<재택수업 13주차 1차 과제1>

(1) $f(x)=\int_0^{\sin x}\sqrt{1+t^2}\,dt$이고 $g(y)=\int_3^{2y} f(x)\,dx$일 때 $g''(\frac{\pi}{6})$을 구하여라.

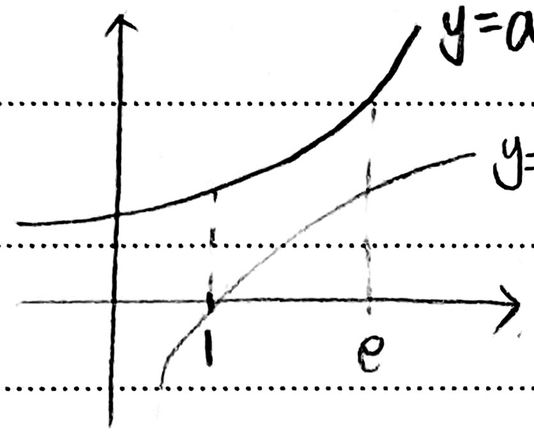
$g'(y)=f(2y)\cdot(2y)'=f(2y)\cdot 2$, $g''(y)=2f'(2y)\cdot 2=4f'(2y)$, $g''(\frac{\pi}{6})=4f'(\frac{\pi}{3})$

$f'(x)=\sqrt{1+\sin^2 x}\cdot(\sin x)'=\sqrt{1+\sin^2 x}\cdot\cos x$

$f'(\frac{\pi}{3})=\sqrt{1+\sin^2(\frac{\pi}{3})}\cdot\cos\frac{\pi}{3}=\sqrt{1+(\frac{\sqrt{3}}{2})^2}\cdot\frac{1}{2}=\sqrt{1+\frac{3}{4}}\cdot\frac{1}{2}=\frac{1}{2}\sqrt{\frac{7}{4}}=\frac{\sqrt{7}}{4}$

따라서 $g''(\frac{\pi}{6})=4f'(\frac{\pi}{3})=4\cdot\frac{\sqrt{7}}{4}=\sqrt{7}$

(2) 곡선 $y=ae^x$, 직선 $x=e$, x축으로 둘러싸인 영역을 $y=\ln x$가 이등분할 때 a의 값을 구하여라.



$\int_1^e ae^x dx = 2\int_1^e \ln x\, dx$ 이다.

$\int_1^e \ln x\, dx = [x\ln x - x]_1^e = (e\ln e - e) - (0-1) = 1$

$\int_1^e ae^x dx = [ae^x]_1^e = 2\int_1^e \ln x\, dx = 2$

$ae^e - ae = ae(e^{e-1}-1) = 2$ $\quad a = \dfrac{2}{e(e^{e-1}-1)}$

(3) $\int_0^2 x\ln x\, dx$가 수렴한다면 수렴값을 구하여라.

$\lim_{M\to 0+}\int_M^2 x\ln x\, dx$, $\quad \int x\ln x\, dx = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$

$\lim_{M\to 0+}\int_M^2 x\ln x\, dx = \lim_{M\to 0+}\left[\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right]_M^2 = \lim_{M\to 0+}\left(2\ln 2 - 1 - \frac{1}{2}M^2\ln M + \frac{1}{4}M^2\right)$

$= 2\ln 2 - 1 - \frac{1}{2}\lim_{M\to 0+} M^2\ln M$ 이다.

$\lim_{M\to 0+} M^2\ln M = \lim_{M\to 0+}\dfrac{\ln M}{\frac{1}{M^2}} \overset{\text{로피탈}}{=} \lim_{M\to 0+}\dfrac{\frac{1}{M}}{-\frac{2}{M^3}} = \lim_{M\to 0+} -\dfrac{M^2}{2} = 0$

따라서 $\lim_{M\to 0+}\int_M^2 x\ln x\, dx = 2\ln 2 - 1 - \frac{1}{2}\lim_{M\to 0+} M^2\ln M = 2\ln 2 - 1$ 이다.