Poverty and Inequality with Complex Survey Data

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Contents

Chapter 1

Introduction

This is a *sample* book written in **Markdown**. You can use anything that Pandoc's Markdown supports, e.g., a math equation $a^2 + b^2 = c^2$.

For now, you have to install the development version of **bookdown** from Github:

```
devtools::install_github("rstudio/bookdown")
```

Remember each Rmd file contains one and only one chapter, and a chapter is defined by the first-level heading `#`.

To compile this example to PDF, you need to install XeLaTeX.

The library convey aims at estimating measures of poverty and income concentration. There are already at least two libraries covering this subject: vardpoor and Laeken. The main difference between the library convey and these two is that the convey strongly hinges on the survey library.

1.1 Installation

- the latest released version from CRAN with

  ```
  install.packages("convey")
  ```

- the latest development version from github with

  ```
  devtools::install_github("djalmapessoa/convey")
  ```

[This may present how to install R, RStudio and required packages. Providing brief information about `survey` and `MonetDBLite` may also be recommended.]

You can label chapter and section titles using `{#label}` after them, e.g., we can reference Chapter 1.1. If you do not manually label them, there will be automatic labels anyway, e.g., Chapter 3.

Figures and tables with captions will be placed in `figure` and `table` environments, respectively.

```
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

Reference a figure by its code chunk label with the `fig:` prefix, e.g., see Figure 1.1. Similarly, you can reference tables generated from `knitr::kable()`, e.g., see Table 1.1.

```
knitr::kable(
  head(iris, 20), caption = 'Here is a nice table!',
```

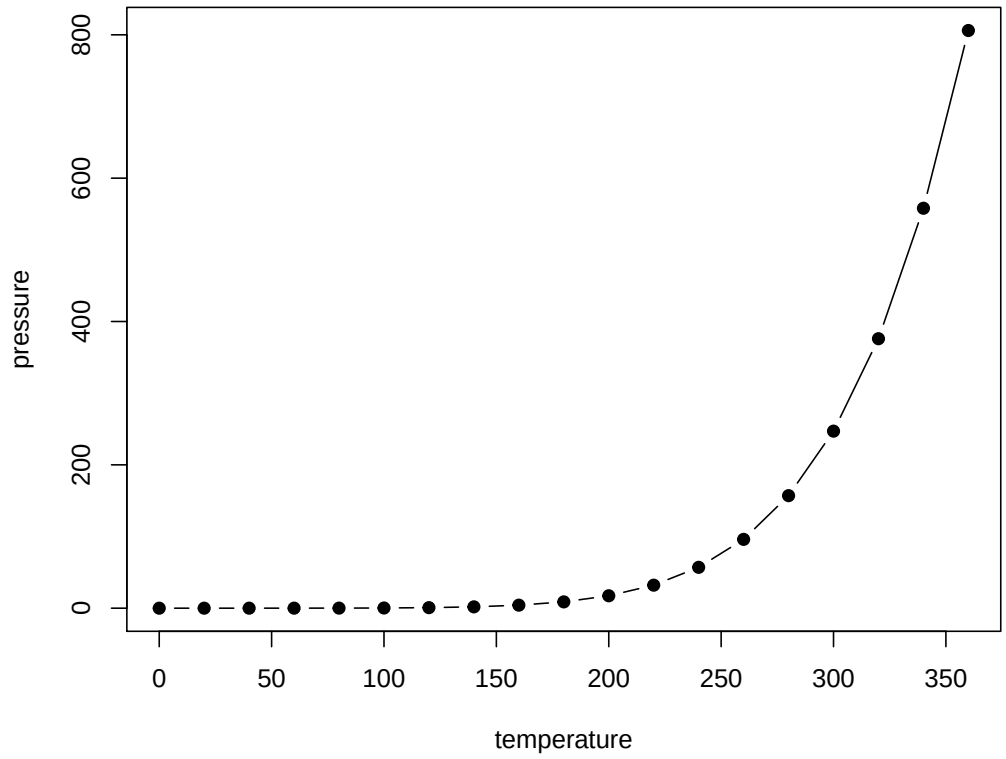

Figure 1.1: Here is a nice figure!

```
  booktabs = TRUE
)
```

You can write citations, too. For example, we are using the **bookdown** package (Xie, 2016) in this sample book, which was built on top of R Markdown and **knitr** (Xie, 2015).

1.2 Complex surveys and statistical inference

[I think we should have a discussion about what is complex survey, its importance and so on. We can use a book Djalma wrote.]

1.3 Linearization

Some measures of poverty and income concentration are defined by non-differentiable functions so that it is not possible to use Taylor linearization to estimate their variances. An alternative is to use **Influence functions** as described in (Deville, 1999) and (Osier, 2009). The library convey implements this methodology to work with `survey.design` objects and also with `svyrep.design` objects.

Some examples of these measures are:

- At-risk-of-poverty threshold: $arpt = .60q_{.50}$ where $q_{.50}$ is the income median;
- At-risk-of-poverty rate $arpr = \frac{\sum_U 1(y_i \leq arpt)}{N}.100$
- Quintile share ratio

Table 1.1: Here is a nice table!

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa
4.6	3.4	1.4	0.3	setosa
5.0	3.4	1.5	0.2	setosa
4.4	2.9	1.4	0.2	setosa
4.9	3.1	1.5	0.1	setosa
5.4	3.7	1.5	0.2	setosa
4.8	3.4	1.6	0.2	setosa
4.8	3.0	1.4	0.1	setosa
4.3	3.0	1.1	0.1	setosa
5.8	4.0	1.2	0.2	setosa
5.7	4.4	1.5	0.4	setosa
5.4	3.9	1.3	0.4	setosa
5.1	3.5	1.4	0.3	setosa
5.7	3.8	1.7	0.3	setosa
5.1	3.8	1.5	0.3	setosa

$qsr = \frac{\sum_U 1(y_i > q_{.80})}{\sum_U 1(y_i \leq q_{.20})}$

- Gini coefficient $1 + G = \frac{2\sum_U (r_i - 1)y_i}{N \sum_U y_i}$ where r_i is the rank of y_i.

Note that it is not possible to use Taylor linearization for these measures because they depend on quantiles and the Gini is defined as a function of ranks. This could be done using the approach proposed by Deville (1999) based upon influence functions.

1.4 Influence function

Let U be a population of size N and M be a measure that allocates mass one to the set composed by one unit, that is $M(i) = M_i = 1$ if $i \in U$ and $M(i) = 0$ if $i \notin U$

Now, a population parameter θ can be expressed as a functional of M $\theta = T(M)$

Examples of such parameters are:

- Total: $Y = \sum_U y_i = \sum_U y_i M_i = \int y dM = T(M)$
- Ratio of two totals: $R = \frac{Y}{X} = \frac{\int y dM}{\int x dM} = T(M)$
- Cumulative distribution function: $F(x) = \frac{\sum_U 1(y_i \leq x)}{N} = \frac{\int 1(y \leq x) dM}{\int dM} = T(M)$

To estimate these parameters from the sample, we replace the measure M by the estimated measure $\hat{M}$ defined by: $\hat{M}(i) = \hat{M}_i = w_i$ if $i \in s$ and $\hat{M}(i) = 0$ if $i \notin s$.

The estimators of the population parameters can then be expressed as functional of the measure $\hat{M}$.

- Total: $\hat{Y} = T(\hat{M}) = \int y d\hat{M} = \sum_s w_i y_i$
- Ratio of totals: $\hat{R} = T(\hat{M}) = \frac{\int y d\hat{M}}{\int x d\hat{M}} = \frac{\sum_s w_i y_i}{\sum_s w_i x_i}$
- Cumulative distribution function: $\hat{F}(x) = T(\hat{M}) = \frac{\int 1(y \leq x) d\hat{M}}{\int d\hat{M}} = \frac{\sum_s w_i 1(y_i \leq x)}{\sum_s w_i}$

1.5 The variance estimator

The variance of the estimator $T(\hat{M})$ can approximated by:

$$Var\left[T(\hat{M})\right] \cong var\left[\sum_s w_i z_i\right]$$

The `linearized` variable z is given by the derivative of the functional:

$$z_k = lim_{t \to 0} \frac{T(M + t\delta_k) - T(M)}{t} = IT_k(M)$$

where, δ_k is the Dirac measure in k: $\delta_k(i) = 1$ if and only if $i = k$.

This **derivative** is called **Influence Function** and was introduced in the area of **Robust Statistics**.

1.6 Influence functions - Examples

- Total:

$$\begin{aligned} IT_k(M) &= lim_{t \to 0} \frac{T(M + t\delta_k) - T(M)}{t} \\ &= lim_{t \to 0} \frac{\int y.d(M + t\delta_k) - \int y.dM}{t} \\ &= lim_{t \to 0} \frac{\int y d(t\delta_k)}{t} = y_k \end{aligned}$$

- Ratio of two totals:

$$\begin{aligned} IR_k(M) = I\left(\frac{U}{V}\right)_k (M) &= \frac{V(M) \times IU_k(M) - U(M) \times IV_k(M)}{V(M)^2} \\ &= \frac{X y_k - Y x_k}{X^2} = \frac{1}{X}(y_k - R x_k) \end{aligned}$$

1.7 Linearization by influence function - Examples

- At-risk-of-poverty threshold:

$$arpt = 0.6 \times m$$

where m is the median income.

$$z_k = -\frac{0.6}{f(m)} \times \frac{1}{N} \times [I(y_k \leq m - 0.5)]$$

- At-risk-of-poverty rate:

$$arpr = \frac{\sum_U I(y_i \leq t)}{\sum_U w_i}.100$$

$$z_k = \frac{1}{N}\left[I(y_k \leq t) - t\right] - \frac{0.6}{N} \times \frac{f(t)}{f(m)}\left[I(y_k \leq m) - 0.5\right]$$

where:

N - population size;

t - at-risk-of-poverty threshold;

y_k - income of person k;

m - median income;

f - income density function;

1.8 Structure of the library

In the library convey, there are some basic functions that produces the linearized variables of some estimates that often enter in the definition of measures of concentration and poverty. For example the `quantile` which is linearized by the function `svyiqalpha`. Other example is the function `svyisq` that linearizes the total below a quantile of the variable.

From the linearized variables of these basic estimates it is possible by using rules of composition, valid for influence functions, to derive the influence function of more complex estimates. By definition the influence function is a Gateaux derivative and the rules rules of composition valid for Gateaux derivatives also hold for Influence Functions.

The following property of Gateaux derivatives was often used in the library convey. Let g be a differentible function of m variables. Suppose we want to compute the influence function of the estimator $g(T_1, T_2, \ldots, T_m)$, knowing the Influence function of the estimators $T_i, i = 1, \ldots, m$. Then the following holds:

$$I(g(T_1, T_2, \ldots, T_m)) = \sum_{i=1}^{m} \frac{\partial g}{\partial T_i} I(T_i)$$

In the library convey this rule is implemented by the function `contrastinf` which uses the R function `deriv` to compute the formal partial derivatives $\frac{\partial g}{\partial T_i}$.

For example, suppose we want to linearize the `Relative median poverty gap`(rmpg), defined as the difference between the at-risk-of-poverty threshold (`arpt`) and the median of incomes less than the `arpt` relative to the `arprt`:

$$rmpg = \frac{arpt - medpoor}{arpt}$$

where `medpoor` is the median of incomes less than `arpt`.

Suppose we know how to linearize `arpt` and `medpoor`, then by applying the function `contrastinf` with

$$g(T_1, T_2) = \frac{(T_1 - T_2)}{T_1}$$

we linearize the `rmpg`.

1.8.1 Examples of use of the library convey

In the following examples we will use the data set `eusilc` contained in the libraries `vardpoor` and `Laeken`.

```
library(vardpoor)
data(eusilc)
```

Next, we create an object of class `survey.design` using the function `svydesign` of the library survey:

```
library(survey)
des_eusilc <- svydesign(ids = ~rb030, strata =~db040,  weights = ~rb050, data = eusilc)
```

Right after the creation of the design object `des_eusilc`, we should use the function `convey_prep` that adds an attribute to the survey design which saves information on the design object based upon the whole sample, needed to work with subset designs.

```
library(convey)
des_eusilc <- convey_prep( des_eusilc )
```

To estimate the `at-risk-of-poverty rate` we use the function `svyarpt`:

```
svyarpr(~eqIncome, design=des_eusilc)
```

```
            arpr     SE
eqIncome 0.14444 0.0028
```

To estimate the `at-risk-of-poverty rate` for domains defined by the variable `db040` we use

```
svyby(~eqIncome, by = ~db040, design = des_eusilc, FUN = svyarpr, deff = FALSE)
```

```
                     db040  eqIncome          se
Burgenland       Burgenland 0.1953984 0.017202243
Carinthia         Carinthia 0.1308627 0.010610622
Lower Austria Lower Austria 0.1384362 0.006517660
Salzburg           Salzburg 0.1378734 0.011579280
Styria               Styria 0.1437464 0.007452360
Tyrol                 Tyrol 0.1530819 0.009880430
Upper Austria Upper Austria 0.1088977 0.005928336
Vienna               Vienna 0.1723468 0.007682826
Vorarlberg       Vorarlberg 0.1653731 0.013754670
```

Using the same data set, we estimate the `quintile share ratio`:

```
# for the whole population
svyqsr(~eqIncome, design=des_eusilc, alpha= .20)
```

```
          qsr     SE
eqIncome 3.97 0.0426
```

```
# for domains
svyby(~eqIncome, by = ~db040, design = des_eusilc,
  FUN = svyqsr, alpha= .20, deff = FALSE)
```

```
                     db040 eqIncome         se
Burgenland       Burgenland 5.008486 0.32755685
Carinthia         Carinthia 3.562404 0.10909726
Lower Austria Lower Austria 3.824539 0.08783599
Salzburg           Salzburg 3.768393 0.17015086
Styria               Styria 3.464305 0.09364800
Tyrol                 Tyrol 3.586046 0.13629739
```

```
Upper Austria Upper Austria 3.668289 0.09310624
Vienna               Vienna 4.654743 0.13135731
Vorarlberg       Vorarlberg 4.366511 0.20532075
```

These functions can be used as S3 methods for the classes `survey.design` and `svyrep.design`.

Let's create a design object of class `svyrep.design` and run the function `convey_prep` on it:

```
des_eusilc_rep <- as.svrepdesign(des_eusilc, type = "bootstrap")
des_eusilc_rep <- convey_prep(des_eusilc_rep)
```

and then use the function `svyarpr`:

```
svyarpr(~eqIncome, design=des_eusilc_rep)
```

```
            arpr     SE
eqIncome 0.14444 0.0032
```

```
svyby(~eqIncome, by = ~db040, design = des_eusilc_rep, FUN = svyarpr, deff = FALSE)
```

```
                     db040  eqIncome se.eqIncome
Burgenland       Burgenland 0.1953984 0.016236045
Carinthia         Carinthia 0.1308627 0.011322973
Lower Austria Lower Austria 0.1384362 0.006671846
Salzburg           Salzburg 0.1378734 0.011012774
Styria               Styria 0.1437464 0.008135005
Tyrol                 Tyrol 0.1530819 0.011701101
Upper Austria Upper Austria 0.1088977 0.005926015
Vienna               Vienna 0.1723468 0.007182969
Vorarlberg       Vorarlberg 0.1653731 0.013064658
```

The functions of the library convey are called in a similar way to the functions in library survey.

It is also possible to deal with missing values by using the argument `na.rm`.

```
# survey.design using a variable with missings
svygini( ~ py010n , design = des_eusilc )
```

```
       gini SE
py010n   NA NA
```

```
svygini( ~ py010n , design = des_eusilc , na.rm = TRUE )
```

```
          gini     SE
py010n 0.64606 0.0036
```

```
# svyrep.design using a variable with missings
# svygini( ~ py010n , design = des_eusilc_rep ) get error
svygini( ~ py010n , design = des_eusilc_rep , na.rm = TRUE )
```

```
          gini     SE
py010n 0.64606 0.0036
```

1.9 FGT indicator

(Foster et al., 1984) proposed a family of indicators to measure poverty.

The class of FGT measures, can be defined as

$$p = \frac{1}{N} \sum_{k \in U} h(y_k, \theta),$$

where

$$h(y_k, \theta) = \left[\frac{(\theta - y_k)}{\theta} \right]^{\gamma} \delta \left\{ y_k \leq \theta \right\},$$

where: θ is the poverty threshold; δ the indicator function that assigns value 1 if the condition $\{y_k \leq \theta\}$ is satisfied and 0 otherwise, and γ is a non-negative constant.

When $\gamma = 0$, p can be interpreted as the ratio of poor people, and for $\gamma \geq 1$, the weight of poor people increases with the value γ, (Foster and all, 1984).

The poverty measure FGT is implemented in the library convey by the function `svyfgt`. The argument `thresh_type` of this function defines the type of poverty threshold adopted. There are three possible choices:

1. `abs` – fixed and given by the argument thresh_value
2. `relq` – a proportion of a quantile fixed by the argument `proportion` and the quantile is defined by the argument `order`.
3. `relm` – a proportion of the mean fixed the argument `proportion`

The quantile and the mean involved in the definition of the threshold are estimated for the whole population. When $\gamma = 0$ and $\theta = .6 * MED$ the measure is equal to the indicator `arpr` computed by the function `svyarpr`.

Next, we give some examples of the function `svyfgt` to estimate the values of the FGT poverty index.

Consider first the poverty threshold fixed ($\gamma = 0$) in the value 10000. The headcount ratio (FGT0) is

```
svyfgt(~eqIncome, des_eusilc, g=0, abs_thresh=10000)
```

```
            fgt0     SE
eqIncome 0.11444 0.0027
```

The poverty gap (FGT1) ($\gamma = 1$) index for the poverty threshold fixed at the same value is

```
svyfgt(~eqIncome, des_eusilc, g=1, abs_thresh=10000)
```

```
             fgt1     SE
eqIncome 0.032085 0.0011
```

To estimate the FGT0 with the poverty threshold fixed at $0.6 * MED$ we fix the argument type_thresh="relq" and use the default values for `percent` and `order`:

```
svyfgt(~eqIncome, des_eusilc, g=0, type_thresh= "relq")
```

```
            fgt0     SE
eqIncome 0.14444 0.0028
```

that matches the estimate obtained by

```
svyarpr(~eqIncome, design=des_eusilc, .5, .6)
```

```
            arpr     SE
eqIncome 0.14444 0.0028
```

To estimate the poverty gap(FGT1) with the poverty threshold equal to $0.6 * MEAN$ we use:

```
svyfgt(~eqIncome, des_eusilc, g=1, type_thresh= "relm")
```

```
             fgt1     SE
eqIncome 0.051187 0.0011
```

djalma, where do these references go on this page? (Berger and Skinner, 2003) and (Osier, 2009) and (Deville, 1999)

Chapter 2

Poverty Indices

[I think this is a good start. I don't think that gender pay gap, quantiles and totals are measures of poverty. Consider another chapter on other wellbeing measures.]

2.1 At Risk of Poverty Ratio and Threshold (svyarpr, svyarpt)

2.2 The Gender Pay Gap (svygpg)

2.3 Quintile Share Ratio (svyqsr)

2.4 Relative Median Income Ratio (svyrmir)

2.5 Relative Median Poverty Gap (svyrmpg)

2.6 Median Income Below the At Risk of Poverty Threshold (svypoormed)

2.7 Foster-Greer-Thorbecke class (svyfgt)

Chapter 3

Inequality Measurement

[Present an introduction to what is inequality].

3.1 Theoretical aspects of inequality

3.1.1 Desirable properties of inequality measures

3.2 Lorenz Curve (svylorenz)

here are the references

(Kovacevic and Binder, 1997) and (Lerman and Yitzhaki, 1989) and (Langel and Tille, 2012)

3.3 Measures derived from the Lorenz Curve

3.3.1 Gini index (svygini)

here are the references

(Osier, 2009) and (Deville, 1999)

3.3.2 Amato index (svyamato)

here are the references

(Barabesi et al., 2016) and (Arnold, 2012)

3.3.3 Zenga Index and Curve (svyzenga, svyzengacurve)

guilherme..this has three references? not just two?

here are the references

(Barabesi et al., 2016) and (Langel and Tille, 2012) and (Deville, 1999)

3.4 Entropy-based Measures

3.4.1 Atkinson index (svyatk)

here are the references

(Langel and Tille, 2012) and (Biewen and Jenkins, 2003)

3.4.2 Generalized Entropy and Decomposition (svygei, svygeidec)

guilherme..this has three references? not just two?

here are the references

(Langel and Tille, 2012) and (Biewen and Jenkins, 2003) and (Shorrocks, 1984)

3.4.3 J-Divergence Entropy and Decomposition (svyjdiv, svyjdivdec)

here are the references

(Shorrocks, 1984) and (Rohde, 2016) and (Biewen and Jenkins, 2003)

3.4.4 Rényi Divergence (svyrenyi)

here are the references

(Langel and Tille, 2012)

Chapter 4

Multidimensional Indices

We have finished a nice book.

4.1 Alkire-Foster Class and Decomposition (svyafc, svyafcdec)

4.2 Bourguignon (1999) inequality class (svybmi)

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