

## Rule M: Effectively dead places and transitions

The Rule M finds and removes effectively dead places and transitions. We define an effectively dead place to be a place that will never gain nor lose tokens. Effectively dead transitions are transitions that are initially disabled (and/or inhibited) by a place that cannot gain (and/or lose) tokens. These places and transitions are found using fixed-point iteration as defined in Algorithm 1.

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### Algorithm 1: Rule M: Effectively dead places and transitions

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**Input:** A net  $N = \langle P, T, \Xi, \Theta, I \rangle$ , initial marking  $M_0$  and CTL\* formula  $\varphi$

**Output:** A reduced net  $N'$  and its initial marking  $M'_0$

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1  $S_{\leq} := P$                                 /* Places that cannot gain tokens */
2  $S_{\geq} := P$                                 /* Places that cannot lose tokens */
3  $F := T$                                     /* Transitions that cannot fire */
4 do
    /* Find transitions that may fire and update sets
    accordingly */
5   foreach  $t \in F$  where
       $\forall p \in P. (\Xi(p, t) \leq M_0(p) \vee p \notin S_{\leq}) \wedge (I(p, t) > M_0(p) \vee p \notin S_{\geq})$ 
    do
6      $F := F \setminus \{t\}$ 
7      $S_{\leq} := S_{\leq} \setminus t^{\Theta}$ 
8      $S_{\geq} := S_{\geq} \setminus \Xi t$ 
9 until  $S_{\leq}$ ,  $S_{\geq}$ , and  $F$  do not change
10  $P' := P \setminus (S_{\leq} \cap S_{\geq} \setminus \text{places}(\varphi))$ 
11  $T' := T \setminus F$ 
12 return  $N' = \langle P', T', \Xi, \Theta, I \rangle$  and  $M_0$ 

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**Theorem 1** *Rule M in Algorithm 1 is correct for CTL\*.*

**Theorem 2** *Rule M supercedes Rule E.*