

## PoW

A proof of work function allows to prove  
that you have spend a certain amount of  
(time/compute resources)

Def  $\text{proof} = \underset{\text{PoW}}{f}(\text{data}, d)$  hard to compute  
difficulty  
 $\text{bool} \leftarrow \underset{\text{easy to compute}}{\text{verify}}(\text{data}, \text{proof})$

Example: For a web api, require a PoW.

→ rate limit.

- a user doing individual request will not notice
- a bot doing tons of requests will notice.  
eg. crawling a social media graph.

Def:  $(\text{hash}, \text{nonce}) \leftarrow \underset{\text{PoW}}{f}(\text{Data})$

verify:  $\text{hash} \stackrel{?}{=} H(\text{Data} \parallel \text{nonce})$

and

first d bits in hash are 0.

difficulty,

Compute f: Try different nonces  
until one works.

Lemma: For two different nonces, the probability  
that they solve PoW is independent.

Thm: If  $p$  is Probability to find a nonce  
then the expected number of trials is  $\frac{1}{p}$

increasing/decreasing  $d \rightarrow$  double/half expected trials needed.

$\rightarrow$  Exercise

Def: better version

$$(h, \text{nonce}) \leftarrow f_{\text{PoW}}(\text{Data})$$

verify:  $h \stackrel{?}{=} H(\text{Data} \parallel \text{nonce})$

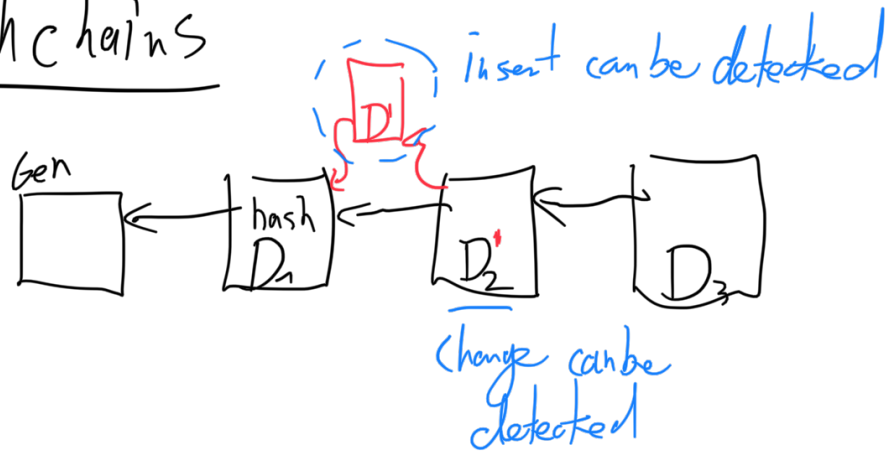
and

$$h \leq d$$

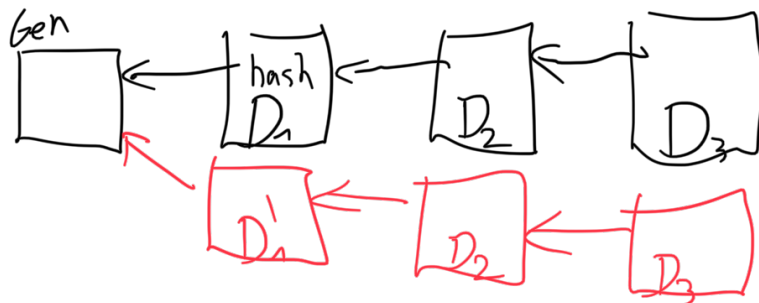
as hexadecimal numbers

Advantage: We can now exactly tune difficulty

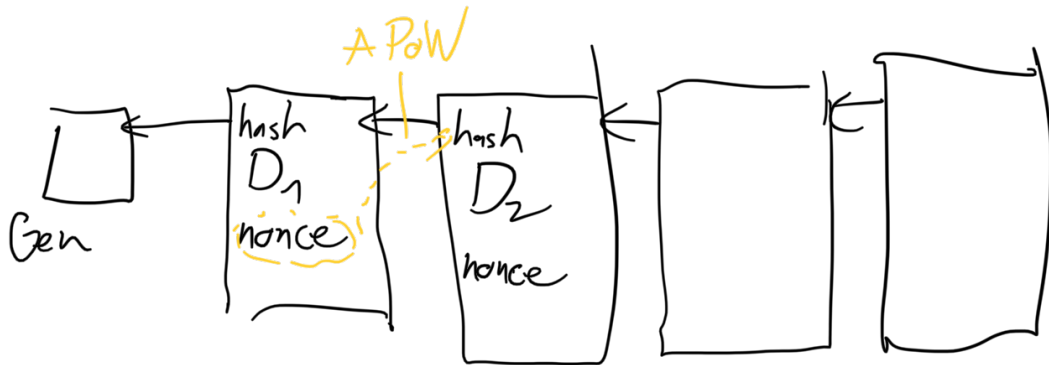
# Hash chains



But can create new chain,



Idea: Add a pow. to every block.

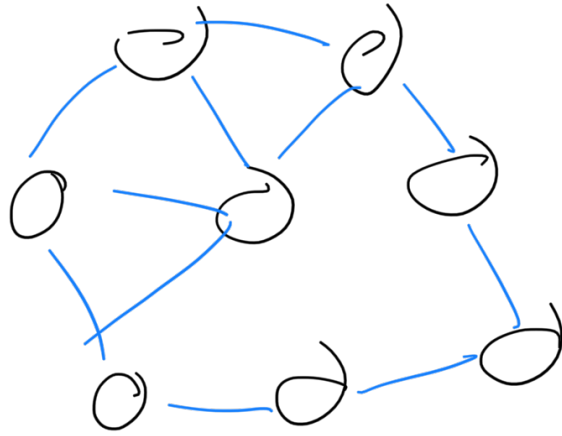


If I want to create new chain  
 ~> recompute all proofs.

Who computes PoW and who creates the block?

A ... bunch of nodes / computers

network - P... - / computing



- all store Blockchain
- all collect transactions into a block
- " try to find a nonce
- the first one to find a nonce publishes his block to everybody.
- all apply transactions in the block

### Censorship resistance:

One node cannot prevent a transaction from ending in a block.

### Fault tolerance:

Individual nodes <sup>may</sup> fail

### Rate control:

Difficulty of PoW ensures slow block

creation.

Conflicting blocks are unlikely:

With large difficulty the probability  
to find two blocks at the same time  
is small.

