# PSA: Markov Sick-Sicker model in R

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- Jalal H, Pechlivanoglou P, Krijkamp E, Alarid-Escudero F, Enns E, Hunink MG. An Overview of R in Health Decision Sciences. Med Decis Making. 2017; 37(3): 735-746. https://journals.sagepub.com/doi/abs/10.1177/0272989X16686559
- Krijkamp EM, Alarid-Escudero F, Enns EA, Jalal HJ, Hunink MGM, Pechlivanoglou P. Microsimulation modeling for health decision sciences using R: A tutorial. Med Decis Making. 2018;38(3):400–22. https://journals.sagepub.com/doi/abs/10.1177/0272989X18754513
- Krijkamp EM, Alarid-Escudero F, Enns E, Pechlivanoglou P, Hunink MM, Jalal H. A Multidimensional Array Representation of State-Transition Model Dynamics. Med Decis Making. 2020 Online first. https://doi.org/10.1177/0272989X19893973
- Alarid-Escudero, F., Krijkamp, E. M., Enns, E. A., Hunink, M. G. M., Pechlivanoglou, P., & Jalal, H. (2020). Cohort state-transition models in R: From conceptualization to implementation. arXiv:2001.07824v1, 1–31. http://arxiv.org/abs/2001.07824
- Alarid-Escudero, F., Enns, E. A., Kuntz, K. M., Michaud, T. L., & Jalal, H. (2019). "Time Traveling
  Is Just Too Dangerous" But Some Methods Are Worth Revisiting: The Advantages of Expected Loss
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```
rm(list = ls())  # clear memory (removes all the variables from the workspace)
```

# 01 Load packages

## 02 Load functions

```
source("Functions.R")
```

# 03 Input model parameters

```
# Strategy names
v_names_str <- c("No Treatment", "Treatment")</pre>
# Number of strategies
n_str <- length(v_names_str)</pre>
# Markov model parameters
    <- 25
age
                                 # age at baseline
max_age <- 55
                                 # maximum age of follow up
     <- max_age - age
                                # time horizon, number of cycles
n_t
       <- c("H", "S1", "S2", "D") # the 4 states of the model: Healthy (H),
v_n
                                 # Sick (S1), Sicker (S2), Dead (D)
      <- length(v n)
n s
                                 # number of health states
# Transition probabilities (per cycle)
     <- 0.005 # probability to die when healthy
p_HD
p_HS1
     <- 0.15
                         # probability to become sick when healthy
                         # probability to become healthy when sick
      <- 0.5
p_S1H
p_S1S2 <- 0.105
                        # probability to become sicker when sick
hr_S1 <- 3
                         # hazard ratio of death in sick vs healthy
hr_S2 <- 10
                         # hazard ratio of death in sicker vs healthy
       <- - log(1 - p_HD) # rate of death in healthy
r HD
p_S1D <- 1 - exp(-r_S1D) # probability to die in sick
p_S2D <- 1 - exp(-r_S2D) # probability to die in sicker
```

```
# Cost and utility inputs
       <- 2000
                 # cost of remaining one cycle in the healthy state
c_H
c S1
        <- 4000 # cost of remaining one cycle in the sick state
c S2
        <- 15000 # cost of remaining one cycle in the sicker state
c_trt <- 12000 # cost of treatment(per cycle)</pre>
       <- 0  # cost of being in the death state
<- 1  # utility when healthy
c D
u_H
u_S1 <- 0.75 # utility when sick
      <- 0.5 # utility when sicker
<- 0 # utility when dead
u S2
\mathtt{u}_{\mathtt{D}}
u_trt <- 0.95 # utility when being treated
# Discounting factor
       <- 0.03 # equal discount of costs and QALYs by 3%
\# calculate discount weights for costs for each cycle based on discount rate d_c
v_dwc <-1 / (1 + d_r) ^ (0:n_t)
\# calculate discount weights for effectiveness for each cycle based on discount rate d_e
v_dwe <-1 / (1 + d_r) ^ (0:n_t)
```

## 04 Define and initialize matrices and vectors

#### 04.1 Cohort trace

```
# create the markov trace matrix M capturing the proportion of the cohort
# in each state at each cycle
m_M_notrt <- m_M_trt <- matrix(NA,</pre>
                                        = n_t + 1, ncol = n_s,
                                dimnames = list(paste("cycle", 0:n_t, sep = " "), v_n))
head(m_M_notrt) # show first 6 rows of the matrix
##
            H S1 S2 D
## cycle O NA NA NA NA
## cycle 1 NA NA NA NA
## cycle 2 NA NA NA NA
## cycle 3 NA NA NA NA
## cycle 4 NA NA NA NA
## cycle 5 NA NA NA NA
# The cohort starts as healthy
m_M_notrt[1, ] <- m_M_trt[1, ] <- c(1, 0, 0, 0) # initiate first cycle of cohort trace
```

#### 04.2 Transition probability matrix

```
## S1 0 0 0 0
## S2 0 0 0 0
## D 0 0 0 0
```

Fill in the transition probability matrix:

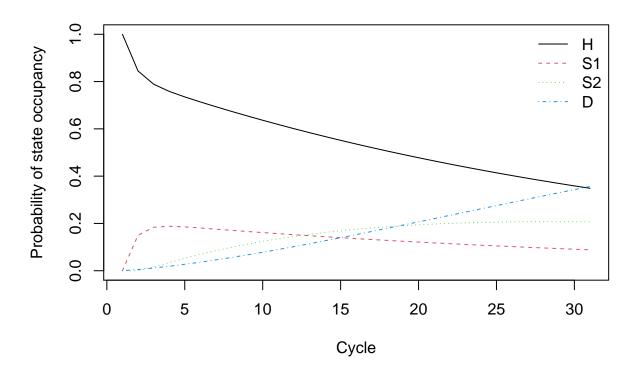
```
# from Healthy
m_P_notrt["H", "H"]
                     <-1 - (p_HS1 + p_HD)
m_P_notrt["H", "S1"] <- p_HS1</pre>
m P notrt["H", "D"]
                       <- p HD
# from Sick
m_P_notrt["S1", "H"] <- p_S1H</pre>
m_P_notrt["S1", "S1"] <- 1 - (p_S1H + p_S1S2 + p_S1D)
m_P_notrt["S1", "S2"] <- p_S1S2</pre>
m_P_notrt["S1", "D"] <- p_S1D</pre>
# from Sicker
m_P_notrt["S2", "S2"] <- 1 - p_S2D</pre>
m_P_notrt["S2", "D"] <- p_S2D</pre>
# from Dead
m_P_notrt["D", "D"] <- 1</pre>
# check rows add up to 1
rowSums(m P notrt)
## H S1 S2 D
## 1 1 1 1
# create transition probability matrix for treatment same as no treatment
m_P_trt <- m_P_notrt</pre>
```

#### 05 Run Markov model

# 06 Compute and Plot Epidemiological Outcomes

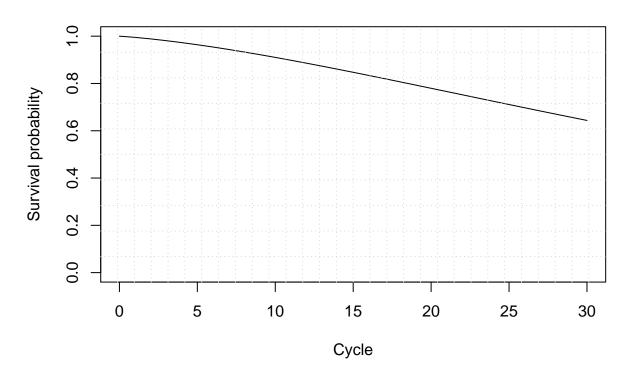
#### 06.1 Cohort trace

## **Cohort Trace**



## 06.2 Overall Survival (OS)

# **Overall Survival**

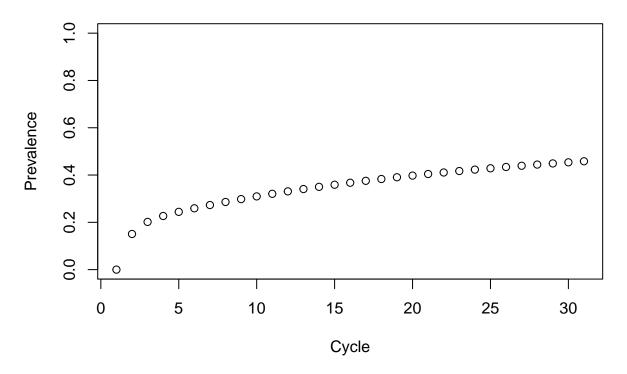


# 06.2.1 Life Expectancy (LE)

```
v_le <- sum(v_os_notrt) # summing probablity of OS over time (i.e. life expectancy)</pre>
```

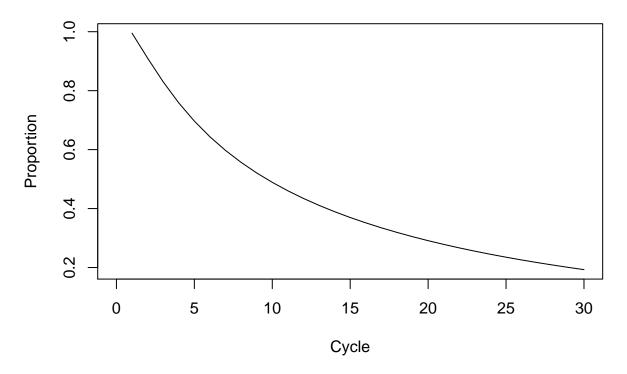
# 06.3 Disease prevalence

# Disease prevalence



# 06.4 Proportion of sick in S1 state

# Proportion of sick in S1 state



# 07 Compute Cost-Effectiveness Outcomes

```
# Vectors with costs and utilities by treatment

v_u_notrt <- c(u_H, u_S1, u_S2, u_D)

v_u_trt <- c(u_H, u_trt, u_S2, u_D)

v_c_notrt <- c(c_H, c_S1, c_S2, c_D)

v_c_trt <- c(c_H, c_S1 + c_trt, c_S2 + c_trt, c_D)
```

## 07.1 Mean Costs and QALYs for Treatment and NO Treatment

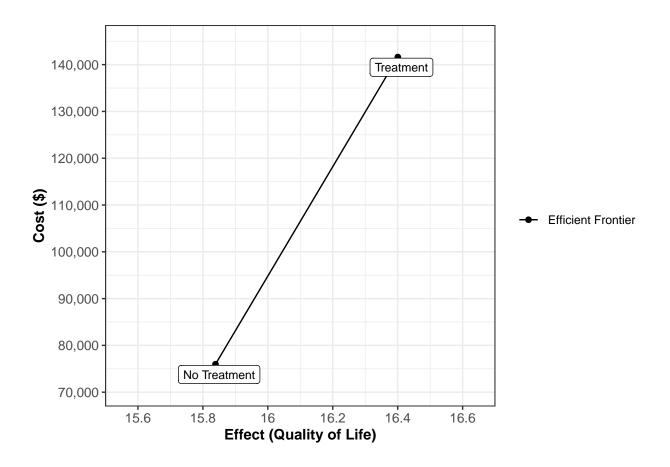
## 07.2 Discounted Mean Costs and QALYs

#### 07.3 Compute ICERs of the Markov model

```
## Strategy Cost Effect Inc_Cost Inc_Effect ICER Status
## 1 No Treatment 75976.15 15.83885 NA NA NA NA
## 2 Treatment 141623.03 16.40041 65646.88 0.5615578 116901.4 ND
```

#### 07.4 Plot frontier of the Markov model

```
plot(df_cea, effect_units = "Quality of Life", xlim = c(15.6, 16.6))
```



# 08 Deterministic Sensitivity Analysis

#### 08.1 List of input parameters

Create list 1\_params\_all with all input probabilities, cost and utilities.

```
l_params_all <- list(</pre>
         = 0.005, # probability to die when healthy
 p_HD
         = 0.15,
                   # probability to become sick when healthy
 p_HS1
                   # probability to become healthy when sick
 p S1H
       = 0.5,
 p_S1S2 = 0.105, # probability to become sicker when sick
 hr_S1
         = 3,
                   # hazard ratio of death in sick vs healthy
 hr_S2 = 10,
                   # hazard ratio of death in sicker vs healthy
 с Н
         = 2000, # cost of remaining one cycle in the healthy state
         = 4000, # cost of remaining one cycle in the sick state
 c S1
         = 15000, # cost of remaining one cycle in the sicker state
 c S2
 c_trt
         = 12000, # cost of treatment(per cycle)
 c_D
         = 0,
                   # cost of being in the death state
                   # utility when healthy
 u_H
         = 1,
 u_S1
         = 0.75,
                   # utility when sick
         = 0.5,
                   # utility when sicker
 u_S2
 u_D
         = 0,
                   # utility when dead
 u_trt
         = 0.95, # utility when treated
         = 0.03,
                   # discount factor for effectiveness
 d_e
         = 0.03
 d_c
                   # discount factor for costs
```

```
# store the parameter names into a vector
v_names_parms <- names(l_params_all)</pre>
```

#### 08.2 Load Sick-Sicker Markov model function

```
source("Functions_markov_sick-sicker.R")
# Test function
calculate_ce_out(l_params_all)

## Strategy Cost Effect NMB
## 1 No Treatment 75976.15 15.83885 1507909
## 2 Treatment 141623.03 16.40041 1498418
```

#### 08.3 One-way sensitivity analysis (OWSA)

```
options(scipen = 999) # disabling scientific notation in R
# dataframe containing all parameters, their basecase values, and the min and
# max values of the parameters of interest
df_params_owsa <- data.frame(pars = c("p_S1S2", "c_trt", "u_S1", "u_trt"),</pre>
                            min = c(0.05, 6000, 0.65, 0.80), # min parameter values
                            \max = c(0.155, 18000, 0.85, 0.98) # max parameter values
owsa nmb <- run owsa det(params range
                                          = df params owsa,
                                                              # dataframe with parameters for owsa
                         params_basecase = l_params_all,
                                                             # list with all parameters
                         nsamp
                                          = 100,
                                                              # number of parameter values
                         FUN
                                         = calculate_ce_out, # function to compute outputs
                         outcomes
                                         = c("NMB"), # output to do the OWSA on
                         strategies
                                          = v_names_str,
                                                             # names of the strategies
                         n_wtp
                                          = 120000)
                                                             # extra argument to pass to FUN
```

## Warning: strategy name 'No Treatment' was converted to 'No.Treatment' for ## compatibility. See ?make.names

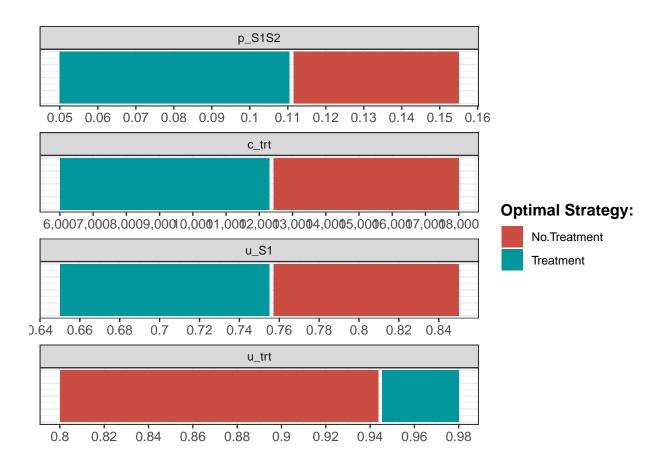
#### 08.3.1 Plot OWSA

```
plot(owsa_nmb, txtsize = 16, n_x_ticks = 5, n_y_ticks = 3,
    facet_scales = "free") +
    theme(legend.position = "bottom",
        axis.text.x = element_text(angle = 45, vjust = 0.5))
```



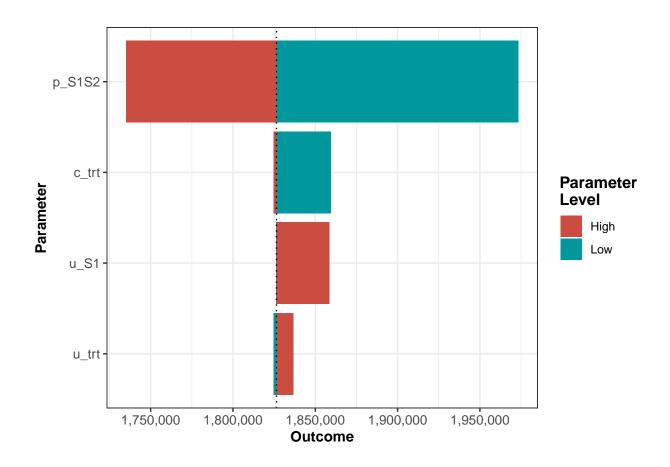
08.3.2 Optimal strategy with OWSA

owsa\_opt\_strat(owsa = owsa\_nmb)



# 08.3.3 Tornado plot

owsa\_tornado(owsa = owsa\_nmb)

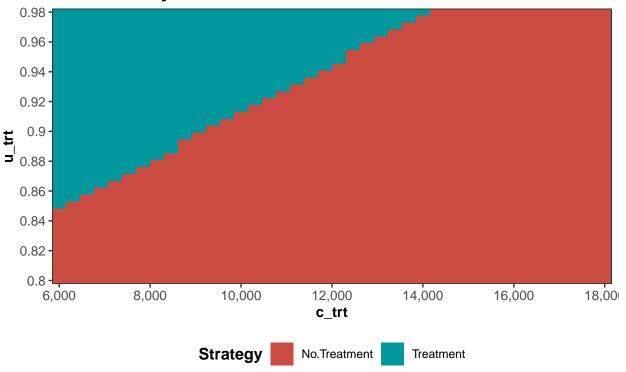


## 08.4 Two-way sensitivity analysis (TWSA)

```
# dataframe containing all parameters, their basecase values, and the min and
# max values of the parameters of interest
df_params_twsa <- data.frame(pars = c("c_trt", "u_trt"),</pre>
                             min = c(6000, 0.80), # min parameter values
                             max = c(18000, 0.98) # max parameter values
                                         = df_params_twsa,
                                                              # dataframe with parameters for twsa
twsa_nmb <- run_twsa_det(params_range</pre>
                         params_basecase = l_params_all,
                                                             # list with all parameters
                                         = 40,
                                                              # number of parameter values
                         nsamp
                                         = calculate_ce_out, # function to compute outputs
                         FUN
                                                             # output to do the twsa on
                         outcomes
                                         = c("NMB"),
                                                            # names of the strategies
                         strategies
                                         = v names str,
                                         = 120000)
                                                             # extra argument to pass to FUN
                         n_wtp
```

#### 08.4.1 Plot TWSA

# Two-way sensitivity analysis Net monetary benefit



# 09 Probabilistic Sensitivity Analysis (PSA)

```
# Function to generate PSA input dataset
generate_psa_params <- function(n_sim = 1000, seed = 071818){</pre>
  set.seed(seed) # set a seed to be able to reproduce the same results
  df_psa <- data.frame(</pre>
    # Transition probabilities (per cycle)
          = rbeta(n_sim, 30, 170), # probability to become sick when healthy
           = rbeta(n_sim, 60, 60) , # probability to become healthy when sick
   p_S1H
   p_S1S2 = rbeta(n_sim, 84, 716) , # probability to become sicker when sick
           = rbeta(n_sim, 10, 1990), # probability to die when healthy
   p_HD
   hr_S1
           = rlnorm(n_sim, log(3), 0.01), # rate ratio of death in S1 vs healthy
           = rlnorm(n_sim, log(10), 0.02), # rate ratio of death in S2 vs healthy
   hr_S2
    # State rewards
    # Costs
    c_H = rgamma(n_sim, shape = 100, scale = 20) , # cost of remaining one cycle in state H
    c_S1 = rgamma(n_sim, shape = 177.8, scale = 22.5), # cost of remaining one cycle in state S1
    c_S2 = rgamma(n_sim, shape = 225, scale = 66.7) , # cost of remaining one cycle in state S2
    c_Trt = rgamma(n_sim, shape = 73.5, scale = 163.3), # cost of treatment (per cycle)
   c D = 0
                                                      , # cost of being in the death state
    # Utilities
   u_H = rbeta(n_sim, shape1 = 200, shape2 = 3) , # utility when healthy
   u_S1 = rbeta(n_sim, shape1 = 130, shape2 = 45) , # utility when sick
```

```
u_S2 = rbeta(n_sim, shape1 = 230, shape2 = 230), # utility when sicker
   u_D = 0
                                              , # utility when dead
   u_Trt = rbeta(n_sim, shape1 = 300, shape2 = 15) , # utility when being treated
   d e = 0.03, # discount factor for effectiveness
       = 0.03 # discount factor for costs
   d_c
   return(df_psa)
}
# Try it
generate_psa_params(10)
         p HS1
                  p S1H
                           p_S1S2
                                       p_HD
                                               hr S1
                                                       hr S2
## 1 0.15132349 0.4820445 0.08485456 0.003973762 3.055410 10.148768 1887.449
## 2 0.12522484 0.4392546 0.11683397 0.004813328 3.005252 9.928044 1966.291
## 3 0.15673720 0.5311020 0.11925841 0.004491843 2.982392 10.021998 2101.059
## 4 0.11276858 0.4493088 0.09393848 0.010646056 2.981918 10.167486 1728.664
## 6 0.20088601 0.4580278 0.10403415 0.004058637 2.971345 10.012715 1969.304
## 7 0.14347796 0.5492245 0.09294048 0.005658464 2.989181 10.001860 1641.192
## 8 0.13149270 0.4899896 0.12838861 0.007354969 2.984080 9.919332 2276.032
## 9 0.15593267 0.5705602 0.09237907 0.004162148 2.988974 10.142331 1717.867
## 10 0.09824029 0.5127637 0.10683148 0.002141140 2.968692 9.749869 2132.549
##
                c S2
                        c Trt c D
                                                       u S2 u D
         c S1
                                      u H
                                              u S1
## 1 4173.600 14607.88 10673.74 0 0.9829120 0.7252701 0.4757751 0 0.9435773
## 2 4188.264 14714.84 13380.61 0 0.9693926 0.7610392 0.4591563
                                                            0 0.9501528
                                                            0 0.9670124
## 3 4578.682 16664.54 12522.58 0 0.9910033 0.7076433 0.5032719
## 4 3395.922 15669.87 13833.65 0 0.9928844 0.7124119 0.5132492 0 0.9649128
## 5 3878.562 15302.70 13154.65 0 0.9748972 0.7344718 0.4895693 0 0.9325969
## 6 3970.522 15303.55 13914.95 0 0.9832065 0.7169406 0.4628316
                                                            0 0.9456881
## 8 3781.956 15614.73 12070.99 0 0.9727485 0.7582961 0.4727426 0 0.9390084
## 9 3448.146 15401.50 11760.17 0 0.9836544 0.7648816 0.4851046 0 0.9537789
0 0.9577708
##
      d_e d_c
## 1 0.03 0.03
## 2 0.03 0.03
## 3 0.03 0.03
## 4 0.03 0.03
## 5 0.03 0.03
## 6 0.03 0.03
## 7 0.03 0.03
## 8 0.03 0.03
## 9 0.03 0.03
## 10 0.03 0.03
# Number of simulations
n_sim <- 1000
# Generate PSA input dataset
df_psa_input <- generate_psa_params(n_sim = n_sim)</pre>
# First six observations
head(df_psa_input)
```

p\_HD

hr\_S1

hr\_S2

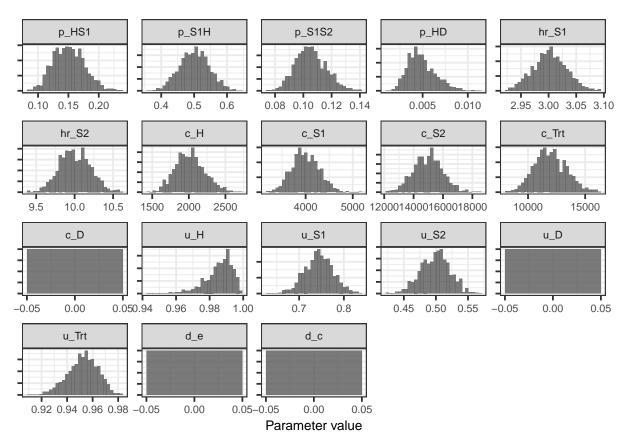
c\_H

##

p\_HS1

p\_S1H p\_S1S2

```
## 1 0.1513235 0.5617628 0.1033817 0.006269864 2.962948 9.820157 1658.447
## 2 0.1252248 0.4380343 0.1152594 0.005371105 3.014068 10.100813 2050.281
## 3 0.1567372 0.5458102 0.1274505 0.004301015 3.018588 9.825985 2094.425
## 4 0.1127686 0.5106953 0.1015742 0.005319105 2.979709 9.697122 2135.411
## 5 0.1429295 0.5277308 0.1020384 0.004225209 3.025490 10.254387 1665.257
## 6 0.2008860 0.5252364 0.1030676 0.008319206 2.979175 10.366400 2049.794
                     c Trt c D
       c S1
              c_S2
                                  u H
                                          u S1
                                                  u S2 u D
                                                            u Trt
0 0.9731185
0 0.9598494
## 4 3726.226 16084.60 10072.741   0 0.9919197 0.7065183 0.5118351   0 0.9357411
0 0.9635901
## 6 3927.001 17060.57 9581.097 0 0.9813330 0.7766346 0.5035428 0 0.9506926
##
    d_e d_c
## 1 0.03 0.03
## 2 0.03 0.03
## 3 0.03 0.03
## 4 0.03 0.03
## 5 0.03 0.03
## 6 0.03 0.03
# Histogram of parameters
ggplot(reshape2::melt(df_psa_input, variable.name = "Parameter",
                 value.name = "Parameter value"),
                 aes(x = `Parameter value`)) +
                 facet_wrap(~Parameter, scales = "free") +
                 geom_histogram(aes(y = ..density..), alpha = 0.8) +
                 scale_x_continuous(breaks = number_ticks(2)) +
                 vlab("") +
                 theme_bw(base_size = 14) +
                 theme(axis.text.y = element_blank(), text = element_text(size=10))
```



## 09.1 Conduct probabilistic sensitivity analysis

```
# Run Markov model on each parameter set of PSA input dataset
for(i in 1:n_sim){
    l_psa_input <- update_param_list(l_params_all, df_psa_input[i,])
    df_out_psa <- calculate_ce_out(l_psa_input)
    df_c[i, ] <- df_out_psa*Cost
    df_e[i, ] <- df_out_psa*Effect
# Display simulation progress
    if(i/(n_sim/10) == round(i/(n_sim/10), 0)) { # display progress every 10%
        cat('\r', paste(i/n_sim * 100, "% done", sep = " "))
    }
}</pre>
```

## 10 % done 20 % done 30 % done 40 % done 50 % done 60 % done 70 % done 80 % done 90 % done 100 % done  $\frac{100}{100}$  % done  $\frac{100}{1$ 

## 09.2 Create PSA object for dampack

## 09.2.1 Save PSA objects

```
save(df_psa_input, df_c, df_e, v_names_str, n_str, l_psa,
    file = "markov_sick-sicker_PSA_dataset.RData")
```

#### 09.3 Create probabilistic analysis graphs

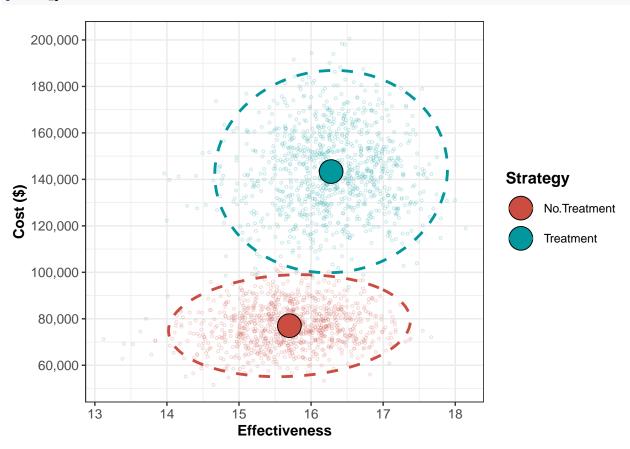
```
load(file = "markov_sick-sicker_PSA_dataset.RData")
```

Vector with willingness-to-pay (WTP) thresholds.

```
v_{wtp} \leftarrow seq(0, 200000, by = 10000)
```

#### 09.3.1 Cost-Effectiveness Scatter plot

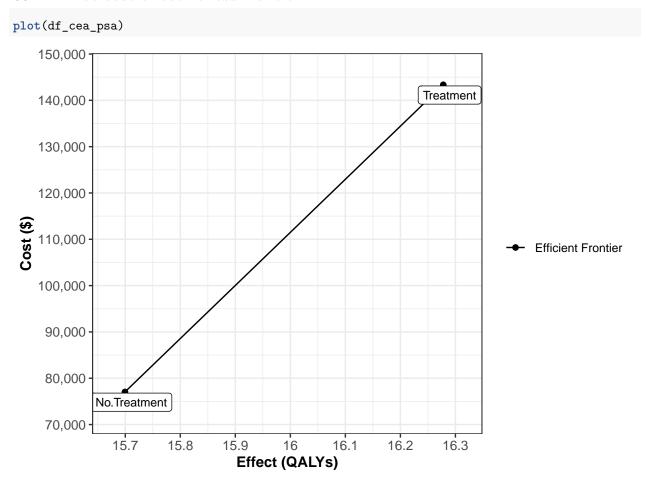
```
plot(l_psa)
```



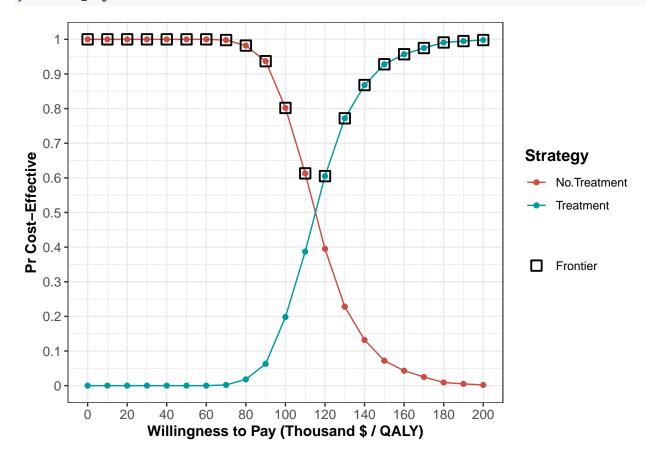
## 09.4 Conduct CEA with probabilistic output

```
# Compute expected costs and effects for each strategy from the PSA
df_out_ce_psa <- summary(1_psa)</pre>
# Calculate incremental cost-effectiveness ratios (ICERs)
df_cea_psa <- calculate_icers(cost</pre>
                                        = df_out_ce_psa$meanCost,
                              effect = df_out_ce_psa$meanEffect,
                              strategies = df_out_ce_psa$Strategy)
df_cea_psa
                      Cost Effect Inc_Cost Inc_Effect
##
                                                            ICER Status
        Strategy
## 1 No.Treatment 77033.8 15.69937
                                                              NA
                                                                     ND
       Treatment 143354.8 16.27791
                                       66321 0.5785414 114634.8
                                                                     ND
# Save CEA table with ICERs
# As .RData
save(df_cea_psa,
     file = "markov sick-sicker probabilistic CEA results.RData")
# As .csv
write.csv(df_cea_psa,
          file = "markov_sick-sicker_probabilistic_CEA_results.csv")
```

#### 09.4.1 Plot cost-effectiveness frontier



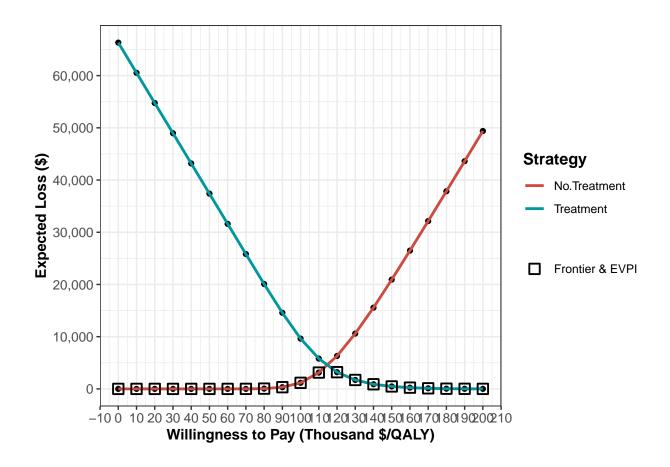
## 09.4.2 Cost-effectiveness acceptability curves (CEACs) and frontier (CEAF)



#### 09.4.3 Expected Loss Curves (ELCs)

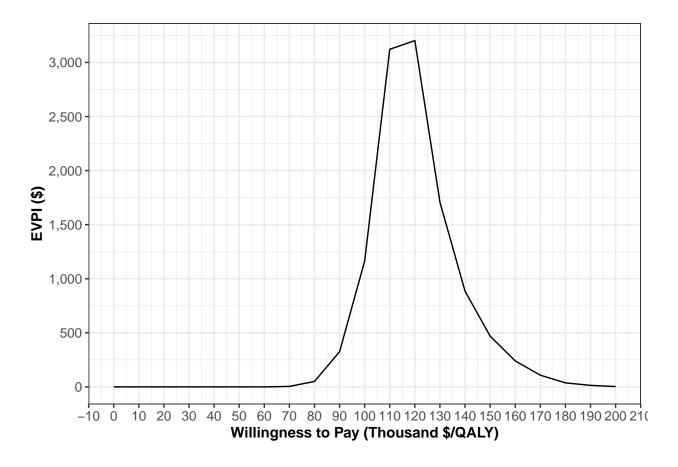
The expected loss is the the quantification of the foregone benefits when choosing a suboptimal strategy given current evidence.

```
elc_obj <- calc_exp_loss(wtp = v_wtp, psa = l_psa)
# ELC plot
plot(elc_obj, log_y = FALSE)</pre>
```



# 09.4.4 Expected value of perfect information (EVPI)

```
evpi <- calc_evpi(wtp = v_wtp, psa = 1_psa)
# EVPI plot
plot(evpi, effect_units = "QALY")</pre>
```



# 07.4.5 Expected value of partial perfect information (EVPPI)

