Poverty and Inequality with Complex Survey Data

Guilherme Jacob, Anthony Damico, and Djalma Pessoa

2016-12-09

Contents

Chapter 1

Introduction

This is a *sample* book written in **Markdown**. You can use anything that Pandoc's Markdown supports, e.g., a math equation $a^2 + b^2 = c^2$.

For now, you have to install the development version of **bookdown** from Github:

```
devtools::install_github("rstudio/bookdown")
```

Remember each Rmd file contains one and only one chapter, and a chapter is defined by the first-level heading `#`.

To compile this example to PDF, you need to install XeLaTeX.

The library convey aims at estimating measures of poverty and income concentration. There are already at least two libraries covering this subject: vardpoor and Laeken. The main difference between the library convey and these two is that the convey strongly hinges on the survey library.

1.1 Installation

`convey` is free and open-source software that runs inside the R environment for statistical computing.

- the latest released version from CRAN with

  ```
  install.packages("convey")
  ```

- the latest development version from github with

  ```
  devtools::install_github("djalmapessoa/convey")
  ```

[This may present how to install R, RStudio and required packages. Providing brief information about `survey` and `MonetDBLite` may also be recommended.]

You can label chapter and section titles using `{#label}` after them, e.g., we can reference Chapter 1.1. If you do not manually label them, there will be automatic labels anyway, e.g., Chapter 3.

Figures and tables with captions will be placed in `figure` and `table` environments, respectively.

```
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

Reference a figure by its code chunk label with the `fig:` prefix, e.g., see Figure 1.1. Similarly, you can reference tables generated from `knitr::kable()`, e.g., see Table 1.1.

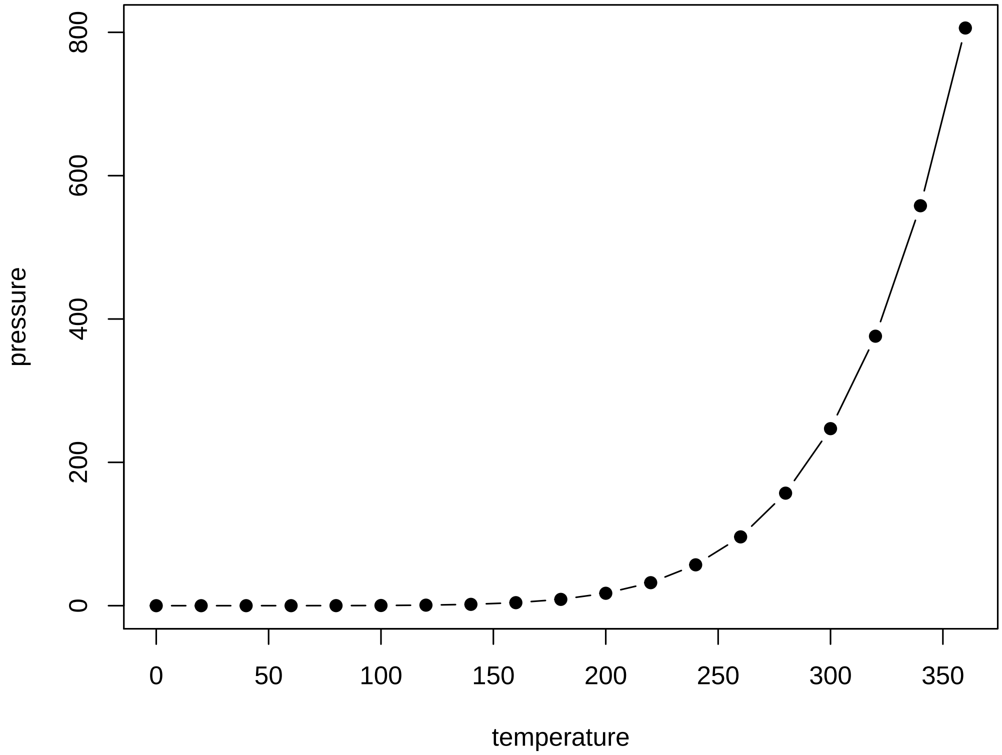

Figure 1.1: Here is a nice figure!

```
knitr::kable(
  head(iris, 20), caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

You can write citations, too. For example, we are using the **bookdown** package (Xie, 2016) in this sample book, which was built on top of R Markdown and **knitr** (Xie, 2015).

1.2 Complex surveys and statistical inference

[I think we should have a discussion about what is complex survey, its importance and so on. We can use a book Djalma wrote.]

1.3 Linearization

Some measures of poverty and income concentration are defined by non-differentiable functions so that it is not possible to use Taylor linearization to estimate their variances. An alternative is to use **Influence functions** as described in (Deville, 1999) and (Osier, 2009). The library convey implements this methodology to work with `survey.design` objects and also with `svyrep.design` objects.

Some examples of these measures are:

- At-risk-of-poverty threshold: $arpt = .60q_{.50}$ where $q_{.50}$ is the income median;
- At-risk-of-poverty rate $arpr = \frac{\sum_U 1(y_i \leq arpt)}{N}.100$

Table 1.1: Here is a nice table!

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa
4.6	3.4	1.4	0.3	setosa
5.0	3.4	1.5	0.2	setosa
4.4	2.9	1.4	0.2	setosa
4.9	3.1	1.5	0.1	setosa
5.4	3.7	1.5	0.2	setosa
4.8	3.4	1.6	0.2	setosa
4.8	3.0	1.4	0.1	setosa
4.3	3.0	1.1	0.1	setosa
5.8	4.0	1.2	0.2	setosa
5.7	4.4	1.5	0.4	setosa
5.4	3.9	1.3	0.4	setosa
5.1	3.5	1.4	0.3	setosa
5.7	3.8	1.7	0.3	setosa
5.1	3.8	1.5	0.3	setosa

- Quintile share ratio

$qsr = \frac{\sum_U 1(y_i > q_{.80})}{\sum_U 1(y_i \leq q_{.20})}$

- Gini coefficient $1 + G = \frac{2\sum_U (r_i - 1) y_i}{N \sum_U y_i}$ where r_i is the rank of y_i.

Note that it is not possible to use Taylor linearization for these measures because they depend on quantiles and the Gini is defined as a function of ranks. This could be done using the approach proposed by Deville (1999) based upon influence functions.

1.4 Influence function

Let U be a population of size N and M be a measure that allocates mass one to the set composed by one unit, that is $M(i) = M_i = 1$ if $i \in U$ and $M(i) = 0$ if $i \notin U$

Now, a population parameter θ can be expressed as a functional of M $\theta = T(M)$

Examples of such parameters are:

- Total: $Y = \sum_U y_i = \sum_U y_i M_i = \int y dM = T(M)$
- Ratio of two totals: $R = \frac{Y}{X} = \frac{\int y dM}{\int x dM} = T(M)$
- Cumulative distribution function: $F(x) = \frac{\sum_U 1(y_i \leq x)}{N} = \frac{\int 1(y \leq x) dM}{\int dM} = T(M)$

To estimate these parameters from the sample, we replace the measure M by the estimated measure $\hat{M}$ defined by: $\hat{M}(i) = \hat{M}_i = w_i$ if $i \in s$ and $\hat{M}(i) = 0$ if $i \notin s$.

The estimators of the population parameters can then be expressed as functional of the measure $\hat{M}$.

- Total: $\hat{Y} = T(\hat{M}) = \int y d\hat{M} = \sum_s w_i y_i$
- Ratio of totals: $\hat{R} = T(\hat{M}) = \frac{\int y d\hat{M}}{\int x d\hat{M}} = \frac{\sum_s w_i y_i}{\sum_s w_i x_i}$
- Cumulative distribution function: $\hat{F}(x) = T(\hat{M}) = \frac{\int 1(y \leq x) d\hat{M}}{\int d\hat{M}} = \frac{\sum_s w_i 1(y_i \leq x)}{\sum_s w_i}$

1.5 The variance estimator

The variance of the estimator $T(\hat{M})$ can approximated by:

$$Var\left[T(\hat{M})\right] \cong var\left[\sum_s w_i z_i\right]$$

The `linearized` variable z is given by the derivative of the functional:

$$z_k = lim_{t \to 0} \frac{T(M + t\delta_k) - T(M)}{t} = IT_k(M)$$

where, δ_k is the Dirac measure in k: $\delta_k(i) = 1$ if and only if $i = k$.

This **derivative** is called **Influence Function** and was introduced in the area of **Robust Statistics**.

1.6 Influence functions - Examples

- Total:

$$\begin{aligned} IT_k(M) &= lim_{t \to 0} \frac{T(M + t\delta_k) - T(M)}{t} \\ &= lim_{t \to 0} \frac{\int y.d(M + t\delta_k) - \int y.dM}{t} \\ &= lim_{t \to 0} \frac{\int yd(t\delta_k)}{t} = y_k \end{aligned}$$

- Ratio of two totals:

$$\begin{aligned} IR_k(M) = I\left(\frac{U}{V}\right)_k (M) &= \frac{V(M) \times IU_k(M) - U(M) \times IV_k(M)}{V(M)^2} \\ = \frac{Xy_k - Yx_k}{X^2} &= \frac{1}{X}(y_k - Rx_k) \end{aligned}$$

1.7 Linearization by influence function - Examples

- At-risk-of-poverty threshold:

$$arpt = 0.6 \times m$$

where m is the median income.

$$z_k = -\frac{0.6}{f(m)} \times \frac{1}{N} \times [I(y_k \leq m - 0.5)]$$

- At-risk-of-poverty rate:

$$arpr = \frac{\sum_U I(y_i \leq t)}{\sum_U w_i}.100$$

$$z_k = \frac{1}{N}\left[I(y_k \leq t) - t\right] - \frac{0.6}{N} \times \frac{f(t)}{f(m)}\left[I(y_k \leq m) - 0.5\right]$$

where:

N - population size;

t - at-risk-of-poverty threshold;

y_k - income of person k;

m - median income;

f - income density function;

1.8 Structure of the library

In the library convey, there are some basic functions that produces the linearized variables of some estimates that often enter in the definition of measures of concentration and poverty. For example the `quantile` which is linearized by the function `svyiqalpha`. Other example is the function `svyisq` that linearizes the total below a quantile of the variable.

From the linearized variables of these basic estimates it is possible by using rules of composition, valid for influence functions, to derive the influence function of more complex estimates. By definition the influence function is a Gateaux derivative and the rules rules of composition valid for Gateaux derivatives also hold for Influence Functions.

The following property of Gateaux derivatives was often used in the library convey. Let g be a differentible function of m variables. Suppose we want to compute the influence function of the estimator $g(T_1, T_2, \ldots, T_m)$, knowing the Influence function of the estimators $T_i, i = 1, \ldots, m$. Then the following holds:

$$I(g(T_1, T_2, \ldots, T_m)) = \sum_{i=1}^{m} \frac{\partial g}{\partial T_i} I(T_i)$$

In the library convey this rule is implemented by the function `contrastinf` which uses the R function `deriv` to compute the formal partial derivatives $\frac{\partial g}{\partial T_i}$.

For example, suppose we want to linearize the `Relative median poverty gap`(rmpg), defined as the difference between the at-risk-of-poverty threshold (`arpt`) and the median of incomes less than the `arpt` relative to the `arprt`:

$$rmpg = \frac{arpt - medpoor}{arpt}$$

where `medpoor` is the median of incomes less than `arpt`.

Suppose we know how to linearize `arpt` and `medpoor`, then by applying the function `contrastinf` with

$$g(T_1, T_2) = \frac{(T_1 - T_2)}{T_1}$$

we linearize the `rmpg`.

1.8.1 Examples of use of the library convey

In the following examples we will use the data set `eusilc` contained in the libraries `vardpoor` and `Laeken`.

```
library(vardpoor)
data(eusilc)
```

Next, we create an object of class `survey.design` using the function `svydesign` of the library survey:

```
library(survey)
des_eusilc <- svydesign(ids = ~rb030, strata =~db040,  weights = ~rb050, data = eusilc)
```

Right after the creation of the design object `des_eusilc`, we should use the function `convey_prep` that adds an attribute to the survey design which saves information on the design object based upon the whole sample, needed to work with subset designs.

```
library(convey)
des_eusilc <- convey_prep( des_eusilc )
```

To estimate the `at-risk-of-poverty rate` we use the function `svyarpt`:

```
svyarpr(~eqIncome, design=des_eusilc)
```

```
            arpr     SE
eqIncome 0.14444 0.0028
```

To estimate the `at-risk-of-poverty rate` for domains defined by the variable `db040` we use

```
svyby(~eqIncome, by = ~db040, design = des_eusilc, FUN = svyarpr, deff = FALSE)
```

```
                      db040  eqIncome          se
Burgenland       Burgenland 0.1953984 0.017202243
Carinthia         Carinthia 0.1308627 0.010610622
Lower Austria Lower Austria 0.1384362 0.006517660
Salzburg           Salzburg 0.1378734 0.011579280
Styria               Styria 0.1437464 0.007452360
Tyrol                 Tyrol 0.1530819 0.009880430
Upper Austria Upper Austria 0.1088977 0.005928336
Vienna               Vienna 0.1723468 0.007682826
Vorarlberg       Vorarlberg 0.1653731 0.013754670
```

Using the same data set, we estimate the `quintile share ratio`:

```
# for the whole population
svyqsr(~eqIncome, design=des_eusilc, alpha= .20)
```

```
          qsr     SE
eqIncome 3.97 0.0426
```

```
# for domains
svyby(~eqIncome, by = ~db040, design = des_eusilc,
  FUN = svyqsr, alpha= .20, deff = FALSE)
```

```
                      db040 eqIncome         se
Burgenland       Burgenland 5.008486 0.32755685
Carinthia         Carinthia 3.562404 0.10909726
Lower Austria Lower Austria 3.824539 0.08783599
Salzburg           Salzburg 3.768393 0.17015086
Styria               Styria 3.464305 0.09364800
Tyrol                 Tyrol 3.586046 0.13629739
Upper Austria Upper Austria 3.668289 0.09310624
Vienna               Vienna 4.654743 0.13135731
Vorarlberg       Vorarlberg 4.366511 0.20532075
```

These functions can be used as S3 methods for the classes `survey.design` and `svyrep.design`.

Let's create a design object of class `svyrep.design` and run the function `convey_prep` on it:

```
des_eusilc_rep <- as.svrepdesign(des_eusilc, type = "bootstrap")
des_eusilc_rep <- convey_prep(des_eusilc_rep)
```

and then use the function `svyarpr`:

```
svyarpr(~eqIncome, design=des_eusilc_rep)
```

```
            arpr     SE
eqIncome 0.14444 0.0025
```

```
svyby(~eqIncome, by = ~db040, design = des_eusilc_rep, FUN = svyarpr, deff = FALSE)
```

```
                      db040  eqIncome se.eqIncome
Burgenland       Burgenland 0.1953984 0.018190284
Carinthia         Carinthia 0.1308627 0.010957303
Lower Austria Lower Austria 0.1384362 0.005961997
Salzburg           Salzburg 0.1378734 0.013242928
Styria               Styria 0.1437464 0.007059837
Tyrol                 Tyrol 0.1530819 0.010066477
Upper Austria Upper Austria 0.1088977 0.005884765
Vienna               Vienna 0.1723468 0.007122467
Vorarlberg       Vorarlberg 0.1653731 0.013509364
```

The functions of the library convey are called in a similar way to the functions in library survey.

It is also possible to deal with missing values by using the argument `na.rm`.

```
# survey.design using a variable with missings
svygini( ~ py010n , design = des_eusilc )
```

```
       gini SE
py010n   NA NA
```

```
svygini( ~ py010n , design = des_eusilc , na.rm = TRUE )
```

```
          gini     SE
py010n 0.64606 0.0036
```

```
# svyrep.design using a variable with missings
# svygini( ~ py010n , design = des_eusilc_rep ) get error
svygini( ~ py010n , design = des_eusilc_rep , na.rm = TRUE )
```

```
          gini     SE
py010n 0.64606 0.0039
```

1.9 FGT indicator

(Foster et al., 1984) proposed a family of indicators to measure poverty.

The class of FGT measures, can be defined as

$$p = \frac{1}{N} \sum_{k \in U} h(y_k, \theta),$$

where

$$h(y_k, \theta) = \left[\frac{(\theta - y_k)}{\theta} \right]^\gamma \delta \left\{ y_k \leq \theta \right\},$$

where: θ is the poverty threshold; δ the indicator function that assigns value 1 if the condition $\{y_k \leq \theta\}$ is satisfied and 0 otherwise, and γ is a non-negative constant.

When $\gamma = 0$, p can be interpreted as the ratio of poor people, and for $\gamma \geq 1$, the weight of poor people increases with the value γ, (Foster and all, 1984).

The poverty measure FGT is implemented in the library convey by the function `svyfgt`. The argument `thresh_type` of this function defines the type of poverty threshold adopted. There are three possible choices:

1. `abs` – fixed and given by the argument thresh_value
2. `relq` – a proportion of a quantile fixed by the argument `proportion` and the quantile is defined by the argument `order`.
3. `relm` – a proportion of the mean fixed the argument `proportion`

The quantile and the mean involved in the definition of the threshold are estimated for the whole population. When $\gamma = 0$ and $\theta = .6 * MED$ the measure is equal to the indicator `arpr` computed by the function `svyarpr`.

Next, we give some examples of the function `svyfgt` to estimate the values of the FGT poverty index.

Consider first the poverty threshold fixed ($\gamma = 0$) in the value 10000. The headcount ratio (FGT0) is

```
svyfgt(~eqIncome, des_eusilc, g=0, abs_thresh=10000)
```

```
            fgt0     SE
eqIncome 0.11444 0.0027
```

The poverty gap (FGT1) ($\gamma = 1$) index for the poverty threshold fixed at the same value is

```
svyfgt(~eqIncome, des_eusilc, g=1, abs_thresh=10000)
```

```
             fgt1     SE
eqIncome 0.032085 0.0011
```

To estimate the FGT0 with the poverty threshold fixed at $0.6 * MED$ we fix the argument type_thresh="relq" and use the default values for `percent` and `order`:

```
svyfgt(~eqIncome, des_eusilc, g=0, type_thresh= "relq")
```

```
            fgt0     SE
eqIncome 0.14444 0.0028
```

that matches the estimate obtained by

```
svyarpr(~eqIncome, design=des_eusilc, .5, .6)
```

```
            arpr     SE
eqIncome 0.14444 0.0028
```

To estimate the poverty gap(FGT1) with the poverty threshold equal to $0.6 * MEAN$ we use:

```
svyfgt(~eqIncome, des_eusilc, g=1, type_thresh= "relm")
```

```
             fgt1     SE
eqIncome 0.051187 0.0011
```

djalma, where do these references go on this page? (Berger and Skinner, 2003) and (Osier, 2009) and (Deville, 1999)

Chapter 2

Poverty Indices

[I think this is a good start. I don't think that gender pay gap, quantiles and totals are measures of poverty. Consider another chapter on other wellbeing measures.] this is a test ## At Risk of Poverty Ratio and Threshold (svyarpr, svyarpt)

here are the references

(Osier, 2009) and (Deville, 1999)

2.1 The Gender Pay Gap (svygpg)

here are the references

(Osier, 2009) and (Deville, 1999)

2.2 Quintile Share Ratio (svyqsr)

here are the references

(Osier, 2009) and (Deville, 1999)

2.3 Relative Median Income Ratio (svyrmir)

here are the references

(Osier, 2009) and (Deville, 1999)

2.4 Relative Median Poverty Gap (svyrmpg)

here are the references

(Osier, 2009) and (Deville, 1999)

2.5 Median Income Below the At Risk of Poverty Threshold (svypoormed)

here are the references

(Osier, 2009) and (Deville, 1999)

2.6 Foster-Greer-Thorbecke class (svyfgt)

here are the references

(Foster et al., 1984) and (Berger and Skinner, 2003)

Chapter 3

Inequality Measurement

[Present an introduction to what is inequality].

3.1 Theoretical aspects of inequality

3.1.1 Desirable properties of inequality measures

3.2 Lorenz Curve (svylorenz)

here are the references

(Kovacevic and Binder, 1997) and (Lerman and Yitzhaki, 1989) and (Langel, 2012)

3.3 Measures derived from the Lorenz Curve

3.3.1 Gini index (svygini)

here are the references

(Osier, 2009) and (Deville, 1999)

3.3.2 Amato index (svyamato)

here are the references

(Barabesi et al., 2016) and (Arnold, 2012)

3.3.3 Zenga Index and Curve (svyzenga, svyzengacurve)

svyzenga: (Barabesi et al., 2016) and (Langel, 2012) and (Deville, 1999)

svyzengacurve: (Barabesi et al., 2016) and (Langel, 2012) and (Deville, 1999) and (Polisicchio and Porro, 2011)

3.4 Entropy-based Measures

3.4.1 Atkinson index (svyatk)

here are the references

(Langel, 2012) and (Biewen and Jenkins, 2003)

3.4.2 Generalized Entropy and Decomposition (svygei, svygeidec)

guilherme..this has three references? not just two?

here are the references

(Langel, 2012) and (Biewen and Jenkins, 2003) and (Shorrocks, 1984)

3.4.3 J-Divergence Entropy and Decomposition (svyjdiv, svyjdivdec)

here are the references

(Shorrocks, 1984) and (Rohde, 2016) and (Biewen and Jenkins, 2003)

3.4.4 Rényi Divergence (svyrenyi)

here are the references

(Langel, 2012)

Chapter 4

Multidimensional Indices

4.1 Alkire-Foster Class and Decomposition (svyafc, svyafcdec)

In November 2015, Christopher Jindra presented at the Oxford Poverty and Human Development Initiative on the Alkire-Foster multidimensional poverty measure. His presentation can be viewed here. The example below reproduces those statistics.

Load and prepare the same data set:

```r
# load the convey package
library(convey)

# load the survey library
library(survey)

# load the stata-style webuse library
library(webuse)

# load the same microdata set used by Jindra in his presentation
webuse("nlsw88")

# coerce that `tbl_df` to a standard R `data.frame`
nlsw88 <- data.frame( nlsw88 )

# create a `collgrad` column
nlsw88$collgrad <-
    factor(
        as.numeric( nlsw88$collgrad ) ,
        label = c( 'not college grad' , 'college grad' ) ,
        ordered = TRUE
      )

# initiate a linearized survey design object
des_nlsw88 <- svydesign( ids = ~1 , data = nlsw88 )

# immediately run the `convey_prep` function on the survey design
des_nlsw88 <- convey_prep(des_nlsw88)
```

Replicate PDF page 9

```
page_nine <-
  svyafc(
    ~ wage + collgrad + hours ,
    design = des_nlsw88 ,
    cutoffs = list( 4, 'college grad' , 26 ) ,
    k = 1/3 , g = 0 ,
    na.rm = TRUE
  )

# M0 and seM0
print( page_nine )
```

```
##      alkire-foster     SE
## [1,]       0.36991 0.0053
```

```
# H seH and A seA
print( attr( page_nine , "extra" ) )
```

```
##        coef          SE
## H 0.8082070 0.008316807
## A 0.4576895 0.004573443
```

Replicate PDF page 10

```
page_ten <- NULL

# loop through every poverty cutoff `k`
for( ks in seq( 0.1 , 1 , .1 ) ){

    this_ks <-
        svyafc(
            ~ wage + collgrad + hours ,
            design = des_nlsw88 ,
            cutoffs = list( 4 , 'college grad' , 26 ) ,
            k = ks ,
            g = 0 ,
            na.rm = TRUE
           )

    page_ten <-
        rbind(
            page_ten ,
            data.frame(
                k = ks ,
                M0 = coef( this_ks ) ,
                seM0 = SE( this_ks ) ,
                H = attr( this_ks , "extra" )[ 1 , 1 ] ,
                seH = attr( this_ks , "extra" )[ 1 , 2 ] ,
                A = attr( this_ks , "extra" )[ 2 , 1 ] ,
                seA = attr( this_ks , "extra" )[ 2 , 2 ]
          )
        )

}
```

```
print( page_ten )
```

```
##      k         MO        seMO          H         seH         A         seA
## 1  0.1 0.36990782 0.005305855 0.80820696 0.008316807 0.4576895 0.004573443
## 2  0.2 0.36990782 0.005305855 0.80820696 0.008316807 0.4576895 0.004573443
## 3  0.3 0.36990782 0.005305855 0.80820696 0.008316807 0.4576895 0.004573443
## 4  0.4 0.18658935 0.006812294 0.25825156 0.009245464 0.7225101 0.005174483
## 5  0.5 0.18658935 0.006812294 0.25825156 0.009245464 0.7225101 0.005174483
## 6  0.6 0.18658935 0.006812294 0.25825156 0.009245464 0.7225101 0.005174483
## 7  0.7 0.04326494 0.004297766 0.04326494 0.004297766 1.0000000 0.000000000
## 8  0.8 0.04326494 0.004297766 0.04326494 0.004297766 1.0000000 0.000000000
## 9  0.9 0.04326494 0.004297766 0.04326494 0.004297766 1.0000000 0.000000000
## 10 1.0 0.04326494 0.004297766 0.04326494 0.004297766 1.0000000 0.000000000
```

still need to replicate PDF page 13

https://github.com/DjalmaPessoa/convey/issues/168

then keep going replicating this

https://github.com/DjalmaPessoa/convey/issues/154

(Alkire and Foster, 2011) and (Sabina Alkire and Ballon, 2015) and (Pacifico and Poge, 2016)

4.2 Bourguignon (1999) inequality class (svybmi)

(Bourguignon, 1999) and (Ana Lugo, 2007)

Bibliography

Alkire, S. and Foster, J. (2011). Counting and multidimensional poverty measurement. *Journal of Public Economics*, 95(7-8):476–487.

Ana Lugo, M. (2007). *Comparing Multidimensional Indices of Inequality: methods and application*, pages 213–236.

Arnold, B. C. (2012). On the amato inequality index. *Statistics and Probability Letters*, 82(8):1504–1506.

Barabesi, L., Diana, G., and Perri, P. F. (2016). Linearization of inequality indices in the design-based framework. *Statistics*, 50(5):1161–1172.

Berger, Y. G. and Skinner, C. J. (2003). Variance estimation for a low income proportion. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 52(4):457–468.

Biewen, M. and Jenkins, S. (2003). Estimation of generalized entropy and atkinson inequality indices from complex survey data. Discussion Papers of DIW Berlin 345, DIW Berlin, German Institute for Economic Research.

Bourguignon, F. (1999). Comment to 'multidimensioned approaches to welfare analysis' by maasoumi, e. In Silber, J., editor, *Handbook of income inequality measurement*, chapter 15, pages 477–484. Kluwer Academic, London.

Deville, J.-C. (1999). Variance estimation for complex statistics and estimators: linearization and residual techniques. *Survey Methodology*, 25(2):193–203.

Foster, J., Greer, J., and Thorbecke, E. (1984). A class of decomposable poverty measures. *Econometrica*, 52(3):761–766.

Kovacevic, M. and Binder, D. (1997). Variance estimation for measures of income inequality and polarization - the estimating equations approach. *Journal of Official Statistics*, 13(1):41–58.

Langel, M. (2012). *Measuring inequality in finite population sampling*. PhD thesis.

Lerman, R. and Yitzhaki, S. (1989). Improving the accuracy of estimates of gini coefficients. *Journal of Econometrics*, 42(1):43–47.

Osier, G. (2009). Variance estimation for complex indicators of poverty and inequality. *Journal of the European Survey Research Association*, 3(3):167–195.

Pacifico, D. and Poge, F. (2016). Mpi: Stata module to compute the alkire-foster multidimensional poverty measures and their decomposition by deprivation indicators and population sub-groups.

Polisicchio, M. and Porro, F. (2011). A comparison between lorenz l(p) curve and zenga i(p) curve. *Statistica Applicata*, 21(3-4):289–301.

Rohde, N. (2016). J-divergence measurements of economic inequality. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 179(3):847–870.

Sabina Alkire, James Foster, S. S. M. E. S. J. M. R. and Ballon, P. (2015). *Multidimensional Poverty Measurement and Analysis*. Oxford University Press. ISBN 9780199689491.

Shorrocks, A. F. (1984). Inequality decomposition by population subgroups. *Econometrica*, 52(6):1369–1385.

Xie, Y. (2015). *Dynamic Documents with R and knitr*. Chapman and Hall/CRC, Boca Raton, Florida, 2nd edition. ISBN 978-1498716963.

Xie, Y. (2016). *bookdown: Authoring Books and Technical Documents with R Markdown*. R package version 0.3.2.