An Efficient Modulation/Coding Scheme for MFSK Systems on Bandwidth Constrained Channels

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***Abstract*—The performance of a bandwidth efficient multiple tone modulation scheme for *M*-ary frequency shift keying (MFSK) is presented in this paper. The use of balanced incomplete block (BIB) designs is proposed to form the signaling frames. On each symbol interval the modulator selects a group of elements from a BIB design and divides its energy into the orthogonal waveforms corresponding to these elements. The multiple tone FSK scheme based on these block designs is shown to increase drastically the bandwidth efficiency of a conventional *M*-ary FSK system. Also, an implicit diversity is incorporated to the modulation scheme. Thus, a performance improvement comparable to using time or frequency diversity is shown on a Rayleigh fading channel and also on an interference channel with Partial Band Gaussian noise. Finally, the multiple tone scheme based on this design is compared to a multiple tone scheme based on Hadamard matrices, suggested by Pieper *et al.* in [7]. It is shown that similar performance is achieved on a fading channel, while an advantage close to 4 dB is obtained for the proposed scheme on an AWGN channel.**

I. Introduction

In a conventional M-ary frequency shift keying (MFSK) system [1]–[3] the signaling set is formed by M orthogonal waveforms. The signaling format is represented by frames of M binary elements, with only one active element (set to "1") per frame and each element corresponding to an input codeword. An extension to this modulation scheme is a multiple tone FSK (MT-FSK) system, on which the modulator divides its energy among $w > 1$ waveforms. The signaling frames are then represented by arrangements of v binary elements, with w active elements per frame and each arrangement corresponding to an input codeword.

The major advantage of this scheme resides on the reduction of the number of orthogonal waveforms required in the signaling set to represent a given alphabet and therefore, in a resultant bandwidth efficiency improvement. For example, if w active elements are arranged in a frame of v elements, then an alphabet of $\binom{v}{w}$ codewords can be represented using a signaling set comprised of only v waveforms. This scheme was initially described by Slepian [4], Schneider [5], and Biglieri *et al.* [6], and referred as FSK permutation modulation. Although this scheme shows an advantage over binary FSK, in terms of the energy per bit required to obtain a given bit error probability on an AWGN channel, its performance on a Rayleigh fading channel, or on a partial band Gaussian noise interference channel is poor. In [7], Pieper *et al.* suggested several techniques to improve the performance of this scheme on a fading channel. Selecting only some of the possible arrangements, the minimum Hamming distance between frames can be increased, thus an implicit diversity can be incorporated to the MT-FSK scheme based on these frames. However, frames obtained by these techniques show a large number of active elements per block, increasing with the alphabet size and degrading the performance of this scheme on an AWGN channel. It has been proved in [7] that the minimum Hamming distance between blocks in the signaling set determines the effective diversity of the system. On the other hand, the number of active elements per block, or block weight, determines the performance degradation of the system in AWGN, when noncoherent detection is used.

In this paper, a method is presented for selecting the arrangements of active elements. The suggested technique is based on a combinatorial construction called *balanced incomplete block design* (BIB design). The resultant frames intersect at most in a single element and the number of active elements per frame is independent of the alphabet size. Thus, the performance is slightly degraded on an AWGN channel, while significant improvements are achieved on Rayleigh fading and also on Partial Band Gaussian noise Interference. Furthermore, significant improvements on the bandwidth efficiency are obtained. Section II describes the basic properties of BIB designs and the MT-FSK based on these designs. The performance of this scheme is presented in Section III. Finally, Section IV compares this performance to that obtained using the techniques suggested in [7].

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II. MT-FSK Systems Based on BIB Designs

1) BIB Designs: A BIB design [8]–[10] is a collection of b blocks, formed by the arrangement of v distinct elements, satisfying the following conditions:

—each block contains w elements,

—each element occurs in r blocks, and

—each pair of elements occurs together in λ blocks.

This arrangement of elements is called a BIB design with parameters (v, b, r, w, λ). There are two basic relations among these parameters,

$$bw = vr, \tag{1.a}$$

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$$\lambda(v - 1) = r(w - 1). \tag{1.b}$$

The first equation counts the total number of single element occurrences, b blocks of w elements, or v elements on r blocks. The second equation counts the number of pairs containing a particular element, $(v - 1)$ pairs repeated on λ blocks, or $(w - 1)$ pairs repeated on r blocks. The number of blocks and the number of repetitions of a particular element are given by

$$b = \lambda \frac{v(v - 1)}{w(w - 1)}, \tag{2.a}$$

$$r = \lambda \frac{v - 1}{w - 1}. \tag{2.b}$$

Here, we consider the particular case, $\lambda = 1$. The BIB designs obtained in this case are known as *Steiner Systems*, and will be referred here as $S_w(v, k)$ where k is the number of input bits represented by the set of blocks. To obtain integer solutions in (2.a) and (2.b), v must satisfy the following conditions:

$$v \equiv 1, \quad \text{or} \quad v \equiv w \bmod w(w - 1). \tag{3}$$

Therefore, only some values of v and w are admissible [11]. Table I shows admissible values of v, for $w = 3, 4$, and 5, required to represent an alphabet of M codewords and compares these values to those required by a conventional MFSK, referred as $O_1(M, k)$, to represent the same alphabet.

By definition of a BIB design, when $\lambda = 1$ a given pair of elements occurs in only one block, therefore for any block B_i, the set of blocks can be partitioned into 3 disjoint subsets,

—B_i.

—$X_0(B_i)$, containing all orthogonal blocks to B_i.

—$X_1(B_i)$, containing all nonorthogonal blocks to B_i (intersecting to B_i on a single element).

It is proved in [10] that the number of blocks in the subsets $X_1(B_i)$ and $X_0(B_i)$, called *block intersection numbers*, are independent of the selected block B_i and are given by

$$x_1 = |X_1(B_i)| = w(r - 1) = \frac{w}{w - 1}(v - w), \tag{4.a}$$

$$x_0 = |X_0(B_i)| = b - x_1 - 1. \tag{4.b}$$

A summary of the parameters of a Steiner BIB design is given in Table II, compared to a conventional MFSK system.

The parameters of a BIB design selected to represent an M-ary alphabet are such that

—the number of blocks b is sufficiently large to represent each input codeword by a block in the Steiner set, and

—v is admissible.

The number of elements per block w determines the performance of the system on an AWGN channel. Here, results for $w = 3$, 4, and 5 are presented. Although higher values of w are possible, the bandwidth advantage of the system is reduced, as will be shown later. It is noted that $w = 2$ results in a design containing all possible combinations of w elements out of v and therefore, in a FSK Permutation Modulation.

TABLE I
NUMBER OF ELEMENTS REQUIRED BY A STEINER BIB DESIGN, $S_w(v, k)$, TO REPRESENT k INPUT BITS

k	$S_3(v,k)$	$S_4(v,k)$	$S_5(v,k)$	$O_1(M,k)$
1	7	13	21	2
2	7	13	21	4
3	9	13	21	8
4	13	16	21	16
5	15	25	41	32
6	21	37	41	64
7	31	40	61	128
8	43	61	81	256
9	57	85	105	512
10	79	112	145	1024

TABLE II
PARAMETERS OF A STEINER BIB DESIGN

Parameter	$S_w(v,k)$	$O_1(M,k)$
Elements	v	$M=2^k$
Blocks	$b=\frac{v(v-1)}{w(w-1)}$	$M=2^k$
Min. Distance	$2(w-1)$	2
Weight	w	1

A MT-FSK based on a BIB design is depicted in Fig. 1(a) and (b). The transmitter selects, according to the input codeword, a block $(1, 2, \cdots, w)$ of w elements from $S_w(v, k)$. Then, a v-ary FSK modulator divides equally its energy into the signals corresponding to the elements forming the selected block, transmitting the block of orthogonal waveforms $(f_1, f_2, \cdots, f_w)$. The receiver consists of a group of v matched filters followed by noncoherent detectors. These v noncoherent detected outputs are combined to obtain a decision variable for each block in the Steiner set. Each decision variable is formed combining the output of the matched filters corresponding to the w active elements forming each block. Finally, the largest decision variable is selected.

Let $B_j = (b_{j1}, b_{j2}, \cdots, b_{jw})$, for $j = 1, 2, \cdots, b$, be a block in the Steiner set. The receiver is based on the square-law combining of the matched filter outputs r_{jk} corresponding to these elements. Therefore, each decision variable is obtained as

$$U_j = \sum_{k=1}^{w} |r_{jk}|^2 \qquad \text{for } j = 1, 2, \cdots, b. \tag{5}$$

Let B_i be the transmitted block, with U_i its decision variable and consider a block B_j $(j \neq i)$, with a decision variable U_j. Two cases are then observed:

—$B_j \in X_0(B_i)$, (B_j has no element in common to B_i).

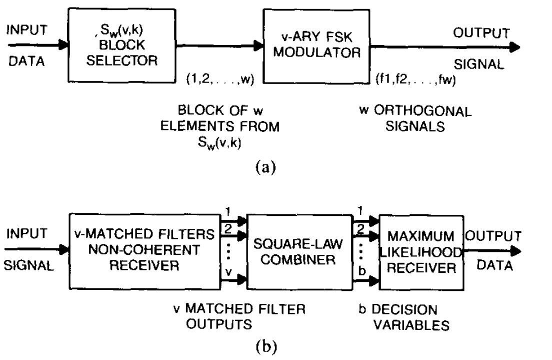


Fig. 1. (a) Steiner MT-FSK transmitter. (b) Steiner MT-FSK receiver.

—$B_j \in X_1(B_i)$, (B_j has only one element is common to B_i).

The number of blocks in each case is given by x_0 and x_1, independent of the transmitted block B_i. This particular intersection property of a Steiner system is used in the next sections to bound the symbol error probability of the proposed MT-FSK scheme.

2) Bandwidth Efficiency: It is known that for a conventional MFSK system, orthogonality in the signaling set is obtained for a frequency separation $\Delta f = 1/T$, with T the symbol interval. Consequently, the bandwidth W required to represent an M-ary alphabet is given by $W = RM/k$ where R is the transmission rate to transmit k bits on the symbol interval T. The bandwidth efficiency R/W for a conventional MFSK system with diversity L, is then given by

$$\frac{R}{W} = \frac{k}{ML}, \tag{6.a}$$

since the same symbol is repeated L times per signaling interval or transmitted over L independent subchannels or branches. For a MT-FSK based on a Steiner BIB design, only v orthogonal waveforms are required in the signaling set. If the transmission rate R is the same, the bandwidth efficiency is then given by

$$\frac{R}{W} = \frac{k}{v}. \tag{6.b}$$

From (6.a) and (6.b) it is seen that the bandwidth efficiency improvement is directly related to the ratio ML/v. Table III shows this ratio using an equivalent diversity $L = w - 1$. In the next section it is shown that a MT-FSK based on $S_w(v, k)$ achieves comparable performance to a conventional MFSK scheme with this diversity.

From (2.a) it is implied that the number of orthogonal waveforms v required by a Steiner MT-FSK to represent an M-ary alphabet is close to $\sqrt{w(w-1)M}$. Therefore, a reduction on the signaling block length is achieved when $w(w-1) < ML^2$, i.e., $S_4(v, k)$ is more efficient than $O_3(M, k)$, for $k > 2$, as can be observed from Table III. Although, an advantage is obtained when the above condition is satisfied, this signaling scheme is not efficient for lower value of k.

TABLE III
SIGNALING FRAMES RATIO (ML/v)

k	$S_3(v,k)/O_2(M,k)$	$S_4(v,k)/O_3(M,k)$	$S_5(v,k)/O_4(M,k)$
1	0.57	0.46	0.38
2	1.14	0.92	0.76
3	1.78	1.85	1.52
4	2.46	3.00	3.05
5	4.27	3.84	3.12
6	6.10	5.19	6.24
7	8.26	9.60	8.39
8	11.91	12.59	12.64
9	17.96	18.07	19.50
10	25.92	27.43	28.25

III. Performance of a MT-FSK Based on BIB Design

1) AWGN Channel: The receiver is based on the square-law combining of the w matched filter outputs corresponding to each block element; thus b decision variables are formed by the receiver. Given that B_i is the transmitted block and U_i its decision variable, a decision error is made when a block $B_j (j \neq i)$ is detected. Let U_j^0 be the decision variable corresponding to $B_j^0 \in X_0(B_i)$ and U_j^1 be the decision variable corresponding to $B_j^1 \in X_1(B_i)$. The symbol error probability can be upper bounded by

$$P_e(\gamma_b, w, M) \leq x_0 P_2(U_j^0 > U_i) + x_1 P_2(U_j^1 > U_i). \tag{7}$$

The first term in (7) corresponds to $B_j^0 \in X_0(B_i)$, and therefore no element is common to both decision variables. The decision error probability, $P_2(U_j^0 > U_i)$, is obtained as

$$P_2(E_0 < 0) = P(U_i - U_j^0 < 0) \tag{8}$$

where the random variable E_0 is defined as $E_0 = \Sigma_{k=1}^{w} (|r_{ik}|^2 - |r_{jk}|^2)$. The second term in (7) corresponds to $B_j^1 \in X_1(B_i)$ and therefore only one element is common to both decision variables. If $b_{iw} = b_{jw}$ is the intersecting element, then the decision error probability, $P_2(U_j^1 > U_i)$ is obtained as

$$P_2(E_1 < 0) = P(U_i - U_j^1 < 0) \tag{9}$$

where the random variable E_1 is defined as $E_1 = \Sigma_{k=1}^{w-1} (|r_{ik}|^2 - |r_{jk}|^2)$. It is shown in [3] that the decision error probability for two orthogonal blocks separated by a Hamming distance $2L$ is given by

$$P_2(\gamma_b, L) = \frac{e^{-k\gamma_b/2}}{2^{2L-1}} \sum_{n=0}^{L-1} \frac{1}{n!} \left(\frac{k\gamma_b}{2}\right)^n \sum_{r=0}^{L-1-n} \binom{2L-1}{r}, \tag{10}$$

where γ_b is the signal-to-noise ratio per bit. Therefore, equations (8) and (9) can be obtained as

$$P_2(E_0 < 0) = P_2(\gamma_b, L = w), \tag{11.a}$$

$$P_2(E_1 < 0) = P_2(\gamma_b, L = w - 1). \tag{11.b}$$

Fig. 2 compares the performance of a MT-FSK based on a $S_w(v, k)$ BIB design against a conventional MFSK with diversity L, referred as $O_L(M, k)$. A performance degradation close to 2 dB is obtained with this scheme in an AWGN channel, however the bandwidth efficiency is largely improved.

2) Rayleigh Fading Channel: For a frequency nonselective slowly fading channel, the signal envelope on each transmitted waveform is modeled as a Rayleigh distributed random variable. It is assumed that each signal is transmitted over independent and identically distributed (iid) subchannels or branches and therefore each received energy is an iid random variable. The decision error probabilities are obtained as the average,

$$P_2(\overline{\gamma_b}, L) = \int_0^{\infty} P_2(\gamma_b, L)p(\gamma_b)d\gamma_b \qquad (12)$$

where $\overline{\gamma_b}$ is the average signal-to-noise ratio per bit. The density function of the combiner output signal-to-noise ratio $p(\gamma_b)$ must be expressed in terms of the branch statistics. If a maximum ratio combiner is used, γ_b is given by the addition of several Rayleigh distributed random variables. The decision error probabilities, $P_2(\gamma_b, L)$, are given in (11.a) and (11.b). Solving (12), it can be shown that for two orthogonal blocks separated by a Hamming distance $2L$, the decision error probability is given by

$$P_2(\overline{\gamma_b}, L) = \left(\frac{1}{2 + \overline{\gamma_c}}\right)^L \sum_{r=0}^{L-1} \binom{L - 1 + r}{r} \left(\frac{1 + \overline{\gamma_c}}{2 + \overline{\gamma_c}}\right)^r \qquad (13)$$

where $\overline{\gamma_c} = \overline{\gamma_b}k/L$ is the signal-to-noise ratio per branch. This result is used in (11.a) with $L = w$, and (11.b) with $L = w - 1$ to obtain the average decision error probabilities. Finally, these equations are replaced in (7) to bound the symbol error probability.

The performance of a Steiner MT-FSK system on a Rayleigh fading channel is shown in Fig. 3, compared to a conventional MFSK system with diversity L. It is observed that the distance between Steiner blocks incorporates an implicit diversity into the system, comparable to a conventional MFSK system with diversity $L = w - 1$, while the bandwidth efficiency is improved.

3) Partial Band Gaussian Interference Channel: A partial band interference channel is modeled as a Gaussian noise jammer that restricts its energy to a fraction ρ ($0 < \rho \leq 1$) of the total spread-spectrum bandwidth. A frequency hopped MFSK (FH/MFSK) system has been proved to be an effective countermeasure to combat this type of interference [12]. In this scheme, several uncorrelated frequency bands or sub-channels are available to the transmitter, each one divided into M orthogonal frequency slots. On each symbol interval the modulator selects one subchannel, according to a random hop pattern, and transmits the MFSK symbol. To maximize its interference action, the jammer also randomly hops the location of the jammed band over the total spread spectrum, however the random hop pattern is only known by the transmitter and the intended receiver. The interference action is maximized considering that the jammer is able to adjust the interfered fraction of the band, such that a maximum degradation is achieved.

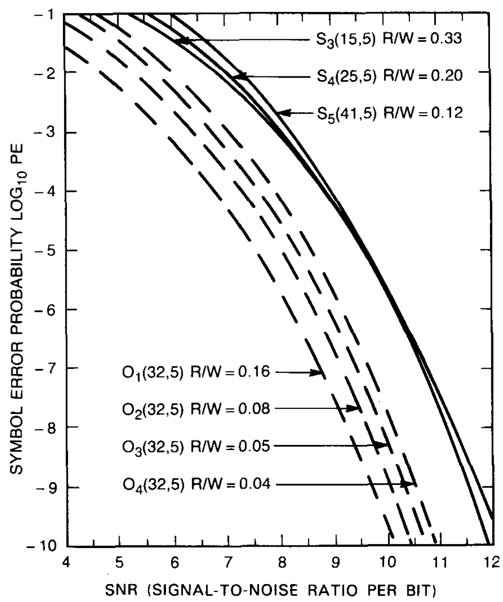


Fig. 2. Symbol error probability versus signal-to-noise ratio per bit (in dB), for a Steiner MT-FSK $S_w(v, k)$ on an AWGN channel, compared to a conventional MFSK with diversity L, $O_L(M, k)$, for $k = 5$.

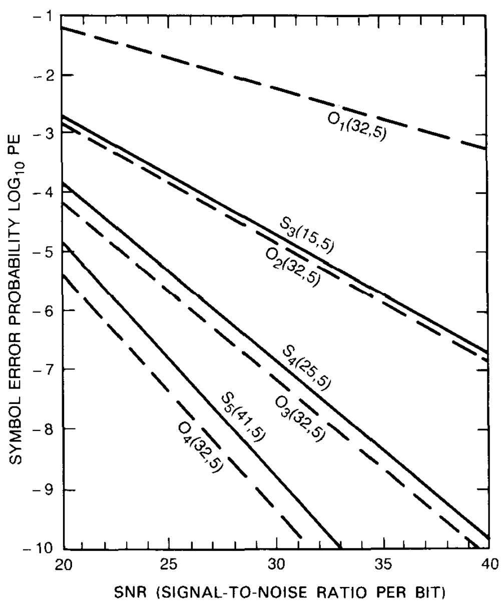


Fig. 3. Symbol error probability versus signal-to-noise ratio per bit (in dB), for a Steiner MT-FSK $S_w(v, k)$ on a Rayleigh fading channel, compared to a conventional MFSK with diversity L, $O_L(M, k)$, for $k = 5$.

The worst case jamming obtained with this strategy has been proved to be an effective interference against an MFSK scheme, since for large signal-to-noise ratio the jammer concentrates its energy within a small region; thus when the proper subchannel is interfered, a symbol error is more likely to occur. Here, a Steiner BIB design is used to obtain the w active elements or cells used by the modulator [13], [14]. These cells are then frequency hopped over the same number of separated and uncorrelated subchannels, the jammer also divides its energy over several subchannels. This FH/MT-FSK scheme is analyzed for a jammer acting in a noiseless background channel and its performance compared to a conventional FH/MFSK system using the same channel. According to this model, each subchannel is interfered with probability ρ, while an interference free transmission occurs on each subchannel with probability $(1 - \rho)$. Then the probability of a simultaneous interference on $j \leq w$ distinct subchannels is given by

$$P_j(\rho) = \binom{w}{j} \rho^j (1 - \rho)^{w-j}. \tag{14}$$

Since a noiseless background channel is assumed here, the receiver is able to know with certainty when a subchannel is interfered. This side information can be implemented by declaring an interfered subchannel when more than one matched filter output is high. If an interference is detected, the jammed subchannel is rejected or erased as an element on the corresponding decision variable. An errorless decision between two blocks can be taken when the distance between these blocks is higher than the minimum Hamming distance of the total set of blocks. When an errorless decision is not possible, the receiver forms a decision variable for each block in the Steiner set, as the square-law combining of the matched filter outputs corresponding to the block elements. Finally, the largest decision variable is decided. The symbol error probability is then given by

$$P_e(\rho\gamma_b, w, M) = \sum_{j=\lfloor d_{\min}/2 \rfloor}^{w} P_j(\rho) P_e(\rho\gamma_b, w, M \mid j) \tag{15}$$

where $P_e(\rho\gamma_b, w, M|j)$ is the symbol error probability, when j subchannel are interfered. For a Steiner set, it has been shown that two blocks intersect at most on a single element, then its minimum Hamming distance is $2(w - 1)$. If $w - 1$ cells are jammed, then any block in $X_1(B_i)$ (besides B_i), constitutes a possible block to be decided, while if all w cells are jammed, any block in the Steiner set is a possible block. In both cases the receiver selects the largest decision variables formed with the metric described before. Therefore, when $w - 1$ cells are interfered the symbol error probability is upper bounded by

$$P_e(\rho\gamma_b, w, M|w - 1) \leq x_1 P_2(U_j^1 > U_i) \tag{16.a}$$

and, when all w transmitted cells are interfered, the symbol error probability is upper bounded by

$$P_e(\rho\gamma_b, w, M|w) \leq x_0 P_2(U_j^0 > U_i) + x_1 P_2(U_j^1 > U_i) \tag{16.b}$$

where the decision error probabilities $P(U_j^0 > U_i)$ and $P(U_j^1 > U_i)$ are computed using (11.a) and (11.b), with γ_b degraded by a factor ρ. The worst case partial band interference probability of a symbol error is finally ob-

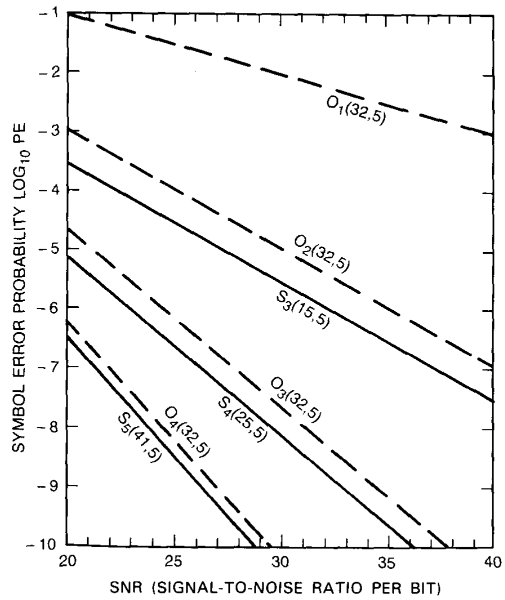


Fig. 4. Symbol error probability versus signal-to-noise ratio per bit (in dB), for a Steiner MT-FSK $S_w(v, k)$ on a partial band Gaussian noise interference channel, compared to a conventional MFSK with time diversity L, $O_L(M, k)$, for $k = 5$.

tained maximizing $P_e(\rho, \gamma_b, w, M)$ over all possible values of ρ,

$$\hat{P}_e(\gamma_b, w, M) = \max_{0 < \rho \leq 1} P_e(\rho\gamma_b, w, M). \tag{17}$$

Results are shown in Fig. 4, compared to a conventional FH/MFSK system with time diversity L. A comparable performance is obtained again, when $L = w - 1$.

IV. Steiner MT-FSK Compared to a Hadamard MT-FSK

Several options to obtain the signaling frames for a multiple tone FSK schemes are also described in [7], such as expurgating a Golay code to obtain constant weight codewords and then using the resultant codewords to obtain the signaling frames. However, a technique based on a normalized Hadamard matrix is shown to be effective against Rayleigh fading since a signaling set with a large Hamming distance is obtained. Here, this alternative design is used to compare the performance of a Steiner MT-FSK scheme.

The signaling blocks for a Hadamard MT-FSK are obtained directly from a Hadamard matrix of order n. Each row of a Hadamard matrix of order n consists of the same number of zeros and ones, also including the all zeros row. If the complements of these rows are considered and the all zeros row is excluded, a total of $b = 2(n - 1)$ blocks of length n, is obtained. It is observed that the number of active elements is $n/2$, and the number of intersecting elements to a given block is $n/4$ (except for its complement). Therefore, the minimum Hamming distance of the set is $n/2$. An appropriate value of n must be selected to obtain the $b \geq M$ blocks required to represent an input alphabet of $M = 2^k$ codewords. The performance

TABLE IV(a)
PERFORMANCE COMPARISON OF A STEINER MT-FSK AND A HADAMARD MT-FSK SYSTEMS, ON AN AWGN CHANNEL

R/W	$H(n,k)$	$S_3(v,k)$	$S_4(v,k)$	$S_5(v,k)$
0.185	10.5	7.2		
0.125	10.3	6.5	6.3	
0.077	10.6		6.0	5.7

TABLE IV(b)
PERFORMANCE COMPARISON OF A STEINER MT-FSK AND A HADAMARD MT-FSK SYSTEMS, ON A RAYLEIGH FADING CHANNEL

R/W	$H(n,k)$	$S_3(v,k)$	$S_4(v,k)$	$S_5(v,k)$
0.185	14.3	14.9		
0.125	12.5	14.3	12.0	
0.077	11.8		11.7	10.8

of this MT-FSK system can be obtained using an analysis similar to that used before. The symbol error probability is then upper bounded by

$$P_e \leq (M - 1)P_2(\gamma_b, L = n/4). \tag{18}$$

The decision error probability is obtained from (10) for an AWGN channel and from (13) for a Rayleigh fading channel, with $L = n/4$.

To compare the performance of a Steiner MT-FSK against a Hadamard MT-FSK, schemes with similar bandwidth efficiency are used, i.e., an $H(20, 5)$ Hadamard BIB design with $R/W = 0.25$, is compared to a $S_3(21, 6)$ Steiner BIB design, with a $R/W = 0.29$. A Reed–Solomon code of rate close to $3/4$ is used to encode both systems. Table IV(a) compares the signal-to-noise ratio per bit required to achieve a symbol error probability of 10^{-5} on an AWGN channel, for both schemes. In Table IV(b), the same comparison is shown for a Rayleigh fading channel.

It is observed that, for a fading channel, the performance of both schemes is similar within 1 dB of difference, however in an AWGN channel the performance of a Steiner MT-FSK is 4 dB better than a Hadamard MT-FSK. The performance of a Hadamard MT-FSK on a Partial Band Gaussian noise interference channel is similar to the results on a fading channel and results are not shown here.

V. CONCLUSIONS

In this paper, we have presented the performance improvements obtained with a MT-FSK scheme based on a balanced incomplete block (BIB) design to select the blocks of multiple waveforms. This signaling scheme is shown to be efficient, in terms of the bandwidth required to represent a given alphabet size. Also, the intersection and distance properties of the combinatorial design used to select the blocks are such that a significant performance improvement is obtained in a Rayleigh fading, and also in an interference channel with partial band Gaussian noise. The performance on an AWGN channel shows a degradation close to 2 dB. We finally compared the performance of the Steiner MT-FSK, with a MT-FSK based on Hadamard matrices [7]. A similar performance is obtained on a Rayleigh fading channel, and performance improvements higher than 4 dB are shown on an AWGN channel.

REFERENCES

[1] M. Schwartz, W. Bennet, and S. Stein, *Communication Systems and Techniques*. New York: Mc-Graw Hill, 1966.

[2] J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering*. New York: Wiley, 1976.

[3] J. G. Proakis, *Digital Communications*. New York: Mc-Graw Hill, 1983.

[4] D. Slepian, "Permutation modulation," *Proc. IEEE*, vol. 53, pp. 228-236, Mar. 1965.

[5] H. L. Schneider, "Data transmission with FSK permutation modulation," *Bell Syst. Tech. J.*, vol. 47, pp. 1131-1138, July-Aug. 1969.

[6] E. Biglieri, V. Castellani, and M. Pent, "Waveform transmission with permutation modulation," *Electron. Lett.*, vol. 5, no. 7, pp. 149-151, Apr. 3, 1969.

[7] J. F. Pieper, J. G. Proakis, R. R. Reed, and J. K. Wolf, "Design of efficient coding and modulation for a Rayleigh fading channel," *IEEE Trans. Inform. Theory*, vol. IT-24, pp. 457-468, July 1978.

[8] M. Hall, Jr., *Combinatorial Theory*. Waltham, MA: Blaisdell, 1967.

[9] I. F. Blake and R. C. Mullin, *The Mathematical Theory of Coding*. New York: Academic, 1975.

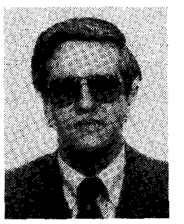
[10] F. G. McWilliams and N. J. Sloane, *The Theory of Error Correcting Codes*. Amsterdam, The Netherlands: North-Holland, 1977.

[11] E. S. Kramer and M. D. Mesner, "Admissible parameters for Steiner systems $S(t, k, v)$, with a table for all $(v - t) \leq 498$," *Utilitas Mathematica*, vol. 7, pp. 211-222, 1975.

[12] M. K. Simon, J. K. Omura, R. A. Scholtz, and B. K. Levitt, *Spread Spectrum Communications*. Rockville MD: Computer Science, 1985.

[13] G. E. Atkin and H. P. Corrales, "Performance analysis of a multiple tone FH/MFSK spread spectrum system," *IEEE Tech. Conf. Tactical Commun.*, Fort Wayne, IN, May 1988, pp. 284-289.

[14] ——, "Performance analysis of a non-orthogonal FH/MFSK system under partial band interference," *Adv. Commun., Contr. Conf.*, Baton Rouge, LA, Oct. 1988, pp. 40-53.



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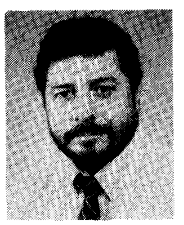

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