# Performance Analysis of mMFSK Frequency Hopping Modulation for Wireless Ad Hoc Networks

*Invited Paper*

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*Abstract-In this paper we analyze performance of wireless ad hoc networks based on mMFSK Frequency Hopping Modulation. This modulation is a modification of FH/MFSK system that includes a multitone MFSK signal which will be designated as mMFSK modulation. In this case the signal energy is split to m separate tones making it more vulnerable to noise and fading but still the overall flow of useful information will be increased. The results demonstrate that under the large range of the signal, channel and interference parameters this system offers better performance than the standard MFSK/FH system.*

## 1. Introduction

Two of the most often used versions of spread spectrum systems are frequency hopping (FH) and direct sequence (DS) configurations. The optimum DS receiver is difficult to implement unless a cellular network infrastructure is used. In this concept mobile units communicate through centralized entities (base - stations) which serve as points of wireless access to a fixed network. In contrast, ad hoc networks represent a different communication concept, which is not dependent on preexisting infrastructure. Thus ad hoc networks can be rapidly deployed, without prior planning and in unknown radio propagation conditions. In ad hoc networks communications between adjacent nodes is allowed, giving rise to multihop routing. The topology of ad hoc networks is distributed. Nodes in an ad hoc network can migrate freely, joining, leaving, and rejoining the network often, without warning, and without disruption to communication between other nodes. The challenges in the design of ad hoc networks stem from the lack of centralized entities (no possibility for centralized processing like multiuser detection in CDMA networks), from the multihop mode of communication, and from the fact that all communication is carried over a wireless medium giving rise to fading and jamming issues. Ad hoc networks have long been considered for military tactical communications under the more familiar name Packet Radio Networks. Recently, however, use of ad hoc networking technology in commercial systems has been investigated. Examples are law enforcement communications, rescue missions, virtual classrooms or local area networks.

The environment described above will demonstrate severe near-far effects, and a communicator may prefer to use a frequency-hopping spread-spectrum (FH-SS) system rather than Direct Sequence modulation which is sensitive to near-far effects. To mitigate this effect power control or multiuser detection must be used, both of which are feasible only in cellular networks.

In order to improve performance in jamming and fading environments FH-SS systems can use diversity. Traditionally, diversity has been achieved using multiple hops per information (or coded) symbol. Such fast hopping makes difficult the synchronization of the carrier phase and, consequently, imposes the use of a noncoherent receiver. Thus, a significant loss in error performance results, due to both noncoherent demodulation and noncoherent combining of the received diversity replicas. Taking into account these losses, and using binary frequency-shift keying (BFSK) modulation, an optimum diversity scheme is analyzed in [1] for the worst case jammer and with side information on noise and jamming levels. Since optimum diversity has more analytical value than practical existence, the error probability is much higher in practice. In order to recover these performance losses, some authors have studied a solution that makes coherent reception feasible; see, e.g., [4, 7, 5]. Frequency diversity as used on Rayleigh fading channels, and which differs from the diversity mentioned above, was proposed in [2] to counter band-limited interference. Such a form of diversity allows one to avoid noncoherent combining loss. In this system, called frequency-diversity spread spectrum (FD-SS), the communication frequency band is partitioned into $N$ disjoint subbands on which $N$ replicas of the signal are simultaneously transmitted. However, since frequency hopping is considered as mandatory in some applications, some solutions combine both FD-SS and FH-SS systems [1]. The main objective is to guarantee coherent demodulation and to avoid noncoherent combining loss.

If coherency is not feasible then a noncoherent solution is the only option. The effect of barrage and partial-band noise jamming on frequency-hopped, M-ary frequency-shift keyed (FH/MFSK) noncoherent receivers, when one or more symbols per hop are transmitted has been examined both for non-fading channels and for Rayleigh fading channels in [9] and [10], respectively. The effect of partialband noise jamming on fast frequency-hopped (FFH) binary frequency-shift keyed (BFSK) noncoherent receivers with diversity has been examined for channels with no fading [11], as has the effect of partial-band noise jamming on FFH/MFSK for Ricean fading channels [12]. The performance degradation resulting from both band and independent multitone jamming of FH/MFSK, where the jamming tones are assumed to correspond to some or all of the possible FH M-ary signaling tones and when thermal and other wideband noise is negligible, is examined in [13 - 15]. The effect of tone interference on noncoherent MFSK when AWGN is not neglected is examined for channels with no fading in [16], and the effect of independent multitone jamming on

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noncoherent FH/BFSK when AWGN is not neglected is examined for channels with no fading in [17].

In this paper we consider a modification of the FH/MFSK system to include a multitone MFSK signal which will be designated as $m$MFSK modulation. In this case the signal energy is split to $m$ separate tones making it more vulnerable to noise and fading but still the overall flow of useful information will be increased. Different versions of this approach are described in [19-25] without in-depth analysis of their performance. The main contribution of this paper is a detailed performance analysis of this system under different channel, network load and jamming conditions. The paper is organized as follows: In Section 2 we present the signal model and performance criteria. In Section 3 performance analysis is given. Selected numerical results are presented in Section 4. These results demonstrate that under the large range of the signal, channel, network load and interference parameters, the new system offers better performance than the standard MFSK/FH system.

## 2. System Model

Standard M-ary FSK modulation uses one out of $M$ frequencies each $T_s$ seconds to transmit a block of $n=\log M$ bits. The optimum receiver has a bank of $M$ matched filters and every $T_s$ seconds makes decision based on the largest output filter sample. This configuration requires coherent demodulation and still is widely considered for practical applications [1-7] due to higher gain in a frequency diversity scheme compared with noncoherent combining. In spite of the combining losses, noncoherent schemes are also being considered for practical applications due to their simplicity [8-15]. Let's suppose now that instead of sending one out of M frequencies we send two frequencies simultaneously. If amplitudes are the same we can form

$$M_2 = \sum_{r=1}^{M-1} r = (M-1)M/2 \tag{1}$$

different combinations and send

$$\begin{aligned} n_2 &= \log(M-1)M/2 = \\ &\log M + \log(M-1) \end{aligned} \tag{2}$$

bits. If $M$ is large $n_2 \approx 2\log M - 1 \approx 2n$ is almost twice as many bits as in the case of simple $M$-ary modulation. The optimum receiver will now have to find the two largest signals at the output of the bank of matched filters. If now instead of two, $m$ out of $M$ frequencies are simultaneously transmitted, we have **mMFSK** modulation. The number of transmitted bits per interval $T_s$ is now further increased. A simple calculus is needed to evaluate capacity improvement in a noise-free channel.

In this paper mMFSK modulation will be considered for frequency hopping systems instead of standard MFSK modulation, hence the name FH/mMFSK modulation. As a performance measure we will be discussing symbol error rate or the system efficiency improvement factor defined as [18]

$$E = [(1-P)n]/[(1-P_0)n_0] \tag{3}$$

where parameters with index zero refer to the standard MFSK modulation. In (3) $n$ is the number of bits per symbol, $P$ is the bit error rate and $(1-P)n$ is the average number of correctly transmitted bits per symbol.

By splitting the available signal power over $m$ frequencies the system will be more error prone but the average number of correctly transmitted bits should be still higher.

## 3. Performance analysis

### 3.1. Error probability for coherent *m*MFSK

The transmitted signals can be represented as

$$\begin{aligned} &u_k(t) = Ae^{j2\pi f_k t} = Au_{ku}(t) \\ &s_k(t) = \mathrm{Re}\{u_k(t)e^{j2\pi f_c t}\} = \\ &A\,\mathrm{Re}\{u_{ku}(t)e^{j2\pi f_c t}\} = As_{ku}(t), \quad k = 1,2,\ldots,M \end{aligned} \tag{4}$$

with

$$\int_0^T u_k(t)u_m(t)dt = \delta_{km} \tag{5}$$

where $u_k(t)$ is the complex envelope of the signal. The energy of these signals is

$$\mathrm{E}_k = \int_0^T s_k^2(t)dt = \tfrac{1}{2}\int_0^T |u_k(t)|^2 dt = A^2\mathrm{E}_u = \mathrm{E} \quad k = 1,2,\ldots,M \tag{6}$$

After frequency down-conversion the received signal envelope is

$$r(t) = \alpha e^{-j\phi}u_k(t) + z(t) \qquad 0 \le t \le T \tag{7}$$

where $\alpha$ is due to channel attenuation, $\phi$ is the phase difference between the input and local signal, and $z(t)$ is Gausian noise. For simplicity, and without losing generality let us assume that the first $m$ frequencies are transmitted, i.e., $s_l(t), \quad l = 1,2,\ldots,m$. The received low-pass equivalent of the signal becomes

$$r(t) = \alpha e^{-j\phi}(u_1(t) + \cdots + u_m(t)) + z(t) \quad 0 \le t \le T \tag{8}$$

The decision variables are now given as

$$U_k = \mathrm{Re}\left\{e^{j\phi}\int_0^T r(t) \;\; u_k^*(t)dt\right\} \; k = 1,2,\ldots,M \tag{9}$$

An optimum receiver will choose the largest one. The receiver block diagram is shown in Fig.1. Parameter $U_k$ can be represented as

$$\begin{aligned} &U_l = 2\alpha\mathcal{E} + N_{lr}, l = 1,2,\ldots,m \\ &U_p = N_{pr}, p = m+1,\ldots,M \end{aligned} \tag{10}$$

were $N_{kr}$ are zero-mean Gaussian variables with variance $\sigma^2 = 2\mathcal{E}N_0$, and $N_0$ is the noise spectral density. Probability density functions (PDFs) for $U_k$ can be represented as

$$p(U_l)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(U_l-2\alpha E)^2/2\sigma^2}\quad l=1,2,\ldots,m$$
$$p(U_p)=\frac{1}{\sqrt{2\pi}\sigma}e^{-U_p^2/2\sigma^2}\quad p=m+1,\ldots,M \tag{11}$$

Probability of correct decision is

$$P_c=P(U_1>U_{m+1},U_1>U_{m+2},\ldots,U_1>U_M)\cdots$$
$$\cdots\cdots P(U_m>U_{m+1},U_m>U_{m+2},\ldots,U_m>U_M)$$
$$P_c=\left[P(U_1>U_{m+1},U_1>U_{m+2},\ldots,U_1>U_M)\right]^m$$

$$P_c=\left[\frac{1}{2^{M-m}\sqrt{\pi}}\int_{-\infty}^{+\infty}e^{-x^2}\left[1+\operatorname{erf}\left(x+\sqrt{\frac{\alpha^2 E}{N_0}}\right)\right]^{M-m}dx\right]^m$$

and the symbol error probability is $P_s=1-P_c$

### 3.2. Error probability for noncoherent *m*MFSK with envelope detector

The transmitted signal, its energies and the equivalent baseband form of the received signal are given again by eqs. 4-8. The receiver will create decision variable

$$U_k=\left|\int_0^T r(t)\;u_k^*(t)dt\right|\quad k=1,2,\ldots,M \tag{14}$$

and choose the $m$ largest ones. The receiver block diagram is shown in Fig.2. If we assume that the transmitted signals are $s_l(t),\quad l=1,2,\ldots,m$, then the received signal is given by eq.8. The decision variables are

$$U_k=\left|\int_0^T r(t)\;u_k^*(t)dt\right|\quad k=1,2,\ldots,M \tag{15}$$

Bearing in mind that signals $u_k(t)$ are orthogonal $U_l$ and $U_p$ are obtained by taking absolute value of the expression defined by eq.10. Parameters $N_{kr}$ are zero mean Gaussian variable having the variance $\sigma^2=2\mathcal{E}N_0$, and $N_0$ is the noise power density. One can show that PDFs for $U_k$ can be expressed as:

$$p(U_l)=\frac{U_l}{2EN_0}\exp\left(-\frac{U_l^2+4\alpha^2E^2}{4EN_0}\right)I_0\left(\frac{\alpha U_l}{N_0}\right)l=1,2,\ldots,m$$
$$p(U_p)=\frac{U_p}{2EN_0}\exp\left(-\frac{U_p^2}{4EN_0}\right)p=m+1,\ldots,M \tag{16}$$

Probability of correct decision is defined in general form by eq.(12) and now becomes

$$P_c=\left[\sum_{n=0}^{M-m}(-1)^n\binom{M-m}{n}\frac{1}{n+1}e^{-\frac{\alpha^2\mathcal{E}}{N_0}\frac{n}{n+1}}\right]^m \tag{17}$$

Finally the symbol error probability is again given as $P_s=1-P_c$. One can show that error probability (eq.20) remain the same if a square-law instead of envelope detector is used. In the presence of other users in the network, jamming and fading, expressions for symbol error rate are further modified. Details are shown in the sequel.

### 3.3. Error probability for coherent FH/*m*MFSK system in ad hoc network

In the presence of MAI (Multiple Access Interference) interference the symbol error rate (SER) for FH/mMFSK system can be represented as

$$P_s=\frac{q}{N}P_s(\mathrm{HJ})+\frac{N-q}{N}P_s(\mathrm{HNJ}) \tag{18}$$

where $q$ is the number users, $N$ is the number of available FH carriers, $P_s$(HJ) is SER when a hop is jammed by other users, and $P_s$(HNJ) is SER when the hop is not jammed.
In the sequel we derive expressions for $P_s$(HJ) and $P_s$(HNJ).

#### A. *Hop is not jammed*

As a starting point in this derivation we use equation (13) which defines SER for coherent *m*MFSK in the presence of Gaussian noise.

$$P_s(\mathrm{HNJ}|a_c)=F_{koh}\left(\frac{a_c^2}{\sigma^2}\right) \tag{19}$$

where $\frac{a_c^2}{\sigma^2}=\frac{\alpha^2\mathcal{E}}{N_0}$ and

$$F_{koh}(z)=1-\left[\frac{1}{2^{M-m}\sqrt{\pi}}\int_{-\infty}^{+\infty}e^{-x^2}\left[1+\operatorname{erf}\left(x+\sqrt{z}\right)\right]^{M-m}dx\right]^m$$

The PDF for the signal amplitude is

$$f_{A_c}(a_c)=\frac{a_c}{\sigma_c^2}\exp\left[-\frac{a_c^2+\alpha_c^2}{2\sigma_c^2}\right]I_0\left(\frac{a_c\alpha_c}{2\sigma_c^2}\right) \tag{21}$$

where $\alpha_c$ and $\sigma_c$ are standard deviation for the steady and diffuse signal component, respectively.
By averaging (19) we get

$$P_s(\mathrm{HNJ})=$$
$$\int_0^\infty P_s(\mathrm{HNJ}\mid a_c)\cdot f_{A_c}(a_c)da_c$$
$$=\int_0^\infty F_{koh}\left(\frac{a_c^2}{\sigma^2}\right)\cdot\frac{a_c}{\sigma_c^2}\exp\left[-\frac{a_c^2+\alpha_c^2}{2\sigma_c^2}\right]I_0\left(\frac{a_c\alpha_c}{2\sigma_c^2}\right)da_c$$
$$=\exp\left(-\frac{\rho_c}{\xi_c}\right)\int_0^\infty F_{koh}\left(x^2\frac{\xi_c}{2}\right)\cdot x\exp\left[-\frac{x^2}{2}\right]I_0\left(x\sqrt{2\frac{\rho_c}{\xi_c}}\right)dx$$

where $\rho_c=\frac{\alpha_c^2}{\sigma^2}$, $\xi_c=\frac{2\sigma_c^2}{\sigma^2}$

### B. Hop is jammed

As there are $m$ jamming tones, when a hop is jammed the jamming signal frequencies may overlap with one useful signal frequency, with two of them, ..., with $m$ of them. Therefore the SER can be expressed as

$$P_s(\text{HJ}) = \frac{1}{\binom{M}{m}} \cdot \sum_{k=0}^{m} \binom{m}{k}\binom{M-m}{m-k} P_s(\text{HJ}/k\,\text{SBJ})$$

where $P_s$(HJ/$k$SBJ) is SER when the hop and $k$ Signal frequency **B**ins are **J**ammed. Without losing generality we can assume that the signal occupies the first $m$ frequency slots and the jamming signal occupies the first $k$, the $(m+1)$th, ...,$(2m-k)$th. Thus we have

$$p(U_l) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(U_l - \sqrt{2}\sqrt{a_c^2 + a_J^2 + 2a_c a_J \cos\theta}\right)^2 / 2\sigma^2} \qquad l = 1,...,k$$

$$p(U_l) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(U_l - \sqrt{2}a_c\right)^2 / 2\sigma^2} \qquad l = k+1,...,m$$

$$p(U_l) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(U_l - \sqrt{2}a_J\right)^2 / 2\sigma^2} \qquad l = m+1,...,2m-k$$

$$p(U_l) = \frac{1}{\sqrt{2\pi}\sigma} e^{-U_l^2 / 2\sigma^2} \qquad l = 2m-k+1,...,M$$

where $a_J$ is the jamming signal amplitude, and $\theta$ is the jamming signal phase relative to the useful signal uniformly distributed in $0$ - $2\pi$ interval.

For given amplitudes of the useful and jamming signal the conditional SER becomes

$$\begin{aligned} &P_s(\text{HJ}/k\,\text{SBJ} \mid a_c, U_1,...,U_m) = \\ &= 1 - \left[P(U_1 > U_{m+1})P(U_1 > U_{m+2}) \cdots P(U_1 > U_M)\right] \times \cdots \\ &\cdots \times \left[P(U_m > U_{m+1})P(U_m > U_{m+2}) \cdots P(U_m > U_M)\right] \end{aligned}$$

Bearing in mind that $p(U_l), l = 1,2,...,k$, $p(U_l),\ l = k+1,...,m$, $p(U_l),\ l = m+1,...,2m-k$ and $p(U_l),\ l = 2m-k+1,...,M$ are the same, we have

$$\begin{aligned} &P_s(\text{HJ}/k\,\text{SBJ} \mid a_c, \theta, U_1) = \\ &1 - \left[(P(U_{k+1} > U_{m+1}))^{m-k}(P(U_{k+1} > U_{2m-k+1}))^{M-2m+k}\right]^{m-k} \times \\ &\times \left[(P(U_1 > U_{m+1}))^{m-k}(P(U_1 > U_{2m-k+1}))^{M-2m+k}\right]^k \end{aligned}$$

where

$$P(U_{k+1} > U_{m+1}) = \int_{-\infty}^{U_{k+1}} p(U_{m+1})dU_{m+1} = \frac{1}{2}\left(1 + erf\left(\frac{U_{k+1} - \sqrt{2}a_J}{\sqrt{2}\sigma}\right)\right)$$

$$P(U_{k+1} > U_{2m-k+1}) = \int_{-\infty}^{U_{k+1}} p(U_{2m-k+1})dU_{2m-k+1} = \frac{1}{2}\left(1 + erf\left(\frac{U_{k+1}}{\sqrt{2}\sigma}\right)\right)$$

$$P(U_1 > U_{m+1}) = \int_{-\infty}^{U_1} p(U_{m+1})dU_{m+1} = \frac{1}{2}\left(1 + erf\left(\frac{U_{k+1} - \sqrt{2}a_J}{\sqrt{2}\sigma}\right)\right)$$

$$P(U_1 > U_{2m-k+1}) = \int_{-\infty}^{U_1} p(U_{2m-k+1})dU_{2m-k+1} = \frac{1}{2}\left(1 + erf\left(\frac{U_1}{\sqrt{2}\sigma}\right)\right)$$

By averaging (25) with respect to $U_1$ and $U_{k+1}$ we have

$$P_s(\text{HJ}/k\,\text{SBJ} \mid a_c, \theta) = F_{kohl}\left(\frac{a_c^2}{\sigma^2}, \frac{a_J^2}{\sigma^2}, \theta\right) \tag{28}$$

where

$$F_{kohl}(y,z,\theta) = 1 - \left[\frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-x^2} F_{erf}\left(x + \sqrt{y} - \sqrt{z}, m-k\right) \times F_{erf}\left(x + \sqrt{y}, M-2m+k\right)dx\right]^{m-k} \times \tag{29}$$

$$\times \left[\frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-x^2} F_{erf}\left(x + \sqrt{y + z + 2\sqrt{yz}\cos\theta} - \sqrt{z}, m-k\right) \times F_{erf}\left(x + \sqrt{y + z + 2\sqrt{yz}\cos\theta}, M-2m+k\right)dx\right]^k$$

and

$$F_{erf}(x,n) = \begin{cases} \left[\frac{1}{2}(1 + \text{erf}(x))\right]^n, & n \geq 0 \\ 1, & n < 0 \end{cases} \tag{30}$$

After averaging eq.28 we get

$$P_s(\text{HJ}/k\,\text{SBJ}) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\infty} F_{kohl}\left(\frac{a_c^2}{\sigma^2}, \frac{a_J^2}{\sigma^2}, \theta\right) f_{A_c}(a_c)\,da_c\,d\theta$$

### 3.4. Error probability for noncoherent FH/mMFSK system in an ad hoc network

In the presence of MAI interference the symbol error rate (SER) for FH/mMFSK system can be represented in general again by eq.18.

### B. Hop is not jammed

As a starting point in this derivation we use eq.17 which defines SER for noncoherent $m$MFSK in the presence of Gaussian noise. It can be written as

$$P_s(\text{HNJ}) = 1 - \sum_k b_k \exp\left(-c_k \frac{a_c^2}{\sigma^2}\right) \tag{32}$$

where $b_k$ and $c_k$ are coefficients that depend on $m$ and $M$.

PDF for the signal amplitude is given by eq.21. By averaging eq.32 with respect to $a_c$ we have:

$$\begin{aligned} P_s(\text{HNJ}) &= \int_0^{\infty} f_{A_c}(a_c) \cdot P_s(\text{HNJ}|a_c)\,da_c \\ &= \sum_k b_k \frac{1}{1 + c_k \xi_c} \exp\left(-\frac{c_k \rho_c}{1 + c_k \xi_c}\right) \end{aligned} \tag{33}$$

*B. Hop is jammed*

As for the coherent detection, SER is given by eq.23. PDFs for $U_i$, $i = 1, \ldots, M$. can be now represented as

$$p(U_l)=\frac{1}{2\sigma^2}\exp\left[-\frac{1}{2}\left(\frac{U_l+2(a_c^2+a_J^2+2a_ca_J\cos\theta)}{\sigma^2}\right)\right]$$
$$I_0\left(\frac{\sqrt{a_c^2+a_J^2+2a_ca_J\cos\theta}\sqrt{2U_l}}{\sigma^2}\right)\qquad l=1,k$$
$$p(U_l)=\frac{1}{2\sigma^2}\exp\left[-\frac{1}{2}\left(\frac{U_l+2a_c^2}{\sigma^2}\right)\right]I_0\left(\frac{a_c\sqrt{2U_l}}{\sigma^2}\right)\qquad l=k+1,\ldots,m \tag{34}$$
$$p(U_l)=\frac{1}{2\sigma^2}\exp\left[-\frac{1}{2}\left(\frac{U_l+2a_J^2}{\sigma^2}\right)\right]I_0\left(\frac{a_J\sqrt{2U_l}}{\sigma^2}\right)\qquad l=m+1,\ldots,2m-k$$
$$p(U_l)=\frac{1}{2\sigma^2}\exp\left[-\frac{U_l}{2\sigma^2}\right]\qquad l=2m-k+1,\ldots,M$$

where $a_J$ is the jamming signal amplitude, and $\theta$ is the jamming signal phase relative to the useful signal, uniformly distributed in $0$ - $2\pi$ interval.

Probability $P$(HJ/$k$SBJ) is given by eq.26, where

$$P(U_{k+1}>U_{m+1})=\int_{-\infty}^{U_{k+1}}p(U_{m+1})dU_{m+1}=1-Q\left(\sqrt{2\frac{a_J^2}{\sigma^2}},\sqrt{\frac{U_{k+1}}{\sigma^2}}\right)$$
$$P(U_{k+1}>U_{2m-k+1})=\int_{-\infty}^{U_{k+1}}p(U_{2m-k+1})dU_{2m-k+1}=1-\exp\left(-\frac{U_{k+1}}{2\sigma^2}\right)$$
$$P(U_1>U_{m+1})=\int_{-\infty}^{U_1}p(U_{m+1})dU_{m+1}=1-Q\left(\sqrt{2\frac{a_J^2}{\sigma^2}},\sqrt{\frac{U_{k+1}}{\sigma^2}}\right)$$
$$P(U_1>U_{2m-k+1})=\int_{-\infty}^{U_1}p(U_{2m-k+1})dU_{2m-k+1}=1-\exp\left(-\frac{U_{k+1}}{2\sigma^2}\right)$$

where $Q(a,b)$ is Marcum's $Q$-function.

By averaging eq.26 with respect to $U_1$ and $U_{k+1}$ we have

$$P_s(\text{HJ}/k\,\text{SBJ}\mid a_c,\theta)=F_{noncoh}\left(\frac{a_c^2}{\sigma^2},\frac{a_J^2}{\sigma^2},\theta\right) \tag{36}$$

where

$$F_{noncoh}(y,z,\theta)=1-\Big[\ \frac{1}{2}\int_0^\infty \begin{matrix}\exp\left(-\frac{x}{2}-y-z-2\sqrt{yz}\cos\theta\right)\\ I_0\left(\sqrt{y-z-2\sqrt{yz}\cos\theta}\sqrt{2x}\right)\end{matrix}\times \tag{37}$$
$$\times F_Q(x,z,m-k)F_{\exp}(x,M-2m+k)dx\ \Big]^k\times$$
$$\times\Big[\ \frac{1}{2}\int_0^\infty \exp\left(-\frac{x}{2}-y\right)I_0\left(\sqrt{y}\sqrt{2x}\right)F_Q(x,z,m-k)$$
$$F_{\exp}(x,M-2m+k)\,dx\ \Big]^{n-k}$$

where

$$F_Q(x,z,n)=\begin{cases}\left[1-Q\left(\sqrt{2z},\sqrt{x}\right)\right]^n, & n\ge 0\\ 1, & n<0\end{cases} \tag{38}$$

$$F_{\exp}(x,n)=\begin{cases}\left[1-\exp\left(-\frac{x}{2}\right)\right]^n, & n\ge 0\\ 1, & n<0\end{cases} \tag{39}$$

After averaging eq.36 we get

$$P_s(\text{HJ}/k\text{SBJ})=\frac{1}{2\pi}\int_0^{2\pi}\int_0^\infty F_{noncoh}\left(\frac{a_c^2}{\sigma^2},\frac{a_J^2}{\sigma^2},\theta\right)f_{A_c}(a_c)da_c d\theta \tag{40}$$

## 4. Numerical results

By introducing multitone MFSK signaling, the energy of the useful signal is split into $m$ frequency bins and symbol error rate is expected to be higher. Nevertheless, for as long as system efficiency is E>1 the new schemes will perform better than the standard MFSK/FH system. In this section we present some numerical results to illustrate under what conditions we can expect E>1.

Efficiency improvement as a function of signal to noise ratio in the presence of MAI interference for coherent FH/mMFSK system is shown in Fig.3. Solid lines represent the case with no MAI interference and dashed lines represent the case with MAI interference with $P_c/P_{mait}$ = -20 dB. This corresponds to $q$=100 additional users of the same power. The values of the other signal parameters are: C-$m$=3, F-$m$=6; b-$M$=8, c- $M$=16, d - $M$=32, $N$=1000, $q$=100.

One can see that for lower $m$ (set C), $E$ becomes higher for lower $E_s/N_0$ but max($E_C$) <max($E_F$).

For each set of curves, max($E$) is higher if $M$ is higher.

In a multihop *ad hoc* network not all signals will reach a certain point with the same level. Relative levels of these signals will depend on the network topology. As an indication of the problem, in Fig. 4 symbol error probability is shown as a function of signal to total interference ratio. The other parameters are as follows. A - $m$=1, B - $m$=2, D - $m$=4; a - $M$=4, b - $M$=8, c - $M$=16, d - $M$=32, $N$=1000, $q$=100, $E_b$ / $N_0$ = 14dB

As it was expected the lower the $m$, the lower the BER, and vice versa. Also, for lower $M$, lower BER is obtained.

Efficiency improvement as a function of signal to noise ratio in the presence of fading and interference is shown in Fig.5.The other parameters are as follows.

coherent detection, $m$ = 2, solid line - no interference, dashed line - interference with $P_c/P_{Jt}$ = -20 dB, A - no fading, B - Rayleigh signal fading, a - $M$=4, b - $M$=8, c - $M$=16, d - $M$=32, $N$=1000, $q$=100.

One can see that efficiency improvement is worse in a fading environment. This is due to the fact that splitting signal power to m frequency bins is more critical in a fading channel.

The same set of curves is shown in Fig. 6 for, $m$ = 4. One can see that for $E_s/N_0$ <10dB, E is much worse for $m$=4 than for $m$=2. On the other hand if $E_s/N_0$ >10dB $E$ becomes much better for $m$=4.

Efficiency improvement as a function of signal to noise ratio in the presence of interference for coherent and noncoherent detection is compared in Fig.7. B - $m$=2, D - $m$=4; a - $M$=4, b - $M$=8, c - $M$=16, $N$=1000, $q$=100

If $E_s/N_0$ >12dB, improvements are larger than one. They are more significant for a coherent system.

For $E_s/N_0$ >18dB, parameter $E$ for both coherent and noncoherent systems become the same.

Fig. 8 represents symbol error probability as a function of signal to total interference ratio for no fading environment. Other parameters are: A - $m$=1, B - $m$=2: $M$=4: $N$=1000,$q$=100,$E_b$ / $N_0$ = 14dB. In a multihop *ad hoc* network for the same number of users $P_c/P_{mait}$ may increase and the system performance will become better. For $q$=100, $P_c/P_{mait}$ =-20dB corresponds to the case when all signals have the same power.

Finally Fig. 9 represents efficiency improvement as a function of signal to noise ratio in the presence of fading for noncoherent detection. The signal parameters are as follows: B - $m$=2, D - $m$=4; b - $M$=8, c - $M$=16; $N$=1000, $q$=100

In this case also, for $E_s/N_0$ >12dB efficiency improvements become larger than one.

In conclusion one can say that FH/mMFSK modulation offers better performance than the standard FH/MFSK signal format. Although for $m$>1 the signal energy is split to $m$ frequency bins, resulting in higher BER, the average number of correctly transmitted bits is increased. To achieve $E$>1, signal to noise ratio must be higher than a threshold value. Depending on the signal parameters this threshold value is between 9 and 14 dB.

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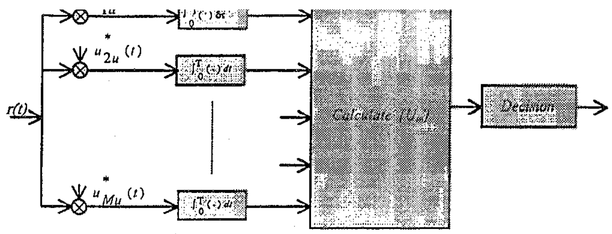


**Fig. 1 Coherent detection**

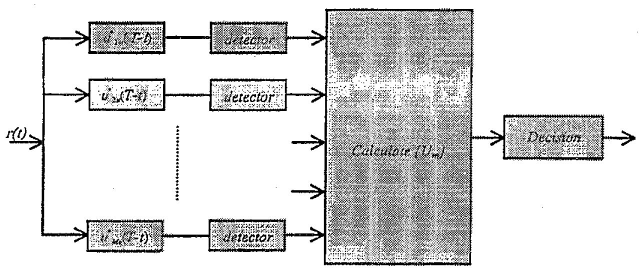


**Fig. 2 Noncoherent detection**

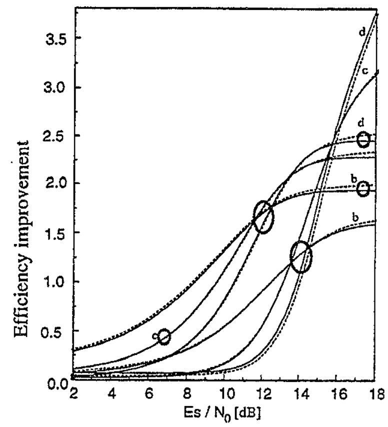


**Fig. 3 Efficiency improvement as a function of signal to noise ratio in the presence of interference**

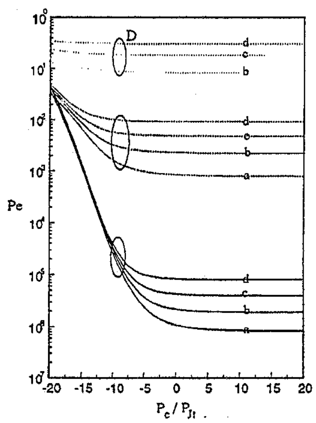


Fig. 4 Symbol error probability as a function of signal to total interference ratio

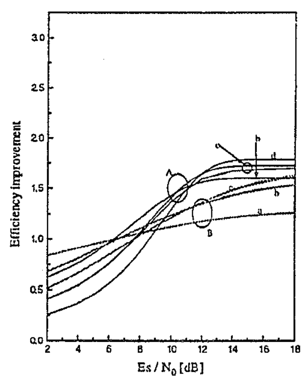


Fig. 5 Efficiency improvement as a function of signal to noise ratio in the presence of fading and interference

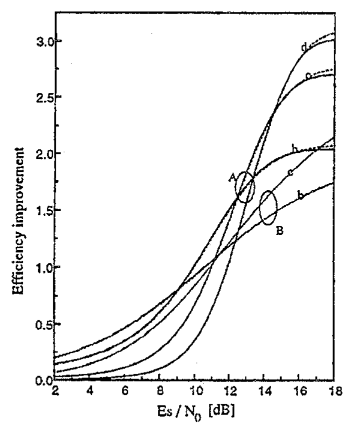


**Fig. 6 Efficiency improvement as a function of signal to noise ratio in the presence of fading and interference**

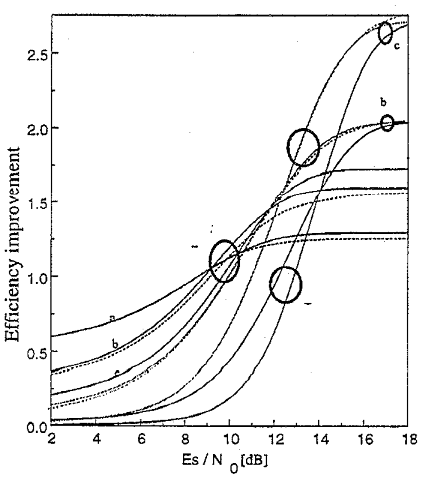


**Fig. 7 Efficiency improvement as a function of signal to noise ratio in the presence of interfence**

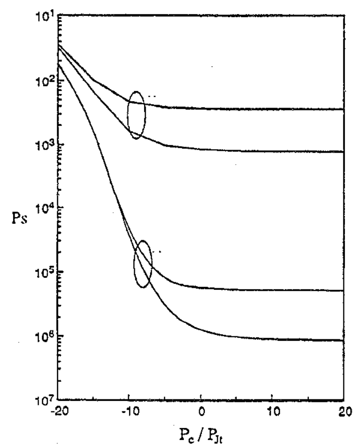


Fig. 8 Symbol error probability as a function of signal to total interference ratio

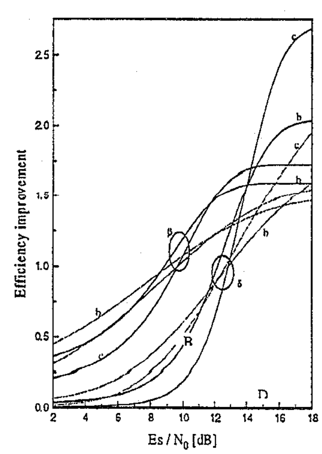


**Fig. 9 Efficiency improvement as a function of signal to noise ratio in the presence of fading**