SA: Markov Sick-Sicker model in R

The DARTH workgroup

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Please cite our publications when using this code:

- Jalal H, Pechlivanoglou P, Krijkamp E, Alarid-Escudero F, Enns E, Hunink MG. An Overview of R in Health Decision Sciences. Med Decis Making. 2017; 37(3): 735-746. https://journals.sagepub.com/doi/abs/10.1177/0272989X16686559
- Krijkamp EM, Alarid-Escudero F, Enns EA, Jalal HJ, Hunink MGM, Pechlivanoglou P. Microsimulation modeling for health decision sciences using R: A tutorial. Med Decis Making. 2018;38(3):400–22. https://journals.sagepub.com/doi/abs/10.1177/0272989X18754513
- Krijkamp EM, Alarid-Escudero F, Enns E, Pechlivanoglou P, Hunink MM, Jalal H. A Multidimensional Array Representation of State-Transition Model Dynamics. Med Decis Making. Online First https://doi.org/10.1177/0272989X19893973

Change `eval` to `TRUE` if you want to knit this document.

```
rm(list = ls())      # clear memory (removes all the variables from the workspace)
```

01 Load packages

```
if (!require('pacman')) install.packages('pacman'); library(pacman) # use this package to conveniently
# load (install if required) packages from CRAN
p_load("here", "dplyr", "devtools", "scales", "ellipse", "ggplot2", "lazyeval", "igraph", "truncnorm",
# load (install if required) packages from GitHub
# install_github("DARTH-git/dampack", force = TRUE) Uncomment if there is a newer version
p_load_gh("DARTH-git/dampack", "DARTH-git/darthtools")
```

02 Load functions

```
# No function needed
```

03 Input model parameters

```
# Strategy names
v_names_str <- c("No Treatment", "Treatment")

# Number of strategies
n_str <- length(v_names_str)

# Markov model parameters
age     <- 25                       # age at baseline
max_age <- 55                       # maximum age of follow up
n_t     <- max_age - age            # time horizon, number of cycles
v_n     <- c("H", "S1", "S2", "D")  # the 4 states of the model: Healthy (H), Sick (S1),
                                    # Sicker (S2), Dead (D)
n_states <- length(v_n)             # number of health states

# Transition probabilities (per cycle)
p_HD    <- 0.005                    # probability to die when healthy
p_HS1   <- 0.15                       # probability to become sick when healthy, conditional on survivi
p_S1H   <- 0.5                        # probability to become healthy when sick, conditional on survivi
p_S1S2  <- 0.105                      # probability to become sicker when sick, conditional on survivin
hr_S1   <- 3                          # hazard ratio of death in sick vs healthy
hr_S2   <- 10                         # hazard ratio of death in sicker vs healthy
r_HD    <- - log(1 - p_HD)          # rate of death in healthy
r_S1D   <- hr_S1 * r_HD               # rate of death in sick
r_S2D   <- hr_S2 * r_HD               # rate of death in sicker
p_S1D   <- 1 - exp(-r_S1D)          # probability to die in sick
p_S2D   <- 1 - exp(-r_S2D)          # probability to die in sicker
```

```r
# Cost and utility inputs
c_H     <- 2000                     # cost of remaining one cycle in the healthy state
c_S1    <- 4000                     # cost of remaining one cycle in the sick state
c_S2    <- 15000                    # cost of remaining one cycle in the sicker state
c_trt   <- 12000                    # cost of treatment (per cycle)
c_D     <- 0                        # cost of being in the death state
u_H     <- 1                        # utility when healthy
u_S1    <- 0.75                     # utility when sick
u_S2    <- 0.5                      # utility when sicker
u_D     <- 0                        # utility when dead
u_trt   <- 0.95                     # utility when being treated

# Discounting factor
d_e  <- d_c <- 0.03                 # equal discount of costs and QALYs by 3%

# calculate discount weights for costs for each cycle based on discount rate d_c
v_dwc <- 1 / (1 + d_e) ^ (0:n_t)
# calculate discount weights for effectiveness for each cycle based on discount rate d_e
v_dwe <- 1 / (1 + d_c) ^ (0:n_t)
```

Create a state-transition diagram of the cohort model

```r
m_P_diag <- matrix(0, nrow = n_states, ncol = n_states, dimnames = list(v_n, v_n))
m_P_diag["H" , "S1"] = ""
m_P_diag["H" , "D" ] = ""
m_P_diag["H" , "H" ] = ""
m_P_diag["S1", "H" ] = ""
m_P_diag["S1", "S2"] = ""
m_P_diag["S1", "D" ] = ""
m_P_diag["S1", "S1"] = ""
m_P_diag["S2", "D" ] = ""
m_P_diag["S2", "S2"] = ""
m_P_diag["D", "D"  ] = ""
layout.fig <- c(3, 1)

plotmat(t(m_P_diag), t(layout.fig), self.cex = 0.5, curve = 0, arr.pos = 0.7,
        latex = T, arr.type = "curved", relsize = 0.9, box.prop = 0.8,
        cex = 0.8, box.cex = 0.9, lwd = 1)
```

04 Define and initialize matrices and vectors

04.1 Cohort trace

```r
# create the markov trace matrix M capturing the proportion of the cohort in each state
# at each cycle
m_M_notrt <- m_M_trt <- matrix(NA,
                               nrow     = n_t + 1, ncol = n_states,
                               dimnames = list(paste("cycle", 0:n_t, sep = " "), v_n))
```

```r
head(m_M_notrt) # show first 6 rows of the matrix

# The cohort starts as healthy
m_M_notrt[1, ] <- m_M_trt[1, ] <- c(1, 0, 0, 0) # initiate first cycle of cohort trace
```

04.2 Transition probability matrix

```r
# create the transition probability matrix for NO treatment
m_P_notrt  <- matrix(0,
                     nrow = n_states,
                     ncol = n_states,
                     dimnames = list(v_n, v_n)) # name the columns and rows of the matrix
m_P_notrt
```

Fill in the transition probability matrix:

```r
# from Healthy
m_P_notrt["H", "H"  ] <- (1 - p_HD) * (1 - p_HS1)
m_P_notrt["H", "S1" ] <- (1 - p_HD) * p_HS1
m_P_notrt["H", "D"  ] <- p_HD
# from Sick
m_P_notrt["S1", "H" ] <- (1 - p_S1D) * p_S1H
m_P_notrt["S1", "S1"] <- (1 - p_S1D) * (1 - (p_S1H + p_S1S2))
m_P_notrt["S1", "S2"] <- (1 - p_S1D) * p_S1S2
m_P_notrt["S1", "D" ] <- p_S1D
# from Sicker
m_P_notrt["S2", "S2"] <- 1 - p_S2D
m_P_notrt["S2", "D" ] <- p_S2D
# from Dead
m_P_notrt["D", "D"  ] <- 1

# Check that transition probabilities are in [0, 1]
check_transition_probability(m_P_notrt, verbose = TRUE)
# Check that all rows sum to 1
check_sum_of_transition_array(m_P_notrt, n_states = n_states, n_t = n_t, verbose = TRUE)

# create transition probability matrix for treatment same as no treatment
m_P_trt <- m_P_notrt
```

05 Run Markov model

```r
for (t in 1:n_t){     # loop through the number of cycles
  m_M_notrt[t + 1, ] <- t(m_M_notrt[t, ]) %*% m_P_notrt  # estimate the Markov trace
                                                         # for the next cycle (t + 1)
  m_M_trt[t + 1, ]   <- t(m_M_trt[t, ])   %*% m_P_trt    # estimate the Markov trace
                                                         # for the next cycle (t + 1)
} # close the loop
```

```r
head(m_M_notrt)  # show the first 6 lines of the matrix
```

06 Compute and Plot Epidemiological Outcomes

06.1 Cohort trace

```r
# create a plot of the data
matplot(m_M_notrt, type = 'l',
        ylab = "Probability of state occupancy",
        xlab = "Cycle",
        main = "Cohort Trace")
# add a legend to the graph
legend("topright", v_n, col = 1:n_states, lty = 1:n_states, bty = "n")
```

06.2 Overall Survival (OS)

```r
# calculate the overall survival (OS) probability for no treatment
v_os_notrt <- 1 - m_M_notrt[, "D"]
# alternative way of calculating the OS probability
v_os_notrt <- rowSums(m_M_notrt[, 1:3])
# create a simple plot showing the OS
plot(0:n_t, v_os_notrt, type = 'l',
     ylim = c(0, 1),
     ylab = "Survival probability",
     xlab = "Cycle",
     main = "Overall Survival")
# add grid
grid(nx = n_t, ny = 10, col = "lightgray", lty = "dotted", lwd = par("lwd"),
     equilogs = TRUE)
```

06.2.1 Life Expectancy (LE)

```r
v_le <- sum(v_os_notrt)  # summing probability of OS over time  (i.e. life expectancy)
```

06.3 Disease prevalence

```r
v_prev <- rowSums(m_M_notrt[, c("S1", "S2")]) / v_os_notrt
plot(v_prev,
     ylim = c(0, 1),
     ylab = "Prevalence",
     xlab = "Cycle",
     main = "Disease prevalence")
```

06.4 Proportion of sick in S1 state

```
v_prop_S1 <- m_M_notrt[, "S1"] / v_prev
plot(0:n_t, v_prop_S1,
     xlab = "Cycle",
     ylab = "Proportion",
     main = "Proportion of sick in S1 state",
     col  = "black", type = "l")
```

07 Compute Cost-Effectiveness Outcomes

```
# Vectors with costs and utilities by treatment
v_u_notrt   <- c(u_H, u_S1,  u_S2, u_D)
v_u_trt     <- c(u_H, u_trt, u_S2, u_D)

v_c_notrt   <- c(c_H, c_S1, c_S2, c_D)
v_c_trt     <- c(c_H, c_S1 + c_trt, c_S2 + c_trt, c_D)
```

07.1 Mean Costs and QALYs for Treatment and NO Treatment

```
v_tu_notrt  <- m_M_notrt   %*%  v_u_notrt
v_tu_trt    <- m_M_trt     %*%  v_u_trt

v_tc_notrt  <- m_M_notrt   %*%  v_c_notrt
v_tc_trt    <- m_M_trt     %*%  v_c_trt
```

07.2 Discounted Mean Costs and QALYs

```
tu_d_notrt  <- t(v_tu_notrt)   %*%  v_dwe
tu_d_trt    <- t(v_tu_trt)     %*%  v_dwe

tc_d_notrt  <- t(v_tc_notrt)   %*%  v_dwc
tc_d_trt    <- t(v_tc_trt)     %*%  v_dwc

# store them into a vector
v_tc_d      <- c(tc_d_notrt, tc_d_trt)
v_tu_d      <- c(tu_d_notrt, tu_d_trt)

# Dataframe with discounted costs and effectiveness
df_ce       <- data.frame(Strategy = v_names_str,
                          Cost     = v_tc_d,
                          Effect   = v_tu_d
                          )
df_ce
```

07.3 Compute ICERs of the Markov model

```
df_cea <- calculate_icers(cost       = df_ce$Cost,
                          effect     = df_ce$Effect,
                          strategies = df_ce$Strategy
                          )
df_cea
```

07.4 Plot frontier of the Markov model

```
plot(df_cea, effect_units = "QALYs", xlim = c(15.6, 16.6))
```

08 Deterministic Sensitivity Analysis

08.1 List of input parameters

Create list `l_params_all` with all input probabilities, cost and utilities.

```
l_params_all <- as.list(data.frame(
  p_HD    = 0.005,  # probability to die when healthy
  p_HS1   = 0.15,   # probability to become sick when healthy, conditional on surviving
  p_S1H   = 0.5,    # probability to become healthy when sick, conditional on surviving
  p_S1S2  = 0.105,  # probability to become sicker when sick, conditional on surviving
  hr_S1   = 3,      # hazard ratio of death in sick vs healthy
  hr_S2   = 10,     # hazard ratio of death in sicker vs healthy
  c_H     = 2000,   # cost of remaining one cycle in the healthy state
  c_S1    = 4000,   # cost of remaining one cycle in the sick state
  c_S2    = 15000,  # cost of remaining one cycle in the sicker state
  c_trt   = 12000,  # cost of treatment(per cycle)
  c_D     = 0,      # cost of being in the death state
  u_H     = 1,      # utility when healthy
  u_S1    = 0.75,   # utility when sick
  u_S2    = 0.5,    # utility when sicker
  u_D     = 0,      # utility when dead
  u_trt   = 0.95,   # utility when treated
  d_e     = 0.03,   # discount factor for effectiveness
  d_c     = 0.03    # discount factor for costs
))

# store the parameter names into a vector
v_names_params <- names(l_params_all)
```

08.2 Load Sick-Sicker Markov model function

```
source("Functions_markov_sick-sicker_sol.R")
# Test function
calculate_ce_out(l_params_all)
```

B ## 08.3 One-way sensitivity analysis (OWSA)

```
options(scipen = 999) # disabling scientific notation in R
# dataframe containing all parameters, their basecase values, and the min and
# max values of the parameters of interest
df_params_owsa <- data.frame(pars = c("p_S1S2", "c_trt", "u_S1", "u_trt"),
                             min  = c(0.05 ,  6000 , 0.65, 0.80),  # min parameter values
                             max  = c(0.155, 18000 , 0.85, 0.98)   # max parameter values
                             )

owsa_nmb  <- run_owsa_det(params_range     = df_params_owsa,    # dataframe with parameters for OWSA
                          params_basecase  = l_params_all,      # list with all parameters
                          nsamp            = 100,               # number of parameter values
                          FUN              = calculate_ce_out,  # function to compute outputs
                          outcomes         = c("NMB"),          # output to do the OWSA on
                          strategies       = v_names_str,       # names of the strategies
                          n_wtp            = 120000)            # extra argument to pass to FUN
```

08.3.1 Plot OWSA

```
plot(owsa_nmb, txtsize = 10, n_x_ticks = 4,
     facet_scales = "free") +
     theme(legend.position = "bottom")
```

08.3.2 Optimal strategy with OWSA

```
owsa_opt_strat(owsa = owsa_nmb, txtsize = 10)
```

08.3.3 Tornado plot

```
owsa_tornado(owsa = owsa_nmb)
```

08.4 Two-way sensitivity analysis (TWSA)

```
# dataframe containing all parameters, their basecase values, and the min and
# max values of the parameters of interest
df_params_twsa <- data.frame(pars = c("c_trt", "u_trt"),
                             min  = c(6000, 0.80),  # min parameter values
                             max  = c(18000, 0.98)  # max parameter values
                             )

twsa_nmb <- run_twsa_det(params_range    = df_params_twsa,    # dataframe with parameters for TWSA
                         params_basecase = l_params_all,      # list with all parameters
                         nsamp           = 40,                # number of parameter values
                         FUN             = calculate_ce_out,  # function to compute outputs
```

```
                          outcomes        = "NMB",          # output to do the TWSA on
                          strategies      = v_names_str,       # names of the strategies
                          n_wtp           = 120000)            # extra argument to pass to FUN
```

l ## 08.4.1 Plot TWSA

```
plot(twsa_nmb)
```

09 Probabilistic Sensitivity Analysis (PSA)

```
# Function to generate PSA input dataset
gen_psa <- function(n_sim = 1000, seed = 071818){
  set.seed(seed) # set a seed to be able to reproduce the same results
  df_psa <- data.frame(
    # Transition probabilities (per cycle)
    p_HS1   = rbeta(n_sim, shape1 = 30, shape2 = 170),     # probability to become sick when healthy
    p_S1H   = rbeta(n_sim, shape1 = 60, shape2 = 60) ,     # probability to become healthy when sick
    p_S1S2  = rbeta(n_sim, shape1 = 84, shape2 = 716),     # probability to become sicker when sick
    p_HD    = rbeta(n_sim, shape1 = 10, shape2 = 1990),    # probability to die when healthy
    hr_S1   = rlnorm(n_sim, meanlog = log(3),  sdlog = 0.01),  # rate ratio of death in S1 vs healthy
    hr_S2   = rlnorm(n_sim, meanlog = log(10), sdlog = 0.02),  # rate ratio of death in S2 vs healthy

    # State rewards
    # Costs
    c_H   = rgamma(n_sim, shape = 100, scale = 20),     # cost of remaining one cycle in state H
    c_S1  = rgamma(n_sim, shape = 177.8, scale = 22.5), # cost of remaining one cycle in state S1
    c_S2  = rgamma(n_sim, shape = 225, scale = 66.7),   # cost of remaining one cycle in state S2
    c_Trt = rgamma(n_sim, shape = 73.5, scale = 163.3), # cost of treatment (per cycle)
    c_D   = 0,                                          # cost of being in the death state

    # Utilities
    u_H   = rbeta(n_sim, shape1 = 200, shape2 = 3),     # utility when healthy
    u_S1  = rbeta(n_sim, shape1 = 130, shape2 = 45),    # utility when sick
    u_S2  = rbeta(n_sim, shape1 = 230, shape2 = 230),   # utility when sicker
    u_D   = 0,                                          # utility when dead
    u_Trt = rbeta(n_sim, shape1 = 300, shape2 = 15),    # utility when being treated
    d_e   = 0.03,                                       # discount factor for effectiveness
    d_c   = 0.03                                        # discount factor for costs
  )
    return(df_psa)
}
# Try it
gen_psa(10)

# Number of simulations
n_sim <- 1000

# Generate PSA input dataset
df_psa_input <- gen_psa(n_sim = n_sim)
# First six observations
```

```r
head(df_psa_input)

# Histogram of parameters
ggplot(melt(df_psa_input, variable.name = "Parameter"), aes(x = value)) +
       facet_wrap(~Parameter, scales = "free") +
       geom_histogram(aes(y = ..density..)) +
       scale_x_continuous(breaks = scales::pretty_breaks(n = 3)) +
       theme_bw(base_size = 16) +
       theme(axis.text = element_text(size=6))

# Initialize dataframes with PSA output
# Dataframe of costs
df_c <- as.data.frame(matrix(0,
                      nrow = n_sim,
                      ncol = n_str))
colnames(df_c) <- v_names_str
# Dataframe of effectiveness
df_e <- as.data.frame(matrix(0,
                      nrow = n_sim,
                      ncol = n_str))
colnames(df_e) <- v_names_str
```

09.1 Conduct probabilistic sensitivity analysis

```r
# Run Markov model on each parameter set of PSA input dataset
for(i in 1:n_sim){
  l_out_temp <- calculate_ce_out(df_psa_input[i, ])
  df_c[i, ] <- l_out_temp$Cost
  df_e[i, ] <- l_out_temp$Effect
  # Display simulation progress
  if(i/(n_sim/10) == round(i/(n_sim/10), 0)) { # display progress every 10%
    cat('\r', paste(i/n_sim * 100, "% done", sep = " "))
  }
}
```

09.2 Create PSA object for dampack

```r
l_psa <- make_psa_obj(cost          = df_c,
                      effectiveness = df_e,
                      parameters    = df_psa_input,
                      strategies    = v_names_str)
```

09.2.1 Save PSA objects

```r
save(df_psa_input, df_c, df_e, v_names_str, n_str, l_psa,
     file = "markov_sick-sicker_PSA_dataset.RData")
```

09.3 Create probabilistic analysis graphs

```
load(file = "markov_sick-sicker_PSA_dataset.RData")
```

Vector with willingness-to-pay (WTP) thresholds.

```
v_wtp <- seq(0, 200000, by = 10000)
```

09.3.1 Cost-Effectiveness Scatter plot

```
plot(l_psa)
```

09.4 Conduct CEA with probabilistic output

```
# Compute expected costs and effects for each strategy from the PSA
df_out_ce_psa <- summary(l_psa)

# Calculate incremental cost-effectiveness ratios (ICERs)
df_cea_psa <- calculate_icers(cost       = df_out_ce_psa$meanCost,
                              effect     = df_out_ce_psa$meanEffect,
                              strategies = df_out_ce_psa$Strategy)
df_cea_psa

# Save CEA table with ICERs
# As .RData
save(df_cea_psa,
     file = "markov_sick-sicker_probabilistic_CEA_results.RData")
# As .csv
write.csv(df_cea_psa,
          file = "markov_sick-sicker_probabilistic_CEA_results.csv")
```

09.4.1 Plot cost-effectiveness frontier

```
plot(df_cea_psa)
```

09.4.2 Cost-effectiveness acceptability curves (CEACs) and frontier (CEAF)

```
ceac_obj <- ceac(wtp = v_wtp, psa = l_psa)
# Regions of highest probability of cost-effectiveness for each strategy
summary(ceac_obj)
# CEAC & CEAF plot
plot(ceac_obj)
```

09.4.3 Expected Loss Curves (ELCs)

The expected loss is the the quantification of the foregone benefits when choosing a suboptimal strategy given current evidence.

```
elc_obj <- calc_exp_loss(wtp = v_wtp, psa = l_psa)
elc_obj
# ELC plot
plot(elc_obj, log_y = FALSE)
```

09.4.4 Expected value of perfect information (EVPI)

```
evpi <- calc_evpi(wtp = v_wtp, psa = l_psa)
# EVPI plot
plot(evpi, effect_units = "QALY")
```