Poverty and Inequality with Complex Survey Data

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Chapter 1

Introduction

The R convey library estimates measures of poverty, inequality, and wellbeing. There are two other R libraries covering this subject, vardpoor and laeken, however, only convey integrates seamlessly with the R survey package.

convey is free and open-source software that runs inside the R environment for statistical computing. Anyone can review and propose changes to the source code for this software. Readers are welcome to propose changes to this book as well.

1.1 Installation

In order to work with the **convey** library, you will need to have R running on your machine. If you have never used R before, you will need to install that software before **convey** can be accessed. Check out FlowingData for a concise list of resources for new R users. Once you have R loaded on your machine, you can install..

• the latest released version from CRAN with

```
install.packages("convey")
```

• the latest development version from github with

```
devtools::install_github("djalmapessoa/convey")
```

1.2 Complex surveys and statistical inference

In this book, we demonstrate how to measure poverty and income concentration in a population based on microdata collected from a complex survey sample. Most surveys administered by government agencies or larger research organizations utilize a sampling design that violates the assumption of simple random sampling (SRS), including:

- 1. Different units selection probabilities;
- 2. Clustering of units:
- 3. Stratification of clusters:
- 4. Reweighting to compensate for missing values and other adjustments.

Therefore, basic unweighted R commands such as mean() or glm() will not properly account for the weighting nor the measures of uncertainty (such as the confidence intervals) present in the dataset. For some examples of publicly-available complex survey data sets, see http://asdfree.com.

Unlike other software, the R convey package does not require that the user specify these parameters throughout the analysis. So long as the svydesign object or svrepdesign object has been constructed properly at the outset of the analysis, the convey package will incorporate the survey design automatically and produce statistics and variances that take the complex sample into account.

1.3 Usage Examples

In the following example, we've loaded the data set eusilc from the R libraries vardpoor and laeken.

```
library(vardpoor)
data(eusilc)
```

Next, we create an object of class survey.design using the function svydesign of the library survey:

```
library(survey)
des_eusilc <- svydesign(ids = ~rb030, strata =~db040, weights = ~rb050, data = eusilc)</pre>
```

Right after the creation of the design object des_eusilc, we should use the function convey_prep that adds an attribute to the survey design which saves information on the design object based upon the whole sample, needed to work with subset designs.

```
library(convey)
des_eusilc <- convey_prep( des_eusilc )</pre>
```

To estimate the at-risk-of-poverty rate, we use the function svyarpt:

```
svyarpr(~eqIncome, design=des_eusilc)
```

```
arpr SE eqIncome 0.14444 0.0028
```

To estimate the at-risk-of-poverty rate across domains defined by the variable db040 we use:

```
svyby(~eqIncome, by = ~db040, design = des_eusilc, FUN = svyarpr, deff = FALSE)
```

```
db040 eqIncome
                 Burgenland 0.1953984 0.017202243
Burgenland
Carinthia
                  Carinthia 0.1308627 0.010610622
Lower Austria Lower Austria 0.1384362 0.006517660
Salzburg
                   Salzburg 0.1378734 0.011579280
Styria
                     Styria 0.1437464 0.007452360
                      Tyrol 0.1530819 0.009880430
Tyrol
Upper Austria Upper Austria 0.1088977 0.005928336
                     Vienna 0.1723468 0.007682826
Vienna
Vorarlberg
                 Vorarlberg 0.1653731 0.013754670
```

Using the same data set, we estimate the quintile share ratio:

```
# for the whole population
svyqsr(~eqIncome, design=des_eusilc, alpha= .20)
```

```
qsr SE
eqIncome 3.97 0.0426

# for domains
svyby(~eqIncome, by = ~db040, design = des_eusilc,
    FUN = svyqsr, alpha= .20, deff = FALSE)
```

```
db040 eqIncome
                Burgenland 5.008486 0.32755685
Burgenland
Carinthia
                 Carinthia 3.562404 0.10909726
Lower Austria Lower Austria 3.824539 0.08783599
Salzburg
                  Salzburg 3.768393 0.17015086
                     Styria 3.464305 0.09364800
Styria
                     Tyrol 3.586046 0.13629739
Tyrol
Upper Austria Upper Austria 3.668289 0.09310624
Vienna
                     Vienna 4.654743 0.13135731
Vorarlberg
                Vorarlberg 4.366511 0.20532075
```

These functions can be used as S3 methods for the classes survey.design and svyrep.design.

Let's create a design object of class svyrep.design and run the function convey_prep on it:

```
des_eusilc_rep <- as.svrepdesign(des_eusilc, type = "bootstrap")
des_eusilc_rep <- convey_prep(des_eusilc_rep)</pre>
```

```
and then use the function svyarpr:
svyarpr(~eqIncome, design=des eusilc rep)
            arpr
                     SE
eqIncome 0.14444 0.0024
svyby(~eqIncome, by = ~db040, design = des_eusilc_rep, FUN = svyarpr, deff = FALSE)
                      db040 eqIncome se.eqIncome
Burgenland
                 Burgenland 0.1953984 0.017387961
Carinthia
                  Carinthia 0.1308627 0.008993253
Lower Austria Lower Austria 0.1384362 0.006213496
                   Salzburg 0.1378734 0.013240320
Salzburg
Styria
                     Styria 0.1437464 0.006841029
Tyrol
                      Tyrol 0.1530819 0.009765381
Upper Austria Upper Austria 0.1088977 0.006211232
Vienna
                     Vienna 0.1723468 0.006902121
Vorarlberg
                 Vorarlberg 0.1653731 0.013740925
```

The functions of the library convey are called in a similar way to the functions in library survey.

It is also possible to deal with missing values by using the argument na.rm.

djalmapessoa_look, where do these references go on this page? (Berger and Skinner, 2003) and (Osier, 2009) and (Deville, 1999)

1.4 Underlying Calculations

djalmapessoa_look, please describe the general purpose of linearization

In the convey library, there are some basic functions that produce the linearized variables needed to measure income concentration and poverty. For example, looking at the income variable in some complex survey dataset, the quantile of that income variable can be linearized by the function convey::svyiqalpha and the sum total below any quantile of the variable is linearized by the function convey::svyisq.

From the linearized variables of these basic estimates, it is possible by using rules of composition, valid for influence functions, to derive the influence function of more complex estimates. By definition the influence function is a Gateaux derivative and the rules rules of composition valid for Gateaux derivatives also hold for Influence Functions.

The following property of Gateaux derivatives was often used in the library convey. Let g be a differentiable function of m variables. Suppose we want to compute the influence function of the estimator $g(T_1, T_2, \ldots, T_m)$, knowing the Influence function of the estimators T_i , $i = 1, \ldots, m$. Then the following holds:

$$I(g(T_1, T_2, \dots, T_m)) = \sum_{i=1}^m \frac{\partial g}{\partial T_i} I(T_i)$$

In the library convey this rule is implemented by the function contrastinf which uses the R function derive to compute the formal partial derivatives $\frac{\partial g}{\partial T_i}$.

For example, suppose we want to linearize the Relative median poverty gap(rmpg), defined as the difference between the at-risk-of-poverty threshold (arpt) and the median of incomes less than the arpt relative to the arprt:

$$rmpg = \frac{arpt - medpoor}{arpt}$$

where medpoor is the median of incomes less than arpt.

Suppose we know how to linearize arpt and medpoor, then by applying the function contrastinf with

$$g(T_1, T_2) = \frac{(T_1 - T_2)}{T_1}$$

we linearize the rmpg.

1.5 The Variance Estimator

djalmapessoa_look please add references to this section

The variance of the estimator $T(\hat{M})$ can approximated by:

$$Var\left[T(\hat{M})\right] \cong var\left[\sum_{s} w_i z_i\right]$$

The linearized variable z is given by the derivative of the functional:

$$z_k = \lim_{t \to 0} \frac{T(M + t\delta_k) - T(M)}{t} = IT_k(M)$$

where, δ_k is the Dirac measure in k: $\delta_k(i) = 1$ if and only if i = k.

This derivative is called Influence Function and was introduced in the area of Robust Statistics.

1.6 Influence Functions

Some measures of poverty and income concentration are defined by non-differentiable functions so that it is not possible to use Taylor linearization to estimate their variances. An alternative is to use **Influence functions** as described in (Deville, 1999) and (Osier, 2009). The convey library implements this methodology to work with survey.design objects and also with svyrep.design objects.

Some examples of these measures are:

- At-risk-of-poverty threshold: $arpt = .60q_{.50}$ where $q_{.50}$ is the income median;
- At-risk-of-poverty rate $arpr = \frac{\sum_{U} 1(y_i \leq arpt)}{N}.100$
- Quintile share ratio

$$qsr = \frac{\sum_{U} 1(y_i > q_{.80})}{\sum_{U} 1(y_i \le q_{.20})}$$

• Gini coefficient $1+G=\frac{2\sum_{U}(r_i-1)y_i}{N\sum_{U}y_i}$ where r_i is the rank of y_i .

Note that it is not possible to use Taylor linearization for these measures because they depend on quantiles and the Gini is defined as a function of ranks. This could be done using the approach proposed by Deville (1999) based upon influence functions.

Let U be a population of size N and M be a measure that allocates mass one to the set composed by one unit, that is $M(i) = M_i = 1$ if $i \in U$ and M(i) = 0 if $i \notin U$

Now, a population parameter θ can be expressed as a functional of M $\theta = T(M)$

Examples of such parameters are:

- Total: $Y = \sum_{U} y_i = \sum_{U} y_i M_i = \int y dM = T(M)$
- Ratio of two totals: $R = \frac{Y}{X} = \frac{\int y dM}{\int x dM} = T(M)$
- Cumulative distribution function: $F(x) = \frac{\sum_{U} 1(y_i \le x)}{N} = \frac{\int 1(y \le x) dM}{\int dM} = T(M)$

To estimate these parameters from the sample, we replace the measure M by the estimated measure \hat{M} defined by: $\hat{M}(i) = \hat{M}_i = w_i$ if $i \in s$ and $\hat{M}(i) = 0$ if $i \notin s$.

The estimators of the population parameters can then be expressed as functional of the measure \hat{M} .

- Total: $\hat{Y} = T(\hat{M}) = \int y d\hat{M} = \sum_{s} w_i y_i$
- Ratio of totals: $\hat{R} = T(\hat{M}) = \frac{\int y d\hat{M}}{\int x d\hat{M}} = \frac{\sum_s w_i y_i}{\sum_s w_i x_i}$
- Cumulative distribution function: $\hat{F}(x) = T(\hat{M}) = \frac{\int 1(y \le x) d\hat{M}}{\int d\hat{M}} = \frac{\sum_s w_i 1(y_i \le x)}{\sum_s w_i}$

1.7 Influence Function Examples

• Total:

$$IT_k(M) = \lim_{t \to 0} \frac{T(M + t\delta_k) - T(M)}{t}$$

$$= \lim_{t \to 0} \frac{\int y \cdot d(M + t\delta_k) - \int y \cdot dM}{t}$$

$$= \lim_{t \to 0} \frac{\int y \cdot d(t\delta_k)}{t} = y_k$$

• Ratio of two totals:

$$IR_k(M) = I\left(\frac{U}{V}\right)_k(M) = \frac{V(M) \times IU_k(M) - U(M) \times IV_k(M)}{V(M)^2}$$
$$= \frac{Xy_k - Yx_k}{X^2} = \frac{1}{X}(y_k - Rx_k)$$

1.8 Examples of Linearization Using the Influence Function

• At-risk-of-poverty threshold:

$$arpt = 0.6 \times m$$

where m is the median income.

$$z_k = -\frac{0.6}{f(m)} \times \frac{1}{N} \times [I(y_k \le m - 0.5)]$$

• At-risk-of-poverty rate:

$$arpr = \frac{\sum_{U} I(y_i \le t)}{\sum_{U} w_i}.100$$

$$z_k = \frac{1}{N} [I(y_k \le t) - t] - \frac{0.6}{N} \times \frac{f(t)}{f(m)} [I(y_k \le m) - 0.5]$$

where:

N - population size;

t - at-risk-of-poverty threshold;

 y_k - income of person k;

m - median income;

f - income density function;

1.9 Replication Designs

 $djalmapessoa_look$, please describe how the software works differently on svrepdesign objects – as compared to svydesign objects

1.10 Decomposition

Some inequality and multidimensional poverty measures can be decomposed. As of December 2016, the decomposition methods in convey are limited to group decomposition.

For instance, the generalized entropy index can be decomposed into between and within group components. This sheds light on a very simple question: of the overall inequality, how much can be explained by inequalities between groups and within groups? Since this measure is additive decomposable, one can get estimates of the coefficients, SEs and covariance between components. For a more practical approach, see (Lima, 2013).

The Alkire-Foster class of multidimensional poverty indices can be decomposed by dimension and groups. This shows how much each group (or dimension) contribute to the overall poverty.

This technique can help understand where and who is more affected by inequality and poverty, contributing to more specific policy and economic analysis.

Chapter 2

Poverty Indices

2.1 At Risk of Poverty Ratio (svyarpr)

For additional usage examples of svyarpr, type ?convey::svyarpr in the R console. here are the references

(Osier, 2009) and (Deville, 1999)

2.2 At Risk of Poverty Threshold (svyarpt)

For additional usage examples of swyarpt, type ?convey::swyarpt in the R console.

here are the references

(Osier, 2009) and (Deville, 1999)

2.3 Relative Median Income Ratio (svyrmir)

For additional usage examples of svyrmir, type ?convey::svyrmir in the R console.

here are the references

(Osier, 2009) and (Deville, 1999)

2.4 Relative Median Poverty Gap (svyrmpg)

For additional usage examples of svyrmpg, type ?convey::svyrmpg in the R console.

here are the references

(Osier, 2009) and (Deville, 1999)

2.5 Median Income Below the At Risk of Poverty Threshold (svy-poormed)

For additional usage examples of svypoormed, type ?convey::svypoormed in the R console.

here are the references

(Osier, 2009) and (Deville, 1999)

2.6 Foster-Greer-Thorbecke class (svyfgt)

(Foster et al., 1984) proposed a family of indicators to measure poverty. This class of FGT measures, can be defined as

$$p = \frac{1}{N} \sum_{k \in U} h(y_k, \theta),$$

where

$$h(y_k, \theta) = \left[\frac{(\theta - y_k)}{\theta}\right]^{\gamma} \delta \{y_k \le \theta\},$$

where: θ is the poverty threshold; δ the indicator function that assigns value 1 if the condition $\{y_k \leq \theta\}$ is satisfied and 0 otherwise, and γ is a non-negative constant.

When $\gamma = 0$, p can be interpreted as the poverty headcount ratio, and for $\gamma \geq 1$, the weight of the income shortfall of the poor to a power γ , (Foster and all, 1984).

The poverty measure FGT is implemented in the library convey by the function svyfgt. The argument thresh type of this function defines the type of poverty threshold adopted. There are three possible choices:

- 1. abs fixed and given by the argument thresh value
- 2. relq a proportion of a quantile fixed by the argument proportion and the quantile is defined by the argument order.
- 3. relm a proportion of the mean fixed the argument proportion

The quantile and the mean involved in the definition of the threshold are estimated for the whole population. When $\gamma = 0$ and $\theta = .6*MED$ the measure is equal to the indicator arpr computed by the function svyarpr.

Next, we give some examples of the function svyfgt to estimate the values of the FGT poverty index.

Consider first the poverty threshold fixed ($\gamma = 0$) in the value 10000. The headcount ratio (FGT0) is

fgt0 SE eqIncome 0.11444 0.0027

The poverty gap (FGT1) ($\gamma = 1$) index for the poverty threshold fixed at the same value is

fgt1 SE eqIncome 0.032085 0.0011

here are the references

(Foster et al., 1984) and (Berger and Skinner, 2003)

To estimate the FGT0 with the poverty threshold fixed at 0.6*MED we fix the argument type_thresh="relq" and use the default values for percent and order:

```
svyfgt(~eqIncome, des_eusilc, g=0, type_thresh= "relq")

fgt0 SE
eqIncome 0.14444 0.0028
that matches the estimate obtained by
svyarpr(~eqIncome, design=des_eusilc, .5, .6)

arpr SE
eqIncome 0.14444 0.0028
To estimate the poverty gap(FGT1) with the poverty threshold equal to 0.6 * MEAN we use:
svyfgt(~eqIncome, des_eusilc, g=1, type_thresh= "relm")

fgt1 SE
eqIncome 0.051187 0.0011
For additional usage examples of svyfgt, type ?convey::svyfgt in the R console.
```

Chapter 3

Inequality Measurement

Another problem faced by societies is inequality. Economic inequality can have several different meanings: income, education, resources, opportunities, wellbeing, etc. Usually, studies on economic inequality focus on income distribution.

Most inequality data comes from censuses and household surveys. Therefore, in order to produce reliable estimates from this samples, appropriate procedures are necessary.

This chapter presents brief presentations on inequality measures, also providing replication examples if possible. It starts with the Lorenz curve and inequality measures derived from it, then the concept of entropy and measures based on it are presented.

3.1 Lorenz Curve (svylorenz)

Though not an inequality measure in itself, the Lorenz curve is a classic instrument of distribution analysis. Basically, it is a function that associates a cumulative share of the population to the share of the total income it owns. In mathematical terms,

$$L(p) = \frac{\int_{-\infty}^{Q_p} y f(y) dy}{\int_{-\infty}^{+\infty} y f(y) dy}$$

where Q_p is the quantile p of the population.

The two extreme distributive cases are

- Perfect equality:
 - Every individual has the same income;
 - Every share of the population has the same share of the income;
 - Therefore, the reference curve is

$$L(p) = p \ \forall p \in [0, 1].$$

- Perfect inequality:
 - One individual concentrates all of society's income, while the other individuals have zero income;
 - Therefore, the reference curve is

$$L(p) = \begin{cases} 0, & \forall p < 1 \\ 1, & \text{if } p = 1. \end{cases}$$

In order to evaluate the degree of inequality in a society, the analyst looks at the distance between the real curve and those two reference curves.

The estimator of this function was derived by (Kovacevic and Binder, 1997):

$$L(p) = \frac{\sum_{i \in S} w_i \cdot y_i \cdot \delta\{y_i \le \widehat{Q}_p\}}{\widehat{Y}}, \ 0 \le p \le 1.$$

Yet, this formula is used to calculate specific points of the curve and their respective SEs. The formula to plot an approximation of the continuous empirical curve comes from (Lerman and Yitzhaki, 1989).

A replication example

In October 2016, (Jann, 2016) released a pre-publication working paper to estimate lorenz and concentration curves using stata. The example below reproduces the statistics presented in his section 4.1.

```
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the stata-style webuse library
library(webuse)

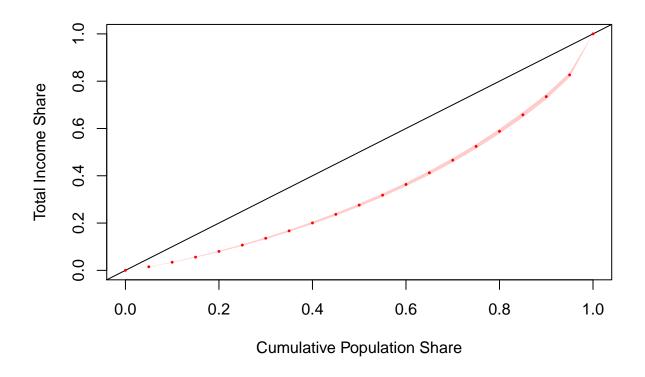
# load the NLSW 1988 data
webuse("nlsw88")

# coerce that `tbl_df` to a standard R `data.frame`
nlsw88 <- data.frame( nlsw88 )

# initiate a linearized survey design object
des_nlsw88 <- svydesign( ids = ~1 , data = nlsw88 )</pre>
```

```
## Warning in svydesign.default(ids = ~1, data = nlsw88): No weights or
## probabilities supplied, assuming equal probability
# immediately run the `convey_prep` function on the survey design
des_nlsw88 <- convey_prep(des_nlsw88)

# estimates lorenz curve
result.lin <- svylorenz( ~wage, des_nlsw88, quantiles = seq( 0, 1, .05 ), na.rm = T )</pre>
```



```
# note: most survey commands in R use Inf degrees of freedom by default
# stata generally uses the degrees of freedom of the survey design.
# therefore, while this extended syntax serves to prove a precise replication of stata
# it is generally not necessary.
section_four_one <-</pre>
    data.frame(
        estimate = coef( result.lin ) ,
        standard_error = SE( result.lin ) ,
        ci_lower_bound =
            coef( result.lin ) +
            SE( result.lin ) *
            qt( 0.025 , degf( subset( des_nlsw88 , !is.na( wage ) ) ) ) ,
        ci upper bound =
            coef( result.lin ) +
            SE( result.lin ) *
            qt( 0.975 , degf( subset( des_nlsw88 , !is.na( wage ) ) ) )
    )
knitr::kable(
  section_four_one , caption = 'Here is a nice table!',
  booktabs = TRUE
```

For additional usage examples of svylorenz, type ?convey::svylorenz in the R console.

	estimate	standard_error	ci_lower_bound	ci_upper_bound
0	0.0000000	0.0000000	0.0000000	0.0000000
0.05	0.0151060	0.0004159	0.0142904	0.0159216
0.1	0.0342651	0.0007021	0.0328882	0.0356420
0.15	0.0558635	0.0010096	0.0538836	0.0578434
0.2	0.0801846	0.0014032	0.0774329	0.0829363
0.25	0.1067687	0.0017315	0.1033732	0.1101642
0.3	0.1356307	0.0021301	0.1314535	0.1398078
0.35	0.1670287	0.0025182	0.1620903	0.1719670
0.4	0.2005501	0.0029161	0.1948315	0.2062687
0.45	0.2369209	0.0033267	0.2303971	0.2434447
0.5	0.2759734	0.0037423	0.2686347	0.2833121
0.55	0.3180215	0.0041626	0.3098585	0.3261844
0.6	0.3633071	0.0045833	0.3543192	0.3722950
0.65	0.4125183	0.0050056	0.4027021	0.4223345
0.7	0.4657641	0.0054137	0.4551478	0.4763804
0.75	0.5241784	0.0058003	0.5128039	0.5355529
0.8	0.5880894	0.0062464	0.5758401	0.6003388
0.85	0.6577051	0.0066148	0.6447333	0.6706769
0.9	0.7346412	0.0068289	0.7212497	0.7480328
0.95	0.8265786	0.0062686	0.8142857	0.8388715
1	1.0000000	0.0000000	1.0000000	1.0000000

Table 3.1: Here is a nice table!

3.2 Gini index (svygini)

The Gini index is an attempt to express the inequality presented in the Lorenz curve as a single number. In essence, it is twice the area between the equality curve and the real Lorenz curve. Put simply:

$$G = 2\left(\int_0^1 pdp - \int_0^1 L(p)dp\right)$$
$$\therefore G = 1 - 2\int_0^1 L(p)dp$$

where G = 0 in case of perfect equality and G = 1 in the case of perfect inequality.

The estimator proposed by (Osier, 2009) is defined as:

$$\widehat{G} = \frac{2\sum_{i \in S} w_i r_i y_i - \sum_{i \in S} w_i y_i}{\widehat{Y}}$$

The linearized formula of \widehat{G} is used to calculate the SE.

For additional usage examples of svygini, type ?convey::svygini in the R console.

3.3 Amato index (svyamato)

The Amato index is also based on the Lorenz curve, but instead of focusing on the area of the curve, it focuses on its length. (Arnold, 2012) proposes a formula not directly based in the Lorenz curve, which (Barabesi et al., 2016) uses to present the following estimator:

$$\widehat{A} = \sum_{i \in S} w_i \left[\frac{1}{\widehat{N}^2} + \frac{y_i^2}{\widehat{Y}^2} \right]^{\frac{1}{2}},$$

which also generates the linearized formula for SE estimation.

The minimum value A assumes is $\sqrt{2}$ and the maximum is 2. In order to get a measure in the interval [0,1], the standardized Amato index \widetilde{A} can be defined as:

$$\widetilde{A} = \frac{A - \sqrt{2}}{2 - \sqrt{2}} \ .$$

For additional usage examples of svyamato, type ?convey::svyamato in the R console.

3.4 Zenga Index and Curve (svyzenga, svyzengacurve)

The Zenga index and its curve were proposed in (Zenga, 2007). As (Polisicchio and Porro, 2011) noticed, this curve derives directly from the Lorenz curve, and can be defined as:

$$Z(p) = 1 - \frac{L(p)}{p} \cdot \frac{1-p}{1-L(p)}.$$

In the convey library, an experimental estimator based on the Lorenz curve is used:

$$\widehat{Z(p)} = \frac{p\widehat{Y} - \widehat{\widetilde{Y}}(p)}{p[\widehat{Y} - \widehat{\widetilde{Y}}(p)]}.$$

In turn, the Zenga index derives from this curve and is defined as:

$$Z = \int_0^1 Z(p)dp.$$

However, its estimators were proposed by (Langel, 2012) and (Barabesi et al., 2016). In this library, the latter is used and is defined as:

$$\widehat{Z} = 1 - \sum_{i \in S} w_i \left[\frac{(\widehat{N} - \widehat{H}_{y_i})(\widehat{Y} - \widehat{K}_{y_i})}{\widehat{N} \cdot \widehat{H}_{y_i} \cdot \widehat{K}_{y_i}} \right]$$

where \hat{N} is the population total, \hat{Y} is the total income, \hat{H}_{y_i} is the sum of incomes below or equal to y_i and \hat{N}_{y_i} is the sum of incomes greater or equal to y_i .

For additional usage examples of svyzenga or svyzengacurve, type ?convey::svyzenga or ?convey::svyzengacurve in the R console.

3.5 Entropy-based Measures

Entropy is a concept derived from information theory, meaning the expected amount of information given the occurrence of an event. Following (Shannon, 1948), given an event y with probability density function $f(\cdot)$, the information content given the occurrence of y can be defined as $g(f(y)) : = -\log f(y)$. Therefore, the expected information or, put simply, the *entropy* is

$$H(f)$$
: $= -E[\log f(y)] = -\int_{-\infty}^{\infty} f(y) \log f(y) dy$

Assuming a discrete distribution, with p_k as the probability of occurring event $k \in K$, the entropy formula takes the form:

$$H = -\sum_{k \in K} p_k \log p_k.$$

The main idea behind it is that the expected amount of information of an event is inversely proportional to the probability of its occurrence. In other words, the information derived from the observation of a rare event is higher than of the information of more probable events.

Using the intuition presented in (Cowell et al., 2009), substituting the density function by the income share of an individual $s(q) = F^{-1}(q) / \int_0^1 F^{-1}(t) dt = y/\mu$, the entropy function becomes the Theil inequality index

$$I_{Theil} = \int_0^\infty \frac{y}{\mu} \log\left(\frac{y}{\mu}\right) dF(y) = -H(s)$$

Therefore, the entropy-based inequality measure increases as a person's income y deviates from the mean μ . This is the basic idea behind entropy-based inequality measures.

3.6 Generalized Entropy and Decomposition (svygei, svygeidec)

Using a generalization of the information function, now defined as $g(f) = \frac{1}{\alpha - 1}[1 - f^{\alpha - 1}]$, the α -class entropy is

$$H_{\alpha}(f) = \frac{1}{\alpha - 1} \left[1 - \int_{-\infty}^{\infty} f(y)^{\alpha - 1} f(y) dy \right].$$

This relates to a class of inequality measures, the Generalized entropy indices, defined as:

$$GE_{\alpha} = \frac{1}{\alpha^2 - \alpha} \int_0^{\infty} \left[\left(\frac{y}{\mu} \right)^{\alpha} - 1 \right] dF(x) = -\frac{-H_{\alpha}(s)}{\alpha}.$$

The parameter α also has an economic interpretation: as α increases, the influence of top incomes upon the index increases. In some cases, this measure takes special forms, such as mean log deviation and the aforementioned Theil index.

In order to estimate it, (Biewen and Jenkins, 2003) proposed the following:

$$GE_{\alpha} = \begin{cases} (\alpha^{2} - \alpha)^{-1} \left[U_{0}^{\alpha - 1} U_{1}^{-\alpha} U_{\alpha} - 1 \right], & \text{if } \alpha \in \mathbb{R} \setminus \{0, 1\} \\ -T_{0} U_{0}^{-1} + \log(U_{1}/U_{0}), & \text{if } \alpha \to 0 \\ T_{1} U_{1}^{-1} - \log(U_{1}/U_{0}), & \text{if } \alpha \to 1 \end{cases}$$

where $U_{\gamma} = \sum_{i \in S} w_i \cdot y_i^{\gamma}$ and $T_{\gamma} = \sum_{i \in S} w_i \cdot y_i^{\gamma} \cdot \log y_i$. since those are all functions of totals, the linearization of the indices are easily achieved using the theorems described in (Deville, 1999).

This class also has several desirable properties, such as additive decomposition. The additive decomposition allows to compare the effects of inequality within and between population groups on the population inequality. Put simply, an additive decomposable index allows for:

$$I_{Total} = I_{Between} + I_{Within}.$$

A replication example

In July 2006, (Jenkins, 2008) presented at the North American Stata Users' Group Meetings on the stata Generalized Entropy Index command. The example below reproduces those statistics.

Load and prepare the same data set:

```
# load the convey package
library(convey)
# load the survey library
library(survey)
# load the foreign library
library(foreign)
# create a temporary file on the local disk
tf <- tempfile()</pre>
# store the location of the presentation file
presentation zip <- "http://repec.org/nasug2006/nasug2006 jenkins.zip"</pre>
# download jenkins' presentation to the temporary file
download.file( presentation_zip , tf , mode = 'wb' )
# unzip the contents of the archive
presentation_files <- unzip( tf , exdir = tempdir() )</pre>
# load the institute for fiscal studies' 1981, 1985, and 1991 data.frame objects
x81 <- read.dta( grep( "ifs81" , presentation_files , value = TRUE ) )
x85 <- read.dta( grep( "ifs85" , presentation_files , value = TRUE ) )
x91 <- read.dta( grep( "ifs91" , presentation files , value = TRUE ) )
# stack each of these three years of data into a single data.frame
x <- rbind( x81 , x85 , x91 )
```

Replicate the author's survey design statement from stata code..

```
. * account for clustering within HHs
. version 8: svyset [pweight = wgt], psu(hrn)
pweight is wgt
psu is hrn
construct an
.. into R code:
```

```
# initiate a linearized survey design object
y <- svydesign( ~ hrn , data = x , weights = ~ wgt )
# immediately run the `convey_prep` function on the survey design
z <- convey_prep( y )</pre>
Replicate the author's subset statement and each of his svygei results..
. svygei x if year == 1981
Warning: x has 20 values = 0. Not used in calculations
Complex survey estimates of Generalized Entropy inequality indices
                                         Number of obs
pweight: wgt
Strata: <one>
                                         Number of strata = 1
PSU: hrn
                                        Number of PSUs = 7459
                                        Population size = 54766261
______
                                     P>|z|
Index | Estimate Std. Err.
                               Z
                                                [95% Conf. Interval]
______
GE(-1) | .1902062 .02474921
                              7.69 0.000
                                               . 1416987
                                                          .2387138
                                               .1088925 .1196777
      | .1142851 .00275138 41.54 0.000
MLD
       | .1116923 .00226489 49.31 0.000
                                               .1072532 .1161314
Theil
GE(2) | .128793 .00330774 38.94 0.000
                                               .1223099 .135276
GE(3)
       1.1739994 .00662015
                               26.28 0.000
                                                .1610242 .1869747
..using R code:
z81 <- subset( z , year == 1981 )
svygei( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = -1 )
##
            gei
## eybhc0 0.19021 0.0247
svygei( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 0 )
                   SE
            gei
## eybhc0 0.11429 0.0028
svygei( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) )
##
            gei
## eybhc0 0.11169 0.0023
svygei( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 2 )
            gei
                   SE
## eybhc0 0.12879 0.0033
svygei( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 3 )
          gei
```

Confirm this replication applies for subsetted objects as well. Compare stata output..

eybhc0 0.174 0.0066

```
. svygei x if year == 1985 & x >= 1
```

Complex survey estimates of Generalized Entropy inequality indices

Index		Estimate	Std. Err.	z	P> z	[95% Conf.	Interval]
GE(-1)	Ċ	.1602358	.00936931	17.10	0.000	.1418723	.1785993
MLD		.127616	.00332187	38.42	0.000	.1211052	.1341267
Theil		.1337177	.00406302	32.91	0.000	.1257543	.141681
GE(2)		.1676393	.00730057	22.96	0.000	.1533304	.1819481
GE(3)		.2609507	.01850689	14.10	0.000	.2246779	.2972235

..to R code:

```
z85 \leftarrow subset(z, year == 1985)
svygei( \sim eybhc0 , subset( z85 , eybhc0 > 1 ) , epsilon = -1 )
                      SE
              gei
## eybhc0 0.16024 0.0094
svygei( ~ eybhc0 , subset( z85 , eybhc0 > 1 ) , epsilon = 0 )
##
              gei
                      SE
## eybhc0 0.12762 0.0033
svygei( ~ eybhc0 , subset( z85 , eybhc0 > 1 ) )
              gei
##
                       SE
## eybhc0 0.13372 0.0041
svygei( ~ eybhc0 , subset( z85 , eybhc0 > 1 ) , epsilon = 2 )
## eybhc0 0.16764 0.0073
svygei( \sim eybhc0 , subset( z85 , eybhc0 > 1 ) , epsilon = 3 )
##
              gei
                      SE
## eybhc0 0.26095 0.0185
```

For additional usage examples of svygei or svygeidec, type ?convey::svygei or ?convey::svygeidec in the R console.

3.7 Rényi Divergence (svyrenyi)

Another measure used in areas like ecology, statistics and information theory is Rényi divergence measure. Using the formula defined in (Langel, 2012), the estimator can be defined as:

$$\widehat{R}_{\alpha} = \begin{cases} \frac{1}{\alpha - 1} \log \left[\widehat{N}^{\alpha - 1} \sum_{i \in S} w_i \cdot \begin{pmatrix} y_k \\ \widehat{Y} \end{pmatrix} \right], & \text{if } \alpha \neq 1, \\ \sum_{i \in S} \frac{w_i y_i}{\widehat{Y}} \log \frac{\widehat{N} y_i}{\widehat{Y}}, & \text{if } \alpha = 1, \end{cases}$$

where α is a parameter with a similar economic interpretation to that of the GE_{α} index.

For additional usage examples of svyrenyi, type ?convey::svyrenyi in the R console.

3.8 J-Divergence and Decomposition (svyjdiv, svyjdivdec)

Proposed by (Rohde, 2016), the J-divergence measure can be seen as the sum of GE_0 and GE_1 , satisfying axioms that, individually, those two indices do not. Using U_{γ} and T_{γ} functions defined in ??, the estimator can be defined as:

$$\widehat{J} = \frac{1}{\widehat{N}} \sum_{i \in S} w_i \left(\frac{y_i - \widehat{\mu}}{\widehat{\mu}} \right) \log \left(\frac{y_i}{\widehat{\mu}} \right)$$

$$\therefore \widehat{J} = \frac{\widehat{T}_1}{\widehat{U}_1} - \frac{\widehat{T}_0}{\widehat{U}_0}$$

Since it is a sum of two additive decomposable measures, J itself is decomposable.

For additional usage examples of svyjdiv or svyjdivdec, type ?convey::svyjdiv or ?convey::svyjdivdec in the R console.

3.9 Atkinson index (svyatk)

Although the original formula was proposed in (Atkinson, 1970), the estimator used here comes from (Biewen and Jenkins, 2003):

$$\widehat{A}_{\epsilon} = \begin{cases} 1 - \widehat{U}_0^{-\epsilon/(1-\epsilon)} \widehat{U}_1^{-1} \widehat{U}_{1-\epsilon}^{1/(1-\epsilon)}, & \text{if } \epsilon \in \mathbb{R}_+ \setminus \{1\} \\ 1 - \widehat{U}_0 \widehat{U}_0^{-1} exp(\widehat{T}_0 \widehat{U}_0^{-1}), & \text{if } \epsilon \to 1 \end{cases}$$

The ϵ is an inequality aversion parameter: as it approaches infinity, more weight is given to incomes in bottom of the distribution.

A replication example

In July 2006, (Jenkins, 2008) presented at the North American Stata Users' Group Meetings on the stata Atkinson Index command. The example below reproduces those statistics.

Load and prepare the same data set:

```
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the foreign library
```

```
27
library(foreign)
# create a temporary file on the local disk
tf <- tempfile()</pre>
# store the location of the presentation file
presentation_zip <- "http://repec.org/nasug2006/nasug2006_jenkins.zip"</pre>
# download jenkins' presentation to the temporary file
download.file( presentation_zip , tf , mode = 'wb' )
# unzip the contents of the archive
presentation_files <- unzip( tf , exdir = tempdir() )</pre>
# load the institute for fiscal studies' 1981, 1985, and 1991 data.frame objects
x81 <- read.dta( grep( "ifs81" , presentation_files , value = TRUE ) )</pre>
x85 <- read.dta( grep( "ifs85" , presentation_files , value = TRUE ) )
x91 <- read.dta( grep( "ifs91" , presentation_files , value = TRUE ) )</pre>
# stack each of these three years of data into a single data.frame
x <- rbind( x81 , x85 , x91 )
Replicate the author's survey design statement from stata code..
. * account for clustering within HHs
. version 8: svyset [pweight = wgt], psu(hrn)
pweight is wgt
psu is hrn
construct an
.. into R code:
```

```
# initiate a linearized survey design object
y <- svydesign( ~ hrn , data = x , weights = ~ wgt )
# immediately run the `convey_prep` function on the survey design
z <- convey_prep( y )</pre>
```

Replicate the author's subset statement and each of his svyatk results with stata...

```
. svyatk x if year == 1981
Warning: x has 20 values = 0. Not used in calculations
Complex survey estimates of Atkinson inequality indices
pweight: wgt
                                         Number of obs = 9752
Strata: <one>
                                         Number of strata = 1
PSU: hrn
                                         Number of PSUs = 7459
                                        Population size = 54766261
Index | Estimate Std. Err. z P>|z| [95% Conf. Interval]
______
A(0.5) | .0543239 .00107583 50.49 0.000 .0522153 .0564324
A(1) | .1079964 .00245424 44.00 0.000 .1031862 .1128066
A(1.5) | .1701794 .0066943 25.42 0.000 .1570588 .1833
```

##

atkinson

SE

```
A(2)
      1 .2755788
                     .02597608
                                10.61
                                       0.000
                                                    .2246666
                                                               .326491
                                 7.39
A(2.5) | .4992701 .06754311
                                                     .366888
                                                             .6316522
                                          0.000
..using R code:
z81 \leftarrow subset(z, year == 1981)
svyatk( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 0.5 )
         atkinson
                     SE
## eybhc0 0.054324 0.0011
svyatk( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) )
##
         atkinson
                     SE
## eybhc0 0.108 0.0025
svyatk( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 1.5 )
         atkinson
                     SE
## eybhc0 0.17018 0.0067
svyatk( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 2 )
##
         atkinson
                    SE
## eybhc0 0.27558 0.026
svyatk( ~ eybhc0 , subset( z81 , eybhc0 > 0 ) , epsilon = 2.5 )
         atkinson
## eybhc0 0.49927 0.0675
Confirm this replication applies for subsetted objects as well, comparing stata code..
. svyatk x if year == 1981 & x >= 1
Complex survey estimates of Atkinson inequality indices
pweight: wgt
                                            Number of obs = 9748
Strata: <one>
                                           Number of strata = 1
PSU: hrn
                                           Number of PSUs = 7457
                                           Population size = 54744234
Index | Estimate Std. Err. z P>|z| [95% Conf. Interval]
A(0.5) | .0540059 .00105011 51.43 0.000
                                                  .0519477 .0560641
A(1) | .1066082 .00223318 47.74 0.000 .1022313 .1109852
A(1.5) | .1638299 .00483069 33.91 0.000
                                                   .154362 .1732979
A(2)
      .2443206 .01425258 17.14 0.000
                                                   .2163861 .2722552
A(2.5) |
                     .04155221
          .394787
                                 9.50 0.000
                                                    .3133461
                                                              .4762278
..to R code:
z81_two \leftarrow subset(z, year == 1981 \& eybhc0 > 1)
svyatk( ~ eybhc0 , z81_two , epsilon = 0.5 )
```

```
## eybhc0 0.054006 0.0011
svyatk( ~ eybhc0 , z81_two )
##
         atkinson
                      SE
## eybhc0 0.10661 0.0022
svyatk( ~ eybhc0 , z81_two , epsilon = 1.5 )
                    SE
         atkinson
## eybhc0 0.16383 0.0048
svyatk( ~ eybhc0 , z81_two , epsilon = 2 )
##
        atkinson
## eybhc0 0.24432 0.0143
svyatk( ~ eybhc0 , z81_two , epsilon = 2.5 )
         atkinson
                      SE
## eybhc0 0.39479 0.0416
```

For additional usage examples of svyatk, type ?convey::svyatk in the R console.

Chapter 4

Wellbeing Measures

djalmapessoa_look do any of the other functions need to be moved to this wellbeing chapter?

4.1 The Gender Pay Gap (svygpg)

For additional usage examples of svygpg, type ?convey::svygpg in the R console. here are the references (Osier, 2009) and (Deville, 1999)

4.2 Quintile Share Ratio (svyqsr)

For additional usage examples of svyqsr, type ?convey::svyqsr in the R console. here are the references (Osier, 2009) and (Deville, 1999)

Chapter 5

Multidimensional Indices

5.1 Alkire-Foster Class and Decomposition (svyafc, svyafcdec)

A replication example

In November 2015, Christopher Jindra presented at the Oxford Poverty and Human Development Initiative on the Alkire-Foster multidimensional poverty measure. His presentation can be viewed here. The example below reproduces those statistics.

Load and prepare the same data set:

```
# load the convey package
library(convey)
# load the survey library
library(survey)
# load the stata-style webuse library
library(webuse)
# load the same microdata set used by Jindra in his presentation
webuse("nlsw88")
# coerce that `tbl_df` to a standard R `data.frame`
nlsw88 <- data.frame( nlsw88 )</pre>
# create a `collgrad` column
nlsw88$collgrad <-
    factor(
        as.numeric( nlsw88$collgrad ) ,
        label = c( 'not college grad' , 'college grad' ) ,
        ordered = TRUE
# initiate a linearized survey design object
des_nlsw88 <- svydesign( ids = ~1 , data = nlsw88 )</pre>
```

```
\# immediately run the `convey_prep` function on the survey design
des_nlsw88 <- convey_prep(des_nlsw88)</pre>
Replicate PDF page 9
page_nine <-
 svyafc(
   ~ wage + collgrad + hours ,
   design = des_nlsw88 ,
   cutoffs = list( 4, 'college grad' , 26 ) ,
   k = 1/3, g = 0,
   na.rm = TRUE
 )
# MO and seMO
print( page_nine )
##
        alkire-foster
## [1,]
              0.36991 0.0053
# H seH and A seA
print( attr( page_nine , "extra" ) )
##
          coef
## H 0.8082070 0.008316807
## A 0.4576895 0.004573443
Replicate PDF page 10
page_ten <- NULL
# loop through every poverty cutoff `k`
for( ks in seq( 0.1 , 1 , .1 ) ){
   this ks <-
       svyafc(
            ~ wage + collgrad + hours ,
            design = des_nlsw88 ,
            cutoffs = list( 4 , 'college grad' , 26 ) ,
            k = ks,
            g = 0,
           na.rm = TRUE
   page_ten <-
       rbind(
            page_ten ,
            data.frame(
                k = ks,
                MO = coef( this_ks ) ,
                seMO = SE( this_ks ) ,
                H = attr( this_ks , "extra" )[ 1 , 1 ] ,
                seH = attr( this_ks , "extra" )[ 1 , 2 ] ,
                A = attr( this_ks , "extra" )[ 2 , 1 ] ,
                seA = attr( this_ks , "extra" )[ 2 , 2 ]
```

k	MO	seMO	Н	seH	A	seA
0.1	0.3699078	0.0053059	0.8082070	0.0083168	0.4576895	0.0045734
0.2	0.3699078	0.0053059	0.8082070	0.0083168	0.4576895	0.0045734
0.3	0.3699078	0.0053059	0.8082070	0.0083168	0.4576895	0.0045734
0.4	0.1865894	0.0068123	0.2582516	0.0092455	0.7225101	0.0051745
0.5	0.1865894	0.0068123	0.2582516	0.0092455	0.7225101	0.0051745
0.6	0.1865894	0.0068123	0.2582516	0.0092455	0.7225101	0.0051745
0.7	0.0432649	0.0042978	0.0432649	0.0042978	1.0000000	0.0000000
0.8	0.0432649	0.0042978	0.0432649	0.0042978	1.0000000	0.0000000
0.9	0.0432649	0.0042978	0.0432649	0.0042978	1.0000000	0.0000000
1.0	0.0432649	0.0042978	0.0432649	0.0042978	1.0000000	0.0000000

Table 5.1: Here is a nice table!

```
knitr::kable(
  page_ten , caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

still need to replicate PDF page 13

https://github.com/DjalmaPessoa/convey/issues/168

then keep going replicating this

https://github.com/DjalmaPessoa/convey/issues/154

For additional usage examples of svyafc or svyafcdec, type ?convey::svyafc or ?convey::svyafcdec in the R console.

(Alkire and Foster, 2011) and (Sabina Alkire and Ballon, 2015) and (Pacifico and Poge, 2016)

5.2 Bourguignon (1999) inequality class (svybmi)

For additional usage examples of svybmi, type ?convey::svybmi in the R console. (Bourguignon, 1999) and (Ana Lugo, 2007)

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