

# **Introduction to Building and Calibrating Simulation Models in R**

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Kuala Lumpur, Malaysia

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# The DARTH Workgroup

- Materials for this course were largely developed by the Decision Analysis in R for Technologies in Health (DARTH) Workgroup
- Goals: To expand knowledge in decision analysis using R and develop educational materials to empower people to construct R-based decision models.

For more information

<http://www.darthworkgroup.com/>

# Attribution and Acknowledgement

- R code provided with this course are yours to reuse and modify

```
#####
# Please cite our publications when using this code
# - Jalal H, Pechlivanoglou P, Krijkamp E, Alarid-Escudero F, Enns E, Hunink MG.
# An Overview of R in Health Decision Sciences. Med Decis Making. 2017; 37(3): 735-746.
# https://journals.sagepub.com/doi/abs/10.1177/0272989X16686559
```



Acknowledgement  
and citation  
information in  
code headers

# Cohort models

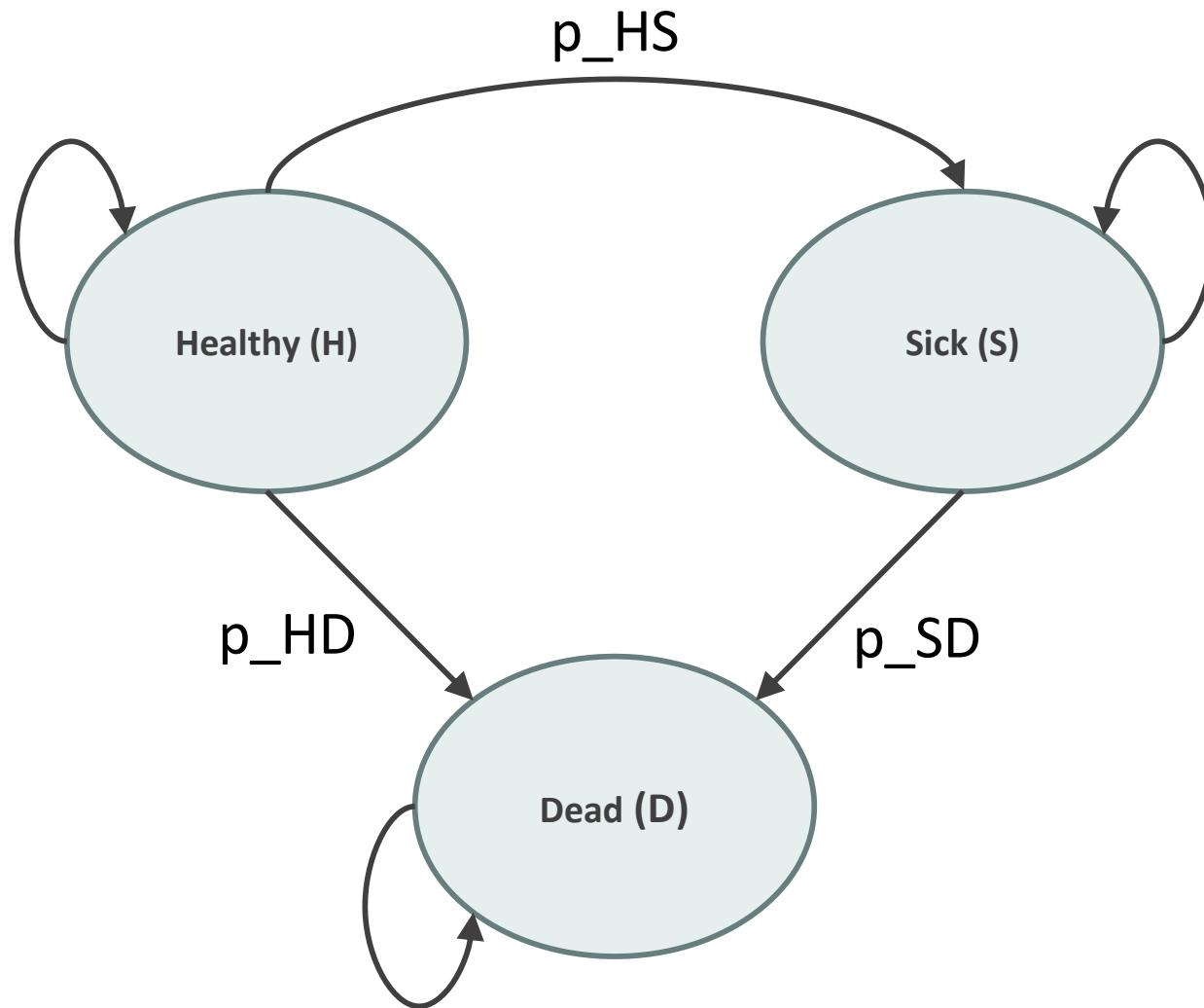
# Cohort State-Transition Models (cSTMs)

- Models where proportions of a cohort occupy states at each point in time (e.g., healthy, sick, stable, progressed, dead).
- Transitions allowed between states with some probability.
- Transitions occur in cycles (months, years etc).
- Each state associated with a value associated with a model outcome (\$, utility).
- Markov assumption: no “memory” within states.

# Building a cohort State-Transition Model

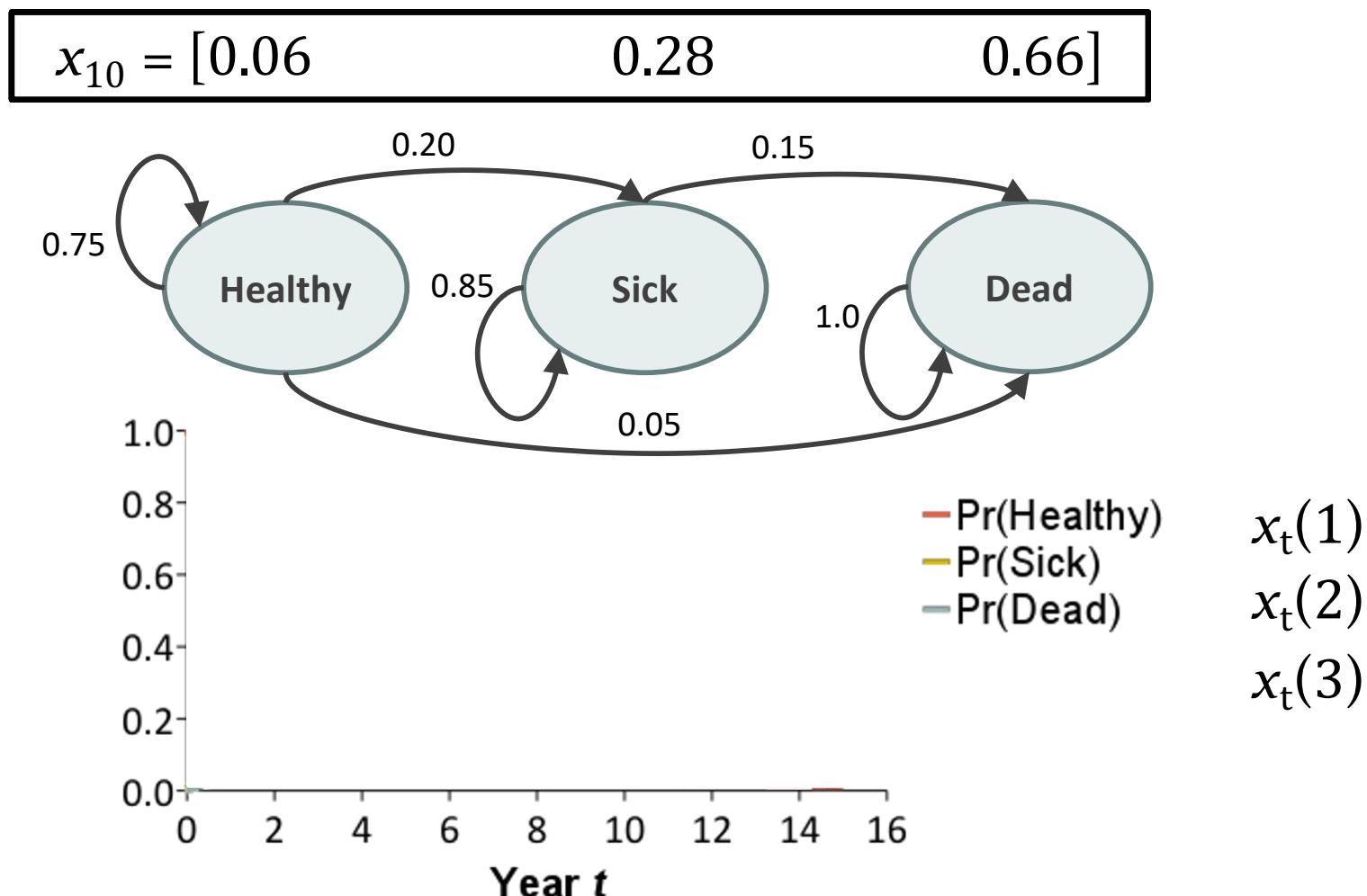
- Determine health states
- Determine transitions
- Choose cycle length
- Estimate transition probabilities
- Estimate state utilities and costs per cycle
- Calculate
- Sensitivity analyses

# Three-State Model



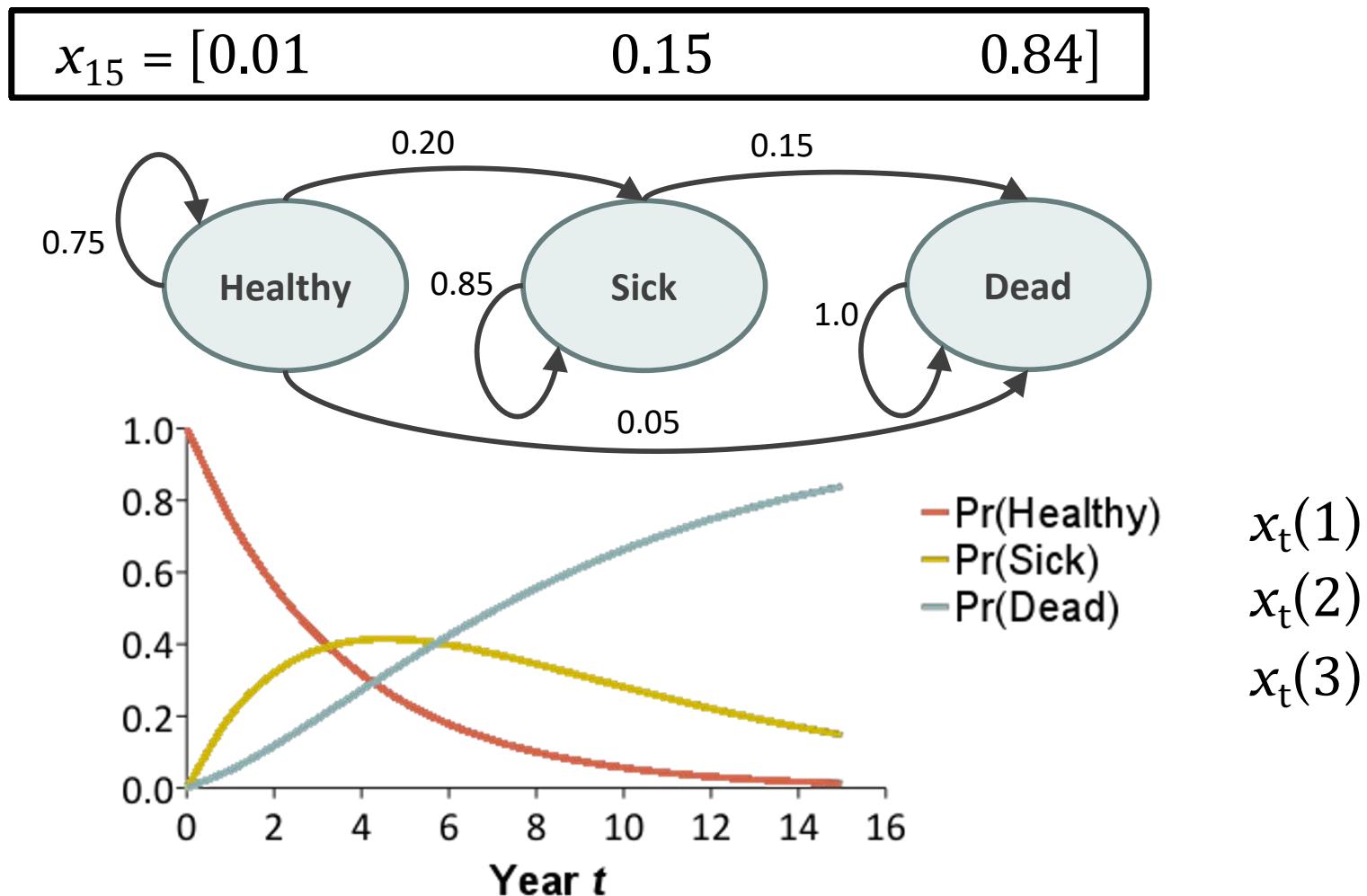
# Trace the Cohort Through Time

- Reflects the distribution of a cohort of patients over a set of health states over time



# Trace the Cohort Through Time

- Reflects the distribution of a cohort of patients over a set of health states over time



# Model as a Transition Matrix

- Summarize transition probabilities as a matrix

		To			
		Healthy	Sick	Dead	
From:	Healthy	0.75	0.20	0.05	
	Sick	0	0.85	0.15	= P
	Dead	0	0	1.0	

- Cohort distribution at next time step calculated through matrix multiplication

$$\begin{bmatrix} \text{--- } x_{t+1} \text{ ---} \end{bmatrix} = \begin{bmatrix} \text{--- } x_t \text{ ---} \end{bmatrix} \begin{bmatrix} & & \\ & P & \\ & & \end{bmatrix}$$

# “Running” the Model

- Summarize transition probabilities as a matrix

		To			
		Healthy	Sick	Dead	
From:	Healthy	0.75	0.20	0.05	
	Sick	0	0.85	0.15	= P
	Dead	0	0	1.0	

- Cohort distribution at next time step calculated through matrix multiplication

$$\begin{bmatrix} \text{--- } x_{t+1} \text{ ---} \end{bmatrix} = \begin{bmatrix} \text{--- } x_t \text{ ---} \end{bmatrix} \begin{bmatrix} 0.75 & 0.20 & 0.05 \\ 0 & 0.85 & 0.15 \\ 0 & 0 & 1.0 \end{bmatrix}$$

# “Running” the Model

- Summarize transition probabilities as a matrix

		To			
		Healthy	Sick	Dead	
From:	Healthy	0.75	0.20	0.05	
	Sick	0	0.85	0.15	
	Dead	0	0	1.0	

=  $P$

- Cohort distribution at next time step calculated through matrix multiplication

$$\begin{bmatrix} x_1 \\ 0.75 & 0.20 & 0.05 \end{bmatrix} = \begin{bmatrix} x_0 \\ 1.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} P \\ 0.75 & 0.20 & 0.05 \\ 0 & 0.85 & 0.15 \\ 0 & 0 & 1.0 \end{bmatrix}$$

# Markov Trace (Life-Years)

- Calculate expected remaining LE, QALE, costs
  - Multiply cohort distribution by state-specific values to calculate expected value at each time
  - Sum expected values over time (discount if desired)

Life-Years: 1.0

1.0

0.0

Time	Healthy	Sick	Dead	E[LYs]
0	1.0	0.0	0.0	--
1	0.75	0.20	0.05	
2	0.56	0.32	0.12	
3	0.42	0.38	0.19	
	...	...	...	

Total life years: 6.77 years  
(Remaining life expectancy)

Sum

\*  $1/(1+r)$

\*  $1/(1+r)^2$

\*  $1/(1+r)^3$

# Markov Trace (Costs)

- Calculate expected remaining LE, QALE, costs
  - Multiply cohort distribution by state-specific values to calculate expected value at each time
  - Sum expected values over time (discount if desired)

Costs:	\$500	\$2,500	\$0	
Time	Healthy	Sick	Dead	E[Costs]
0	1.0	0.0	0.0	--
1	0.75	0.20	0.05	
2	0.56	0.32	0.12	
3	0.42	0.38	0.19	
	...	...	...	

Total costs: \$11,557

(Total remaining lifetime costs)

14

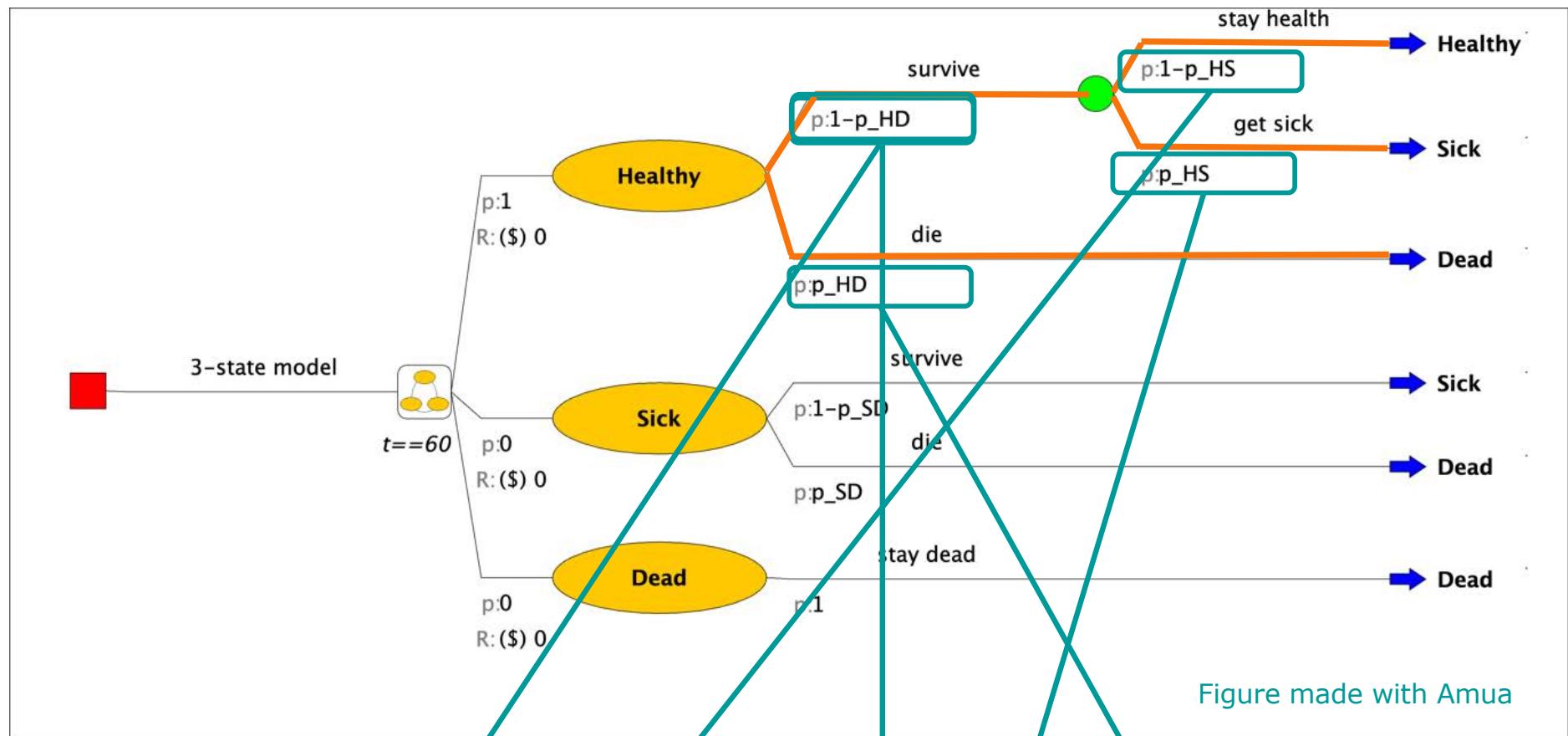
Sum

\*  $1/(1+r)$

\*  $1/(1+r)^2$

\*  $1/(1+r)^3$

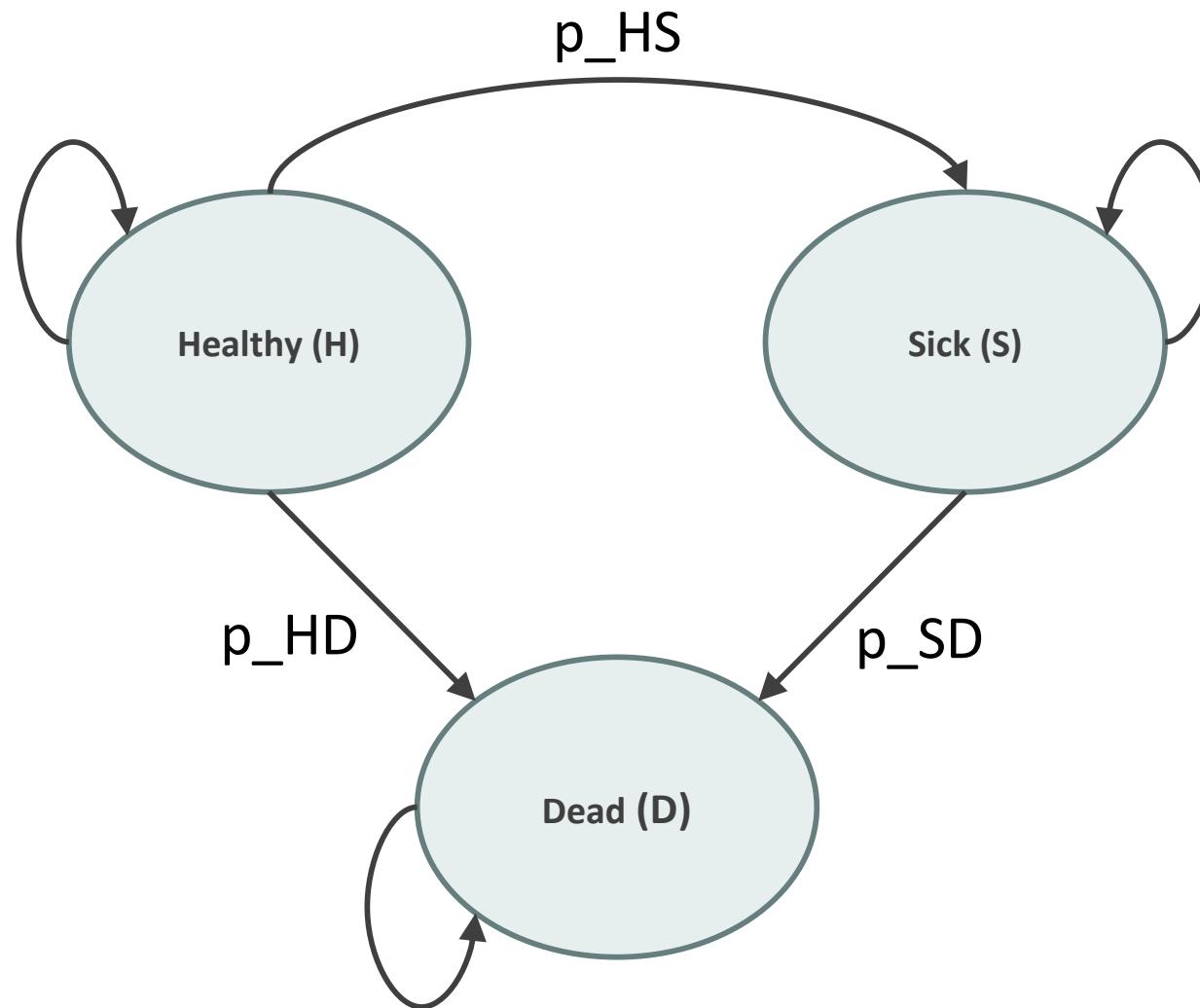
# Transition Probabilities



From/To	Healthy	Sick	Dead
Healthy	$(1-p_{HD}) * (1-p_{HS})$	$(1-p_{HD}) * p_{HS}$	$p_{HD}$
...			

# Conceptualizing the Markov model in R

# Simple state transition model in R



# Simple state transition model

## Model input:

$p_{HS}$ : transition probability from *Healthy* to *Sick*

$p_{HD}$ : transition probability *Healthy* to *Dead*

$p_{SD}$ : transition probability *Sick* to *Dead*

$c_H$ : cost of being in state *Healthy*

$c_S$ : cost of being in state *Sick*

$e_H$ : outcomes associated with state *Healthy*

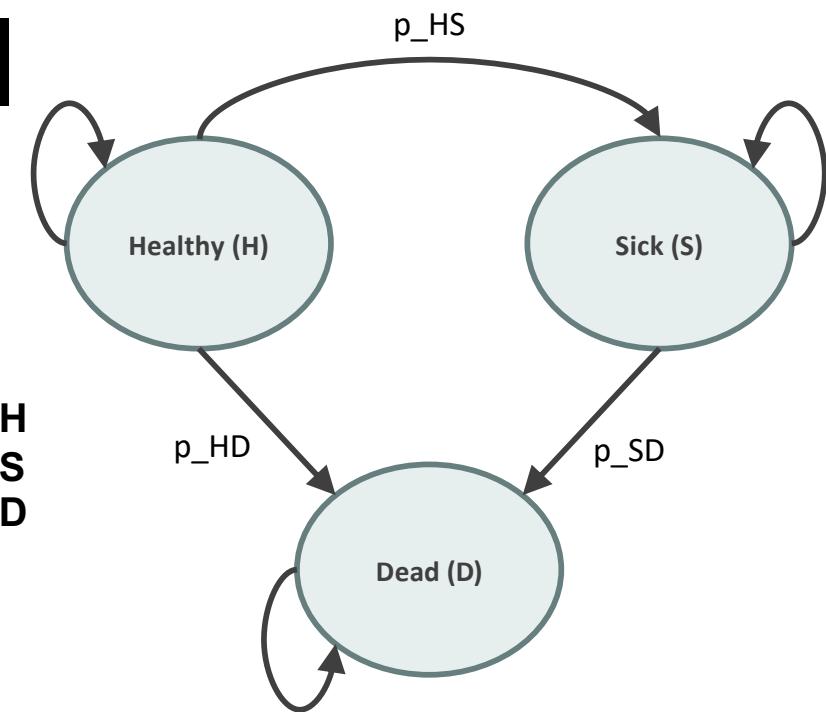
$e_S$ : outcomes associated with state *Sick*

No cost or disutility associated with death

# Matrix Implementation of the Markov Model

Transition probability matrix

$$P = \begin{bmatrix} 1 - p_{HS} - p_{HD} & p_{HS} & p_{HD} \\ 0 & 1 - p_{SD} & p_{SD} \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{smallmatrix} H \\ S \\ D \end{smallmatrix}$$



Vector of cycle's cost/outcomes

$$C = \begin{bmatrix} c_H \\ c_S \\ 0 \end{bmatrix} \quad \begin{smallmatrix} H \\ S \\ D \end{smallmatrix} \quad E = \begin{bmatrix} e_H \\ e_S \\ 0 \end{bmatrix} \quad \begin{smallmatrix} H \\ S \\ D \end{smallmatrix}$$

# Matrix Implementation of the Markov Model

Create the  $t \times 3$  matrix  $M$  that will store the proportion of the cohort at each state and cycle:

At  $t = 0$  :

$$M_0 = [1, 0, 0]$$

For  $t < T$  :

$$M_{t+1} = M_t P$$

$$[H_t \quad S_t \quad D_t] \begin{bmatrix} 1 - p_{HS} - p_{HD} & p_{HS} & p_{HD} \\ 0 & 1 - p_{SD} & p_{SD} \\ 0 & 0 & 1 \end{bmatrix}$$

# Calculating total costs & effects

Total effects (TE):

$$E = M e$$

$$TE = \mathbf{1}_T E$$

Total costs (TC):

$$C = M c$$

$$TC = \mathbf{1}_T C$$

$\mathbf{1}_T$ :  $1 \times T$  vector of ones

# Calculating total costs & effects (discounted)

Total effects (TE):  $E = M e$

$$TE = dw_T E$$

Total costs (TC):

$$C = M c$$
$$TC = dw_T C$$

$$dw_T = \frac{1}{(1 + d)^{0:T}}$$

# Time-dependence

- **Since start of the model**
  - Transition probabilities often depend on age
    - Background mortality
    - Risk of developing disease or experiencing an event
  - In other words, matrix  $P$  is not the same every cycle
- 
- **Depending on state residence**
  - Some transition probabilities depend on time since an event, not age
    - e.g., The risk of developing recurrence among newly diagnosed cancer patients declines with time

# Time-dependence since model start

- Transition probabilities often depend on time since model start
  - Background mortality
  - Risk of developing disease or experiencing an event
- In other words, matrix  $P$  is not the same every cycle
- Replace matrix  $P$  with matrices  $P_t$ , where  $t$  is time since model start

# Time-varying probabilities in R

- We create a 3D array,  $a\_P$ , that stores a collection of time-varying transition matrices,  $P_t$ , in the third dimension
  - For example,  $a\_P$  for the Sick-Sicker Markov model:

$$\mathbf{a\_P} = \begin{bmatrix} p_{[H,H,n_t]} & p_{[H,S1_1,n_t]} & p_{[H,S1_2,n_t]} & \cdots & p_{[H,S1_T,n_t]} & p_{[H,S2,n_t]} & p_{[H,D,n_t]} \\ p_{[H,H,2]} & p_{[H,S1_1,2]} & p_{[H,S1_2,2]} & \cdots & p_{[H,S1_T,2]} & p_{[H,S2,2]} & p_{[H,D,2]} \\ p_{[H,H,1]} & p_{[H,S1_1,1]} & p_{[H,S1_2,1]} & \cdots & p_{[H,S1_T,1]} & p_{[H,S2,1]} & p_{[H,D,1]} \\ p_{[S1_1,H,1]} & p_{[S1_1,S1_1,1]} & p_{[S1_1,S1_2,1]} & \cdots & p_{[S1_1,S1_T,1]} & p_{[S1_1,S2,1]} & p_{[S1_1,D,1]} \\ p_{[S1_2,H,1]} & p_{[S1_2,S1_1,1]} & p_{[S1_2,S1_2,1]} & \cdots & p_{[S1_2,S1_T,1]} & p_{[S1_2,S2,1]} & p_{[S1_2,D,1]} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ p_{[S1_T,H,1]} & p_{[S1_T,S1_1,1]} & p_{[S1_T,S1_2,1]} & \cdots & p_{[S1_T,S1_T,1]} & p_{[S1_T,S2,1]} & p_{[S1_T,D,1]} \\ p_{[S2,H,1]} & p_{[S2,S1_1,1]} & p_{[S2,S1_2,1]} & \cdots & p_{[S2,S1_T,1]} & p_{[S2,S2,1]} & p_{[S2,D,1]} \\ p_{[D,H,1]} & p_{[D,S1_1,1]} & p_{[D,S1_2,1]} & \cdots & p_{[D,S1_T,1]} & p_{[D,S2,1]} & p_{[D,D,1]} \end{bmatrix}$$

# R session

## State-Transition Model & Computing CEA Results

# R tools



- Programming Language/Interpreter: R
- Integrated Development Environment: R Studio
- Extensions: Various Packages/Libraries

The screenshot shows the RStudio interface with the following components:

- Code Editor:** The "flights-example.R" script contains R code for data manipulation and visualization. It includes calls to `library(nycflights13)`, `ggplot`, and `geom\_boxplot` to create a box plot of flights by weekday.
- Environment View:** Shows the "daily" dataset with 365 observations and 3 variables: date (Date), n (int), and wday (Ord.factor).
- Console View:** Displays the R command history, including the execution of the script and the resulting output.
- Plots View:** A box plot titled "Number of 2013 New York Flights Each Weekday" showing the distribution of flights per weekday.

# Starting our script

- We clear out the environment
- We load libraries that we need (and we may need to install these first)

```
3 rm(list = ls())      # clear memory (removes all the variables from the workspace)
4
5 # 01 Load packages
6
7 library(devtools)
8 install_github("DARTH-git/darthtools", force = TRUE)
9 p_load_gh("DARTH-git/darthtools")
10 library(darthtools)
11 library(dampack)
12 library(reshape2)
13 library(diagram)
```

# R scripts can also load other functions as needed

- In our first example, we do not have another script file

```
15 # 02 Load functions  
16  
17 # all functions are in the darthtools package
```

- But if we need to do this we could add code like

```
source("path/load_data.R")  
source("path/analysis.R")
```

# Define model inputs (1)

```
# 03 Model input

n_time_horizon_yr <- 60                                # time horizon (in years)
cycle_length      <- 1                                    # cycle length in years (use 1/12 for monthly)
n_cycles          <- n_time_horizon_yr / cycle_length    # number of cycles
v_names_cycles    <- paste("cycle", 0:n_cycles)          # cycle names
v_names_states    <- c("Healthy", "Sick", "Dead")        # state names
n_states          <- length(v_names_states)                # number of health states

### Discounting factors
d_c <- 0.03 # annual discount rate for costs
d_e <- 0.03 # annual discount rate for QALYs

### Strategies
v_names_str       <- c("Standard of Care",             # store the strategy names
                        "Treatment A",
                        "Treatment B")
n_str             <- length(v_names_str)                  # number of strategies

### Within-cycle correction (WCC) using Simpson's 1/3 rule
v_wcc <- gen_wcc(n_cycles = n_cycles, method = "Simpson1/3")

### Transition probabilities
p_HD_yr           <- 0.01 # annual probability of dying when healthy
p_SD_yr           <- 0.10 # annual probability of dying when sick
# Annual probability of becoming sick from Healthy, conditional on surviving cycle
p_HS_yr_SoC       <- 0.05 # under standard of care
p_HS_yr_trtA     <- 0.04 # under treatment A
p_HS_yr_trtB     <- 0.02 # under treatment B
```

# Define model inputs (2)

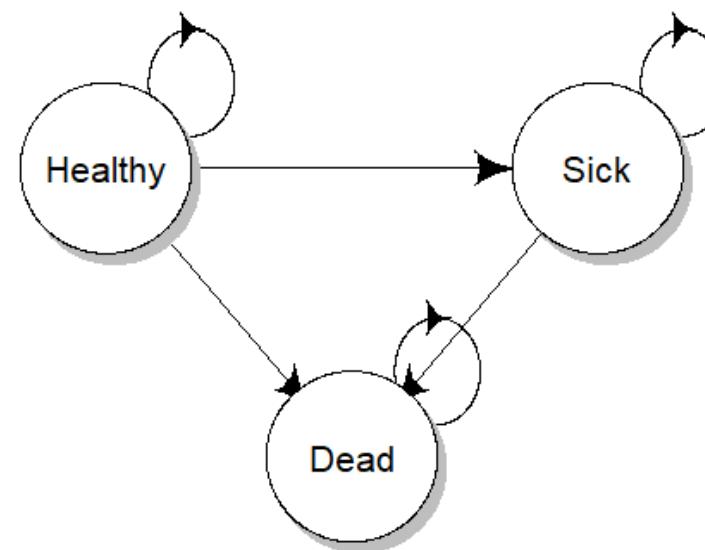
```
51  ### State rewards
52
53  ##### Costs
54  c_H_yr      <- 400    # cost of one year in healthy state
55  c_S_yr      <- 1000   # cost of one year in sick state
56  c_D_yr      <- 0      # cost of one year in dead state
57  c_trtA_yr   <- 800    # cost of treatment A (per year) in healthy state
58  c_trtB_yr   <- 1500   # cost of treatment B (per year) in healthy state
59  ##### Utilities
60  u_H         <- 1      # utility when healthy
61  u_S         <- 0.5    # utility when sick
62  u_D         <- 0      # utility when dead
63
64 ## 03.2 calculate internal model parameters
65
66  ##### Cycle-specific discount weight for costs and effects
67  v_dwc       <- 1 / ((1 + (d_e * cycle_length)) ^ (0:n_cycles))
68  v_dwe       <- 1 / ((1 + (d_c * cycle_length)) ^ (0:n_cycles))
69
70  ##### Calculate cycle-specific transition probabilities
71  p_HD <- 1 - exp(-(-log(1 - p_HD_yr) * cycle_length)) # probability of dying from Healthy
72  p_SD <- 1 - exp(-(-log(1 - p_SD_yr) * cycle_length)) # probability of dying from Sick
73  # probability of becoming sick from Healthy
74  p_HS_SoC  <- 1 - exp(-(-log(1 - p_HS_yr_SoC) * cycle_length)) # Standard of Care
75  p_HS_trtA <- 1 - exp(-(-log(1 - p_HS_yr_trtA) * cycle_length)) # Treatment A
76  p_HS_trtB <- 1 - exp(-(-log(1 - p_HS_yr_trtB) * cycle_length)) # Treatment B
```

# Define model inputs (3)

```
78  ### Calculate cycle-specific state rewards
79  ##### Costs
80  v_c_SoC  <- c(c_H_yr, c_S_yr, c_D_yr) * cycle_length          # Standard of Care
81  v_c_trtA <- c(c_H_yr + c_trtA_yr, c_S_yr, c_D_yr) * cycle_length # Treatment A
82  v_c_trtB <- c(c_H_yr + c_trtB_yr, c_S_yr, c_D_yr) * cycle_length # Treatment B
83  ##### QALYs
84  v_q_SoC  <- c(u_H, u_S, u_D) * cycle_length # Standord of Care
85  v_q_trtA <- v_q_trtB <- v_q_SoC           # Treatments A and B have same utilities as SoC
86
```

# Using R to make a state-transition diagram

```
87 # 04 Construct state-transition models
88
89 m_P_diag <- matrix(0, nrow = n_states, ncol = n_states,
90                      dimnames = list(v_names_states, v_names_states))
91 m_P_diag["Healthy", "Sick"]      = ""
92 m_P_diag["Healthy", "Dead"]      = ""
93 m_P_diag["Healthy", "Healthy"]   = ""
94 m_P_diag["Sick", "Dead"]         = ""
95 m_P_diag["Sick", "Sick"]         = ""
96 m_P_diag["Dead", "Dead"]         = ""
97 layout.fig <- c(2, 1)
98 plotmat(t(m_P_diag), t(layout.fig), self.cex = 0.5, curve = 0,
99          latex = T, arr.type = "curved", relsize = 0.85, box.prop = 0.8,
100         cex = 0.8, box.cex = 0.7, lwd = 1)
```



# Proportions starting in each state and Markov trace

```
103 ## 04.1 Initial state vector
104
105 # All starting healthy
106 v_m_init <- c("Healthy" = 1, "Sick" = 0, "Dead" = 0)
107 v_m_init
108
109 ## 04.2 Initialize cohort traces
110
111 ### Initialize cohort trace for SoC
112 m_M_SoC <- matrix(0,
113                     nrow = (n_cycles + 1), ncol = n_states,
114                     dimnames = list(v_names_cycles, v_names_states))
115 # Store the initial state vector in the first row of the cohort trace
116 m_M_SoC[1, ] <- v_m_init
117
118 ## Initialize cohort traces for treatments A and B
119 # Structure and initial states are the same as for SoC
120 m_M_trtA <- m_M_trtB <- m_M_SoC
```

# Fill transition matrix

```
122 ## 04.3 Create transition probability arrays
123
124 ## Create transition probability matrices for strategy SOC
125 ### Initialize transition probability matrix
126 # All transitions to a non-death state are assumed to be conditional on survival
127 m_P_SoC <- matrix(0,
128                     nrow = n_states, ncol = n_states,
129                     dimnames = list(v_names_states, v_names_states))
130 ### Fill in matrix
131 # from Healthy
132 m_P_SoC["Healthy", "Healthy"] <- (1 - p_HD) * (1 - p_HS_SoC)
133 m_P_SoC["Healthy", "Sick"] <- (1 - p_HD) * p_HS_SoC
134 m_P_SoC["Healthy", "Dead"] <- p_HD
135 # from Sick
136 m_P_SoC["Sick", "Sick"] <- 1 - p_SD
137 m_P_SoC["Sick", "Dead"] <- p_SD
138 # from Dead
139 m_P_SoC["Dead", "Dead"] <- 1
140
141 ## Treatment A
142 # Start with same matrix as SOC, but replace parameters that differ for trtA
143 m_P_trtA <- m_P_SoC
144 m_P_trtA["Healthy", "Healthy"] <- (1 - p_HD) * (1 - p_HS_trtA)
145 m_P_trtA["Healthy", "Sick"] <- (1 - p_HD) * p_HS_trtA
146
147 ## Treatment B
148 # Start with same matrix as SOC, but replace parameters that differ for trtB
149 m_P_trtB <- m_P_SoC
150 m_P_trtB["Healthy", "Healthy"] <- (1 - p_HD) * (1 - p_HS_trtB)
151 m_P_trtB["Healthy", "Sick"] <- (1 - p_HD) * p_HS_trtB
152
```

# Are transition matrices valid?

```
153 ## Check if transition probability matrices are valid
154 ### Check that transition probabilities are in [0, 1]
155 check_transition_probability(m_P_SoC, verbose = TRUE)
156 check_transition_probability(m_P_trtA, verbose = TRUE)
157 check_transition_probability(m_P_trtB, verbose = TRUE)
158 ### Check that all rows sum to 1
159 check_sum_of_transition_array(m_P_SoC, n_states = n_states, n_cycles = n_cycles, verbose = TRUE)
160 check_sum_of_transition_array(m_P_trtA, n_states = n_states, n_cycles = n_cycles, verbose = TRUE)
161 check_sum_of_transition_array(m_P_trtB, n_states = n_states, n_cycles = n_cycles, verbose = TRUE)
```

Console Terminal × Background Jobs ×

R 4.1.3 · C:/Users/jeremygf/Dropbox/SharedWork/Courses/AdvancedDSMethods/CURRENT\_YEAR/WORKSHOPS/INFECTIOUS\_DISEASE\_MODEL\_WORKSHOP/N

```
>
> ## Check if transition probability matrices are valid
> ### Check that transition probabilities are in [0, 1]
> check_transition_probability(m_P_SoC, verbose = TRUE)
[1] "Valid transition probabilities"
> check_transition_probability(m_P_trtA, verbose = TRUE)
[1] "Valid transition probabilities"
> check_transition_probability(m_P_trtB, verbose = TRUE)
[1] "Valid transition probabilities"
> ### Check that all rows sum to 1
> check_sum_of_transition_array(m_P_SoC, n_states = n_states, n_cycles = n_cycles, verbose = TRUE)
[1] "This is a valid transition matrix"
> check_sum_of_transition_array(m_P_trtA, n_states = n_states, n_cycles = n_cycles, verbose = TRUE)
[1] "This is a valid transition matrix"
> check_sum_of_transition_array(m_P_trtB, n_states = n_states, n_cycles = n_cycles, verbose = TRUE)
[1] "This is a valid transition matrix"
```

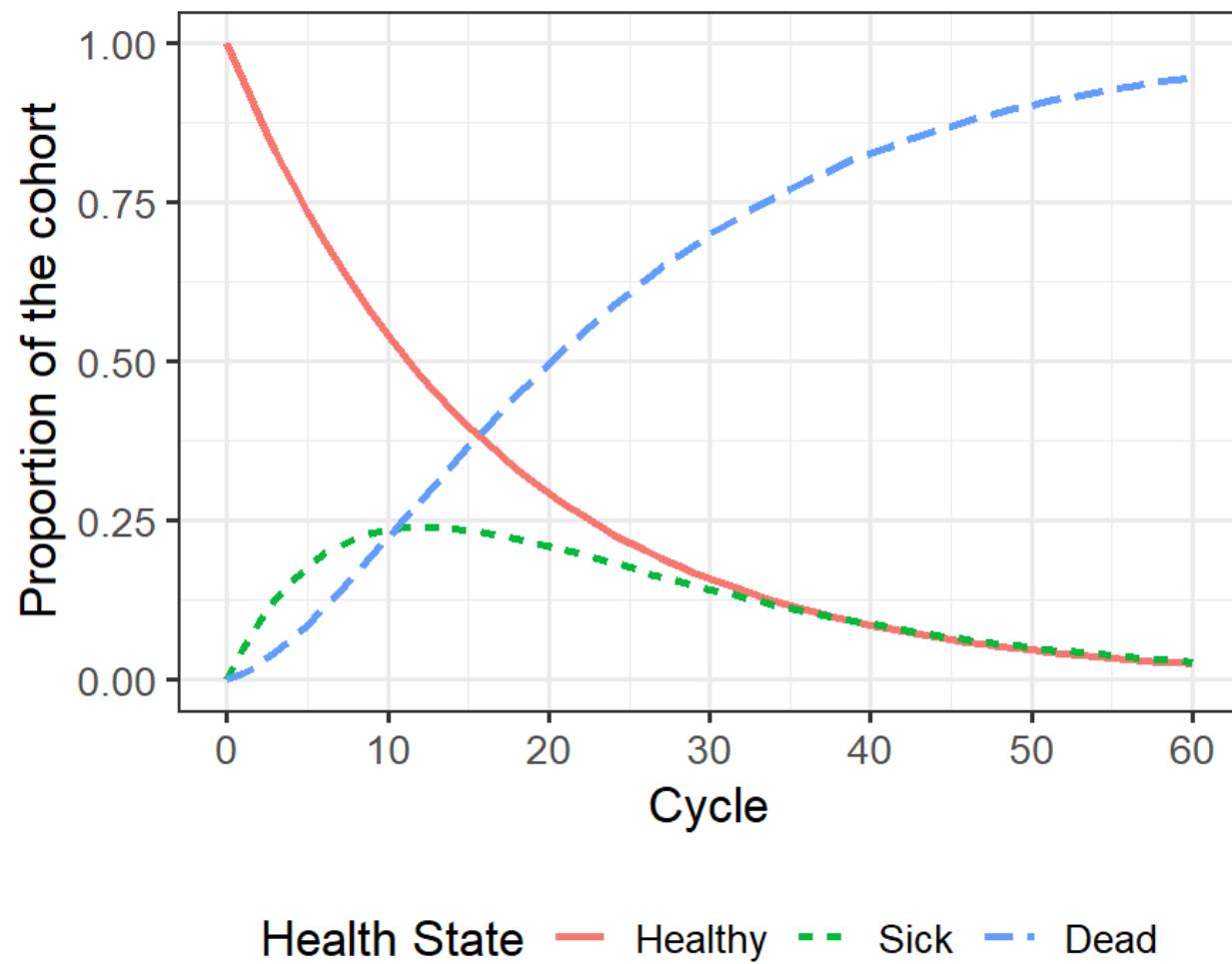
# Run the model for each strategy

```
163 # 05 Run cohort state transition model  
164  
165 ## Loop over time  
166 # Calculating cohort state based on previous state and transition matrix  
167 for (t in 1:n_cycles){  
168   # For SoC  
169   m_M_SoC [t + 1, ] <- m_M_SoC [t, ] %*% m_P_SoC  
170   # For treatment A  
171   m_M_trtA[t + 1, ] <- m_M_trtA[t, ] %*% m_P_trtA  
172   # For treatment B  
173   m_M_trtB[t + 1, ] <- m_M_trtB[t, ] %*% m_P_trtB  
174 }
```

```
> m_M_trtB  
          Healthy      Sick      Dead  
cycle 0 1.0000000 0.00000000 0.00000000  
cycle 1 0.9702000 0.01980000 0.01000000  
cycle 2 0.9412880 0.03702996 0.02168200  
cycle 3 0.9132377 0.05196447 0.03479788  
cycle 4 0.8860232 0.06485013 0.04912670  
cycle 5 0.8596197 0.07590837 0.06447194  
cycle 6 0.8340030 0.08533800 0.08065898  
cycle 7 0.8091497 0.09331746 0.09753281  
cycle 8 0.7850371 0.10000688 0.11495605  
cycle 9 0.7616430 0.10554993 0.13280711  
cycle 10 0.7389460 0.11007547 0.15097853
```

# Plot Model trace

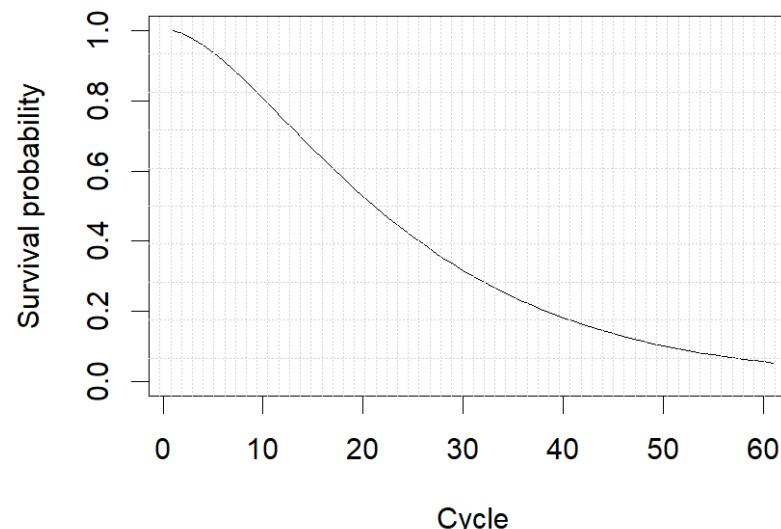
```
178 # 06 Plot Outputs  
179  
180 ## 06.1 Plot the cohort trace for strategies SoC  
181  
182 plot_trace(m_M_SoC)
```



# Survival curves & life expectancy

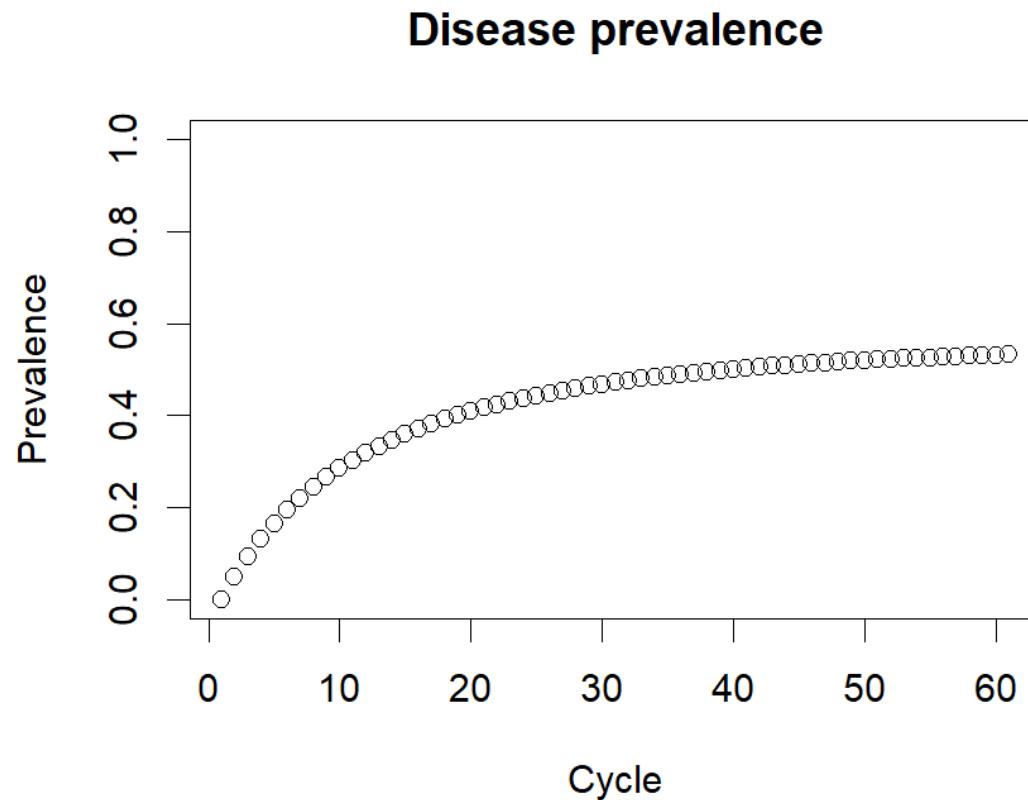
```
184 ## 06.2 Overall survival (OS)
185
186 #Print the overall survival for the Standard of Care
187
188 v_os_SoC <- 1 - m_M_SoC[, "Dead"]    # calculate the overall survival (OS) probability
189 v_os_SoC <- rowSums(m_M_SoC[, 1:2])  # alternative way of calculating the OS probability
190
191 plot(v_os_SoC, type = 'l',
192       ylim = c(0, 1),
193       ylab = "Survival probability",
194       xlab = "Cycle",
195       main = "Overall Survival") # create a simple plot showing the OS
196
197 # add grid
198 grid(nx = n_cycles, ny = 10, col = "lightgray", lty = "dotted", lwd = par("lwd"),
199       equilogs = TRUE)
200
201 ## 06.2.1 Life Expectancy (LE)
202
203 le_SoC <- sum(v_os_SoC) # summing probability of OS over time (i.e. life expectancy)
```

**Overall Survival**



# Disease prevalence

```
205 ## 06.2.2 Disease prevalence  
206  
207 v_prev <- m_M_SoC[, "Sick"]/v_os_SoC  
208 plot(v_prev,  
209     ylim = c(0, 1),  
210     ylab = "Prevalence",  
211     xlab = "Cycle",  
212     main = "Disease prevalence")  
213
```



# State rewards (initialize values) (1)

```
214 # 07 State Rewards
215
216 ## Scale by the cycle length
217 # Vector of state utilities under strategy SoC
218 v_u_SoC    <- c(H = u_H,
219                 S = u_S,
220                 D = u_D) * cycle_length
221 # Vector of state costs under strategy SoC
222 v_c_SoC    <- c(H = c_H_yr,
223                 S = c_S_yr,
224                 D = c_D_yr) * cycle_length
225 # Vector of state utilities under treatment A
226 v_u_trtA   <- c(H = u_H,
227                 S = u_S,
228                 D = u_D) * cycle_length
229 # Vector of state costs under treatment A
230 v_c_trtA   <- c(H = c_H_yr + c_trtA_yr,
231                 S = c_S_yr,
232                 D = c_D_yr) * cycle_length
233 # Vector of state utilities under treatment B
234 v_u_trtB   <- c(H = u_H,
235                 S = u_S,
236                 D = u_D) * cycle_length
237 # Vector of state costs under treatment B
238 v_c_trtB   <- c(H = c_H_yr + c_trtB_yr,
239                 S = c_S_yr,
240                 D = c_D_yr) * cycle_length
```

# State rewards (initialize values) (2)

```
242 ## Store state rewards
243 # Store the vectors of state utilities for each strategy in a list
244 l_u <- list(SQ = v_u_SoC,
245           A = v_u_trtA,
246           B = v_u_trtB)
247 # Store the vectors of state cost for each strategy in a list
248 l_c <- list(SQ = v_c_SoC,
249           A = v_c_trtA,
250           B = v_c_trtB)
251
252 # assign strategy names to matching items in the lists
253 names(l_u) <- names(l_c) <- v_names_str
```

# State rewards (apply values to trace)

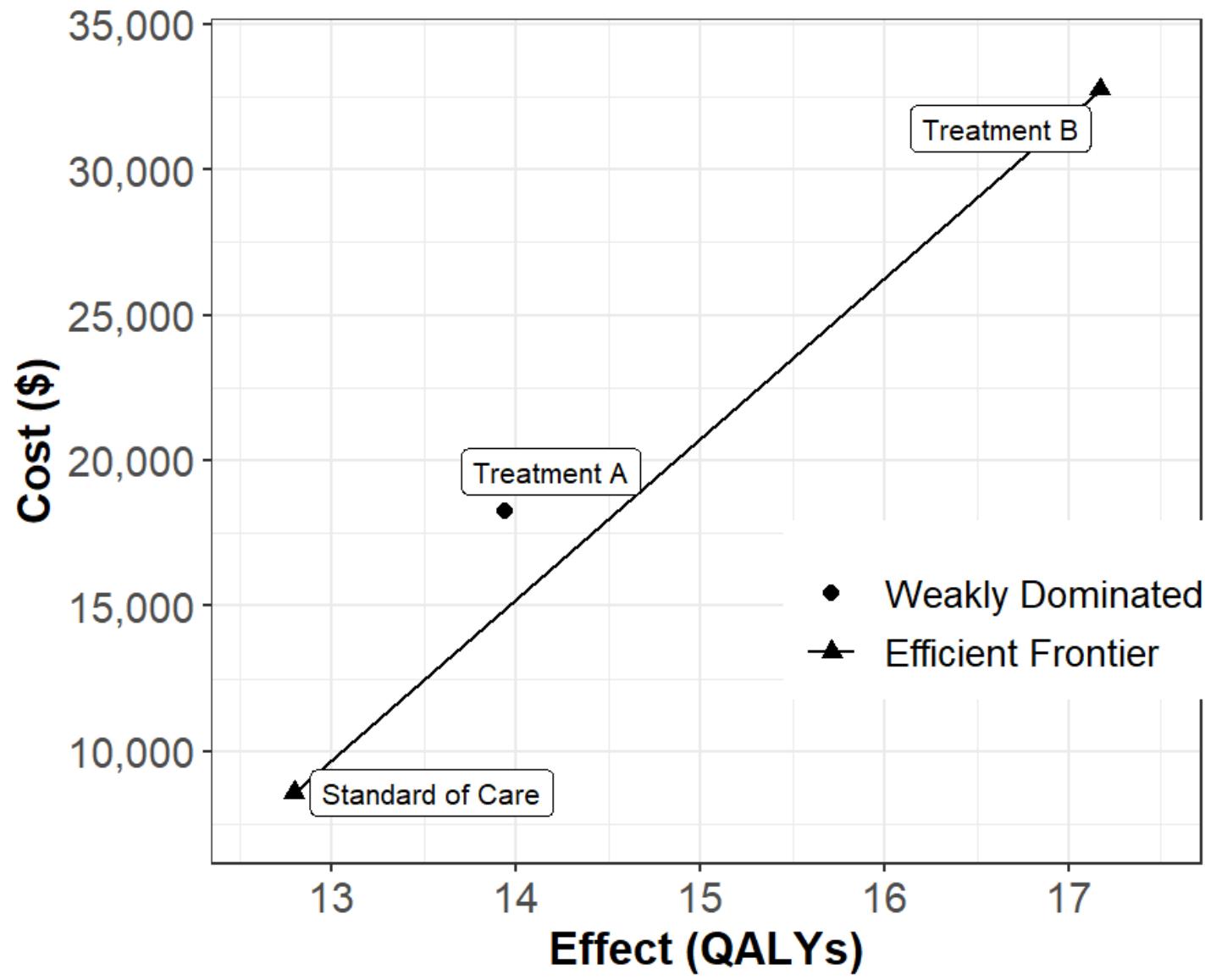
```
255 # 08 Compute expected outcomes  
256  
257  
258 # Create empty vectors to store total costs and QALYs  
259 v_tot_cost <- v_tot_qaly <- vector(mode = "numeric", length = n_str)  
260 names(v_tot_qaly) <- names(v_tot_cost) <- v_names_str  
261  
262  
263 ### Expected discounted cost for each strategy  
264 v_tot_cost["Standard of Care"] <- t(m_M_SoC %*% v_c_SoC) %*% (v_dwe * v_wcc)  
265 v_tot_cost["Treatment A"] <- t(m_M_trtA %*% v_c_trtA) %*% (v_dwe * v_wcc)  
266 v_tot_cost["Treatment B"] <- t(m_M_trtB %*% v_c_trtB) %*% (v_dwe * v_wcc)  
267  
268 ### Expected discounted QALYs for each strategy  
269 v_tot_qaly["Standard of Care"] <- t(m_M_SoC %*% v_q_SoC) %*% (v_dwe * v_wcc)  
270 v_tot_qaly["Treatment A"] <- t(m_M_trtA %*% v_q_trtA) %*% (v_dwe * v_wcc)  
271 v_tot_qaly["Treatment B"] <- t(m_M_trtB %*% v_q_trtB) %*% (v_dwe * v_wcc)
```

# CEA (1)

```
273 # 09 Cost-effectiveness analysis (CEA)
274
275 ## Incremental cost-effectiveness ratios (ICERs)
276 df_cea <- calculate_icers(cost      = v_tot_cost,
277                           effect     = v_tot_qaly,
278                           strategies = v_names_str)
279 df_cea
280
281 ## CEA table in proper format
282 table_cea <- format_table_cea(df_cea)
283 table_cea
284
285 ## CEA frontier
286 plot(df_cea, label = "all", txtsize = 14) +
287   expand_limits(x = max(table_cea$QALYs) + 0.1) +
288   theme(legend.position = c(0.8, 0.3))
```

	Strategy	Costs (\$)	QALYs	Incremental Costs (\$)	Incremental QALYs	ICER (\$/QALY)	Status
Standard of Care	Standard of Care	8,575	12.79	<NA>	NA	<NA>	ND
Treatment B	Treatment B	32,745	17.17	24,171	4.37	5,526	ND
Treatment A	Treatment A	18,272	13.94	<NA>	NA	<NA>	ED

## CEA (2)



# Sensitivity Analysis

- Vary input parameters within plausible ranges
- For which values is each strategy optimal?
- Deterministic sensitivity analysis (DSA)
  - One-way analysis: vary one parameter, hold rest fixed
  - Two-way analysis: vary two parameters, hold rest fixed
- **Probabilistic sensitivity analysis (PSA)**
  - Simultaneously vary input parameters by randomly sampling from appropriate probability distributions
  - How often is each alternative cost-effective?
  - What strategy has the highest **expected** net benefit?

# Sensitivity Analyses (Deterministic) (1)

```
290 # 10 Deterministic Sensitivity Analysis (DSA)
291
292 ## Load model, CEA and PSA functions
293 source('Functions_cSTM_3state.R')
```

```
| calculate_ce_out <- function(l_params_all, n_wtp = 10000 ,verbose= FALSE){
```

```
| generate_psa_params <- function(n_sim = 1000, seed = 071818){
```

# Sensitivity Analyses (Deterministic) (2)

```
295 ## 10.1 Model input for SA
296
297 l_params_all <- list(
298   # Transition probabilities
299   # probability of dying
300   p_HD_yr      = 0.01, # annual probability of dying when healthy
301   p_SD_yr      = 0.10, # annual probability of dying when sick
302   # Annual probability of becoming sick from Healthy, conditional on surviving cycle
303   p_HS_yr_SoC  = 0.05, # under standard of care
304   p_HS_yr_trtA = 0.04, # under treatment A
305   p_HS_yr_trtB = 0.02, # under treatment B
306
307   ### State rewards
308
309   ##### Costs
310   c_H_yr       = 400, # cost of one year in healthy state
311   c_S_yr       = 1000, # cost of one year in sick state
312   c_D_yr       = 0,   # cost of one year in dead state
313   c_trtA_yr   = 800, # cost of treatment A (per year) in healthy state
314   c_trtB_yr   = 1500, # cost of treatment B (per year) in healthy state
315   ##### Utilities
316   u_H          = 1,   # utility when healthy
317   u_S          = 0.5, # utility when sick
318   u_D          = 0,   # utility when dead
319   # Discount rates
320   d_e          = 0.03, # discount rate per cycle equal discount of costs and QALYs by 3%
321   d_c          = 0.03, # discount rate per cycle equal discount of costs and QALYs by 3%
322   # Time horizon
323   n_time_horizon_yr = 60,           # time horizon (in years)
324   cycle_length    = 1               # cycle length in years (use 1/12 for monthly)
325
326 )
```

# Sensitivity Analyses (Deterministic) (3)

```
328 #**Test model functions**
329 #
330 #A function is defined in the `Functions_cSTM_3state.R` file.
331 #The first is the `calculate_ce_out()` function which runs the decision model
332 #and uses the resulting Markov trace to compute the total costs, QALYs,
333 #and net monetary benefit (NMB) for each strategy, returning a data frame of
334 #costs and QALYs.
335 #
336
337 # Try the calculate_ce_out() function
338 df_ce <- calculate_ce_out(l_params_all)
339 df_ce
340
341 # Get strategies names (will be used to label plots)
342 v_names_str <- df_ce$Strategy
343 n_str <- length(v_names_str)
344
```

> df\_ce

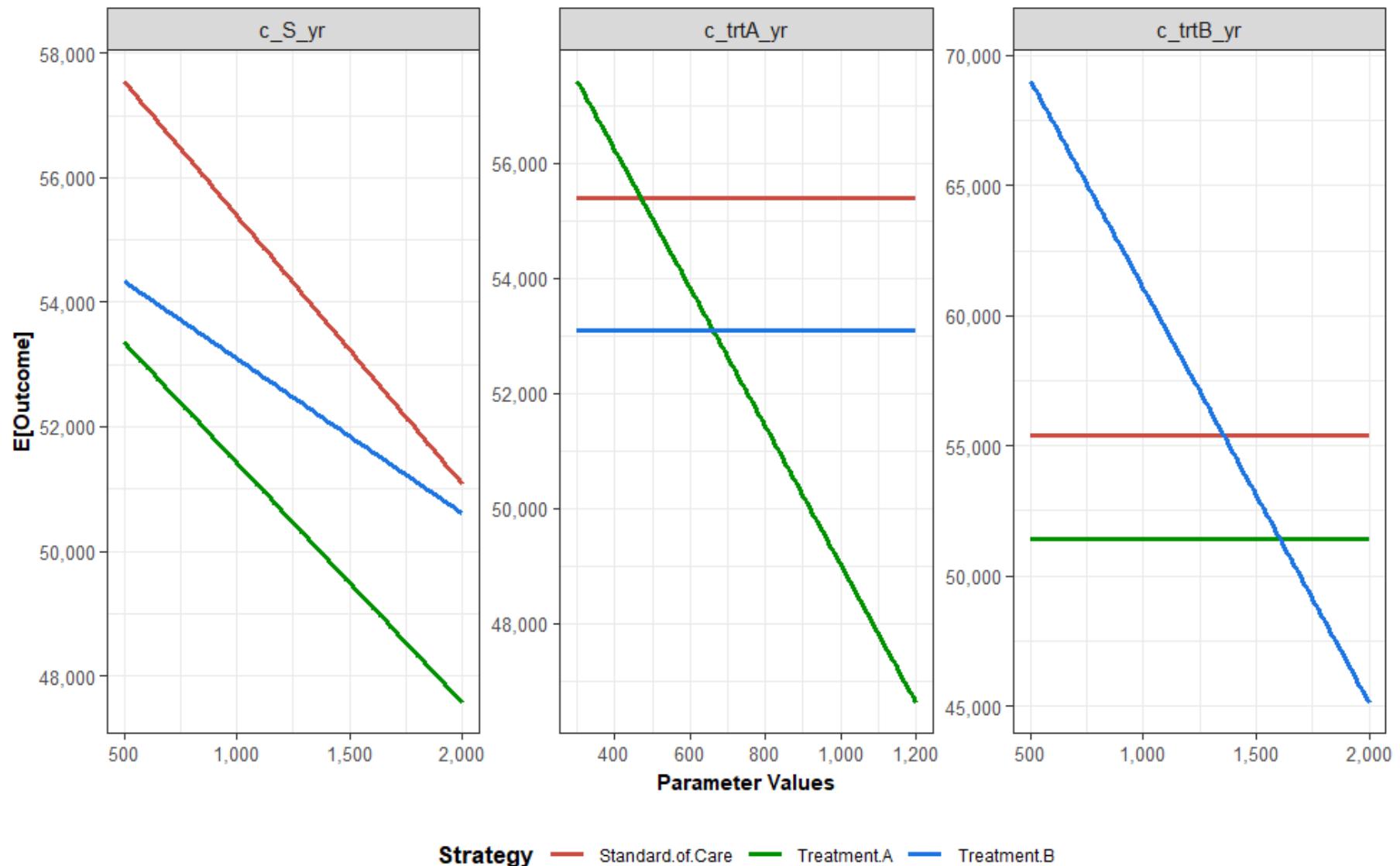
	Strategy	Cost	Effect	NMB
Standard of Care	Standard of Care	8574.738	12.79462	119371.4
Treatment A	Treatment A	18272.100	13.94028	121130.7
Treatment B	Treatment B	32745.314	17.16887	138943.4

	Strategy	Costs (\$)	QALYs	Incremental Costs (\$)	Incremental QALYs	ICER (\$/QALY)	Status
Standard of Care	Standard of Care	8,575	12.79		<NA>	NA	<NA> ND
Treatment B	Treatment B	32,745	17.17		24,171	4.37	5,526 ND
Treatment A	Treatment A	18,272	13.94		<NA>	NA	<NA> ED

# Sensitivity Analyses (Deterministic) (4)

```
345 ## 10.2 One-way sensitivity analysis (OWSA)
346
347 options(scipen = 999) # disabling scientific notation in R
348 # dataframe containing all parameters, their base case values, and the min and
349 # max values of the parameters of interest
350 df_params_owsa <- data.frame(pars = c("c_trtA_yr", "c_trtB_yr", "c_S_yr"),
351                               min = c(300, 500, 500), # min parameter values
352                               max = c(1200, 2000, 2000) # max parameter values
353 )
354 owsa_nmb <- run_owsa_det(params_range      = df_params_owsa,
355                           params_basecase   = l_params_all,
356                           nsamp             = 100,
357                           FUN               = calculate_ce_out,
358                           outcomes          = c("NMB"),
359                           strategies        = v_names_str,
360                           n_wtp              = 5000) # dataframe with parameters for
# list with all parameters
# number of parameter values
# function to compute outputs
# output to do the OWSA on
# names of the strategies
# extra argument to pass to FUN
361
362 plot(owsa_nmb, txtsize = 10, n_x_ticks = 4,
363       facet_scales = "free") +
364       theme(legend.position = "bottom")
365
```

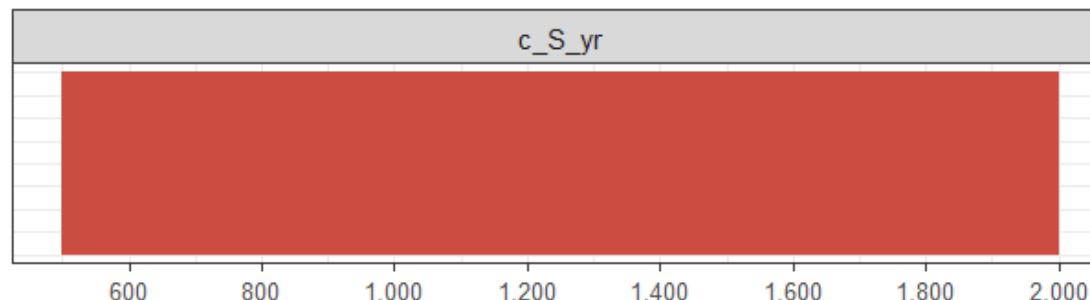
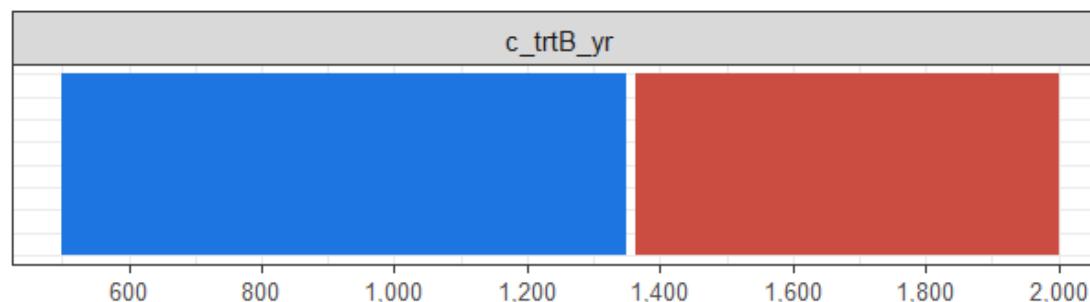
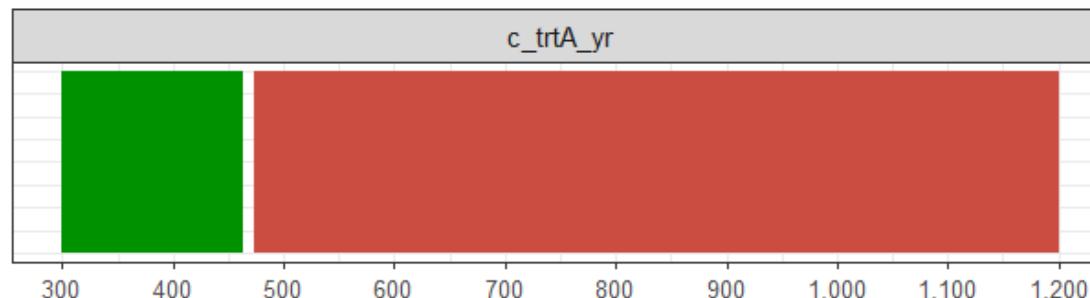
# Sensitivity Analyses (Deterministic) (5)



# Sensitivity Analyses (Deterministic) (6)

```
### 10.2.1 Optimal strategy with owsa
```

```
owsa_opt_strat(owsa = owsa_nmb, txtsize = 10)
```



**Optimal Strategy:**

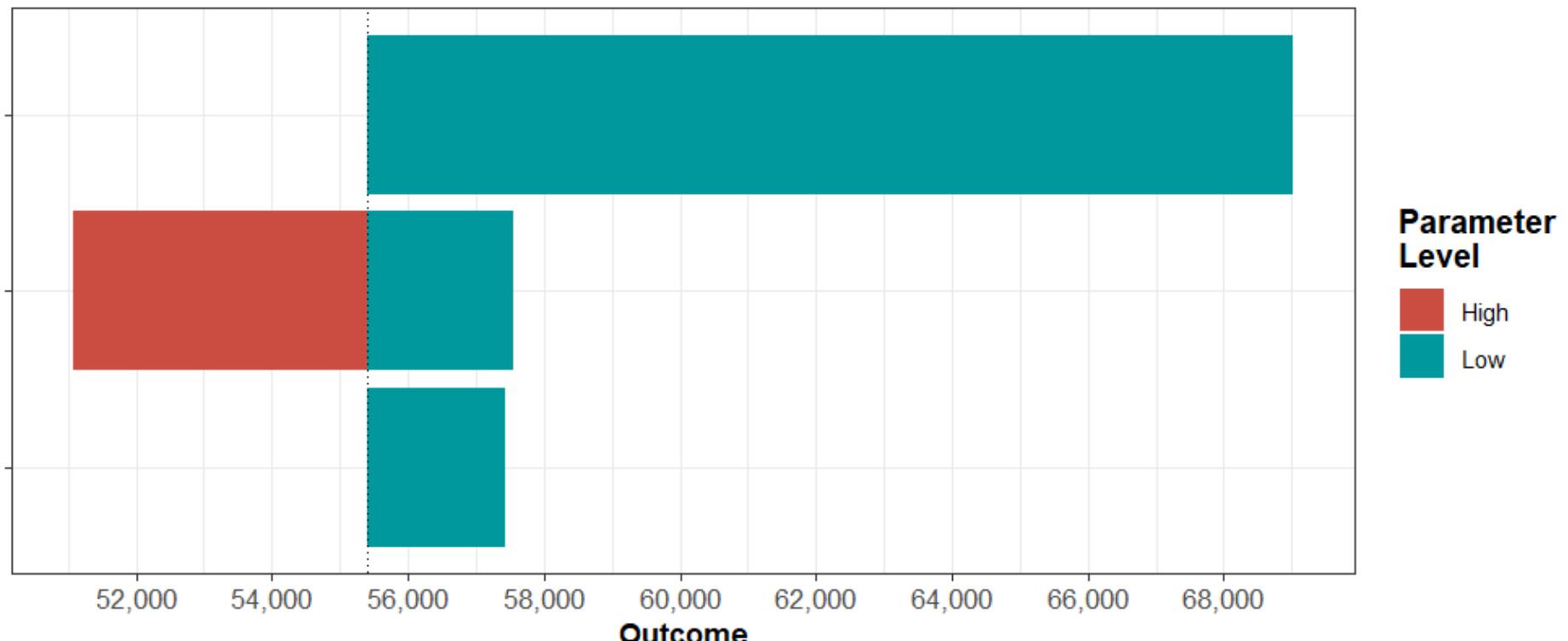
- █ Standard.of.Care
- █ Treatment.A
- █ Treatment.B

# Sensitivity Analyses (Deterministic) (6)

```
370 ### 10.2.2 Tornado plot
371
372 tornado <- owsa_tornado(owsa = owsa_nmb)
373 tornado + labs(title="Tornado plot",
374   subtitle="c_trtA_yr: min = €300, max = €1,200\n c_trtB_yr: min = €500, max = €2,000\n c_S_yr: min = €500, max = €2,000",
375   x = "Outcome (Net Monetary Benefit in €)") +
376 theme(plot.subtitle=element_text(size=12, face="plain", color="black"))
```

## Tornado plot

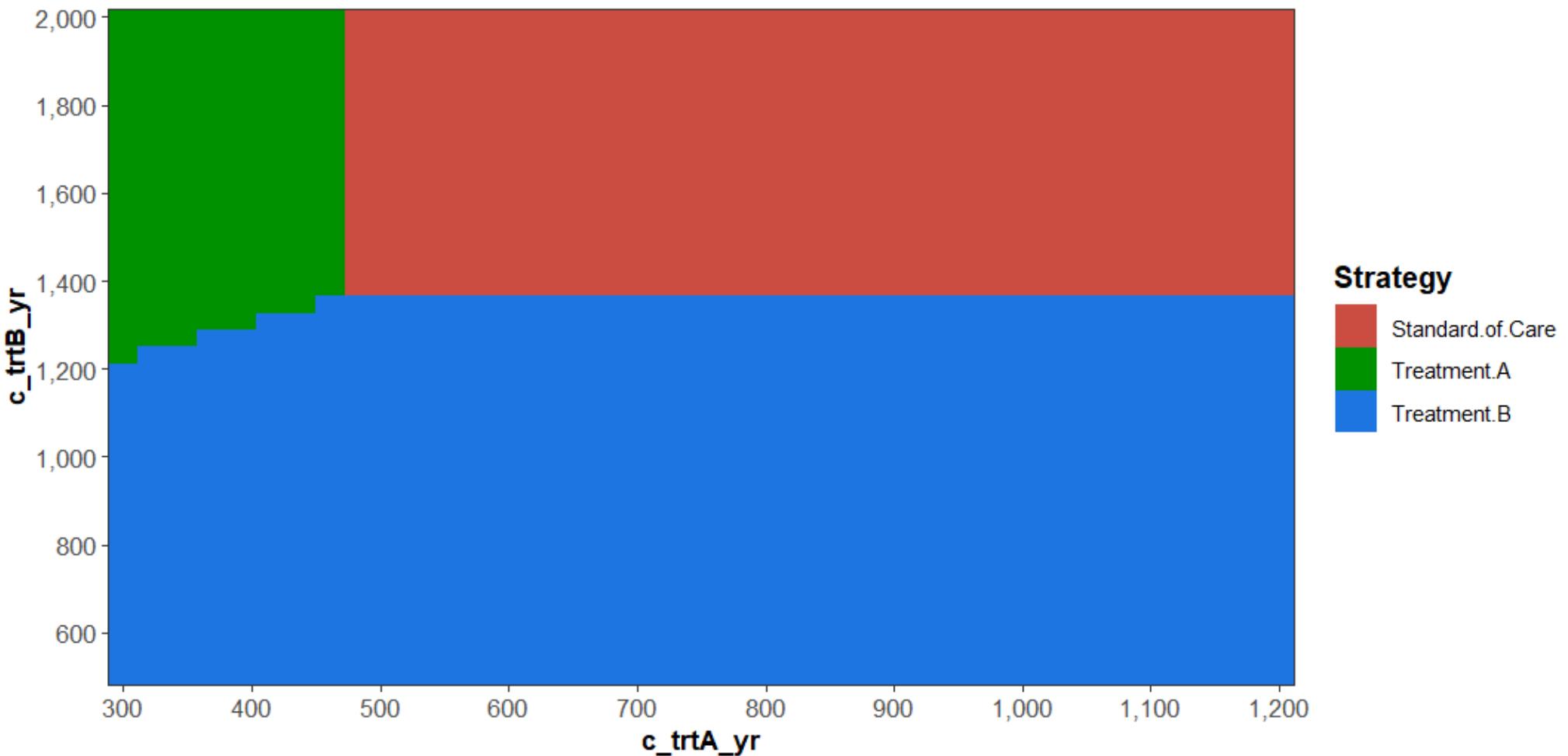
c\_trtA\_yr: min = €300, max = €1,200  
c\_trtB\_yr: min = €500, max = €2,000  
c\_S\_yr: min = €500, max = €2,000



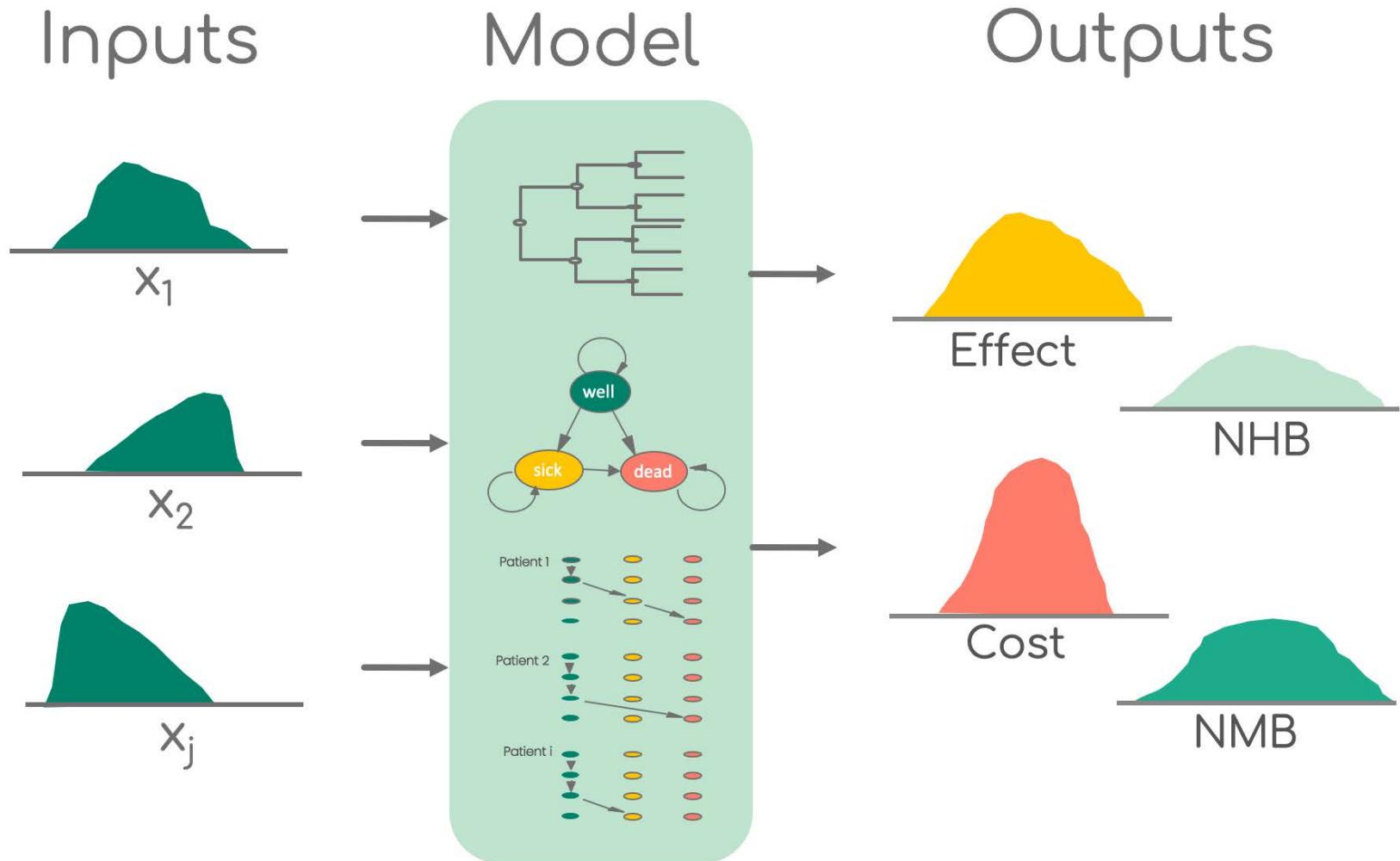
# Sensitivity Analyses (Deterministic) (7)

```
378 ## 10.3 Two-way sensitivity analysis (TWSA)
379
380 # dataframe containing all parameters, their basecase values, and the min and
381 # max values of the parameters of interest
382 df_params_twsa <- data.frame(pars = c("c_trtA_yr", "c_trtB_yr"),
383                               min = c(300, 500), # min parameter values
384                               max = c(1200, 2000) # max parameter values
385                               )
386
387 twsa_nmb <- run_twsa_det(params_range      = df_params_twsa,          # dataframe with parameters for TWSA
388                             params_basecase = l_params_all,        # list with all parameters
389                             nsamp           = 40,                  # number of parameter values
390                             FUN              = calculate_ce_out,    # function to compute outputs
391                             outcomes         = "NMB",            # output to do the TWSA on
392                             strategies       = v_names_str,       # names of the strategies
393                             n_wtp            = 5000)             # extra argument to pass to FUN
394
395 ### 10.3.1 Plot TWSA
396
397 plot(twsa_nmb)
```

# Sensitivity Analyses (Deterministic) (8)

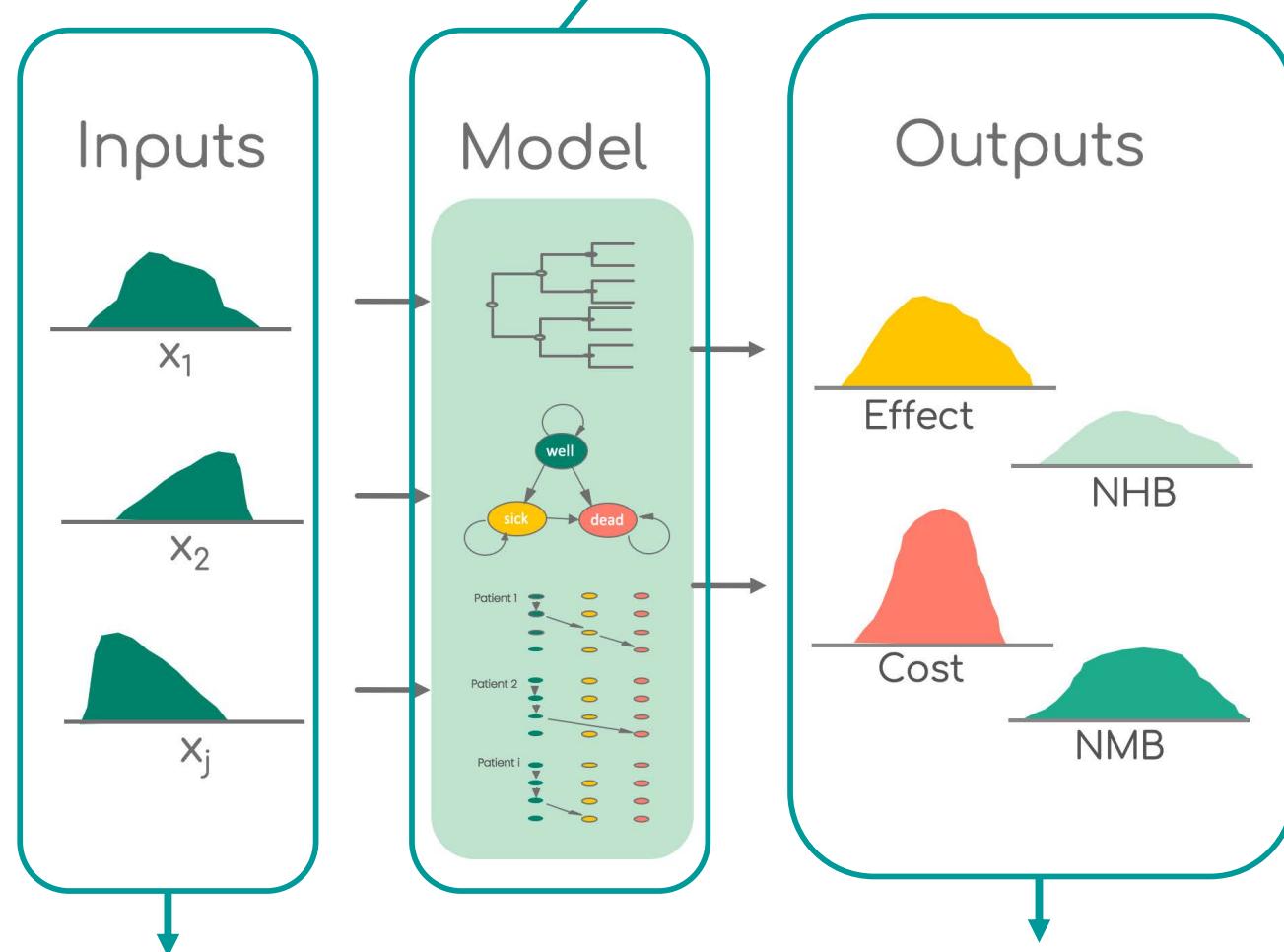


# Probabilistic Sensitivity Analysis



NHB: Net health benefit ; NMB: Net monetary benefit

# PSA & R



**List or data set** with parameter values for each iteration

**Function** which runs the model using the parameter values from the list

**Data frame** with model outcome estimates for each iteration

# Sensitivity Analyses (Probabilistic) (9)

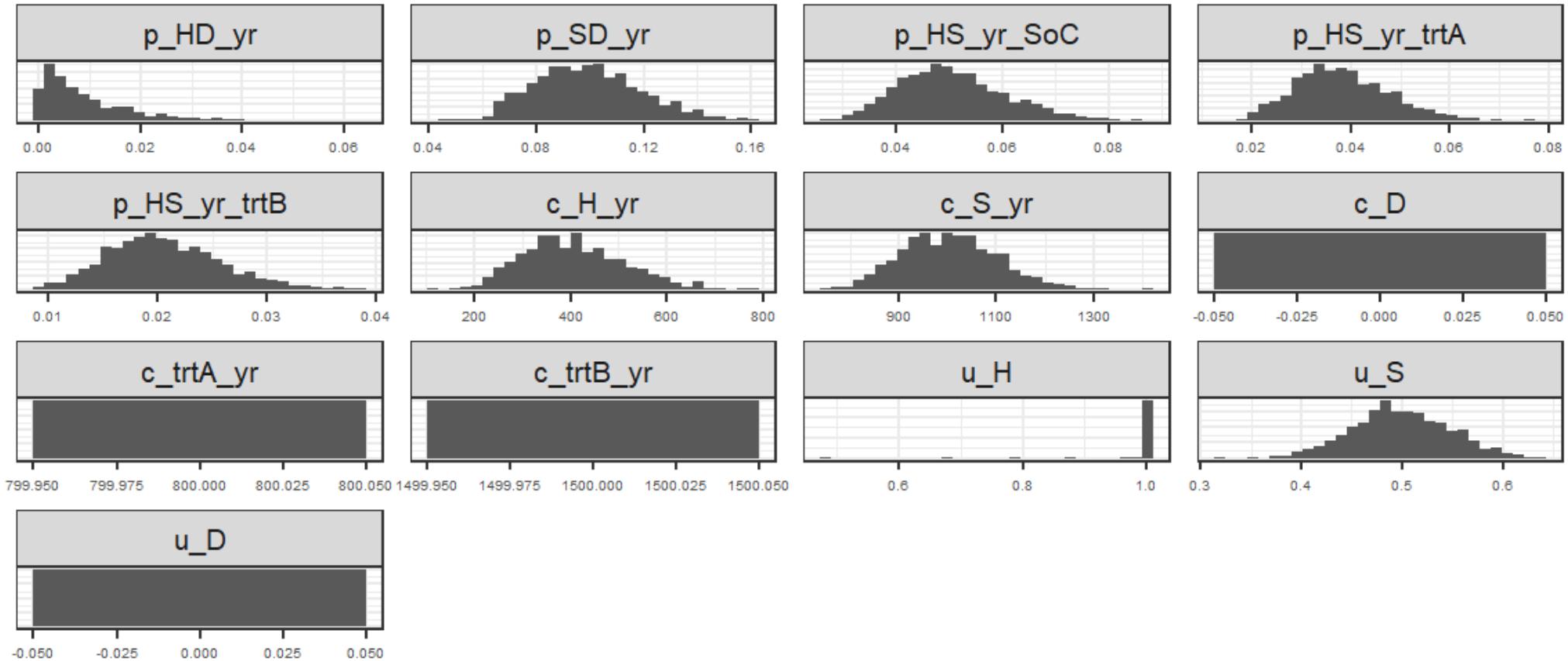
```
399 # 11 Probabilistic Sensitivity Analysis (PSA)
400
401 ## 11.1 Model input
402
403 # Store the parameter names into a vector
404 v_names_params <- names(l_params_all)
405
406 ## Test functions to generate CE outcomes and PSA dataset
407 # Test function to compute CE outcomes
408 calculate_ce_out(l_params_all)
409
410 # Test function to generate PSA input dataset
411 generate_psa_params(10)
```

```
> generate_psa_params(10)
   p_HD_yr    p_SD_yr p_HS_yr_SoC p_HS_yr_trtA p_HS_yr_trtB    c_H_yr    c_S_yr c_D c_trtA_yr c_trtB_yr u_H      u_S u_D
1 0.0008905932 0.07962620 0.04080578 0.07084322 0.01716266 390.7068 984.6521 0     800    1500 1 0.5399242 0
2 0.0092979184 0.14336539 0.06172540 0.05135318 0.02601948 393.7788 820.5960 0     800    1500 1 0.5418116 0
3 0.0069371243 0.08285488 0.04973326 0.03317056 0.01537997 388.4159 1138.0161 0     800    1500 1 0.5599530 0
4 0.0148746692 0.12408528 0.03997063 0.04320045 0.01843236 348.6654 858.9336 0     800    1500 1 0.5384445 0
5 0.0008164595 0.09565414 0.07481755 0.05311687 0.01753576 383.0443 1066.2746 0     800    1500 1 0.5669157 0
6 0.0144162389 0.13613101 0.03950458 0.03384405 0.01839549 337.9317 1056.6110 0     800    1500 1 0.5479511 0
7 0.0291361361 0.10579557 0.05236064 0.01984253 0.02530839 375.8565 1061.5311 0     800    1500 1 0.4594113 0
8 0.0072422302 0.06572414 0.04388866 0.07142634 0.01843345 443.4329 1193.5129 0     800    1500 1 0.4863897 0
9 0.0173178373 0.12304579 0.04945120 0.04120239 0.02108065 265.5241 800.1153 0     800    1500 1 0.5303334 0
10 0.0038339205 0.12801141 0.04737147 0.03291192 0.02603099 474.4756 958.2335 0     800    1500 1 0.5053532 0
```

# Sensitivity Analyses (Probabilistic) (10)

```
413 ## Generate PSA dataset
414 # Number of simulations
415 n_sim <- 1000
416
417 # Generate PSA input dataset
418 df_psa_input <- generate_psa_params(n_sim = n_sim)
419 # First six observations
420 head(df_psa_input)
421
422 ### Histogram of parameters
423 ggplot(melt(df_psa_input, variable.name = "Parameter"), aes(x = value)) +
  facet_wrap(~Parameter, scales = "free") +
  geom_histogram(aes(y = after_stat(density))) +
  ylab("") +
  theme_bw(base_size = 16) +
  theme(axis.text = element_text(size = 6),
        axis.title.x = element_blank(),
        axis.title.y = element_blank(),
        axis.text.y = element_blank(),
        axis.ticks.y = element_blank())
433
```

# Sensitivity Analyses (Probabilistic) (11)



# Sensitivity Analyses (Probabilistic) (12)

```
434 ## 11.2 Run PSA
435
436 # Initialize data.frames with PSA output
437 # data.frame of costs
438 df_c <- as.data.frame(matrix(0,
439                         nrow = n_sim,
440                         ncol = n_str))
441 colnames(df_c) <- v_names_str
442 # data.frame of effectiveness
443 df_e <- as.data.frame(matrix(0,
444                         nrow = n_sim,
445                         ncol = n_str))
446 colnames(df_e) <- v_names_str
447
448 # Conduct probabilistic sensitivity analysis
449 # Run Markov model on each parameter set of PSA input dataset
450 n_time_init_psa_series <- Sys.time()
451 for (i in 1:n_sim) { # i <- 1
452   l_psa_input <- update_param_list(l_params_all, df_psa_input[i,])
453   # Outcomes
454   l_out_ce_temp <- calculate_ce_out(l_psa_input)
455   df_c[i, ] <- l_out_ce_temp$Cost
456   df_e[i, ] <- l_out_ce_temp$Effect
457   # Display simulation progress
458   if (i/(n_sim/100) == round(i/(n_sim/100), 0)) { # display progress every 5%
459     cat('\r', paste(i/n_sim * 100, "% done", sep = " "))
460   }
461 }
```

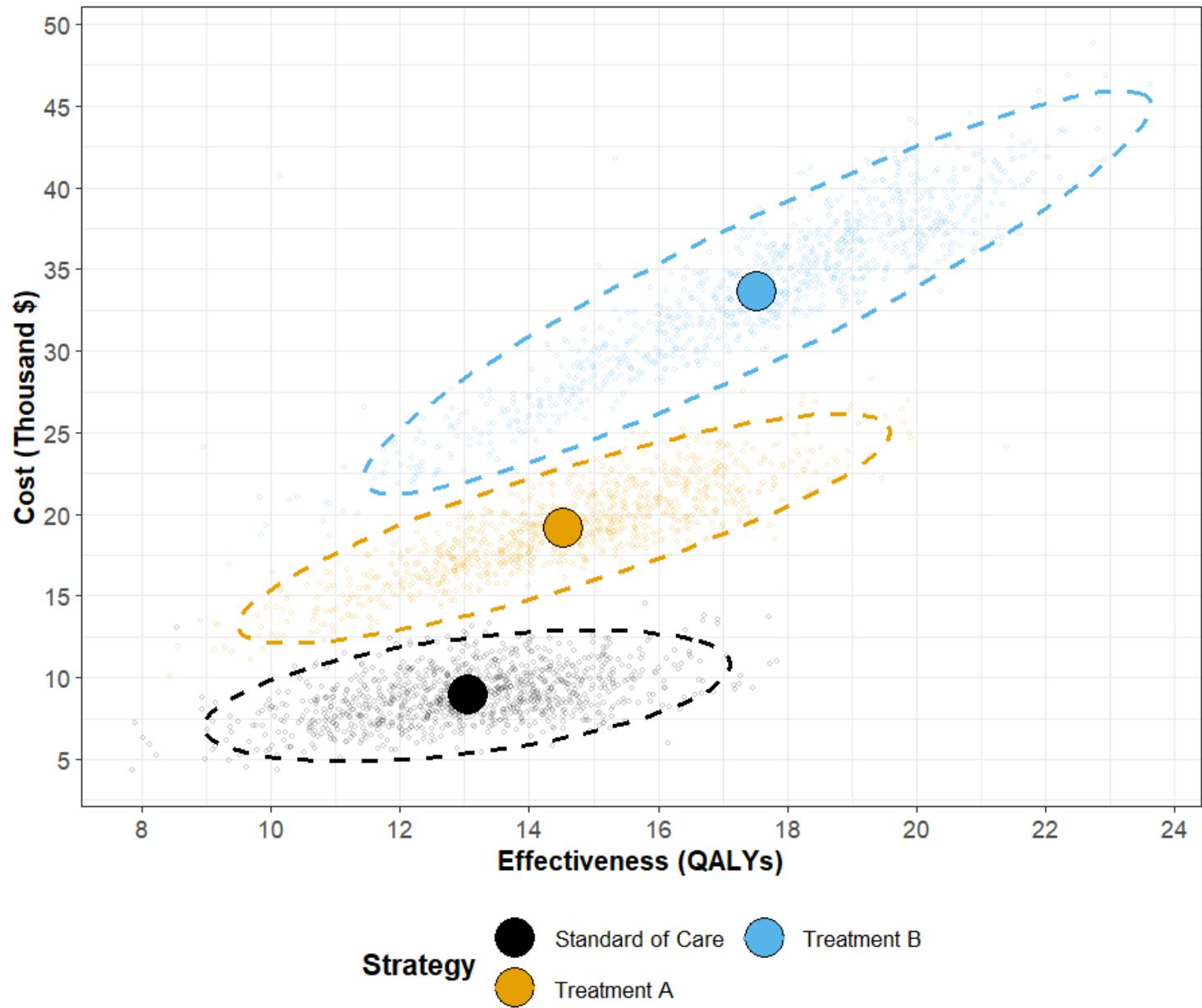
# Sensitivity Analyses (Probabilistic) (13)

```
462 n_time_end_psa_series <- Sys.time()
463 n_time_total_psa_series <- n_time_end_psa_series - n_time_init_psa_series
464 print(paste0("PSA with ", scales::comma(n_sim), " simulations run in series in ",
465                 round(n_time_total_psa_series, 2), " ",
466                 units(n_time_total_psa_series)))
+
+           units(n_time_total_psa_series)))
[1] "PSA with 1,000 simulations run in series in 9.96 secs"
> |
```

# Sensitivity Analyses (Probabilistic) (14)

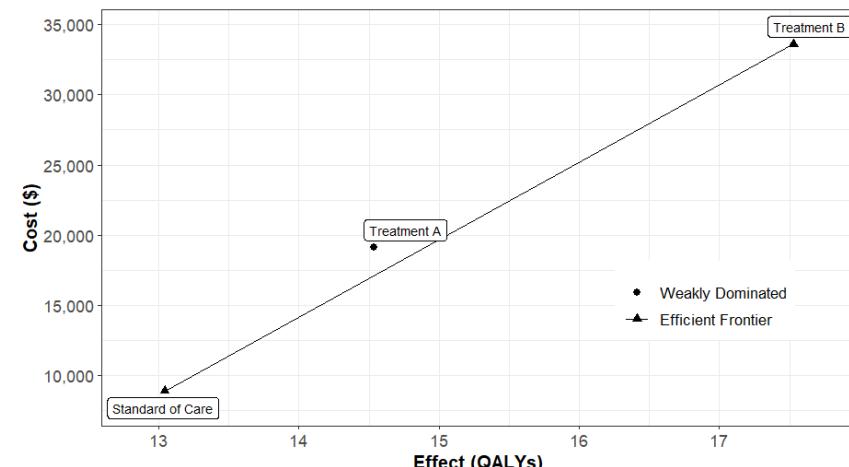
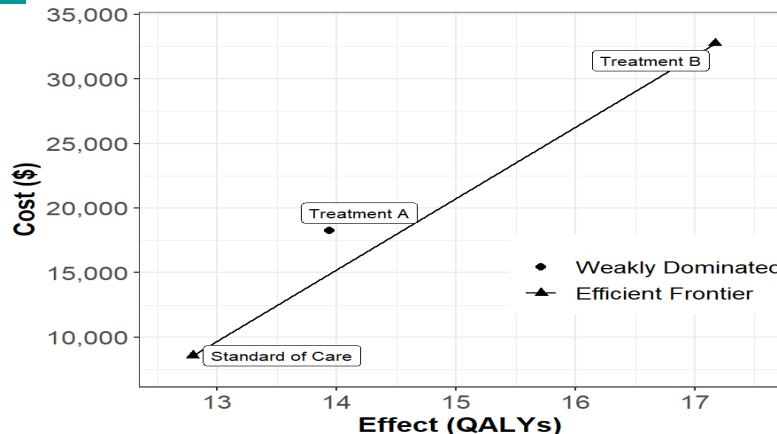
```
468 # 11.3 Visualize PSA results for CEA
469
470 ### Create PSA object
471 l_psa <- make_psa_obj(cost      = df_c,
472                         effectiveness = df_e,
473                         parameters   = df_psa_input,
474                         strategies   = v_names_str)
475 l_psa$strategies <- v_names_str
476 colnames(l_psa$effectiveness) <- v_names_str
477 colnames(l_psa$cost) <- v_names_str
478
479 # Vector with willingness-to-pay (WTP) thresholds.
480 v_wtp <- seq(0, 30000, by = 1000)
481
482 ## 11.3.1 Cost-Effectiveness Scatter plot
483
484 ### Cost-Effectiveness Scatter plot
485 txtsize <- 13
486 gg_scatter <- plot_psa(l_psa, txtsize = txtsize) +
487   ggthemes::scale_color_colorblind() +
488   ggthemes::scale_fill_colorblind() +
489   scale_y_continuous("Cost (Thousand $)",
490                       breaks = number_ticks(10),
491                       labels = function(x) x/1000) +
492   xlab("Effectiveness (QALYs)") +
493   guides(col = guide_legend(nrow = 2)) +
494   theme(legend.position = "bottom")
495 gg_scatter
```

# Sensitivity Analyses (Probabilistic) (15)



# Sensitivity Analyses (Probabilistic) (16)

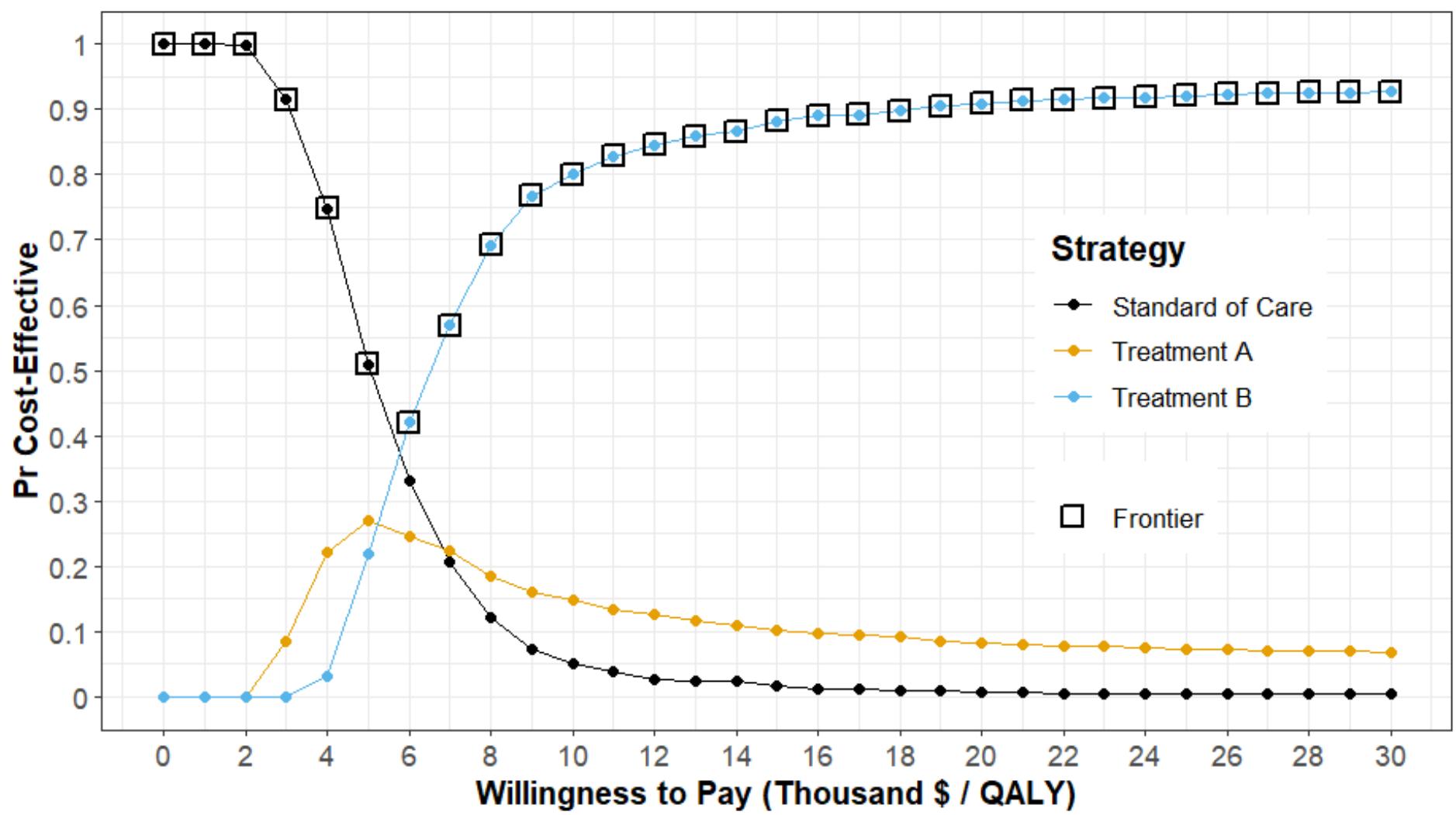
```
497 ## 11.3.2 Incremental cost-effectiveness ratios (ICERs) with probabilistic output
498
499 ### Incremental cost-effectiveness ratios (ICERs) with probabilistic output
500 # Compute expected costs and effects for each strategy from the PSA
501 df_out_ce_psa <- summary(l_psa)
502 df_cea_psa <- calculate_icers(cost      = df_out_ce_psa$meanCost,
503                                 effect     = df_out_ce_psa$meanEffect,
504                                 strategies = df_out_ce_psa$Strategy)
505 df_cea_psa
506
507 ## 11.3.3 Plot cost-effectiveness frontier with probabilistic output
508
509 ### Plot cost-effectiveness frontier with probabilistic output
510 plot_icers(df_cea_psa, label = "all", txtsize = txtsize) +
511   expand_limits(x = max(table_cea$QALYs) + 0.1) +
512   theme(legend.position = c(0.8, 0.3))
```



# Sensitivity Analyses (Probabilistic) (17)

```
514 ## 11.3.4 Cost-effectiveness acceptability curves (CEACs) and frontier (CEAF)
515
516 ### Cost-effectiveness acceptability curves (CEACs) and frontier (CEAF)
517 ceac_obj <- ceac(wtp = v_wtp, psa = l_psa)
518 # Regions of highest probability of cost-effectiveness for each strategy
519 summary(ceac_obj)
520 # CEAC & CEA plot
521 gg_ceac <- plot_ceac(ceac_obj, txtsize = txtsize, xlim = c(0, NA), n_x_ticks = 14) +
522   ggthemes::scale_color_colorblind() +
523   ggthemes::scale_fill_colorblind() +
524   theme(legend.position = c(0.8, 0.48))
525 gg_ceac
526
```

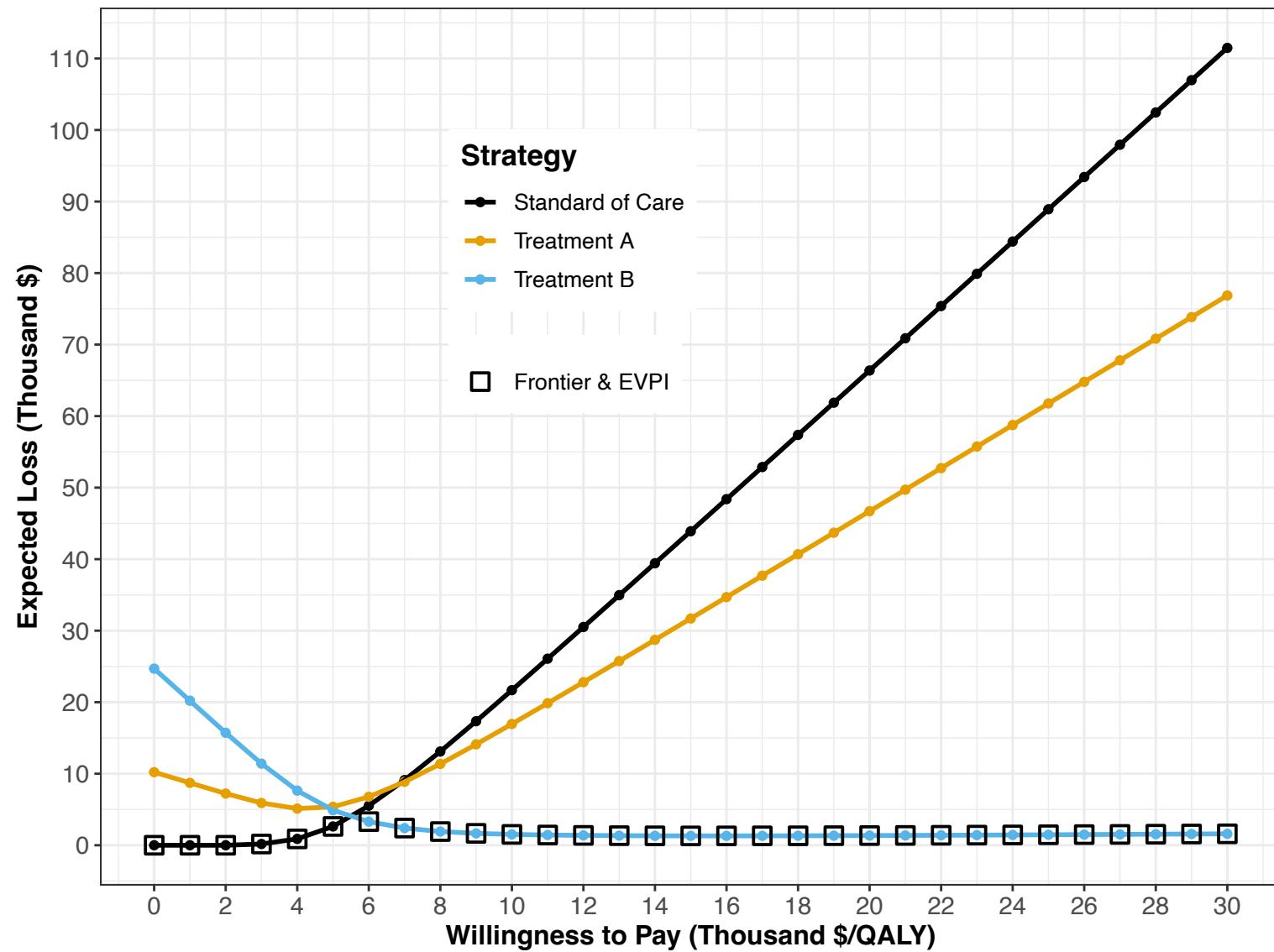
# Sensitivity Analyses (Probabilistic) (18)



# Advanced Topics: Expected Loss and Value of Information

```
527 ## 11.3.5 Expected Loss Curves (ELCs)
528
529 ### Expected Loss Curves (ELCs)
530 elc_obj <- calc_exp_loss(wtp = v_wtp, psa = l_psa)
531 elc_obj
532
533 # ELC plot
534 gg_elc <- plot_exp_loss(elc_obj, log_y = FALSE,
535                         txtsize = txtsize, xlim = c(0, NA), n_x_ticks = 14,
536                         col = "full") +
537                         ggthemes::scale_color_colorblind() +
538                         ggthemes::scale_fill_colorblind() +
539                         # geom_point(aes(shape = as.name("Strategy"))) +
540                         scale_y_continuous("Expected Loss (Thousand $)",
541                                             breaks = number_ticks(10),
542                                             labels = function(x) x/1000) +
543                         theme(legend.position = c(0.4, 0.7),)
544 gg_elc
545
546 ## 11.3.6 Expected value of perfect information (EVPI)
547
548 ### Expected value of perfect information (EVPI)
549 evpi <- calc_evpi(wtp = v_wtp, psa = l_psa)
550 # EVPI plot
551 gg_evpi <- plot_evpi(evpi, effect_units = "QALY",
552                         txtsize = txtsize, xlim = c(0, NA), n_x_ticks = 14) +
553                         scale_y_continuous("EVPI (Thousand $)",
554                                             breaks = number_ticks(10),
555                                             labels = function(x) x/1000)
556 gg_evpi
```

# Advanced Topics: Expected Loss and Value of Information



# Use the ConVOI website to learn more about these topics (<https://convoy-group.org>)

The screenshot shows the homepage of the ConVOI website. At the top is the ConVOI logo with the tagline "Collaborative Network For Value of Information". Below the logo is a section titled "What is a Value of Information analysis?". Underneath this is a large heading "Useful resources for Value of Information". Below the heading are five collapsed sections: "Reading resources", "Video resources", "Vol tools", "ConVOI GitHub", and "ConVOI training and courses".

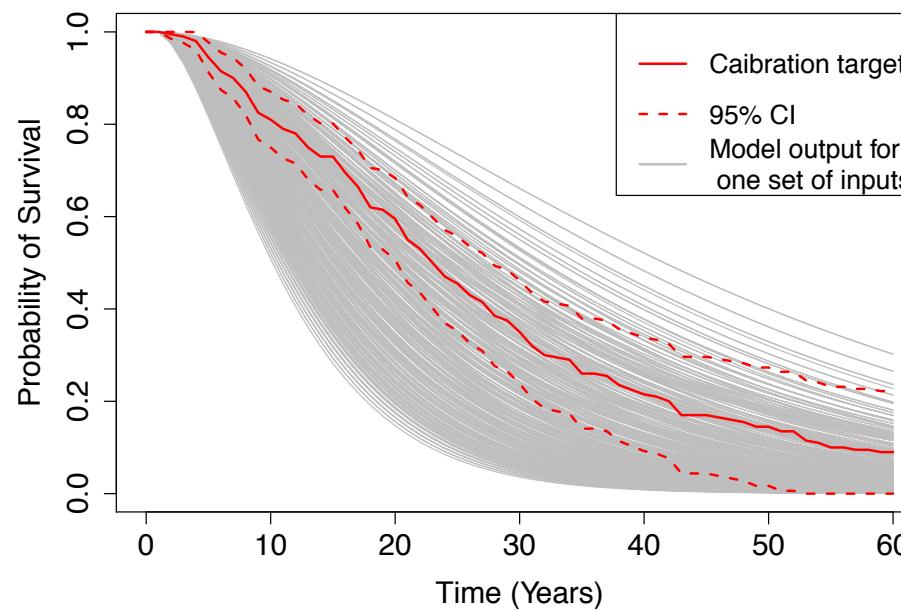
The screenshot shows the "ConVOI Members" page. It features a grid of 20 member profiles, each with a small photo, the member's name, and a "Read more" link. The members are arranged in four rows of five. The names listed are: Natalia Kunst, Anna Heath, Hawre Jalal, Fernando Alarid-Escudero, Gianluca Baio; Alan Brennan, Doug Coyle, David Glynn, Jeremy D. Goldhaber-Fiebert, Sabine Grimm; Chris Jackson, Erik Koffijberg, Nicolas Menzies, Claire Rothery, Mark Strong; Howard Thom, Haitham Tuffaha, Nicky Welton, Ed Wilson, Michael Fairley.

# R session

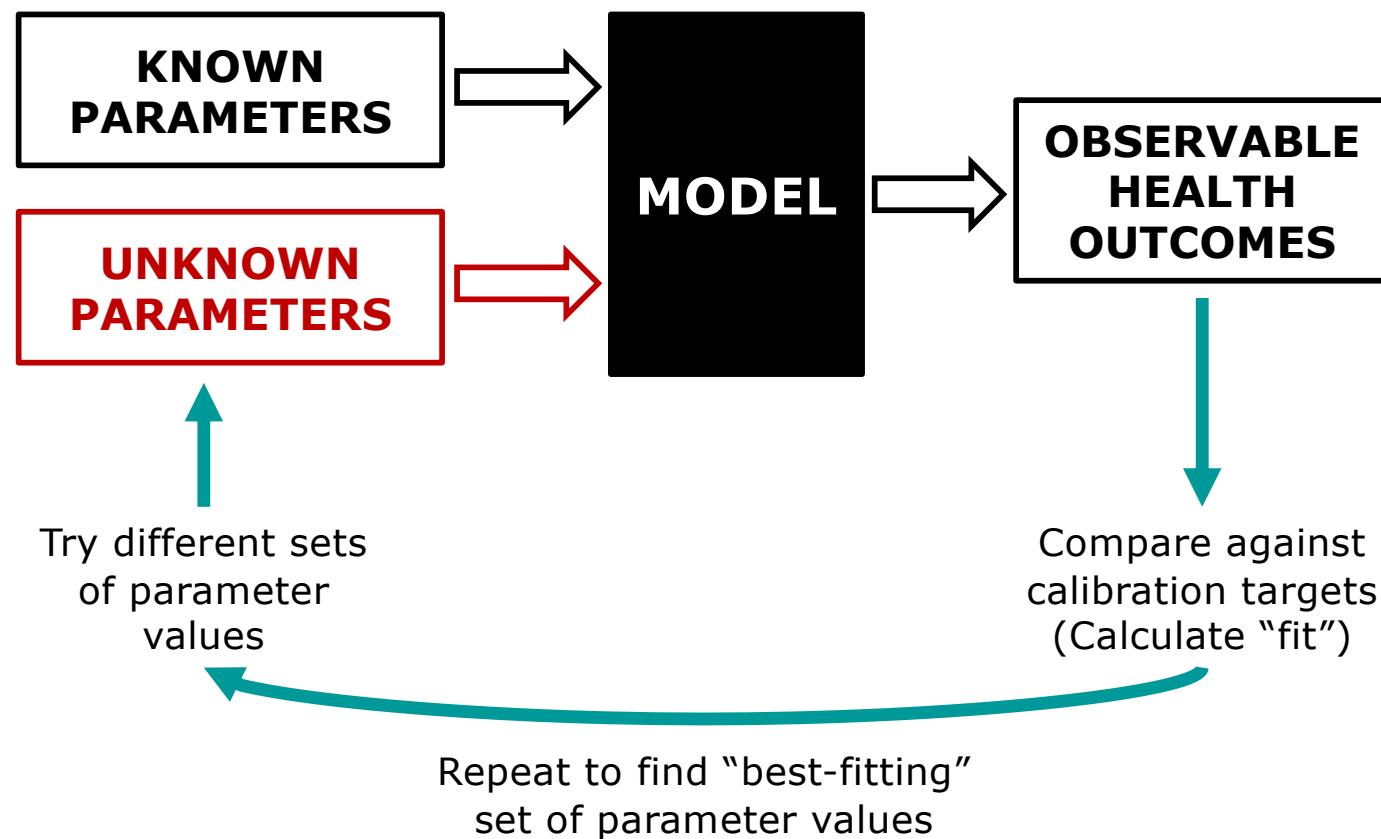
## Model Calibration

# Calibration definition

- Process of adjusting model input parameter values to match data on an outcome of interest (e.g., survival, prevalence, or incidence)
- Outcome(s) of interest = “calibration targets”



# Calibration process



# Calibration targets

- Empirical data to be replicated by the model
- Summary statistics (e.g. mean age of cancer diagnosis) or series of observations (e.g. age-specific incidence)
- Can calibrate to multiple targets (e.g. survival and prevalence) simultaneously
- Model should be able to output the outcomes of interest

# Calculating “fit”

- Goodness-of-Fit (GoF) is the quantitative measure of how the model is replicating the target data
- Different ways to measure GoF
  - Distance
  - Likelihood
- Notation
  - $M$  : a mathematical model (e.g., Markov model)
  - $\theta$  : Set of  $K$  parameters to be calibrated
  - $\phi = M(\theta)$  : Model output for parameter set  $\theta$
  - $y$  : Values of  $T$  calibration targets

# Distance GoF measures

- Sum of squared errors

$$SSE(\theta) = \sum_{i=1}^T (y_i - M_i(\theta))^2$$

- Weighted sum of squared errors
  - Assign different weight to different targets ( $w_i$ )
  - Often,  $w_i = \frac{1}{\sigma^2}$

$$WSSE(\theta) = \sum_{i=1}^T w_i (y_i - M_i(\theta))^2$$

# Likelihood as GoF

- How likely is the observed data to have come from a model  $M$  with set of parameter values  $\theta$  ?
- Assuming targets are independent, overall likelihood is the product of individual likelihoods

$$L(y|M(\theta)) = \prod_{i=1}^T L_i(y_i|M(\theta))$$

- Generally, we work with log-likelihood

$$\mathcal{L}(y|M(\theta)) = \sum_{i=1}^T \log L_i(y_i|M_i(\theta))$$

# Commonly used likelihoods

- **Normal distribution**

$$\log L(y_i, \sigma_i | M_i(\theta)) = -\frac{1}{2} \log(2\pi\sigma_i^2) - \frac{1}{\sigma_i^2} (y_i - M_i(\theta))^2$$

- In R: `dnorm(x=yi, mean=Mi(θ), sd=σi, log=T)`

- **Binomial distribution**

$$\log L(y_i, n_i | M_i(\theta)) = \log \binom{n_i}{y_i} + y_i \log(M_i(\theta)) + (n_i - y_i) \log(1 - M_i(\theta))$$

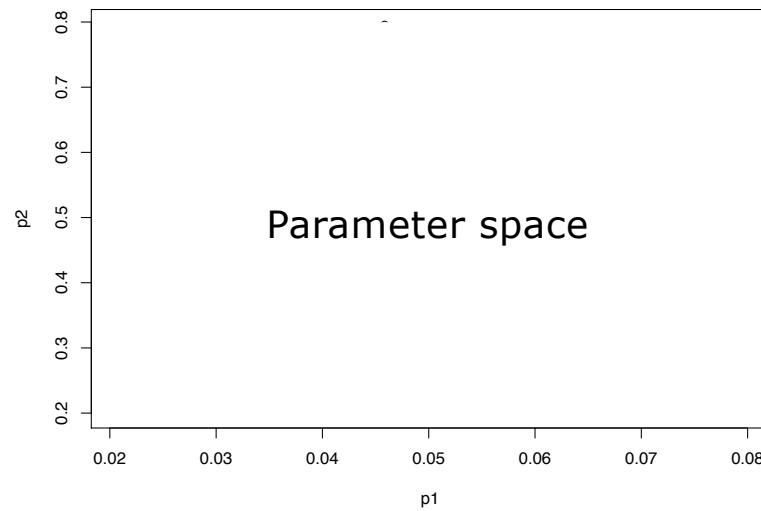
- In R: `dbinom(x=yi, size=ni, prob=Mi(θ), log=T)`

- **Multinomial distribution**

- In R: `dmultinom(x=yi, prob=Mi(θ), log=T)`

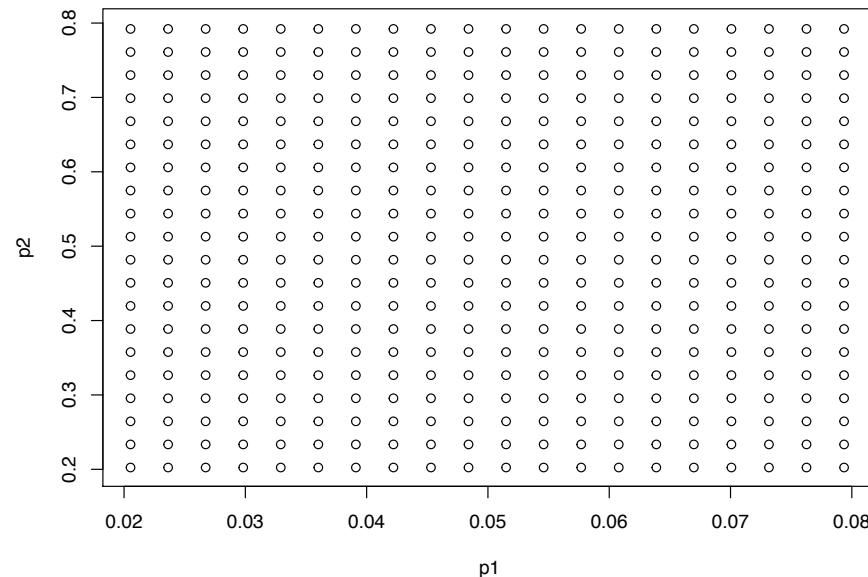
# Search Strategy

- Define plausible ranges for parameter whose values are unknown
- Use a search strategy to “search” through the input parameter space
  - Run the model for sets of parameter values generated by search strategy



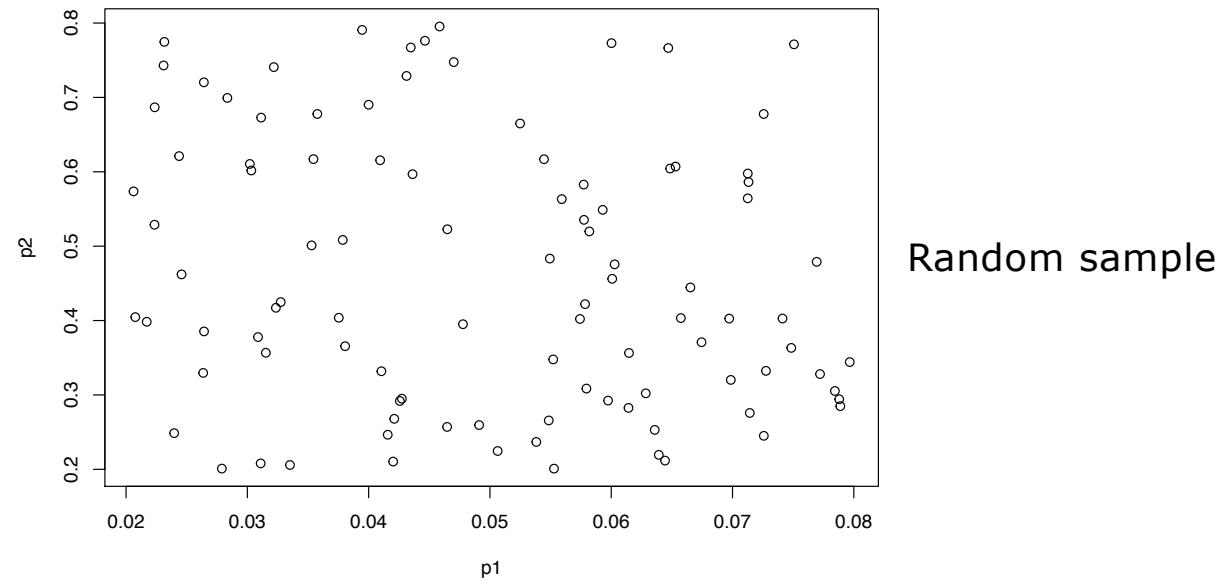
# Grid Search

- Run model for all possible combinations of parameter values
- Often infeasible



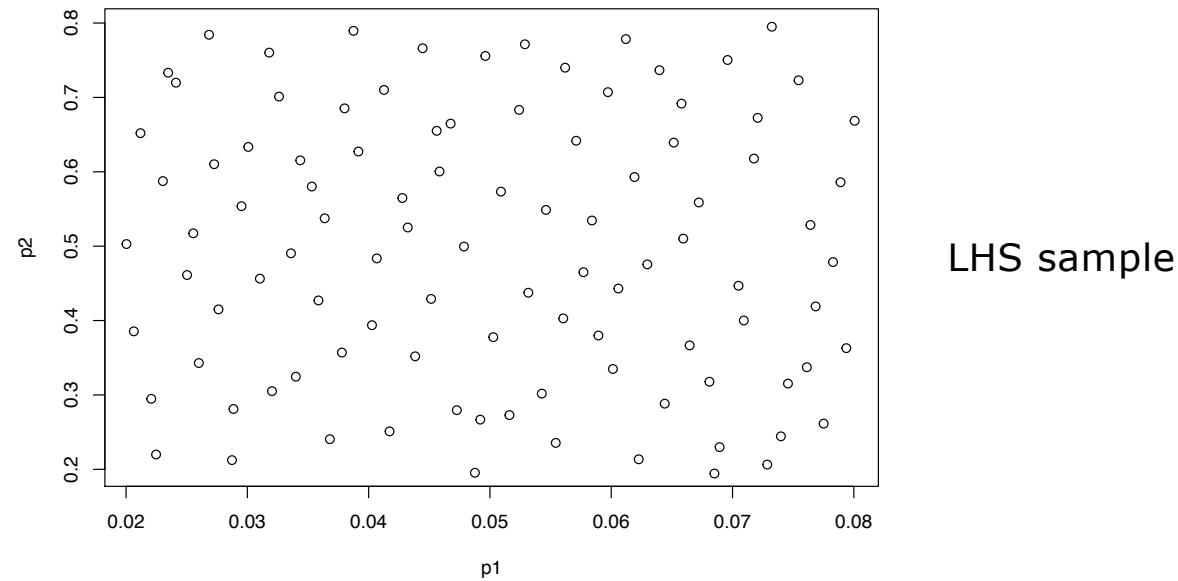
# Random Search

- Randomly sample a large number of parameter value sets from probabilistic distributions
- Use "Latin hypercube sampling" (LHS) to ensure sample captures the full parameter space



# Random Search

- Randomly sample a large number of parameter value sets from probabilistic distributions
- Use "Latin hypercube sampling" (LHS) to ensure sample captures the full parameter space

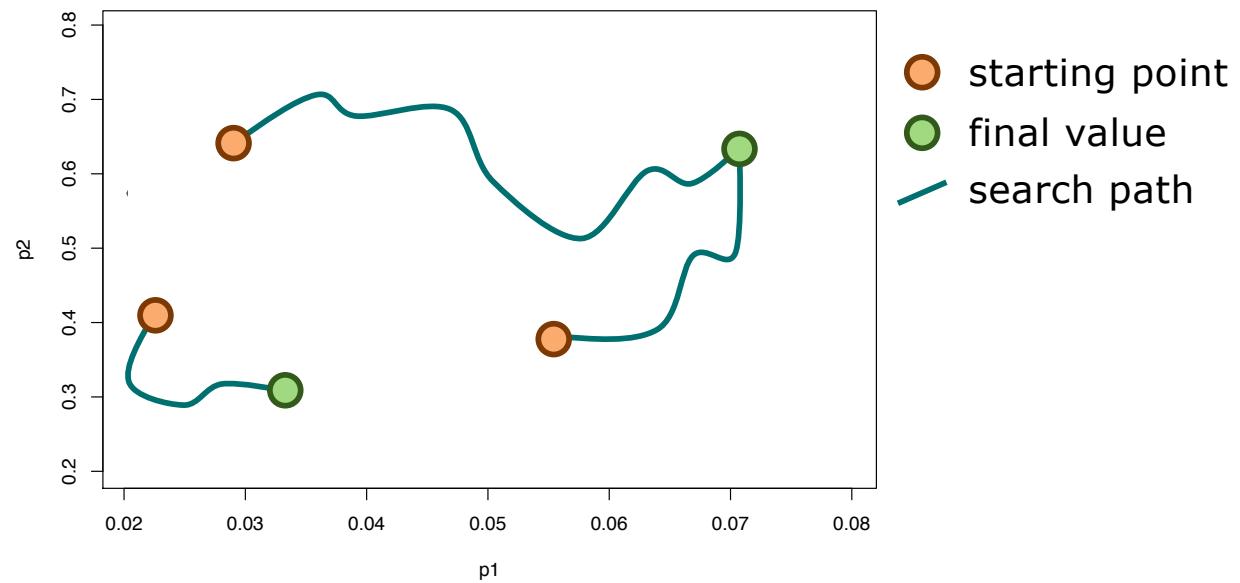


# Iterative Search

- Use fits of past input values to determine which input values to try next
- Directed methods
  - Nelder-Mead (simplex method)
  - Gradient-descent and others
- Meta-heuristic algorithms
  - Genetic algorithms
  - Simulated annealing

# Nelder-Mead Algorithm

- Downhill simplex method
- Must be run multiple times for different starting points to avoid local extrema



Example:  
Calibrating a 3-state cancer model

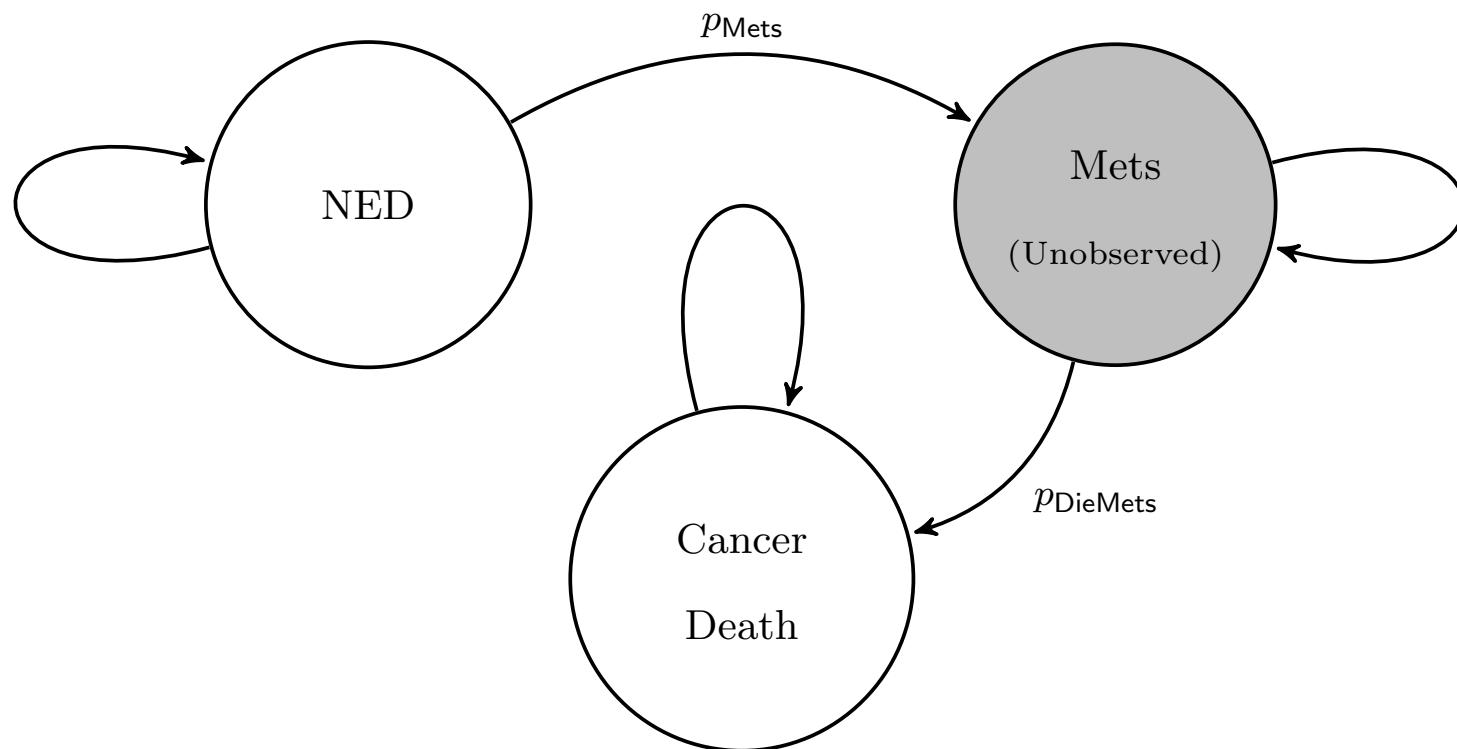
# 3-state cancer model

- Relative survival as reported by the Surveillance, Epidemiology, and End Results (SEER) Program, represents cancer survival in the absence of other causes of death
- Cancer Markov models often have a distant metastasis state, a state not directly observed in SEER, from which cancer deaths are presumed to occur
- These models can then be used to evaluate interventions that affect transitions to and from this state

Alarid-Escudero F, Maclehose RF, Peralta Y, Kuntz KM, & Enns, EA. Non-identifiability in model calibration and implications for medical decision making. *Medical Decision Making*, 2018;38(7):810–21.

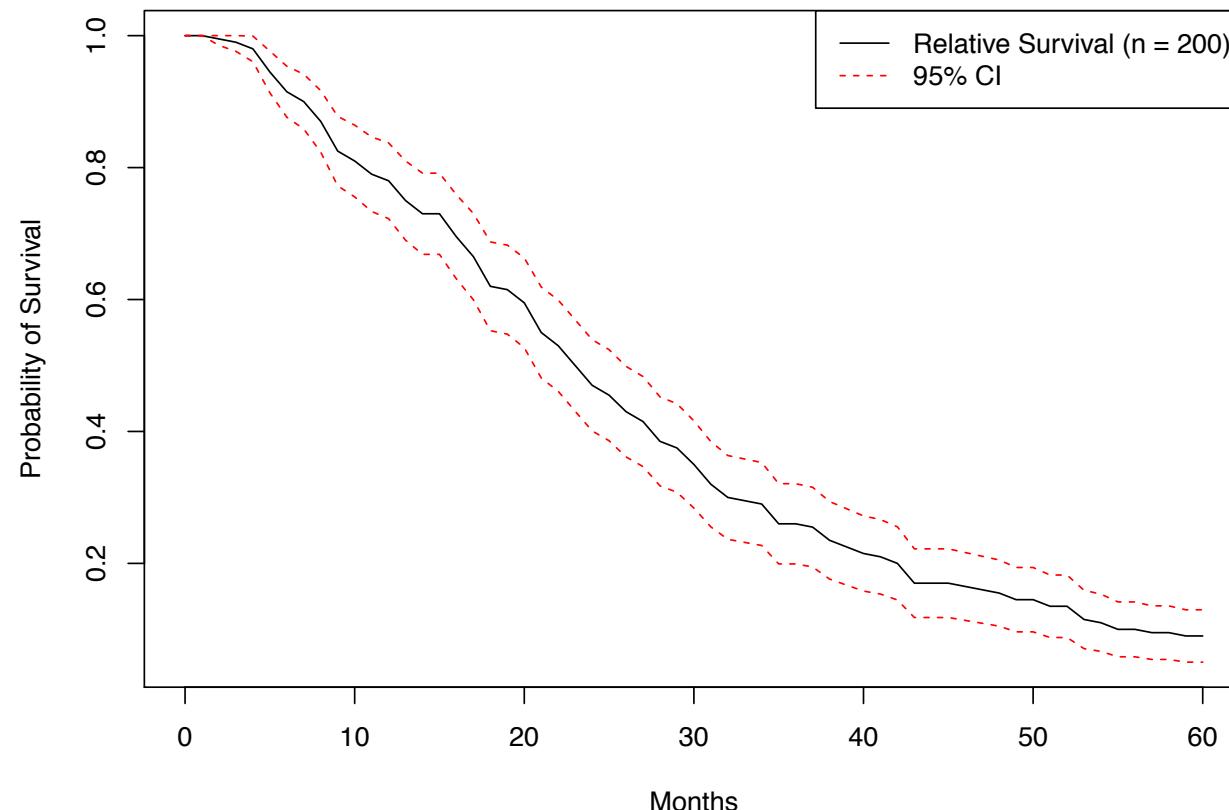
# 3-state cancer model

- Two unknown transition probabilities
  - p.Mets: Monthly risk of developing distant recurrence, range = [0.04, 0.16]
  - p.DieMets: Monthly risk of cancer death, range = [0.04, 0.16]



# Target: Relative survival

- Data frame “CRS\_targets\$Surv” stored in data file “CRS\_CalibTargets.RData”



# Calibration R code template

```
### Load calibration targets ###  
  
### Load model as a function ###  
  
### Specify calibration parameters ###  
  
### Calibrate ###  
  
### Explore best-fitting input sets ###
```

# R Session

# Initialize and load libraries (00\_prep.R)

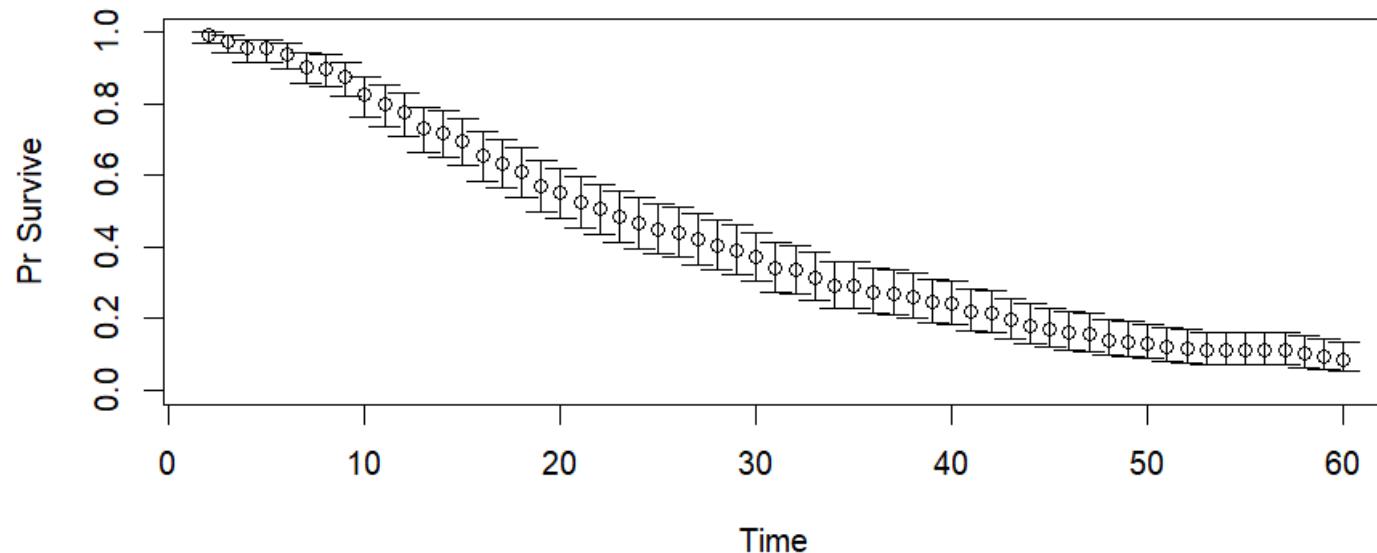
```
1  ### Run all code in this script to download the desired R packages
2  ### from either CRAN or GitHub
3
4
5  # first download and use this package to conveniently install other packages
6  if (!require('pacman')) install.packages('pacman'); library(pacman)
7
8  # load (install if required) packages from CRAN
9  p_load("devtools", "lhs", "plotrix", "psych", "DEoptim", "matrixStats", "scatterplot3d")
10
11 # Install IMIS
12 devtools::install_version("IMIS", version = "0.1", repos = "http://cran.us.r-project.org")
13
```

# Random Search (CRS\_Calib\_RandomSearch.R)

```
32 - #####  
33 - ##### Load packages and function files ####  
34 - #####  
35 # calibration functionality  
36 library(lhs)  
37  
38 # visualization  
39 library(plotrix)  
40 library(psych)  
41  
42  
43 - #####  
44 - ##### Load target data ####  
45 - #####  
46 load("CRS_CalibTargets.RData")  
47 lst_targets <- CRS_targets  
48
```

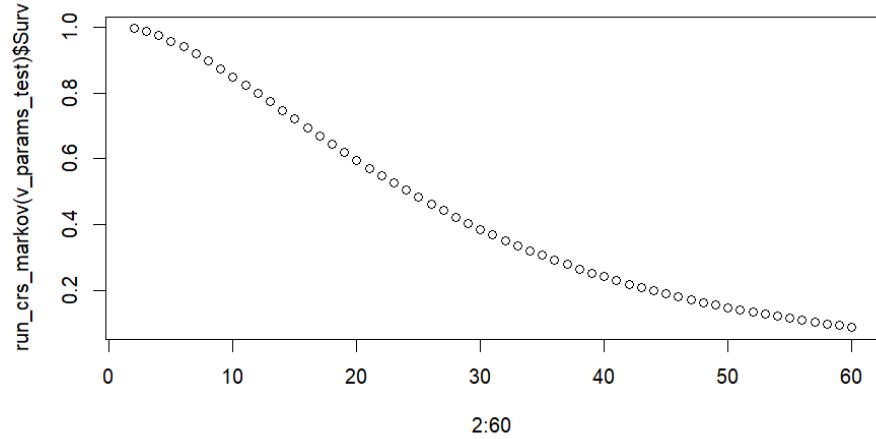
# Plot targets

```
49 # Plot the targets
50
51 # TARGET 1: Survival ("Surv")
52 plotrix:::plotCI(x = 1st_targets$Surv$time, y = 1st_targets$Surv$value,
53                   ui = 1st_targets$Surv$ub,
54                   li = 1st_targets$Surv$lb,
55                   ylim = c(0, 1),
56                   xlab = "Time", ylab = "Pr Survive")
57
58 # TARGET 2: (if you had more...)
59 # plotrix:::plotCI(x = 1st_targets$Target2$time, y = 1st_targets$Target2$value,
60 #                     ui = 1st_targets$Target2$ub,
61 #                     li = 1st_targets$Target2$lb,
62 #                     ylim = c(0, 1),
63 #                     xlab = "Time", ylab = "Target 2")
64
```



# Model as function

```
6 ###### Load model as a function #####
7 ##### Load model as a function #####
8 # - inputs are parameters to be estimated through calibration
9 # - outputs correspond to the target data
10 # Check that it works
11 v_params_test <- c(p_Mets = 0.10, p_DieMets = 0.05)
12 plot(y= run_crs_markov(v_params_test)$Surv, x=2:60) # It works!
```



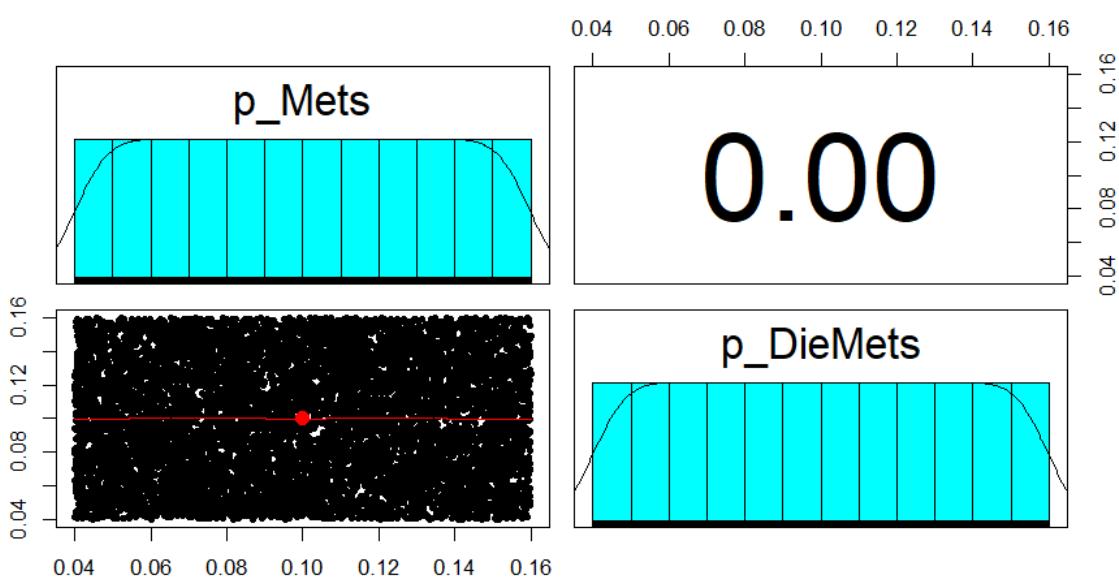
```
1 ##### CRS Markov model in a function #####
2 run_crs_markov <- function(v_params) {
3   with(as.list(v_params), {
4     ## Markov model parameters
5     n_t <- 60                                # time horizon, number of cycles
6     v_n <- c("NED", "Mets", "Death") # the 3 states of the model
7     n_s <- length(v_n)                      # number of health states
8
9     # Transition probabilities
10    # p_Mets      = 0.10                  # probability to become sicker when sick
11    # p_DieMets   = 0.05                  # hazard ratio of death in sick vs healthy
12
13    ##### INITIALIZATION #####
14    # create the cohort trace
15    m_M <- matrix(NA, nrow = n_t + 1 ,
16                  ncol = n_s,
17                  dimnames = list(0:n_t, v_n)) # create Markov trace (n_t + 1 b
18
19    m_M[1, ] <- c(1, 0, 0)                  # initialize Markov trace
20
21    # create transition probability matrix for NO treatment
22    m_P <- matrix(0,
23                  nrow = n_s,
24                  ncol = n_s,
25                  dimnames = list(v_n, v_n))
26    # fill in the transition probability array
27    ### From NED
28    m_P["NED", "NED"] <- 1 - (p_Mets)
29    m_P["NED", "Mets"] <- p_Mets
30    m_P["NED", "Death"] <- 0                 # Not allowed to die from cancer in NED
31    ### From Mets
32    m_P["Mets", "NED"] <- 0
33    m_P["Mets", "Mets"] <- 1 - (p_DieMets)
34    m_P["Mets", "Death"] <- p_DieMets
35    ### From Death
36    m_P["Death", "Death"] <- 1
37
38    # check rows add up to 1
39    if (!isTRUE(all.equal(as.numeric(rowSums(m_P)), as.numeric(rep(1, n_s))))) {
40      stop("This is not a valid transition Matrix")
41    }
42
43    ##### PROCESS #####
44
45    for (t in 1:n_t){                         # throughout the number of cycles
46      m_M[t + 1, ] <- m_M[t, ] %*% m_P # estimate the Markov trace for cycle t
47    }
48
49    ##### EPIDEMIOLOGICAL OUTPUT #####
50    ##### Overall Survival (OS) #####
51    v_os <- 1 - m_M[, "Death"] # calculate the overall survival (OS) probability
52
53    ##### RETURN OUTPUT #####
54    out <- list(Surv = v_os[-c(1:2)])
55
56    return(out)
57  }
58}
59}
```

# Set up calibration parameters

```
79 ######
80 ##### Specify calibration parameters #####
81 #####
82 # Specify seed (for reproducible sequence of random numbers)
83 set.seed(072218)
84
85 # number of random samples
86 n_samp <- 10000
87
88 # names and number of input parameters to be calibrated
89 v_param_names <- c("p_Mets", "p_DieMets")
90 n_param <- length(v_param_names)
91
92 # range on input search space
93 lb <- c(p_Mets = 0.04, p_DieMets = 0.04) # lower bound
94 ub <- c(p_Mets = 0.16, p_DieMets = 0.16) # upper bound
95
96 # number of calibration targets
97 v_target_names <- c("Surv")
98 n_target <- length(v_target_names)
99
```

# Calibrate – Priors (1)

```
101 #####  
102 ##### Calibrate! #####  
103 #####  
104 # record start time of calibration  
105 t_init <- Sys.time()  
106  
107 ### Generate a random sample of input values ###  
108  
109 # Sample unit Latin Hypercube  
110 m_lhs_unit <- randomLHS(n_samp, n_param)  
111  
112 # Rescale to min/max of each parameter  
113 m_param_samp <- matrix(nrow=n_samp, ncol=n_param)  
114 for (i in 1:n_param){  
115   m_param_samp[,i] <- qunif(m_lhs_unit[,i],  
116                               min = lb[i],  
117                               max = ub[i])  
118 }  
119 colnames(m_param_samp) <- v_param_names  
120  
121 # view resulting parameter set samples  
122 pairs.panels(m_param_samp)
```



# Calibrate – Run model w/ priors (2)

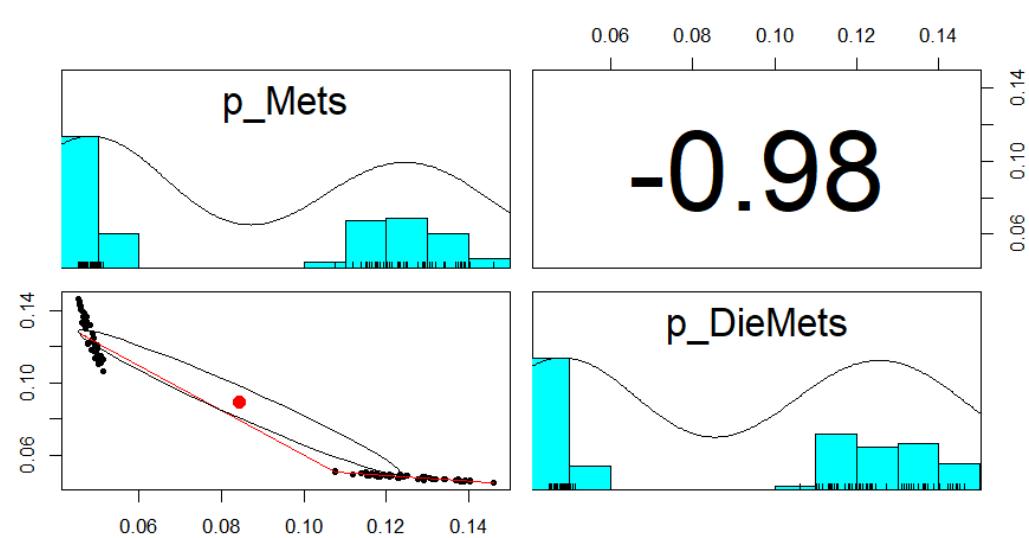
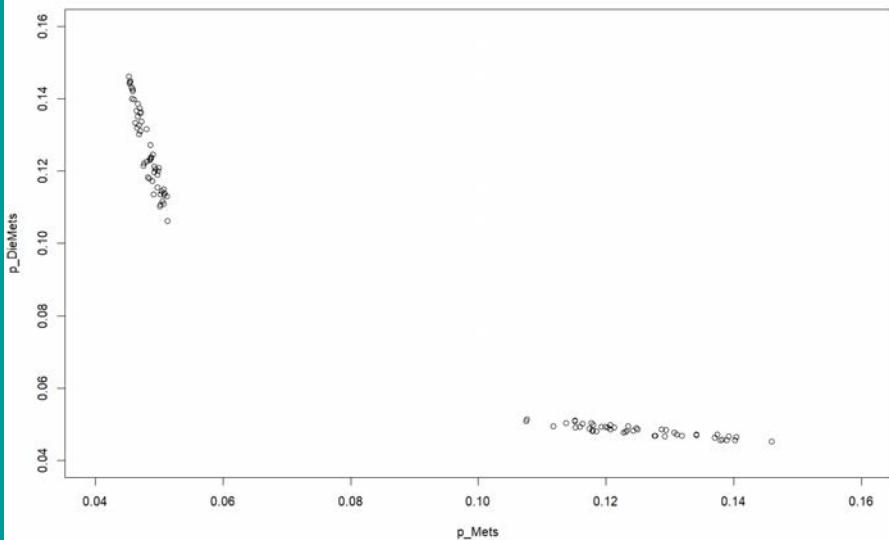
```
125  ### Run the model for each set of input values ###
126
127 # initialize goodness-of-fit vector
128 m_GOF <- matrix(nrow = n_samp, ncol = n_target)
129 colnames(m_GOF) <- paste0(v_target_names, "_fit")
130
131 # loop through sampled sets of input values
132 for (j in 1:n_samp){
133
134     ### Run model for a given parameter set  ###
135     model_res <- run_crs_markov(v_params = m_param_samp[j, ])
136
137
138     ### Calculate goodness-of-fit of model outputs to targets  ###
139
140     # TARGET 1: Survival ("Surv")
141     # log likelihood
142     m_GOF[j,1] <- sum(dnorm(x = 1st_targets$Surv$value,
143                           mean = model_res$Surv,
144                           sd = 1st_targets$Surv$se,
145                           log = T))
146
147     # weighted sum of squared errors (alternative to log likelihood)
148     # w <- 1/(1st_targets$Surv$se^2)
149     # m_GOF[j,1] <- -sum(w*(1st_targets$Surv$value - v_res)^2)
150
151
152     # TARGET 2: (if you had more...)
153     # log likelihood
154     # m_GOF[j,2] <- sum(dnorm(x = 1st_targets$Target2$value,
155                           #                         mean = model_res$Target2,
156                           #                         sd = 1st_targets$Target2$se,
157                           #                         log = T))
158
159
160 } # End loop over sampled parameter sets
```

# Calibrate – Compute GOF for each (3)

```
163 ### Combine fits to the different targets into single GOF ####
164 # can give different targets different weights
165 v_weights <- matrix(1, nrow = n_target, ncol = 1)
166 # matrix multiplication to calculate weight sum of each GOF matrix row
167 v_GOF_overall <- c(m_GOF%*%v_weights)
168 # Store in GOF matrix with column name "Overall"
169 m_GOF <- cbind(m_GOF,Overall_fit=v_GOF_overall)
170
171 # Calculate computation time
172 comp_time <- Sys.time() - t_init
173
```

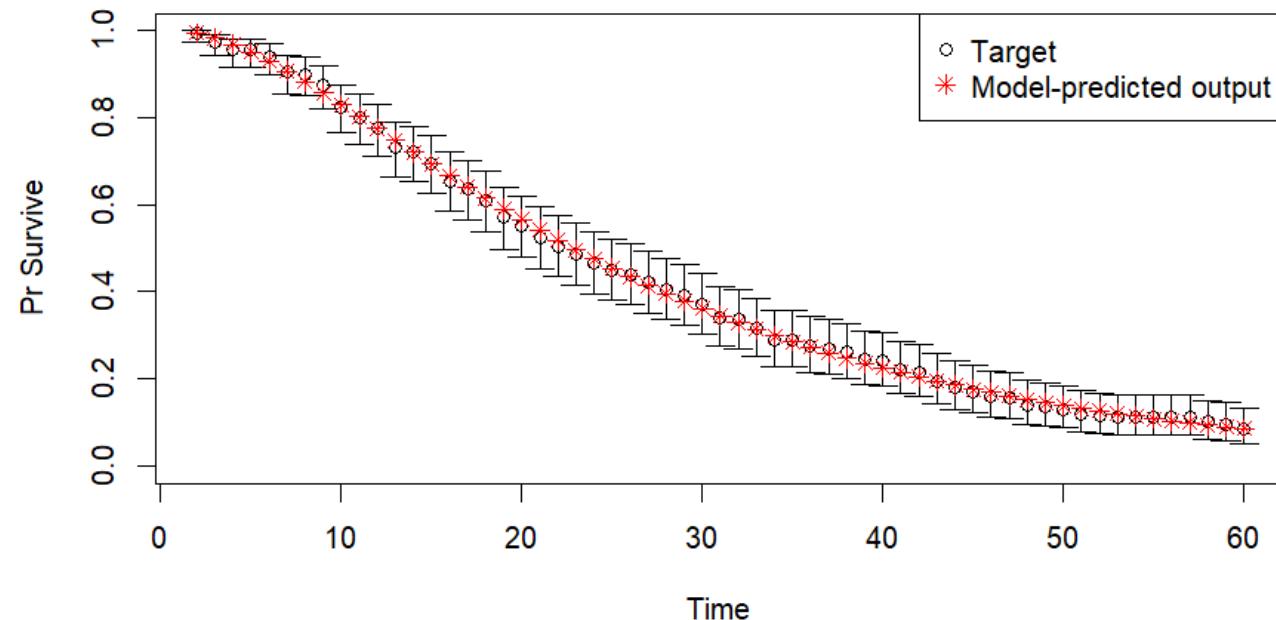
# Calibrate – Posteriors (4)

```
174 - ##### Exploring best-fitting input sets #####
175 - ##### Exploring best-fitting input sets #####
176 - #####
177
178 # Arrange parameter sets in order of fit
179 m_calib_res <- cbind(m_param_samp,m_GOF)
180 m_calib_res <- m_calib_res[order(-m_calib_res[, "Overall_fit"] ),]
181
182 # Examine the top 10 best-fitting sets
183 m_calib_res[1:10,]
184
185 # Plot the top 100 (top 10%)
186 plot(m_calib_res[1:100,1],m_calib_res[1:100,2],
187       xlim=c(lb[1],ub[1]),ylim=c(lb[2],ub[2]),
188       xlab = colnames(m_calib_res)[1],ylab = colnames(m_calib_res)[2])
189
190 # Pairwise comparison of top 100 sets
191 pairs.panels(m_calib_res[1:100,v_param_names])
```



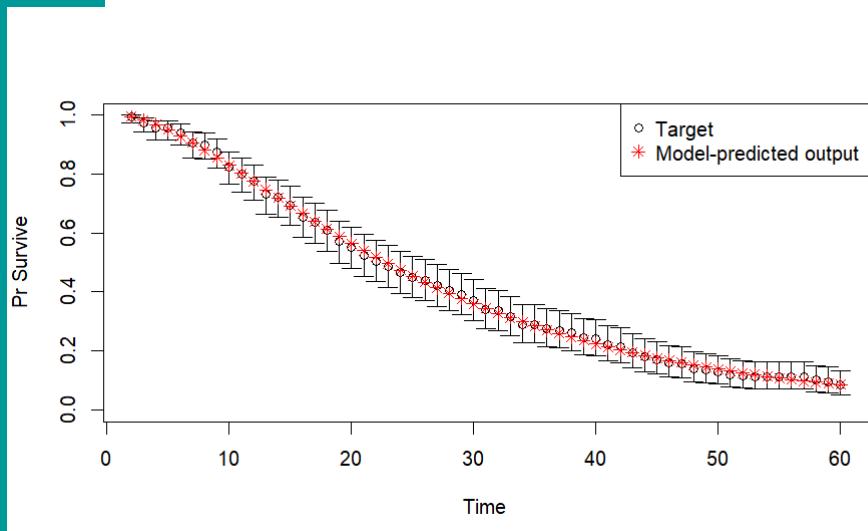
# Calibrate – Compute GOF for each (5)

```
193 ### Plot model-predicted output at best set vs targets ###
194 v_out_best <- run_crs_markov(m_calib_res[1,])
195
196 # TARGET 1: Survival ("Surv")
197 plotrix:::plotCI(x = 1st_targets$Surv$time, y = 1st_targets$Surv$value,
198                 ui = 1st_targets$Surv$ub,
199                 li = 1st_targets$Surv$lb,
200                 ylim = c(0, 1),
201                 xlab = "Time", ylab = "Pr Survive")
202 points(x = 1st_targets$Surv$time,
203         y = v_out_best$Surv,
204         pch = 8, col = "red")
205 legend("topright",
206         legend = c("Target", "Model-predicted output"),
207         col = c("black", "red"), pch = c(1, 8))
```

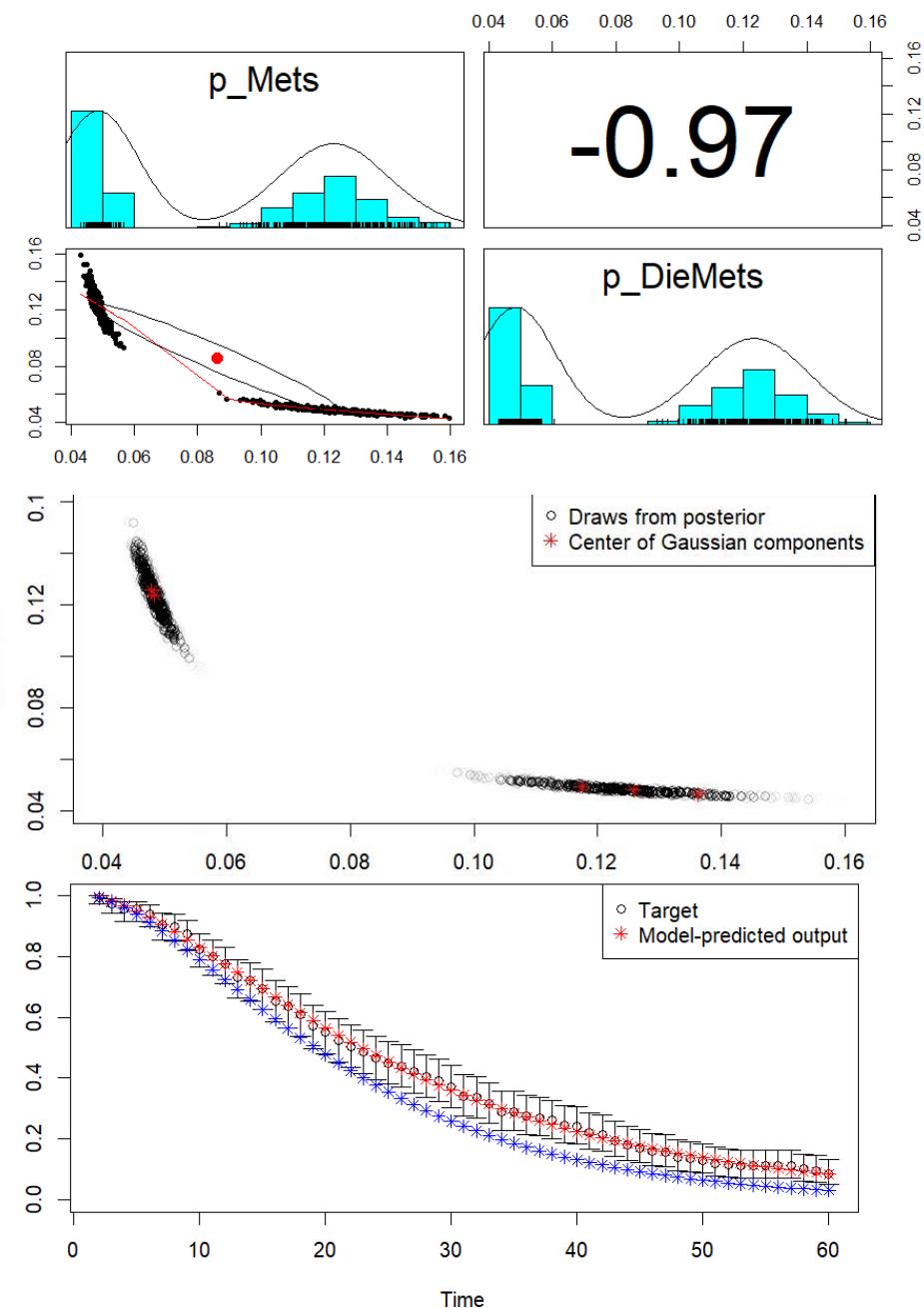


# Advanced Calibration

R CRS\_Calib\_NelderMead



R CRS\_Calib\_IMIS





# R session

## Microsimulation

# Paper on Microsimulation in R

*Tutorial*



## **Microsimulation Modeling for Health Decision Sciences Using R: A Tutorial**

**Eline M. Krijkamp, Fernando Alarid-Escudero, Eva A. Enns,  
Hawre J. Jalal, M. G. Myriam Hunink, and Petros Pechlivanoglou**

*Medical Decision Making*  
2018, Vol. 38(3) 400–422  
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DOI: [10.1177/0272989X18754513](https://doi.org/10.1177/0272989X18754513)  
[journals.sagepub.com/home/mdm](http://journals.sagepub.com/home/mdm)  
The SAGE logo features a stylized dollar sign (\$) symbol followed by the word "SAGE" in a bold, black, sans-serif font.

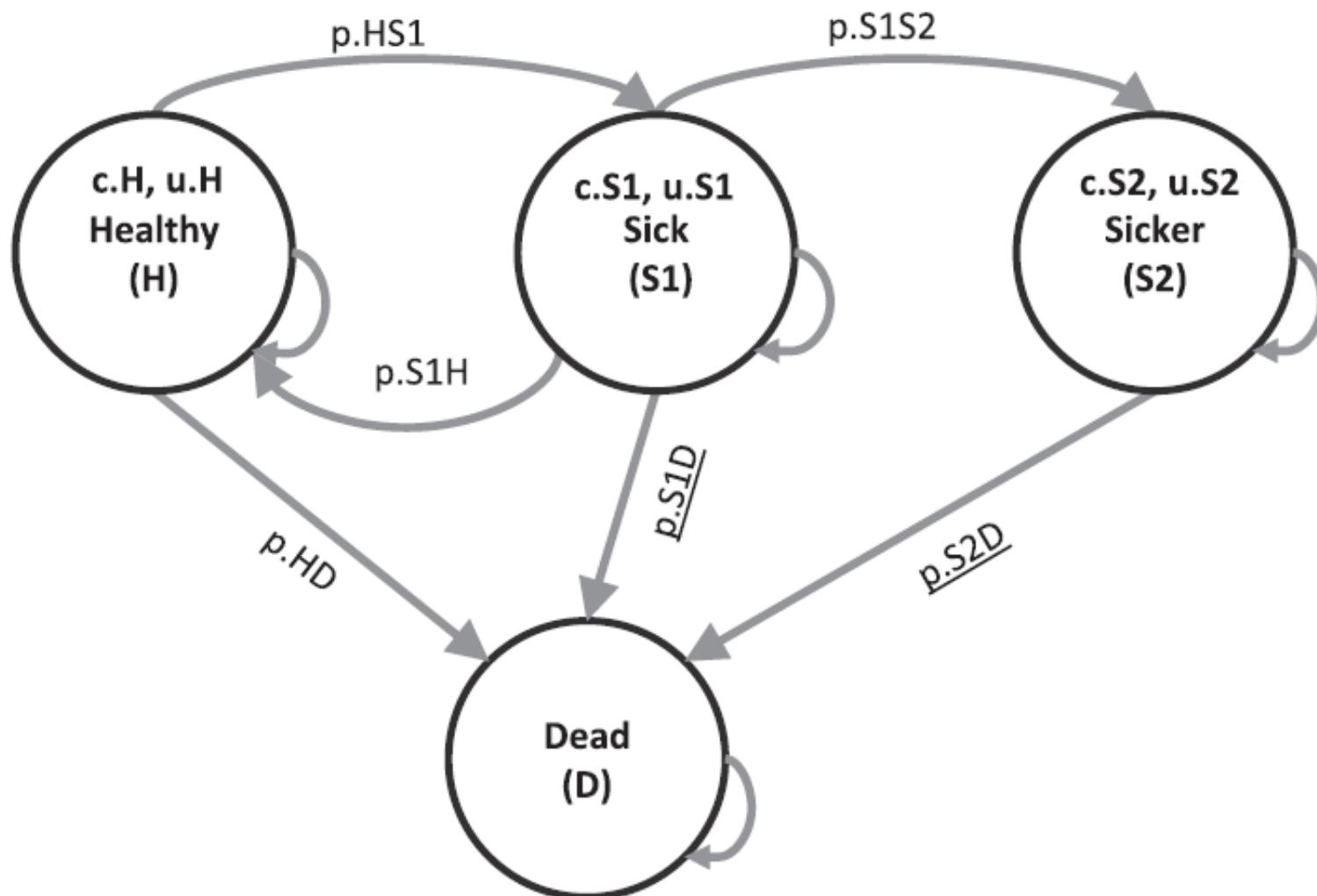
Two biggest differences from previous R code:

1. What happens inside model function: each individual of a specific # of people is simulated instead proportions of a cohort without a real defined size
2. Analysis must deal with Monte Carlo noise (first-order uncertainty) due to simulating a finite sample

# Microsimulation Algorithm

1. The individual starts the simulation in an initial health state and is assigned a cost and health outcome value associated with staying in this initial state for one cycle, taking into consideration any individual-level characteristics.
2. During each cycle, the individual's probability to transition to a different health state (or remain in the current health state) in the next cycle is assigned based on the health states previously occupied and the individual's characteristics.
3. The health state to which the individual will transition to in the next cycle is sampled from a categorical distribution based on the probability of transitioning to each possible health state. The individual will then either transition to a new health state or remain in the same health state at the end of the cycle.
4. Each health state is associated with a particular cost or health outcome value attributed to remaining in the health state for one cycle. This could represent the costs associated with a health state and in a cost-utility context, the utility of remaining in a certain health state for one cycle. State-specific costs and health outcome values can depend on the individual's characteristics (age, gender, etc.) as well as on past transitions of the individual. Transition values, one-time costs and one-time changes in health outcomes associated with the transition, may apply.
5. By aggregating all the state and transition values over the model's cycles (and applying discounting if needed) we can estimate the (discounted) total model outcomes for that individual's lifetime.

# Sick-Sicker Model



# Simple Microsimulation – Inputs

```
25 # Model input
26 n.i <- 100000          # number of simulated individuals
27 n.t <- 30              # time horizon, 30 cycles
28 v.n <- c("H","S1","S2","D") # the model states: Healthy (H), Sick (S1), Sicker (S2), Dead (D)
29 n.s <- length(v.n)      # the number of health states
30 v.M_1 <- rep("H", n.i)    # everyone begins in the healthy state
31 d.c <- d.e <- 0.03       # equal discounting of costs and QALYs by 3%
32 v.Trt <- c("No Treatment", "Treatment") # store the strategy names
33
34 # Transition probabilities (per cycle)
35 p.HD <- 0.005           # probability to die when healthy
36 p.HS1 <- 0.15            # probability to become sick when healthy
37 p.S1H <- 0.5              # probability to become healthy when sick
38 p.S1S2 <- 0.105          # probability to become sicker when sick
39 rr.S1 <- 3                # rate ratio of death when sick vs healthy
40 rr.S2 <- 10               # rate ratio of death when sicker vs healthy
41 r.HD <- -log(1 - p.HD)   # rate of death when healthy
42 r.S1D <- rr.S1 * r.HD     # rate of death when sick
43 r.S2D <- rr.S2 * r.HD     # rate of death when sicker
44 p.S1D <- 1 - exp(- r.S1D) # probability to die when sick
45 p.S2D <- 1 - exp(- r.S2D) # probability to die when sicker
46
47 # Cost and utility inputs
48 c.H <- 2000             # cost of remaining one cycle healthy
49 c.S1 <- 4000             # cost of remaining one cycle sick
50 c.S2 <- 15000            # cost of remaining one cycle sicker
51 c.Trt <- 12000           # cost of treatment (per cycle)
52
53 u.H <- 1                 # utility when healthy
54 u.S1 <- 0.75             # utility when sick
55 u.S2 <- 0.5              # utility when sicker
56 u.Trt <- 0.95            # utility when sick(er) and being treated
```

# Simple Microsimulation – MicroSim function (1)

```
63 MicroSim <- function(v.M_1, n.i, n.t, v.n, d.c, d.e,
64   TR.out = TRUE, TS.out = TRUE, Trt = FALSE, seed = 1) {
65 # Arguments:
66 # v.M_1: vector of initial states for individuals
67 # n.i: number of individuals
68 # n.t: total number of cycles to run the model
69 # v.n: vector of health state names
70 # d.c: discount rate for costs
71 # d.e: discount rate for health outcome (QALYs)
72 # TR.out: should the output include a microsimulation trace? (default is TRUE)
73 # TS.out: should the output include a matrix of transitions between states? (default is TRUE)
74 # Trt: are the n.i individuals receiving treatment? (scalar with a Boolean value, default is FALSE)
75 # seed: starting seed number for random number generator (default is 1)
76 # Makes use of:
77 # Probs: function for the estimation of transition probabilities
78 # Costs: function for the estimation of cost state values
79 # Effs: function for the estimation of state specific health outcomes (QALYs)
80
```

# Simple Microsimulation – MicroSim function (2)

```
81 v.dwc <- 1 / (1 + d.c) ^ (0:n.t)    # calculate the cost discount weight based on the discount rate d.c
82 v.dwe <- 1 / (1 + d.e) ^ (0:n.t)    # calculate the QALY discount weight based on the discount rate d.e
83
84 # create the matrix capturing the state name/costs/health outcomes for all individuals at each time point
85 m.M <- m.C <- m.E <- matrix(nrow = n.i, ncol = n.t + 1,
86                               dimnames = list(paste("ind", 1:n.i, sep = " "),
87                               paste("cycle", 0:n.t, sep = " ")))
88
89 m.M[, 1] <- v.M_1                      # indicate the initial health state
90
91 for (i in 1:n.i) {
92   set.seed(seed + i)                    # set the seed for every individual for the random number generator
93   m.C[i, 1] <- Costs(m.M[i, 1], Trt)  # estimate costs per individual for the initial health state conditional on treatment
94   m.E[i, 1] <- Effs (m.M[i, 1], Trt) # estimate QALYs per individual for the initial health state conditional on treatment
95
96 for (t in 1:n.t) {
97   v.p <- Probs(m.M[i, t])            # calculate the transition probabilities at cycle t
98
99   m.M[i, t + 1] <- sample(v.n, prob = v.p, size = 1) # sample the next health state and store that state in matrix m.M
100  m.C[i, t + 1] <- Costs(m.M[i, t + 1], Trt)    # estimate costs per individual during cycle t + 1 conditional on treatment
101  m.E[i, t + 1] <- Effs( m.M[i, t + 1], Trt)   # estimate QALYs per individual during cycle t + 1 conditional on treatment
102
103 } # close the loop for the time points
104 if(i/100 == round(i/100,0)) {          # display the progress of the simulation
105   cat('\r', paste(i/n.i * 100, "% done", sep = " "))
106 }
107 } # close the loop for the individuals
108
109 tc <- m.C %*% v.dwc                # total (discounted) cost per individual
110 te <- m.E %*% v.dwe                # total (discounted) QALYs per individual
111
112 tc_hat <- mean(tc)                 # average (discounted) cost
113 te_hat <- mean(te)                 # average (discounted) QALYs
```

# Simple Microsimulation – MicroSim function (3)

```
115 if (TS.out == TRUE) { # create a matrix of transitions across states
116   TS <- paste(m.M, cbind(m.M[, -1], NA), sep = "->") # transitions from one state to the other
117   TS <- matrix(TS, nrow = n.i)
118   rownames(TS) <- paste("Ind", 1:n.i, sep = " ") # name the rows
119   colnames(TS) <- paste("Cycle", 0:n.t, sep = " ") # name the columns
120 } else {
121   TS <- NULL
122 }
123
124 if (TR.out == TRUE) { # create a trace from the individual trajectories
125   TR <- t(apply(m.M, 2, function(x) table(factor(x, levels = v.n, ordered = TRUE))))
126   TR <- TR / n.i # create a distribution trace
127   rownames(TR) <- paste("Cycle", 0:n.t, sep = " ") # name the rows
128   colnames(TR) <- v.n # name the columns
129 } else {
130   TR <- NULL
131 }
132
133 results <- list(m.M = m.M, m.C = m.C, m.E = m.E, tc = tc, te = te,
134                   tc_hat = tc_hat, te_hat = te_hat,
135                   TS = TS, TR = TR) # store the results from the simulation in a list
136 return(results) # return the results
137 } # end of the MicroSim function
138
```

# Simple Microsimulation – Probs function

```
140 ##### Probability function
141 # The Probs function that updates the transition probabilities of every cycle is shown below.
142
143 Probs <- function(M_it) {
144   # M_it:    health state occupied by individual i at cycle t (character variable)
145
146   v.p.it <- rep(NA, n.s)      # create vector of state transition probabilities
147   names(v.p.it) <- v.n       # name the vector
148
149   # update v.p.it with the appropriate probabilities
150   v.p.it[M_it == "H"] <- c(1 - p.HS1 - p.HD, p.HS1, 0, p.HD)          # transition probabilities when healthy
151   v.p.it[M_it == "S1"] <- c(p.S1H, 1 - p.S1H - p.S1S2 - p.S1D, p.S1S2, p.S1D)  # transition probabilities when sick
152   v.p.it[M_it == "S2"] <- c(0, 0, 1 - p.S2D, p.S2D)                      # transition probabilities when sicker
153   v.p.it[M_it == "D"] <- c(0, 0, 0, 1)                                     # transition probabilities when dead
154   ifelse(sum(v.p.it) == 1, return(v.p.it), print("Probabilities do not sum to 1")) # return the transition probabilities
155 }
```

# Simple Microsimulation – Costs function

```
161 Costs <- function (M_it, Trt = FALSE) {  
162   # M_it: health state occupied by individual i at cycle t (character variable)  
163   # Trt:  is the individual being treated? (default is FALSE)  
164  
165   c.it <- 0                                # by default the cost for everyone is zero  
166   c.it[M_it == "H"]  <- c.H                # update the cost if healthy  
167   c.it[M_it == "S1"] <- c.S1 + c.Trt * Trt  # update the cost if sick conditional on treatment  
168   c.it[M_it == "S2"] <- c.S2 + c.Trt * Trt  # update the cost if sicker conditional on treatment  
169   return(c.it)                             # return the costs  
170 }
```

# Simple Microsimulation – Effs function

```
173  ### Health outcome function
174  # The Effs function to update the utilities at every cycle.
175
176 + Effs <- function (M_it, Trt = FALSE, cl = 1) {
177  # M_it: health state occupied by individual i at cycle t (character variable)
178  # Trt:  is the individual treated? (default is FALSE)
179  # cl:   cycle length (default is 1)
180
181  u.it <- 0                      # by default the utility for everyone is zero
182  u.it[M_it == "H"]   <- u.H      # update the utility if healthy
183  u.it[M_it == "S1"]  <- Trt * u.Trt + (1 - Trt) * u.S1 # update the utility if sick conditional on treatment
184  u.it[M_it == "S2"]  <- u.S2      # update the utility if sicker
185  QALYs <- u.it * cl            # calculate the QALYs during cycle t
186  return(QALYs)                 # return the QALYs
187 + }
```

# Simple Microsimulation – Running the model and computing results

```
190 ##### Run the simulation #####
191 sim_no_trt <- MicroSim(v.M_1, n.i, n.t, v.n, d.c, d.e, Trt = FALSE) # run for no treatment
192 sim_trt <- MicroSim(v.M_1, n.i, n.t, v.n, d.c, d.e, Trt = TRUE) # run for treatment
193 ##### Cost-effectiveness analysis #####
194
195 # store the mean costs (and the MCSE) of each strategy in a new variable v.C (vector costs)
196 v.C <- c(sim_no_trt$tc_hat, sim_trt$tc_hat)
197 se.C <- c(sd(sim_no_trt$tc), sd(sim_trt$tc)) / sqrt(n.i)
198 # store the mean QALYs (and the MCSE) of each strategy in a new variable v.E (vector health outcomes)
199 v.E <- c(sim_no_trt$te_hat, sim_trt$te_hat)
200 se.E <- c(sd(sim_no_trt$te), sd(sim_trt$te)) / sqrt(n.i)
201
202 delta.C <- v.C[2] - v.C[1] # calculate incremental costs
203 delta.E <- v.E[2] - v.E[1] # calculate incremental QALYs
204 se.delta.E <- sd(sim_trt$te - sim_no_trt$te) / sqrt(n.i) # Monte Carlo squared error (MCSE) of incremental costs
205 se.delta.C <- sd(sim_trt$tc - sim_no_trt$tc) / sqrt(n.i) # Monte Carlo squared error (MCSE) of incremental QALYs
206 ICER <- delta.C / delta.E # calculate the ICER
207 results <- c(delta.C, delta.E, ICER) # store the values in a new variable
```

# Simple Microsimulation – Running the model and computing results

```
209 # Create full incremental cost-effectiveness analysis table
210 table_micro <- data.frame(
211   c(round(v.C, 0), ""),
212   c(round(se.C, 0), ""),
213   c(round(v.E, 3), ""),
214   c(round(se.E, 3), ""),
215   c("", round(delta.C, 0), ""),
216   c("", round(se.delta.C, 0), ""),
217   c("", round(delta.E, 3), ""),
218   c("", round(se.delta.E, 3), ""),
219   c("", round(ICER, 0), ""))
220 )
221 rownames(table_micro) <- c(v.Trt, "* are MCSE values") # name the rows
222 colnames(table_micro) <- c("Costs", "*", "QALYs", "*", "Incremental Costs", "*", "QALYs Gained", "*", "ICER") # name the columns
223 table_micro # print the table
224
```

	Costs	*	QALYs	*	Incremental Costs	*	QALYs Gained	*	ICER
No Treatment	75996	183	15.823	0.016					
Treatment	141644	343	16.384	0.016		65648	164	0.561	0.001 117087

\* are MCSE values

# Sick-Sicker Model

**Table 1** Input Parameters for the Illustrative Microsimulation Model

Parameter	R name	Value
Time horizon ( $nt$ )	n.t	30 y
Cycle length ( $cl$ )	c1	1 y
Number of simulated individuals ( $ni$ )	n.i	100,000
Names of health states ( $n$ )	v.n	H, S1, S2, D
Annual discount rate (costs/QALYs) ( $dc/de$ )	d.c/d.e	3%
Annual transition probabilities		
- Disease onset (H to S1)	p.HS1	0.15
- Recovery (S1 to H)	p.S1H	0.5
- Disease progression (S1 to S2)	p.S1S2	0.105
Annual risks of death		
- H to D	p.HD	0.005
- Rate ratio of death in S1 v. healthy	rr.S1	3
- Rate ratio of death in S2 v. healthy	rr.S2	10
Annual costs		
- Healthy individuals	c.H	\$2,000
- Sick individuals in S1	c.S1	\$4,000
- Sick individuals in S2	c.S2	\$15,000
- Annual treatment cost per sick individual (S1 and S2)	c.Trt	\$12,000
Utility weights		
- Healthy individuals	u.H	1.00
- Sick individuals in S1	u.S1	0.75
- Sick individuals in S2	u.S2	0.50
Intervention effect		
- Utility for treated individuals in S1 (SD)	u.Trt	0.95
Time varying extension of Sick-Sicker model (Figure 1)		
- Treatment effect modifier at baseline	v.x	Uniform (0.95, 1.05)
- Utility decrement of treated sick individuals with every additional year of being sick (S1 and S2)	ru.S1S2	0.03
- Proportional increase of the mortality rate with every additional year of being sick/sicker (S1 and S2)	rp.S1S2	0.2

# Extended Microsimulation

**Same inputs as with simple Microsimulation but now with individual heterogeneity of effect and dwell-time effects on mortality and utility of being sick/sicker**

```
47 rp.S1S2 <- 0.2          # increase of the mortality rate with every additional year being sick  
59 ru.S1S2 <- 0.03        # decrease in utility of treated sick individuals with every  
60                      # additional year being sick/sicker  
61 v.x      <- runif(n.i, 0.95, 1.05) # vector capturing individuals' effect modifier at baseline
```

# Extended Microsimulation – MicroSim function (1)

**Same MicroSim function as before except now**

```
65 # The MicroSim functions for the extended microsimulation of the 'Sick-Sicker' model
66 # keeps track of what happens to each individual during each cycle.
67 MicroSim <- function(v.M_1, n.i, n.t, v.n, X = NULL, d.c, d.e,
68 #                                         TR.out = TRUE, TS.out = TRUE, Trt = FALSE, seed = 1) {
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99 # create the dur variable that stores the number of consecutive cycles the individual occupies either when sick or sicker
100 dur <- 0 # the individual start without history
101 m.C[i, 1] <- Costs(m.M[i, 1], Trt) # estimate costs per individual for the initial health state conditional on treatment
102 m.E[i, 1] <- Effs(m.M[i, 1], dur, Trt, X = X[i]) # estimate QALYs per individual for the initial health state conditional
103
104 for (t in 1:n.t) {
105   v.p <- Probs(m.M[i, t], dur) # calculate the transition probabilities at cycle t conditional on the duration of
106
107   m.M[i, t + 1] <- sample(v.n, prob = v.p, size = 1) # sample the new health state and store that state in matrix m.M
108   m.C[i, t + 1] <- Costs(m.M[i, t + 1], Trt) # estimate the cost per individual during cycle t + 1 conditional on trea
109   m.E[i, t + 1] <- Effs(m.M[i, t + 1], dur, Trt, X = X[i]) # estimate the utility per individual during cycle t + 1 co
110
111 if (m.M[i, t + 1] == "S1" | m.M[i, t + 1] == "S2") { # expression to identify sick/sicker individuals
112   dur <- dur + 1 # update the duration of being sick/sicker
113 } else {
114   dur <- 0} # reset duration variable
115
116 } # close the loop for the time points
117 if(i/100 == round(i/100,0)) { # display the progress of the simulation
118   cat('\r', paste(i/n.i * 100, "% done", sep = ""))
119 }
120 } # close the loop for the individuals
```

# Extended Microsimulation – MicroSim function (2)

Same MicroSim function now individual characteristics X are used as is duration in Sick/Sicker

```
65 # The MicroSim functions for the extended microsimulation of the 'Sick-Sicker' model
66 # keeps track of what happens to each individual during each cycle.
67 MicroSim <- function(v.M_1, n.i, n.t, v.n, X = NULL, d.c, d.e,
68 #                                     TR.out = TRUE, TS.out = TRUE, Trt = FALSE, seed = 1) {
99
100
101
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108
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111
112
113
114
115
116
117
118
119
120
```

```
99      # create the dur variable that stores the number of consecutive cycles the individual occupies either when sick or sicker
100     dur <- 0                                # the individual start without history
101     m.C[i, 1] <- Costs(m.M[i, 1], Trt)      # estimate costs per individual for the initial health state conditional on treatment
102     m.E[i, 1] <- Effs(m.M[i, 1], dur, Trt, X = X[i]) # estimate QALYs per individual for the initial health state conditional
103
104     for (t in 1:n.t) {
105       v.p <- Probs(m.M[i, t], dur)           # calculate the transition probabilities at cycle t conditional on the duration of
106
107       m.M[i, t + 1] <- sample(v.n, prob = v.p, size = 1) # sample the new health state and store that state in matrix m.M
108       m.C[i, t + 1] <- Costs(m.M[i, t + 1], Trt)        # estimate the cost per individual during cycle t + 1 conditional on trea
109       m.E[i, t + 1] <- Effs(m.M[i, t + 1], dur, Trt, X = X[i]) # estimate the utility per individual during cycle t + 1 co
110
111     if (m.M[i, t + 1] == "S1" | m.M[i, t + 1] == "S2") { # expression to identify sick/sicker individuals
112       dur <- dur + 1    # update the duration of being sick/sicker
113     } else {
114       dur <- 0}          # reset duration variable
115
116   } # close the loop for the time points
117   if(i/100 == round(i/100,0)) {                # display the progress of the simulation
118     cat('\r', paste(i/n.i * 100, "% done", sep = ""))
119   }
120 } # close the loop for the individuals
```

# Extended Microsimulation – Probs function

```
150 ##### Probability function
151 # The Probs function that updates the transition probabilities of every cycle is shown below.
152
153 Probs <- function(M_it, dur) {
154   # M_it: health state occupied by individual i at cycle t (character variable)
155   # dur: the duration of being sick (sick/sicker)
156
157   v.p.it <- rep(NA, n.s)      # create vector of state transition probabilities
158   names(v.p.it) <- v.n       # name the vector
159
160   # update probabilities of death after first converting them to rates and applying the rate ratio
161   r.S1D <- - log(1 - p.S1D)
162   r.S2D <- - log(1 - p.S2D)
163   p.S1D <- 1 - exp(- r.S1D * (1 + dur * rp.S1S2)) # calculate p.S1D conditional on duration of being sick/sicker
164   p.S2D <- 1 - exp(- r.S2D * (1 + dur * rp.S1S2)) # calculate p.S2D conditional on duration of being sick/sicker
165
166   # update v.p.it with the appropriate probabilities
167   v.p.it[M_it == "H"] <- c(1 - p.HS1 - p.HD, p.HS1, 0, p.HD)          # transition probabilities when healthy
168   v.p.it[M_it == "S1"] <- c(p.S1H, 1 - p.S1H - p.S1S2 - p.S1D, p.S1S2, p.S1D) # transition probabilities when sick
169   v.p.it[M_it == "S2"] <- c(0, 0, 1 - p.S2D, p.S2D)                      # transition probabilities when sicker
170   v.p.it[M_it == "D"] <- c(0, 0, 0, 1)                                     # transition probabilities when dead
171   ifelse(sum(v.p.it) == 1, return(v.p.it), print("Probabilities do not sum to 1")) # return the transition probabilities or
172 }
```

# Extended Microsimulation – Effs function

```
189 ### Health outcome function
190 # The Effs function to update the utilities at every cycle for the extended microsimulation.
191
192 Effs <- function (M_it, dur, Trt = FALSE, cl = 1, X = NULL) {
193   # M_it: health state occupied by individual i at cycle t (character variable)
194   # dur: the duration of being sick/sicker
195   # Trt: is the individual being treated? (default is FALSE)
196   # cl: the cycle length (default = 1 )
197   # X: the vector or matrix of individual characteristics (optional)
198
199   u.it <- 0           # by default the utility for everyone is zero
200   u.it[M_it == "H"] <- u.H # update the utility if healthy
201   u.it[M_it == "S1"] <- X * Trt * (u.Trt - dur * ru.S1S2) + (1 - Trt) * u.s1 # update the utility if sick conditional on treatment
202   u.it[M_it == "S2"] <- u.S2 # update the utility if sicker
203   QALYs <- u.it * cl      # calculate the QALYs during cycle t
204   return(QALYs)
205 }
```

	Costs	*	QALYs	*	Incremental Costs	*	QALYs Gained	*	ICER
No Treatment	62667	120	15.279	0.017					
Treatment	117455	231	15.785	0.017		54787	117	0.507	0.001 108090
* are MCSE values									

# Extended Microsimulation – Running the model and computing results

```
209 # Create full incremental cost-effectiveness analysis table
210 table_micro <- data.frame(
211   c(round(v.C, 0), ""),
212   c(round(se.C, 0), ""),
213   c(round(v.E, 3), ""),
214   c(round(se.E, 3), ""),
215   c("", round(delta.C, 0), ""),
216   c("", round(se.delta.C, 0), ""),
217   c("", round(delta.E, 3), ""),
218   c("", round(se.delta.E, 3), ""),
219   c("", round(ICER, 0), ""))
220 )
221 rownames(table_micro) <- c(v.Trt, "* are MCSE values") # name the rows
222 colnames(table_micro) <- c("Costs", "*", "QALYs", "*", "Incremental Costs", "*", "QALYs Gained", "*", "ICER") # name the columns
223 table_micro # print the table
224
```

	Costs	*	QALYs	*	Incremental Costs	*	QALYs Gained	*	ICER
No Treatment	62667	120	15.279	0.017					
Treatment	117455	231	15.785	0.017		54787	117	0.507	0.001 108090

\* are MCSE values

	Costs	*	QALYs	*	Incremental Costs	*	QALYs Gained	*	ICER
No Treatment	75996	183	15.823	0.016					
Treatment	141644	343	16.384	0.016		65648	164	0.561	0.001 117087

\* are MCSE values

# Wrap-up

# DARTH Publications

## An Overview of R in Health Decision Sciences

Hawre Jalal, MD, PhD, Petros Pechlivanoglou, MSc, PhD, Eline Krijkamp, MSc, Fernando Alarid-Escudero, MSc, Eva Enns, MS, PhD, M. G. Myriam Hunink, MD, PhD

### Microsimulation Modeling for Health Decision Sciences Using R: A Tutorial

Eline M. Krijkamp, Fernando Alarid-Escudero, Eva A. Enns, Hawre J. Jalal, M. G. Myriam Hunink, and Petros Pechlivanoglou



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Brief Report

### A Multidimensional Array Representation of State-Transition Model Dynamics

Eline M. Krijkamp<sup>1</sup>, Fernando Alarid-Escudero<sup>1</sup>, Eva A. Enns, Petros Pechlivanoglou, M.G. Myriam Hunink, Alan Yang<sup>1</sup>, and Hawre J. Jalal<sup>1</sup>



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#### PRACTICAL APPLICATION



### A Need for Change! A Coding Framework for Improving Transparency in Decision Modeling

Fernando Alarid-Escudero<sup>1</sup>, Eline M. Krijkamp<sup>2</sup>, Petros Pechlivanoglou<sup>3</sup>, Hawre Jalal<sup>4</sup>, Szu-Yu Zoe Kao<sup>5</sup>, Alan Yang<sup>6</sup>, Eva A. Enns<sup>5</sup>

Tutorial

### An Introductory Tutorial on Cohort State-Transition Models in R Using a Cost-Effectiveness Analysis Example

Fernando Alarid-Escudero<sup>1</sup>, Eline Krijkamp<sup>1</sup>, Eva A. Enns, Alan Yang<sup>1</sup>, M. G. Myriam Hunink, Petros Pechlivanoglou, and Hawre Jalal<sup>1</sup>

Tutorial

### A Tutorial on Time-Dependent Cohort State-Transition Models in R Using a Cost-Effectiveness Analysis Example

Fernando Alarid-Escudero<sup>1</sup>, Eline Krijkamp<sup>1</sup>, Eva A. Enns<sup>1</sup>, Alan Yang<sup>1</sup>, M. G. Myriam Hunink, Petros Pechlivanoglou, and Hawre Jalal<sup>1</sup>



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