

# Hybrid DS/FH-CDMA System Employing MT-FSK Modulation for Mobile Radio\*

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## Abstract

This paper presents an analytical evaluation of a direct-sequence/ frequency-hopped code division multiple access (DS/FH-CDMA) system employing non-coherent multiple tone frequency shift keying (MT-FSK) modulation in a Rayleigh fading environment. Multiple-tones per symbol rather than single tone per symbol (as in conventional MFSK) is used to provide diversity in a frequency selective channel. In the performance evaluations, multiple access interference (MAI) has been taken into account and the capacity for uncoded as well as convolutionally coded systems have been calculated.

## 1 Introduction

In CDMA when an active user is not speaking, the interference experienced by other users is reduced. Therefore CDMA can take better advantage of the voice activity factor than TDMA or FDMA. Also, CDMA is more tolerant of co-channel and adjacent channel interference. Hence CDMA is a good candidate for mobile communications.

CDMA systems which employ direct sequence spread spectrum (DS-SS) techniques can provide path diversity for faded signals [1]. However, DS is sensitive to the near-far effect. Frequency-hopped (FH) spread spectrum systems are more resistant to the near-far effect but usually their associated capacity is low. Hybrid DS/FH-CDMA systems are examined as a means of combining the advantages of both systems while avoiding their disadvantages [2-5]. In [5], a hybrid DS/FH-CDMA system employing fast frequency hopping (FFH) and M-ary frequency shift keying (MFSK) modulation is presented. The system discussed in this paper is the similar to the one described in [5] the major difference being that here wideband multiple tone frequency shift keying (MT-FSK) is the modulation scheme used. Thus by using multiple-tones per symbol rather than single tone per symbol frequency diversity is introduced to combat frequency selective fading.

## 2 MT-FSK Modulation

MT-FSK is a multiple tone modulation scheme in which energy is transmitted simultaneously over  $w$  orthogonal frequencies out of  $v$  possible frequencies. Thus it can convey at most  $\log_2 \binom{v}{w}$  bits of information per character. It has the potential to be more bandwidth efficient than conventional MFSK.<sup>1</sup> MT-FSK is an application of a more general system called permutation modulation [6].

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<sup>1</sup>MFSK is a special case of MT-FSK where  $w = 1$  and  $v = M$ .

## 2.1 Permutation Modulation

To describe the principle of permutation modulation, suppose we have a finite number of light emitting diodes (LEDs)  $l_1, l_2, \dots, l_w$ , and an alphabet  $a_1, a_2, \dots, a_z$  that we wish to represent by a combination of illuminated LEDs. If  $z = v$ , we can represent each letter in the alphabet by illuminating only one LED per symbol. If  $z > v$ , we can illuminate a pair or more LEDs to represent each letter. In general,  $\binom{v}{w}$  letters can be represented by only  $v$  LEDs by representing each letter by a combination of  $w$  illuminated LEDs. In the preceding example instead of switching on LEDs, if the information was conveyed by transmitting sinusoidal waveforms, the resulting modulation becomes MT-FSK.

## 2.2 Design of Efficient MT-FSK Waveforms

MT-FSK signalling employing permutation modulation theory may be more bandwidth efficient than 16 or 32FSK, but not more power efficient for a modest number of tones. An example of this is that  $\binom{16}{2}$  MT-FSK can transmit 120 possible symbols using half the number of frequency tones as 32FSK, yet 32FSK has about a 1dB power advantage. This is because in  $\binom{16}{2}$  MT-FSK, 16 symbols share the same frequency tone, therefore 1 symbol differs from 30 other symbols by 1 tone. This reduces the pairwise probability of error between adjacent symbols to that of MFSK, with only half the symbol energy (since the other half of the symbol energy is contained by the shared tone). Therefore it seems logical to suggest that MT-FSK using permutation modulation will not be as power efficient as MFSK in AWGN or flat fading channels.

However, instead of all of the 120 possible permutations suppose that only a fraction of them are actually used to represent an alphabet. Then we can introduce a redundancy into the modulation itself, giving us larger distances between symbols, and possibly some power gain. Furthermore, multiple tone systems inherently provide diversity to combat frequency selective fading.

In this paper, we design two waveforms. The first one is obtained by expurgating the (7,4) Hamming code to select a subset of equal weight codewords. The codewords of a (7,4) Hamming code are shown in Table 1. This method proves to be cumbersome when we wish to have a large signalling set. Therefore the second waveform is designed using the **Balanced Incomplete Block Design (BIB)** [7].

Let each position in a codeword represents a specific frequency. For example the first bit represents  $f_1$ , the second bit represents  $f_2$  etc. Also a 0 means that the frequency it represents is unused while a 1 indicates the frequency is

Table 1 : (7,4) Hamming code.

0000000	0001101
1101000	1100101
0110100	0111001
1011100	1010001
0011010	0010111
1110010	0100011
0101110	1001011
1000110	1111111

used in that specific waveform.

In order to obtain equal energy waveforms, we must use equal weight codewords. Let us examine all codewords of weight 3. The design of the waveforms using these codewords is shown in Table 2.

Table 2 : Equal energy waveforms derived from (7,4) Hamming codewords of weight 3.

Codeword	Waveform
1101000	$\cos 2\pi f_1 t + \cos 2\pi f_2 t + \cos 2\pi f_4 t$
0110100	$\cos 2\pi f_2 t + \cos 2\pi f_3 t + \cos 2\pi f_5 t$
0011010	$\cos 2\pi f_3 t + \cos 2\pi f_4 t + \cos 2\pi f_6 t$
1000110	$\cos 2\pi f_1 t + \cos 2\pi f_5 t + \cos 2\pi f_6 t$
0001101	$\cos 2\pi f_4 t + \cos 2\pi f_5 t + \cos 2\pi f_7 t$
1010001	$\cos 2\pi f_1 t + \cos 2\pi f_3 t + \cos 2\pi f_7 t$
0100011	$\cos 2\pi f_2 t + \cos 2\pi f_6 t + \cos 2\pi f_7 t$

We can see from Table 2 that 7 waveforms are obtained by using all (7,4) Hamming codewords of weight 3. However, for binary communications it is desirable to have  $k = \log_2 M$  where  $M$  is the number of symbols (waveforms) in the set. In this case, we must set  $M = 4$ . By eliminating all waveforms with  $\cos 2\pi f_7 t$ , we obtain 4 waveforms which employ 6 different frequencies. We will denote this signalling set as (6,3) MT-FSK. The waveforms are shown in Table 3 where each symbol of (6,3) MT-FSK shares only one frequency with every other symbol in the set.

Table 3 : (6,3) MT-FSK waveforms.

Symbol	Waveform
1	$\cos 2\pi f_1 t + \cos 2\pi f_2 t + \cos 2\pi f_4 t$
2	$\cos 2\pi f_2 t + \cos 2\pi f_3 t + \cos 2\pi f_5 t$
3	$\cos 2\pi f_3 t + \cos 2\pi f_4 t + \cos 2\pi f_6 t$
4	$\cos 2\pi f_1 t + \cos 2\pi f_5 t + \cos 2\pi f_6 t$

Using the BIB design detailed in [7], an 8-ary signalling set is designed. This signalling set employs 8 frequency tones, and each symbol is represented by 3 frequency tones. Thus it is referred to as (8,3) MT-FSK. The waveforms used in (8,3) MT-FSK are shown in Table 4.

Table 4 : (8,3) MT-FSK waveforms.

Symbol	Waveform
1	$\cos 2\pi f_1 t + \cos 2\pi f_2 t + \cos 2\pi f_4 t$
2	$\cos 2\pi f_2 t + \cos 2\pi f_3 t + \cos 2\pi f_5 t$
3	$\cos 2\pi f_3 t + \cos 2\pi f_4 t + \cos 2\pi f_6 t$
4	$\cos 2\pi f_4 t + \cos 2\pi f_5 t + \cos 2\pi f_7 t$
5	$\cos 2\pi f_5 t + \cos 2\pi f_6 t + \cos 2\pi f_8 t$
6	$\cos 2\pi f_1 t + \cos 2\pi f_3 t + \cos 2\pi f_8 t$
7	$\cos 2\pi f_1 t + \cos 2\pi f_6 t + \cos 2\pi f_7 t$
8	$\cos 2\pi f_2 t + \cos 2\pi f_7 t + \cos 2\pi f_8 t$

### 2.3 Performance of Wideband MT-FSK in Rayleigh Fading

For simplicity, we will analyse the performance of (6,3) MT-FSK then we will extend the results to include (8,3)

MT-FSK. Suppose we choose a minimum frequency tone separation  $\Delta f$  that is greater than the coherence bandwidth  $(\Delta f)_c$  of the channel. Then, for example, the fading process encountered by tone  $f_2$  is independent of the fading process seen by  $f_1$  or  $f_3$ . The fading process encountered by frequency tone  $f_i$  will be denoted as  $\alpha_i$ .

The optimal receiver for (6,3) MT-FSK is shown in Figure 1. The received signal is sent to 6 different matched filters, each filter matched to a different frequency tone. The outputs of these matched filters are squared and summed accordingly, and 4 decision variables are formed. The analysis assumes that the fading process is sufficiently slow that it can be considered constant over the duration of the signalling interval.

If symbol 1 is transmitted, the decision variables at the receiver are [8]:

$$\begin{aligned} U_1 &= |2\mathcal{E}\alpha_1 + N_1|^2 + |2\mathcal{E}\alpha_2 + N_2|^2 + |2\mathcal{E}\alpha_4 + N_4|^2 \\ U_2 &= |2\mathcal{E}\alpha_2 + N_2|^2 + |N_3|^2 + |N_5|^2 \\ U_3 &= |N_3|^2 + |2\mathcal{E}\alpha_4 + N_4|^2 + |N_6|^2 \\ U_4 &= |2\mathcal{E}\alpha_1 + N_1|^2 + |N_5|^2 + |N_6|^2 \end{aligned} \quad (1)$$

where  $\mathcal{E}$  is the energy per frequency tone ( $\mathcal{E} = \frac{E_s}{w}$ , where  $E_s$  is the total symbol energy),  $N_i$  are 0 mean noise random variables at the output of filter  $i$  with variance  $N_o W$ .  $N_o$  is the noise spectral density and  $W$  is the equivalent bandwidth of each matched filter.

The symbol error rate,  $P_M$ , can be found using the union bound

$$P_M < (M-1)P_2(D) \quad (2)$$

where  $D$  is the equivalent diversity order of the modulation scheme, and  $P_2(D)$  is given by

$$P_2(D) = P[U_2 > U_1] = P[U'_2 > U'_1] \quad (3)$$

$U'_1$  and  $U'_2$  are formed by eliminating the common term shared by  $U_1$  and  $U_2$ . Therefore

$$\begin{aligned} U'_1 &= |2\mathcal{E}\alpha_1 + N_1|^2 + |2\mathcal{E}\alpha_4 + N_4|^2 \\ U'_2 &= |N_3|^2 + |N_5|^2 \end{aligned} \quad (4)$$

It is shown in [8] that for the decision variables described by eq. 4,  $P_2(D)$  is given by

$$P_2(D) = \left(\frac{1-\mu}{2}\right)^D \sum_{i=0}^{D-1} \binom{D-1+i}{i} \left(\frac{1+\mu}{2}\right)^i \quad (5)$$

where  $D = w - 1 = 2$  and  $\mu$  is given by

$$\mu = \left(\frac{\bar{\mathcal{E}}}{N_o}\right) / \left(2 + \frac{\bar{\mathcal{E}}}{N_o}\right) \quad (6)$$

where  $\frac{\bar{\mathcal{E}}}{N_o}$  is the average energy to noise spectral density ratio per frequency tone. Therefore

$$\frac{\bar{\mathcal{E}}}{N_o} = \frac{\bar{E}_s}{N_o w} = \frac{k \bar{E}_b}{w N_o} \quad (7)$$

For (8,3) MT-FSK, each waveform shares 1 tone with 6 other waveforms and is completely orthogonal to the remaining waveform. Therefore, the symbol error rate performance of (8,3) MT-FSK is given by

$$P_M < 6P_2(D = w - 1) + P_2(D = w) \quad (8)$$

The bit error rate,  $P_b$  is

$$P_b = \frac{2^{k-1}}{2^k - 1} P_M \quad (9)$$

The bit error rate performance of wideband MT-FSK in Rayleigh fading is shown in Figure 2. For low  $E_b/N_o$ , (6,3) MT-FSK performs better than (8,3) MT-FSK. However, at high  $E_b/N_o$  (8,3) MT-FSK surpasses (6,3) MT-FSK. This is consistent with the behaviour of error correcting codes.

### 3 Hybrid DS/FH-CDMA System

The hybrid DS/FH-CDMA system discussed in this paper is shown in Figure 3. Wideband MT-FSK is the employed modulation scheme. The constraint is that the minimum frequency tone spacing be greater than the channel coherence bandwidth ( $\Delta f > (\Delta f)_c$ ). The output of the modulator is shown by point (a) in Figure 3. The signal is then multiplied by a pseudonoise (PN) sequence which has a rate of  $R_c$  chips/sec. By choosing the chip rate to be a multiple of  $R_s$ , the null-to-null bandwidth of one "spread" frequency tone is  $W_{ss} = 2R_c$ . We can select a value for  $R_c$  such that this bandwidth is less than the coherence bandwidth of the channel ( $2R_c < (\Delta f)_c$ ). We have the DS spread signal at point (b) in Figure 3. The signal is then hopped at a rate  $R_h$ . In other words, the "spread" signal is upconverted by a carrier whose value changes every  $1/R_h$  seconds. We wish to hop the signal  $L$  times per symbol to obtain additional diversity. Therefore  $R_h = LR_s$ , and  $R_h < R_c$ . This signal is transmitted over the channel at point (c) along with signals from other users. Thus assuming a large number of users, the entire available spectrum will contain components from different users.

At the receiver, the received signal is dehopped, then despread. Assuming that the locally generated replicas of the two PN sequences are perfectly aligned with those at the transmitter, the original signal corrupted by noise and multiple access interference (MAI) is obtained. Because the signal was transmitted using  $L$  carriers, it can be demodulated at the hop rate. Therefore, this will act as a repetition code, with the advantage that each equivalent symbol has undergone independent fading processes.

#### 3.1 Performance of Hybrid DS/FH-CDMA Employing MT-FSK

Let us first assume that there is no hopping, i.e.,  $L = 1$ . Therefore all users share the same  $v$  frequencies. If (6,3) MT-FSK is the modulation employed by this system then the equivalent decision variables from eq. 4 become

$$\begin{aligned} U'_1 &= |2\mathcal{E}\alpha_1 + N_1 + I_1|^2 + |2\mathcal{E}\alpha_4 + N_4 + I_4|^2 \\ U'_2 &= |N_3 + I_3|^2 + |N_5 + I_5|^2 \end{aligned} \quad (10)$$

where  $I_i$  is the effect of the MAI seen at the output of the  $i$ th filter. It has been shown in [2] that for DS-CDMA systems employing wideband FSK modulation schemes  $N_i + I_i$  can be approximated by a Gaussian random variable with variance  $N'_o W$  where

$$N'_o = N_o + \frac{2(U-1)}{v^2} \sum_{i=1}^v \sum_{j=1}^v \frac{E_b R_b R_c}{(2R_c)^2 + (\pi \Delta f)^2 (i-j)^2} \quad (11)$$

where  $U$  is the number of simultaneous users. Previously, we stated that  $2R_c \leq (\Delta f)_c \leq \Delta f$ . Therefore  $2R_c = \rho \Delta f$ , where  $\rho \geq 1$ . Thus eq. 11 becomes

$$\begin{aligned} N'_o &= N_o \left[ 1 + \frac{(U-1) \frac{E_b R_b}{N_o R_c}}{2v^2} \sum_{i=1}^v \sum_{j=1}^v \frac{1}{1 + (\pi \rho)^2 (i-j)^2} \right] \\ &= N_o \left[ 1 + (U-1) \frac{E_b R_b}{N_o R_c} \sigma(v, \rho) \right] \end{aligned} \quad (12)$$

where

$$\sigma(v, \rho) = \frac{1}{2v^2} \sum_{i=1}^v \sum_{j=1}^v \frac{1}{1 + (\pi \rho)^2 (i-j)^2} \quad (13)$$

The average bit energy to equivalent noise spectral density ratio is

$$\frac{\bar{E}_b}{N'_o} = \frac{E_b/N_o}{1 + (U-1) \frac{E_b R_b}{N_o R_c} \sigma(v, \rho)} \quad (14)$$

The symbol error rate for (6,3) MT-FSK in a DS-CDMA system can be found from eqs. 2, 5 and 6 by replacing  $\bar{E}_b/N_o$  by  $\bar{E}_b/N'_o$ . By extension, the symbol error rate of (8,3) MT-FSK in this system is given by eq. 8 with  $\bar{E}_b/N_o$  replaced by  $\bar{E}_b/N'_o$ .

The total bandwidth of the system without hopping is  $W_{tot} = 2R_c + (v-1)\Delta f = W_{ph}$ . The bandwidth efficiency of this system is:

$$\eta = \frac{U_{max} R_b}{W_{tot}} \quad (15)$$

where  $U_{max}$  is the maximum number of users the system can support. We define  $\gamma'_b$  as the average bit energy to equivalent noise spectral density when the number of users simultaneously accessing the channel is  $U_{max}$ . Therefore

$$\gamma'_b = \frac{E_b/N_o}{1 + (U_{max}-1) \frac{E_b R_b}{N_o R_c} \sigma(v, \rho)} \quad (16)$$

Rearranging eq. 15, we find

$$\begin{aligned} U_{max} &= \frac{\frac{E_b}{N_o} - \gamma'_b}{\frac{E_b R_b}{N_o R_c} \gamma'_b \sigma(v, \rho)} + 1 \\ &\approx \frac{R_c}{\gamma'_b R_b \sigma(v, \rho)} \quad \text{for } \frac{E_b}{N_o} \gg \gamma'_b \end{aligned} \quad (17)$$

Therefore the bandwidth efficiency is

$$\begin{aligned} \eta &\approx \frac{R_c}{\gamma'_b \sigma(v, \rho) [2R_c + (v-1)\Delta f]} \\ &\approx \frac{1}{2\gamma'_b \sigma(v, \rho) [1 + (v-1)\rho]} \end{aligned} \quad (18)$$

Now let us investigate the effect of hopping on this system. Each symbol is hopped into each of  $L$  independent channels. The total bandwidth  $W_{tot} = L W_{ph} = L[2R_c + (v-1)\Delta f]$ . At the receiver, the  $L$  symbols are combined using square-law combining. The equivalent decision variables become

$$U_m^* = U_m^{(1)} + U_m^{(2)} + \dots + U_m^{(L)} \quad (19)$$

For these decision variables,  $P_2(D)$  shown in eq. 5 becomes  $P_2(LD)$  and  $\frac{E_b}{N_o}$  must be replaced by  $\frac{E_b}{N_o L}$ . In other words, we have simply multiplied the inherent diversity of the modulation scheme by  $L$ .

The bit error rate performance of (6,3) MT-FSK and (8,3) MT-FSK are shown in Figure 4 for  $L = 1, 2$  and 3. Note that increasing the diversity order causes a degradation in bit error rate performance for small  $E_b/N_o$ , while the performance improves greatly for large  $E_b/N_o$ .

The values of  $\gamma'_b$  required to provide a bit error rate performance of  $10^{-3}$  are shown in Table 5 for different orders of diversity. As can be expected, (8,3) MT-FSK outperforms (6,3) MT-FSK for all  $L$ .

Table 5 :  $\gamma'_b$  required for a bit error rate of  $10^{-3}$ .

	$L=1$	$L=2$	$L=3$
(6,3) MT-FSK	20.5 dB	16.1 dB	15.0 dB
(8,3) MT-FSK	20.0 dB	15.1 dB	13.8 dB

For a hopped system, on average, the MAI seen by a signal comes from  $1/L$  of the remaining users. Therefore the equivalent noise spectral density becomes

$$N'_o = N_o \left[ 1 + \frac{U-1}{L} \frac{E_b R_b}{N_o R_c} \sigma(v, \rho) \right] \quad (20)$$

Therefore we can show that  $U_{max}$  increases by a factor of  $L$  for a hopped system ( $L > 1$ ) compared to a system

without hopping ( $L = 1$ ). However, the total bandwidth is also increased by a factor of  $L$ ; therefore the bandwidth efficiency for a hybrid DS/FH-CDMA system is also given by eq. 18. The only difference between the two is that as  $L$  increases,  $\gamma'_b$  decreases, thus improving the bandwidth efficiency.

A typical land mobile channel has a delay spread of roughly  $5 \mu\text{s}$  [8], hence the coherence bandwidth  $(\Delta f)_c \approx 200 \text{ kHz}$ . Therefore we use  $2R_c \leq 200 \text{ kchips/sec}$ , and  $\Delta f \geq 200 \text{ kHz}$ . Typical voice codecs operate at 4.8 to 9.6 kbps. Let us examine the case when  $R_b = 9.6 \text{ kbps}$ . Then a good choice for  $R_c$  is 96 kchips/sec. We also choose  $\Delta f = 211.2 \text{ kHz}$ . Therefore  $\rho = 1.1$ .

For (6,3) MT-FSK,  $v = 6$ , therefore  $\sigma(v, \rho) = 0.098$ , and for (8,3) MT-FSK,  $\sigma(v, \rho) = 0.074$ . The values for  $\gamma'_b$  are taken from Table 5. The bandwidth efficiency of the hybrid DS/FH-CDMA system employing MT-FSK is shown in Table 6 for both modulation schemes and different orders of diversity.

Table 6 : Bandwidth efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing uncoded MT-FSK in Rayleigh fading.

	$L = 1$	$L = 2$	$L = 3$
(6,3) MT-FSK	0.0092	0.0194	0.0250
(8,3) MT-FSK	0.0078	0.0240	0.0324

The results obtained in Table 6 are quite low. We can improve these results by employing error control coding.

### 3.2 Performance of Hybrid DS/FH-CDMA System with Error Control Coding

The performance of MT-FSK at low  $E_b/N_o$  is poor. Therefore, the use of low rate codes should be avoided, because the channel bit (as opposed to information bit) energy to noise spectral density ratio is  $E_{cb}/N_o = rE_b/N_o$  where  $r < 1$ . Therefore to have modest  $E_{cb}/N_o$ ,  $r$  should not be much less than 1 (say 3/4 or greater).

We consider a rate 3/4 constraint length 6 convolutional code and assume an interleaver which fully randomizes channel errors is included. Nonbinary BCH codes are also applicable to this system, however, this will be analysed in future work.

The performance of a rate  $v/n$  convolutional code using soft decision Viterbi decoding in Rayleigh fading is given by the following equation [8,9]

$$P_b < \frac{1}{v} \sum_{d=d_{free}}^{\infty} w_d P(d) \quad (21)$$

where  $w_d$  is the number of different paths (codewords) which are a distance  $d$  from the all-zero path. A list of  $w_d$  for different convolutional codes is given in [9:page 403]. The free distance of the code is denoted by  $d_{free}$  and is equal to 6 for the rate 3/4 constraint length 6 convolutional code [9].  $P(d)$  is given by:

$$P(d) = p^d \sum_{x=0}^{d-1} \binom{d-1+x}{x} (1-p)^x \quad (22)$$

where  $p$  is the raw bit error rate seen by the decoder. In other words  $p = P_b(rE_b/N_o)$  of the modulation scheme under consideration. For the rate 3/4 constraint length 6 convolutional code, to obtain a bit error rate of  $10^{-3}$ ,  $p = 0.0365$ .

Table 7 shows the values of  $\gamma'_b$  required to obtain  $P_b = 10^{-3}$  for MT-FSK with rate 3/4 constraint length 6 coding in Rayleigh fading for different orders of diversity.

Table 7 :  $\gamma'_b$  required for a bit error rate of  $10^{-3}$ .

	$L = 1$	$L = 2$	$L = 3$
(6,3) MT-FSK	13.2 dB	12.0 dB	11.9 dB
(8,3) MT-FSK	12.9 dB	11.4 dB	11.0 dB

The coding gain obtained from this code at  $P_b = 10^{-3}$  is about 7 dB for  $L = 1$  and between 2 and 4 dB for  $L = 2$  or 3. The bandwidth efficiency of the hybrid DS/FH-CDMA system employing rate 3/4 constraint length 6 convolutional coding is shown in Table 8.

Table 8 : Bandwidth efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing MT-FSK and rate 3/4 constraint length 6 convolutional coding.

	$L = 1$	$L = 2$	$L = 3$
(6,3) MT-FSK	0.0376	0.0495	0.0506
(8,3) MT-FSK	0.0398	0.0563	0.0617

Comparing Table 8 to Table 6, we see a significant improvement in the bandwidth efficiency when coding is used. However, there is still much room for improvement.

## 4 Conclusions

In this paper we presented a hybrid DS/FH-CDMA system which employs MT-FSK modulation. We showed that by increasing the hop rate to take advantage of the inherent frequency diversity, we can improve the bandwidth efficiency of the system. We also showed that error control coding can further improve the bandwidth efficiency. In our example, we employed a rate 3/4 constraint length 6 convolutional code. However, due to the modulation scheme's poor performance at low  $E_b/N_o$ , it might be advantageous to explore higher rate nonbinary block codes, such as BCH codes to further improve the performance of this system.

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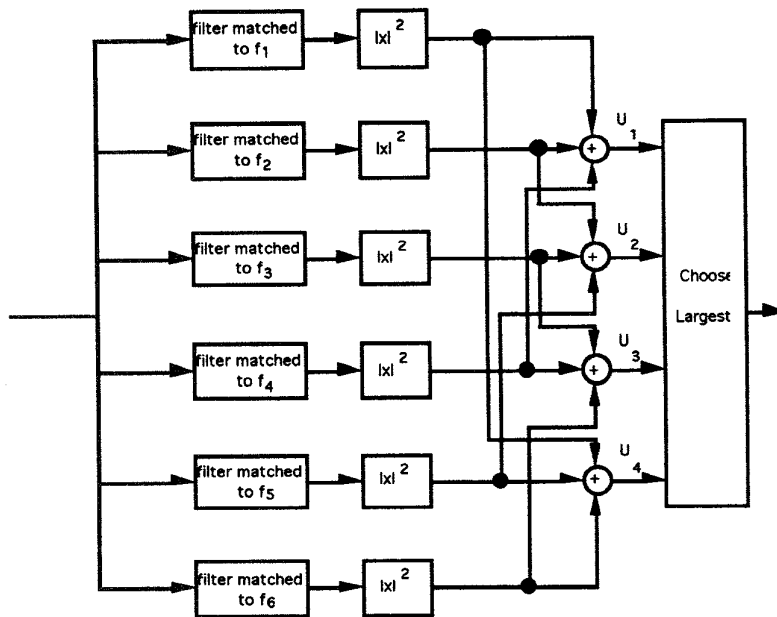


Fig. 1: Optimal receiver for (6,3) MT-FSK.

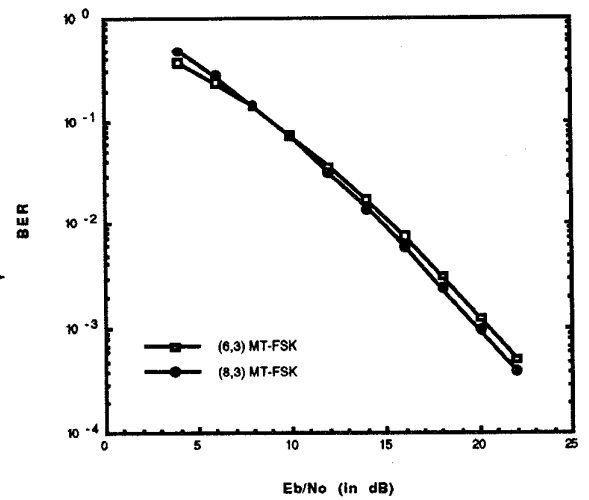


Fig. 2: Bit error performance of wideband MT-FSK in slow Rayleigh fading.

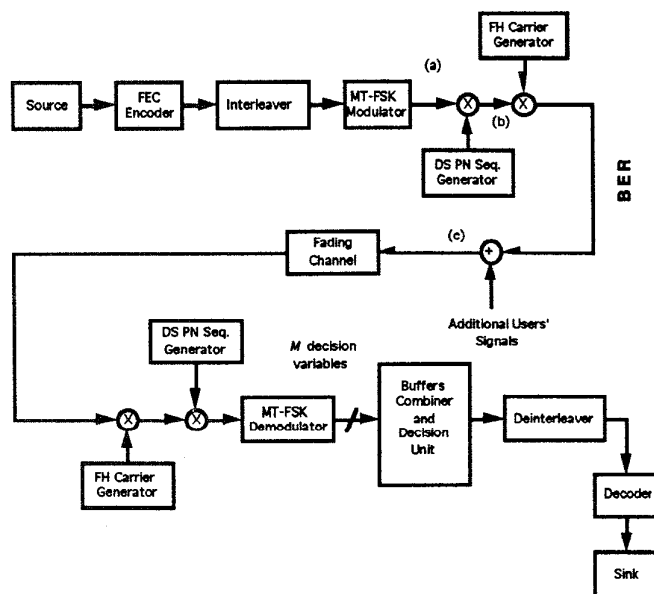


Fig. 3: Hybrid DS/FH-CDMA system employing MT-FSK.

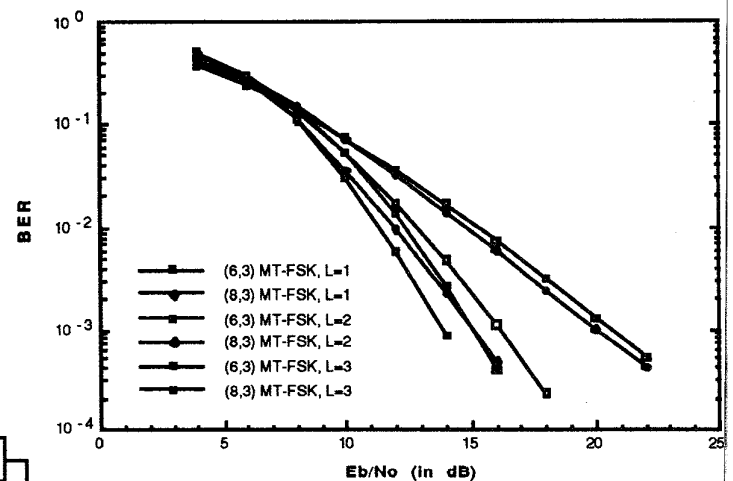


Fig. 4: Performance of (6,3) and (8,3) MT-FSK in Rayleigh fading with diversity.