

Poverty and Inequality with Complex Survey Data

Guilherme Jacob, Anthony Damico, and Djalma Pessoa

2017-02-06

Contents

1	Introduction	5
1.1	Installation	5
1.2	Complex surveys and statistical inference	5
1.3	Usage Examples	6
1.4	Underlying Calculations	8
1.5	The Variance Estimator	8
1.6	Influence Functions	9
1.7	Influence Function Examples	10
1.8	Examples of Linearization Using the Influence Function	10
1.9	Replication Designs	10
1.10	Decomposition	11
2	Poverty Indices	13
2.1	At Risk of Poverty Ratio (svyarpr)	13
2.2	At Risk of Poverty Threshold (svyarpt)	16
2.3	Relative Median Income Ratio (svyrmir)	19
2.4	Relative Median Poverty Gap (svyrmprg)	19
2.5	Median Income Below the At Risk of Poverty Threshold (svypoormed)	22
2.6	Foster-Greer-Thorbecke class (svyfgt)	25
3	Inequality Measurement	31
3.1	Lorenz Curve (svylorenz)	31
3.2	Gini index (svygini)	34
3.3	Amato index (svyamato)	37
3.4	Zenga Index and Curve (svyzenga, svyzengacurve)	38
3.5	Entropy-based Measures	38
3.6	Generalized Entropy and Decomposition (svygei, svygeidec)	39
3.7	Rényi Divergence (svyrenyi)	42
3.8	J-Divergence and Decomposition (svyjdiv, svyjdivdec)	42
3.9	Atkinson index (svyatk)	43
4	Wellbeing Measures	47
4.1	The Gender Pay Gap (svygp)	47
4.2	Quintile Share Ratio (svyqsr)	50
5	Multidimensional Indices	55
5.1	Alkire-Foster Class and Decomposition (svyafc, svyafcdec)	55
5.2	Bourguignon-Chakravarty (2003) multidimensional poverty class	62
5.3	Bourguignon (1999) inequality class (svybmi)	62

Chapter 1

Introduction

The R `convey` library estimates measures of poverty, inequality, and wellbeing. There are two other R libraries covering this subject, `vardpoor` and `laeken`, however, only `convey` integrates seamlessly with the R survey package.

`convey` is free and open-source software that runs inside the R environment for statistical computing. Anyone can review and propose changes to the source code for this software. Readers are welcome to propose changes to this book as well.

1.1 Installation

In order to work with the `convey` library, you will need to have R running on your machine. If you have never used R before, you will need to install that software before `convey` can be accessed. Check out `FlowingData` for a concise list of resources for new R users. Once you have R loaded on your machine, you can install..

- the latest released version from CRAN with

```
install.packages("convey")
```

- the latest development version from github with

```
devtools::install_github("djalmapessoa/convey")
```

1.2 Complex surveys and statistical inference

In this book, we demonstrate how to measure poverty and income concentration in a population based on microdata collected from a complex survey sample. Most surveys administered by government agencies or larger research organizations utilize a sampling design that violates the assumption of simple random sampling (SRS), including:

1. Different units selection probabilities;
2. Clustering of units;
3. Stratification of clusters;
4. Reweighting to compensate for missing values and other adjustments.

Therefore, basic unweighted R commands such as `mean()` or `glm()` will not properly account for the weighting nor the measures of uncertainty (such as the confidence intervals) present in the dataset. For some examples of publicly-available complex survey data sets, see <http://asdfree.com>.

Unlike other software, the R `convey` package does not require that the user specify these parameters throughout the analysis. So long as the `svydesign` object or `svrepdesign` object has been constructed properly at the outset of the analysis, the `convey` package will incorporate the survey design automatically and produce statistics and variances that take the complex sample into account.

1.3 Usage Examples

In the following example, we've loaded the data set `eusilc` from the R libraries `vardpoor` and `laeken`.

```
library(vardpoor)
data(eusilc)
```

Next, we create an object of class `survey.design` using the function `svydesign` of the library `survey`:

```
library(survey)
des_eusilc <- svydesign(ids = ~rb030, strata = ~db040, weights = ~rb050, data = eusilc)
```

Right after the creation of the design object `des_eusilc`, we should use the function `convey_prep` that adds an attribute to the survey design which saves information on the design object based upon the whole sample, needed to work with subset designs.

```
library(convey)
des_eusilc <- convey_prep( des_eusilc )
```

To estimate the at-risk-of-poverty rate, we use the function `svyarprt`:

```
svyarprt(~eqIncome, design=des_eusilc)
```

```
      arpr      SE
eqIncome 0.14444 0.0028
```

To estimate the at-risk-of-poverty rate across domains defined by the variable `db040` we use:

```
svyby(~eqIncome, by = ~db040, design = des_eusilc, FUN = svyarprt, deff = FALSE)
```

```
      db040 eqIncome      se
Burgenland   Burgenland 0.1953984 0.017202243
Carinthia     Carinthia 0.1308627 0.010610622
Lower Austria Lower Austria 0.1384362 0.006517660
Salzburg      Salzburg 0.1378734 0.011579280
Styria        Styria 0.1437464 0.007452360
Tyrol         Tyrol 0.1530819 0.009880430
Upper Austria Upper Austria 0.1088977 0.005928336
Vienna        Vienna 0.1723468 0.007682826
Vorarlberg    Vorarlberg 0.1653731 0.013754670
```

Using the same data set, we estimate the quintile share ratio:

```
# for the whole population
svyqsr(~eqIncome, design=des_eusilc, alpha= .20)
```

```
      qsr      SE
eqIncome 3.97 0.0426
```

```
# for domains
svyby(~eqIncome, by = ~db040, design = des_eusilc,
      FUN = svyqsr, alpha= .20, deff = FALSE)
```

	db040	eqIncome	se
Burgenland	Burgenland	5.008486	0.32755685
Carinthia	Carinthia	3.562404	0.10909726
Lower Austria	Lower Austria	3.824539	0.08783599
Salzburg	Salzburg	3.768393	0.17015086
Styria	Styria	3.464305	0.09364800
Tyrol	Tyrol	3.586046	0.13629739
Upper Austria	Upper Austria	3.668289	0.09310624
Vienna	Vienna	4.654743	0.13135731
Vorarlberg	Vorarlberg	4.366511	0.20532075

These functions can be used as S3 methods for the classes `survey.design` and `svyrep.design`.

Let's create a design object of class `svyrep.design` and run the function `convey_prep` on it:

```
des_eusilc_rep <- as.svrepdesign(des_eusilc, type = "bootstrap")
des_eusilc_rep <- convey_prep(des_eusilc_rep)
```

and then use the function `svyarpr`:

```
svyarpr(~eqIncome, design=des_eusilc_rep)
```

	arpr	SE
eqIncome	0.14444	0.0026

```
svyby(~eqIncome, by = ~db040, design = des_eusilc_rep, FUN = svyarpr, deff = FALSE)
```

	db040	eqIncome	se.eqIncome
Burgenland	Burgenland	0.1953984	0.019022326
Carinthia	Carinthia	0.1308627	0.009883582
Lower Austria	Lower Austria	0.1384362	0.007635893
Salzburg	Salzburg	0.1378734	0.012294944
Styria	Styria	0.1437464	0.007847486
Tyrol	Tyrol	0.1530819	0.009287302
Upper Austria	Upper Austria	0.1088977	0.004870420
Vienna	Vienna	0.1723468	0.006546435
Vorarlberg	Vorarlberg	0.1653731	0.015004881

The functions of the library `convey` are called in a similar way to the functions in library `survey`.

It is also possible to deal with missing values by using the argument `na.rm`.

```
# survey.design using a variable with missings
svygini( ~ py010n , design = des_eusilc )
```

	gini	SE
py010n	NA	NA

```
svygini( ~ py010n , design = des_eusilc , na.rm = TRUE )
```

	gini	SE
py010n	0.64606	0.0036

```
# svyrep.design using a variable with missings
svygini( ~ py010n , design = des_eusilc_rep )
```

	gini	SE
py010n	NA	NA

```
svygini( ~ py010n , design = des_eusilc_rep , na.rm = TRUE )
```

```

      gini      SE
py010n 0.64606 0.0035

```

djalmapessoa_look, where do these references go on this page? (Berger and Skinner, 2003) and (Osier, 2009) and (Deville, 1999)

1.4 Underlying Calculations

djalmapessoa_look, please describe the general purpose of linearization

In the `convey` library, there are some basic functions that produce the linearized variables needed to measure income concentration and poverty. For example, looking at the income variable in some complex survey dataset, the `quantile` of that income variable can be linearized by the function `convey::svyiqalpha` and the sum total below any quantile of the variable is linearized by the function `convey::svyisq`.

From the linearized variables of these basic estimates, it is possible by using rules of composition, valid for influence functions, to derive the influence function of more complex estimates. By definition the influence function is a Gateaux derivative and the rules rules of composition valid for Gateaux derivatives also hold for Influence Functions.

The following property of Gateaux derivatives was often used in the library `convey`. Let g be a differentiable function of m variables. Suppose we want to compute the influence function of the estimator $g(T_1, T_2, \dots, T_m)$, knowing the Influence function of the estimators $T_i, i = 1, \dots, m$. Then the following holds:

$$I(g(T_1, T_2, \dots, T_m)) = \sum_{i=1}^m \frac{\partial g}{\partial T_i} I(T_i)$$

In the library `convey` this rule is implemented by the function `contrastinf` which uses the R function `deriv` to compute the formal partial derivatives $\frac{\partial g}{\partial T_i}$.

For example, suppose we want to linearize the `Relative median poverty gap`(`rmpg`), defined as the difference between the at-risk-of-poverty threshold (`arpt`) and the median of incomes less than the `arpt` relative to the `arpt`:

$$rmpg = \frac{arpt - medpoor}{arpt}$$

where `medpoor` is the median of incomes less than `arpt`.

Suppose we know how to linearize `arpt` and `medpoor`, then by applying the function `contrastinf` with

$$g(T_1, T_2) = \frac{(T_1 - T_2)}{T_1}$$

we linearize the `rmpg`.

1.5 The Variance Estimator

djalmapessoa_look please add references to this section

The variance of the estimator $T(\hat{M})$ can approximated by:

$$Var \left[T(\hat{M}) \right] \cong var \left[\sum_s w_i z_i \right]$$

The **linearized** variable z is given by the derivative of the functional:

$$z_k = \lim_{t \rightarrow 0} \frac{T(M + t\delta_k) - T(M)}{t} = IT_k(M)$$

where, δ_k is the Dirac measure in k : $\delta_k(i) = 1$ if and only if $i = k$.

This **derivative** is called **Influence Function** and was introduced in the area of **Robust Statistics**.

1.6 Influence Functions

Some measures of poverty and income concentration are defined by non-differentiable functions so that it is not possible to use Taylor linearization to estimate their variances. An alternative is to use **Influence functions** as described in (Deville, 1999) and (Osier, 2009). The convey library implements this methodology to work with `survey.design` objects and also with `svyrep.design` objects.

Some examples of these measures are:

- At-risk-of-poverty threshold: $arpt = .60q_{.50}$ where $q_{.50}$ is the income median;
- At-risk-of-poverty rate $arpr = \frac{\sum_U 1(y_i \leq arpt)}{N} \cdot 100$
- Quintile share ratio

$$qsr = \frac{\sum_U 1(y_i > q_{.80})}{\sum_U 1(y_i \leq q_{.20})}$$

- Gini coefficient $1 + G = \frac{2 \sum_U (r_i - 1)y_i}{N \sum_U y_i}$ where r_i is the rank of y_i .

Note that it is not possible to use Taylor linearization for these measures because they depend on quantiles and the Gini is defined as a function of ranks. This could be done using the approach proposed by Deville (1999) based upon influence functions.

Let U be a population of size N and M be a measure that allocates mass one to the set composed by one unit, that is $M(i) = M_i = 1$ if $i \in U$ and $M(i) = 0$ if $i \notin U$

Now, a population parameter θ can be expressed as a functional of M $\theta = T(M)$

Examples of such parameters are:

- Total: $Y = \sum_U y_i = \sum_U y_i M_i = \int y dM = T(M)$
- Ratio of two totals: $R = \frac{Y}{X} = \frac{\int y dM}{\int x dM} = T(M)$
- Cumulative distribution function: $F(x) = \frac{\sum_U 1(y_i \leq x)}{N} = \frac{\int 1(y \leq x) dM}{\int dM} = T(M)$

To estimate these parameters from the sample, we replace the measure M by the estimated measure \hat{M} defined by: $\hat{M}(i) = \hat{M}_i = w_i$ if $i \in s$ and $\hat{M}(i) = 0$ if $i \notin s$.

The estimators of the population parameters can then be expressed as functional of the measure \hat{M} .

- Total: $\hat{Y} = T(\hat{M}) = \int y d\hat{M} = \sum_s w_i y_i$
- Ratio of totals: $\hat{R} = T(\hat{M}) = \frac{\int y d\hat{M}}{\int x d\hat{M}} = \frac{\sum_s w_i y_i}{\sum_s w_i x_i}$
- Cumulative distribution function: $\hat{F}(x) = T(\hat{M}) = \frac{\int 1(y \leq x) d\hat{M}}{\int d\hat{M}} = \frac{\sum_s w_i 1(y_i \leq x)}{\sum_s w_i}$

1.7 Influence Function Examples

- Total:

$$\begin{aligned} IT_k(M) &= \lim_{t \rightarrow 0} \frac{T(M + t\delta_k) - T(M)}{t} \\ &= \lim_{t \rightarrow 0} \frac{\int y.d(M + t\delta_k) - \int y.dM}{t} \\ &= \lim_{t \rightarrow 0} \frac{\int yd(t\delta_k)}{t} = y_k \end{aligned}$$

- Ratio of two totals:

$$\begin{aligned} IR_k(M) &= I\left(\frac{U}{V}\right)_k(M) = \frac{V(M) \times IU_k(M) - U(M) \times IV_k(M)}{V(M)^2} \\ &= \frac{Xy_k - Yx_k}{X^2} = \frac{1}{X}(y_k - Rx_k) \end{aligned}$$

1.8 Examples of Linearization Using the Influence Function

- At-risk-of-poverty threshold:

$$arpt = 0.6 \times m$$

where m is the median income.

$$z_k = -\frac{0.6}{f(m)} \times \frac{1}{N} \times [I(y_k \leq m - 0.5)]$$

- At-risk-of-poverty rate:

$$arpr = \frac{\sum_U I(y_i \leq t)}{\sum_U w_i} . 100$$

$$z_k = \frac{1}{N} [I(y_k \leq t) - t] - \frac{0.6}{N} \times \frac{f(t)}{f(m)} [I(y_k \leq m) - 0.5]$$

where:

N - population size;

t - at-risk-of-poverty threshold;

y_k - income of person k ;

m - median income;

f - income density function;

1.9 Replication Designs

djalmapessoa_look, please describe how the software works differently on svrepdesign objects – as compared to svydesign objects

1.10 Decomposition

Some inequality and multidimensional poverty measures can be decomposed. As of December 2016, the decomposition methods in **convey** are limited to group decomposition.

For instance, the generalized entropy index can be decomposed into between and within group components. This sheds light on a very simple question: of the overall inequality, how much can be explained by inequalities between groups and within groups? Since this measure is additive decomposable, one can get estimates of the coefficients, SEs and covariance between components. For a more practical approach, see (Lima, 2013).

The Alkire-Foster class of multidimensional poverty indices can be decomposed by dimension and groups. This shows how much each group (or dimension) contribute to the overall poverty.

This technique can help understand where and who is more affected by inequality and poverty, contributing to more specific policy and economic analysis.

Chapter 2

Poverty Indices

2.1 At Risk of Poverty Ratio (svyarpr)

here are the references

(Osier, 2009) and (Deville, 1999)

A replication example

The R `vardpoor` package (Breidaks et al., 2016), created by researchers at the Central Statistical Bureau of Latvia, includes a `arpr` coefficient calculation using the ultimate cluster method. The example below reproduces those statistics.

Load and prepare the same data set:

```
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the vardpoor library
library(vardpoor)

# load the synthetic european union statistics on income & living conditions
data(eusilc)

# make all column names lowercase
names( eusilc ) <- tolower( names( eusilc ) )

# add a column with the row number
dati <- data.table(IDd = 1 : nrow(eusilc), eusilc)

# calculate the arpr coefficient
# using the R vardpoor library
varpoord_arpr_calculation <-
  vardpoord(

    # analysis variable
```

```

Y = "eqincome",

# weights variable
w_final = "rb050",

# row number variable
ID_level1 = "IDd",

# strata variable
H = "db040",

N_h = NULL ,

# clustering variable
PSU = "rb030",

# data.table
dataset = dati,

# arpr coefficient function
type = "linarpr"

)

```

```
## NULL
```

```
# all calculations produced by vardpoor::linarpr
```

```
varpoord_arpr_calculation$all_result
```

```
##      type respondent_count n_nonzero pop_size      value value_eu      var
## 1: ARPR              14827      14824  8182222 14.44422      NA 0.07586194
##      se      rse      cv absolute_margin_of_error
## 1: 0.2754305 0.01906856 1.906856              0.5398338
##      relative_margin_of_error CI_lower CI_upper      S2_y_HT      S2_y_ca
## 1:              3.737369 13.90438 14.98405 1.639976e-11 1.639976e-11
##      S2_res var_srs_HT var_cur_HT var_srs_ca deff_sam deff_est
## 1: 1.639976e-11 0.07391608 0.07586194 0.07391608 1.026325      1
##      deff
## 1: 1.026325
```

```
# construct a survey.design
# using our recommended setup
```

```
des_eusilc <-
```

```
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,
    data = eusilc
  )

```

```
# immediately run the convey_prep function on it
```

```
des_eusilc <- convey_prep( des_eusilc )
```

```
# coefficients do match
```

```
varpoord_arpr_calculation$all_result$value
```

```
## [1] 14.44422
coef( svyarpr( ~ eqincome , des_eusilc ) ) * 100

## eqincome
## 14.44422
# variances do not match exactly
attr( svyarpr( ~ eqincome , des_eusilc ) , 'var' ) * 10000
```

```
## eqincome
## eqincome 0.07599778
varpoord_arpr_calculation$all_result$var
```

```
## [1] 0.07586194
# standard errors do not match exactly
varpoord_arpr_calculation$all_result$se
```

```
## [1] 0.2754305
SE( svyarpr( ~ eqincome , des_eusilc ) ) * 100
```

```
## eqincome
## eqincome 0.2756769
```

By default, the `convey::svyarpr` function comes close to the results of `vardpoor::linarpr`. However, the measures of uncertainty do not line up, because `library(vardpoor)` defaults to the ultimate cluster method. This can be replicated with an alternative setup of the `survey.design` object.

```
# within each strata, sum up the weights
cluster_sums <- aggregate( eusilc$rb050 , list( eusilc$db040 ) , sum )

# name the within-strata sums of weights the `cluster_sum`
names( cluster_sums ) <- c( "db040" , "cluster_sum" )

# merge this column back onto the data.frame
eusilc <- merge( eusilc , cluster_sums )

# construct a survey.design
# with the fpc using the cluster sum
des_eusilc_ultimate_cluster <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,
    data = eusilc ,
    fpc = ~ cluster_sum
  )

# again, immediately run the convey_prep function on the `survey.design`
des_eusilc_ultimate_cluster <- convey_prep( des_eusilc_ultimate_cluster )

# matches
attr( svyarpr( ~ eqincome , des_eusilc_ultimate_cluster ) , 'var' ) * 10000

## eqincome
## eqincome 0.07586194
```

```
varpoord_arpr_calculation$all_result$var

## [1] 0.07586194
# matches
varpoord_arpr_calculation$all_result$se

## [1] 0.2754305
SE( svyarpr( ~ eqincome , des_eusilc_ultimate_cluster ) ) * 100

##           eqincome
## eqincome 0.2754305
```

For additional usage examples of `svyarpr`, type `?convey::svyarpr` in the R console.

2.2 At Risk of Poverty Threshold (`svyarpt`)

here are the references

(Osier, 2009) and (Deville, 1999)

A replication example

The R `vardpoor` package (Breidaks et al., 2016), created by researchers at the Central Statistical Bureau of Latvia, includes a `arpt` coefficient calculation using the ultimate cluster method. The example below reproduces those statistics.

Load and prepare the same data set:

```
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the vardpoor library
library(vardpoor)

# load the synthetic european union statistics on income & living conditions
data(eusilc)

# make all column names lowercase
names( eusilc ) <- tolower( names( eusilc ) )

# add a column with the row number
dati <- data.table(IDd = 1 : nrow(eusilc), eusilc)

# calculate the arpt coefficient
# using the R vardpoor library
varpoord_arpt_calculation <-
  vardpoord(

    # analysis variable
    Y = "eqincome",
```



```

# weights variable
w_final = "rb050",

# row number variable
ID_level1 = "IDd",

# strata variable
H = "db040",

N_h = NULL ,

# clustering variable
PSU = "rb030",

# data.table
dataset = dati,

# arpt coefficient function
type = "linarpt"

)

```

```
## NULL
```

```

# all calculations produced by vardpoor::linarpt
varpoord_arpt_calculation$all_result

```

```

##      type respondent_count n_nonzero pop_size   value value_eu   var
## 1: ARPT           14827      14824  8182222 10859.24      NA 2559.442
##      se      rse      cv absolute_margin_of_error
## 1: 50.59093 0.004658793 0.4658793           99.1564
##      relative_margin_of_error CI_lower CI_upper      S2_y_HT      S2_y_ca
## 1:           0.9131066 10760.08 10958.39 5.557052e-07 5.557052e-07
##      S2_res var_srs_HT var_cur_HT var_srs_ca deff_sam deff_est
## 1: 5.557052e-07 2504.644 2559.442 2504.644 1.021879      1
##      deff
## 1: 1.021879

```

```

# construct a survey.design
# using our recommended setup

```

```

des_eusilc <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,
    data = eusilc
  )

```

```

# immediately run the convey_prep function on it
des_eusilc <- convey_prep( des_eusilc )

```

```

# coefficients do match
varpoord_arpt_calculation$all_result$value

```

```
## [1] 10859.24
```

```
coef( svyarpt( ~ eqincome , des_eusilc ) )
```

```
## eqincome
## 10859.24
```

```
# variances do not match exactly
```

```
attr( svyarpt( ~ eqincome , des_eusilc ) , 'var' )
```

```
## eqincome
## eqincome 2564.027
```

```
varpoord_arpt_calculation$all_result$var
```

```
## [1] 2559.442
```

```
# standard errors do not match exactly
```

```
varpoord_arpt_calculation$all_result$se
```

```
## [1] 50.59093
```

```
SE( svyarpt( ~ eqincome , des_eusilc ) )
```

```
## eqincome
## eqincome 50.63622
```

By default, the `convey::svyarpt` function comes close to the results of `vardpoor::linarpt`. However, the measures of uncertainty do not line up, because `library(vardpoor)` defaults to the ultimate cluster method. This can be replicated with an alternative setup of the `survey.design` object.

```
# within each strata, sum up the weights
```

```
cluster_sums <- aggregate( eusilc$rb050 , list( eusilc$db040 ) , sum )
```

```
# name the within-strata sums of weights the `cluster_sum`
```

```
names( cluster_sums ) <- c( "db040" , "cluster_sum" )
```

```
# merge this column back onto the data.frame
```

```
eusilc <- merge( eusilc , cluster_sums )
```

```
# construct a survey.design
```

```
# with the fpc using the cluster sum
```

```
des_eusilc_ultimate_cluster <-
```

```
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,
    data = eusilc ,
    fpc = ~ cluster_sum
  )
```

```
# again, immediately run the convey_prep function on the `survey.design`
```

```
des_eusilc_ultimate_cluster <- convey_prep( des_eusilc_ultimate_cluster )
```

```
# matches
```

```
attr( svyarpt( ~ eqincome , des_eusilc_ultimate_cluster ) , 'var' )
```

```
## eqincome
## eqincome 2559.442
```

```
varpoord_arpt_calculation$all_result$var

## [1] 2559.442

# matches
varpoord_arpt_calculation$all_result$se

## [1] 50.59093

SE( svyarpt( ~ eqincome , des_eusilc_ultimate_cluster ) )

##           eqincome
## eqincome 50.59093
```

For additional usage examples of `svyarpt`, type `?convey::svyarpt` in the R console.

2.3 Relative Median Income Ratio (svyrmir)

For additional usage examples of `svyrmir`, type `?convey::svyrmir` in the R console.

here are the references

(Osier, 2009) and (Deville, 1999)

2.4 Relative Median Poverty Gap (svyrmprg)

here are the references

(Osier, 2009) and (Deville, 1999)

A replication example

The R `vardpoor` package (Bredakis et al., 2016), created by researchers at the Central Statistical Bureau of Latvia, includes a `rmpg` coefficient calculation using the ultimate cluster method. The example below reproduces those statistics.

Load and prepare the same data set:

```
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the vardpoor library
library(vardpoor)

# load the synthetic european union statistics on income & living conditions
data(eusilc)

# make all column names lowercase
names( eusilc ) <- tolower( names( eusilc ) )

# add a column with the row number
```

```

dati <- data.table(IDd = 1 : nrow(eusilc), eusilc)

# calculate the rmpg coefficient
# using the R vardpoor library
varpoord_rmpg_calculation <-
  varpoord(

    # analysis variable
    Y = "eqincome",

    # weights variable
    w_final = "rb050",

    # row number variable
    ID_level1 = "IDd",

    # strata variable
    H = "db040",

    N_h = NULL ,

    # clustering variable
    PSU = "rb030",

    # data.table
    dataset = dati,

    # rmpg coefficient function
    type = "linrmpg"

  )

## NULL

# all calculations produced by vardpoor::linrmpg
varpoord_rmpg_calculation$all_result

##      type respondent_count n_nonzero pop_size  value value_eu      var
## 1: RMPG           14827      14824  8182222 18.9286        NA 0.3316454
##      se      rse      cv absolute_margin_of_error
## 1: 0.5758866 0.03042416 3.042416           1.128717
##      relative_margin_of_error CI_lower CI_upper      S2_y_HT      S2_y_ca
## 1:           5.963025 17.79988 20.05731 7.025364e-11 7.025364e-11
##      S2_res var_srs_HT var_cur_HT var_srs_ca deff_sam deff_est
## 1: 7.025364e-11 0.3166433 0.3316454 0.3166433 1.047378      1
##      deff
## 1: 1.047378

# construct a survey.design
# using our recommended setup
des_eusilc <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,

```

```

    data = eusilc
  )

# immediately run the convey_prep function on it
des_eusilc <- convey_prep( des_eusilc )

# coefficients do match
varpoord_rmpg_calculation$all_result$value

## [1] 18.9286

coef( svyrmpg( ~ eqincome , des_eusilc ) ) * 100

## eqincome
## 18.9286

# variances do not match exactly
attr( svyrmpg( ~ eqincome , des_eusilc ) , 'var' ) * 10000

##          eqincome
## eqincome 0.332234
varpoord_rmpg_calculation$all_result$var

## [1] 0.3316454

# standard errors do not match exactly
varpoord_rmpg_calculation$all_result$se

## [1] 0.5758866

SE( svyrmpg( ~ eqincome , des_eusilc ) ) * 100

##          eqincome
## eqincome 0.5763974

```

By default, the `convey::svyrmpg` function comes close to the results of `vardpoor::linrmpg`. However, the measures of uncertainty do not line up, because `library(vardpoor)` defaults to the ultimate cluster method. This can be replicated with an alternative setup of the `survey.design` object.

```

# within each strata, sum up the weights
cluster_sums <- aggregate( eusilc$rb050 , list( eusilc$db040 ) , sum )

# name the within-strata sums of weights the `cluster_sum`
names( cluster_sums ) <- c( "db040" , "cluster_sum" )

# merge this column back onto the data.frame
eusilc <- merge( eusilc , cluster_sums )

# construct a survey.design
# with the fpc using the cluster sum
des_eusilc_ultimate_cluster <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,
    data = eusilc ,
    fpc = ~ cluster_sum
  )

```

```

)

# again, immediately run the convey_prep function on the `survey.design`
des_eusilc_ultimate_cluster <- convey_prep( des_eusilc_ultimate_cluster )

# matches
attr( "svyrmprg" ) = ~ eqincome , des_eusilc_ultimate_cluster ) , 'var' ) * 10000

##           eqincome
## eqincome 0.3316454
varpoord_rmpg_calculation$all_result$var

## [1] 0.3316454
# matches
varpoord_rmpg_calculation$all_result$se

## [1] 0.5758866
SE( svyrmprg( ~ eqincome , des_eusilc_ultimate_cluster ) ) * 100

##           eqincome
## eqincome 0.5758866

```

For additional usage examples of `svyrmprg`, type `?convey::svyrmprg` in the R console.

2.5 Median Income Below the At Risk of Poverty Threshold (`svy-poormed`)

here are the references

(Osier, 2009) and (Deville, 1999)

A replication example

The R `vardpoor` package (Breibaks et al., 2016), created by researchers at the Central Statistical Bureau of Latvia, includes a poormed coefficient calculation using the ultimate cluster method. The example below reproduces those statistics.

Load and prepare the same data set:

```

# load the convey package
library(convey)

# load the survey library
library(survey)

# load the vardpoor library
library(vardpoor)

# load the synthetic european union statistics on income & living conditions
data(eusilc)

# make all column names lowercase

```

```

names( eusilc ) <- tolower( names( eusilc ) )

# add a column with the row number
dati <- data.table(IDd = 1 : nrow(eusilc), eusilc)

# calculate the poormed coefficient
# using the R vardpoor library
varpoord_poormed_calculation <-
  varpoord(

    # analysis variable
    Y = "eqincome",

    # weights variable
    w_final = "rb050",

    # row number variable
    ID_level1 = "IDd",

    # strata variable
    H = "db040",

    N_h = NULL ,

    # clustering variable
    PSU = "rb030",

    # data.table
    dataset = dati,

    # poormed coefficient function
    type = "linpoormed"

  )

## NULL

# all calculations produced by vardpoor::linpoormed
varpoord_poormed_calculation$all_result

##      type respondent_count n_nonzero pop_size   value value_eu   var
## 1: POORMED           14827     14824  8182222 8803.735     NA 5302.086
##      se      rse      cv absolute_margin_of_error
## 1: 72.81542 0.008270969 0.8270969           142.7156
##      relative_margin_of_error CI_lower CI_upper   S2_y_HT   S2_y_ca
## 1:           1.62108 8661.019 8946.451 1.123565e-06 1.123565e-06
##      S2_res var_srs_HT var_cur_HT var_srs_ca deff_sam deff_est
## 1: 1.123565e-06  5064.071   5302.086   5064.071 1.047001      1
##      deff
## 1: 1.047001

# construct a survey.design
# using our recommended setup
des_eusilc <-
  svydesign(

```

```

    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,
    data = eusilc
  )

# immediately run the convey_prep function on it
des_eusilc <- convey_prep( des_eusilc )

# coefficients do match
varpoord_poormed_calculation$all_result$value

## [1] 8803.735

coef( svypoormed( ~ eqincome , des_eusilc ) )

## eqincome
## 8803.735

# variances do not match exactly
attr( svypoormed( ~ eqincome , des_eusilc ) , 'var' )

##          eqincome
## eqincome  5311.47
varpoord_poormed_calculation$all_result$var

## [1] 5302.086

# standard errors do not match exactly
varpoord_poormed_calculation$all_result$se

## [1] 72.81542

SE( svypoormed( ~ eqincome , des_eusilc ) )

##          eqincome
## eqincome 72.87983

```

By default, the `convey::svypoormed` function comes close to the results of `vardpoor::linpoormed`. However, the measures of uncertainty do not line up, because `library(vardpoor)` defaults to the ultimate cluster method. This can be replicated with an alternative setup of the `survey.design` object.

```

# within each strata, sum up the weights
cluster_sums <- aggregate( eusilc$rb050 , list( eusilc$db040 ) , sum )

# name the within-strata sums of weights the `cluster_sum`
names( cluster_sums ) <- c( "db040" , "cluster_sum" )

# merge this column back onto the data.frame
eusilc <- merge( eusilc , cluster_sums )

# construct a survey.design
# with the fpc using the cluster sum
des_eusilc_ultimate_cluster <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,

```



```

    weights = ~ rb050 ,
    data = eusilc ,
    fpc = ~ cluster_sum
  )

# again, immediately run the convey_prep function on the `survey.design`
des_eusilc_ultimate_cluster <- convey_prep( des_eusilc_ultimate_cluster )

# matches
attr( svypoormed( ~ eqincome , des_eusilc_ultimate_cluster ) , 'var' )

##           eqincome
## eqincome 5302.086

varpoord_poormed_calculation$all_result$var

## [1] 5302.086

# matches
varpoord_poormed_calculation$all_result$se

## [1] 72.81542

SE( svypoormed( ~ eqincome , des_eusilc_ultimate_cluster ) )

##           eqincome
## eqincome 72.81542

```

For additional usage examples of `svypoormed`, type `?convey::svypoormed` in the R console.

2.6 Foster-Greer-Thorbecke class (svyfgt)

(Foster et al., 1984) proposed a family of indicators to measure poverty. This class of *FGT* measures, can be defined as

$$p = \frac{1}{N} \sum_{k \in U} h(y_k, \theta),$$

where

$$h(y_k, \theta) = \left[\frac{(\theta - y_k)}{\theta} \right]^\gamma \delta \{y_k \leq \theta\},$$

where: θ is the poverty threshold; δ the indicator function that assigns value 1 if the condition $\{y_k \leq \theta\}$ is satisfied and 0 otherwise, and γ is a non-negative constant.

When $\gamma = 0$, p can be interpreted as the poverty headcount ratio, and for $\gamma \geq 1$, the weight of the income shortfall of the poor to a power γ , (Foster and all, 1984).

The poverty measure FGT is implemented in the library `convey` by the function `svyfgt`. The argument `thresh_type` of this function defines the type of poverty threshold adopted. There are three possible choices:

1. **abs** – fixed and given by the argument `thresh_value`
2. **relq** – a proportion of a quantile fixed by the argument `proportion` and the quantile is defined by the argument `order`.
3. **relm** – a proportion of the mean fixed the argument `proportion`

The quantile and the mean involved in the definition of the threshold are estimated for the whole population. When $\gamma = 0$ and $\theta = .6 * MED$ the measure is equal to the indicator `arpr` computed by the function `svyarpr`.

Next, we give some examples of the function `svyfgt` to estimate the values of the FGT poverty index.

Consider first the poverty threshold fixed ($\gamma = 0$) in the value 10000. The headcount ratio (FGT0) is

```
svyfgt(~eqincome, des_eusilc, g=0, abs_thresh=10000)
```

```
      fgt0      SE
eqincome 0.11444 0.0027
```

The poverty gap (FGT1) ($\gamma = 1$) index for the poverty threshold fixed at the same value is

```
svyfgt(~eqincome, des_eusilc, g=1, abs_thresh=10000)
```

```
      fgt1      SE
eqincome 0.032085 0.0011
```

To estimate the FGT0 with the poverty threshold fixed at $0.6 * MED$ we fix the argument `type_thresh="relq"` and use the default values for `percent` and `order`:

```
svyfgt(~eqincome, des_eusilc, g=0, type_thresh= "relq")
```

```
      fgt0      SE
eqincome 0.14444 0.0028
```

that matches the estimate obtained by

```
svyarpr(~eqincome, design=des_eusilc, .5, .6)
```

```
      arpr      SE
eqincome 0.14444 0.0028
```

To estimate the poverty gap (FGT1) with the poverty threshold equal to $0.6 * MEAN$ we use:

```
svyfgt(~eqincome, des_eusilc, g=1, type_thresh= "relm")
```

```
      fgt1      SE
eqincome 0.051187 0.0011
```

A replication example

In July 2006, (Jenkins, 2008) presented at the North American Stata Users' Group Meetings on the stata Atkinson Index command. The example below reproduces those statistics.

In order to match the results in (Jenkins, 2008) using the `svyfgt` function from the convey library, the poverty threshold was considered absolute despite being directly estimated from the survey sample. This effectively treats the variance of the estimated poverty threshold as zero; `svyfgt` does not account for the uncertainty of the poverty threshold when the level has been stated as absolute with the `abs_thresh=` parameter. In general, we would instead recommend using either `relq` or `relm` in the `type_thresh=` parameter in order to account for the added uncertainty of the poverty threshold calculation. This example serves only to show that `svyfgt` behaves properly as compared to other software.

Load and prepare the same data set:

```
# load the convey package
library(convey)

# load the survey library
library(survey)
```

```

# load the foreign library
library(foreign)

# create a temporary file on the local disk
tf <- tempfile()

# store the location of the presentation file
presentation_zip <- "http://repec.org/nasug2006/nasug2006_jenkins.zip"

# download jenkins' presentation to the temporary file
download.file( presentation_zip , tf , mode = 'wb' )

# unzip the contents of the archive
presentation_files <- unzip( tf , exdir = tempdir() )

# load the institute for fiscal studies' 1981, 1985, and 1991 data.frame objects
x81 <- read.dta( grep( "ifs81" , presentation_files , value = TRUE ) )
x85 <- read.dta( grep( "ifs85" , presentation_files , value = TRUE ) )
x91 <- read.dta( grep( "ifs91" , presentation_files , value = TRUE ) )

# NOTE: we recommend using ?convey::svyarpt rather than this unweighted calculation #

# calculate 60% of the unweighted median income in 1981
unwtd_arpt81 <- quantile( x81$eybhc0 , 0.5 ) * .6

# calculate 60% of the unweighted median income in 1985
unwtd_arpt85 <- quantile( x85$eybhc0 , 0.5 ) * .6

# calculate 60% of the unweighted median income in 1991
unwtd_arpt91 <- quantile( x91$eybhc0 , 0.5 ) * .6

# stack each of these three years of data into a single data.frame
x <- rbind( x81 , x85 , x91 )

```

Replicate the author's survey design statement from stata code..

```

. ge poor = (year==1981)*(x < $z_81) + (year==1985)*(x < $z_85) + (year==1991)*(x < $z_91)
. * account for clustering within HHs
. svyset hrn [pweight = wgt]

```

.. into R code:

```

# initiate a linearized survey design object
y <- svydesign( ~ hrn , data = x , weights = ~ wgt )

# immediately run the `convey_prep` function on the survey design
z <- convey_prep( y )

```

Replicate the author's headcount ratio results with stata..

```

. svy: mean poor if year == 1981
(running mean on estimation sample)

```

Survey: Mean estimation

```

Number of strata =      1      Number of obs      =    9772

```

```
Number of PSUs = 7476      Population size = 5.5e+07
                          Design df      = 7475
```

		Linearized		
		Mean	Std. Err.	[95% Conf. Interval]
poor		.1410125	.0044859	.132219 .149806

```
. svy: mean poor if year == 1985
(running mean on estimation sample)
```

Survey: Mean estimation

```
Number of strata = 1      Number of obs = 8991
Number of PSUs = 6972    Population size = 5.5e+07
                          Design df      = 6971
```

		Linearized		
		Mean	Std. Err.	[95% Conf. Interval]
poor		.137645	.0046531	.1285235 .1467665

```
. svy: mean poor if year == 1991
(running mean on estimation sample)
```

Survey: Mean estimation

```
Number of strata = 1      Number of obs = 6468
Number of PSUs = 5254    Population size = 5.6e+07
                          Design df      = 5253
```

		Linearized		
		Mean	Std. Err.	[95% Conf. Interval]
poor		.2021312	.0062077	.1899615 .2143009

..using R code:

```
headcount_81 <-
  svyfgt(
    ~ eybhc0 ,
    subset( z , year == 1981 ) ,
    g = 0 ,
    abs_thresh = unwt_d_arpt81
  )

headcount_81
```

```
##          fgt0      SE
```

```
## eybhc0 0.14101 0.0045
confint( headcount_81 , df = degf( subset( z , year == 1981 ) ) )
```

```
##          2.5 %    97.5 %
## eybhc0 0.1322193 0.1498057
headcount_85 <-
  svyfgt(
    ~ eybhc0 ,
    subset( z , year == 1985 ) ,
    g = 0 ,
    abs_thresh = unwtd_arpt85
  )
```

```
headcount_85
```

```
##          fgt0      SE
## eybhc0 0.13764 0.0047
confint( headcount_85 , df = degf( subset( z , year == 1985 ) ) )
```

```
##          2.5 %    97.5 %
## eybhc0 0.1285239 0.1467661
headcount_91 <-
  svyfgt(
    ~ eybhc0 ,
    subset( z , year == 1991 ) ,
    g = 0 ,
    abs_thresh = unwtd_arpt91
  )
```

```
headcount_91
```

```
##          fgt0      SE
## eybhc0 0.20213 0.0062
confint( headcount_91 , df = degf( subset( z , year == 1991 ) ) )
```

```
##          2.5 % 97.5 %
## eybhc0 0.1899624 0.2143
```

Confirm this replication applies for the normalized poverty gap as well, comparing stata code..

```
. ge ngap = poor*($z_81- x)/$z_81 if year == 1981
```

```
. svy: mean ngap if year == 1981
(running mean on estimation sample)
```

Survey: Mean estimation

Number of strata =	1	Number of obs =	9772
Number of PSUs =	7476	Population size =	5.5e+07
		Design df =	7475

```
-----
          |          Linearized
          |          Mean   Std. Err.   [95% Conf. Interval]
```

```
-----+-----
ngap | .0271577 .0013502 .0245109 .0298044
-----
```

..to R code:

```
norm_pov_81 <-
  svyfgt(
    ~ eybhc0 ,
    subset( z , year == 1981 ) ,
    g = 1 ,
    abs_thresh = unwtd_arpt81
  )
```

```
norm_pov_81
```

```
##          fgt1      SE
## eybhc0 0.027158 0.0014
```

```
confint( norm_pov_81 , df = degf( subset( z , year == 1981 ) ) )
```

```
##          2.5 %      97.5 %
## eybhc0 0.02451106 0.02980428
```

For additional usage examples of `svyfgt`, type `?convey::svyfgt` in the R console.

here are the references

(Foster et al., 1984) and (Berger and Skinner, 2003)

Chapter 3

Inequality Measurement

Another problem faced by societies is inequality. Economic inequality can have several different meanings: income, education, resources, opportunities, wellbeing, etc. Usually, studies on economic inequality focus on income distribution.

Most inequality data comes from censuses and household surveys. Therefore, in order to produce reliable estimates from this samples, appropriate procedures are necessary.

This chapter presents brief presentations on inequality measures, also providing replication examples if possible. It starts with the Lorenz curve and inequality measures derived from it, then the concept of entropy and measures based on it are presented.

3.1 Lorenz Curve (svylorenz)

Though not an inequality measure in itself, the Lorenz curve is a classic instrument of distribution analysis. Basically, it is a function that associates a cumulative share of the population to the share of the total income it owns. In mathematical terms,

$$L(p) = \frac{\int_{-\infty}^{Q_p} yf(y)dy}{\int_{-\infty}^{+\infty} yf(y)dy}$$

where Q_p is the quantile p of the population.

The two extreme distributive cases are

- Perfect equality:
 - Every individual has the same income;
 - Every share of the population has the same share of the income;
 - Therefore, the reference curve is

$$L(p) = p \quad \forall p \in [0, 1].$$

- Perfect inequality:
 - One individual concentrates all of society's income, while the other individuals have zero income;
 - Therefore, the reference curve is

$$L(p) = \begin{cases} 0, & \forall p < 1 \\ 1, & \text{if } p = 1. \end{cases}$$

In order to evaluate the degree of inequality in a society, the analyst looks at the distance between the real curve and those two reference curves.

The estimator of this function was derived by (Kovacevic and Binder, 1997):

$$L(p) = \frac{\sum_{i \in S} w_i \cdot y_i \cdot \delta\{y_i \leq \hat{Q}_p\}}{\hat{Y}}, \quad 0 \leq p \leq 1.$$

Yet, this formula is used to calculate specific points of the curve and their respective SEs. The formula to plot an approximation of the continuous empirical curve comes from (Lerman and Yitzhaki, 1989).

A replication example

In October 2016, (Jann, 2016) released a pre-publication working paper to estimate lorenz and concentration curves using stata. The example below reproduces the statistics presented in his section 4.1.

```
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the stata-style webuse library
library(webuse)

# load the NLSW 1988 data
webuse("nlsw88")

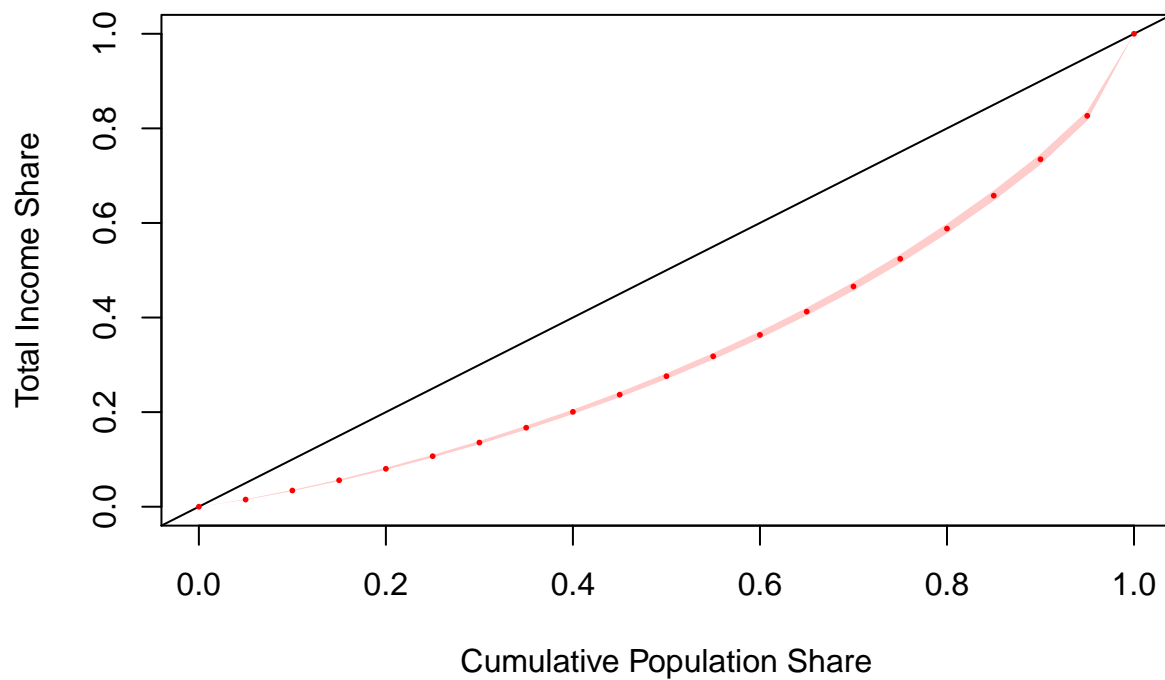
# coerce that `tbl_df` to a standard R `data.frame`
nlsw88 <- data.frame( nlsw88 )

# initiate a linearized survey design object
des_nlsw88 <- svydesign( ids = ~1 , data = nlsw88 )

## Warning in svydesign.default(ids = ~1, data = nlsw88): No weights or
## probabilities supplied, assuming equal probability

# immediately run the `convey_prep` function on the survey design
des_nlsw88 <- convey_prep(des_nlsw88)

# estimates lorenz curve
result.lin <- svylorenz( ~wage, des_nlsw88, quantiles = seq( 0, 1, .05 ), na.rm = T )
```

```
# note: most survey commands in R use Inf degrees of freedom by default
# stata generally uses the degrees of freedom of the survey design.
# therefore, while this extended syntax serves to prove a precise replication of stata
# it is generally not necessary.
section_four_one <-
  data.frame(
    estimate = coef( result.lin ) ,
    standard_error = SE( result.lin ) ,
    ci_lower_bound =
      coef( result.lin ) +
      SE( result.lin ) *
      qt( 0.025 , degf( subset( des_nlsw88 , !is.na( wage ) ) ) ) ,
    ci_upper_bound =
      coef( result.lin ) +
      SE( result.lin ) *
      qt( 0.975 , degf( subset( des_nlsw88 , !is.na( wage ) ) ) )
  )

knitr::kable(
  section_four_one , caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

For additional usage examples of `svylorenz`, type `?convey::svylorenz` in the R console.

Table 3.1: Here is a nice table!

	estimate	standard_error	ci_lower_bound	ci_upper_bound
0	0.0000000	0.0000000	0.0000000	0.0000000
0.05	0.0151060	0.0004159	0.0142904	0.0159216
0.1	0.0342651	0.0007021	0.0328882	0.0356420
0.15	0.0558635	0.0010096	0.0538836	0.0578434
0.2	0.0801846	0.0014032	0.0774329	0.0829363
0.25	0.1067687	0.0017315	0.1033732	0.1101642
0.3	0.1356307	0.0021301	0.1314535	0.1398078
0.35	0.1670287	0.0025182	0.1620903	0.1719670
0.4	0.2005501	0.0029161	0.1948315	0.2062687
0.45	0.2369209	0.0033267	0.2303971	0.2434447
0.5	0.2759734	0.0037423	0.2686347	0.2833121
0.55	0.3180215	0.0041626	0.3098585	0.3261844
0.6	0.3633071	0.0045833	0.3543192	0.3722950
0.65	0.4125183	0.0050056	0.4027021	0.4223345
0.7	0.4657641	0.0054137	0.4551478	0.4763804
0.75	0.5241784	0.0058003	0.5128039	0.5355529
0.8	0.5880894	0.0062464	0.5758401	0.6003388
0.85	0.6577051	0.0066148	0.6447333	0.6706769
0.9	0.7346412	0.0068289	0.7212497	0.7480328
0.95	0.8265786	0.0062686	0.8142857	0.8388715
1	1.0000000	0.0000000	1.0000000	1.0000000

3.2 Gini index (svygini)

The Gini index is an attempt to express the inequality presented in the Lorenz curve as a single number. In essence, it is twice the area between the equality curve and the real Lorenz curve. Put simply:

$$G = 2 \left(\int_0^1 p dp - \int_0^1 L(p) dp \right)$$

$$\therefore G = 1 - 2 \int_0^1 L(p) dp$$

where $G = 0$ in case of perfect equality and $G = 1$ in the case of perfect inequality.

The estimator proposed by (Osier, 2009) is defined as:

$$\hat{G} = \frac{2 \sum_{i \in S} w_i r_i y_i - \sum_{i \in S} w_i y_i}{\hat{Y}}$$

The linearized formula of \hat{G} is used to calculate the SE.

A replication example

The R `vardpoor` package (Breibaks et al., 2016), created by researchers at the Central Statistical Bureau of Latvia, includes a gini coefficient calculation using the ultimate cluster method. The example below reproduces those statistics.

Load and prepare the same data set:

```
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the vardpoor library
library(vardpoor)

# load the synthetic european union statistics on income & living conditions
data(eusilc)

# make all column names lowercase
names( eusilc ) <- tolower( names( eusilc ) )

# add a column with the row number
dati <- data.table(IDd = 1 : nrow(eusilc), eusilc)

# calculate the gini coefficient
# using the R vardpoor library
varpoord_gini_calculation <-
  vardpoord(

    # analysis variable
    Y = "eqincome",

    # weights variable
    w_final = "rb050",

    # row number variable
    ID_level1 = "IDd",

    # strata variable
    H = "db040",

    N_h = NULL ,

    # clustering variable
    PSU = "rb030",

    # data.table
    dataset = dati,

    # gini coefficient function
    type = "lingini"

  )
```

```
## NULL
```

```
# all calculations produced by vardpoor::lingini
varpoord_gini_calculation$all_result
```

```
##      type respondent_count n_nonzero pop_size      value value_eu      var
```

```
## 1: GINI          14827      14824  8182222 26.49652 26.48962 0.03783931
##           se           rse           cv absolute_margin_of_error
## 1: 0.1945233 0.007341467 0.7341467           0.3812587
##           relative_margin_of_error CI_lower CI_upper      S2_y_HT      S2_y_ca
## 1:           1.438901 26.11526 26.87778 8.120785e-12 8.120785e-12
##           S2_res var_srs_HT var_cur_HT var_srs_ca deff_sam deff_est
## 1: 8.120785e-12 0.03660155 0.03783931 0.03660155 1.033817      1
##           deff
## 1: 1.033817
```

```
# construct a survey.design
# using our recommended setup
des_eusilc <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,
    data = eusilc
  )

# immediately run the convey_prep function on it
des_eusilc <- convey_prep( des_eusilc )

# coefficients do match
varpoord_gini_calculation$all_result$value
```

```
## [1] 26.49652
```

```
coef( svygini( ~ eqincome , des_eusilc ) ) * 100
```

```
## eqincome
```

```
## 26.49652
```

```
# variances do not match exactly
attr( svygini( ~ eqincome , des_eusilc ) , 'var' ) * 10000
```

```
##           eqincome
```

```
## eqincome 0.03790739
```

```
varpoord_gini_calculation$all_result$var
```

```
## [1] 0.03783931
```

```
# standard errors do not match exactly
varpoord_gini_calculation$all_result$se
```

```
## [1] 0.1945233
```

```
SE( svygini( ~ eqincome , des_eusilc ) ) * 100
```

```
##           eqincome
```

```
## eqincome 0.1946982
```

By default, the `convey::svygini` function comes close to the results of `vardpoor::lingini`. However, the measures of uncertainty do not line up, because `library(vardpoor)` defaults to the ultimate cluster method. This can be replicated with an alternative setup of the `survey.design` object.

```
# within each strata, sum up the weights
cluster_sums <- aggregate( eusilc$rb050 , list( eusilc$db040 ) , sum )
```

```

# name the within-strata sums of weights the `cluster_sum`
names( cluster_sums ) <- c( "db040" , "cluster_sum" )

# merge this column back onto the data.frame
eusilc <- merge( eusilc , cluster_sums )

# construct a survey.design
# with the fpc using the cluster sum
des_eusilc_ultimate_cluster <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,
    data = eusilc ,
    fpc = ~ cluster_sum
  )

# again, immediately run the convey_prep function on the `survey.design`
des_eusilc_ultimate_cluster <- convey_prep( des_eusilc_ultimate_cluster )

# matches
attr( svygini( ~ eqincome , des_eusilc_ultimate_cluster ) , 'var' ) * 10000

##           eqincome
## eqincome 0.03783931
varpoord_gini_calculation$all_result$var

## [1] 0.03783931
# matches
varpoord_gini_calculation$all_result$se

## [1] 0.1945233
SE( svygini( ~ eqincome , des_eusilc_ultimate_cluster ) ) * 100

##           eqincome
## eqincome 0.1945233

```

For additional usage examples of `svygini`, type `?convey::svygini` in the R console.

3.3 Amato index (svyamato)

The Amato index is also based on the Lorenz curve, but instead of focusing on the area of the curve, it focuses on its length. (Arnold, 2012) proposes a formula not directly based in the Lorenz curve, which (Barabesi et al., 2016) uses to present the following estimator:

$$\hat{A} = \sum_{i \in S} w_i \left[\frac{1}{\widehat{N}^2} + \frac{y_i^2}{\widehat{Y}^2} \right]^{\frac{1}{2}},$$

which also generates the linearized formula for SE estimation.

The minimum value A assumes is $\sqrt{2}$ and the maximum is 2. In order to get a measure in the interval $[0, 1]$, the standardized Amato index \tilde{A} can be defined as:

$$\tilde{A} = \frac{A - \sqrt{2}}{2 - \sqrt{2}}.$$

For additional usage examples of `svyamato`, type `?convey::svyamato` in the R console.

3.4 Zenga Index and Curve (`svyzenga`, `svyzengacurve`)

The Zenga index and its curve were proposed in (Zenga, 2007). As (Polisicchio and Porro, 2011) noticed, this curve derives directly from the Lorenz curve, and can be defined as:

$$Z(p) = 1 - \frac{L(p)}{p} \cdot \frac{1 - p}{1 - L(p)}.$$

In the `convey` library, an experimental estimator based on the Lorenz curve is used:

$$\widehat{Z(p)} = \frac{p\hat{Y} - \hat{\hat{Y}}(p)}{p[\hat{Y} - \hat{\hat{Y}}(p)]}.$$

In turn, the Zenga index derives from this curve and is defined as:

$$Z = \int_0^1 Z(p) dp.$$

However, its estimators were proposed by (Langel, 2012) and (Barabesi et al., 2016). In this library, the latter is used and is defined as:

$$\hat{Z} = 1 - \sum_{i \in S} w_i \left[\frac{(\hat{N} - \hat{H}_{y_i})(\hat{Y} - \hat{K}_{y_i})}{\hat{N} \cdot \hat{H}_{y_i} \cdot \hat{K}_{y_i}} \right]$$

where \hat{N} is the population total, \hat{Y} is the total income, \hat{H}_{y_i} is the sum of incomes below or equal to y_i and \hat{N}_{y_i} is the sum of incomes greater or equal to y_i .

For additional usage examples of `svyzenga` or `svyzengacurve`, type `?convey::svyzenga` or `?convey::svyzengacurve` in the R console.

3.5 Entropy-based Measures

Entropy is a concept derived from information theory, meaning the expected amount of information given the occurrence of an event. Following (Shannon, 1948), given an event y with probability density function $f(\cdot)$, the information content given the occurrence of y can be defined as $g(f(y)) = -\log f(y)$. Therefore, the expected information or, put simply, the *entropy* is

$$H(f) = -E[\log f(y)] = -\int_{-\infty}^{\infty} f(y) \log f(y) dy$$

Assuming a discrete distribution, with p_k as the probability of occurring event $k \in K$, the entropy formula takes the form:

$$H = - \sum_{k \in K} p_k \log p_k.$$

The main idea behind it is that the expected amount of information of an event is inversely proportional to the probability of its occurrence. In other words, the information derived from the observation of a rare event is higher than of the information of more probable events.

Using the intuition presented in (Cowell et al., 2009), substituting the density function by the income share of an individual $s(q) = F^{-1}(q) / \int_0^1 F^{-1}(t)dt = y/\mu$, the entropy function becomes the Theil inequality index

$$I_{Theil} = \int_0^\infty \frac{y}{\mu} \log \left(\frac{y}{\mu} \right) dF(y) = -H(s)$$

Therefore, the entropy-based inequality measure increases as a person's income y deviates from the mean μ . This is the basic idea behind entropy-based inequality measures.

3.6 Generalized Entropy and Decomposition (svygei, svygeidec)

Using a generalization of the information function, now defined as $g(f) = \frac{1}{\alpha-1}[1 - f^{\alpha-1}]$, the α -class entropy is

$$H_\alpha(f) = \frac{1}{\alpha-1} \left[1 - \int_{-\infty}^\infty f(y)^{\alpha-1} f(y) dy \right].$$

This relates to a class of inequality measures, the Generalized entropy indices, defined as:

$$GE_\alpha = \frac{1}{\alpha^2 - \alpha} \int_0^\infty \left[\left(\frac{y}{\mu} \right)^\alpha - 1 \right] dF(x) = -\frac{H_\alpha(s)}{\alpha}.$$

The parameter α also has an economic interpretation: as α increases, the influence of top incomes upon the index increases. In some cases, this measure takes special forms, such as mean log deviation and the aforementioned Theil index.

In order to estimate it, (Biewen and Jenkins, 2003) proposed the following:

$$GE_\alpha = \begin{cases} (\alpha^2 - \alpha)^{-1} [U_0^{\alpha-1} U_1^{-\alpha} U_\alpha - 1], & \text{if } \alpha \in \mathbb{R} \setminus \{0, 1\} \\ -T_0 U_0^{-1} + \log(U_1/U_0), & \text{if } \alpha \rightarrow 0 \\ T_1 U_1^{-1} - \log(U_1/U_0), & \text{if } \alpha \rightarrow 1 \end{cases}$$

where $U_\gamma = \sum_{i \in S} w_i \cdot y_i^\gamma$ and $T_\gamma = \sum_{i \in S} w_i \cdot y_i^\gamma \cdot \log y_i$. since those are all functions of totals, the linearization of the indices are easily achieved using the theorems described in (Deville, 1999).

This class also has several desirable properties, such as additive decomposition. The additive decomposition allows to compare the effects of inequality within and between population groups on the population inequality. Put simply, an additive decomposable index allows for:

$$I_{Total} = I_{Between} + I_{Within}.$$

A replication example

In July 2006, (Jenkins, 2008) presented at the North American Stata Users' Group Meetings on the stata Generalized Entropy Index command. The example below reproduces those statistics.

Load and prepare the same data set:

```
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the foreign library
library(foreign)

# create a temporary file on the local disk
tf <- tempfile()

# store the location of the presentation file
presentation_zip <- "http://repec.org/nasug2006/nasug2006_jenkins.zip"

# download jenkins' presentation to the temporary file
download.file( presentation_zip , tf , mode = 'wb' )

# unzip the contents of the archive
presentation_files <- unzip( tf , exdir = tempdir() )

# load the institute for fiscal studies' 1981, 1985, and 1991 data.frame objects
x81 <- read.dta( grep( "ifs81" , presentation_files , value = TRUE ) )
x85 <- read.dta( grep( "ifs85" , presentation_files , value = TRUE ) )
x91 <- read.dta( grep( "ifs91" , presentation_files , value = TRUE ) )

# stack each of these three years of data into a single data.frame
x <- rbind( x81 , x85 , x91 )
```

Replicate the author's survey design statement from stata code..

```
. * account for clustering within HHs
. version 8: svyset [pweight = wgt], psu(hrn)
pweight is wgt
psu is hrn
construct an

.. into R code:
```

```
# initiate a linearized survey design object
y <- svydesign( ~ hrn , data = x , weights = ~ wgt )

# immediately run the `convey_prep` function on the survey design
z <- convey_prep( y )
```

Replicate the author's subset statement and each of his svygei results..

```
. svygei x if year == 1981
```

Warning: x has 20 values = 0. Not used in calculations

Complex survey estimates of Generalized Entropy inequality indices

```
pweight: wgt          Number of obs   = 9752
Strata: <one>         Number of strata = 1
PSU: hrn              Number of PSUs   = 7459
                      Population size  = 54766261
```

Index	Estimate	Std. Err.	z	P> z	[95% Conf. Interval]	
GE(-1)	.1902062	.02474921	7.69	0.000	.1416987	.2387138
MLD	.1142851	.00275138	41.54	0.000	.1088925	.1196777
Theil	.1116923	.00226489	49.31	0.000	.1072532	.1161314
GE(2)	.128793	.00330774	38.94	0.000	.1223099	.135276
GE(3)	.1739994	.00662015	26.28	0.000	.1610242	.1869747

..using R code:

```
z81 <- subset( z , year == 1981 )

svygei( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = -1 )
```

```
##          gei      SE
## eybhc0 0.19021 0.0247
```

```
svygei( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 0 )
```

```
##          gei      SE
## eybhc0 0.11429 0.0028
```

```
svygei( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) )
```

```
##          gei      SE
## eybhc0 0.11169 0.0023
```

```
svygei( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 2 )
```

```
##          gei      SE
## eybhc0 0.12879 0.0033
```

```
svygei( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 3 )
```

```
##          gei      SE
## eybhc0 0.174 0.0066
```

Confirm this replication applies for subsetted objects as well. Compare stata output..

```
. svygei x if year == 1985 & x >= 1
```

Complex survey estimates of Generalized Entropy inequality indices

```
pweight: wgt          Number of obs   = 8969
Strata: <one>         Number of strata = 1
PSU: hrn              Number of PSUs   = 6950
                      Population size  = 55042871
```

Index	Estimate	Std. Err.	z	P> z	[95% Conf. Interval]	
GE(-1)	.1602358	.00936931	17.10	0.000	.1418723	.1785993

MLD		.127616	.00332187	38.42	0.000	.1211052	.1341267
Theil		.1337177	.00406302	32.91	0.000	.1257543	.141681
GE(2)		.1676393	.00730057	22.96	0.000	.1533304	.1819481
GE(3)		.2609507	.01850689	14.10	0.000	.2246779	.2972235

..to R code:

```
z85 <- subset( z , year == 1985 )

svygei( ~ eybhc0 , subset( z85 , eybhc0 > 1 ) , epsilon = -1 )

##           gei      SE
## eybhc0 0.16024 0.0094

svygei( ~ eybhc0 , subset( z85 , eybhc0 > 1 ) , epsilon = 0 )

##           gei      SE
## eybhc0 0.12762 0.0033

svygei( ~ eybhc0 , subset( z85 , eybhc0 > 1 ) )

##           gei      SE
## eybhc0 0.13372 0.0041

svygei( ~ eybhc0 , subset( z85 , eybhc0 > 1 ) , epsilon = 2 )

##           gei      SE
## eybhc0 0.16764 0.0073

svygei( ~ eybhc0 , subset( z85 , eybhc0 > 1 ) , epsilon = 3 )

##           gei      SE
## eybhc0 0.26095 0.0185
```

For additional usage examples of `svygei` or `svygeidec`, type `?convey::svygei` or `?convey::svygeidec` in the R console.

3.7 Rényi Divergence (svyrenyi)

Another measure used in areas like ecology, statistics and information theory is Rényi divergence measure. Using the formula defined in (Langel, 2012), the estimator can be defined as:

$$\hat{R}_\alpha = \begin{cases} \frac{1}{\alpha-1} \log \left[\hat{N}^{\alpha-1} \sum_{i \in S} w_i \cdot \left(\frac{y_i}{\bar{Y}} \right)^\alpha \right], & \text{if } \alpha \neq 1, \\ \sum_{i \in S} \frac{w_i y_i}{\bar{Y}} \log \frac{\hat{N} y_i}{\bar{Y}}, & \text{if } \alpha = 1, \end{cases}$$

where α is a parameter with a similar economic interpretation to that of the GE_α index.

For additional usage examples of `svyrenyi`, type `?convey::svyrenyi` in the R console.

3.8 J-Divergence and Decomposition (svyjdiv, svyjdivdec)

Proposed by (Rohde, 2016), the J-divergence measure can be seen as the sum of GE_0 and GE_1 , satisfying axioms that, individually, those two indices do not. Using U_γ and T_γ functions defined in ??, the estimator can be defined as:

$$\hat{J} = \frac{1}{\hat{N}} \sum_{i \in S} w_i \left(\frac{y_i - \hat{\mu}}{\hat{\mu}} \right) \log \left(\frac{y_i}{\hat{\mu}} \right)$$

$$\therefore \hat{J} = \frac{\hat{T}_1}{\hat{U}_1} - \frac{\hat{T}_0}{\hat{U}_0}$$

Since it is a sum of two additive decomposable measures, J itself is decomposable.

For additional usage examples of `svyjdiv` or `svyjdivdec`, type `?convey::svyjdiv` or `?convey::svyjdivdec` in the R console.

3.9 Atkinson index (svyatk)

Although the original formula was proposed in (Atkinson, 1970), the estimator used here comes from (Biewen and Jenkins, 2003):

$$\hat{A}_\epsilon = \begin{cases} 1 - \hat{U}_0^{-\epsilon/(1-\epsilon)} \hat{U}_1^{-1} \hat{U}_{1-\epsilon}^{1/(1-\epsilon)}, & \text{if } \epsilon \in \mathbb{R}_+ \setminus \{1\} \\ 1 - \hat{U}_0 \hat{U}_0^{-1} \exp(\hat{T}_0 \hat{U}_0^{-1}), & \text{if } \epsilon \rightarrow 1 \end{cases}$$

The ϵ is an inequality aversion parameter: as it approaches infinity, more weight is given to incomes in bottom of the distribution.

A replication example

In July 2006, (Jenkins, 2008) presented at the North American Stata Users' Group Meetings on the stata Atkinson Index command. The example below reproduces those statistics.

Load and prepare the same data set:

```
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the foreign library
library(foreign)

# create a temporary file on the local disk
tf <- tempfile()

# store the location of the presentation file
presentation_zip <- "http://repec.org/nasug2006/nasug2006_jenkins.zip"

# download jenkins' presentation to the temporary file
download.file( presentation_zip , tf , mode = 'wb' )

# unzip the contents of the archive
presentation_files <- unzip( tf , exdir = tempdir() )

# load the institute for fiscal studies' 1981, 1985, and 1991 data.frame objects
x81 <- read.dta( grep( "ifs81" , presentation_files , value = TRUE ) )
```

```
x85 <- read.dta( grep( "ifs85" , presentation_files , value = TRUE ) )
x91 <- read.dta( grep( "ifs91" , presentation_files , value = TRUE ) )

# stack each of these three years of data into a single data.frame
x <- rbind( x81 , x85 , x91 )
```

Replicate the author's survey design statement from stata code..

```
. * account for clustering within HHs
. version 8: svyset [pweight = wgt], psu(hrn)
pweight is wgt
psu is hrn
construct an
```

.. into R code:

```
# initiate a linearized survey design object
y <- svydesign( ~ hrn , data = x , weights = ~ wgt )

# immediately run the `convey_prep` function on the survey design
z <- convey_prep( y )
```

Replicate the author's subset statement and each of his svyatk results with stata..

```
. svyatk x if year == 1981
```

Warning: x has 20 values = 0. Not used in calculations

Complex survey estimates of Atkinson inequality indices

```
pweight: wgt                      Number of obs    = 9752
Strata: <one>                     Number of strata = 1
PSU: hrn                          Number of PSUs   = 7459
                                   Population size  = 54766261
```

Index	Estimate	Std. Err.	z	P> z	[95% Conf. Interval]	
A(0.5)	.0543239	.00107583	50.49	0.000	.0522153	.0564324
A(1)	.1079964	.00245424	44.00	0.000	.1031862	.1128066
A(1.5)	.1701794	.0066943	25.42	0.000	.1570588	.1833
A(2)	.2755788	.02597608	10.61	0.000	.2246666	.326491
A(2.5)	.4992701	.06754311	7.39	0.000	.366888	.6316522

..using R code:

```
z81 <- subset( z , year == 1981 )

svyatk( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 0.5 )
```

```
##          atkinson      SE
## eybhc0 0.054324 0.0011
```

```
svyatk( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) )
```

```
##          atkinson      SE
## eybhc0    0.108 0.0025
```

```
svyatk( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 1.5 )
```

```
##          atkinson      SE
## eybhc0  0.17018 0.0067
```

```
svyatk( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 2 )
```

```
##          atkinson      SE
## eybhc0  0.27558 0.026
```

```
svyatk( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 2.5 )
```

```
##          atkinson      SE
## eybhc0  0.49927 0.0675
```

Confirm this replication applies for subsetted objects as well, comparing stata code..

```
. svyatk x if year == 1981 & x >= 1
```

Complex survey estimates of Atkinson inequality indices

```
pweight: wgt          Number of obs    = 9748
Strata: <one>          Number of strata = 1
PSU: hrn              Number of PSUs    = 7457
                     Population size    = 54744234
```

Index	Estimate	Std. Err.	z	P> z	[95% Conf. Interval]	
A(0.5)	.0540059	.00105011	51.43	0.000	.0519477	.0560641
A(1)	.1066082	.00223318	47.74	0.000	.1022313	.1109852
A(1.5)	.1638299	.00483069	33.91	0.000	.154362	.1732979
A(2)	.2443206	.01425258	17.14	0.000	.2163861	.2722552
A(2.5)	.394787	.04155221	9.50	0.000	.3133461	.4762278

..to R code:

```
z81_two <- subset( z , year == 1981 & eybhc0 > 1 )
```

```
svyatk( ~ eybhc0 , z81_two , epsilon = 0.5 )
```

```
##          atkinson      SE
## eybhc0  0.054006 0.0011
```

```
svyatk( ~ eybhc0 , z81_two )
```

```
##          atkinson      SE
## eybhc0  0.10661 0.0022
```

```
svyatk( ~ eybhc0 , z81_two , epsilon = 1.5 )
```

```
##          atkinson      SE
## eybhc0  0.16383 0.0048
```

```
svyatk( ~ eybhc0 , z81_two , epsilon = 2 )
```

```
##          atkinson      SE
## eybhc0  0.24432 0.0143
```

```
svyatk( ~ eybhc0 , z81_two , epsilon = 2.5 )
```

```
##          atkinson      SE  
## eybhc0  0.39479 0.0416
```

For additional usage examples of `svyatk`, type `?convey::svyatk` in the R console.

Chapter 4

Wellbeing Measures

djalmapessoa_look do any of the other functions need to be moved to this wellbeing chapter?

4.1 The Gender Pay Gap (svygpg)

here are the references

(Osier, 2009) and (Deville, 1999)

A replication example

The R `vardpoor` package (Breidaks et al., 2016), created by researchers at the Central Statistical Bureau of Latvia, includes a gpg coefficient calculation using the ultimate cluster method. The example below reproduces those statistics.

Load and prepare the same data set:

```
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the vardpoor library
library(vardpoor)

# load the synthetic european union statistics on income & living conditions
data(eusilc)

# make all column names lowercase
names( eusilc ) <- tolower( names( eusilc ) )

# coerce the gender variable to numeric 1 or 2
eusilc$one_two <- as.numeric( eusilc$rb090 == "female" ) + 1

# add a column with the row number
dati <- data.table(IDd = 1 : nrow(eusilc), eusilc)
```

```

# calculate the gpg coefficient
# using the R vardpoor library
varpoord_gpg_calculation <-
  varpoord(

    # analysis variable
    Y = "eqincome",

    # weights variable
    w_final = "rb050",

    # row number variable
    ID_level1 = "IDd",

    # strata variable
    H = "db040",

    N_h = NULL ,

    # clustering variable
    PSU = "rb030",

    # data.table
    dataset = dati,

    # gpg coefficient function
    type = "lingpg" ,

    # gender variable
    gender = "one_two"
  )

```

```
## NULL
```

```

# all calculations produced by vardpoor::lingpg
varpoord_gpg_calculation$all_result

```

```

##      type respondent_count n_nonzero pop_size      value value_eu      var
## 1:  GPG              14827      14824  8182222 7.645389      NA 0.6482346
##      se      rse      cv absolute_margin_of_error
## 1: 0.8051301 0.1053092 10.53092      1.578026
##      relative_margin_of_error CI_lower CI_upper      S2_y_HT      S2_y_ca
## 1:      20.64023 6.067363 9.223415 1.388432e-10 1.388432e-10
##      S2_res var_srs_HT var_cur_HT var_srs_ca deff_sam deff_est
## 1: 1.388432e-10 0.6257864 0.6482346 0.6257864 1.035872      1
##      deff
## 1: 1.035872

```

```

# construct a survey.design
# using our recommended setup
des_eusilc <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,

```



```

    data = eusilc
  )

# immediately run the convey_prep function on it
des_eusilc <- convey_prep( des_eusilc )

# coefficients do match
varpoord_gpg_calculation$all_result$value

## [1] 7.645389

coef( svygp( ~ eqincome , des_eusilc , sex = ~ rb090 ) ) * 100

## eqincome
## 7.645389

# variances do not match exactly
attr( svygp( ~ eqincome , des_eusilc , sex = ~ rb090 ) , 'var' ) * 10000

##          eqincome
## eqincome 0.6493911
varpoord_gpg_calculation$all_result$var

## [1] 0.6482346

# standard errors do not match exactly
varpoord_gpg_calculation$all_result$sse

## [1] 0.8051301

SE( svygp( ~ eqincome , des_eusilc , sex = ~ rb090 ) ) * 100

##          eqincome
## eqincome 0.8058481

```

By default, the `convey::svygp` function comes close to the results of `vardpoor::lingpg`. However, the measures of uncertainty do not line up, because `library(vardpoor)` defaults to the ultimate cluster method. This can be replicated with an alternative setup of the `survey.design` object.

```

# within each strata, sum up the weights
cluster_sums <- aggregate( eusilc$rb050 , list( eusilc$db040 ) , sum )

# name the within-strata sums of weights the `cluster_sum`
names( cluster_sums ) <- c( "db040" , "cluster_sum" )

# merge this column back onto the data.frame
eusilc <- merge( eusilc , cluster_sums )

# construct a survey.design
# with the fpc using the cluster sum
des_eusilc_ultimate_cluster <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,
    data = eusilc ,
    fpc = ~ cluster_sum
  )

```

```

)

# again, immediately run the convey_prep function on the `survey.design`
des_eusilc_ultimate_cluster <- convey_prep( des_eusilc_ultimate_cluster )

# matches
attr( "svyggp" ) ~ eqincome , des_eusilc_ultimate_cluster , sex = ~ rb090 ) , 'var' ) * 10000

##           eqincome
## eqincome 0.6482346
varpoord_gpg_calculation$all_result$var

## [1] 0.6482346

# matches
varpoord_gpg_calculation$all_result$se

## [1] 0.8051301
SE( svyggp( ~ eqincome , des_eusilc_ultimate_cluster , sex = ~ rb090 ) ) * 100

##           eqincome
## eqincome 0.8051301

```

For additional usage examples of `svyggp`, type `?convey::svyggp` in the R console.

4.2 Quintile Share Ratio (svyqsr)

here are the references

(Osier, 2009) and (Deville, 1999)

A replication example

The R `vardpoor` package (Bredaks et al., 2016), created by researchers at the Central Statistical Bureau of Latvia, includes a `qsr` coefficient calculation using the ultimate cluster method. The example below reproduces those statistics.

Load and prepare the same data set:

```

# load the convey package
library(convey)

# load the survey library
library(survey)

# load the vardpoor library
library(vardpoor)

# load the synthetic european union statistics on income & living conditions
data(eusilc)

# make all column names lowercase
names( eusilc ) <- tolower( names( eusilc ) )

```

```

# add a column with the row number
dati <- data.table(IDd = 1 : nrow(eusilc), eusilc)

# calculate the qsr coefficient
# using the R varpoor library
varpoord_qsr_calculation <-
  varpoord(

    # analysis variable
    Y = "eqincome",

    # weights variable
    w_final = "rb050",

    # row number variable
    ID_level1 = "IDd",

    # strata variable
    H = "db040",

    N_h = NULL ,

    # clustering variable
    PSU = "rb030",

    # data.table
    dataset = dati,

    # qsr coefficient function
    type = "linqsr"

  )

## NULL

# all calculations produced by varpoor::linqsr
varpoord_qsr_calculation$all_result

##      type respondent_count n_nonzero pop_size      value value_eu      var
## 1:   QSR             14827      14824  8182222 3.970004 3.971415 0.001807323
##      se      rse      cv absolute_margin_of_error
## 1: 0.04251263 0.01070846 1.070846             0.08332321
##      relative_margin_of_error CI_lower CI_upper      S2_y_HT      S2_y_ca
## 1:             2.098819 3.886681 4.053328 3.840034e-13 3.840034e-13
##      S2_res var_srs_HT var_cur_HT var_srs_ca deff_sam deff_est
## 1: 3.840034e-13 0.001730759 0.001807323 0.001730759 1.044238      1
##      deff
## 1: 1.044238

# construct a survey.design
# using our recommended setup
des_eusilc <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,

```

```

    weights = ~ rb050 ,
    data = eusilc
  )

# immediately run the convey_prep function on it
des_eusilc <- convey_prep( des_eusilc )

# coefficients do match
varpoord_qsr_calculation$all_result$value

## [1] 3.970004

coef( svyqsr( ~ eqincome , des_eusilc ) )

## eqincome
## 3.970004

# variances do not match exactly
attr( svyqsr( ~ eqincome , des_eusilc ) , 'var' )

##
## eqincome
## eqincome 0.001810537
varpoord_qsr_calculation$all_result$var

## [1] 0.001807323

# standard errors do not match exactly
varpoord_qsr_calculation$all_result$sse

## [1] 0.04251263

SE( svyqsr( ~ eqincome , des_eusilc ) )

##
## eqincome
## eqincome 0.04255041

```

By default, the `convey::svyqsr` function comes close to the results of `vardpoor::linqsr`. However, the measures of uncertainty do not line up, because `library(vardpoor)` defaults to the ultimate cluster method. This can be replicated with an alternative setup of the `survey.design` object.

```

# within each strata, sum up the weights
cluster_sums <- aggregate( eusilc$rb050 , list( eusilc$db040 ) , sum )

# name the within-strata sums of weights the `cluster_sum`
names( cluster_sums ) <- c( "db040" , "cluster_sum" )

# merge this column back onto the data.frame
eusilc <- merge( eusilc , cluster_sums )

# construct a survey.design
# with the fpc using the cluster sum
des_eusilc_ultimate_cluster <-
  svydesign(
    ids = ~ rb030 ,
    strata = ~ db040 ,
    weights = ~ rb050 ,
    data = eusilc ,

```

```

    fpc = ~ cluster_sum
  )

# again, immediately run the convey_prep function on the `survey.design`
des_eusilc_ultimate_cluster <- convey_prep( des_eusilc_ultimate_cluster )

# matches
attr( "svyqsr" ) = ~ eqincome , des_eusilc_ultimate_cluster ) , 'var' )

##           eqincome
## eqincome 0.001807323
varpoord_qsr_calculation$all_result$var

## [1] 0.001807323
# matches
varpoord_qsr_calculation$all_result$se

## [1] 0.04251263
SE( svyqsr( ~ eqincome , des_eusilc_ultimate_cluster ) )

##           eqincome
## eqincome 0.04251263

```

For additional usage examples of `svyqsr`, type `?convey::svyqsr` in the R console.

Chapter 5

Multidimensional Indices

Inequality and poverty can be seen as multidimensional concepts, combining several livelihood characteristics. Usual approaches take into account income, housing, sanitation, etc.

In order to transform these different measures from into meaningful numbers, economic theory builds on the idea of utility functions. Utility is a measure of well-being, assigning a “well-being score” to a vector of characteristics. Depending on the utility function, the analyst may allow for substitutions among characteristics: for instance, someone with a slightly lower income, but with access to sanitation, can have a higher wellbeing than someone with a higher income, but without access to sanitation. This depends on the set of weights given to the set of attributes.

Most measures below follow from this kind of two-step procedure: (1) estimating individual scores from an individual’s set of characteristics; then (2) aggregating those individual scores into a single measure for the population.

The following section will present a measure of multidimensional poverty and a measure of multidimensional inequality, describing the main aspects of the theory and estimation procedures of each.

5.1 Alkire-Foster Class and Decomposition (svyafc, svyafcdec)

This class of measures are defined in (Alkire and Foster, 2011), using what is called the “dual cutoff” approach. This method applies a cutoffs to define dimensional deprivations and another cutoff for multidimensional deprivation.

To analyze a population of n individuals across d achievement dimensions, the first step of the method is applying a FGT-like transformation to each dimension, defined as

$$g_{ij}^{\alpha} = \left(\frac{z_j - x_{ij}}{z_j} \right)^{\alpha}$$

where i is an observation index, j is a dimension index and α is an exponent weighting the deprivation intensity. If $\alpha = 0$, then g_{ij}^0 becomes a binary variable, assuming value 1 if person i is deprived in dimension j and 0 otherwise. The $n \times d$ matrix G^{α} will be referred to as *deprivation matrix*.

Each dimension receives a weight w_j , so that the weighted sum of multidimensional deprivation is the matrix multiplication of G^{α} by the $j \times 1$ vector $W = [w_j]$. The $n \times 1$ vector $C^{\alpha} = [c_i^{\alpha}]$ is the weighted sum of dimensional deprivation scores, i.e.,

$$c_i^\alpha = \sum_{j \in d} w_j g_{ij}^\alpha$$

The second cutoff is defining those considered to be multidimensionally poor. Assuming that $\sum_{j \in d} w_j = 1$, the multidimensional cutoff k belongs to the interval $(0, 1]$. If $c_i^0 \geq k$, then this person is considered multidimensionally poor. The *censored vector of deprivation sums* $C^\alpha(k)$ is defined as

$$C^\alpha(k) = \left[c_{ij}^\alpha \cdot \delta(c_{ij}^0 \geq k) \right],$$

where $\delta(A)$ is an indicator function, taking value 1 if condition A is true and 0 otherwise. If $k \geq \min w_j$, this is called the “union approach”, where a person is considered poor if she is poor in at least one dimension. On the other extreme, the “intersection approach” happens when $k = 1$, meaning that a person is considered poor if she is poor in all dimensions.

The average of vector $C^0(k)$ returns the multidimensional headcount ratio. For the multidimensional FGT class, a general measure can be defined as

$$M^\alpha = \frac{1}{n} \sum_{i \in n} \sum_{j \in d} w_j g_{ij}^\alpha(k), \alpha \geq 0,$$

where $g_{ij}^\alpha(k) = g_{ij}^\alpha \cdot \delta(c_i^0 \geq k)$.

For inferential purposes, since this variable is actually the average of scores $\sum_{j \in d} w_j g_{ij}^\alpha(k)$, the linearization is straightforward.

The Alkire-Foster index is both dimensional and subgroup decomposable. This way, it is possible to analyze how much each dimension or group contribute to the general result. The overall poverty measure can be seen as the weighted sum of each group’s poverty measure, as in the formula below:

$$M^\alpha = \sum_{l \in L} \frac{n_l}{n} M_l^\alpha$$

where l is one of L groups.

Also, the overall poverty index can be expressed across dimensions as

$$M^\alpha = \sum_{j \in d} w_j \left[\frac{1}{n} \sum_{i \in n} g_{ij}^\alpha(k) \right].$$

Since those functions are linear combinations of ratios and totals, it is also possible to calculate standard errors for such measures.

A replication example

In November 2015, Christopher Jindra presented at the Oxford Poverty and Human Development Initiative on the Alkire-Foster multidimensional poverty measure. His presentation can be viewed [here](#). The example below reproduces those statistics.

Load and prepare the same data set:


```

# load the convey package
library(convey)

# load the survey library
library(survey)

# load the stata-style webuse library
library(webuse)

# load the same microdata set used by Jindra in his presentation
webuse("nlsw88")

# coerce that `tbl_df` to a standard R `data.frame`
nlsw88 <- data.frame( nlsw88 )

# create a `collgrad` column
nlsw88$collgrad <-
  factor(
    as.numeric( nlsw88$collgrad ) ,
    label = c( 'not college grad' , 'college grad' ) ,
    ordered = TRUE
  )

# coerce `married` column to factor
nlsw88$married <-
  factor(
    nlsw88$married ,
    levels = 0:1 ,
    labels = c( "single" , "married" )
  )

# initiate a linearized survey design object
des_nlsw88 <- svydesign( ids = ~1 , data = nlsw88 )

# immediately run the `convey_prep` function on the survey design
des_nlsw88 <- convey_prep(des_nlsw88)

```

Replicate PDF page 9

```

page_nine <-
  svyafc(
    ~ wage + collgrad + hours ,
    design = des_nlsw88 ,
    cutoffs = list( 4, 'college grad' , 26 ) ,
    k = 1/3 , g = 0 ,
    na.rm = TRUE
  )

# MO and seMO
print( page_nine )

```

```

##      alkire-foster      SE
## [1,]      0.36991 0.0053

```

```

# H seH and A seA
print( attr( page_nine , "extra" ) )

##          coef          SE
## H 0.8082070 0.008316807
## A 0.4576895 0.004573443

Replicate PDF page 10
page_ten <- NULL

# loop through every poverty cutoff `k`
for( ks in seq( 0.1 , 1 , .1 ) ){

  this_ks <-
    svyafc(
      ~ wage + collgrad + hours ,
      design = des_nls88 ,
      cutoffs = list( 4 , 'college grad' , 26 ) ,
      k = ks ,
      g = 0 ,
      na.rm = TRUE
    )

  page_ten <-
    rbind(
      page_ten ,
      data.frame(
        k = ks ,
        MO = coef( this_ks ) ,
        seMO = SE( this_ks ) ,
        H = attr( this_ks , "extra" )[ 1 , 1 ] ,
        seH = attr( this_ks , "extra" )[ 1 , 2 ] ,
        A = attr( this_ks , "extra" )[ 2 , 1 ] ,
        seA = attr( this_ks , "extra" )[ 2 , 2 ]
      )
    )
}

```

Replicate PDF page 13

```

page_thirteen <- NULL

# loop through every poverty cutoff `k`
for( ks in c( 0.5 , 0.75 , 1 ) ){

  this_ks <-
    svyafc(
      ~ wage + collgrad + hours ,
      design = des_nls88 ,
      cutoffs = list( 4 , 'college grad' , 26 ) ,
      k = ks ,
      g = 0 ,
      dimw = c( 0.5 , 0.25 , 0.25 ) ,
      na.rm = TRUE
    )
}

```

Table 5.1: PDF Page 10 Replication

	k	MO	seMO	H	seH	A	seA
alkire-foster	0.1	0.3699078	0.0053059	0.8082070	0.0083168	0.4576895	0.0045734
alkire-foster1	0.2	0.3699078	0.0053059	0.8082070	0.0083168	0.4576895	0.0045734
alkire-foster2	0.3	0.3699078	0.0053059	0.8082070	0.0083168	0.4576895	0.0045734
alkire-foster3	0.4	0.1865894	0.0068123	0.2582516	0.0092455	0.7225101	0.0051745
alkire-foster4	0.5	0.1865894	0.0068123	0.2582516	0.0092455	0.7225101	0.0051745
alkire-foster5	0.6	0.1865894	0.0068123	0.2582516	0.0092455	0.7225101	0.0051745
alkire-foster6	0.7	0.0432649	0.0042978	0.0432649	0.0042978	1.0000000	0.0000000
alkire-foster7	0.8	0.0432649	0.0042978	0.0432649	0.0042978	1.0000000	0.0000000
alkire-foster8	0.9	0.0432649	0.0042978	0.0432649	0.0042978	1.0000000	0.0000000
alkire-foster9	1.0	0.0432649	0.0042978	0.0432649	0.0042978	1.0000000	0.0000000

Table 5.2: PDF Page 13 Replication

	k	MO	seMO	H	seH	A	seA
alkire-foster	0.50	0.1913470	0.0069137	0.2689563	0.0093668	0.7114428	0.0068474
alkire-foster1	0.75	0.1489741	0.0066918	0.1842105	0.0081889	0.8087167	0.0052160
alkire-foster2	1.00	0.0432649	0.0042978	0.0432649	0.0042978	1.0000000	0.0000000

```

    )

page_thirteen <-
  rbind(
    page_thirteen ,
    data.frame(
      k = ks ,
      MO = coef( this_ks ) ,
      seMO = SE( this_ks ) ,
      H = attr( this_ks , "extra" )[ 1 , 1 ] ,
      seH = attr( this_ks , "extra" )[ 1 , 2 ] ,
      A = attr( this_ks , "extra" )[ 2 , 1 ] ,
      seA = attr( this_ks , "extra" )[ 2 , 2 ]
    )
  )
}

```

Replicate PDF page 16

```

page_sixteen <- NULL

# loop through every alpha value `g`
for( gs in 0:3 ){

  this_gs <-
    svyafc(
      ~ wage + collgrad + hours ,
      design = des_nls88 ,
      cutoffs = list( 4, 'college grad' , 26 ) ,

```

Table 5.3: PDF Page 16 Replication

	g	MO	seMO
alkire-foster	0	0.3699078	0.0053059
alkire-foster1	1	0.2859332	0.0033708
alkire-foster2	2	0.2676266	0.0031164
alkire-foster3	3	0.2616335	0.0030531

```

k = 1/3 ,
g = gs ,
na.rm = TRUE
)

page_sixteen <-
  rbind(
    page_sixteen ,
    data.frame(
      g = gs ,
      MO = coef( this_gs ) ,
      seMO = SE( this_gs )
    )
  )
}

```

Replicate $k=1/3$ rows of PDF page 17 and 19

```

svyafcdec(
  ~ wage + collgrad + hours ,
  design = des_nls88 ,
  cutoffs = list( 4 , 'college grad' , 26 ) ,
  k = 1/3 ,
  g = 0 ,
  na.rm = TRUE
)

```

```

## $overall
##      alkire-foster      SE
## [1,]      0.36991 0.0053
##
## $`raw headcount ratio`
##      raw headcount      SE
## wage      0.19492 0.0084
## collgrad   0.76316 0.0090
## hours      0.15165 0.0076
##
## $`censored headcount ratio`
##      cens. headcount      SE
## wage      0.19492 0.0084
## collgrad   0.76316 0.0090
## hours      0.15165 0.0076
##
## $`percentual contribution per dimension`
##      dim. % contribution      SE

```

```
## wage          0.17564 0.0061
## collgrad      0.68770 0.0077
## hours         0.13666 0.0059
```

Replicate PDF pages 21 and 22

```
svyafdec(
  ~ wage + collgrad + hours ,
  subgroup = ~married ,
  design = des_nls88 ,
  cutoffs = list( 4 , 'college grad' , 26 ) ,
  k = 1/3 ,
  g = 0 ,
  na.rm = TRUE
)
```

```
## $overall
##      alkire-foster      SE
## [1,]      0.36991 0.0053
##
## $`raw headcount ratio`
##      raw headcount      SE
## wage          0.19492 0.0084
## collgrad      0.76316 0.0090
## hours         0.15165 0.0076
##
## $`censored headcount ratio`
##      cens. headcount      SE
## wage          0.19492 0.0084
## collgrad      0.76316 0.0090
## hours         0.15165 0.0076
##
## $`percentual contribution per dimension`
##      dim. % contribution      SE
## wage          0.17564 0.0061
## collgrad      0.68770 0.0077
## hours         0.13666 0.0059
##
## $`subgroup alkire-foster estimates`
##      alkire-foster      SE
## single        0.35414 0.0088
## married       0.37867 0.0066
##
## $`percentual contribution per subgroup`
##      grp. % contribution      SE
## single        0.34204 0.012
## married       0.65796 0.012
```

For additional usage examples of `svyafc` or `svyafdec`, type `?convey::svyafc` or `?convey::svyafdec` in the R console.

(Alkire and Foster, 2011) and (Sabina Alkire and Ballon, 2015) and (Pacifico and Poge, 2016)

5.2 Bourguignon-Chakravarty (2003) multidimensional poverty class

A class of poverty measures is proposed in (Bourguignon and Chakravarty, 2003), using a cross-dimensional function that assigns values to each set of dimensionally normalized poverty gaps. It can be defined as:

$$BCh = \sum_{i \in n} \left[\left(\sum_{j \in d} w_j x_{ij} \right)^{\frac{1}{\theta}} \right]^{\alpha}, \theta > 0, \alpha > 0$$

where x_{ij} being the normalized poverty gap of dimension j for observation i , w_j is the weight of dimension j , θ and α are parameters of the function.

The parameter θ is the elasticity of substitution between the normalized gaps. In another words, θ defines the order of the weighted generalized mean across achievement dimensions. For instance, when $\theta = 1$, the cross-dimensional aggregation becomes the weighted average of all dimensions. As θ increases, the importance of the individual's most deprived dimension increases. As (Maria Casilda Lasso de la Vega and Diez, 2009) points out, it also weights the inequality among deprivations. In its turn, α works as society's poverty-aversion measure parameter. In another words, as α increases, more weight is given to the most deprived individuals. Similar to θ , when $\alpha = 1$, BCh is the average of the weighted deprivation scores.

5.3 Bourguignon (1999) inequality class (svybmi)

For additional usage examples of `svybmi`, type `?convey::svybmi` in the R console.

(Bourguignon, 1999) and (Ana Lugo, 2007)

Bibliography

- Alkire, S. and Foster, J. (2011). Counting and multidimensional poverty measurement. *Journal of Public Economics*, 95(7-8):476–487.
- Ana Lugo, M. (2007). *Comparing Multidimensional Indices of Inequality: methods and application*, pages 213–236.
- Arnold, B. C. (2012). On the amato inequality index. *Statistics and Probability Letters*, 82(8):1504–1506.
- Atkinson, A. B. (1970). On the measurement of inequality. *Journal of Economic Theory*, 2(3):244–263.
- Barabesi, L., Diana, G., and Perri, P. F. (2016). Linearization of inequality indices in the design-based framework. *Statistics*, 50(5):1161–1172.
- Berger, Y. G. and Skinner, C. J. (2003). Variance estimation for a low income proportion. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 52(4):457–468.
- Biewen, M. and Jenkins, S. (2003). Estimation of generalized entropy and atkinson inequality indices from complex survey data. Discussion Papers of DIW Berlin 345, DIW Berlin, German Institute for Economic Research.
- Bourguignon, F. (1999). Comment to 'multidimensioned approaches to welfare analysis' by maasoumi, e. In Silber, J., editor, *Handbook of income inequality measurement*, chapter 15, pages 477–484. Kluwer Academic, London.
- Bourguignon, F. and Chakravarty, S. R. (2003). The measurement of multidimensional poverty. *The Journal of Economic Inequality*, 1.
- Breidaks, J., Liberts, M., and Ivanova, S. (2016). vardpoor: Estimation of indicators on social exclusion and poverty and its linearization, variance estimation. R package version 0.8.0.
- Cowell, F. A., Flachaire, E., and Bandyopadhyay, S. (2009). Goodness-of-fit: An economic approach. Economics Series Working Papers 444, University of Oxford, Department of Economics.
- Deville, J.-C. (1999). Variance estimation for complex statistics and estimators: linearization and residual techniques. *Survey Methodology*, 25(2):193–203.
- Foster, J., Greer, J., and Thorbecke, E. (1984). A class of decomposable poverty measures. *Econometrica*, 52(3):761–766.
- Jann, B. (2016). Estimating Lorenz and concentration curves in Stata. University of Bern Social Sciences Working Papers 15, University of Bern, Department of Social Sciences.
- Jenkins, S. (2008). Estimation and interpretation of measures of inequality, poverty, and social welfare using stata. North american stata users' group meetings 2006, Stata Users Group.
- Kovacevic, M. and Binder, D. (1997). Variance estimation for measures of income inequality and polarization - the estimating equations approach. *Journal of Official Statistics*, 13(1):41–58.

- Langel, M. (2012). *Measuring inequality in finite population sampling*. PhD thesis.
- Lerman, R. and Yitzhaki, S. (1989). Improving the accuracy of estimates of gini coefficients. *Journal of Econometrics*, 42(1):43–47.
- Lima, L. C. F. (2013). The Persistent Inequality in the Great Brazilian Cities: The Case of Brasília. MPRA Papers 50938, University of Brasília.
- Maria Casilda Lasso de la Vega, A. U. and Diez, H. (2009). The Bourguignon and Chakravarty multidimensional poverty family: A characterization. Technical report.
- Osier, G. (2009). Variance estimation for complex indicators of poverty and inequality. *Journal of the European Survey Research Association*, 3(3):167–195.
- Pacifico, D. and Poge, F. (2016). Mpi: Stata module to compute the alkire-foster multidimensional poverty measures and their decomposition by deprivation indicators and population sub-groups.
- Polisicchio, M. and Porro, F. (2011). A comparison between lorenz $l(p)$ curve and zenga $i(p)$ curve. *Statistica Applicata*, 21(3-4):289–301.
- Rohde, N. (2016). J-divergence measurements of economic inequality. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 179(3):847–870.
- Sabina Alkire, James Foster, S. S. M. E. S. J. M. R. and Ballon, P. (2015). *Multidimensional Poverty Measurement and Analysis*. Oxford University Press. ISBN 9780199689491.
- Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal*, 27(3):379–423.
- Zenga, M. (2007). Inequality curve and inequality index based on the ratios between lower and upper arithmetic means. *Statistica e Applicazioni*, 1(4):3–27.