인하대학교
INHA UNIVERSITY

Electromagnetics 1 (ICE2003)
-- Ch. 2. Coulomb's Law and Electric Field Intensity --

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Chapter Outline

- Coulomb's Experimental Law 소개
- Electric Field Intensity (**E**) 개념 이해
- Coulomb's Law를 이용하여 몇 가지 단순한 전하 분포에서의 **E** 계산 연습
  - 점전하 (하나, 여러 개)
  - 선전하 (무한/유한길이, 직선/원형)
  - 면전하 (Infinite plane, Disk of finite radius)
  - (부피전하) → 다음 chapter까지 연기
- Electric Field 가시화 - Streamline

Electricity

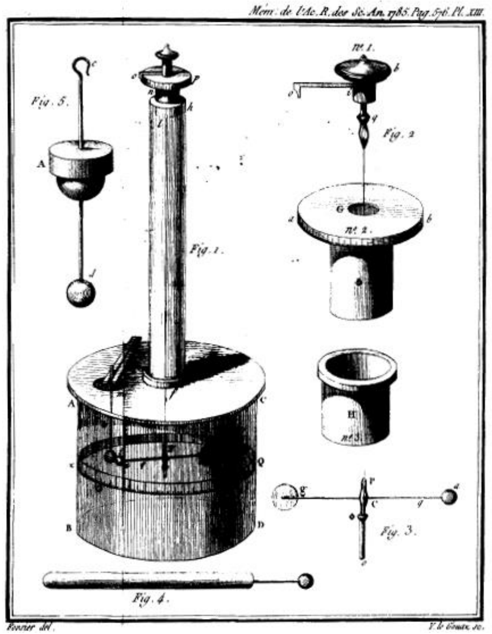
https://en.wikipedia.org/wiki/Amber

Electricity

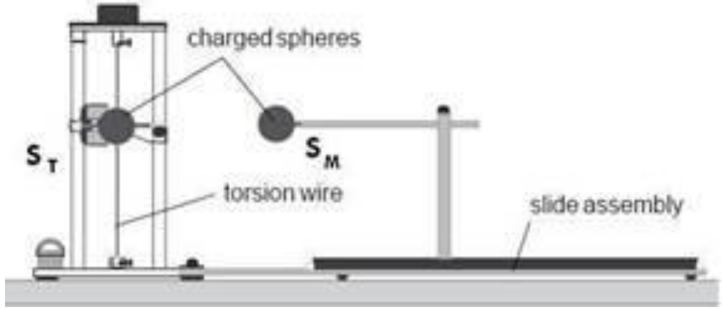
https://en.wikipedia.org/wiki/Electric_field

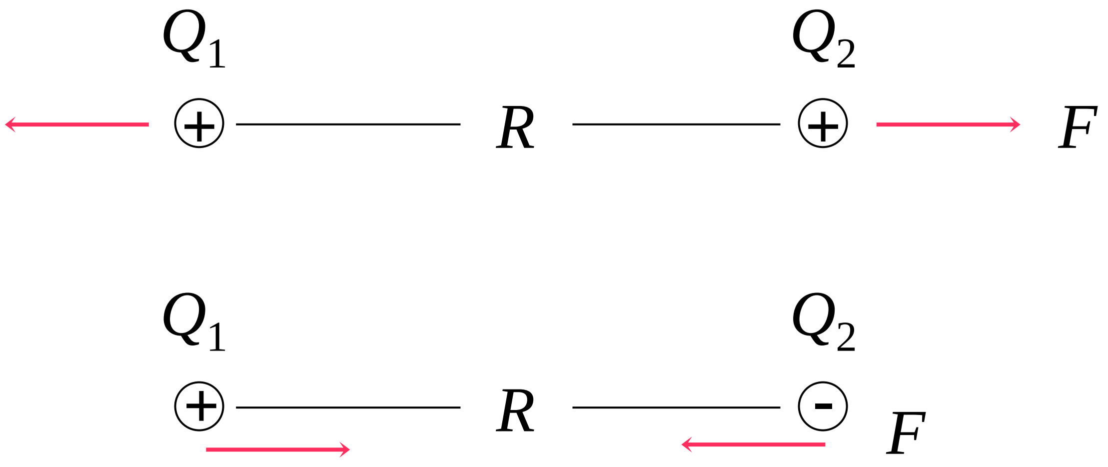
https://en.wikipedia.org/wiki/Amber

Coulomb's Experimental Law



https://en.wikipedia.org/wiki/Coulomb%27s_law





- Force of repulsion, *F*, occurs when charges have the same sign.
- Charges attract when of opposite sign

$$F \propto \frac{Q_1 Q_2}{R^2}$$

Coulomb's Experimental Law

$$\boxed{F = k\frac{Q_1 Q_2}{R^2}}$$

where $k = \dfrac{1}{4\pi\epsilon_0}$

$$\boxed{\epsilon_0 = 8.854 \times 10^{-12} \doteq \frac{1}{36\pi} 10^{-9}\ \text{F/m}}$$

Permittivity of free space

F : Newton (N)

Q : Coulomb (C)

R : Meter (m)

- Charge of an electron = -e = -1.602×10^{-19}(C)
- 1C = charge of 6.24×10^{18} electrons

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$$4000mAh = 4000 \times 10^{-3} \times 3600 As = 14400C$$

Coulomb's Experimental Law

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$$\epsilon_0 = 8.854 \times 10^{-12} \doteq \frac{1}{36\pi}10^{-9}\ \text{F/m}$$

Permittivity of free space

F : Newton (N)

Q : Coulomb (C)

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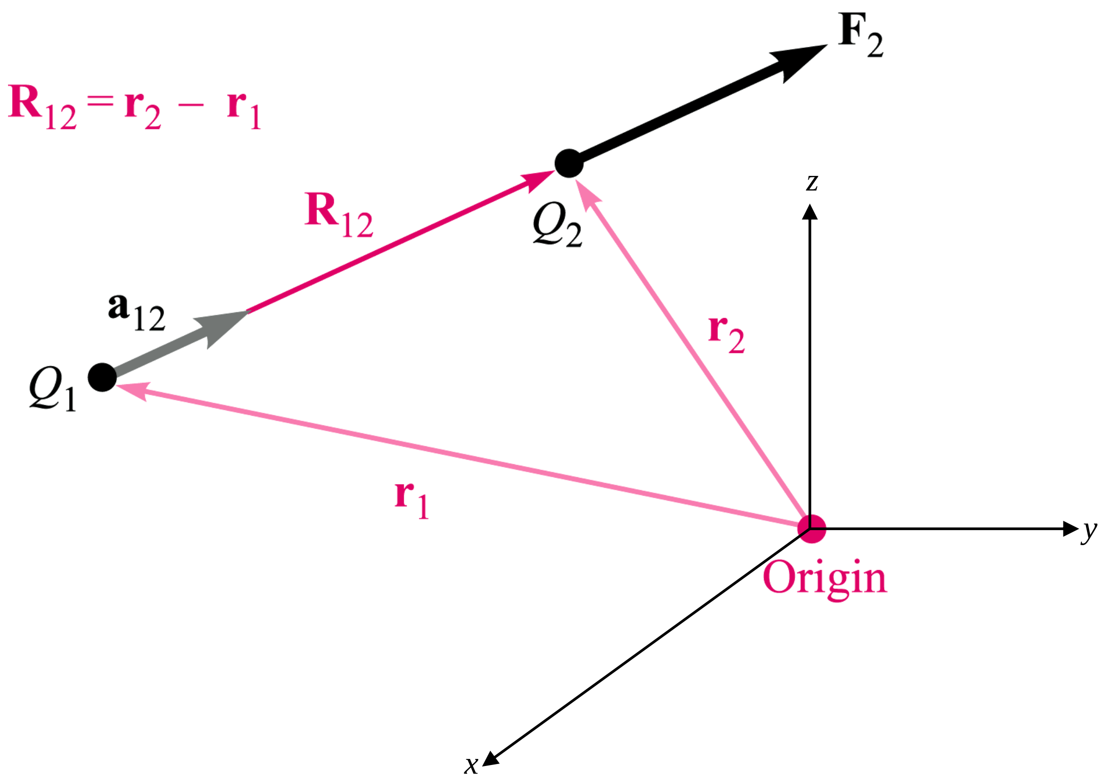
- Charge of an electron = -e = -1.602 x 10^{-19}(C)
- 1C = charge of 6.24 x 10^{18} electrons

Proton mass = 1.67 x 10^{-27} (Kg)
Earth mass = 5.97 x 10^{24} (Kg)

$$F = G\frac{M_1 M_2}{R^2}$$

Coulomb's Force with Charges Off-Origin



$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12}$$

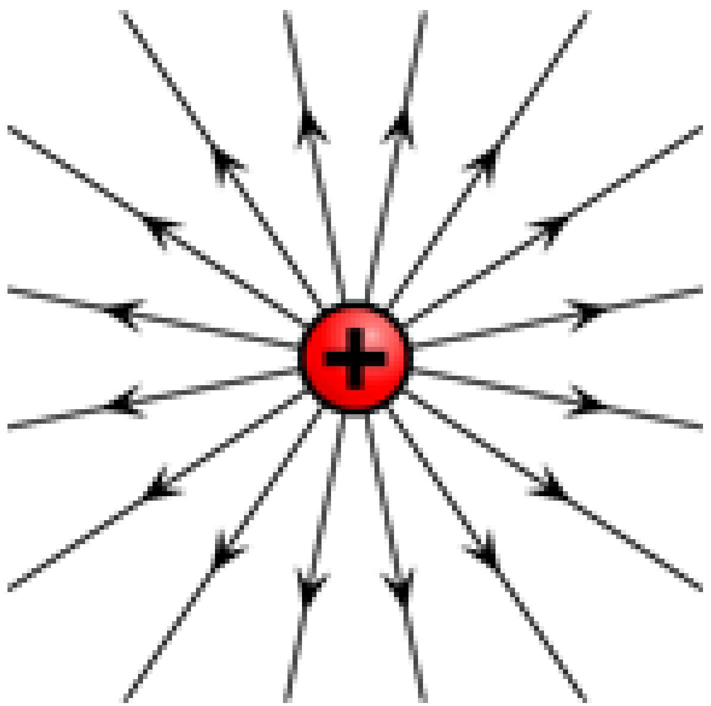
$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

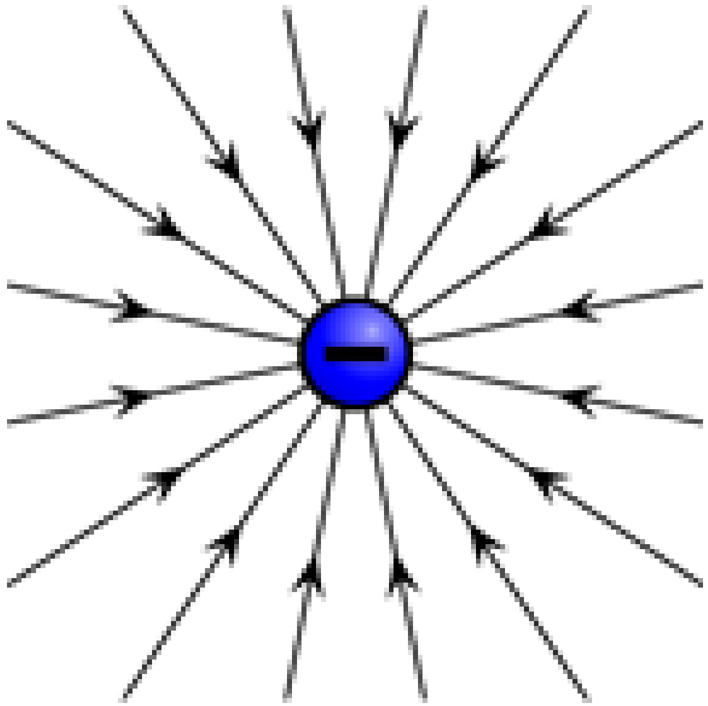
$$\boxed{\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_o |\mathbf{r}_2 - \mathbf{r}_1|^2}\frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}}$$

Coulomb's Force with Charges Off-Origin

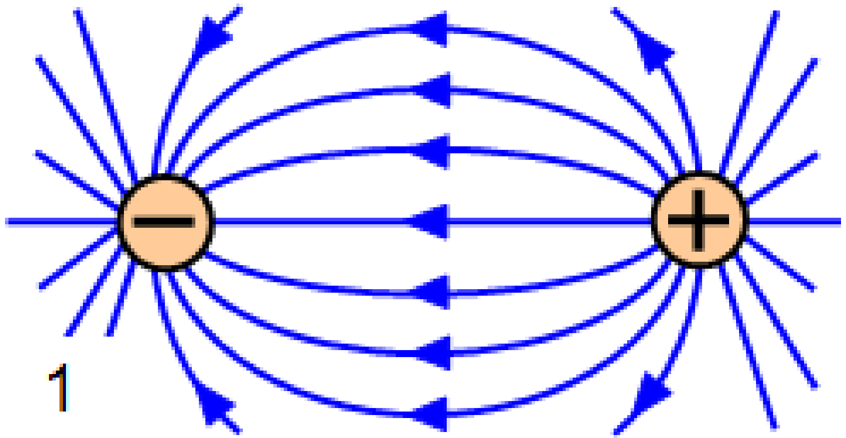
- Let us illustrate the use of the vector form of Coulomb's law by locating a charge of $Q_1=3\times10^{-4}C$ at M(1,2,3) and a charge of $Q_2=-10^{-4}C$ at N(2,0,5) in a vacuum. We desire the force exerted on Q_2 by Q_1

Electric Field

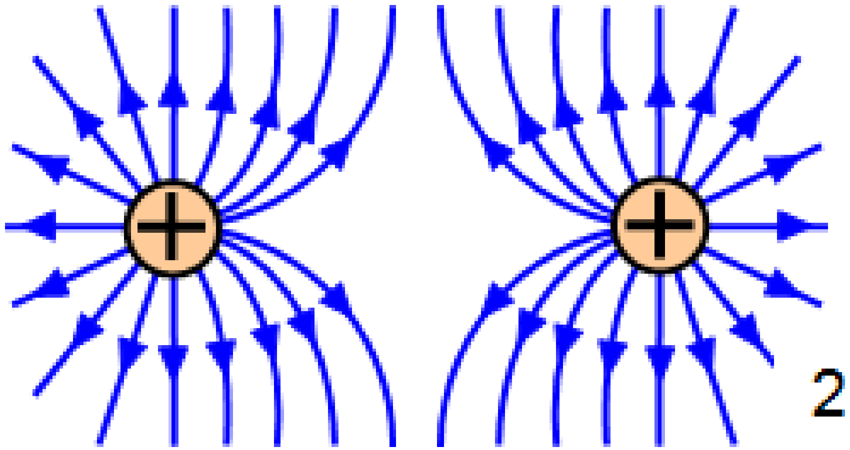




Electric Field



1



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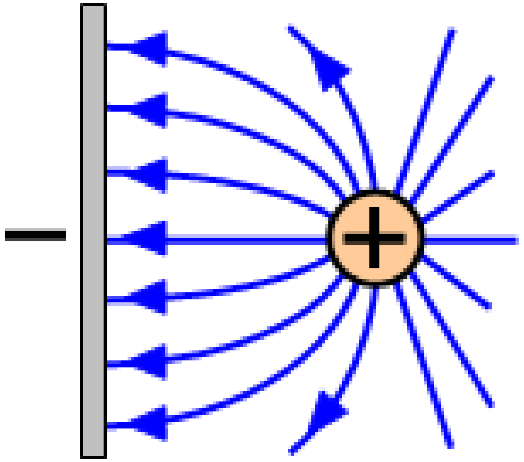

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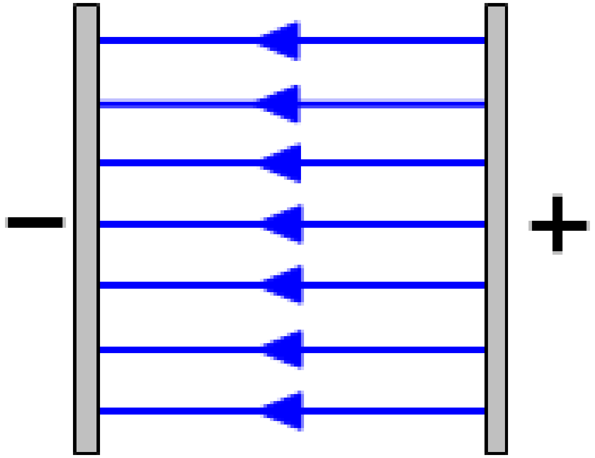

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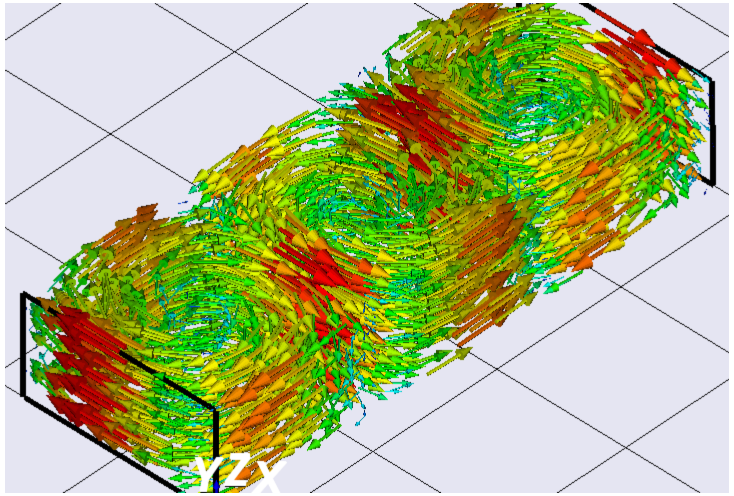
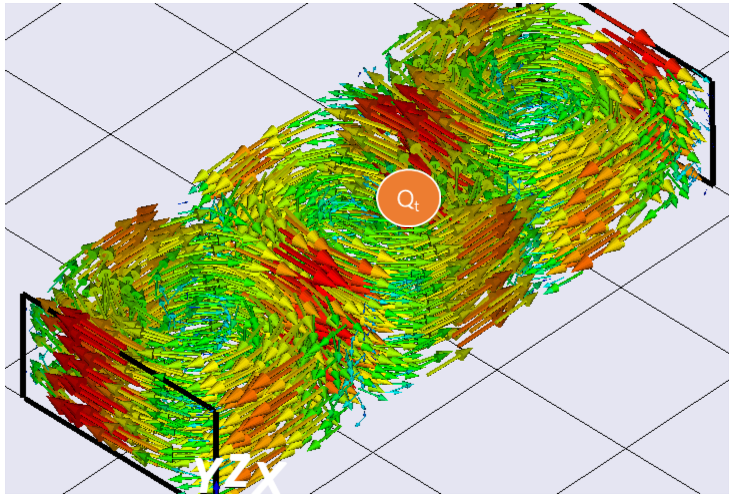


http://www.petr-lorenz.com/emgine/

http://www.schoolphysics.co.uk/age11-14/Electricity%20and%20magnetism/Electrostatics/text/Electric_fields/index.html

Electric Field Intensity



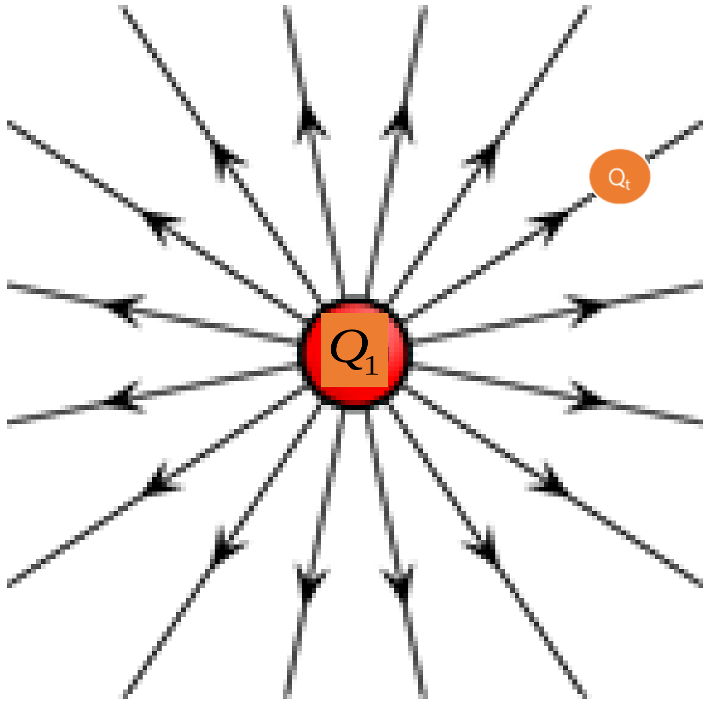
Consider the force acting on a test charge, Q_t

$$\mathbf{F}_\mathbf{t}$$

The *electric field intensity* is defined as the force per unit test charge, or

$$\boxed{\mathbf{E} = \frac{\mathbf{F}_\mathbf{t}}{Q_t} \qquad \text{N/C (or V/m)}}$$

Electric Field Intensity



Consider the force acting on a test charge, Q_t , arising from charge Q_1:

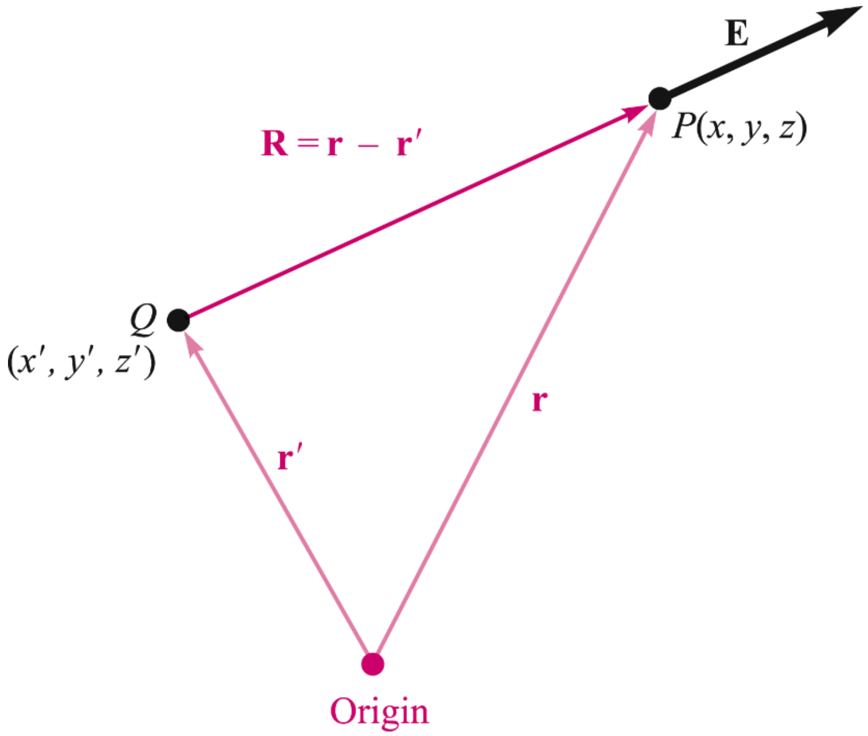
$$\mathbf{F}_t = \frac{Q_1 Q_t}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t}$$

where a_{1t} is the unit vector directed from Q_1 to Q_t

The *electric field intensity* is defined as the force per unit test charge, or

$$\boxed{\mathbf{E_1} = \frac{\mathbf{F_t}}{Q_t} = \frac{Q_1}{4\pi\varepsilon_o R_{1t}^2}\mathbf{a_{1t}} \qquad \text{N/C (or V/m)}}$$

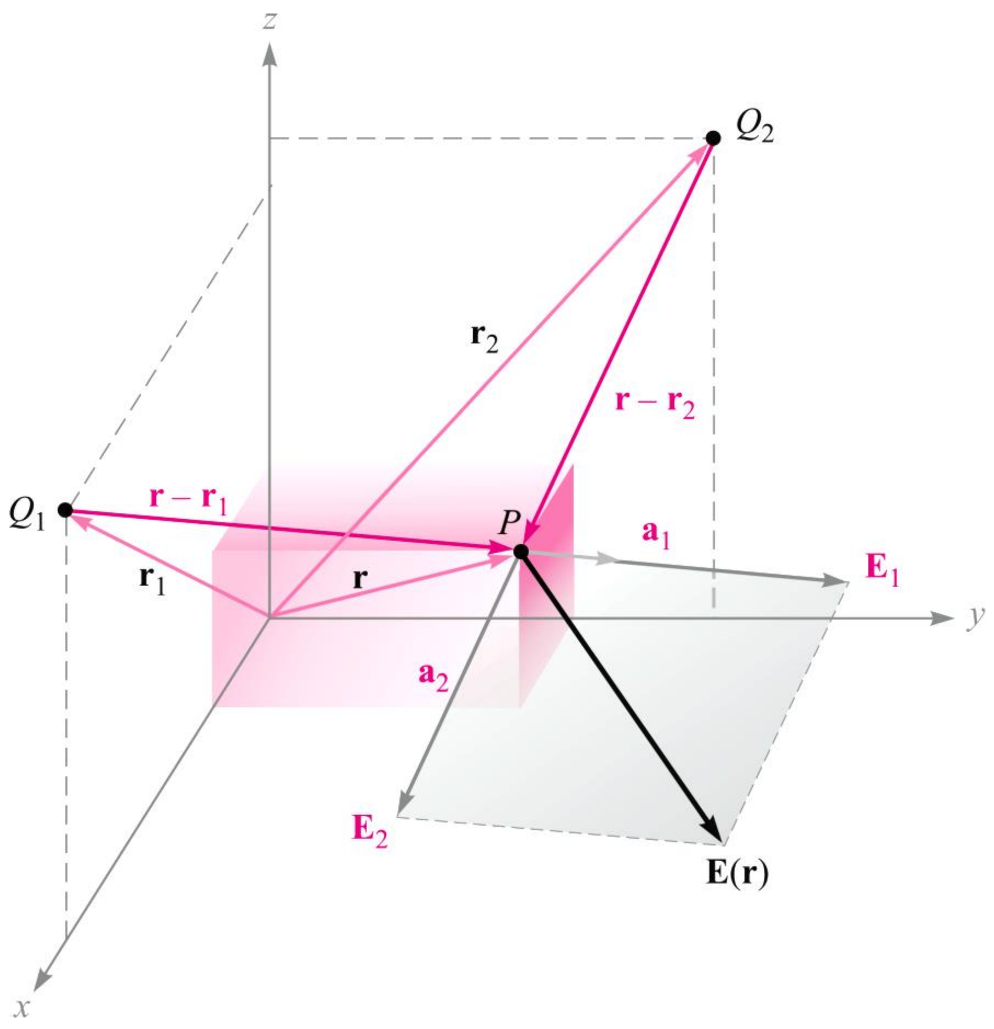
Electric field of a charge off-origin



$$\mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^2}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} = \frac{Q(\mathbf{r}-\mathbf{r}')}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^3}$$

$$= \frac{Q[(x-x')\mathbf{a}_x + (y-y')\mathbf{a}_y + (z-z')\mathbf{a}_z]}{4\pi\epsilon_0[(x-x')^2+(y-y')^2+(z-z')^2]^{3/2}}$$

Superposition of Fields from Two point charges

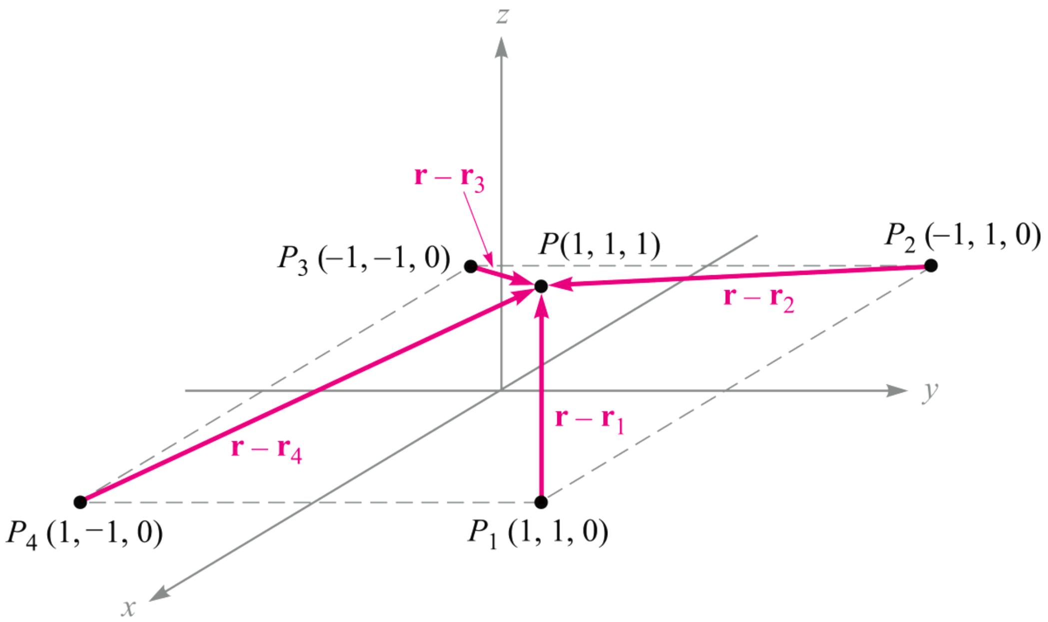


$$\mathbf{E}(\mathbf{r}) = \frac{Q_1}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{Q_2}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2$$

For n charges:

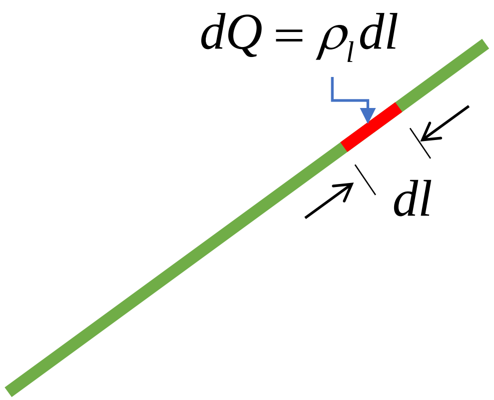
$$\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_m|^2}\mathbf{a}_m$$

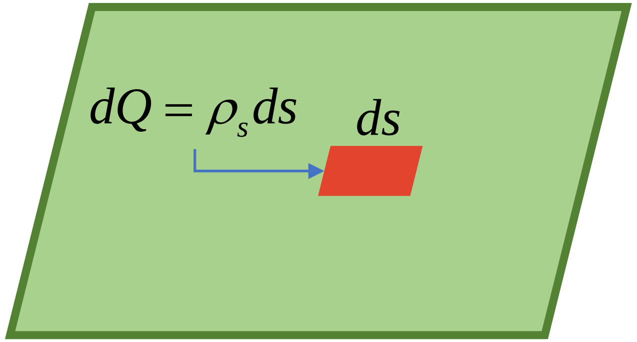
Example

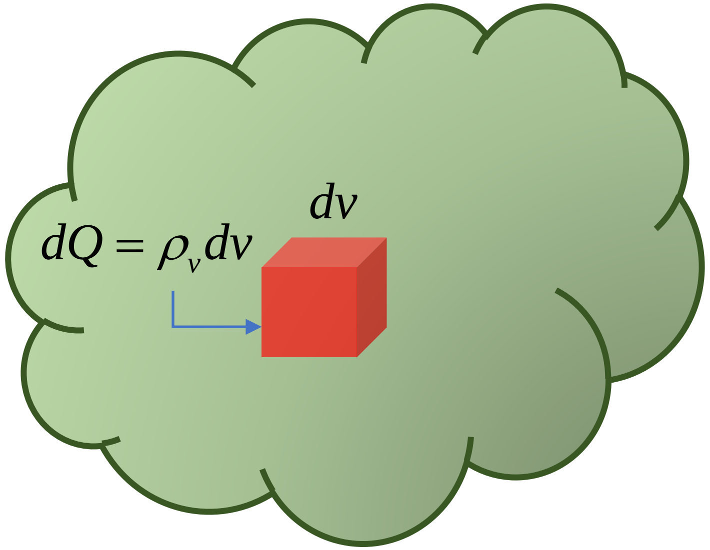


Charge & Charge Density

- **Line charge** Q
- **Surface charge** Q
- **Volume charge** Q







Line charge density

$$\rho_l = \lim_{\Delta l \to 0} \frac{\Delta Q}{\Delta l} \quad \text{(C/m)}$$

$$Q = \int \rho_l dl$$

Surface charge density

$$\rho_s = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s} \quad (\text{C/m}^2)$$

$$Q = \int \rho_s ds$$

Volume charge density

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v} \quad (\text{C/m}^3)$$

$$Q = \int \rho_v dv$$

Example

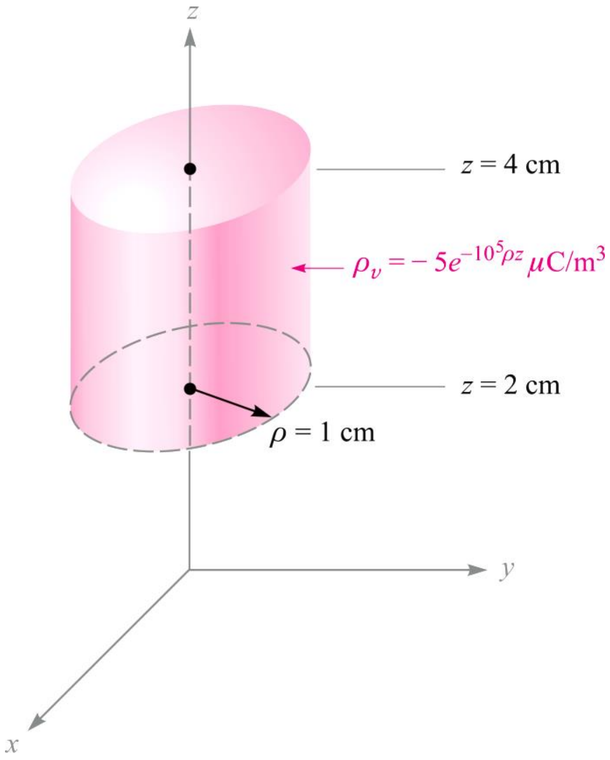
Find the charge contained within a 2-cm length of the electron beam shown below, in which the charge density is $\rho_v = -5 \times 10^{-6} e^{-10^5 \rho z}$ C/m^2

$$Q = \int_{0.02}^{0.04} \int_{0}^{2\pi} \int_{0}^{0.01} -5 \times 10^{-6} e^{-10^5 \rho z} \rho \, d\rho \, d\phi \, dz$$

$$= \int_{0.02}^{0.04} \int_{0}^{0.01} -10^{-5} \pi e^{-10^5 \rho z} \rho \, d\rho \, dz$$

$$= \int_{0}^{0.01} \left(\frac{-10^{-5}\pi}{-10^5 \rho} e^{-10^5 \rho z} \rho \, d\rho \right)_{z=0.02}^{z=0.04}$$

$$= \int_{0}^{0.01} -10^{-5} \pi (e^{-2000\rho} - e^{-4000\rho}) d\rho$$



Example

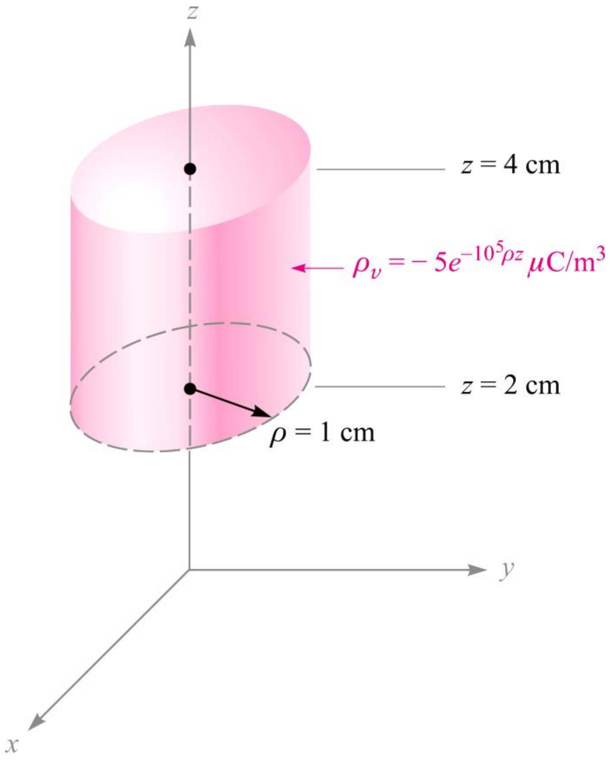
Find the charge contained within a 2-cm length of the electron beam shown below, in which the charge density is $\rho_v = -5 \times 10^{-6} e^{-10^5 \rho z}\ \mathrm{C/m^2}$

$$Q = \int_0^{0.01} -10^{-5}\pi \left(e^{-2000\rho} - e^{-4000\rho}\right) d\rho$$

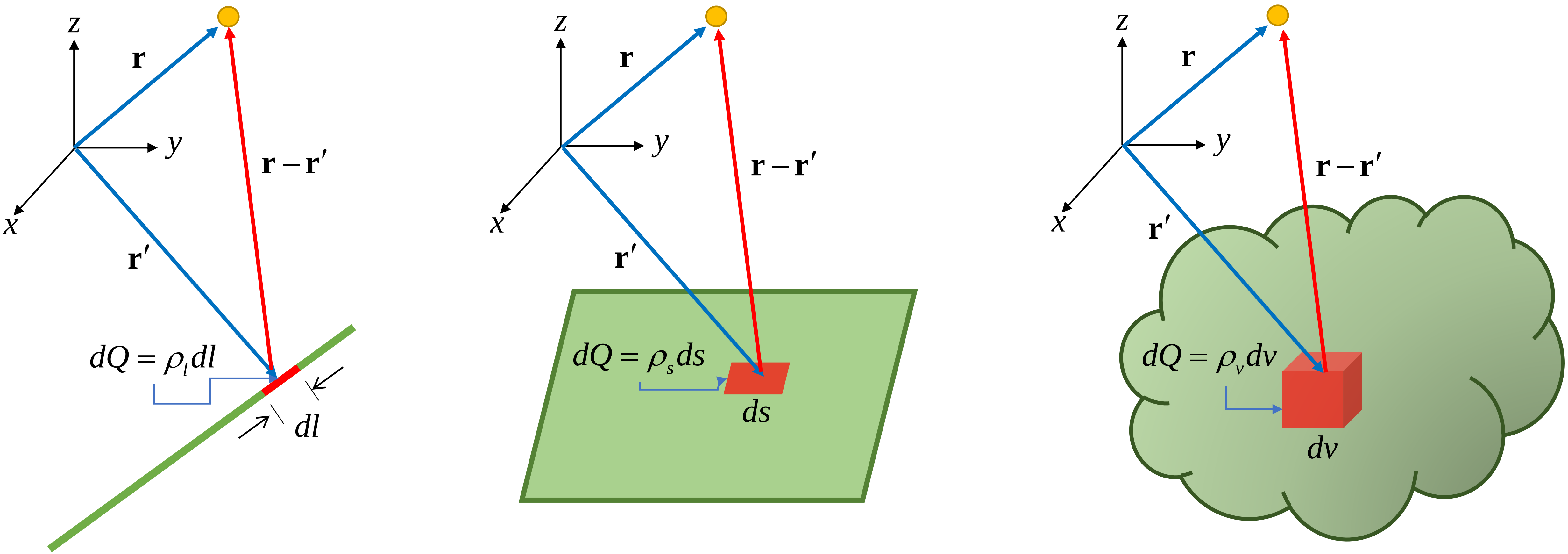
$$= -10^{-10}\pi \left(\frac{e^{-2000\rho}}{-2000} - \frac{e^{-4000\rho}}{-4000}\right)_0^{0.01}$$

$$= -10^{-10}\pi \left(\frac{1}{2000} - \frac{1}{4000}\right)$$

$$= -0.07854\mathrm{pC}$$

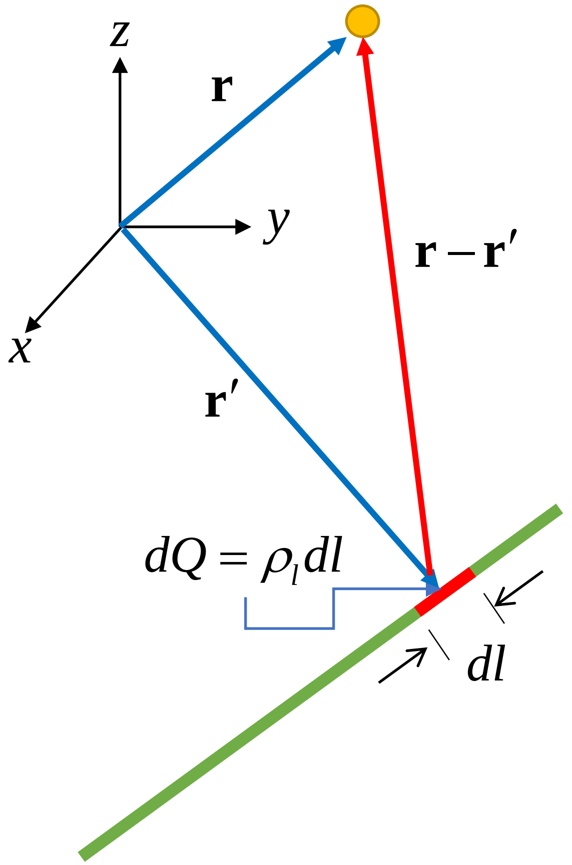


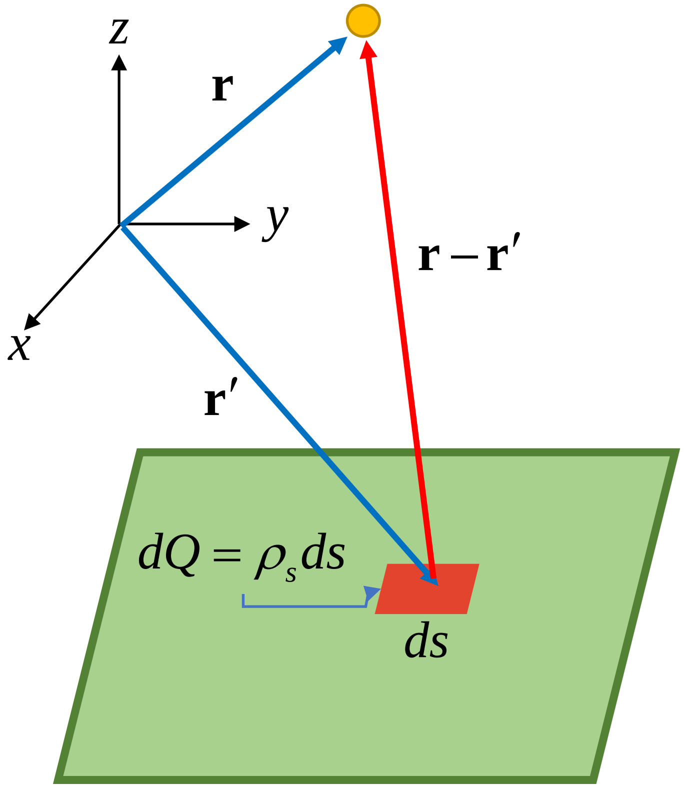
Electric field from line/surface/volume charge distribution

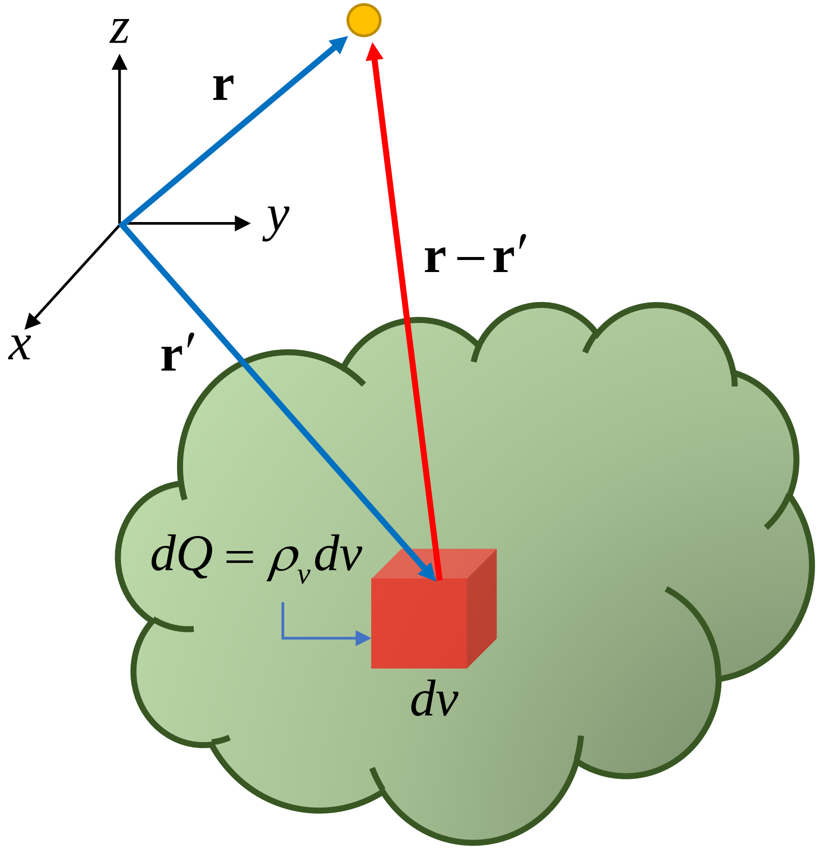


$$\Delta\mathbf{E}(\mathbf{r}) = \frac{\Delta Q}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^2}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

Electric field from line/surface/volume charge distribution







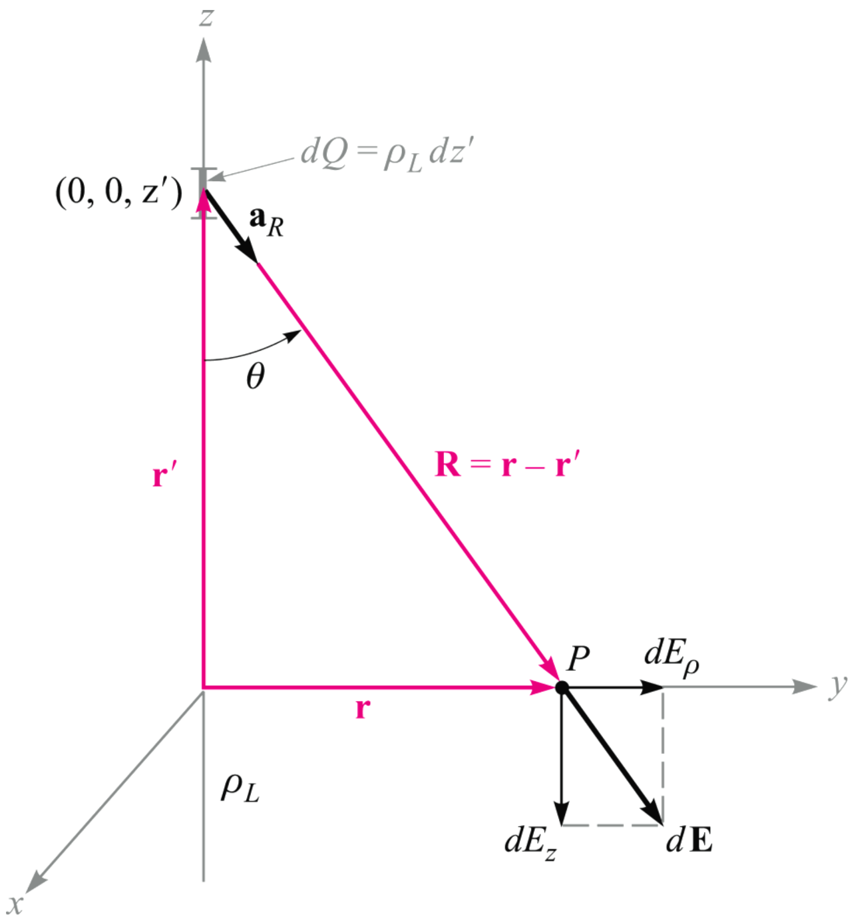
$$\mathbf{E}(\mathbf{r}) = \int \frac{\rho_l(\mathbf{r}')\,dl'}{4\pi\varepsilon_o |\mathbf{r}-\mathbf{r}'|^2} \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$$\mathbf{E}(\mathbf{r}) = \int \frac{\rho_s(\mathbf{r}')\,ds'}{4\pi\varepsilon_o |\mathbf{r}-\mathbf{r}'|^2} \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$$\mathbf{E}(\mathbf{r}) = \int \frac{\rho_v(\mathbf{r}')\,dv'}{4\pi\varepsilon_o |\mathbf{r}-\mathbf{r}'|^2} \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

Line charge electric field

Line charge of constant density ρ_L Coul/m lies along the entire z axis.



At point P, the electric field arising from charge dQ on the z axis is:

$$d\mathbf{E} = \frac{\rho_L dz'(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|^3}$$

where $\quad \mathbf{r} = y\mathbf{a}_y = \rho\mathbf{a}_\rho$

and $\quad \mathbf{r}' = z'\mathbf{a}_z$

so that $\quad \mathbf{r} - \mathbf{r}' = \rho\mathbf{a}_\rho - z'\mathbf{a}_z$

Therefore $\quad d\mathbf{E} = \dfrac{\rho_L dz'(\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$

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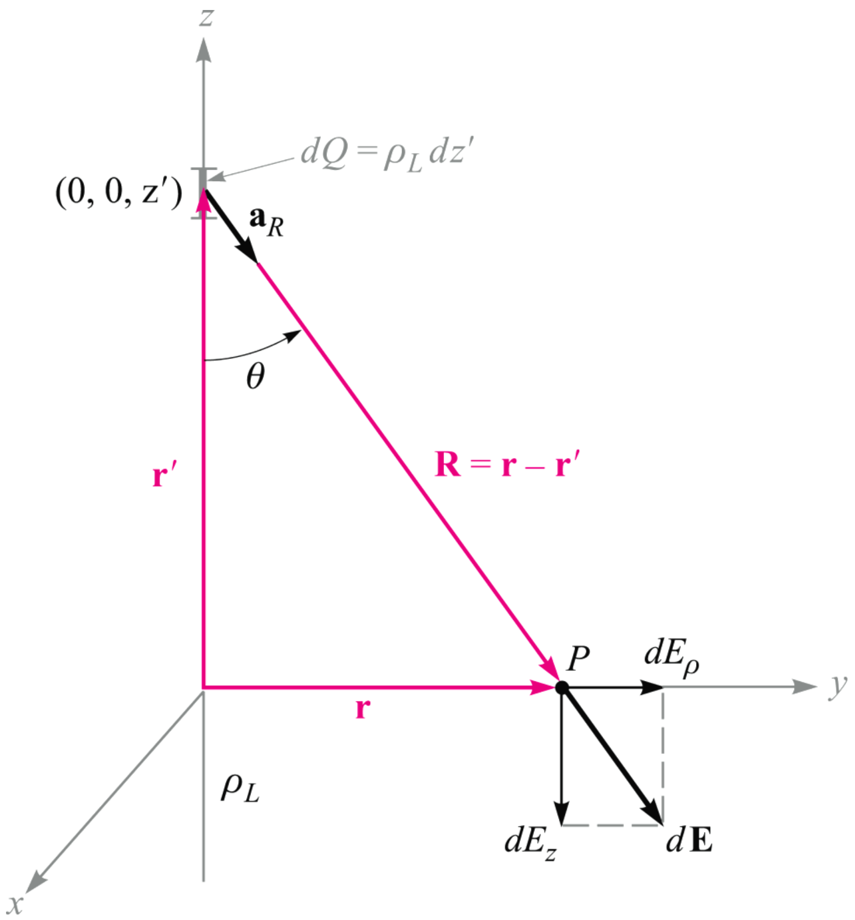


We have: $$d\mathbf{E} = \frac{\rho_L dz'(\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$$

By symmetry, only a radial component is present:

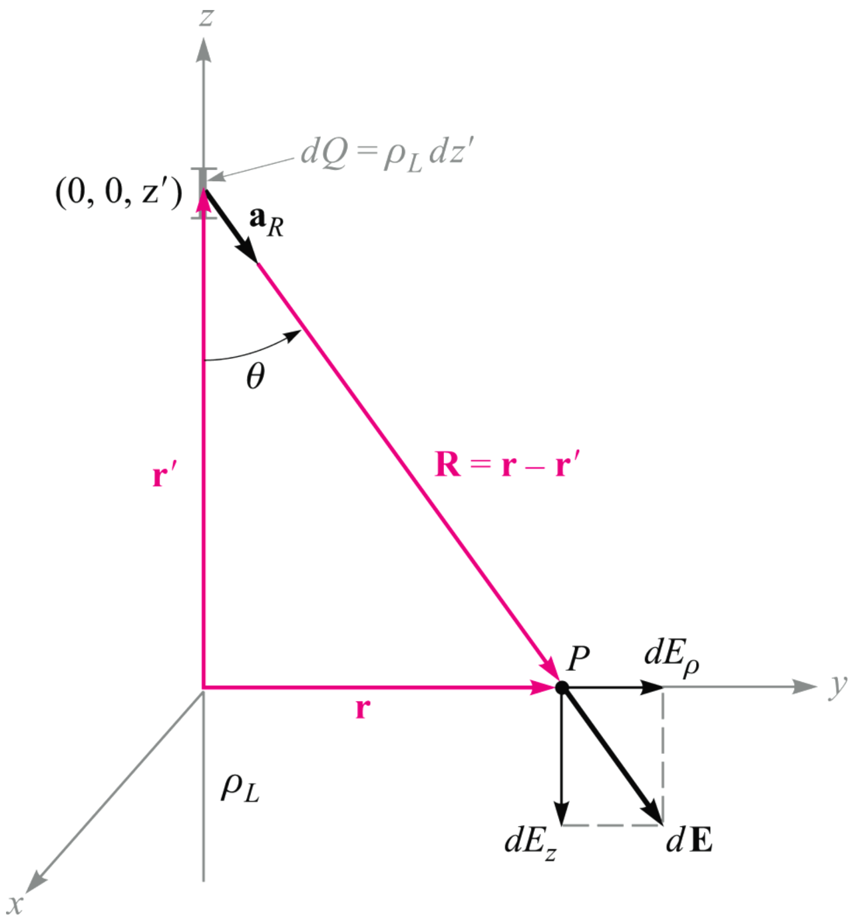
$$dE_\rho = \frac{\rho_L \rho dz'}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$$

$$E_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho dz'}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$$

$$= \frac{\rho_L}{4\pi\epsilon_0}\rho\left(\frac{1}{\rho^2}\frac{z'}{\sqrt{\rho^2 + z'^2}}\right)_{-\infty}^{\infty}$$

Line charge electric field

Line charge of constant density ρ_L Coul/m lies along the entire z axis.

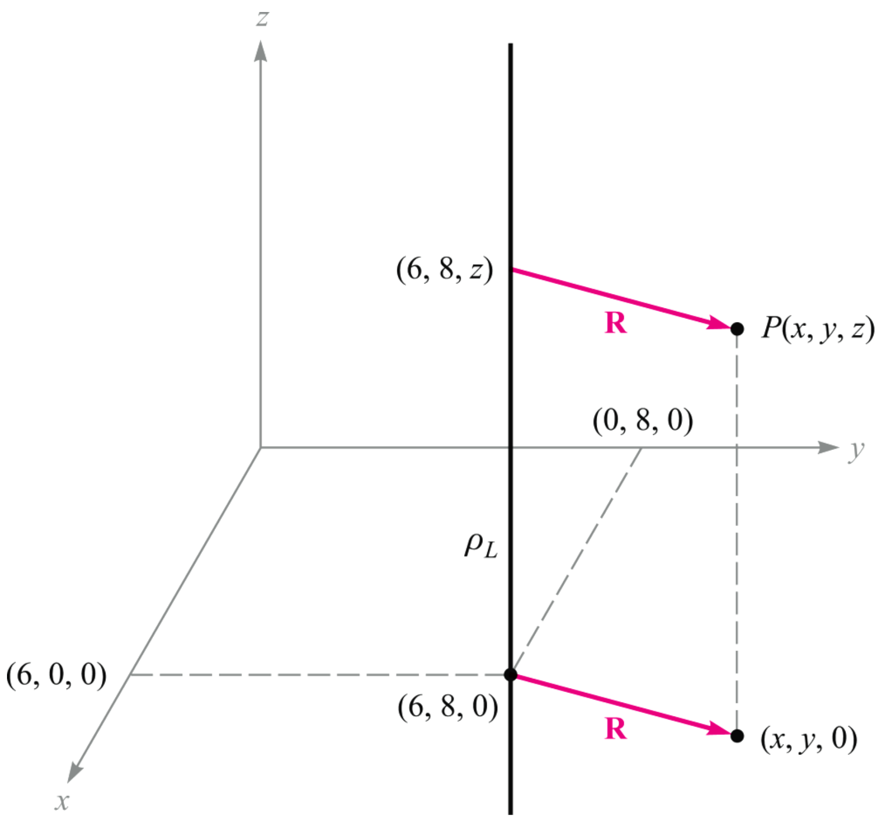


$$E_\rho = \frac{\rho_L}{4\pi\epsilon_0}\rho\left(\frac{1}{\rho^2}\frac{z'}{\sqrt{\rho^2 + z'^2}}\right)_{-\infty}^{\infty}$$

$$= \frac{\rho_L}{2\pi\epsilon_0\rho}$$

$$\boxed{\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\mathbf{a}_\rho}$$

Off-axis line charge



With the line displaced to (6,8), the field becomes:

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\sqrt{(x-6)^2+(y-8)^2}}\mathbf{a}_R$$

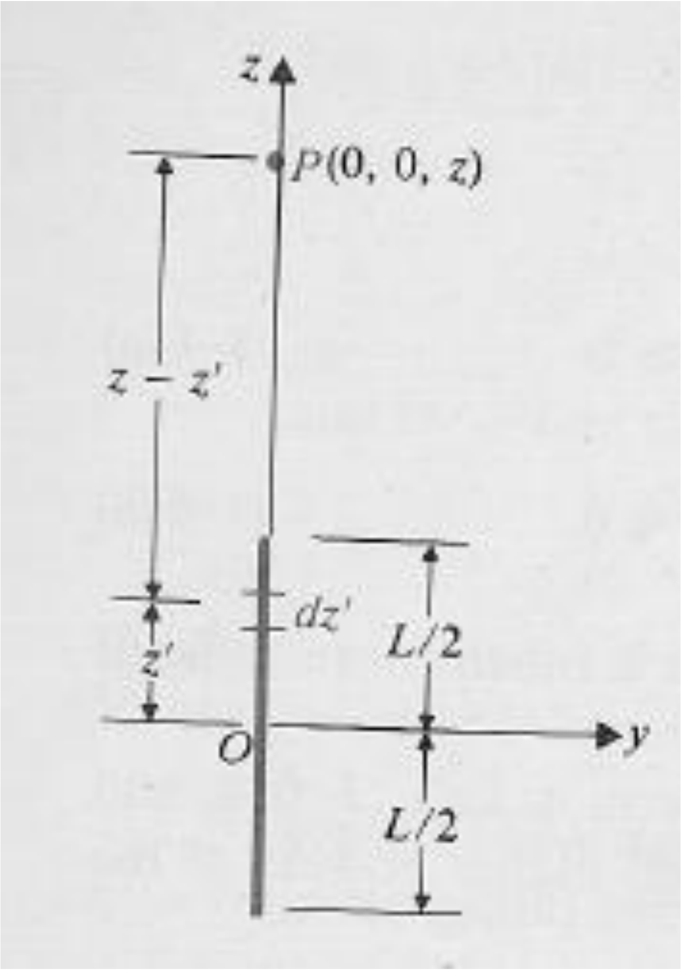
where

$$\mathbf{a}_R = \frac{\mathbf{R}}{|\mathbf{R}|} = \frac{(x-6)\mathbf{a}_x+(y-8)\mathbf{a}_y}{\sqrt{(x-6)^2+(y-8)^2}}$$

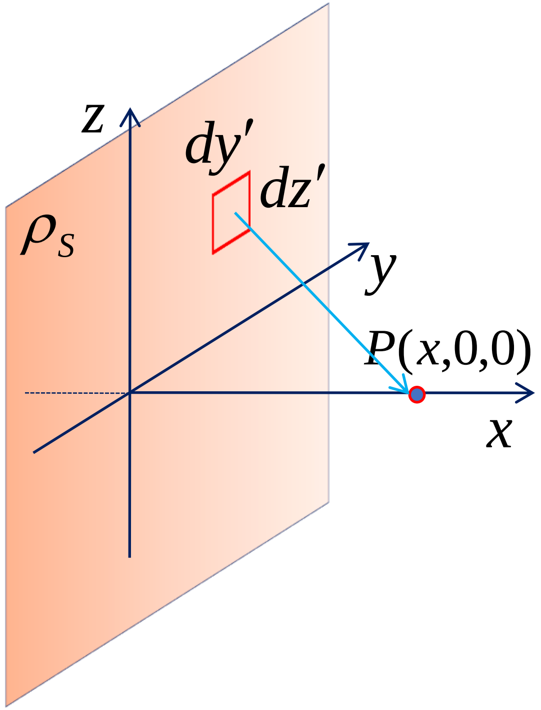
Finally:

$$\boxed{\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0}\frac{(x-6)\mathbf{a}_x+(y-8)\mathbf{a}_y}{(x-6)^2+(y-8)^2}}$$

Finite line charge



Sheet charge field



$$d\mathbf{E} = \frac{\rho_S dS}{4\pi\varepsilon_o |\mathbf{r}-\mathbf{r}'|^2} \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$$dS = dy'dz' \qquad \mathbf{r} = x\mathbf{a_x}$$

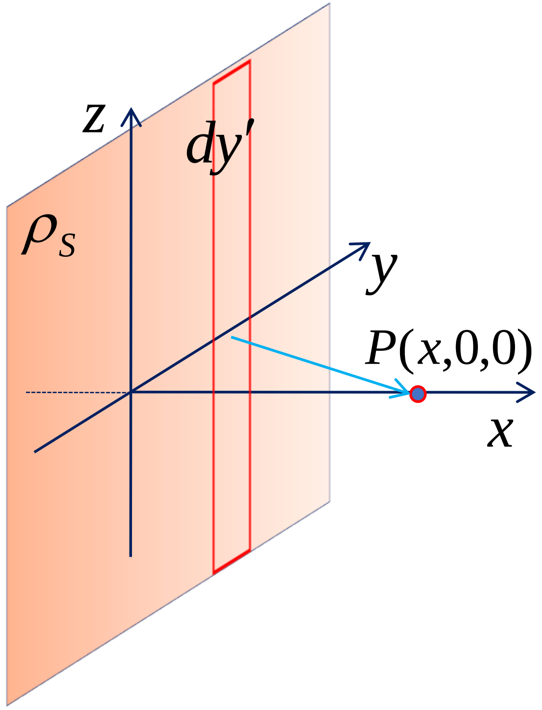
$$\mathbf{r}' = y'\mathbf{a_y} + z'\mathbf{a_z}$$

$$d\mathbf{E} = \frac{\rho_S dy'dz'}{4\pi\varepsilon_o \left(x^2 + y'^2 + z'^2\right)} \frac{x\mathbf{a_x} - y'\mathbf{a_y} - z'\mathbf{a_z}}{\sqrt{x^2 + y'^2 + z'^2}}$$

$$\mathbf{E} = \int \int_{-\infty}^{\infty} \frac{\rho_S}{4\pi\varepsilon_o \left(x^2 + y'^2 + z'^2\right)} \frac{x\mathbf{a_x} - y'\mathbf{a_y} - z'\mathbf{a_z}}{\sqrt{x^2 + y'^2 + z'^2}} dy'dz'$$

$$\boxed{\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}\mathbf{a_x} = \frac{\rho_S}{2\varepsilon_o}\mathbf{a_N}}$$

Sheet charge field

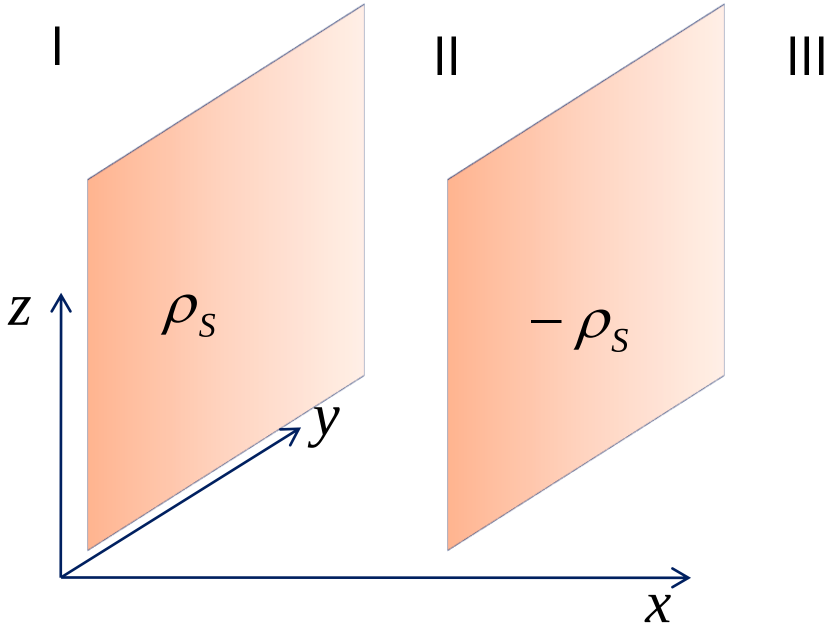


$$d\mathbf{E} = \frac{\rho_S dy'}{2\pi\varepsilon_o\sqrt{x^2+y'^2}}\frac{x\mathbf{a_x} - y'\mathbf{a_y}}{\sqrt{x^2+y'^2}}$$

$$\mathbf{E} = \int_{-\infty}^{\infty}\frac{\rho_S dy'}{2\pi\varepsilon_o\sqrt{x^2+y'^2}}\frac{x\mathbf{a_x} - y'\mathbf{a_y}}{\sqrt{x^2+y'^2}}$$

$$\boxed{\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}\mathbf{a_x} = \frac{\rho_S}{2\varepsilon_o}\mathbf{a_N}}$$

Sheet charge field



$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}\mathbf{a}_x = \frac{\rho_S}{2\varepsilon_o}\mathbf{a}_N$$

- Region I

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}(-\mathbf{a}_x) + \frac{-\rho_S}{2\varepsilon_o}(-\mathbf{a}_x) = 0$$

- Region II

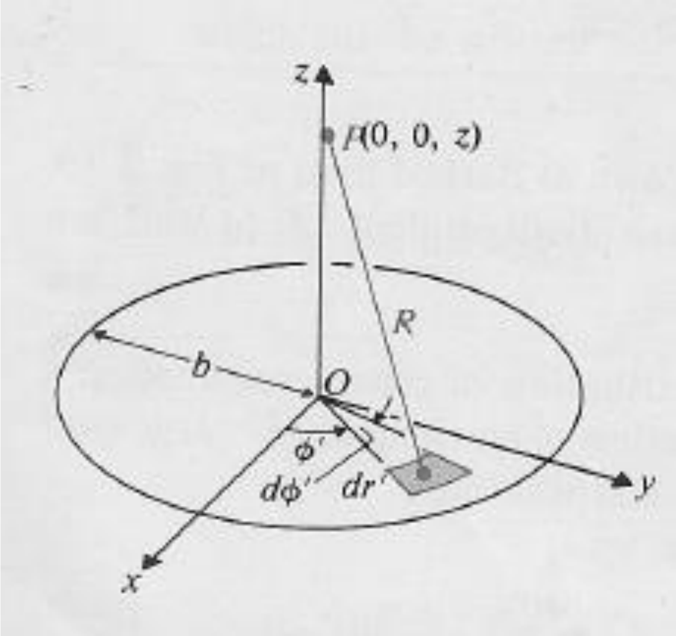
$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}\mathbf{a}_x + \frac{-\rho_S}{2\varepsilon_o}(-\mathbf{a}_x) = \frac{\rho_S}{\varepsilon_o}\mathbf{a}_x$$

- Region III

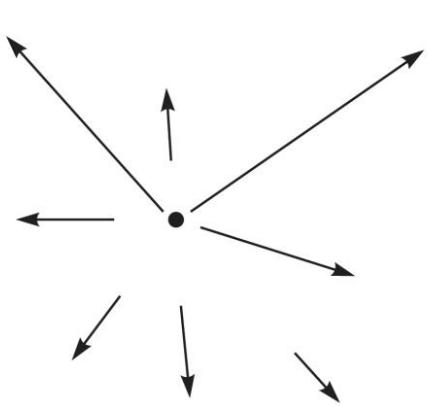
$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}\mathbf{a}_x + \frac{-\rho_S}{2\varepsilon_o}\mathbf{a}_x = 0$$

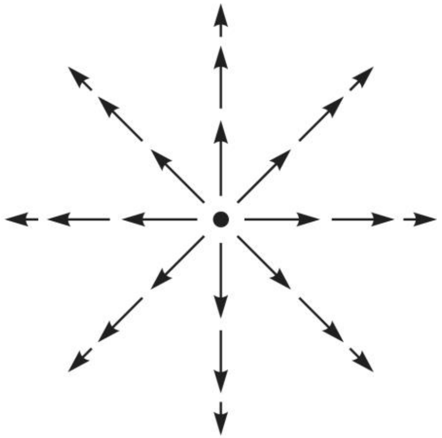
Example

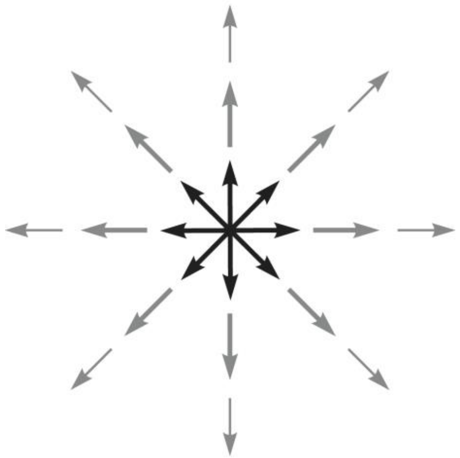
EXAMPLE 3–9 Obtain a formula for the electric field intensity on the axis of a circular disk of radius b that carries a uniform surface charge density ρ_s.

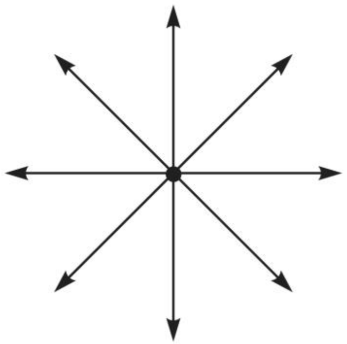


Type of field visualization


(a)


(b)


(c)


(d)

Chapter Summary

- Coulomb's Experimental Law 소개
- Electric Field Intensity (**E**) 개념 이해
- Coulomb's Law를 이용하여 몇 가지 단순한 전하 분포에서의 **E** 계산 연습
  - 점전하 (하나, 여러 개)
  - 선전하 (무한/유한길이, 직선/원형)
  - 면전하 (Infinite plane, Disk of finite radius)
  - (부피전하) → 다음 chapter까지 연기
- Electric Field 가시화 - Streamline