MathApp Documentation

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1 Introduction

Welcome to MathApp. This is an app that I made to explore some fun pieces of mathematics. Use one of the functionalities and the answer will be output in the response area on the bottom. Please note that for floating point numbers, the decimal point (.) is used and not a comma.

1.1 Contents

Currently, the app contains the following:

- Summing all integers in a certain interval [n,m].
- Calculating factorials for very large numbers.
- Solving the geometric series (i.e. $1 + x + x^2 + x^3 + \dots$ for $x \in (-1, 1)$).
- Calculating the number e, by solving $(1 + 1/n)^n$ for large values of n (i.e. taking $n \to \infty$).
- Calculating simple derivatives from user input.
- Encrypting and decrypting messages to share with your friends via making and solving a pseudo-random system of equations.
- An actual calculator that requires a string as input.
- Factorize numbers into prime factors.
- Calculate binomial coefficients.
- Unit conversions, currently supported: Fahrenheit-Celsius, pounds-kilograms, miles-kilometers and vice versa.
- Randomly generated math problems to work on your ability to solve systems of equations.
- Currency converter that's automatically updated.
- Basic linear algebra.

2 Integer Summation

The integer summation solves the equation

$$\sum_{n=i}^{m} n, \tag{1}$$

where, $m, n, i \in \mathbb{Z}^+$. In other words, summing all integers from i to m. In the backend, this is being done via the equation

$$\sum_{n=i}^{m} n = \frac{(i+m)(m-i+1)}{2}, \tag{2}$$

which is the famous solution invented by Gauss.

2.1 How to use

In the app, you're supposed to give the starting integer and stopping integer. Put these in the fields that say "start (integer)" and "end (integer)" and then press the "Sum" button.

3 Factorial

Factorial of an integer is defined as

$$n! = n(n-1)(n-2)(n-3)\ldots 3 \cdot 2 \cdot 1, \tag{3}$$

where $n \in \mathbb{Z}^+$.

3.1 How to use

The computer evaluates this just as a loop and will only take integers as an input. The Gamma function is not taken in consideration here. Enter the value of n into the field that says "factorial (integer)" and press the "Factorial" button.

4 Geometric series

A geometric series is defined by the infinite sum

$$\sum_{k=0}^{\infty} r^k, \tag{4}$$

where $r \in (-1, 1)$. A commonly used name for r is the common ratio. If any number outside of the allowed interval is entered as the common ratio then the series will diverge, otherwise it will converge to a finite value. This value can be calculated easily via

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}. \tag{5}$$

Proof:
Let $a \in \mathbb{R}$ and

$$s = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}, \tag{6}$$

which is the geometric series for the first n terms. Now multiply both left and right with the common ratio r. This gives

$$rs = ar + ar^2 + ar^3 + ar^4 + \ldots + ar^n. \tag{7}$$

Subtracting equation 7 from equation 6 gives

$$s - rs = a - ar^n. \tag{8}$$

Rearranging gives

$$s = \frac{a - ar^n}{1-r}. \tag{9}$$

Now taking the limit of $n \to \infty$ in equation 9 gives

$$s = \lim_{n\to\infty} \frac{a - ar^n}{1-r} = \frac{a}{1-r}, \tag{10}$$

if and only if $r \in (-1, 1)$, otherwise the limit will diverge. In the case that $a = 1$, as in our original case, equation 10 turns into

$$s = \frac{1}{1-r}. \tag{11}$$

$\blacksquare$

4.1 How to use

The backend of the application uses equation 11 to compute the infinite sum in equation 4. In order to use it, fill in your number in the field that says "Value of r" and press the "Geometric series" button. Make sure that the input r satisfies $-1 < r < 1$, otherwise the series will diverge and the app will tell you that it does. Remember to use a decimal point, and not a comma. In other words, 0.321 is a valid input, where 0,321 is not.

5 Definition of e

The number e is defined by the limit

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n . \tag{12}$$

This gives e a numerical value of

$$e = 2.71828182845904\ldots \tag{13}$$

The number e is a famous transcendental number. Transcendental means that it is not the root solution to a non-zero polynomial with rational coefficients. However, this number is mostly known for its exponential function being the derivative of itself. In other words

$$\frac{d}{dx} e^x = e^x . \tag{14}$$

5.1 How to use

The computer solves equation 12, but takes n as an input instead of taking the limit to infinity. Enter the value of n in the field that says "Value of n (integer)" and press the button that says "Get e". This will solve the equation and give an approximation of e, that becomes better as the value of n increases.

6 Derivatives

The derivative of a function measures the rate of change of a function in a given infinitesimal point. The change of a function $y = f(x)$, between points (x_1, y_1) and (x_2, y_2) is given by

$$\Delta f(x) = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} . \tag{15}$$

If the point (x_2, y_2) is redefined as point (x_1, y_1) plus an offset h, then equation 15 becomes

$$\Delta f(x) = \frac{f(x_1 + h) - f(x_1)}{h} . \tag{16}$$

In order to get the difference in the infinitesimal point (x_2, y_2), the limit of $h \to 0$ has to be taken, which gives

$$\frac{df(x_1)}{dx} = \lim_{h\to 0} \frac{f(x_1 + h) - f(x_1)}{h} . \tag{17}$$

Of course, x_2 can be any point where the curve is differentiable, so x_2 can be substituted by x as in

$$\frac{df(x)}{dx} = \lim_{h\to 0} \frac{f(x + h) - f(x)}{h} , \tag{18}$$

which is the definition of the derivative.

6.1 Rules of Derivatives

The definition of the derivative has given rise to a set of rules for taking derivatives.

6.1.1 Power rule

The power rule is defined as

$$\frac{d}{dx}x^n = nx^{n-1}, \tag{19}$$

where $n \in \mathbb{R}$. This follows from the definition:

$$\frac{d}{dx}x^n = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h} = \frac{x^n + h^n + c.t. - x^n}{h} = \frac{h^n + c.t.}{h}, \tag{20}$$

where $c.t.$ represent cross terms from the multiplication, defined by

$$c.t. = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}h^k, \tag{21}$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}. \tag{22}$$

Solving each separate term gives

$$\frac{d}{dx}x^n = \lim_{h\to 0}\left\{\frac{h^n}{h} + \frac{1}{h}\sum_{k=0}^{n}\binom{n}{k}x^{n-k}h^k\right\} \tag{23}$$

$$= \lim_{h\to 0}\left\{h^{n-1} + \sum_{k=0}^{n}\binom{n}{k}x^{n-k}h^{k-1}\right\}. \tag{24}$$

Now the limit can be taken safely and only the term with $k = 1$ will survive from the sum, which gives

$$\frac{d}{dx}x^n = \binom{n}{1}x^{n-1} = nx^{n-1} \tag{25}$$

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6.1.2 Sum rule

The sum rule is defined by

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x). \tag{26}$$

This follows from the definition via

$$\begin{aligned}
\frac{d}{dx}(f(x)+g(x)) &= \lim_{h\to 0}\frac{f(x+h)+g(x+h)-f(x)-g(x)}{h} \\
&= \lim_{h\to 0}\frac{f(x+h)-f(x)+g(x+h)-g(x)}{h} \\
&= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\
&= \frac{d}{dx}f(x)+\frac{d}{dx}g(x) \qquad (27)
\end{aligned}$$

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6.1.3 Product rule

The product rule is defined as

$$\frac{d}{dx}\{f(x)g(x)\} = g(x)\frac{d}{dx}f(x)+f(x)\frac{d}{dx}g(x). \qquad (28)$$

This follows from the definition of derivatives

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}, \qquad (29)$$

now add and subtract the term $f(x)g(x+h)$, which gives

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}\frac{f(x+h)g(x+h)+f(x)g(x+h)-f(x)g(x+h)-f(x)g(x)}{h}. \qquad (30)$$

Combining the terms with $g(x+h)$ and $f(x)$ gives,

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}\frac{g(x+h)[f(x+h)-f(x)]+f(x)[g(x+h)-g(x)]}{h}. \qquad (31)$$

This can be rewritten to

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}g(x+h)\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}f(x)\frac{g(x+h)-g(x)}{h}. \qquad (32)$$

Note that, because $g(x)$ is continuous, we can write $\lim_{h\to 0}g(x+h)=g(x)$. So equation 32 can be rewritten to

$$\frac{d}{dx}\{f(x)g(x)\} = g(x)\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+f(x)\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}. \qquad (33)$$

Then using the definition of derivatives, this gives

$$\frac{d}{dx}\{f(x)g(x)\} = g(x)\frac{d}{dx}f(x)+f(x)\frac{d}{dx}g(x). \qquad (34)$$

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6.1.4 Chain Rule

The chain rule is defined by

$$\frac{d}{dx}f(g(x)) = \frac{df(g(x))}{d(g(x))}\frac{dg(x)}{dx}. \tag{35}$$

This follows from the definition of derivatives by

$$\begin{aligned}
\frac{d}{dx}f(g(x)) &= \lim_{h\to 0}\frac{f(g(x+h)) - f(g(x))}{h} \\
&= \lim_{h\to 0}\frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)}\frac{g(x+h) - g(x)}{h} \\
&= \lim_{h\to 0}\frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)}\lim_{h\to 0}\frac{g(x+h) - g(x)}{h} \\
&= \frac{df(g(x))}{d(g(x))}\frac{dg(x)}{dx}.
\end{aligned} \tag{36}$$

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6.1.5 Standard Derivatives

A lot of functions have well documented standard derivatives. Table 1 shows a list of some standard derivatives that are used by MathApp.

Function	**Derivative**
e^x	e^x
b^x	$b^x \ln(b)$
$\ln(x)$	$1/x$
$\log_b(x)$	$1/(x\ln(b))$
x^x	$x^x \ln(x+1)$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\sec(x)$	$\tan(x)\sec(x)$
$\csc(x)$	$-\cot(x)\csc(x)$
$\cot(x)$	$-\csc^2(x)$

Table 1: Standard derivatives of some common functions. Note that the letter b is used for any base number other than e.

6.2 How to use

In order to use the derivative function in MathApp, the user has to determine which rule of differentiation has to be used. In order to do this, the following rules of syntax have been put into place.

- For using the power rule or exponential functions, use the $\wedge$ (SHIFT+6) operator. Correct syntax would be $x \wedge 2$ or $2 \wedge x$ for power and exponential rules.

- The product rule is called using the $*$ operator. Correct syntax would be $\sin(x) * \cos(x)$. However, never use the product rule for a case like $x(x+1)$. In such a case, always multiply it out yourself first and use $x^2 + x$ or $x \wedge 2 + x$ instead.

- The quotient rule does not exist. It is actually a product rule or a power rule. Therefore, do not use $1/x$, instead use $x \wedge -1$. Also do not use $\sin(x)/x$, but use $\sin(x) * x \wedge -1$. In the case of $1/(x^2 + x)$, use $\{x \wedge 2 + x\} \wedge -1$. This will use the chain rule.

- In order to use the chain rule, surround the function $g(x)$, from $f(g(x))$, with $\{$ and $\}$. In other words, $\sin(\{x \wedge 2\})$ is correct syntax for $\sin(x^2)$.

- In order to get the derivative of $x/(x^2 + x)$, grouping can be used in the following way: $x * (\{x \wedge 2 - x\} \wedge -1)$. Notice the use of the product rule and chain rule instead of the quotient rule. Also notice the use of parenthesis.

- Note that table 1 shows all of the included special functions. They can be called as written, e.g. $\sin(x)$ and $e \wedge x$ are correct syntax.

7 Encryption/Decryption

The encryption algorithm in MathApp is based on a polynomial of degree n (P_n) passing exactly though $n+1$ points. This can be used to share a secret word or sentence with a group of $n+1$ friends and insure that the string can only be decrypted when all friends are together.
Let's say we have a random string we want to encrypt:

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We can convert this into numbers according to the ASCII table and to make sure we always have 3 digits for each character, we add 101 to the value of each character and an additional value every time the position of a character in the string increases, in this case 1 is picked. However, we may also pick larger numbers to add. Next we concatenate all the numbers we get in order. This will give us the following number:

$$a_0 = 169199200214137189217219210217212222. \tag{37}$$

Now consider the following first degree polynomial,

$$y = a_0 + a_1 x \tag{38}$$

and note that the value of a_0 is the same in eq. 38 as in eq. 37. Next, pick a random value for a_1 and find two points at e.g. $x = 1$ and $x = 2$ that lie on this line. These will be your encrypted numbers. Only somebody who knows both numbers can figure out what the original number a_0 was that you started with. This can of course be extended to higher order equations, e.g.

$$y = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n. \tag{39}$$

Note that for a polynomial of degree n, we need (at least) $n + 1$ points to describe it. In all cases, a_0 needs to be the original number (eq. 37) and all the others can be picked at random. As was done with the linear equation, pick random x values for every point and solve equation 39 for each x value and save the output. Now you can store every number on a separate location and only when someone has all the numbers, will they be able to solve the original message. So, if you and your friend have a secret, you can do this and give one number to your friend. Then keep one number yourself. Now neither of you can decrypt the message on your own, but you can solve it when you are together.

8 Decryption

I will describe the specific case first and then a general case.

8.1 Specific case

In order to decrypt a certain message, remember that we have two points. Let's say the points P_1 and P_2 are defined by the following coordinates:

$$\begin{aligned} P_1 &= (-98, -5515727087651319714542069912182090674), \\ P_2 &= (44, 2721615084561893350088736775376082910). \end{aligned} \tag{40}$$

Since there are only two points, our best guess is that the original was formed using a first degree polynomial (eq. 38). So in order to get the value for a_0, we need to derive the value of a_1 from the given points. This is defined by:

$$a_1 = \frac{\Delta y}{\Delta x} = \frac{P_2^{(y)} - P_1^{(y)}}{P_2^{(x)} - P_1^{(x)}}, \tag{41}$$

where $P_n^{(y,x)}$ refers to the y and x coordinate of the n'th point, respectively. Then filling in all the values gives

$$a_1 = 58009451916994458201625399208156152. \tag{42}$$

Then a_0 will be

$$a_0 = P_1^{(y)} - a_1 = 1691992002141371892172192102172122222. \tag{43}$$

Note that each character will be represented by three digits and that the ASCII value can be generated by subtracting 101, and also another 1 for each character, starting at 0. Doing this will get our original string back, which was

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8.2 General case

Let's say we are given any number of points, defined by

$$\{P_1 = (x_1, y_1), P_2 = (x_2, y_2), ..., P_n = (x_n, y_n)\}. \tag{44}$$

These points define the n'th degree polynomial, defined by

$$y(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n. \tag{45}$$

In order to decrypt the message, a_0 must be found. Combining eqs. 44 and 45 gives a system of equations with n unknowns and n equations defined by

$$\begin{aligned}
&y_1 = a_0 + a_1 x_1 + a_2 x_1^2 + ... + a_n x_1^n \\
&y_2 = a_0 + a_1 x_2 + a_2 x_2^2 + ... + a_n x_2^n \\
&\vdots \\
&y_n = a_0 + a_1 x_n + a_2 x_n^2 + ... + a_n x_n^n.
\end{aligned} \tag{46}$$

Note that all the values for x_n and y_n are known from the points defined in eq. 44. Now solve the system of equations shown in equation 46. This should give a value for a_0 that gives the numerical representation of the original string. Once again, this number should be split into three digits for each character and can then be converted by subtracting 101 and using the ASCII table.