# Weibull transition probabilities

## 0.1 Weibull hazard

To add a state-residence dependency to the simulation-time-dependent Sick-Sicker model defined above, we assume the risk of progression from S1 to S2 increases as a function of the time $\tau = 1, \ldots, n_{\text{tunnels}}$ the cohort remains in the S1 state. This increase follows a Weibull hazard function, $h(\tau)$, defined as

$$h(\tau) = \gamma\lambda(\lambda\tau)^{\gamma-1},$$

with a corresponding cumulative hazard, $H(\tau)$,

$$H(\tau) = (\lambda\tau)^{\gamma},$$

where $\lambda$ and $\gamma$ are the scale and shape parameters of the Weibull function, respectively.

## 0.2 Weibull transition probability

To derive a transition probability from S1 to S2 as a function as a function of the time the cohort spends in S1, $p_{[S1_\tau,S2,\tau]}$, from $H(\tau)$, we use the following equation[@Diaby2014]

$$p_{[S1_\tau,S2,\tau]} = 1 - \exp\left(H(\tau-1) - H(\tau)\right) \tag{1}$$

Substituting the Weibull cumulative hazard in Equation (1), the transition probability is

$$p_{[S1_\tau,S2,\tau]} = 1 - \exp\left((\lambda(\tau-1))^{\gamma} - (\lambda\tau)^{\gamma}\right)$$

and simplifies to

$$p_{[S1_\tau,S2,\tau]} = 1 - \exp\left(\lambda^{\gamma}((\tau-1)^{\gamma} - \tau^{\gamma})\right)$$

## 0.3 Effectiveness of a treatment as a hazard ratio

To account for the effectiveness of a treatment, we multiply the hazard ratio of the treatment by the difference of the cumulative hazards.

$$p\_treat_{[S1_\tau,S2,\tau]} = 1 - \exp\left(hr \cdot ((\lambda(\tau-1))^{\gamma} - (\lambda\tau)^{\gamma})\right)$$

or

$$p_{[S1_\tau,S2,\tau]} = 1 - \exp\left(hr \cdot \lambda^{\gamma}((\tau-1)^{\gamma} - \tau^{\gamma})\right),$$

which is equivalent to transform $p_{[S1_\tau,S2,\tau]}$ to a vector of rates, $r_{[S1_\tau,S2,\tau]}$, and multiply it by the hazard ratio of the treatment, $hr$. Then, we transform back to probabilities to produce $p\_treat_{[S1_\tau,S2,\tau]}$, a transition probabilities that accounts for the duration of S1 state-residence under the treatment.