On Approaching Noncoherent Channel Capacity Using Nonbinary IRA Code and Multi-tone FSK

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***Abstract*—Noncoherent underwater acoustic communication faces the problems of the high requirement of the signal noise ratio per bit and the inconsistency of performances between the simulation and the experiment. Noncoherent channel capacity based on composite fading model is studied, and a novel scheme of code and modulation approaching the noncoherent capacity is present. In the scheme, multi-tone frequency shift keying modulation is adopted, whose performance in the view of the channel capacity is obviously better than the traditional frequency shift keying modulation. Irregular nonbinary repeat-accumulate codes are used as the error correction code, with the optimized degree distribution. The processing in the receiver is independent of the channel fading status and the power estimations of the signal and the noise. In the performance simulations under the composite fading channel, the difference between the signal noise ratio requirement at a 10% frame error rate and that from the channel capacity curve is less than 2 dB. The relation of the signal noise ratio and the frame error rate is measured in a shallow ocean experiment at the distance of nearly 5 km, and the difference between the experiment curve and the simulation is less than 0.2 dB.**

I. Introduction

Underwater acoustic communication channel has the characteristics of time-varying, serious fading and low signal-to-noise ratio (SNR). When the channel is severe or the training symbols are not sufficient, the estimation of the channel amplitude and phase may not fitting the need of the coherent communication. In the multicarrier noncoherent communication, the frequency-selective channel is converted into multiple sub-channels of flat fading as it is similar in coherent OFDM system and the difference is that channel estimation is not necessary in the noncoherent system. Therefore, the noncoherent communication is the most popular signaling method for acoustic modem products, especially when there is only one element for receiving. Although the SNR threshold of the noncoherent communication is less than that of the coherent one, the comparison on SNR per bit is on the contrary. The SNR per bit requirement in current research about noncoherent underwater acoustic communication is usually larger than 10 dB[1,2,3], with a large gap to the Shannon capacity[4]. In this paper, the performance limitation under different fading distributions models of noncoherent channel is studied, and the scheme of code and modulation approaching the limitation is also studied.

The underwater channel is usually modeled as small-scale fading, such as the additive white Gaussian (AWGN) channel, Rician fading channel and the Rayleigh fading channel, and their difference is the proportion of the direct path in the total paths. Among these fading models, Rayleigh fading is most severe and usually used in simulation of noncoherent acoustic systems [1,3]. However, given a specific bit error rate (BER) level (10^{-5}), the SNR requirement in actual experiments especially in the long-distance horizontal channel, is several dB higher [5] than that in simulation under the Rayleigh model. The small-scale fading model cannot fit the severe underwater acoustic channel. In [5], both large-scale and small-scale fading are used to fit the envelop distribution of the high-frequency acoustic channel. The large-scale and small-scale fading are modeled as lognormal distribution and Rayleigh distribution respectively [5], and their combination is approximated by the K-distribution [6]. In [7], the performance of the frequency shift keying (FSK) modulation under K-distribution fading channel is studied and verified by underwater acoustic experiment data. In [8], K distribution model of underwater channel is used to optimize the code rate at different distances, where the convolutional code with a hard-decision Viterbi decoder is adopted.

The cutoff rate of the FSK modulation with different orders for underwater channel is simulated in [1], and the convolutional code of large constrain length with a sequential detection decoder is used to approach the limits of the cutoff rate. It is verified that in the coherent communication, the SNR requirement of the cutoff rate curve is several dB higher than the channel capacity [14], and the iterative-decoding codes, such as turbo codes, low-density parity-check (LDPC) codes or repeat-accumulate codes, can break the limitation of the cutoff rate of coherent modulations. The discrete masses of non-equal-probability as the channel input can be used to calculate the capacity of the noncoherent Rayleigh fading channel [4]. When the SNR is not very large, for example less than 10 dB, two masses are enough, with the null signal included. We use non-equal-probability two masses as the input of the K-distribution fading channel to find the numerical results of

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the capacity. As the elements of ordinary error-correction codes are equal-probability, the non-equal-probability mapping cannot be used as a practical modulation scheme, and a novel modulation scheme better than FSK is presented in the paper.

It is well proved that in coherent communication system, when nonbinary modulation is selected, both bit-interleaved coded modulation with iterative decoding (BICM-ID) and nonbinary channel codes can approach the channel capacity, and optimized degree distributions of channel codes in these methods can further improve the performances. Turbo coded BICM-ID is used in [10] for orthogonal FSK modulations under noncoherent fading channels, and performance is further improved in [11] by using binary irregular repeat-accumulate (IRA) codes. The superiority of nonorthogonal modulations against the orthogonal ones is analyzed in the view of channel capacity in [12], especially in low SNR region.

In this paper, we calculate the capacities of the K-distribution fading channel without specified modulations, and that of the orthogonal or nonorthogonal modulation, among which the superiority of the nonorthogonal modulation in severe fading channel is obvious. We present a scheme of the nonbinary IRA code and the two-tone FSK for underwater communication, and introduce the optimization method of the degree distribution of the nonbinary IRA code. In the simulation under severe channel model, the SNR requirement of the proposed scheme is only 2 dB higher than the channel capacity without specified modulations. The collection of channel amplitudes in the shallow ocean experiment is fitted by K-distribution. The difference between the SNR-FER (frame error rate) curve of the actual channel and that of the simulation under the fitted K-distribution channel is within 0.2 dB.

II. Numerical Capacities of Noncoherent Channels

A. Channel Fading Models

The amplitude of frequency-bins in the severe acoustic channel is supposed to be memoryless in both the frequency and time domain, and an interleaver ensures the correctness of this hypothesis. The transmission signal at certain time and frequency slot is X, and the amplitude of the received signal is written as $Y = \|XH + N\|$, where the noise N is zero-mean circularly-symmetric complex Gaussian random variable, with variance of σ_n^2, and its probability density function (PDF) is

$$p_(n) = \frac{1}{\pi\sigma_n^2}\exp\left\{-\frac{\|n\|^2}{\sigma_n^2}\right\}. \tag{1}$$

H is the channel amplitude with a PDF of $p_H(h)$. We have that $H \geq 0$ and $\mathrm{E}\{\|H\|^2\} = 1$. When the channel is not fading, we have that $p_H(h) = \delta(h)$. When it is Rayleigh fading, we have

$$p_H(h) = \frac{h}{b^2}\exp\left\{-\frac{h^2}{2b^2}\right\} \tag{2}$$

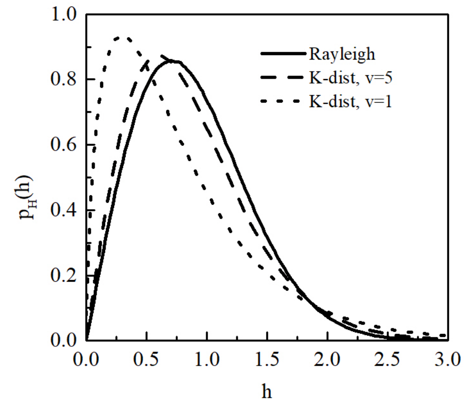

Fig. 1. PDFs of channel fading amplitude.

where $b = \frac{1}{\sqrt{2}}$. When it is K-distribution fading channel,

$$p_H(h) = \frac{4}{\sqrt{\alpha}\Gamma(v)}\left(\frac{h}{\sqrt{\alpha}}\right)^v K_{v-1}\left(\frac{2z}{\sqrt{\alpha}}\right), \tag{3}$$

where v is the shaping parameter, $\alpha = \frac{1}{v}$, and $K_{v-1}()$ is the second modified Bessel function. At $v \to \infty$, K-distribution is approximated by Rayleigh distribution. Fig. 1 shows three PDFs of channel amplitude. K-distribution with a small v parameter is fading most severe.

B. Channel Capacities with two masses as input

The discrete masses of non-equal-probability as the channel input can be used to calculate the capacity of the noncoherent Rayleigh fading channel [7]. When the SNR is not very large, for example less than 10 dB, two masses are enough, with the null signal included. Supposed that the channel input $X \in \{0, 1\}$ with the probabilities of $P_X(0) = P_0$, and $P_X(1) = 1 - P_0$, the channel capacity can be written as

$$\begin{aligned} &C(P_X(0) = P_0) \\ &= \sum_{x=0,1}\int_{y\geq 0} P_X(x)\, p_{Y|X}(y|x)\log_2\frac{p_{Y|X}(y|x)}{p_Y(y)}\mathrm{d}y, \end{aligned} \tag{4}$$

where $p_Y(y) = \sum_{x=0,1} P_X(x)\, p_{Y|X}(y|x)$ is post-probability function and shown in Fig. 2.

The channel capacity can be further written as a mathematical expectation,

$$C(P_0) = \mathrm{E}_{X,H}\left\{\log_2\frac{p_{Y|X}(y|x)}{p_y(y)}\right\}, \tag{5}$$

and can be numerically calculated by averaging plenty of random samples. The channel capacity at optimized input distribution is

$$C_{\mathrm{opt}} = \max_{P_0} C(P_X(0) = P_0). \tag{6}$$

The SNR per bit is written as $\frac{E_b}{N_0} = \frac{1-P_0}{\sigma_n^2 C}$. Fig. 3 shows the relationship of the channel utility and the SNR per bit.

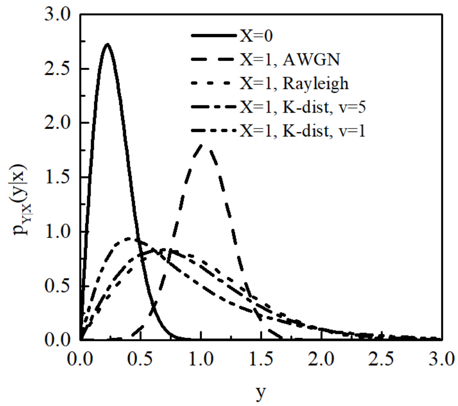

Fig. 2. Post probabilities of received carrier amplitude.

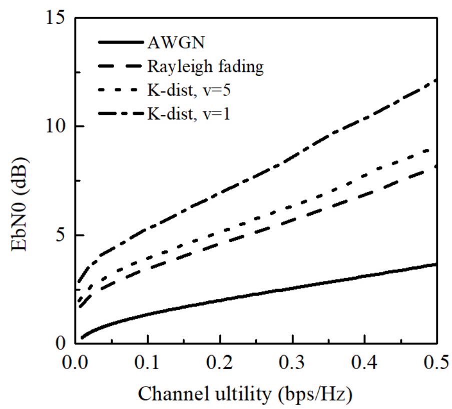

Fig. 3. Channel capacities of optimized distribution of two mass points in the noncoherent fading channels.

C. Capacities of Orthogonal and Nonorthogonal Modulations

Ordinary channel codes can be applied only when the channel input (modulation symbols) are of equal-probability. We make N channel slots as a block, then the transmission vector is written as $\mathbf{X} = [X_0, \cdots, X_{N-1}]$, and the received vector is written as $\mathbf{Y}$. We select M out of N slots to transmit equal amplitude signals, and we have $\sum\limits_{i=0}^{N-1} X_i = M$ and $X_i \in \{0,1\}$. Therefore, the number of possible selections is $\binom{N}{M} = \frac{M!}{M!(N-M)!}$, and we use 2^T out of all the $\binom{N}{M}$ possibilities as the block modulation set. The elements in the set is of equal-probability. The according channel capacity is written as

$$C = \frac{1}{N}\mathrm{E}_{\mathbf{X},\mathbf{H}}\left\{\log_2 \frac{p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|\mathbf{x})}{p_{\mathbf{y}}(\mathbf{y})}\right\}, \tag{7}$$

where $p_{\mathbf{Y}}(\mathbf{y}) = 2^{-T}\sum_{x_i\in\Omega} p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|\mathbf{x}_i)$, and

$$p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|\mathbf{x}) = \prod_{i=0}^{N-1} p_{Y_i|X_i}(y_i|x_i). \tag{8}$$

When $N = 4$, $M = 1$, $T = 2$, the block modulation is ordinary 4FSK. When $N = 12$, $M = 2$, $T = 6$, it means 2 out of 12 FSK, and two tones are used in each block. The 2 out of 12 FSK is nonorthogonal. Fig. 4 shows the capacities of these modulations under different channel fading models. It can be seen that 2 out of 12 FSK is always better than 4FSK. When the channel is K-distribution fading with $v = 1$ and the channel utility is at 1/6 bps/Hz, the 2 out of 12 FSK modulation is only 0.22 dB apart from the optimized two-mass distribution.

III. Proposed Scheme of Code and Modulation under Severe Channel

A. Nonbinary IRA Code

The RA code has the advantage of low encoding complexity compared with the LDPC code. Irregular non-systematic RA code has better performance than the regular systematic RA code. Nonbinary IRA code can be obtained by weighting random non-zero variables on the repeater output [13].

Proposed scheme of noncoherent underwater acoustic communication is shown in Fig. 5. The IRA encoder is based on q-ary Galois Field (GF), and the input length of the encoder is K. The K varibles on GF(q) are repeated with varying times and turned into N variables. The range of the repeat times is $\{d_1, d_2, \cdots d_J\}$. The percentage of the repeater's output under a degree of $d_j\,(1 \le j \le J)$ is λ_j, and $\sum\limits_{j=1}^{J}\lambda_j = 1$. Then the repeater's output are weighted by non-zeros random variables taken from GF(q), and interleaved. To reduce the complexity, we choose the rate-1 accumulator. Therefore, the IRA encoder's rate is written as $r = \frac{K}{N} = \sum\limits_{j=1}^{J}\frac{\lambda_j}{d_j}$. We set $q = 64$ to match the modulation order. The encoded vector is modulated by 2 out of 12 FSK. The whole channel is divided into 120 orthogonal subcarriers, and ten modulation symbols are transmitted concurrently. After adding cyclic-prefix (CP) and synchronization signals, the waveform carrying source information is transmitted through the acoustic channel. In the receiver, after synchronization and time-to-frequency transformation, the soft decisions of the modulated symbol can be obtained by summation on the square-law values of the supported carriers.

B. Max-sum Decoder

The iterative decoder of the IRA code is shown in Fig. 5. We use the max-sum form of the iterative decoder, as it is robust and has less complexity. Estimation of the channel fading distribution and SNR are need to obtain exact symbol probabilities, and the estimation is not robust if the channel status is varying. The max-sum decoder, which is not sensitive to the scaling of the probabilities, can be fed with the soft symbol decisions based on square-law summation.

The accumulator in the RA encoder is a kind of convolutional code, and can be decoded by nonbinary BCJR (Bahl-Cocke-Jelinek-Raviv) algorithm[9]. The exact log-likelihood

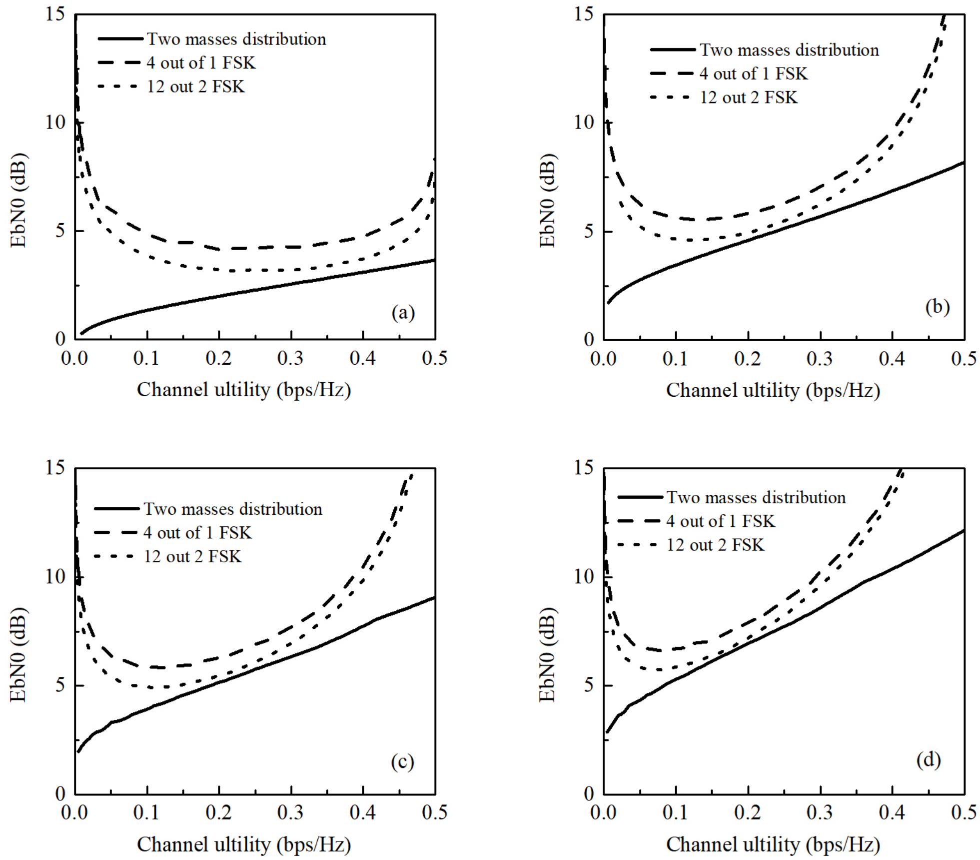

Fig. 4. Channel capacities of 4FSK and 2 out of 12 FSK: (a) AWGN channel; (b) Rayleigh fading channel; (c) K-distribution channel, v=5; (d) K-distribution channel, v=1.

ratio (LLR) of the GF summation in BJCR algorithm is written as

$$L_{X+Y}(\gamma) = \log\left\{\sum_{\alpha+\beta=\gamma} \exp\left\{L_X(\alpha) + L_Y(\beta)\right\}\right\}, \tag{9}$$

and is simplified in max-sum form as

$$L_{X+Y}(\gamma) \approx \max_{\alpha+\beta=\gamma} \left(L_X(\alpha) + L_Y(\beta)\right). \tag{10}$$

Supposed that the variable X of the RA encode is repeated d times, and in the repeater's decoder the i-th LLR input of X is $L_{X_i}(\alpha_i)$, then the i-th LLR output is

$$L'_{X_i}(\alpha) = \sum_{j \neq i} L_{X_j}(\alpha). \tag{11}$$

C. Optimization of the Degree of the IRA Code

We use three degrees in the repeater, that is $J = 3$, and $d_1 = 2,\ d_2 = 3,\ d_3 = 6$. The code rate $r = 1/3$, and we have $\lambda_3 = 1 - 2\lambda_2$, $\lambda_6 = \lambda_2$. The optimization of the code's degree distribution is to minimize FER by changing λ_2 . In the simulation, we set $K = 320$ and a maximum iteration of 20. Fig. 6 shows the FERs at varying λ_2 under

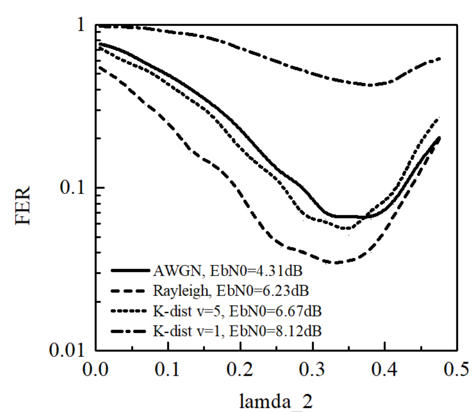

Fig. 6. FER performances of different degree distributions.

different fading channels each with a specific SNR. We choose $\lambda_2 = 0.38$ in the following simulations and experiments, as FER is minimized in the most severe channel.

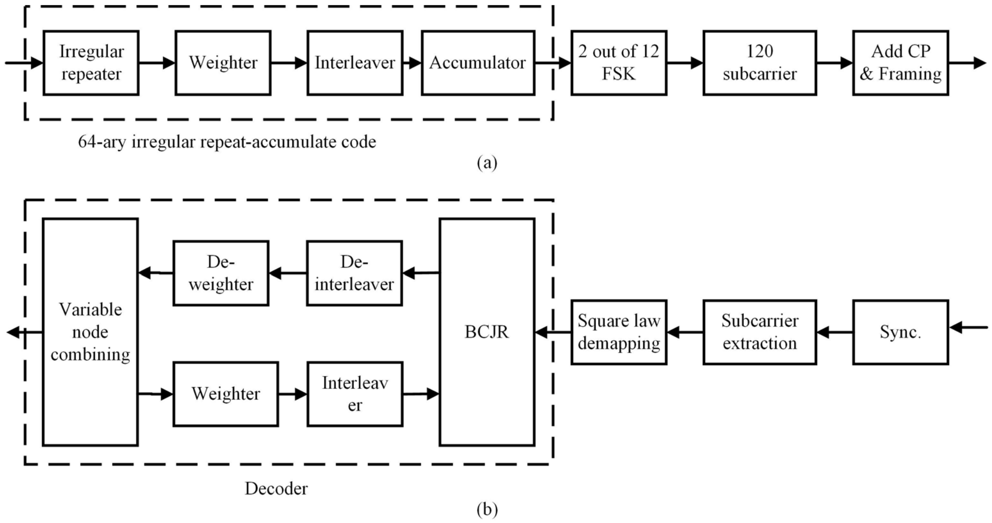

Fig. 5. Flow chart of underwater acoustic noncoherent communication: (a) transmitter; (b) receiver.

D. Performances under Fading Channel

Fig. 7 shows the performances of simulations under various fading channels at a maximum iteration of 20. The IRA code rate is 1/3, and the channel utility of modulation is 1/2, and therefore the channel utility of coded modulation is 1/6 bps/Hz. Under the AWGN noncoherent channel, the threshold (FER < 0.1) of SNR per bit in our scheme is 4.27 dB, which is even lower than the limitation of 4FSK modulation (4.44 dB). Under the severe channel of K-distribution fading with v=1, the SNR per bit requirement is only 8.7 dB.

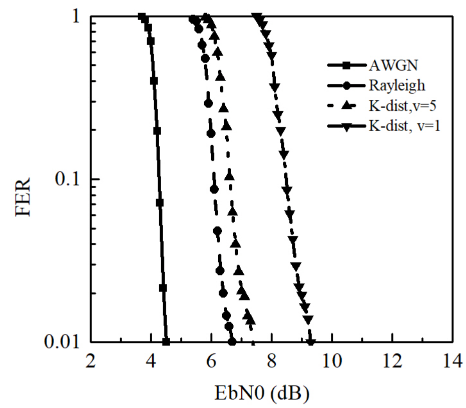

Fig. 7. FER performances of different fading channels with optimized degree distribution.

IV. Shallow Ocean Experiment

Shallow ocean experiment was carried out at the South China Sea in April 2014. The transmitting and receiving transducers were deployed at 15 m depth, and the water depth is 70 m. The communication bandwidth was 6˜10 kHz, divided into 120 subcarriers. The sound speed profile is shown in Fig. 8. At the distance of more than 2000 m, there is hardly no direct path, and the sound channel is formed by reflecting from the sea bottom. Fig. 9 shows the histograms of subcarrier channel amplitudes from the experiment at different distance, and shows fitting results under K-distribution. The histogram at the distance of 4600 m, which is fitted with a lower v, is fading more seriously than that of 5100 m, To obtain the FER curve, we superimpose random noise samples onto the waveform, as the SNRs of the collected waveforms are much higher than decoding requirement. Fig. 10 shows the FER performances of the waveforms' decoding results and the simulations under the fitted K-distribution channel. In can be seen that at both distances, performance differences between the actual channel and the K-distribution fading channel is less than 0.2 dB. That verifies the K-distribution fading assumption of severe underwater channel in the simulation of coded modulation.

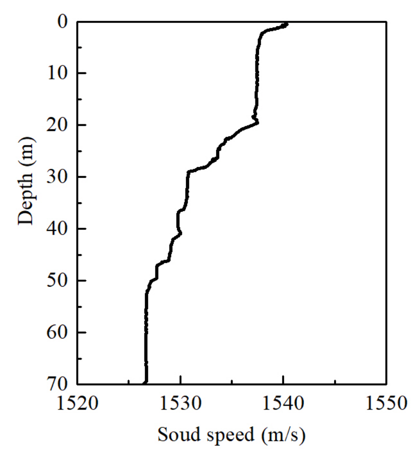

Fig. 8. Sound speed profile during the South China Sea experiment.

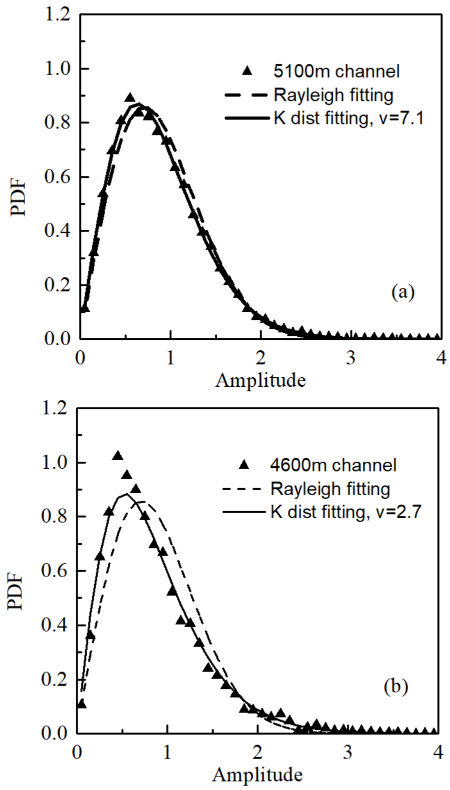

Fig. 9. PDF estimations of amplitude of frequency bins of collect data and curve fittings by Rayleigh distribution and K-distribution(a) Distance = 5100 m; (b) distance = 4600 m.

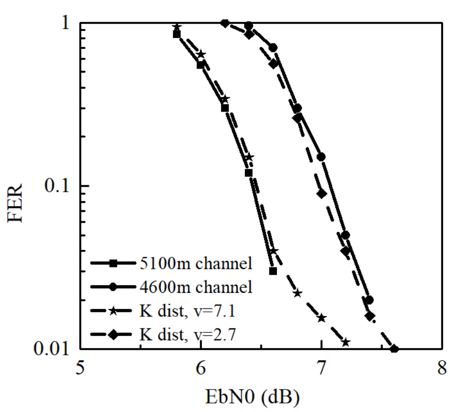

Fig. 10. Performance comparisons of the experiment and the simulation.

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