MathApp Documentation

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1 Introduction

Welcome to MathApp. This is an app that I made to explore some fun pieces of mathematics. Use one of the functionalities and the answer will be output in the response area on the bottom. Please note that for floating point numbers, the decimal point (.) is used and not a comma.

1.1 Contents

Currently, the app contains the following:

- Summing all integers in a certain interval [n,m].
- Calculating factorials for very large numbers.
- Solving the geometric series (i.e. $1 + x + x^2 + x^3 + \dots$ for $x \in (-1, 1)$).
- Calculating the number e, by solving $(1 + 1/n)^n$ for large values of n (i.e. taking $n \to \infty$).
- Calculating simple derivatives from user input.
- Encrypting and decrypting messages to share with your friends via making and solving a pseudo-random system of equations.
- Unit conversions, currently supported: Fahrenheit-Celsius, pounds-kilograms, miles-kilometers and vice versa.
- Randomly generated math problems to work on your ability to solve systems of equations.
- An actual calculator that requires a string as input.
- Currency converter that's automatically updated.
- Basic linear algebra.
- Factorize numbers into prime factors.
- Calculate binomial coefficients.

2 Integer Summation

The integer summation solves the equation

$$\sum_{n=i}^{m} n, \tag{1}$$

where, $m, n, i \in \mathbb{Z}^+$. In other words, summing all integers from i to m. In the backend, this is being done via the equation

$$\sum_{n=i}^{m} n = \frac{(i+m)(m-i+1)}{2}, \tag{2}$$

which is the famous solution invented by Gauss.

2.1 How to use

In the app, you're supposed to give the starting integer and stopping integer. Put these in the fields that say "start (integer)" and "end (integer)" and then press the "Sum" button.

3 Factorial

Factorial of an integer is defined as

$$n! = n(n-1)(n-2)(n-3)\dots 3 \cdot 2 \cdot 1, \tag{3}$$

where $n \in \mathbb{Z}^+$.

3.1 How to use

The computer evaluates this just as a loop and will only take integers as an input. The Gamma function is not taken in consideration here. Enter the value of n into the field that says "factorial (integer)" and press the "Factorial" button.

4 Geometric series

A geometric series is defined by the infinite sum

$$\sum_{k=0}^{\infty} r^k, \tag{4}$$

where $r \in (-1, 1)$. A commonly used name for r is the common ratio. If any number outside of the allowed interval is entered as the common ratio then the series will diverge, otherwise it will converge to a finite value. This value can be calculated easily via

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}. \tag{5}$$

Proof:
Let $a \in \mathbb{R}$ and

$$s = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}, \tag{6}$$

which is the geometric series for the first n terms. Now multiply both left and right with the common ratio r. This gives

$$rs = ar + ar^2 + ar^3 + ar^4 + \ldots + ar^n. \tag{7}$$

Subtracting equation 7 from equation 6 gives

$$s - rs = a - ar^n. \tag{8}$$

Rearranging gives

$$s = \frac{a - ar^n}{1-r}. \tag{9}$$

Now taking the limit of $n \to \infty$ in equation 9 gives

$$s = \lim_{n\to\infty} \frac{a - ar^n}{1-r} = \frac{a}{1-r}, \tag{10}$$

if and only if $r \in (-1, 1)$, otherwise the limit will diverge. In the case that $a = 1$, as in our original case, equation 10 turns into

$$s = \frac{1}{1-r}. \tag{11}$$

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4.1 How to use

The backend of the application uses equation 11 to compute the infinite sum in equation 4. In order to use it, fill in your number in the field that says "Value of r" and press the "Geometric series" button. Make sure that the input r satisfies $-1 < r < 1$, otherwise the series will diverge and the app will tell you that it does. Remember to use a decimal point, and not a comma. In other words, 0.321 is a valid input, where 0,321 is not.

5 Definition of e

The number e is defined by the limit

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n . \tag{12}$$

This gives e a numerical value of

$$e = 2.71828182845904\ldots \tag{13}$$

The number e is a famous transcendental number. Transcendental means that it is not the root solution to a non-zero polynomial with rational coefficients. However, this number is mostly known for its exponential function being the derivative of itself. In other words

$$\frac{d}{dx} e^x = e^x . \tag{14}$$

5.1 How to use

The computer solves equation 12, but takes n as an input instead of taking the limit to infinity. Enter the value of n in the field that says "Value of n (integer)" and press the button that says "Get e". This will solve the equation and give an approximation of e, that becomes better as the value of n increases.

6 Derivatives

The derivative of a function measures the rate of change of a function in a given infinitesimal point. The change of a function $y = f(x)$, between points (x_1, y_1) and (x_2, y_2) is given by

$$\Delta f(x) = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} . \tag{15}$$

If the point (x_2, y_2) is redefined as point (x_1, y_1) plus an offset h, then equation 15 becomes

$$\Delta f(x) = \frac{f(x_1 + h) - f(x_1)}{h} . \tag{16}$$

In order to get the difference in the infinitesimal point (x_2, y_2), the limit of $h \to 0$ has to be taken, which gives

$$\frac{df(x_1)}{dx} = \lim_{h\to 0} \frac{f(x_1 + h) - f(x_1)}{h} . \tag{17}$$

Of course, x_2 can be any point where the curve is differentiable, so x_2 can be substituted by x as in

$$\frac{df(x)}{dx} = \lim_{h\to 0} \frac{f(x + h) - f(x)}{h} , \tag{18}$$

which is the definition of the derivative.

6.1 Rules of Derivatives

The definition of the derivative has given rise to a set of rules for taking derivatives.

6.1.1 Power rule

The power rule is defined as

$$\frac{d}{dx}x^n = nx^{n-1}, \tag{19}$$

where $n \in \mathbb{R}$. This follows from the definition:

$$\frac{d}{dx}x^n = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h} = \frac{x^n + h^n + c.t. - x^n}{h} = \frac{h^n + c.t.}{h}, \tag{20}$$

where $c.t.$ represent cross terms from the multiplication, defined by

$$c.t. = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}h^k, \tag{21}$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}. \tag{22}$$

Solving each separate term gives

$$\frac{d}{dx}x^n = \lim_{h\to 0}\left\{\frac{h^n}{h} + \frac{1}{h}\sum_{k=0}^{n}\binom{n}{k}x^{n-k}h^k\right\} \tag{23}$$

$$= \lim_{h\to 0}\left\{h^{n-1} + \sum_{k=0}^{n}\binom{n}{k}x^{n-k}h^{k-1}\right\}. \tag{24}$$

Now the limit can be taken safely and only the term with $k = 1$ will survive from the sum, which gives

$$\frac{d}{dx}x^n = \binom{n}{1}x^{n-1} = nx^{n-1} \tag{25}$$

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6.1.2 Sum rule

The sum rule is defined by

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x). \tag{26}$$

This follows from the definition via

$$\begin{aligned}
\frac{d}{dx}(f(x)+g(x)) &= \lim_{h\to 0}\frac{f(x+h)+g(x+h)-f(x)-g(x)}{h} \\
&= \lim_{h\to 0}\frac{f(x+h)-f(x)+g(x+h)-g(x)}{h} \\
&= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\
&= \frac{d}{dx}f(x)+\frac{d}{dx}g(x) \qquad (27)
\end{aligned}$$

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6.1.3 Product rule

The product rule is defined as

$$\frac{d}{dx}\{f(x)g(x)\} = g(x)\frac{d}{dx}f(x)+f(x)\frac{d}{dx}g(x). \qquad (28)$$

This follows from the definition of derivatives

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}, \qquad (29)$$

now add and subtract the term $f(x)g(x+h)$, which gives

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}\frac{f(x+h)g(x+h)+f(x)g(x+h)-f(x)g(x+h)-f(x)g(x)}{h}. \qquad (30)$$

Combining the terms with $g(x+h)$ and $f(x)$ gives,

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}\frac{g(x+h)[f(x+h)-f(x)]+f(x)[g(x+h)-g(x)]}{h}. \qquad (31)$$

This can be rewritten to

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}g(x+h)\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}f(x)\frac{g(x+h)-g(x)}{h}. \qquad (32)$$

Note that, because $g(x)$ is differentiable and therefore continuous, $\lim_{h\to 0} g(x+h) = g(x)$. So equation 32 can be rewritten to

$$\frac{d}{dx}\{f(x)g(x)\} = g(x)\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+f(x)\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}. \qquad (33)$$

Then using the definition of derivatives, this gives

$$\frac{d}{dx}\{f(x)g(x)\} = g(x)\frac{d}{dx}f(x)+f(x)\frac{d}{dx}g(x). \qquad (34)$$

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6.1.4 Chain Rule

The chain rule is defined by

$$\frac{d}{dx}f(g(x)) = \frac{df(g(x))}{d(g(x))}\frac{dg(x)}{dx}. \tag{35}$$

This follows from the definition of derivatives by

$$\begin{aligned}
\frac{d}{dx}f(g(x)) &= \lim_{h\to 0}\frac{f(g(x+h)) - f(g(x))}{h} \\
&= \lim_{h\to 0}\frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)}\frac{g(x+h) - g(x)}{h} \\
&= \lim_{h\to 0}\frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)}\lim_{h\to 0}\frac{g(x+h) - g(x)}{h} \\
&= \frac{df(g(x))}{d(g(x))}\frac{dg(x)}{dx}. \qquad (36)
\end{aligned}$$

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