MathApp Documentation

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November 4, 2020
Version 0.13

Contents

1 Introduction

Welcome to MathApp. This is an app that I made to explore some fun pieces of mathematics. Use one of the functionalities and the answer will be output in the response area on the bottom. Please note that for floating point numbers, the decimal point (.) is used and not a comma. The code used is available at GitHub. This document also contains some mathematical background information on the subjects in MathApp. This was added to make MathApp more accessible to people without a strong background in mathematics. Each section has a "How to use" subsection where usage and syntax are discussed.

1.1 Contents

Currently, the app contains the following:

- Summing all integers in a certain interval [n,m].
- Calculating factorials for very large numbers.
- Solving the geometric series (i.e. $1 + x + x^2 + x^3 + \ldots$ for $x \in (-1, 1)$).
- Calculating the number e, by solving $(1 + 1/n)^n$ for large values of n (i.e. taking $n \to \infty$).
- Calculating simple derivatives from user input.
- Encrypting and decrypting messages to share with your friends via making and solving a pseudo-random system of equations.
- An actual calculator that requires a string as input.
- Factorize numbers into prime factors.
- Calculate binomial coefficients.
- Unit conversions, currently supported: Fahrenheit-Celsius, pounds-kilograms, miles-kilometers and vice versa.
- Currency converter that's automatically updated.
- Randomly generated math problems to work on your ability to solve systems of equations.
- Basic linear algebra.

2 Integer Summation

The integer summation solves the equation

$$\sum_{n=i}^{m} n, \tag{1}$$

where, $m, n, i \in \mathbb{Z}^+$. In other words, summing all integers from i to m. In the backend, this is being done via the equation

$$\sum_{n=i}^{m} n = \frac{(i+m)(m-i+1)}{2}, \tag{2}$$

which is the famous solution invented by Gauss.

2.1 How to use

In the app, you're supposed to give the starting integer and stopping integer. Put these in the fields that say "start (integer)" and "end (integer)" and then press the "Sum" button.

3 Factorial

Factorial of an integer is defined as

$$n! = n(n-1)(n-2)(n-3)\dots 3 \cdot 2 \cdot 1, \tag{3}$$

where $n \in \mathbb{Z}^+$.

3.1 How to use

The computer evaluates this just as a loop and will only take integers as an input. The Gamma function is not taken in consideration here. Enter the value of n into the field that says "factorial (integer)" and press the "Factorial" button.

4 Geometric Series

A geometric series is defined by the infinite sum

$$\sum_{k=0}^{\infty} r^k, \tag{4}$$

where $r \in (-1, 1)$. A commonly used name for r is the common ratio. If any number outside of the allowed interval is entered as the common ratio then the series will diverge, otherwise it will converge to a finite value. This value can be calculated easily via

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}. \tag{5}$$

Proof:
Let $a \in \mathbb{R}$ and

$$s = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}, \tag{6}$$

which is the geometric series for the first n terms. Now multiply both left and right with the common ratio r. This gives

$$rs = ar + ar^2 + ar^3 + ar^4 + \ldots + ar^n. \tag{7}$$

Subtracting equation 7 from equation 6 gives

$$s - rs = a - ar^n. \tag{8}$$

Rearranging gives

$$s = \frac{a - ar^n}{1-r}. \tag{9}$$

Now taking the limit of $n \to \infty$ in equation 9 gives

$$s = \lim_{n\to\infty} \frac{a - ar^n}{1-r} = \frac{a}{1-r}, \tag{10}$$

if and only if $r \in (-1, 1)$, otherwise the limit will diverge. In the case that $a = 1$, as in our original case, equation 10 turns into

$$s = \frac{1}{1-r}. \tag{11}$$

■

4.1 How to use

The backend of the application uses equation 11 to compute the infinite sum in equation 4. In order to use it, fill in your number in the field that says "Value of r" and press the "Geometric series" button. Make sure that the input r satisfies $-1 < r < 1$, otherwise the series will diverge and the app will tell you that it does. Remember to use a decimal point, and not a comma. In other words, 0.321 is a valid input, where 0,321 is not.

5 Definition of e

The number e is defined by the limit

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n. \tag{12}$$

This gives e a numerical value of

$$e = 2.71828182845904\ldots \tag{13}$$

The number e is a famous transcendental number. Transcendental means that it is not the root solution to a non-zero polynomial with rational coefficients. However, this number is mostly known for its exponential function being the derivative of itself. In other words

$$\frac{d}{dx}e^x = e^x. \tag{14}$$

5.1 How to use

The computer solves equation 12, but takes n as an input instead of taking the limit to infinity. Enter the value of n in the field that says "Value of n (integer)" and press the button that says "Get e". This will solve the equation and give an approximation of e, that becomes better as the value of n increases.

6 Derivatives

The derivative of a function measures the rate of change of a function in a given infinitesimal point. The change of a function $y = f(x)$, between points (x_1, y_1) and (x_2, y_2) is given by

$$\Delta f(x) = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}. \tag{15}$$

If the point (x_2, y_2) is redefined as point (x_1, y_1) plus an offset h, then equation 15 becomes

$$\Delta f(x) = \frac{f(x_1 + h) - f(x_1)}{h}. \tag{16}$$

In order to get the difference in the infinitesimal point (x_2, y_2), the limit of $h \to 0$ has to be taken, which gives

$$\frac{df(x_1)}{dx} = \lim_{h\to 0}\frac{f(x_1 + h) - f(x_1)}{h}. \tag{17}$$

Of course, x_2 can be any point where the curve is differentiable, so x_2 can be substituted by x as in

$$\frac{df(x)}{dx} = \lim_{h\to 0}\frac{f(x + h) - f(x)}{h}, \tag{18}$$

which is the definition of the derivative.

6.1 Rules of Derivatives

The definition of the derivative has given rise to a set of rules for taking derivatives.

6.1.1 Power rule

The power rule is defined as

$$\frac{d}{dx}x^n = nx^{n-1}, \tag{19}$$

where $n \in \mathbb{R}$. This follows from the definition:

$$\frac{d}{dx}x^n = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h} = \frac{x^n + h^n + c.t. - x^n}{h} = \frac{h^n + c.t.}{h}, \tag{20}$$

where $c.t.$ represent cross terms from the multiplication, defined by

$$c.t. = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}h^k, \tag{21}$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}. \tag{22}$$

Solving each separate term gives

$$\frac{d}{dx}x^n = \lim_{h\to 0}\left\{\frac{h^n}{h} + \frac{1}{h}\sum_{k=0}^{n}\binom{n}{k}x^{n-k}h^k\right\} \tag{23}$$

$$= \lim_{h\to 0}\left\{h^{n-1} + \sum_{k=0}^{n}\binom{n}{k}x^{n-k}h^{k-1}\right\}. \tag{24}$$

Now the limit can be taken safely and only the term with $k = 1$ will survive from the sum, which gives

$$\frac{d}{dx}x^n = \binom{n}{1}x^{n-1} = nx^{n-1} \tag{25}$$

■

6.1.2 Sum rule

The sum rule is defined by

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x). \tag{26}$$

This follows from the definition via

$$\begin{aligned}
\frac{d}{dx}(f(x)+g(x)) &= \lim_{h\to 0}\frac{f(x+h)+g(x+h)-f(x)-g(x)}{h} \\
&= \lim_{h\to 0}\left\{\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right\} \\
&= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\
&= \frac{d}{dx}f(x)+\frac{d}{dx}g(x) \qquad (27)
\end{aligned}$$

■

6.1.3 Product rule

The product rule is defined as

$$\frac{d}{dx}\{f(x)g(x)\} = g(x)\frac{d}{dx}f(x)+f(x)\frac{d}{dx}g(x). \qquad (28)$$

This follows from the definition of derivatives

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}, \qquad (29)$$

now add and subtract the term $f(x)g(x+h)$, which gives

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}\frac{f(x+h)g(x+h)+f(x)g(x+h)-f(x)g(x+h)-f(x)g(x)}{h}. \qquad (30)$$

Combining the terms with $g(x+h)$ and $f(x)$ gives,

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}\frac{g(x+h)[f(x+h)-f(x)]+f(x)[g(x+h)-g(x)]}{h}. \qquad (31)$$

This can be rewritten to

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0}g(x+h)\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}f(x)\frac{g(x+h)-g(x)}{h}. \qquad (32)$$

Note that, because $g(x)$ is continuous, we can write $\lim_{h\to 0}g(x+h)=g(x)$. So equation 32 can be rewritten to

$$\frac{d}{dx}\{f(x)g(x)\} = g(x)\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+f(x)\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}. \qquad (33)$$

Then using the definition of derivatives, this gives

$$\frac{d}{dx}\{f(x)g(x)\} = g(x)\frac{d}{dx}f(x)+f(x)\frac{d}{dx}g(x). \qquad (34)$$

■

6.1.4 Chain Rule

The chain rule is defined by

$$\frac{d}{dx}f(g(x)) = \frac{df(g(x))}{d(g(x))}\frac{dg(x)}{dx}. \tag{35}$$

This follows from the definition of derivatives by

$$\begin{aligned}
\frac{d}{dx}f(g(x)) &= \lim_{h\to 0}\frac{f(g(x+h)) - f(g(x))}{h} \\
&= \lim_{h\to 0}\frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)}\frac{g(x+h) - g(x)}{h} \\
&= \lim_{h\to 0}\frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)}\lim_{h\to 0}\frac{g(x+h) - g(x)}{h} \\
&= \frac{df(g(x))}{d(g(x))}\frac{dg(x)}{dx}.
\end{aligned} \tag{36}$$

■

6.1.5 Standard Derivatives

A lot of functions have well documented standard derivatives. Table 1 shows a list of some standard derivatives that are used by MathApp.

Function	**Derivative**
e^x	e^x
b^x	$b^x \ln(b)$
$\ln(x)$	$1/x$
$\log_b(x)$	$1/(x\ln(b))$
x^x	$x^x \ln(x+1)$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\sec(x)$	$\tan(x)\sec(x)$
$\csc(x)$	$-\cot(x)\csc(x)$
$\cot(x)$	$-\csc^2(x)$

Table 1: Standard derivatives of some common functions. Note that the letter b is used for any base number other than e and $\ln(x)$ refers to the natural logarithm.

6.2 How to use

In order to use the derivative function in MathApp, the user has to determine which rule of differentiation has to be used. In order to do this, the following rules of syntax have been put into place.

- For using the power rule or exponential functions, use the $\wedge$ (SHIFT+6) operator. Correct syntax would be $x \wedge 2$ or $2 \wedge x$ for power and exponential rules.
- The product rule is called using the $*$ operator. Correct syntax would be $\sin(x) * \cos(x)$. However, never use the product rule for a case like $x(x+1)$. In such a case, always multiply it out yourself first and use $x^2 + x$ or $x \wedge 2 + x$ instead.
- The quotient rule does not exist. It is actually a product rule or a power rule. Therefore, do not use $1/x$, instead use $x \wedge -1$. Also do not use $\sin(x)/x$, but use $\sin(x) * x \wedge -1$. In the case of $1/(x^2 + x)$, use $\{x \wedge 2 + x\} \wedge -1$. This will use the chain rule.
- In order to use the chain rule, surround the function $g(x)$, from $f(g(x))$, with $\{$ and $\}$. In other words, $\sin(\{x \wedge 2\})$ is correct syntax for $\sin(x^2)$.
- In order to get the derivative of $x/(x^2 + x)$, grouping can be used in the following way: $x * (\{x \wedge 2 - x\} \wedge -1)$. Notice the use of the product rule and chain rule instead of the quotient rule. Also notice the use of parenthesis.
- Note that table 1 shows all of the included special functions. They can be called as written, e.g. $\sin(x)$ and $e \wedge x$ are correct syntax.

7 Encryption/Decryption

7.1 Encryption

The encryption algorithm in MathApp is based on a polynomial of degree n (P_n) passing exactly though $n+1$ points. This can be used to share a secret word or sentence with a group of $n+1$ friends and insure that the string can only be decrypted when all friends are together.
Let's say we have a random string we want to encrypt:

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We can convert this into numbers according to the ASCII table and to make sure we always have 3 digits for each character, we add 101 to the value of each character and an additional value every time the position of a character in the string increases, in this case 1 is picked. However, we may also pick larger numbers to add. Next we concatenate all the numbers we get in order.

This will give us the following number:

$$a_0 = 16919920021413718921721921021721222. \tag{37}$$

Now consider the following first degree polynomial,

$$y = a_0 + a_1 x \tag{38}$$

and note that the value of a_0 is the same in eq. 38 as in eq. 37. Next, pick a random value for a_1 and find two points at e.g. $x = 1$ and $x = 2$ that lie on this line. These will be your encrypted numbers. Only somebody who knows both numbers can figure out what the original number a_0 was that you started with. This can of course be extended to higher order equations, e.g.

$$y = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n. \tag{39}$$

Note that for a polynomial of degree n, we need (at least) $n + 1$ points to describe it. In all cases, a_0 needs to be the original number (eq. 37) and all the others can be picked at random. As was done with the linear equation, pick random x values for every point and solve equation 39 for each x value and save the output. Now you can store every number on a separate location and only when someone has all the numbers, will they be able to solve the original message. So, if you and your friend have a secret, you can do this and give one number to your friend. Then keep one number yourself. Now neither of you can decrypt the message on your own, but you can solve it when you are together.

7.2 Decryption

I will describe the specific case first and then a general case.

7.2.1 Specific case

In order to decrypt a certain message, remember that we have two points. Let's say the points P_1 and P_2 are defined by the following coordinates:

$$\begin{aligned} P_1 &= (-98, -5515727087651319714542069912182090674), \\ P_2 &= (44, 2721615084561893350088736775376082910). \end{aligned} \tag{40}$$

Since there are only two points, our best guess is that the original was formed using a first degree polynomial (eq. 38). So in order to get the value for a_0, we need to derive the value of a_1 from the given points. This is defined by:

$$a_1 = \frac{\Delta y}{\Delta x} = \frac{P_2^{(y)} - P_1^{(y)}}{P_2^{(x)} - P_1^{(x)}}, \tag{41}$$

where $P_n^{(y,x)}$ refers to the y and x coordinate of the n'th point, respectively. Then filling in all the values gives

$$a_1 = 58009451916994458201625399208156152. \tag{42}$$

Then a_0 will be

$$a_0 = P_1^{(y)} - a_1 = 169199200214137189217219210217212222. \tag{43}$$

Note that each character will be represented by three digits and that the ASCII value can be generated by subtracting 101, and also another 1 for each character, starting at 0. Doing this will get our original string back, which was

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```

7.2.2 General case

Let's say we are given any number of points, defined by

$$\{P_1 = (x_1, y_1), P_2 = (x_2, y_2), ..., P_n = (x_n, y_n)\}. \tag{44}$$

These points define the n'th degree polynomial,

$$y(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n. \tag{45}$$

In order to decrypt the message, a_0 must be found. Combining eqs. 44 and 45 gives a system of equations with n unknowns (coefficients a_i) and n equations defined by

$$\begin{aligned} y_1 &= a_0 + a_1 x_1 + a_2 x_1^2 + ... + a_n x_1^n \\ y_2 &= a_0 + a_1 x_2 + a_2 x_2^2 + ... + a_n x_2^n \\ &\vdots \\ y_n &= a_0 + a_1 x_n + a_2 x_n^2 + ... + a_n x_n^n. \end{aligned} \tag{46}$$

Note that all the values for x_n and y_n are known from the points defined in eq. 44. Now solve the system of equations shown in equation 46. This should give a value for a_0 that gives the numerical representation of the original string. Once again, this number should be split into three digits for each character and can then be converted by subtracting $\{101 + (\text{characterPosition})\}$ and using the ASCII table. Solving of a set of n equations with n unknowns can be done by MathApp itself (see: section 14) or with a Python script. It can even be done by hand and MathApp has a feature where the user can practice this ability (section 13).

Many other additions can still be made to the encryption and decryption algorithms, and might be done in the future.

7.3 How to use

In order to use the encryption algorithm provided by MathApp, first click the "Encryption" button to go to the encryption screen.

7.3.1 Encryption

Fill in the string to encrypt in the field that says “Secret message” and fill in the degree of the encryption in the field that says “Degree (integer)”. The degree of the encryption should be one less than the number of points that are required. For example, entering the value 1 will give 2 points, entering the value 2 will give 3 points, and so on. Finally, click the button that says “Get encryption”. The output will give the numbers to share and means to test it. In order to use the auto generated Python script, the Sympy[1] package is required. Sympy can be installed using

```
pip install sympy
```

7.3.2 Decryption

In order to get the decrypted number from MathApp itself, enter the points in the field that says “point”. The input must follow the following syntax:

```
1: (xvalue1,yvalue1);
2: (xvalue2,yvalue2);
3: (xvalue3,yvalue3);
...
```

There are no exceptions to this rule. Always make sure that each line starts with a number, followed by a colon (:) and ends with a semicolon (;). To get the decrypted number, click the “Decrypt” button. To translate the number into the original message, enter the number that was obtained, from either Python or MathApp itself, in the field that says “Number to decrypt” and click the “Get message” button.

8 Calculator

The calculator is a feature in MathApp that interprets an input string as a sum and gives the answer as output. All the regular rules of arithmetic apply here. It comes with the following set of syntax rules:

- Input is always read from left to right.
- Symbols used:
  - () for parentheses.
  - $\wedge$ for power.
  - * for multiplication.
  - / for division.
  - + for addition.

[1] Sympy can be found at `https://www.sympy.org/en/index.html`

  - $-$ for subtraction.
- Special numbers (case insensitive):
  - pi for number π.
  - e for number e.
  - phi for golden ratio $\phi = \frac{1+\sqrt{5}}{2}$.
- Special functions have input arguments between { and }.
- Special functions are
  - sin, cos, tan, cot, sec, csc
  - sinh, cosh, tanh
  - asin, acos, atan
  - sqrt, cbrt
  - exp
  - ! (factorial, usage: $n!$ where $n \in \mathbb{Z}^+$. Note: does not use { and }).
- Contains radio buttons labeled "DEG" and "RAD" for using degrees and radians respectively.
- Using sin{pi/2} is incorrect syntax. Use sin{(pi/2)} instead. This will make the computer evaluate pi/2 first before taking the sine of it. Otherwise this will give the error: "Unknown input! For input string: "p""

Enter the sum in the field that says "Calculator". To evaluate the sum, press the "Calculate" button. The calculator also has an option to copy the answer to the clipboard. To do this simply click the "Copy answer" button that shows up after the answer is calculated. This button will be in the response area after a calculation is done.

9 Prime Number Factorization

Every positive integer is either a prime or can be written as a unique product of primes. A prime number is a number that can only divide itself and 1. The first 25 prime numbers are: {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}. All these numbers have no other divisors, but themselves and 1. All other positive integers are called composite numbers. An example of a composite number is 12, because $12 = 4 \times 3 = 2^2 \times 3$. This means that 2 and 3 are the only prime numbers that can divide 12 and 2 can divide it twice. This property give prime numbers high importance in mathematics, because the primes act like "building blocks" for composite numbers.

When it comes to checking for prime numbers, it is only necessary to check odd numbers, because all primes are odd, with the exception of 2. Furthermore, for a number n it is only necessary to check up to $\sqrt{n}$ for any prime factors. It is possible to get a prime factor of n that is larger than $\sqrt{n}$, but these can easily be found using the smaller prime factors. An example of this is 15. 15 has the prime factors 3 and 5, but $\sqrt{15} \approx 3.873$. Once 15 is divided by 3, only a factor of 5 will be left.

9.1 How to use

First click on the "Factorizations" button in order to go to the correct screen as highlighted in figure 1. Next, the screen shown in figure 2 will come up.

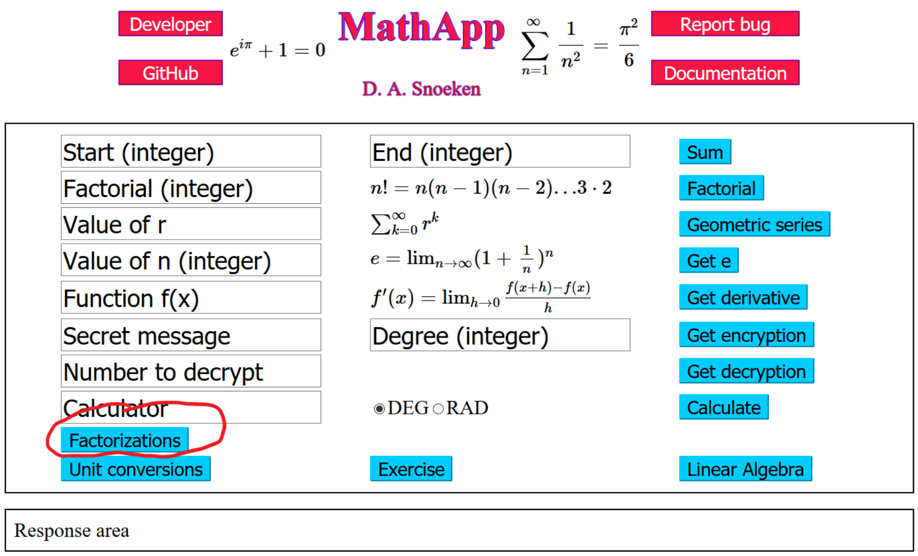

Figure 1: Click this button to go to the factorization screen.

Then enter the number to be factorized in the field that is highlighted in red and press the "Factorize" button. Then the response area will show what the prime factorization is of that number, or, if the number is prime, that it is a prime number. The code in the backend can handle arbitrarily large numbers. However, it will take longer to check extremely large numbers, keep that in mind. The complexity of the algorithm for integer n is roughly $\mathcal{O}(\sqrt{n})$.

MathApp also has a functionality where a number is checked to see if it is prime. This does not factor the number, it just checks all possible factors up to $\sqrt{n}$ and breaks as soon as it finds one. This will not return as much information, but is a lot faster than the actual factorization algorithm. Using this function is recommended over the complete factorization. This function

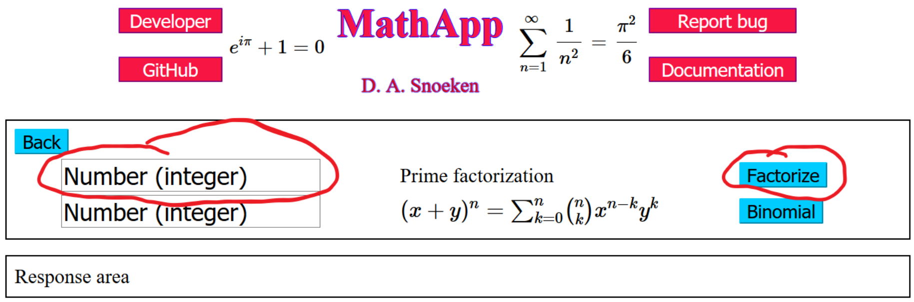

Figure 2: Factorization screen, enter the number in the highlighted field and press the "Factorize" button.

can be used via the "Factorizations" screen by clicking the "Is it prime?" button and entering a number in the field on the same line.

10 Binomial coefficients

Binomial coefficients are the coefficients that arise when expanding the power of a sum of two numbers or variables. For example, in

$$(x+y)^2 = x^2 + 2xy + y^2 \tag{47}$$

the binomial coefficients are 1 (for x^2), 2 (for xy) and 1 (for y^2). In general, for any power $n \in \mathbb{Z}^+$, these equations satisfy

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}. \tag{48}$$

Here, $k \in \mathbb{Z}^+$ and $\binom{n}{k}$ is the binomial coefficient and defined by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \tag{49}$$

where the ! mark symbolizes the factorial function, defined by

$$n! = n(n-1)(n-2)\dots 3 \cdot 2 \cdot 1. \tag{50}$$

The symbol $\binom{n}{k}$ symbolizes that there are $\binom{n}{k}$ ways to pick a set of k elements from a set of n elements. This is also why $\binom{n}{k}$ is usually pronounced as "n *choose* k".

10.1 How to use

MathApp allows the user to use binomial coefficients to expand $(x+y)^n$ for any power $n \in \mathbb{Z}^+$. Note that for negative powers:

$$(x+y)^{-n} = \frac{1}{(x+y)^n} = \left\{ \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \right\}^{-1}. \tag{51}$$

So MathApp does not take any negative numbers for n, because conversion from positive powers to negative powers is trivial.

In order to navigate to the factorization screen, click the "Factorizations" button as shown in figure 1. Then enter the value for n in the highlighted field in figure 3 and press the "binomial" button.

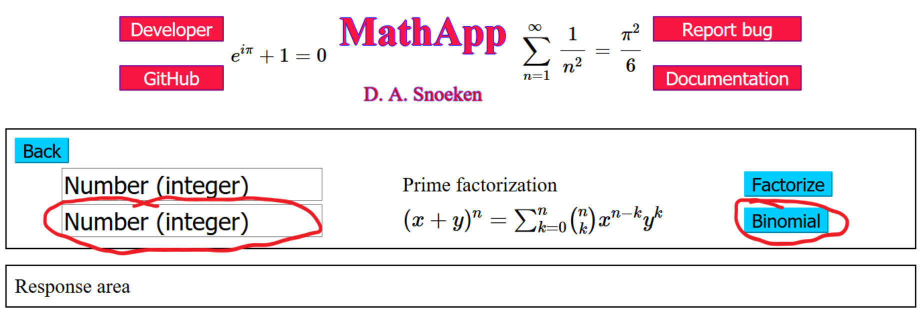

Figure 3: Factorization screen, enter the number in the highlighted field and press the "Binomial" button.

11 Unit Conversions

MathApp allows the user to quickly convert between some common United States customary units and the metric system. Currently it is able to convert between units in table 2.

Quantity	Metric	US system
Temperature	^oC	^oF
Mass	kg	lbs
Distance	km	miles
Distance	cm	feet + inch

Table 2: Unit conversions currently supported by MathApp.

11.1 How to use

In order to use the unit conversion, first go to the conversion screen by clicking the "Unit conversion" button. Then just click one of the buttons that show up to get the desired conversion.

12 Currency converter

MathApp allows for the conversion between many different currencies. This feature works via a Jsoup webcrawler and gets conversion data from The Money Converter. Currently MathApp can convert the following currencies: EUR, USD, AUD, CAD, CHF, CNY, DKK, GBP, HKD, ILS, INR, JPY, KRW, NZD, PLN, RUB, SEK.

12.1 How to use

In order to use the currency converter, first go to the unit conversion screen by clicking the "Unit conversion" button. Then click the "Currency converter" button. Use the field to give a certain amount of a currency to convert and select the "from" and "to" currency for the conversion. The "switch" button can be used to flip them around. Then press the "Convert" button. The webcrawler will give the amount of money back and a timestamp for when The Money Converter was last updated. Note that, in order to prevent spamming The Money Converter with bots and potentially getting the IP blocked, it is only possible to click the "Convert" button once. After clicking it once, it will be deactivated, and the "Re-enable button" button has to be clicked. It will take about a second for it to be reactivated. This is done on purpose to prevent spamming.

13 Random problems

MathApp has a feature that allows the user to randomly generate a problem, where the user has to find a polynomial that exactly fits a certain number of points. This essentially comes down to solving a system of n equations with n unknowns. A strategy of solving such a problem can be as follows:

Consider the system of equations:

$$A + B = 7, \tag{52}$$

$$A - 2B = 1. \tag{53}$$

Here, A can be expressed in terms of B, by rewriting the system to

$$A + B = 7, \tag{54}$$

$$A = 1 + 2B. \tag{55}$$

Then use the second equation to rewrite the first to

$$1 + 2B + B = 7. \tag{56}$$

Rewrite again to

$$1 + 3B = 7. \tag{57}$$

This gives

$$3B = 6. \tag{58}$$

Meaning that $B = 2$. Now using the value of B, it is possible to get A from either the first or second equation. Lets use the first here, so

$$A + 2 = 7 \tag{59}$$

and therefore $A = 5$. So the solution is $A = 5$ and $B = 2$, which solves both equations:

$$5 + 2 = 7, \tag{60}$$

$$5 - 2 \times 2 = 1. \tag{61}$$

13.1 How to use

In order to use the random problem feature, first click the "Exercise" button. Then a window will show up where the user can enter the difficulty of the problem. This field requires a positive integer as input and sets the degree of the polynomial the user is supposed to find. Referring to the problem presented in section 13, this number has to be 1 less than the number of unknowns and equations the user wishes to solve. Once a number has been entered in this field, click the "Set Degree" button. Next the program will ask the user to enter the polynomial coefficients. The polynomial coefficients always have to be entered in the order of increasing powers of x. In other words, the order should be

$$a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n = y, \tag{62}$$

where a_i represent the coefficients entered by the user. Once the answer has been entered, click the "Submit" button and MathApp will tell the user if the answer was correct or not. MathApp will only tell the user if the answer was correct or not, and will not say where any mistakes happened.

14 Linear Algebra

Linear algebra is one of the most important parts of mathematics. Providing not just mathematicians, but also scientists and engineers with extremely powerful tools for solving linear equations and study their transformation properties.[2] These transformations include, for example, translations and rotations in spaces of potentially infinitely many dimensions.

This section will first contain some theoretical background on linear algebra, section 14.5 contains an explanation on how to use linear algebra on MathApp.

14.1 Vectors and Matrices

Vectors and matrices are central to linear algebra. Vectors are essentially arrays of numbers, whereas matrices are arrays of vectors. A general example of a column vector $\mathbf{v}$ is

$$\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} . \tag{63}$$

Here, vector $\mathbf{v}$ has n elements labeled v_i, for the i'th element. Of course, row vectors are also allowed and an example of a row vector $\mathbf{r}$ is

$$\mathbf{r} = \begin{pmatrix} r_1, r_2, \ldots, r_n \end{pmatrix} . \tag{64}$$

Where $\mathbf{r}$ has n elements, labeled r_i for the i'th element. An example of a matrix $\mathbf{A}$ is

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1m} \\ a_{21} & a_{22} & \ldots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nm} \end{pmatrix} . \tag{65}$$

Matrix $\mathbf{A}$ has a total of $n \times m$ elements (labeled a_{ij}) and is also referred to as an "$n \times m$ matrix". Also note that the notation for vectors is a bold face lower case letter, for matrices is a bold face capital letter and for elements is a lower case letter with indices.

14.2 Linear Transformations

Under construction...

14.3 Operations

Some of the basic operations associated with vectors and matrices are, for example, addition and subtraction.[3]

[2] Weisstein, Eric W. "Linear Algebra." From MathWorld–A Wolfram Web Resource. `https://mathworld.wolfram.com/LinearAlgebra.html`

[3] Brown, William C. (1991), Matrices and vector spaces, New York, NY: Marcel Dekker, ISBN 978-0-8247-8419-5

14.3.1 Addition and Subtraction

In order to add or subtract a matrix or vector, both matrices or vectors must have the same dimensions. If that is the case, the addition and subtraction will be done element-wise. In other words,

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}. \tag{66}$$

Exactly the same principle holds for subtraction. Sometimes the notation

$$\mathbf{A}_{ij} + \mathbf{B}_{ij} = (\mathbf{A}+\mathbf{B})_{ij} \tag{67}$$

is used to refer to the addition of matrices. The subscript i and j refer to the addition of matrices being done element-wise.

14.3.2 Multiplication

For multiplication of matrices, the inner dimensions of the matrices have to match. What this means is, that a $n \times m$ matrix can only be multiplied with a $p \times q$ matrix if and only if $m = p$. This also has to be in the correct order: $(n \times m) \cdot (p \times q)$. Where $(p \times q) \cdot (n \times m)$ will only work if and only if $q = n$. This is also a hint that, for matrices, multiplication is generally not commutative. However, there are exceptions to this rule. This means that in most cases

$$\mathbf{AB} \neq \mathbf{BA}. \tag{68}$$

Square matrices of dimensions $n \times n$ can always be multiplied together, irregardless of the order (as long as they are both the same size). However the resulting answers can be different, depending on that order.

The same multiplication rules apply to vectors. Vectors can be multiplied as

$$\begin{pmatrix} v_1, v_2, v_3 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = v_1 w_1 + v_2 w_2 + v_3 w_3. \tag{69}$$

This is commonly referred to as the dot product of two vectors and for two vectors $\mathbf{v}$ and $\mathbf{w}$ is commonly written as

$$\mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^{n} v_i w_i = v_i w^i, \tag{70}$$

where both $\mathbf{v}$ and $\mathbf{w}$ are of length n. The notation $v_i w^i$ is called the Einstein summation convention, where a lower index and an upper index, of the same index, indicate summation over that index. This is sometimes used to prevent having to write a lot of summation symbols, and the upper index is not to be confused with raising a power. This really is nothing more than a notational convention.[4]

The multiplication of matrices is essentially a lot of these dot products, where the rows of the first matrix are multiplied with the columns of the second matrix. The result is another matrix. If a matrix of dimensions $n \times m$ and $m \times q$ are multiplied together, then the dimensions of the resulting matrix will be $n \times q$. For example:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{pmatrix} \tag{71}$$

shows that the multiplication of a 3×2 matrix and a 2×3 matrix results in a 3×3 matrix. Equation 71 also shows how matrix multiplication works. Another notation for matrix multiplication is

$$(\mathbf{AB})_{jk} = \sum_{i=1}^{m} a_{ji} b_{ik} = a_i^j b_k^i, \tag{72}$$

where $(\mathbf{AB})_{jk}$ refers to element j, k from the product between matrices $\mathbf{A}$ and $\mathbf{B}$, which have elements a_{ji} and b_{ik} respectively. Matrices can also be raised to a power, this is equivalent to multiplying with itself.

Matrices can also be multiplied with numbers, these are referred to as "scalars". These multiplications happen element-wise:

$$c(\mathbf{A})_{ij} = (c\mathbf{A})_{ij}, \tag{73}$$

where $c \in \mathbb{R}$.

14.3.3 Transposition

Transposition of a matrix is flipping the matrix over the diagonal. During transposition, the rows of a matrix will turn into the columns of a matrix and vice versa. In practice, transposition looks like

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{\mathrm{T}} = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix}. \tag{74}$$

Note that the diagonal elements (a_{11} and a_{22}) do not change position, but the off-diagonal elements (a_{12} and a_{21}) are switched after transposition. Using the more general, element-wise notation, transposition can be written as

$$(\mathbf{A})_{ij}^{\mathrm{T}} = (\mathbf{A})_{ji}. \tag{75}$$

[4] Further reading: `https://en.wikipedia.org/wiki/Einstein_notation`

14.3.4 Row echelon forms

The row echelon form of a matrix is a specific shape of a matrix, which can be achieved via Gaussian elimination. There are two different types of row echelon forms, referred to by "Row Echelon Form" (REF) and "Reduced Row Echelon Form" (RREF). A matrix is in REF if all rows that only contain 0 are at the bottom and if the pivot of each row is to the right of the row above it. The pivot of a row is the first number in each row that is not equal to 0. Sometimes, people like to say that the pivot should always be equal to 1, but this is not true. For example, the matrix

$$\mathbf{R} = \begin{pmatrix} 3 & 2 & 0 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{76}$$

is in REF. Note that, even though $\mathbf{R}$ was a square matrix, row reduction is allowed for non-square matrices as well. The RREF form of matrix $\mathbf{R}$ can be generated by making sure that each row starts with a 1 (referred to as a leading 1) and that every *column* that contains a leading 1 has a 0 everywhere else. This means that the RREF of matrix $\mathbf{R}$ is

$$\text{RREF}(\mathbf{R}) = \begin{pmatrix} 1 & 0 & -\frac{4}{3} & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{77}$$

Note that the 3^{rd} column in equation 77 still has a 2 in the 2^{nd} row and now has $-\frac{4}{3}$ in the 1^{st} row. This was obtained via Gaussian elimination of the rows.

Gaussian elimination is a method that is generally used for solving a system of equations. It allows for 3 different kind of actions to be done to the rows of a matrix. These are (along with their symbols):

- Adding one row to another row. ($R_n \to R_n + cR_m$, where $c \in \mathbb{R}$)
- Multiplying a row with a non-zero scalar. ($R_n \to cR_n$, where $c \in \mathbb{R}$)
- Switch the position of rows. ($R_n \leftrightarrow R_m$)

For example, in order to bring the matrix

$$\begin{pmatrix} 3 & 7 & 1 \\ 2 & 1 & 1 \end{pmatrix} \tag{78}$$

into RREF, first multiply row 1 with $\frac{1}{3}$ ($R_1 \to \frac{1}{3}R_1$). This gives

$$\begin{pmatrix} 1 & \frac{7}{3} & \frac{1}{3} \\ 2 & 1 & 1 \end{pmatrix}. \tag{79}$$

Now subtract row 1 twice from row 2 and set this to be the new row 2 ($R_2 \to R_2 - 2R_1$). This gives

$$\begin{pmatrix} 1 & \frac{7}{3} & \frac{1}{3} \\ 0 & -\frac{11}{3} & \frac{1}{3} \end{pmatrix}. \tag{80}$$

Note that it is in REF now. Now multiply row 2 with $-\frac{3}{11}$ ($R_2 \to -\frac{3}{11}R_2$). This gives

$$\begin{pmatrix} 1 & \frac{7}{3} & \frac{1}{3} \\ 0 & 1 & -\frac{1}{11} \end{pmatrix}. \tag{81}$$

Now add row 2 $-\frac{7}{3}$ times to row 1 ($R_1 \to R_1 - \frac{7}{3}R_2$). This gives

$$\begin{pmatrix} 1 & 0 & \frac{6}{11} \\ 0 & 1 & -\frac{1}{11} \end{pmatrix}. \tag{82}$$

Now it is in RREF. This also means that the system of equations:

$$\begin{aligned} 3x + 7y &= 1 \\ 2x + y &= 1 \end{aligned} \tag{83}$$

has the solution $x = \frac{6}{11}$ and $y = -\frac{1}{11}$.

14.4 Square matrices

Square matrices have been introduced before. However, since there are several operations that only apply to square matrices, this type of matrix deserves its own section.

A matrix is considered square when it has the same number of rows as columns and a matrix with dimensions $(n \times n)$ is called a square matrix of order n. Square matrices play an important role in linear transformations and have certain properties that only apply to square matrices, such as the trace (section 14.4.2) and the determinant(section 14.4.3). [5,6,7]

14.4.1 Special square matrices

There exist three special kinds of square matrices. These are the diagonal, upper triangular and lower triangular matrices. A diagonal matrix is a matrix

[5] Brown, William C. (1991), Matrices and vector spaces, New York, NY: Marcel Dekker, ISBN 978-0-8247-8419-5

[6] Horn, Roger A.; Johnson, Charles R. (1985), Matrix Analysis, Cambridge University Press, ISBN 978-0-521-38632-6

[7] Mirsky, Leonid (1990), An Introduction to Linear Algebra, Courier Dover Publications, ISBN 978-0-486-66434-7

which only has elements that are non-zero on the diagonal. For example, the order n square matrix $\mathbf{D}$ is diagonal,

$$\mathbf{D} = \begin{pmatrix} d_{11} & 0 & \dots & 0 \\ 0 & d_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & d_{nn} \end{pmatrix}, \tag{84}$$

because only the elements that satisfy d_{ii} are non-zero.

In the case of a triangular matrix, also the elements either above (upper triangular) or below (lower triangular) are non-zero. An example of an order n upper triangular matrix $\mathbf{U}$ is

$$\mathbf{U} = \begin{pmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ 0 & u_{22} & \dots & u_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & u_{nn} \end{pmatrix}, \tag{85}$$

where elements $u_{ij} \neq 0$. An example of an order n lower triangular matrix $\mathbf{L}$ is

$$\mathbf{L} = \begin{pmatrix} l_{11} & 0 & \dots & 0 \\ l_{21} & l_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \dots & l_{nn} \end{pmatrix}, \tag{86}$$

where elements $l_{ij} \neq 0$. Note that the transpose (section 14.3.3) of an upper triangular matrix is a lower angular matrix, and vice versa. The diagonal matrix remains the same under transposition.

14.4.2 Trace

The trace of a matrix is defined as the sum of the elements on the main diagonal. Given a square matrix $\mathbf{A}$, the trace of $\mathbf{A}$ is defined as

$$\operatorname{tr}(\mathbf{A}) = \sum_{i=0}^{n} a_{ii}. \tag{87}$$

14.4.3 Determinant

The determinant is an extremely important property of square matrices. Given a matrix

$$\mathbf{A} = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}, \tag{88}$$

corresponding to the vectors

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{89}$$

and

$$\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \tag{90}$$

that define a linear transformation (section 14.2), then the determinant of $\mathbf{A}$ (equation 88) is the area of the parallelogram that has vectors $\mathbf{a}$ and $\mathbf{b}$ as sides. This is shown in figure 4. The idea of the determinant being the area of

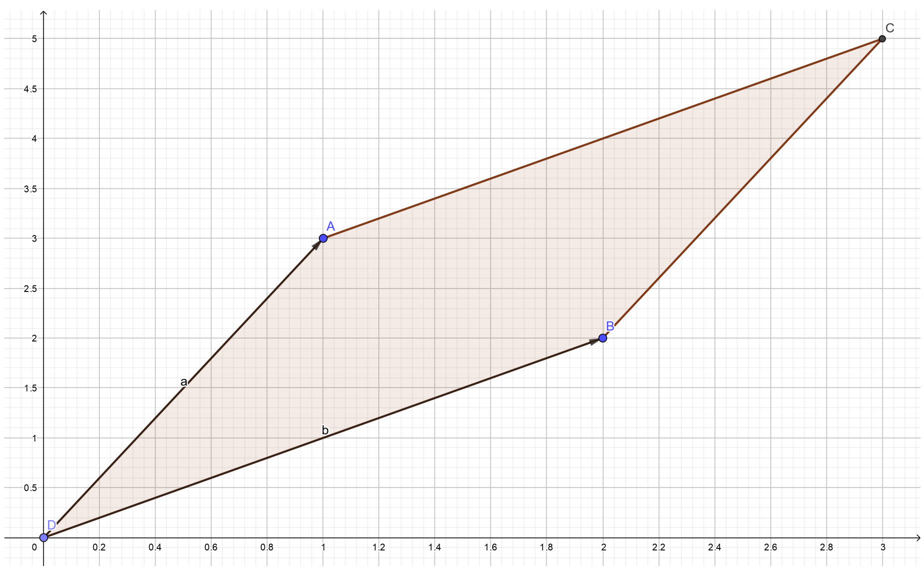

Figure 4: The determinant of a (2×2) matrix is defined by the area of the parallelogram defined by vectors $\mathbf{a}$ and $\mathbf{b}$. The parallelogram is the colored area made by points A, B, C (C = A + B) and D (origin). This image was made using Geogebra.

the parallelogram shown in figure 4 generalizes to other dimensions as well. Of course in 3 dimensions, it no longer describes an area, but rather a volume.

The determinant of a 2×2 matrix is computed via

$$\det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}, \tag{91}$$

where the matrix with straight vertical lines instead of parentheses is another notation for a determinant. This method does not generalize to higher dimensions, however the method used for (3×3) matrices does generalize to higher dimensions. A (3×3) determinant is calculated using

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = b_{11} \begin{vmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{vmatrix} - b_{12} \begin{vmatrix} b_{21} & b_{23} \\ b_{31} & b_{33} \end{vmatrix} + b_{13} \begin{vmatrix} b_{21} & b_{22} \\ b_{31} & b_{32} \end{vmatrix}. \tag{92}$$

This shows that a single (3×3) determinant is a sum of 3 (2×2) determinants. Naturally a single (4×4) determinant is the sum of 4 (3×3) determinants, again each of the (3×3) determinants are the sum of 3 (2×2) determinants. It is obvious that the algorithm becomes very complicated as the dimensions of the matrix increase.

MathApp avoids this method of computation and instead uses its own row echelon form (REF) algorithm to first bring the matrix into REF (section 14.3.4). This is useful, because the determinant does not change when rows are added to each other. However, it does change when a row is multiplied by a constant (by the constant factor) or if two rows are switched (change of sign, i.e. multiplied by -1). However, it is not strictly necessary to use the constant multiplication or row switching for the purpose of calculating a determinant. Only using the operation of adding rows together is enough. This fact becomes extremely useful for calculating higher dimensional determinants. Changing the matrix in equation 92 to upper triangular (sec 14.4.1), as would be achieved by REF, gives

$$\begin{aligned} \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{vmatrix} &= c_{11} \begin{vmatrix} c_{22} & c_{23} \\ 0 & c_{33} \end{vmatrix} - c_{12} \begin{vmatrix} 0 & c_{23} \\ 0 & c_{33} \end{vmatrix} + c_{13} \begin{vmatrix} 0 & c_{22} \\ 0 & 0 \end{vmatrix} \\ &= c_{11}(c_{22}c_{33} - 0) - c_{12}(0 - 0) + c_{13}(0 - 0) \\ &= c_{11}c_{22}c_{33}, \end{aligned} \tag{93}$$

which shows that the determinant becomes the product of the diagonal elements. This method also generalizes to all other dimensions. Any triangular or diagonal matrix (sec 14.4.1) will give its determinant when the diagonal elements are multiplied together.

The determinant has the following properties for a random square matrix of size n $\mathbf{A}$:

- $\det(\mathbf{A}) = \det(\mathbf{A}^{\mathrm{T}})$ (transposition is described in section 14.3.3).
- $\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B}) = \det(\mathbf{B})\det(\mathbf{A}) = \det(\mathbf{BA})$, where $\mathbf{B}$ is a square matrix of size n.
- If $\mathbf{A}$ is invertible then $\det(\mathbf{A}^{-1}) = \det(\mathbf{A})^{-1}$.
- $\det(\mathbf{A}^k) = \det(\mathbf{A})^k$, for $k \in \mathbb{Z}^+$ (or $k \in \mathbb{R}$ if $\mathbf{A}$ is diagonalizable).
- If one or more of the rows or columns of $\mathbf{A}$ is entirely made up out of zeros, then the determinant of $\mathbf{A}$ is 0.

- If two or more rows or columns of $\mathbf{A}$ are the same, or are different by a constant multiple, then the determinant of $\mathbf{A}$ is 0.

14.4.4 Inverse

Under construction...

14.5 How to use

In order to use linear algebra in MathApp, first click the "Linear Algebra" button. To create a matrix, use the "Matrix" field to define the matrix and click the "Set Matrix" button. The correct syntax for matrices is that all columns are separated by comma's and rows are separated by semicolons. It is important that the matrix is written down row by row. For example, in order to create the matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \tag{94}$$

the correct syntax would be

```
1,2;3,4
```

where it should be noted that there is no semicolon at the end, because there is not a new row after that point. Once a matrix has been set, the new matrix will be printed along all the other matrices that are in memory, if there are any. Each matrix is given an ID, which is also printed. Matrices can be removed or printed by ID, if the appropriate buttons are used.

In order to use the matrix for certain operations, click the "To Operations" button. The operations screen is also using the ID's given to the matrices when they were created. The "Print all matrices" button will print all matrices along with their ID's on screen. A function can be called by filling in the ID in the appropriate field and clicking the button on the same line. For a description on what each button actually does behind the scenes, refer to sections 14.3 and 14.4.