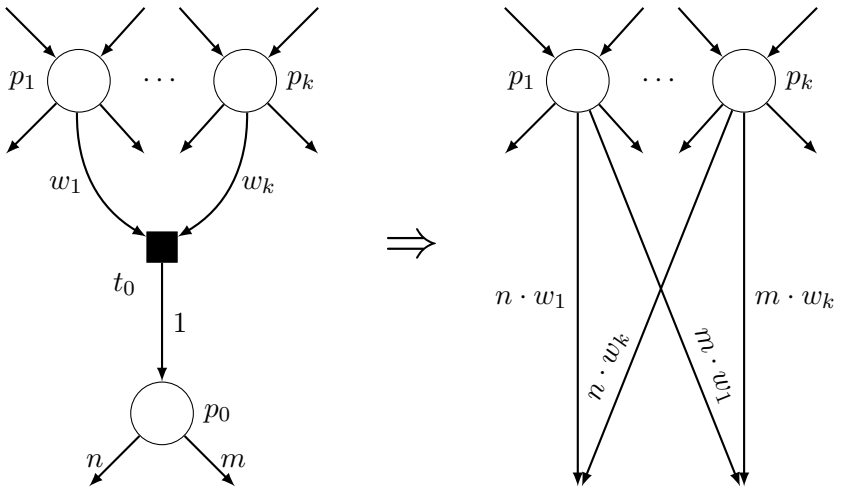
## Rule A: Sequential transition removal

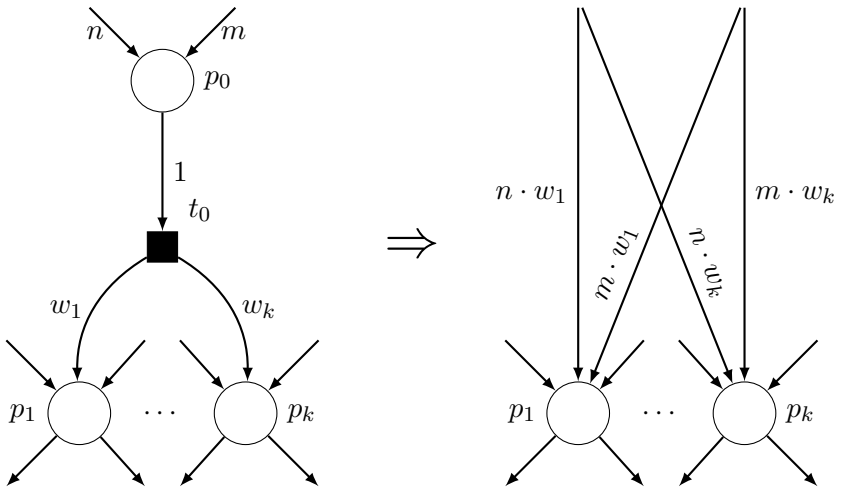
Rule A merges sequential transitions, i.e. a transition and another transition that must precede or follow it. Rule A is equivalent to a pre (or post) agglomeration with exactly one producer (or consumer) with a weight of 1. The two variants of Rule A can be seen in Figure 1 and Figure 2.

**Theorem 1** *The two variants of Rule A in Figure 1 and Figure 2 are both correct for LTL\X.*




| Precondition | Update |
|---|---|
| Fix $p_0$ and $t_0$ where ${}^\bullet t_0 = \{p_1, \ldots, p_k\}$ s.t.: | |
| A1) $t_0^\bullet = \{p_0\}$ and $\boxplus(t_0, p_0) = 1$ | UA1) For all $t \in p_0^\bullet$ and all $p \in \{p_1, \ldots, p_k\}$ set $\boxminus'(p,t) := \boxminus(p,t) + \boxminus(p_0,t) \cdot \boxminus(p,t_0)$ |
| A2) ${}^\bullet p_0 = \{t_0\}$ and $p_0 \notin \{p_1, \ldots, p_k\}$ | |
| A3) $p_0^\circ = p_1^\circ = \cdots = p_k^\circ = {}^\circ t_0 = \emptyset$ | UA2) Remove $p_0$ and $t_0$ |
| A4) $\{p_0, p_1, \ldots, p_k\} \cap places(\varphi) = \emptyset$ | |
| A5) $M_0(p_0) = 0$ | |

Figure 1: Rule A: Sequential transition removal (pre)



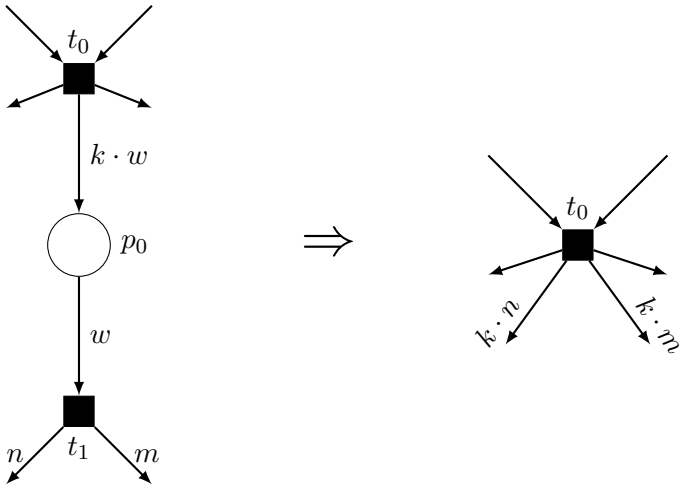

| Precondition | Update |
|---|---|
| Fix $p_0$ and $t_0$ where $t_0^\bullet = \{p_1, \ldots, p_k\}$ s.t.: | |
| A1) ${}^\bullet t_0 = \{p_0\}$ and $\boxminus(p_0, t_0) = 1$ | UA1) For all $p \in \{p_1, \ldots, p_k\}$ change the initial marking s.t. $M_0'(p) := M_0(p) + M_0(p_0) \cdot \boxplus(t_0, p)$ |
| A2) $p_0^\bullet = \{t_0\}$ and $p_0 \notin \{p_1, \ldots, p_k\}$ | |
| A3) $p_0^\circ = p_1^\circ = \cdots = p_k^\circ = {}^\circ t_0 = \emptyset$ | UA2) For all $t \in {}^\bullet p_0$ and all $p \in \{p_1, \ldots, p_k\}$ set $\boxplus'(t, p) := \boxplus(t, p) + \boxplus(t, p_0) \cdot \boxplus(t_0, p)$ |
| A4) $\{p_0, p_1, \ldots, p_k\} \cap places(\varphi) = \emptyset$ | UA3) Remove $p_0$ and $t_0$ |

Figure 2: Rule A: Sequential transition removal (post)

## Rule B: Sequential place removal

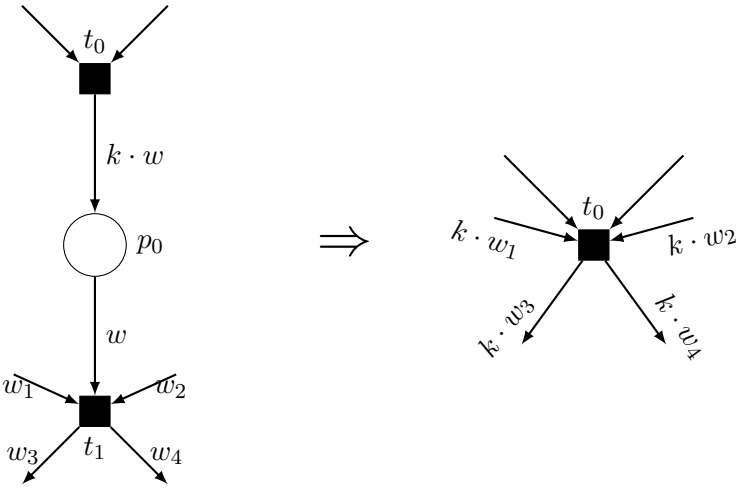
Rule B merges two transitions surrounding a place with no other transitions than the two. Rule B is equivalent to an agglomeration with exactly one producer and one consumer, but allow them to have different weights. Hence, there is a pre- and post-agglomeration variant of Rule B defined in Figure **??** and Figure **??**, respectively.

**Theorem 2** *The two variants of Rule B in Figure* **??** *and Figure* **??** *are both correct for LTL\X.*




| Precondition | Update |
|---|---|
| Fix $p_0$ and $t_0, t_1$ where $t_0 \neq t_1$ s.t.: | |
| B1) ${}^\bullet p_0 = \{t_0\}, p_0^\bullet = \{t_1\}, {}^\bullet t_1 = \{p_0\}$ | UB1) For all $p \in P \setminus \{p_0\}$ set $M_0'(p) := M_0(p) + \lfloor M_0(p_0) / \boxminus (p_0, t_1) \rfloor \cdot \boxplus(t_1, p)$ |
| B2) $\boxplus(t_0, p_0) = k \cdot \boxminus(p_0, t_1)$ for $k \geq 1$ | UB2) For all $p \in P \setminus \{p_0\}$ set $\boxplus'(t_0, p) := \boxplus(t_0, p) + k \cdot \boxplus(t_1, p)$ |
| B3) $p_0^\circ = {}^\circ t_0 = {}^\circ t_1 = \emptyset$ | |
| B4) $p_0 \notin \mathit{places}(\varphi)$ | UB3) Remove $p_0$ and $t_1$ |
| B5) $p^\circ = \emptyset$ and $p \notin \mathit{places}(\varphi)$ for all $p \in t_1^\bullet$ | |

Figure 3: Rule B: Sequential place removal (pre)



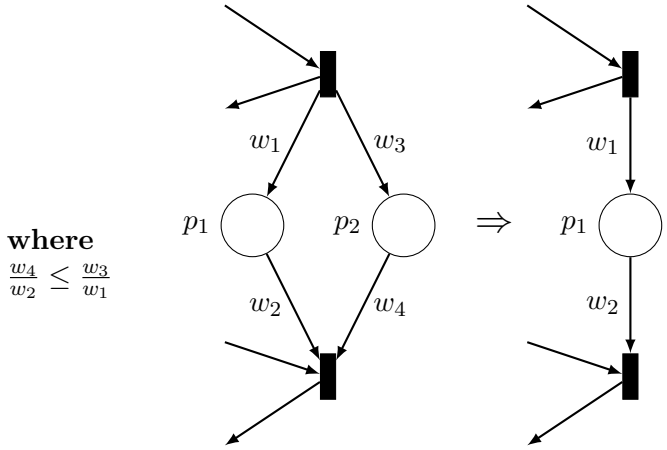

| Precondition | Update |
|---|---|
| Fix $p_0$ and $t_0, t_1$ where $t_0 \neq t_1$ s.t.: | |
| B1) ${}^\bullet p_0 = \{t_0\}, p_0^\bullet = \{t_1\}, t_0^\bullet = \{p_0\}$ | UB1) For all $p \in P \setminus \{p_0\}$ set $\boxminus'(p, t_0) := \boxminus(p, t_0) + k \cdot \boxminus(p, t_1)$ |
| B2) $\boxplus(t_0, p_0) = k \cdot \boxminus(p_0, t_1)$ for $k \geq 1$ | UB2) For all $p \in P \setminus \{p_0\}$ set $\boxplus'(t_0, p) := \boxplus(t_0, p) + k \cdot \boxplus(t_1, p)$ |
| B3) $p_0^\circ = {}^\circ t_0 = {}^\circ t_1 = \emptyset$ | |
| B4) $p_0 \notin \mathit{places}(\varphi)$ and $M_0(p_0) = 0$ | UB3) Remove $p_0$ and $t_1$ |
| B5) $p^\circ = \emptyset$ and $p \notin \mathit{places}(\varphi)$ for all $p \in {}^\bullet t_0$ | |

Figure 4: Rule B: Sequential place removal (post)

## Rule C: Parallel Places

When two places are parallel to each other and one may accumulate tokens, Rule C will remove it. See Figure **??**. By convention $\min \emptyset = -\infty$ and $\max \emptyset = \infty$. The fraction $d$ describes how fast tokens can be consumed from $p_2$ compared to $p_1$, while $f$ describes how slow tokens can be fed to $p_2$ compared to $p_1$. If $d \leq f$ then $p_2$ is always fed faster than it is emptied compared to $p_1$, which means $p_2$ can be removed, since it will always be $p_1$ which is missing tokens and disables their consumers.

**Theorem 3** *Rule C shown in Figure* **??** *are correct for CTL*.*

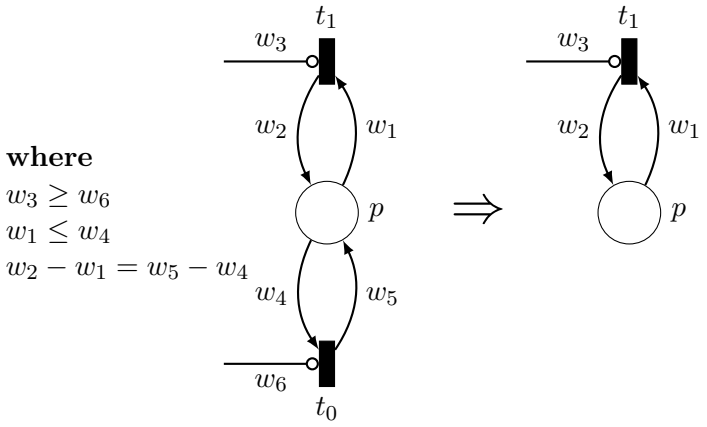



| Precondition | Update |
|---|---|
| Fix places $p_1$ and $p_2$ s.t.:<br>C1) $p_2 \notin places(\varphi)$<br>C2) $p_2^\circ = \emptyset$<br>C3) $p_1^\bullet \neq \emptyset$<br>C4) $p_1^\bullet \supseteq p_2^\bullet$<br>C5) ${}^\bullet p_1 \subseteq {}^\bullet p_2$<br>C6) $M(p_2) \geq M(p_1) \cdot d$<br>C7) $d \leq f$<br>where<br>$d = \max_{t \in p_1^\bullet} \frac{\boxminus(p_2, t)}{\boxminus(p_1, t)}$<br>$f = \min_{t \in {}^\bullet p_1} \frac{\boxplus(t, p_2)}{\boxplus(t, p_1)}$ | UC1) Remove $p_2$ |

Figure 5: Rule C: Parallel places

## Rule L: Dominated Transition

Rule L removes transitions that have the same effect as another transition, but with more preconditions. Since both transitions lead to the same state, we can therefore remove the one with the higher preconditions and use the other instead. See the formal description in Figure 3.




| Precondition | Update |
|---|---|
| Fix transition $t_1$ and $t_0$ s.t.:<br>L1) $I(t_1) \geq I(t_0)$<br>L2) $\boxminus(t_1) \leq \boxminus(t_0)$<br>L3) $E(t_1) = E(t_0)$ | UL1) Remove $t_0$ |

Figure 6: Rule L: Dominated Transition

**Theorem 4** *Rule L in Figure 3 is correct for CTL*.*

## Rule M: Effectively dead places and transitions

The Rule M finds and removes effectively dead places and transitions. We define an effectively dead place to be a place that will never gain nor lose tokens. Effectively dead transitions are transitions that are initially disabled (and/or inhibited) by a place that cannot gain (and/or lose) tokens. These places and transitions are found using fixed-point iteration as defined in Algorithm 1.

**Algorithm 1:** Rule M: Effectively dead places and transitions

**Input:** A net $N = \langle P, T, \boxminus, \boxplus, I \rangle$, initial marking $M_0$ and CTL* formula $\varphi$

**Output:** A reduced net $N'$ and its initial marking $M'_0$

1 $S_{\leq} := P$ /* Places that cannot gain tokens */
2 $S_{\geq} := P$ /* Places that cannot lose tokens */
3 $F := T$ /* Transitions that cannot fire */
4 **do**
  /* Find transitions that may fire and update sets accordingly */
5  **foreach** $t \in F$ *where* $\forall p \in P.(\boxminus(p,t) \leq M_0(p) \vee p \notin S_{\leq}) \wedge (I(p,t) > M_0(p) \vee p \notin S_{\geq})$ **do**
6  $F := F \setminus \{t\}$
7  $S_{\leq} := S_{\leq} \setminus t^{\boxplus}$
8  $S_{\geq} := S_{\geq} \setminus {}^{\boxminus}t$
9 **until** $S_{\leq}$, $S_{\geq}$, *and* $F$ *do not change*
10 $P' := P \setminus (S_{\leq} \cap S_{\geq} \setminus places(\varphi))$
11 $T' := T \setminus F$
12 **return** $N' = \langle P', T', \boxminus, \boxplus, I \rangle$ *and* $M_0$

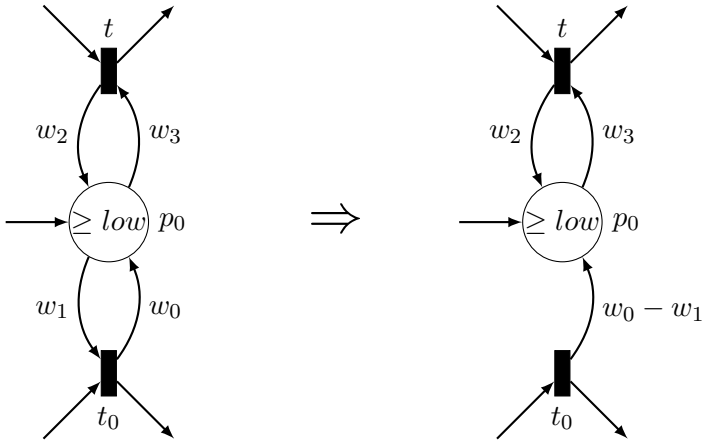
**Theorem 5** *Rule M in Algorithm 1 is correct for CTL*.*

**Theorem 6** *Rule M supercedes Rule E.*

## Rule N: Redundant arc removal

The lower bound number of tokens at a place $p_0$ is given by the minimum of the initial marking and the number of tokens returned by any consuming transition with a negative effect on $p_0$. Using the lower bound we can then find transitions, which are never disabled by $p_0$ and remove the transition's dependency on $p_0$, since it is unnecessary, as long as we maintain the effect of firing the transition.

**where**
$w_3 > w_2 \geq low$
$w_0 \geq low \geq w_1$



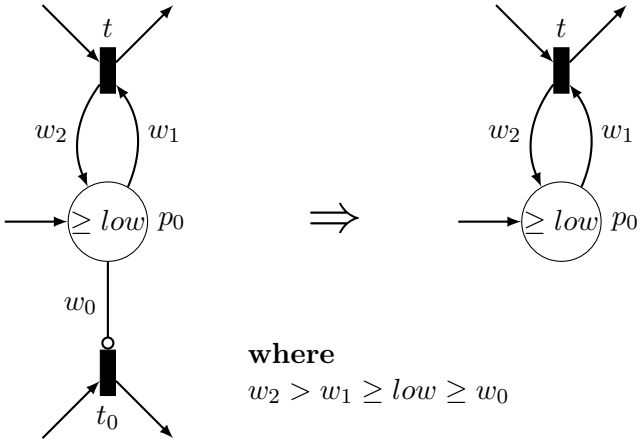

| Precondition | Update |
|---|---|
| Fix place $p_0$ and transition $t_0$ s.t.: | |
| N1) $t_0 \in p_0^\bullet \setminus p_0^{\boxminus}$ | UN1) Set $\boxplus(p_0, t_0) := \boxplus(p_0, t_0) - \boxminus(p_0, t_0)$ |
| N2) $\boxminus(p_0, t_0) \leq low$ | UN2) Set $\boxminus(p_0, t_0) := 0$ |
| where | |
| $low = \min\{M_0(p_0)\} \cup \{\boxplus(p_0, t) \mid t \in p_0^{\boxminus}\}$ | |

Figure 7: Rule N: Redundant arc removal

**Theorem 7** *Rule N in Figure 4 is correct for CTL*.*

## Rule O: Inhibited transition

We can find the lower bound of tokens at a place $p_0$. Any inhibitor arc from $p_0$ with a weight smaller than the lower bound always inhibits the given transition, which means that the transition can be removed. See Figure 5 for a formal description of Rule O.



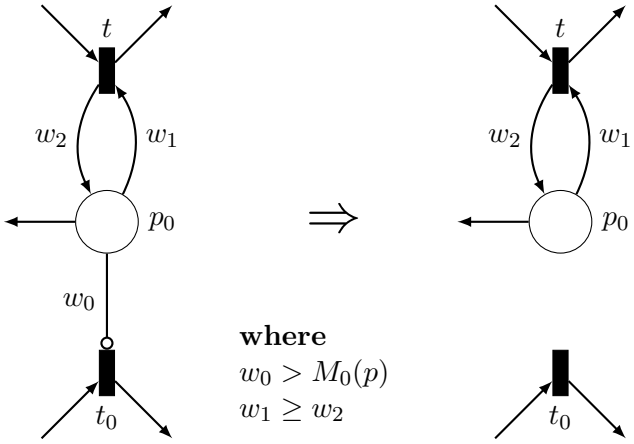

| Precondition | Update |
|---|---|
| Fix place $p_0$ and transition $t_0$ s.t.:<br>O1) $t_0 \in p_0^\circ$<br>O2) $I(p_0, t_0) \leq low$<br>where<br>$low = \min\{M_0(p_0)\} \cup \{\boxplus(p_0, t) \mid t \in p_0^{\boxminus}\}$ | UO1) Remove $t_0$. |

Figure 8: Rule O: Inhibited transition

**Theorem 8** *Rule O in Figure 5 is correct for CTL**

## Rule P: Redundant inhibitor arc

Sometimes we can find an upper bound on the number of tokens at a place $p_0$. This upper bound is given by the initial marking if all transitions have a non-positive effect on $p_0$. Any inhibitor arc from $p_0$ with a weight higher than the upper bound of $p_0$ therefore never inhibits, which means the inhibitor arc can be removed. See Figure 6 for a formal description of Rule P.



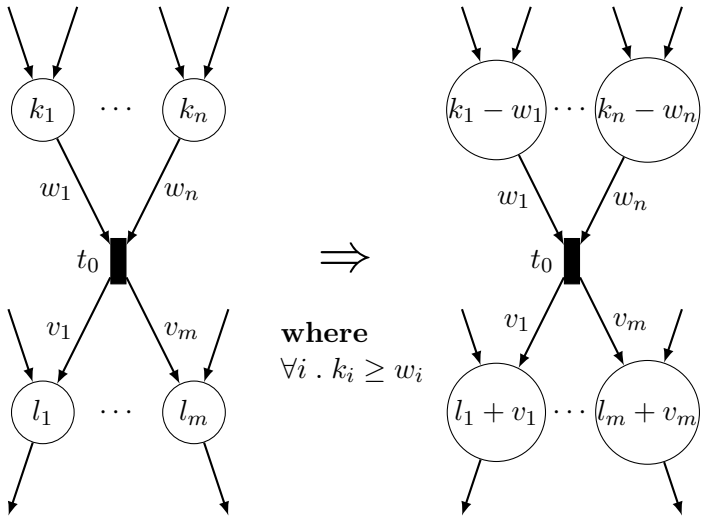

| Precondition | Update |
|---|---|
| Fix place $p_0$ and transition $t_0$ s.t.:<br>P1) $t_0 \in p_0^\circ$<br>P2) $I(p_0, t_0) > M_0(p_0)$<br>P3) ${}^\boxplus p_0 = \emptyset$ | UP1) $I(p_0, t_0) = \infty$. |

Figure 9: Rule P: Redundant inhibitor arc

**Theorem 9** *Rule P in Figure 6 is correct for CTL*.*

## Rule Q: Preemptive transition firing

Rule Q evaluates transitions that are initially enabled and are the only consumer of all places in its pre set. The formal description of Rule Q can be found in Figure 7. Remark that Rule Q can potentially put tokens into places which will prevent other reductions. Furthermore, it can be applied infinitely if $\boxminus(t_0) \leq \boxplus(t_0)$, or if the Petri net contains a loop.




| Precondition | Update |
|---|---|
| Fix transition $t_0$ s.t.:<br>Q1) $({}^\bullet t)^\bullet = \{t_0\}$<br>Q2) $\boxminus(t_0) \leq M_0 < I(t_0)$<br>Q3) $({}^\bullet t_0 \cup t_0^\bullet) \cap places(\varphi) = \emptyset$<br>Q4) $({}^\bullet t_0)^\circ = (t_0^\bullet)^\circ = \emptyset$ | UQ1) $M_0 := M_0 + E(t_0)$. |

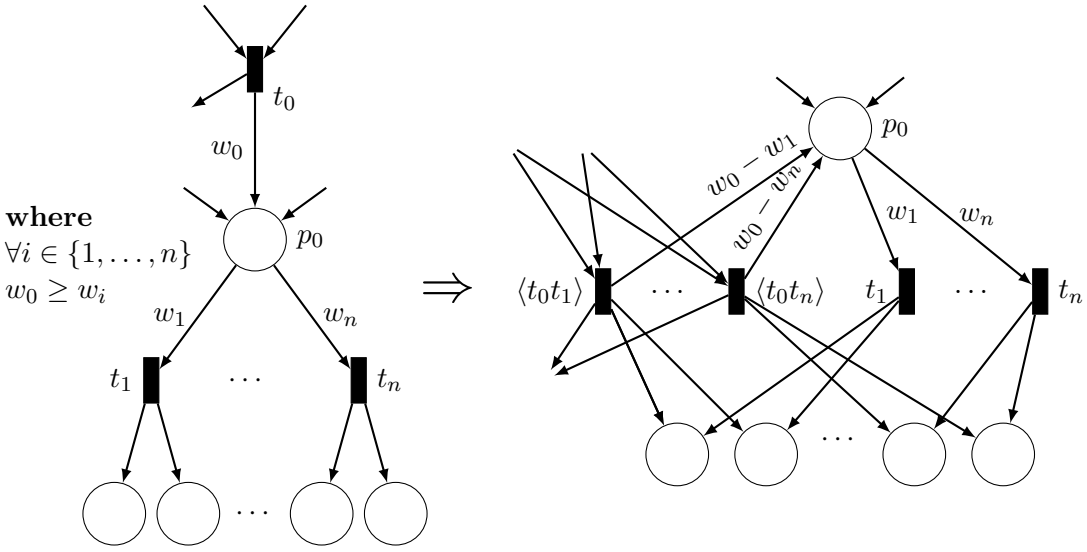
Figure 10: Rule Q: Preemptive transition firing

**Theorem 10** *Rule Q in Figure 7 is correct for CTL\X.*

## Rule R: Atomic post-agglomerable producer

Rule R is similar to a post agglomeration rule and a formal description of Rule R is in Figure 8. In Rule R we look for a place $p_0$ with a producer $t_0$ such that $t_0$ can always be followed by a firing of any consumer of $p_0$ without inhibiting other transitions or affecting places in $places(\varphi)$. The producer $t_0$ is then replaced with new transitions, one for each consumer, and these new transitions combine the effect of firing $t_0$ and the given consumer. Similarly to an agglomeration rule, Rule R removes interleavings despite potentially increasing the size of the Petri net. However, Rule R is more general, since it only operates on one producer at a time and leaves $p_0$ untouched, allowing tokens in $p_0$ in the initial marking, which a post agglomeration does not. Additionally, Rule R does not require the weights of the arcs to and from the agglomerated place to be equal, making R usable in many cases.

**Theorem 11** *Rule R in Figure 8 is correct for LTL\X.*




| Precondition | Update |
|---|---|
| Fix place $p_0$ and transition $t_0$ s.t.: | |
| R1) $t_0 \in {}^\bullet p_0 \wedge p_0^\bullet \neq \emptyset$ | UR1) For each transition $t \in p_0^\bullet$ create a transition $\langle t_0 t \rangle$ with the following arcs: |
| R2) ${}^\bullet p_0 \cap p_0^\bullet = \emptyset$ | $\boxminus(\langle t_0 t \rangle) = \boxminus(t_0)$ |
| R3) $p_0^\circ = {}^\circ(p_0^\bullet) = (({p_0^\bullet})^\bullet)^\circ = \emptyset$ | $\boxplus(\langle t_0 t \rangle) = \boxplus(t_0) + \boxplus(t) - \boxminus(t)$ |
| R4) $(\{p_0\} \cup (p_0^\bullet)^\bullet) \cap places(\varphi) = \emptyset$ | $I(\langle t_0 t \rangle) = I(t_0)$ |
| R5) ${}^\bullet(p_0^\bullet) = \{p_0\}$ | UR2) Remove $t_0$ |
| R6) $\boxplus(t_0, p_0) \geq \boxminus(p_0, t)$ for all $t \in p_0^\bullet$ | |

Figure 11: Rule R: Atomic post-agglomerable producer