbie.
c. There are also 5 other Nash equilibriums in this game: (X,X,X), (Y,Y,Y), (Z,Z,Z), (Z,Z,X) and (Z,Y,Y). Argue why these strategy profiles are Nash equilibriums.

You probably noticed that voting games may have peculiar Nash equilibriums, such as the equilibriums where all voters vote for the same candidate. The principle is as simple as it is discouraging: once *everybody else* votes for the same candidate, no matter how unpopular this candidate is, *you are entirely indifferent.* So *you* might vote for this particular unpopular candidate as well.

You may be inclined to think that this peculiar result stems from the fact that individuals are only allowed to vote for one candidate and that a more refined procedure in which voters can express their preferences in more detail might give us a better performance. Let us proceed and see whether this thought holds out under more critical scrutiny.

Jean-Charles de Borda
1733-1799

[**] Draws can be settled in many ways. The particular way we deal here with draws is quite rigorous, but this is simply done for ease of exposition. It keeps the arguments simple; other ways to settle draws will not make the problems at hand vanish conveniently.

A more refined, voting procedure is the *Borda scoring rule.* In the Borda scoring rule, each voter can give points to a candidate (comparable to the voting procedure in the Eurovision Song Contest). The points to be divided are 1, 2 and 3, each to be assigned to one of the three candidates[††]. After the voters assigned points to the candidates, the total number of points for each candidate is computed and the candidate with the highest number of points is appointed as the representative. Again, the procedure considers a draw to be a deadlock, in which case no representative is chosen. This is considered as a particularly bad outcome by all voters.

Example: Borda count

Suppose that eleven individuals, Abbie, Bo, Chris, Doris, Esther, Finn, Gary, Harry, Iris, Jo and Kylie, who are members of a jury, have to vote for a song to represent their college in a song contest. There are three candidates, namely Little Glitches, M.A.D band, and N-Tyce.

The preferences of the eleven individuals over the three candidates are given by a *ranking.* These rankings are shown in the following table.

ranking	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**
1	L	L	L	L	L	M	M	N	N	N	N
2	N	N	M	M	M	N	N	M	M	M	M
3	M	M	N	N	N	L	L	L	L	L	L

Borda count. 1^{st} = 5 points, 2^{nd} = 3 points, 3^{rd} = 0 points

Exercise 7)

a. Calculate the total number of points for each candidate and indicate which of the candidates will be appointed as the representative.
b. Consider Esther (Little Glitch > M.A.D band > N-Tyce). Could she change the outcome more to her liking by wilfully misreporting her preference?

The plethora of Nash equilibriums in voting models (whether it be simple voting or a more sophisticated scoring rule such as the Borda rule), especially the persistence of the unanimous vote anomaly, might be interpreted as a failure of the predictive power of Nash equilibrium in these models. However, perhaps a more realistic way to view these strange results is that aggregation of preferences into a single choice is simply a problematic endeavour and that consequently the problematic nature of choice aggregation is reflected (highlighted?) by our inability to design voting procedures where Nash equilibrium has decent predictive behaviour.

[††] Note that indeed the voters can now express their preferences much better in comparison with the previous voting rule where voters could only give one point to one (their preferred?) candidate and zero points to all others.

Case study 3. An Experiment: It's a Small World After All

In 1967, Stanley Milgram, a Harvard professor, conducted an experiment. He chose two target persons, the wife of a divinity student in Sharon, Massachusetts and a stock broker in Boston. Then he sent letters to randomly chosen residents of Wichita and Omaha asking them to participate in a study of social contact in American society. The letter contained a photograph, name and address and other information about one of the target persons. Participants were asked to mail the letter directly to the target person if they knew him or her on a personal basis, i.e. if they had previously met the target person and knew each other on a first name basis. If the participant did not know the target person on a personal basis, they were asked not to try and contact him/her directly. Instead, they were asked to send the letter to a personal acquaintance that was more likely than them to know the target person. They were allowed to send it to a friend, relative or acquaintance, but it had to be someone they knew on a first name basis.

Surprisingly, it did not take long before letters started to arrive at the targets. In some cases they arrived at the target person within one or two steps, whereas some chains were up to nine or ten steps long. In the end, the average path length turned out to be around 5.5 or 6 and the researchers concluded that there are 'six degrees of separation' between the citizens of the US.

Nowadays we have the internet and websites such as Facebook and LinkedIn. They allow us to communicate with people all around the globe. An application on Facebook named 'Six Degrees' calculated the degrees of separation between different people. The average separation for all users of the application was 5.7 degrees, whereas the maximum degree of separation was 12. So apparently, the degrees of separation are not decreased with the emergence of sites like Facebook.

Many first semester students are trying to find their way at UCM. During the introduction week, they get a lot of information and meet many new people to become familiar with. For those students, the first period that follows the introduction is just as overwhelming. In addition to attending the courses and studying lots of new and interesting topics, they need to figure out procedures such as course registration and academic advising.

Due to the open curriculum and students participating in courses at different times in their program, they also meet the fellow students who are in their second or third year at UCM. This has a benefit as well. Since those students have been around a lot longer, they are valuable sources of information on UCM and its rules and regulations, procedures and people. They may even help out in establishing a quick link to people such as the UCM student representatives in the Board of Studies and Universalis who are acquainted with several staff members that are in the management team, faculty council and curriculum committee.

Let's do a little experiment related to this. Let one student start by drawing a dot on the board, which represents him/her. Then five dots should be drawn that represent fellow UCM-ers that s/he is most familiar and/or in contact with. Make sure that dots are linked by means of lines in case people know each other. Then another student does the same but also links to the existing network on the board, i.e. in case s/he shares the same fellow UCM-er. Keep doing this until at least three students have contributed their part to drawing the network.

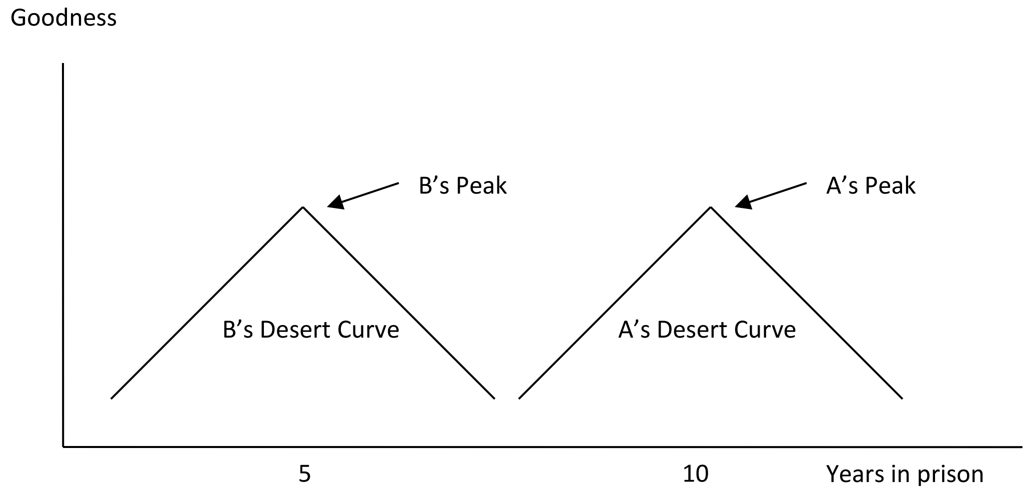
Since modelling is also about explaining and predicting: what will this drawing probably look like in three years, when all the first semester students in the tutorial group will be in their sixth semester?

Case study 4. Who to Let Go?

The state of California is bankrupt. In a desperate attempt to save money, it has decided to spend less on its prison system by letting prisoners go free before they have served their entire sentences. But this requires the state to develop a model to decide who to let out when. In making that model the state must meet its goal for saving money but it must also respect judges' decisions concerning how much punishment individuals deserve for their crimes and be fair to individual prisoners.

Saving Money
One thing to consider is the capacity of the prison system. Putting people in jail is expensive, and given limited financial means, there is a maximum amount of prison years that society has at its disposal for punishment. For example, perhaps society has only allocated enough money for 11 prison years, which may be divided over two criminals, Criminal A, the murderer, and Criminal B, the rapist. This means society could put one of them in jail for 10 years, and the other for one year. But it could also put both of them in jail for 5.5 years, or one for 4 and the other for 7. To think about this aspect of the problem, one might consider the following graph.

Desert
The idea behind the justice system is that people who are convicted of crimes deserve to go to jail for a certain amount of time. Judges examine individual cases and determine how much time in prison individual criminals deserve. Obviously, from the perspective of justice, the best outcome would be for everyone to go to prison for the exact time they deserve. If they spend more or less time in prison this would be less good from the perspective of desert. This aspect of the problem can also be expressed in a graph.



The roof-like line is a criminal’s desert-curve. It represents how good it would be if this individual spent a certain amount of time in prison. The amount of jail time one deserves is represented by the highest point in the graph, or the peak. For example, a judge may have determined that murderer A deserves to go to prison for 10 years, while rapist B deserves to be incarcerated for 5. If they spend more or less time in prison than this optimum, this would be less good, and so the curve slopes downwards from the peak on both sides. The more an individual deviates from his peak, the worse the outcome is, and the lower the relevant point on the graph. One question that one will have to think about is the slope of the desert curves. Are they the same for everyone? Or do they have different slopes depending on how severe their crimes were? Maybe one thinks that the more severe the crime the worse it is to be let out early. This is called bell-motion desert. Also, are these curves straight or curved? Perhaps it is not too bad if criminals get out a little early, but the more they deviate from what they deserve, the quicker things get worse. This is called curved desert.

Fairness

A third issue that needs to be incorporated in this model is a concern for fairness, that all cases should be treated equally. For example, one solution to the problem in the example would be to let murderer A serve the full 10 years, and let rapist B serve only 1 year. But this seems quite unfair. Another option would be to put them both in jail for 5.5 years. But this seems unfair too, as B must serve more than his entire sentence while A is let out after approximately half of his jail term.

All these three aspects of the problem need to be incorporated into one model to come up with a system for letting prisoners out early that saves enough money, is optimal from the perspective of desert, and is fair. And once one has such a model, one can use it to think about other situations when one is allocating scarce resources over individuals who deserve different amounts in a fair way. For example, one might use it to make cutbacks in people’s salaries in a company when it must reduce its wage bill. And there are many other cases where a model of this sort might be helpful.

Appendix. Overview resources

The following book is considered the main course book for COR1005 Theory Construction and Modelling Techniques:

Jaccard, J., & Jacoby, J. (2010). *Theory construction and model building skills: A practical guide for social scientists.* New York: Guilford.

It can be bought in the academic bookstore and found in the UM library and UCM Reading Room.

The other publications mentioned below can be found in the e-reader for which a link is provided in the Student Portal for this course, unless indicated otherwise.

Resources indicated with a * provide the theoretical framework for the task. The other resources are illustrative examples of the application of the models in science. Both are mandatory, relevant and required for the exam. However, your approach in studying them should be different. This will be explained during the course.

Please note that some e-readers may not be legible in some applications. It has come to our attention that for example a few Apple/Mac applications experience issues with presenting the e-readers on screen appropriately. This is not due to the e-reader but a characteristic of the application you use. Try and find another one.

Task 1. Naïve Theories?

*Introductory Lecture – L. Bevers and W. van Dellen

*Jaccard, J., & Jacoby, J. (2010). *Theory construction and model building skills: A practical guide for social scientists.* New York: Guilford. (Chapters 1, 2 and 3)

Silvert, W. (2001). Modelling as a discipline. *Int. J. General Systems, 30(3)*, 261-282

Task 2. Modelling UCM – The Dean's Dilemma (1)

*Jaccard, J., & Jacoby, J. (2010). *Theory construction and model building skills: A practical guide for social scientists.* New York: Guilford. (Chapters 5 and 6)

Jaccard, J., & Jacoby, J. (2010). *Theory construction and model building skills: A practical guide for social scientists.* New York: Guilford. (Chapter 4)

Task 3. Cause and Effect – The Dean's Dilemma (2)

Ajzen, I., Joyce, N., Sheikh, S., and Cote, N. G. (2011). Knowledge and the prediction of behavior: The role of information accuracy in the Theory of Planned Behavior. *Basic and Applied Social Psychology, 33,* 101-117

Cheung, S.F., Chan, D.K.-S., & Wong, S.-Y. (1999). Reexamining the Theory of Planned Behavior in understanding wastepaper recycling. *Environment and Behavior, 31(5),* 587-612.

*Jaccard, J., & Jacoby, J. (2010). *Theory construction and model building skills: A practical guide for social scientists.* New York: Guilford. (Chapter 7)

Task 4. Numbers, Numbers and More Numbers

*Jaccard, J., & Jacoby, J. (2010). *Theory construction and model building skills: A practical guide for social scientists.* New York: Guilford. (Chapter 8)

*Tannenbaum, P. (2010). The mathematics of population growth. In: *Excursions in Modern Mathematics (pp.474-487)*. New Jersey: Pearson Prentice Hall

*Lecture Models of Growth and Decay – L. Bevers and W. van Dellen

Case study 1. Bunnies and Foxes, Regimes and Protesters

Francisco, R.A. (1995). The relationship between coercion and protest: An empirical evaluation in three coercive states. *The Journal of Conflict Resolution, 39(2),* 263-282

*Jaccard, J., & Jacoby, J. (2010). *Theory construction and model building skills: A practical guide for social scientists.* New York: Guilford. (Chapter 8 – function transformations)

*Lecture Models of Growth and Decay – L. Bevers and W. van Dellen

Robbins, J. (2004). Lessons from the wolf. *Scientific American, June 2004*, 76-81

*Tannenbaum, P. (2010). The mathematics of population growth. In: *Excursions in Modern Mathematics (pp.474-487)*. New Jersey: Pearson Prentice Hall

Zukerman, W. (2009) Australia's battle with the bunny. *ABC Science*

Case study 2. Game Theory

No readings required. For additional information, see the links to the videos, and the chapters listed below (both books can be found in the UCM Reading Room).

https://www.khanacademy.org/economics-finance-domain/microeconomics/nash-equilibrium-tutorial/nash-eq-tutorial/v/prisoners-dilemma-and-nash-equilibrium

https://www.khanacademy.org/economics-finance-domain/microeconomics/nash-equilibrium-tutorial/nash-eq-tutorial/v/more-on-nash-equilibrium

Perloff, J.M. (2009). Game Theory. In *Microeconomics* (pp. 477-503). New Jersey : Person Prentice Hall

Pindyck, R.S. and Rubinfeld, D.L. (2009). Game Theory and Competitive Strategy. In *Microeconomics* (pp. 479-516). New Jersey: Pearson Prentice Hall

Case study 3. An Experiment: It's a Small World After All

*Barabasi, A.L. & Bonabeau, E. (2003). Scale-free networks. *Scientific American, May 2003*, 60-69

Buzsaki, G. (2007). The structure of consciousness. *Nature*, 446, 267

Couzin, J. (2009). Friendship as a health factor. Science, 232, 454-457

Dodds, P.S., Muhamad, R. & Watts, D.J. (2003). An experimental study of search in global social networks. *Science*, *301*, 827-829.

*Haythornthwaite, C. (1996). Social network analysis: An approach and technique for the study of information exchange. *Library and Information Science Research, 18,* 323-342

*Lecture Network Systems – W. van Dellen and L. Bevers

Case study 4. Who to Let Go?

*Dekker T.J. (2008). Desert and distributive efficiency. *Ethics and Economics, 5(2)*, 1-23; can be retrieved from: https://papyrus.bib.umontreal.ca/xmlui/bitstream/handle/1866/3405/2008v5n2_DEKKER.pdf

*Jaccard, J., & Jacoby, J. (2010). *Theory construction and model building skills: A practical guide for social scientists.* New York: Guilford. (Chapter 8 – function transformations)

*Kagan, S. (2003). Comparative Desert. In S. Olsaretti (Ed.), *Desert and Justice* (pp. 93-122). Oxford: Oxford University Press; can be retrieved from: https://campuspress.yale.edu/shellykagan/files/2016/03/Comparative-Desert-23iyp5c.pdf

*Lecture Desert – T. Dekker