Experiment 6 — 9/9/2025

Implement Gradient Descent & Backpropagation

Aim: To implement a deep neural network demonstrating the use of forward Propagation, backpropagation and gradient descent.

Objectives:

- → Understand the XOR problem as a non linearly separable dataset.
- → Perform backpropagation to compute gradients.
- → Visualize the training loss and decision boundary of the network.

Pseudocode:

- → Import PyTorch
- → Define XOR dataset
- → Initialize hidden layer
- → Initialize output layer
- → Use sigmoid
- → Define the forward function
- → Initialize Model:

  model = XORNet()

  loss_function = MSE Loss()
- → Set number of epochs
- → Train the model — For each epoch
  - → Forward pass
  - → Compute Loss
  - → Backpropagation: loss.backward
  - → update weights
  - → Reset gradients
- → End.

Pseudocode: Plotting Decision Boundary

→ Import libraries

→ create a grid of points covering input space

→ Plot decision boundary using contourf

→ blue = 0, red = 1

→ end.

Observation

epoch 0, Loss: 0.2595

epoch 1000, Loss: 0.2497

:

epoch 9000, Loss: 0.0194

:

epoch 19000, Loss: 0.0024

tensor([[0.0494],
[0.9517],
[0.9560],
[0.455]])

loss is computed using $L = \frac{1}{N}\sum_{i=1}^{N}(\underset{\text{predicted}}{\hat{y_i}} - \underset{\text{true}}{y_i})^2$

gradients using

$$\frac{\partial L}{\partial w} = (\hat{y} - y)\cdot\hat{y}\cdot(1-\hat{y})\cdot a_{prev}$$

a_{prev} ↓ input to this weight.

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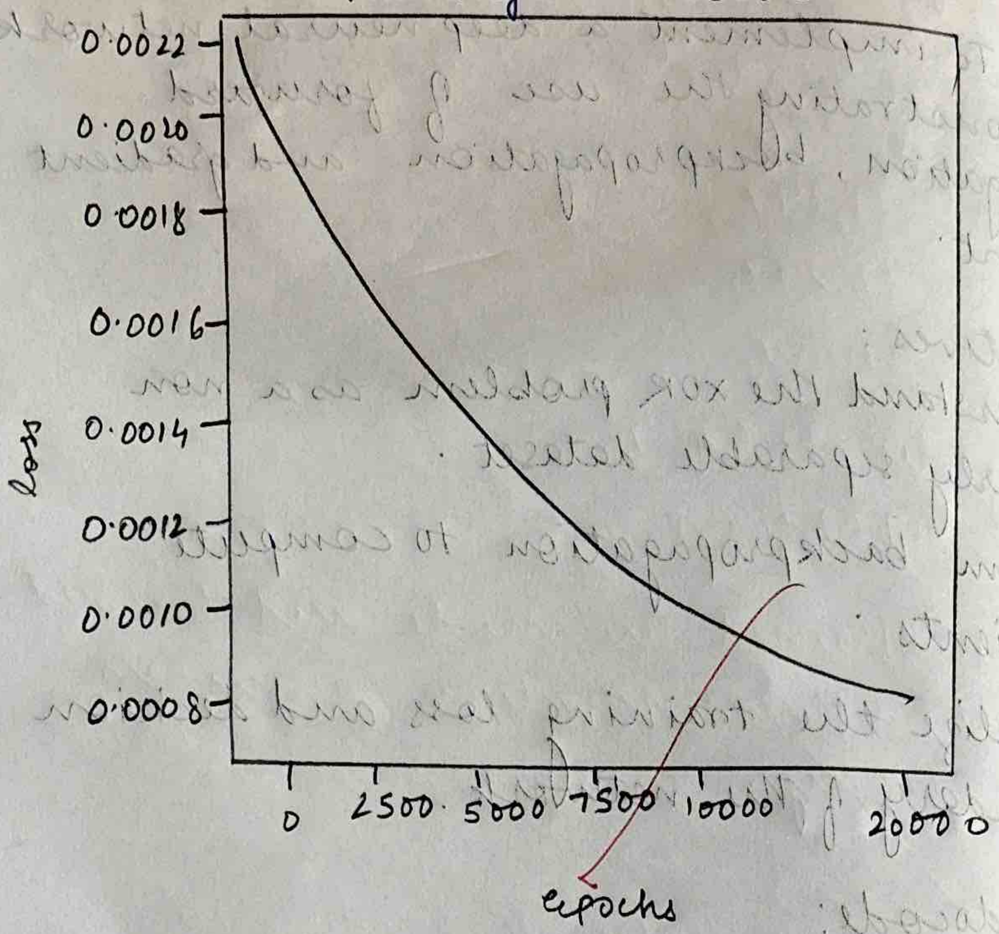
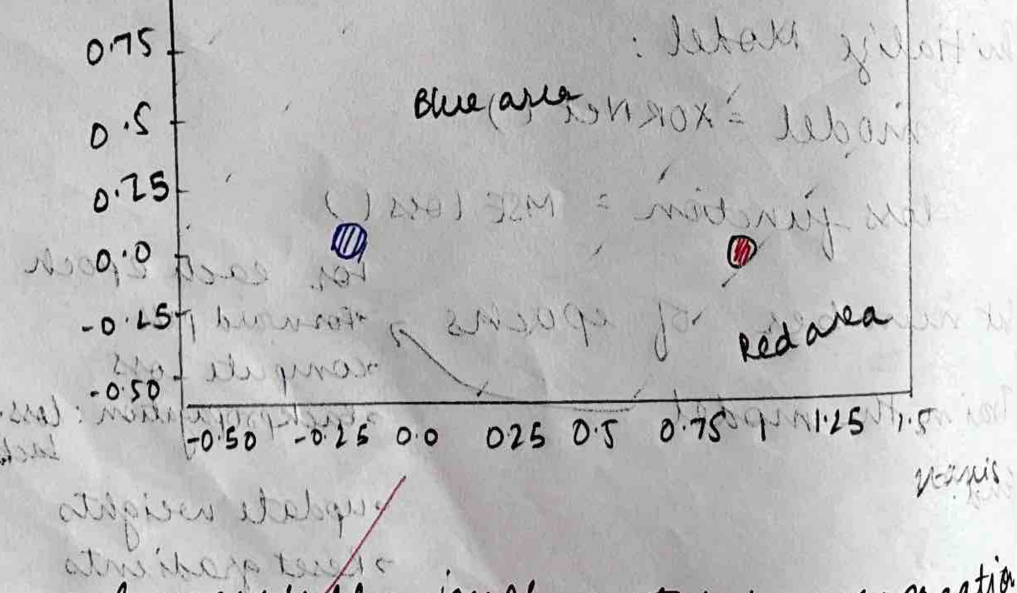
$$W = W - \eta\frac{\partial L}{\partial W}$$

Training Loss Curve



Decision Boundary of Neural Network



Result: Successfully implemented backpropagation to train a deep neural network.

```python
import torch
import torch.nn as nn
import torch.optim as optim

# Dataset (XOR problem)
X = torch.tensor([[0,0],[0,1],[1,0],[1,1]], dtype=torch.float32)
y = torch.tensor([[0],[1],[1],[0]], dtype=torch.float32)

# Define a simple neural network
class XORNet(nn.Module):
    def __init__(self):
        super(XORNet, self).__init__()
        self.hidden = nn.Linear(2, 4)   # input layer -> hidden
        self.output = nn.Linear(4, 1)  # hidden -> output
        self.sigmoid = nn.Sigmoid()

    def forward(self, x):
        x = self.sigmoid(self.hidden(x))
        x = self.sigmoid(self.output(x))
        return x

# Initialize model, loss, and optimizer
model = XORNet()
criterion = nn.MSELoss()
optimizer = optim.SGD(model.parameters(), lr=0.1)

# Training loop
epochs = 20000
for epoch in range(epochs):
    # Forward pass
    outputs = model(X)
    loss = criterion(outputs, y)
```

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```python
    # Backward pass + update
    optimizer.zero_grad()   # reset gradients
    loss.backward()         # compute gradients
    optimizer.step()        # update weights

    if epoch % 1000 == 0:
        print(f"Epoch {epoch}, Loss: {loss.item():.4f}")

# Final predictions
print("Final predictions:")
print(model(X).detach())
```

```
Epoch 0, Loss: 0.2595
Epoch 1000, Loss: 0.2497
Epoch 2000, Loss: 0.2492
Epoch 3000, Loss: 0.2483
Epoch 4000, Loss: 0.2458
Epoch 5000, Loss: 0.2361
Epoch 6000, Loss: 0.1899
Epoch 7000, Loss: 0.0877
Epoch 8000, Loss: 0.0361
Epoch 9000, Loss: 0.0194
Epoch 10000, Loss: 0.0125
Epoch 11000, Loss: 0.0089
Epoch 12000, Loss: 0.0069
Epoch 13000, Loss: 0.0055
Epoch 14000, Loss: 0.0046
Epoch 15000, Loss: 0.0039
Epoch 16000, Loss: 0.0034
Epoch 17000, Loss: 0.0030
Epoch 18000, Loss: 0.0027
Epoch 19000, Loss: 0.0024
Final predictions:
tensor([[0.0494],
        [0.9517],
        [0.9560]
```

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```
Epoch 19000, Loss: 0.0024
Final predictions:
tensor([[0.0494],
        [0.9517],
        [0.9560],
        [0.0455]])
```

In [6]:

```python
import matplotlib.pyplot as plt

losses = []

for epoch in range(epochs):
    outputs = model(X)
    loss = criterion(outputs, y)

    optimizer.zero_grad()
    loss.backward()
    optimizer.step()

    losses.append(loss.item())

# Plot loss curve
plt.plot(losses)
plt.xlabel("Epochs")
plt.ylabel("Loss")
plt.title("Training Loss Curve")
plt.show()
```

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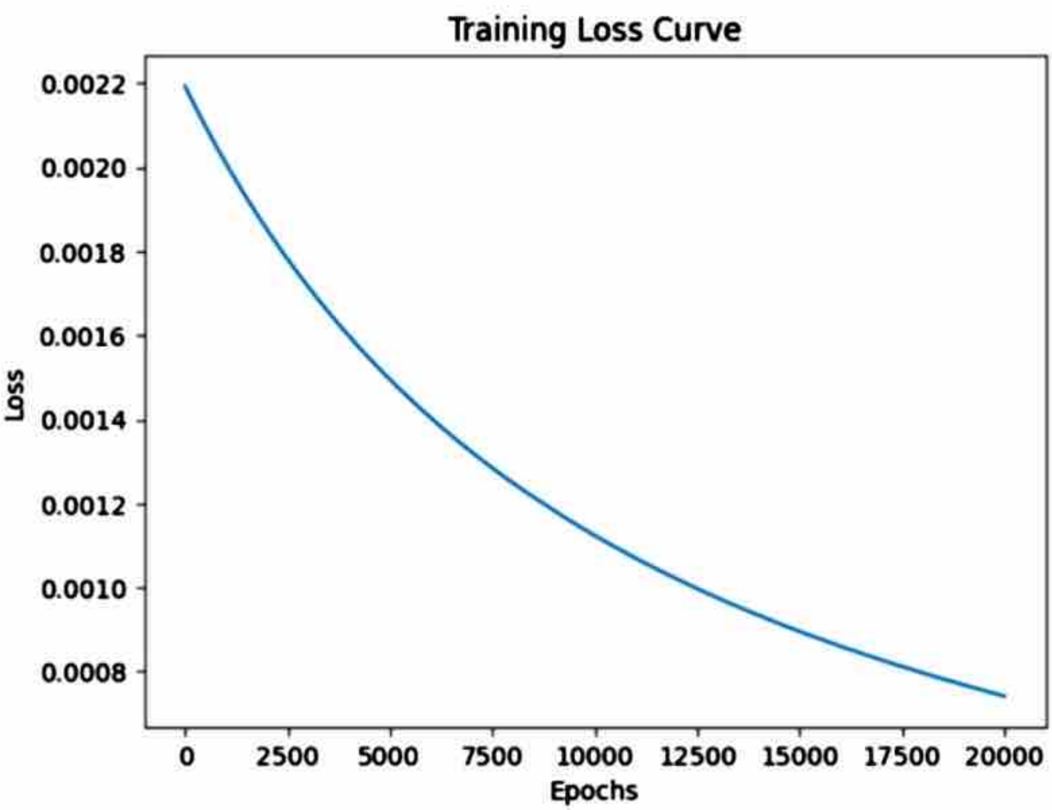

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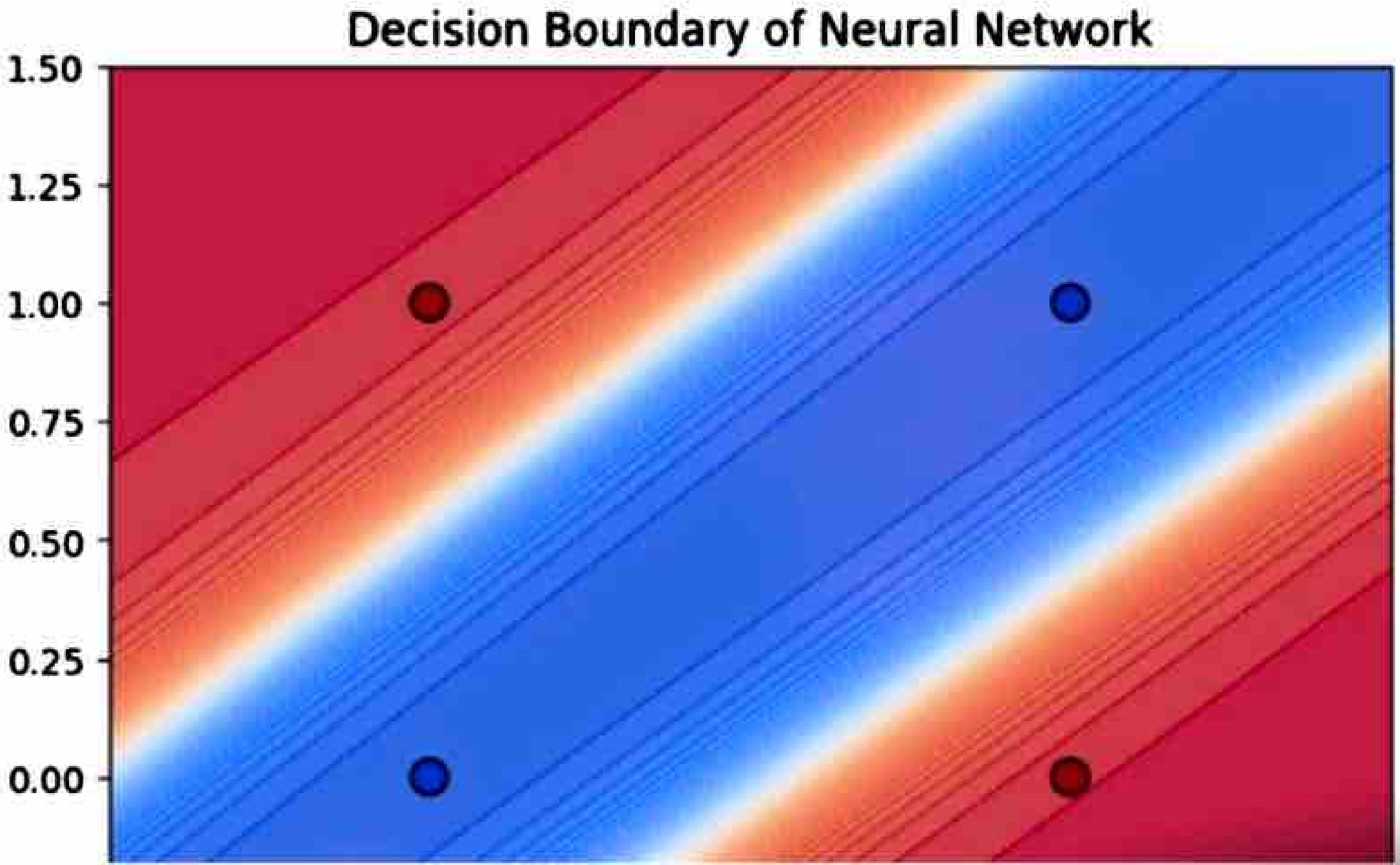
In [7]:

```python
import numpy as np
```

```python
xx, yy = np.meshgrid(np.linspace(-0.5, 1.5, 200),
                     np.linspace(-0.5, 1.5, 200))
grid = torch.tensor(np.c_[xx.ravel(), yy.ravel()], dtype=torch.float32)

# Get predictions
with torch.no_grad():
    Z = model(grid).numpy()
Z = Z.reshape(xx.shape)

# Plot decision boundary
plt.contourf(xx, yy, Z, levels=50, cmap="coolwarm", alpha=0.7)
plt.scatter(X[:,0], X[:,1], c=y[:,0], edgecolor='k', s=100, cmap="coolwarm")
plt.title("Decision Boundary of Neural Network")
plt.show()
```



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