

Cohort State-Transition Models

Decision Modeling for Public Health

November 2020

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Cohort State-Transition Modeling Overview

Cohort State-Transition Models

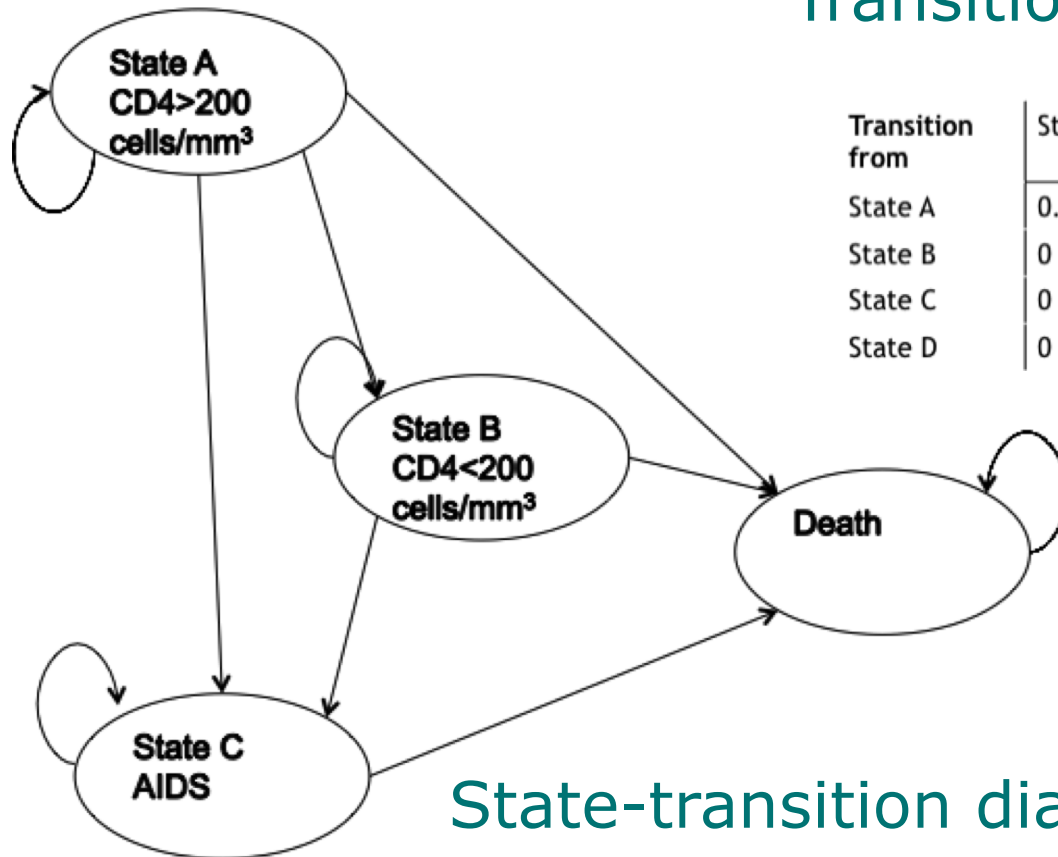
- Model that describes how a cohort is distributed across health states over time
 - e.g., healthy, sick, stable, progressed, dead
- Allow transitions between states with some probability
- Transitions are modeled in discrete time steps (e.g. weekly, monthly, yearly)

Cohort Model Assumptions

- Health states are mutually exclusive, collectively exhaustive
- Within a given health state, population is homogeneous
- Markovian assumption: transition probabilities depends only on current health state (“memoryless”)

Simple State-Transition Model of HIV Progression

Transition probability matrix



Transition from	Transition to			
	State A	State B	State C	State D
State A	0.721	0.202	0.067	0.01
State B	0	0.581	0.407	0.012
State C	0	0	0.75	0.25
State D	0	0	0	1

State-transition diagram

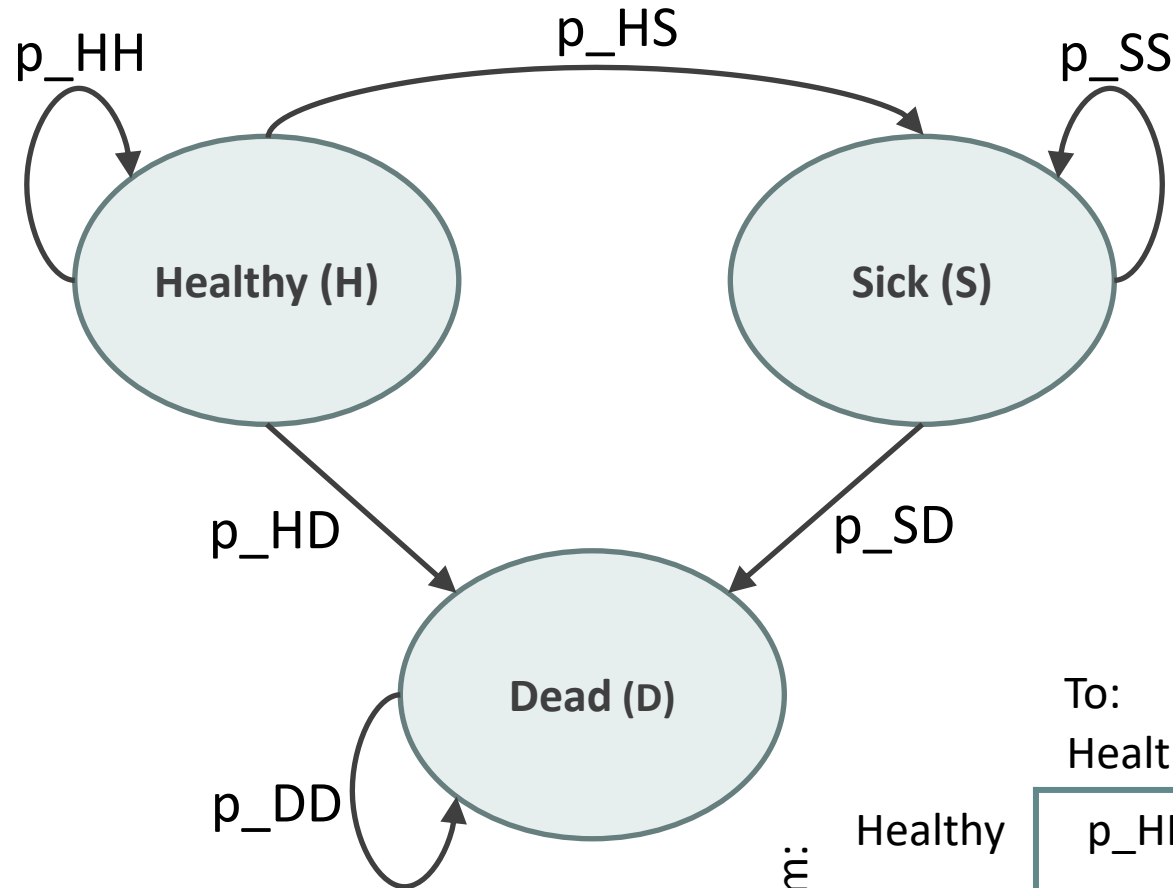
Cycle	State A	State B	State C	State D	Total
0	1000	0	0	0	1000
	1000×0.721	1000×0.202	1000×0.067	1000×0.01	
	↓	↓	↓	↓	
1	721	202	67	10	1000
2	520	263	181	36	1000
3	375	258	277	90	1000
4	270	226	338	166	1000
5	195	186	363	256	1000
6	140	147	361	351	1000
7	101	114	340	445	1000
8	73	87	308	532	1000
9	53	65	271	611	1000
10	38	48	234	680	1000
11	27	36	197	739	1000
12	20	26	164	789	1000
13	14	19	135	831	1000
14	10	14	110	865	1000
15	7	10	89	893	1000
16	5	7	72	916	1000
17	4	5	57	934	1000
18	3	4	45	948	1000
19	2	3	36	959	1000
20	1	2	28	968	1000

Cohort "Trace"

Number or proportion
of individuals at each
time step

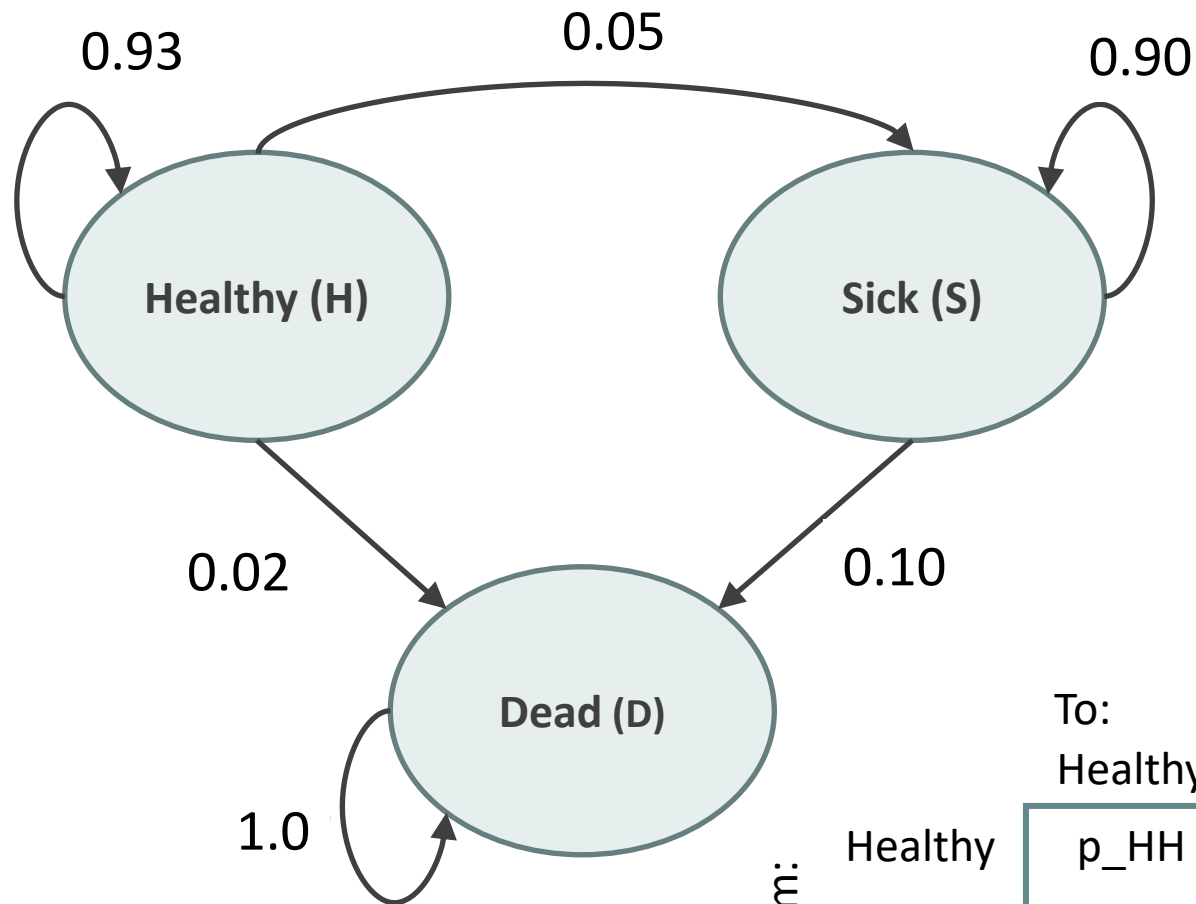
Simple 3-State Example

Three-State Model



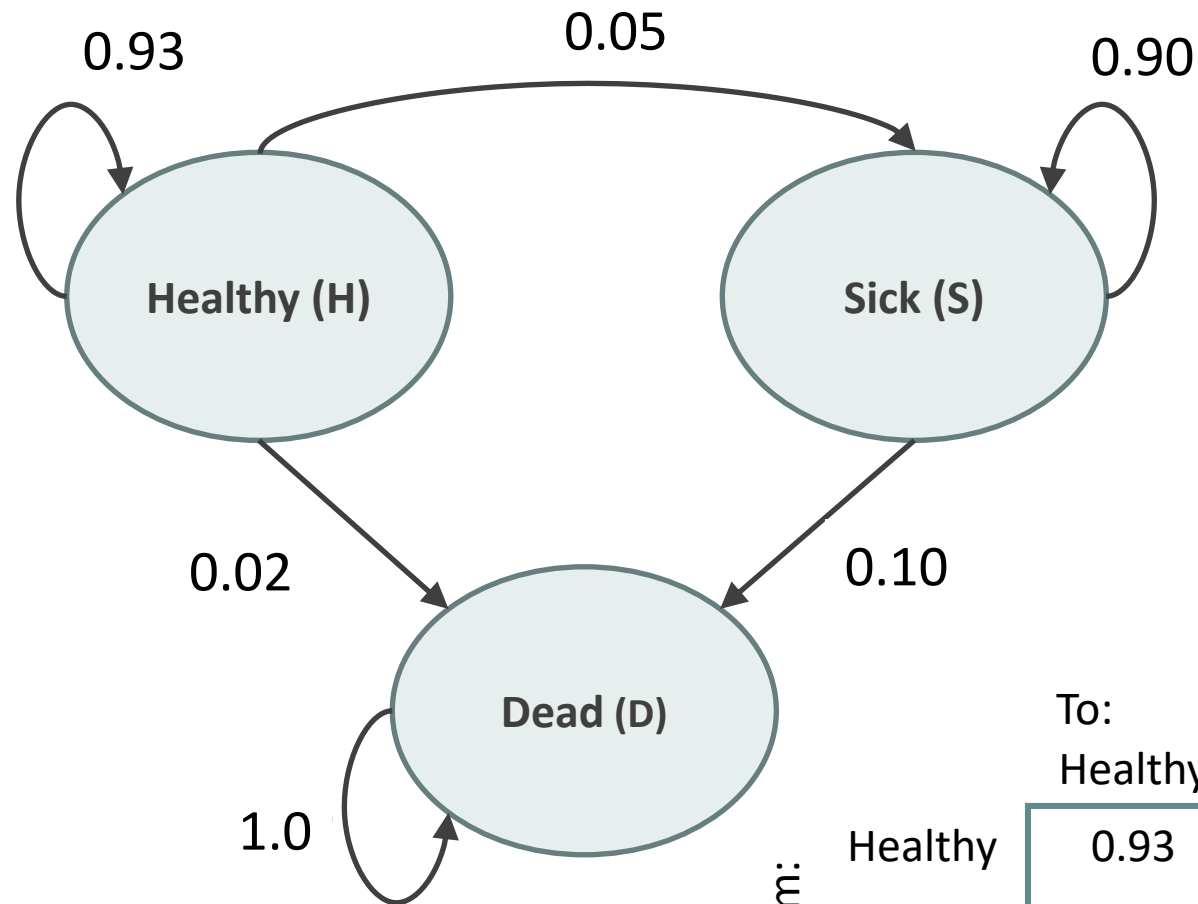
From:	To:		
	Healthy	Sick	Dead
Healthy	p_{HH}	p_{HS}	p_{HD}
Sick	0	p_{SS}	p_{SD}
Dead	0	0	p_{DD}

Three-State Model



From:	To:		
	Healthy	Sick	Dead
Healthy	p_{HH}	p_{HS}	p_{HD}
Sick	0	p_{SS}	p_{SD}
Dead	0	0	p_{DD}

Three-State Model



From:	To:		
	Healthy	Sick	Dead
Healthy	0.93	0.05	0.02
Sick	0	0.90	0.10
Dead	0	0	1.0

Simulate the Cohort over Time

- Initial cohort distribution:

$$x_0 = [1.0 \quad 0.0 \quad 0.0] \quad (\text{everyone starts healthy})$$

- Cohort distribution at next time step is calculated through matrix multiplication

$$\left[\text{---} x_{t+1} \text{---} \right] = \left[\text{---} x_t \text{---} \right] \begin{bmatrix} \text{Transition} \\ \text{Probability} \\ \text{Matrix} \\ P \end{bmatrix}$$

Simulate the Cohort over Time

- Initial cohort distribution:

$$x_0 = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} \quad (\text{everyone starts healthy})$$

- Cohort distribution at next time step is calculated through matrix multiplication

$$\begin{bmatrix} x_1 \end{bmatrix} = \begin{bmatrix} x_0 \\ 1.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} 0.93 & 0.05 & 0.02 \\ 0 & 0.90 & 0.10 \\ 0 & 0 & 1.0 \end{bmatrix}$$

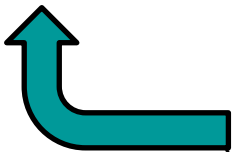
Simulate the Cohort over Time

- Initial cohort distribution:

$$x_0 = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} \quad (\text{everyone starts healthy})$$

- Cohort distribution at next time step is calculated through matrix multiplication

$$\begin{matrix} x_1 \\ \begin{bmatrix} 0.93 \end{bmatrix} \end{matrix} = \begin{matrix} x_0 \\ \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} \end{matrix} \begin{bmatrix} 0.93 & 0.05 & 0.02 \\ 0 & 0.90 & 0.10 \\ 0 & 0 & 1.0 \end{bmatrix}$$


 $(0.93)(1.0) + (0.0)(0.0) + (0.0)(0.0)$

$$(p_{Healthy,Healthy}) * \Pr(healthy) + (p_{Sick,Healthy}) * \Pr(sick) + (p_{Dead,Healthy}) * \Pr(dead)$$

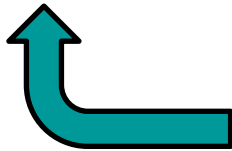
Simulate the Cohort over Time

- Initial cohort distribution:

$$x_0 = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} \quad (\text{everyone starts healthy})$$

- Cohort distribution at next time step is calculated through matrix multiplication

$$\begin{bmatrix} x_1 \\ 0.93 & 0.05 \end{bmatrix} = \begin{bmatrix} x_0 \\ 1.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} 0.93 & 0.05 & 0.02 \\ 0 & 0.90 & 0.10 \\ 0 & 0 & 1.0 \end{bmatrix}$$


 $(0.05)(1.0) + (0.90)(0.0) + (0.0)(0.0)$

Simulate the Cohort over Time

- Initial cohort distribution:

$$x_0 = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} \quad (\text{everyone starts healthy})$$

- Cohort distribution at next time step is calculated through matrix multiplication

$$\begin{matrix} x_1 \\ \begin{bmatrix} 0.93 & 0.05 & 0.02 \end{bmatrix} \end{matrix} = \begin{matrix} x_0 \\ \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} \end{matrix} \begin{bmatrix} 0.93 & 0.05 & 0.02 \\ 0 & 0.90 & 0.10 \\ 0 & 0 & 1.0 \end{bmatrix}$$

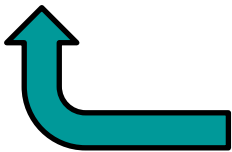
Simulate the Cohort over Time

- Initial cohort distribution:

$$x_0 = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} \quad (\text{everyone starts healthy})$$

- Cohort distribution at next time step is calculated through matrix multiplication

$$\begin{matrix} & x_2 & \\ \begin{bmatrix} 0.87 & & \end{bmatrix} & = & \begin{matrix} x_1 \\ \begin{bmatrix} 0.93 & 0.05 & 0.02 \end{bmatrix} \end{matrix} \begin{bmatrix} 0.93 & 0.05 & 0.02 \\ 0 & 0.90 & 0.10 \\ 0 & 0 & 1.0 \end{bmatrix} \end{matrix}$$

 $(0.93)(0.93) + (0.0)(0.05) + (0.0)(0.02)$

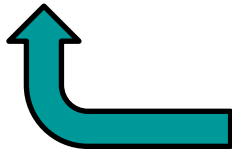
Simulate the Cohort over Time

- Initial cohort distribution:

$$x_0 = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} \quad (\text{everyone starts healthy})$$

- Cohort distribution at next time step is calculated through matrix multiplication

$$\begin{bmatrix} x_2 & & \\ 0.87 & 0.09 & \end{bmatrix} = \begin{bmatrix} x_1 & & \\ 0.93 & 0.05 & 0.02 \end{bmatrix} \begin{bmatrix} 0.93 & 0.05 & 0.02 \\ 0 & 0.90 & 0.10 \\ 0 & 0 & 1.0 \end{bmatrix}$$


 $(0.05)(0.93) + (0.90)(0.05) + (0.0)(0.02)$


Simulate the Cohort over Time

- Initial cohort distribution:

$$x_0 = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} \quad (\text{everyone starts healthy})$$

- Cohort distribution at next time step is calculated through matrix multiplication

$$\begin{matrix} & x_2 & \\ \begin{bmatrix} 0.87 & 0.09 & 0.04 \end{bmatrix} & = & \begin{matrix} x_1 \\ \begin{bmatrix} 0.93 & 0.05 & 0.02 \end{bmatrix} \end{matrix} \begin{bmatrix} 0.93 & 0.05 & 0.02 \\ 0 & 0.90 & 0.10 \\ 0 & 0 & 1.0 \end{bmatrix} \end{matrix}$$

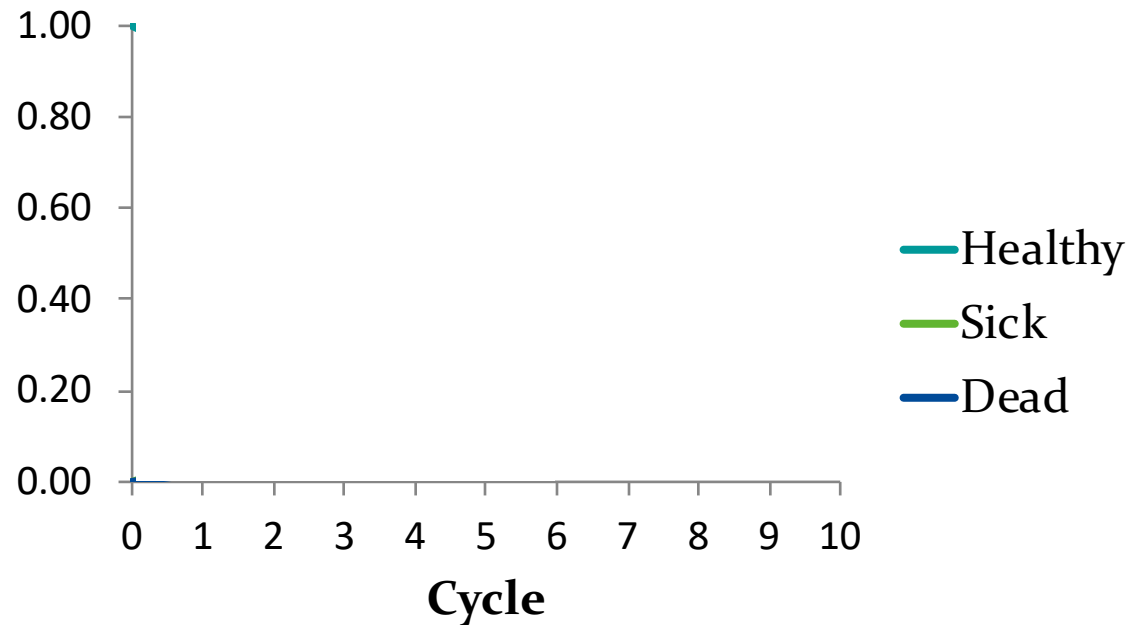

 $(0.02)(0.93) + (0.10)(0.05) + (1.0)(0.02)$

Simulate the Cohort over Time

- Cohort distribution over time:

$$x_4 = [0.75 \quad 0.15 \quad 0.10]$$

- Cohort trace:



Calculating Model Outcomes

Expected outcomes

- Expected remaining life-years, quality-adjusted life-years (QALYs), costs
 - Generally calculated over lifetime
- Multiply cohort distribution by state-specific values to calculate expected value at each cycle
- Sum expected values over time
 - discount if desired

Remaining Life-Expectancy

- State-specific values: 1 if alive, 0 if dead

State values: 1.0 1.0 0.0

Time	Healthy	Sick	Dead	E[LYs]
0	1.0	0.0	0.0	--
1	0.93	0.05	0.02	
2	0.87	0.09	0.04	
3	0.80	0.13	0.07	
	

Total life years: 21.0 years

Remaining QALE

- Remaining quality-adjusted life-expectancy (QALE)
- State-specific values: utility of that state

State values: 0.8 0.5 0.0

Time	Healthy	Sick	Dead	E[QALYs]
0	1.0	0.0	0.0	--
1	0.93	0.05	0.02	
2	0.87	0.09	0.04	
3	0.80	0.13	0.07	
	

Total QALYs: 14.7 QALYs

Remaining costs

- Remaining lifetime health care costs
- State-specific values: cost of spending one cycle in that health state

State values: \$400 \$1,000 \$0

Time	Healthy	Sick	Dead	E[costs]
0	1.0	0.0	0.0	--
1	0.93	0.05	0.02	
2	0.87	0.09	0.04	
3	0.80	0.13	0.07	
	

Total health care costs: \$12,531

Discounting

- In cost-effectiveness analysis, costs and QALYs are discounted at some rate, r
- At each cycle t , multiply expected outcome by the discount factor: $\frac{1}{(1+r)^t}$

State values:

0.8

0.5

0.0

Time	Healthy	Sick	Dead	E[QALYs]
0	1.0	0.0	0.0	--
1	0.93	0.05	0.02	0.77
2	0.87	0.09	0.04	0.74
3	0.80	0.13	0.07	0.71
	

Sum

* $1/(1+r)$

* $1/(1+r)^2$

* $1/(1+r)^3$



Total QALYs: 14.7 QALYs

Total *discounted* (3% per year) QALYs: 10.2 QALYs