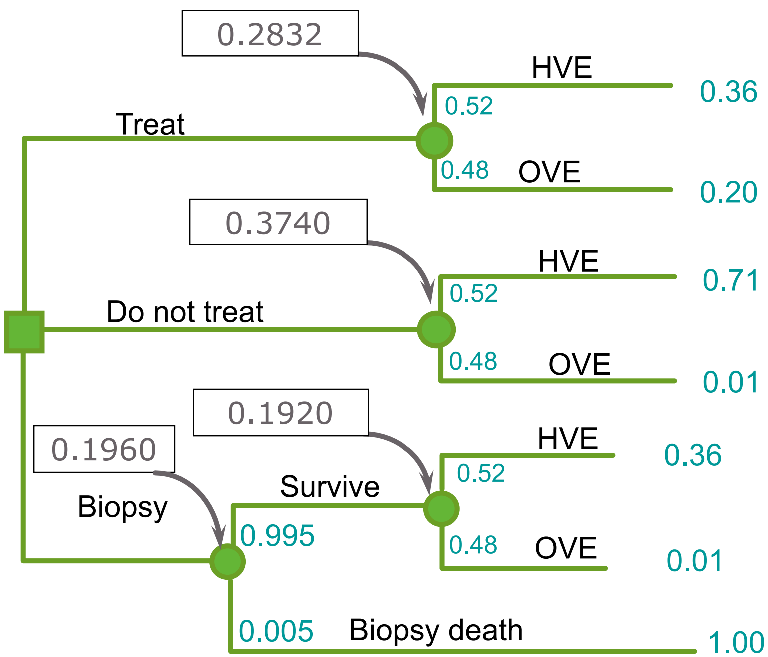
Matrix Algebra

DARTH workgroup

Why matrix algebra in decision analysis?

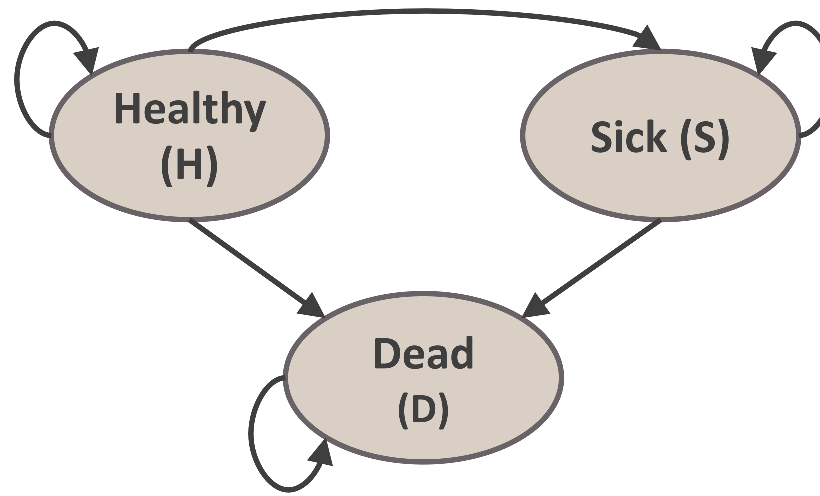
Calculations in decision analysis involve a lot of multiplying and adding one of the strengths of matrix algebra

Decision tree



Folding back method in decision trees: Involves multiplication and addition.

State-transition model



Estimation of proportion of a cohort in a specific health states also involves multiplication and addition.

people in sick =

$(p_{Sick,Sick})$*Pr(*Sick*) + $(p_{Healthy,\ Sick})$*Pr(*Healthy*) + $(p_{Dead,\ sick})$*Pr(*Dead*)

Matrix Addition and Subtraction

- Matrix addition and subtraction are ***element-wise*** operations.
- Only matrices with the same dimensions can be added/subtracted

$$
\begin{pmatrix} 1 & 1 \\ 5 & 8 \\ -2 & 3 \\ 4 & 0 \\ 1 & -6 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 9 & 1 \\ -2 & -3 \\ 3 & 1 \\ 7 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 14 & 9 \\ -4 & 0 \\ 7 & 1 \\ 8 & -4 \end{pmatrix}
$$

In R

```
> matA <- matrix(rep(1.5, 6),
+                nrow = 2,
+                ncol = 3)
>
> matB <- matrix(1:10,
+                nrow = 2,
+                ncol = 5)
>
> matC <- matrix(rep(1:5, 2),
+                nrow = 2,
+                ncol = 5)
>
> matB + matC
     [,1] [,2] [,3] [,4] [,5]
[1,]    2    6   10    9   13
[2,]    4    8    7   11   15
> matA + matB
Error in matA + matB : non-conformable arrays
```

This works similar for subtractions

Matrix Addition and Subtraction (2)

$n \times m$ $\quad$ $m \times p$

3×3 $\quad$ 3×3

$$\begin{array}{c}C\\ \begin{bmatrix} 1.50 & 0.40 & 0.1 \\ 0 & 1.70 & 0.3 \\ 0 & 0 & 2.0 \end{bmatrix}\end{array} = \begin{array}{c}A\\ \begin{bmatrix} 0.75 & 0.20 & 0.05 \\ 0 & 0.85 & 0.15 \\ 0 & 0 & 1.0 \end{bmatrix}\end{array} + \begin{array}{c}B\\ \begin{bmatrix} 0.75 & 0.20 & 0.05 \\ 0 & 0.85 & 0.15 \\ 0 & 0 & 1.0 \end{bmatrix}\end{array}$$

In R: `m_A + m_B`

This works similar for subtractions

Matrix Multiplication

Multiple a matrix by a number
Each element in the matrix multiplied with that number

$$2 \times \begin{pmatrix} 8 & 0 & 2 & 2 \\ -2 & 5 & 3 & -5 \\ 5 & 7 & -3 & 0 \\ 3 & -1 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 16 & 0 & 4 & 4 \\ -4 & 10 & 6 & -10 \\ 10 & 14 & -6 & 0 \\ 6 & -2 & 2 & -4 \end{pmatrix}$$

In R

```
> matB <- matrix(1:10, nrow = 2,ncol = 5)
> matB
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    3    5    7    9
[2,]    2    4    6    8   10
```

→

```
> 2 * matB
     [,1] [,2] [,3] [,4] [,5]
[1,]    2    6   10   14   18
[2,]    4    8   12   16   20
```

Matrix Multiplication (2)

Matrix Element-wise Multiplication

$$\underset{C}{\begin{bmatrix} 0.56 & 0.04 & .0025 \\ 0 & 0.72 & .023 \\ 0 & 0 & 1 \end{bmatrix}} = \underset{A}{\begin{bmatrix} 0.75 & 0.20 & 0.05 \\ 0 & 0.85 & 0.15 \\ 0 & 0 & 1.0 \end{bmatrix}} * \underset{B}{\begin{bmatrix} 0.75 & 0.20 & 0.05 \\ 0 & 0.85 & 0.15 \\ 0 & 0 & 1.0 \end{bmatrix}}$$

In R: `m_A * m_B`

This works similar for divisions

Matrix Multiplication (3)

Multiple a matrix by a matrix

$$\begin{pmatrix} 8 & 0 & 2 & 2 \\ -2 & 5 & 3 & 1 \end{pmatrix} \times \begin{pmatrix} 8 & 0 & 2 \\ -2 & 5 & 3 \\ 5 & 7 & -3 \\ 3 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 80 & 12 & 12 \\ -8 & 45 & 3 \end{pmatrix}$$

2×4 $\quad$ 4×3 $\quad$ 2×3

Matrix multiplication requires the first matrix to have the same number of *columns* and the number of *rows* in the second matrix

$= 8 \times 8 + 0 \times (-2) + 2 \times 5 + 2 \times 3 = 80$

$= 8 \times 0 + 0 \times 5 + 2 \times 7 + 2 \times (-1) = 12$

Matrix Multiplication in R

- The standard multiplication operator in R * gives *element-wise* multiplication

```
> matB
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    3    5    7    9
[2,]    2    4    6    8   10
> matC
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    3    5    2    4
[2,]    2    4    1    3    5
```

```
> matB * matC
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    9   25   14   36
[2,]    4   16    6   24   50
```

- Matrix multiplication is achieved using the %*% operator

```
> matD <- matrix(c(8, -2, 0, 5, 2, 3, 2, 1), nrow = 2, ncol = 4)
> matE <- matrix(c(8, -2, 5, 3, 0, 5, 7, -1, 2, 3, -3, 1), nrow = 4, ncol = 3)
> matD %*% matE
     [,1] [,2] [,3]
[1,]   80   12   12
[2,]   -8   45    3
```

Matrix Transpose

- The transpose of a matrix is where the first *row* of the original matrix becomes the first *column* of the transposed matrix
- This is sometimes required to match the dimensions of matrices for calculations

$$B = \begin{pmatrix} 1 & 1 \\ 5 & 8 \\ -2 & 3 \\ 4 & 0 \\ 1 & -6 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 5 & -2 & 4 & 1 \\ 1 & 8 & 3 & 0 & -6 \end{pmatrix}$$

In R

```
> matB
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    3    5    7    9
[2,]    2    4    6    8   10
> t(matB)
     [,1] [,2]
[1,]    1    2
[2,]    3    4
[3,]    5    6
[4,]    7    8
[5,]    9   10
```