Engineering Circuits Analysis (ICE2002)

Chapter 6. Inductance, Capacitance, and Mutual Inductance - Part1/2/3

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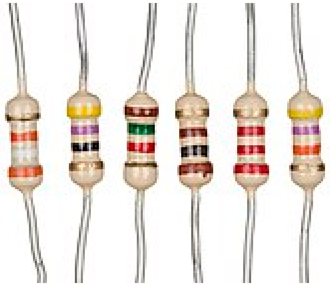
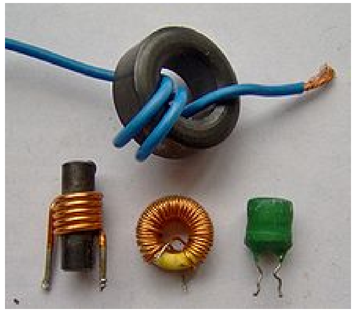
Voltage & current sources

Resistors



Capacitors

Inductors



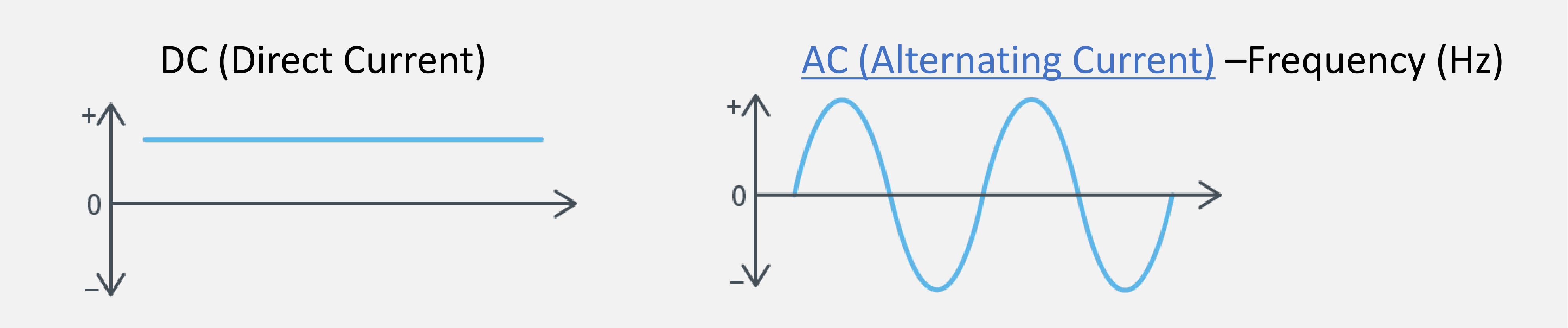
Circuit Elements

- **5 ideal basic circuit elements**
  - Voltage source
  - Current source

  Active elements, capable of generating electric energy

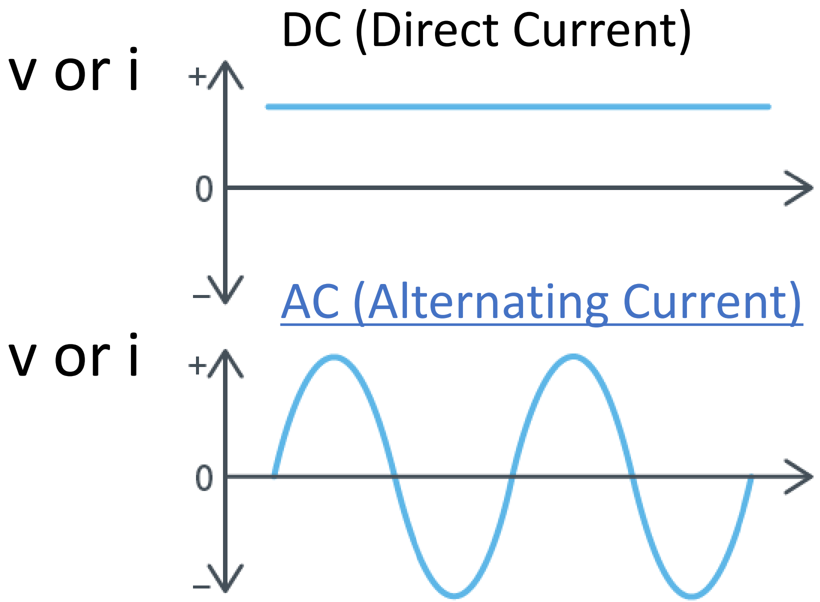
  - Resistor
  - Inductor
  - Capacitor

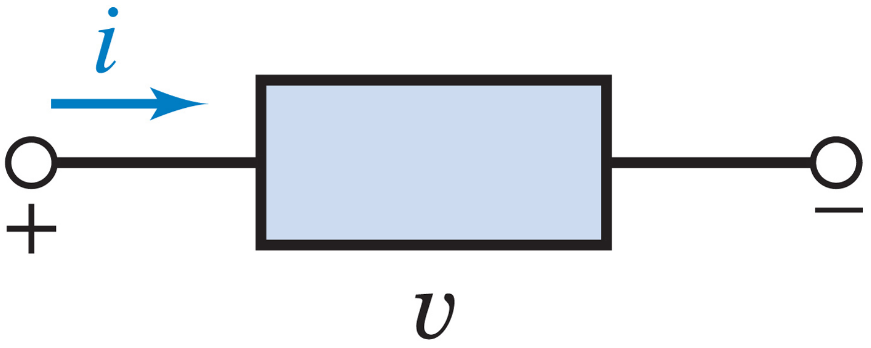
  Passive elements , incapable of generating electric energy



In Chapter 6,

- AC voltage sources, current sources, and inductors/capacitors can be described by plotting the voltage (v)/ current (i) as a function of current (i)/ voltage (v).



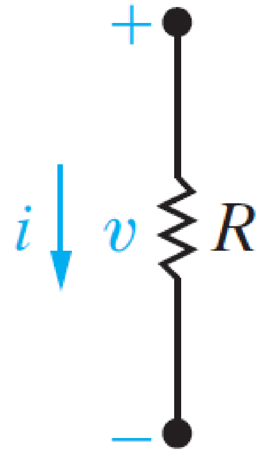


Review Chapter 2: Resistor and Ohm's Law

- Ohm's law establishes the proportionality of voltage and current in a resistor. It states that the voltage across a resistor is directly proportional to the current I flowing through the resistor.

$$v = Ri$$

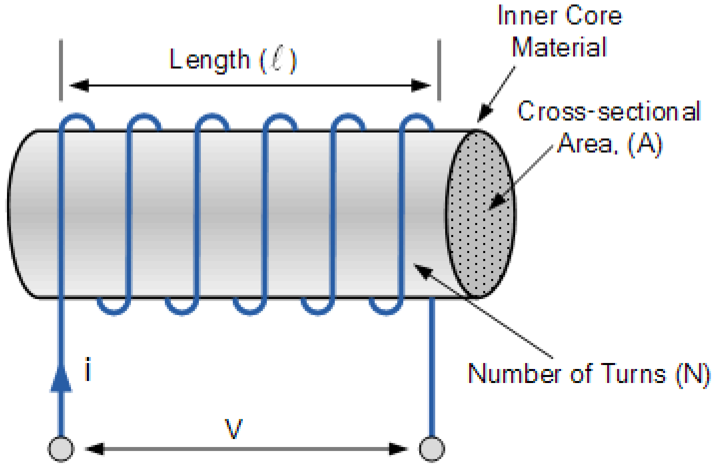
where
v = the voltage in volts,
i = the current in amperes,
R = the resistance in ohms.

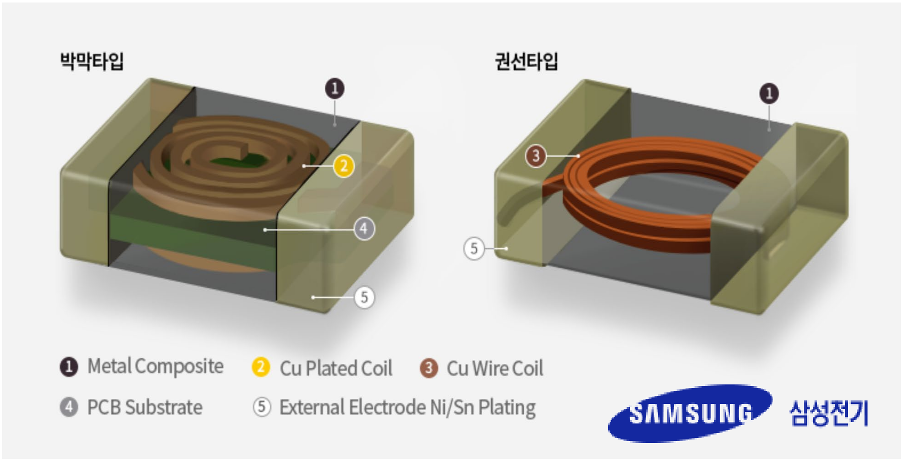


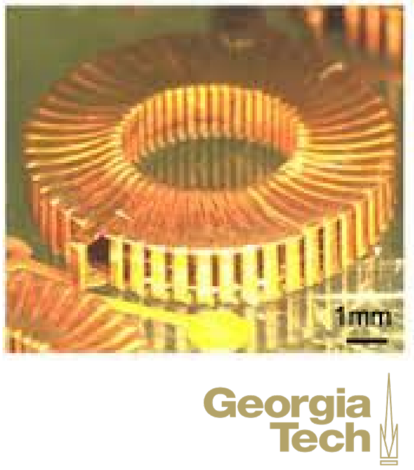
$v = iR$

Inductor

- An inductor is a passive element designed to store energy in its magnetic field.
- It is a coil of wire wound around supporting magnetic/or non-magnetic core material.



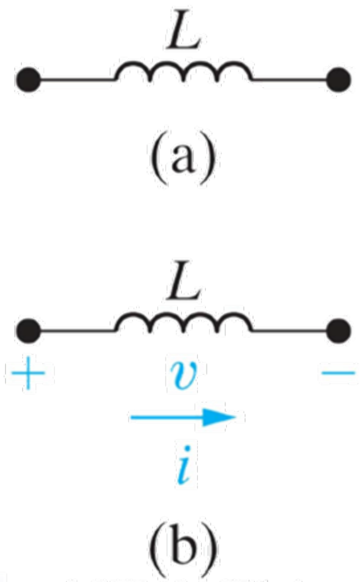




Ref) https://www.electronics-tutorials.ws/inductor/inductor.html

Inductor

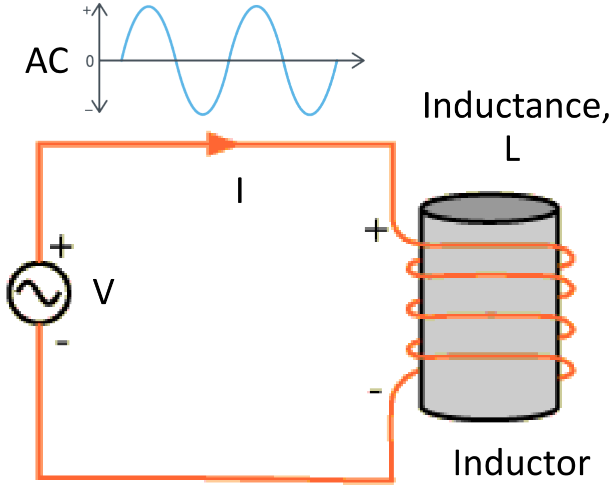
- Inductance is a linear circuit parameter that relates the voltage induced by a time-varying magnetic field to the current producing the field.



The graphic symbol for an inductor with an inductance of henrys [H].

Assigning reference voltage and current to the inductor following the passive sign convention.

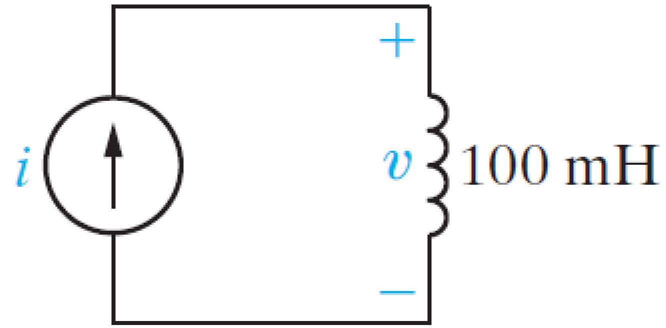
$$v = L\frac{di}{dt}$$



Where
v is measured in volts [V], L in henrys [H], i in amperes [A], and t in seconds [s].

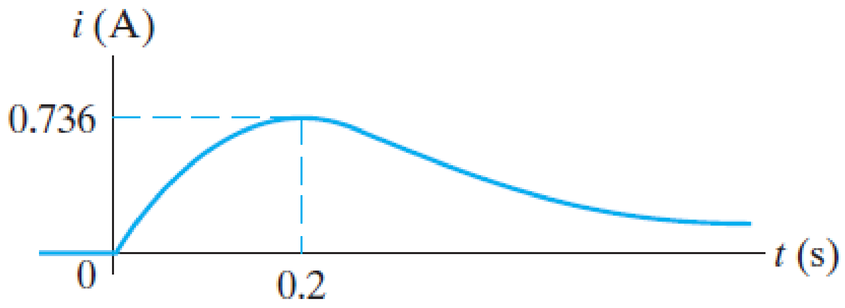
Example 6.1

Q. The independent current source in the circuit generates the pulse current.

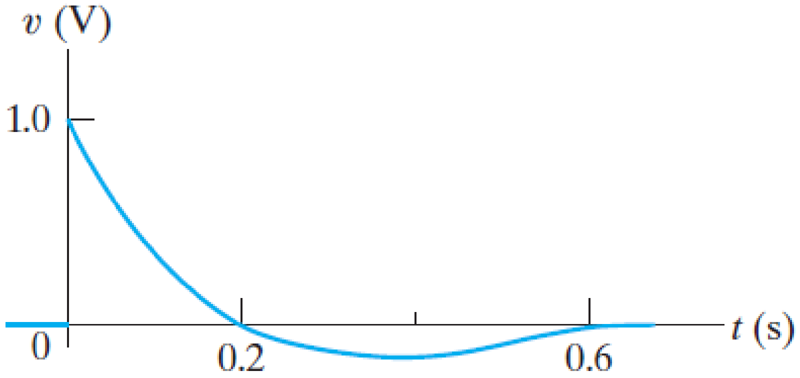


$$i = 0, \qquad t < 0$$

$$i = 10te^{-5t}\,\text{A}, \qquad t > 0$$



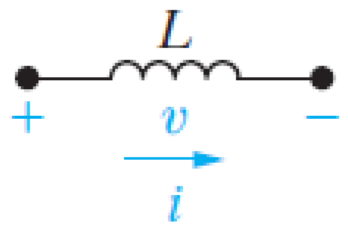
Current waveform



Voltage waveform

Voltage, Current, Power, Energy in an Inductor

- Inductor I (current) - V (voltage) equation



$$v = L\frac{di}{dt}$$

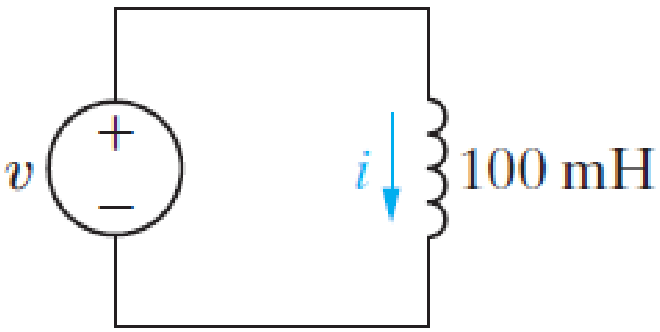
where
v is measured in volts [V], *L* in henrys [H],
i in amperes [A], and t in seconds [s].

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v dt + i(t_0)$$

where
$i(t_0)$ is the value of the inductor current at the time when we initiate the integration, namely, t_0.

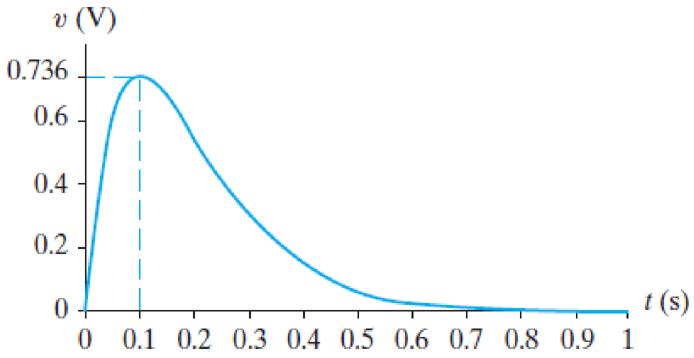
Example 6.2

Q. The independent voltage source in the circuit below generates the voltage pulse.

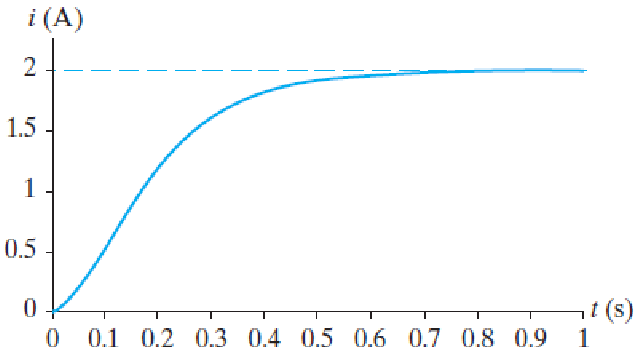


$$v = 0, \qquad t < 0$$

$$v = 20te^{-10t}\ \text{V}, \quad t > 0$$



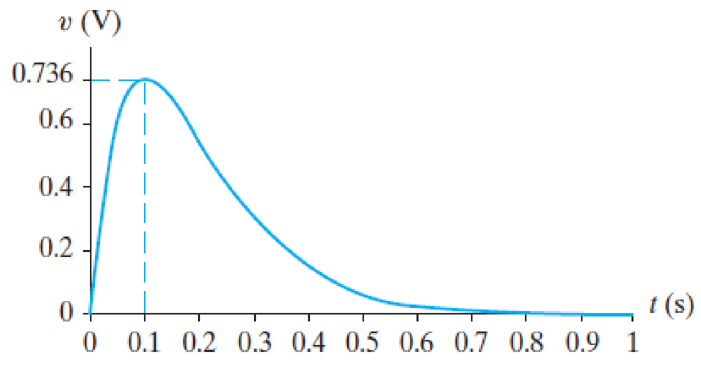
Voltage waveform



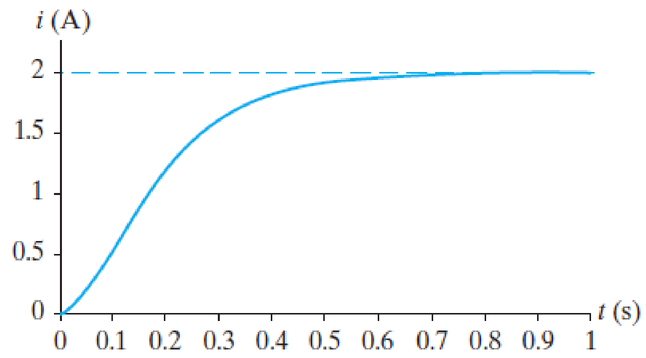
Current waveform

Example 6.2

Q. The independent voltage source in the circuit below generates the voltage pulse.



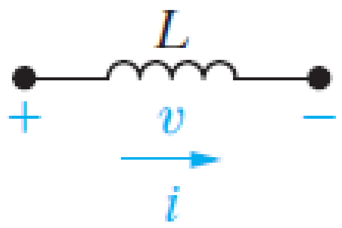
Voltage waveform



Current waveform

Voltage, Current, Power, Energy in an Inductor

- Power (P) and Energy (w) in an Inductor



P and w

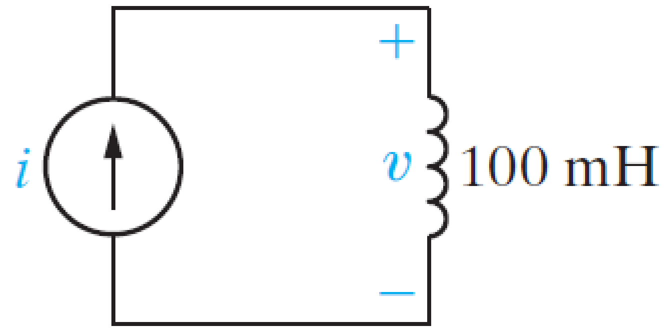
$$P = \left(L\frac{di}{dt}\right)i \quad [W]$$

$$P = v\left(\frac{1}{L}\int_{t0}^{t} v\,d\tau + i(t0)\right) \quad [W]$$

$$w = \frac{1}{2}Li^2 \quad [J]$$

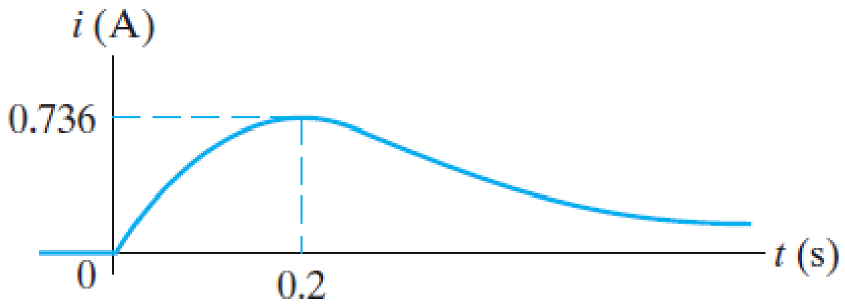
Example 6.3

Q. Find i, v, P and w of the circuit below

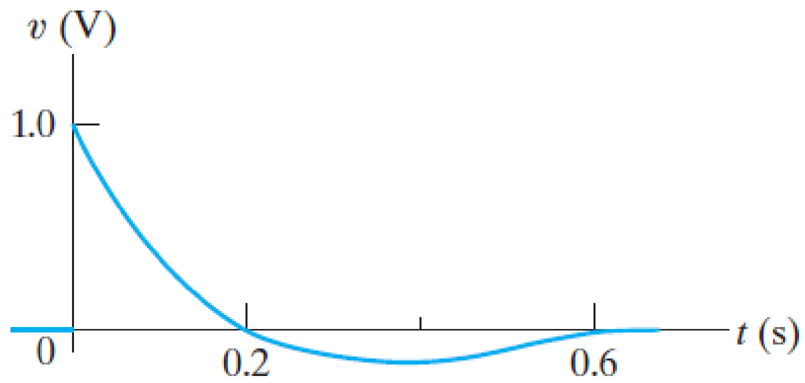


$$i = 0, \qquad t < 0$$

$$i = 10te^{-5t}\text{A}, \qquad t > 0$$



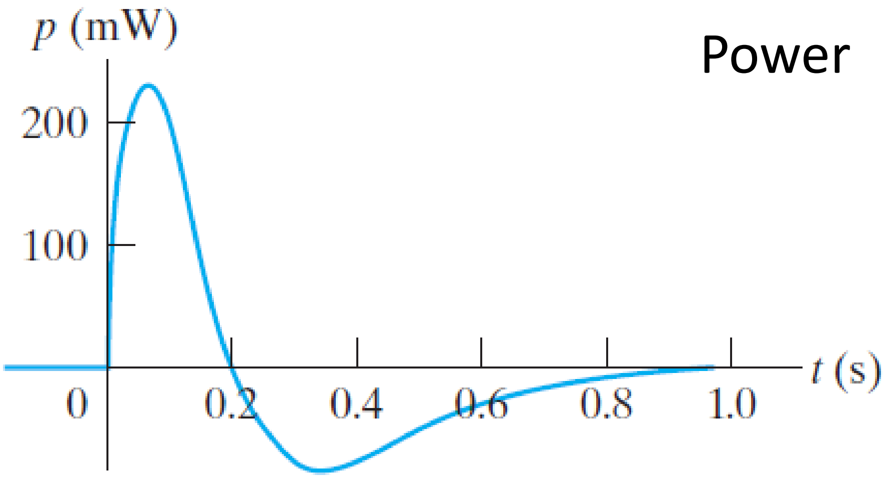
Current waveform

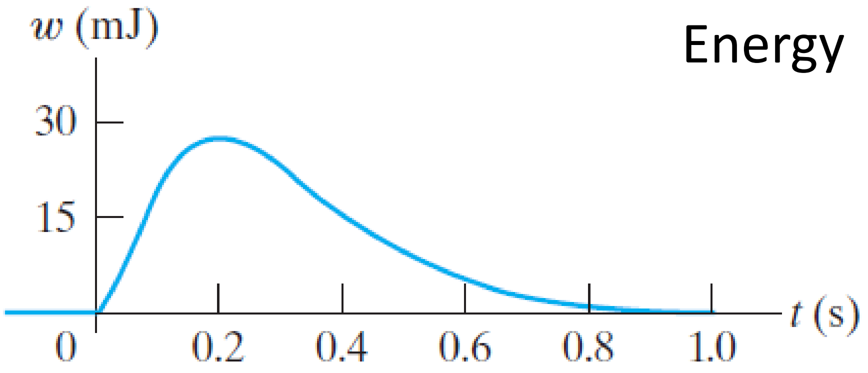


Voltage waveform

Example 6.3

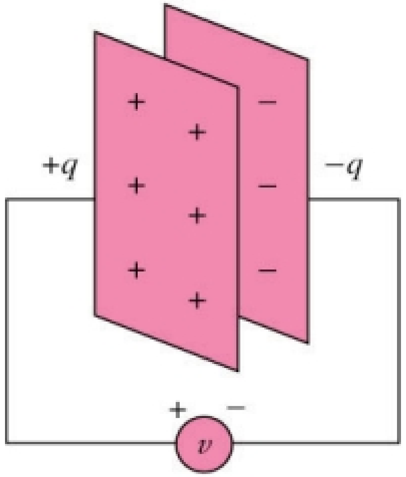
Q. Find i, v, P and w of the circuit below

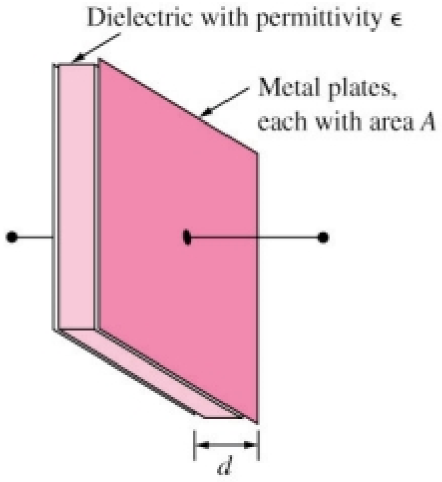




Capacitor

- A capacitor is a passive element designed to store energy in its electric field.
- It consists of two conducting plates separated by an insulator (or dielectric).



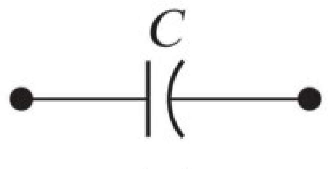


$$q = C\,v \quad \text{and} \quad C = \frac{\varepsilon A}{d}$$

- *where*
  - *ε is the permittivity of the dielectric material between the plates,*
  - *A is the surface area of each plate,*
  - *d is the distance between the plates.*
- *Unit: F, pF (10^{-12}), nF (10^{-9}), and μF (10^{-6})*

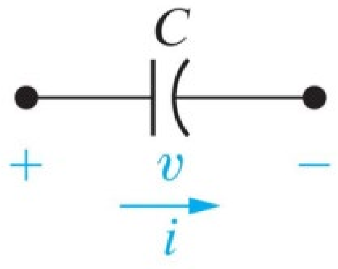
Capacitor

- Capacitance is a linear circuit parameter that relates the current induced by a time-varying electric field to the voltage producing the field.



(a)

The graphic symbol for a capacitor with a capacitance of farads [F].

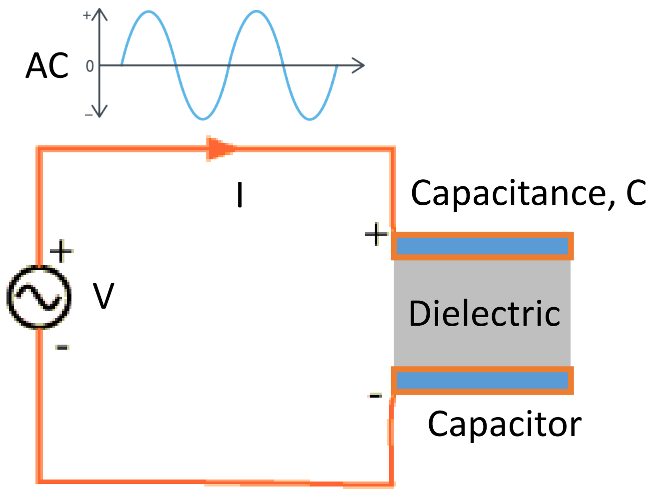


(b)

Assigning reference voltage and current to the capacitor, following the passive sign convention.

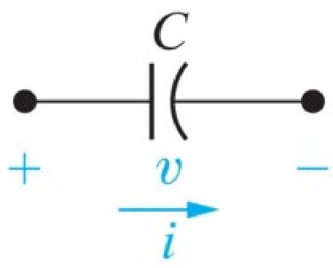
$$i = C\frac{dv}{dt}$$

where
i in amperes [A], C in farads [F],
v is measured in volts [V],
and t in seconds [s].



Voltage, Current, Power, Energy in a Capacitor

- Capacitor I (current) - V (voltage) equation



$$i = C\frac{dv}{dt}$$

where
i in amperes [A], *C* in farads [F],
v is measured in volts [V],
and t in seconds [s].

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i dt + v(t_0)$$

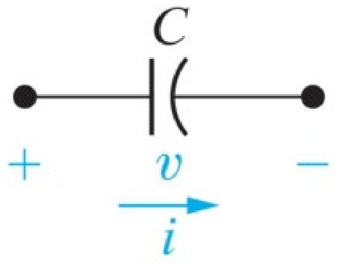
where
$v(t_0)$ is the value of the capacitor voltage at the time when we initiate the integration, namely, t_0.

Voltage, Current, Power, Energy in an Inductor

- Power (P) and Energy (w) in an Inductor

$$P = v\left(C\frac{dv}{dt}\right) \quad [W]$$

$$P = \left(\frac{1}{C}\int_{t0}^{t} i d\tau + v(t_0)\right) i \quad [W]$$



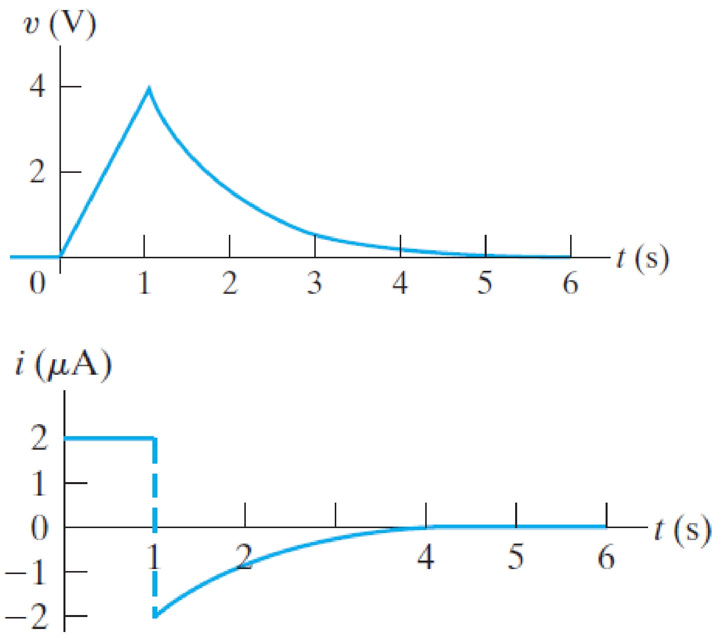
P and w

$$w = \frac{1}{2}Cv^2 \quad [J]$$

Example 6.4

Q. The voltage across the terminals of a 0.5 μF capacitor is:

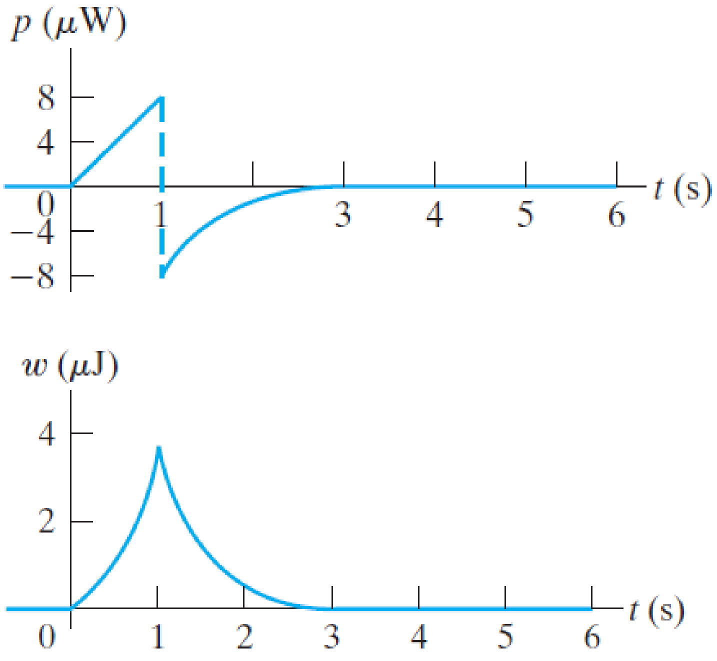
$$v(t) = \begin{cases} 0[V] & t \le 0s \\ 4t[V] & 0s \le t \le 1s \\ 4e^{-(t-1)}[V] & t \ge 1s \end{cases}$$



Example 6.4

Q. The voltage across the terminals of a 0.5 μF capacitor is:

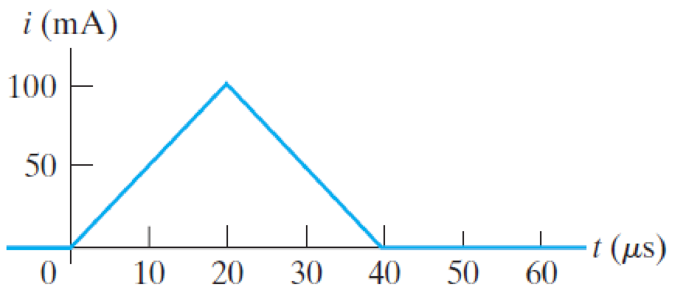
$$v(t) = \begin{cases} 0[V] & t \le 0s \\ 4t[V] & 0s \le t \le 1s \\ 4e^{-(t-1)}[V] & t \ge 1s \end{cases}$$

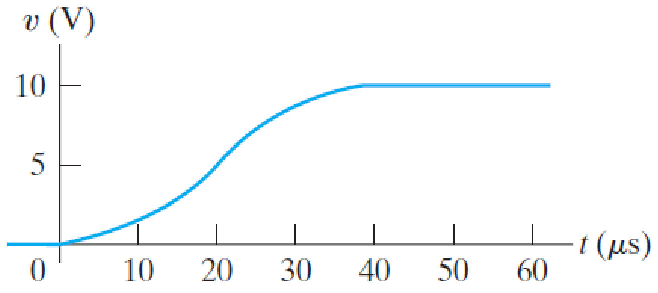


Example 6.4

Q. The current across the terminals of a 0.2 μF capacitor is:

$$
i(t) = \begin{cases} 0[A] & t \le 0 \\ 5000t[A] & 0s \le t \le 20\mu s \\ 0.2 - 5000t[A] & 20\mu s \le t \le 40\mu s \\ 0[A] & t \ge 40\mu s \end{cases}
$$

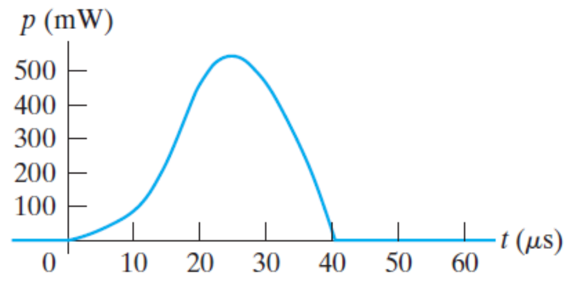


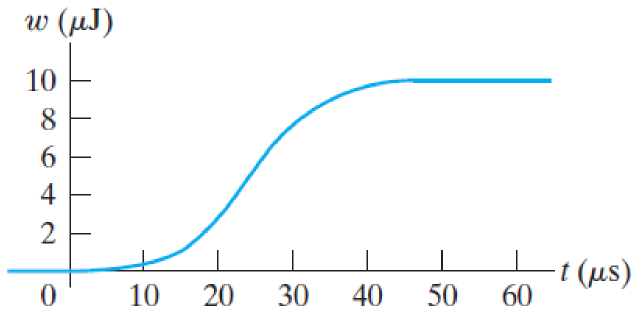


Example 6.4

Q. The current across the terminals of a 0.2 μF capacitor is:

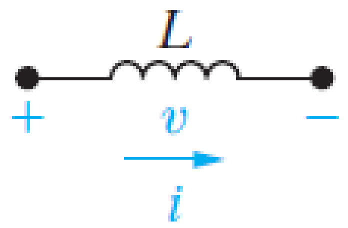
$$
i(t) = \begin{cases} 0[A] & t \le 0 \\ 5000t[A] & 0s \le t \le 20\mu s \\ 0.2 - 5000t[A] & 20\mu s \le t \le 40\mu s \\ 0[A] & t \ge 40\mu s \end{cases}
$$





V, I, P, W in an Inductor

- Inductor I (current) - V (voltage) equation



$$v = L\frac{di}{dt}$$

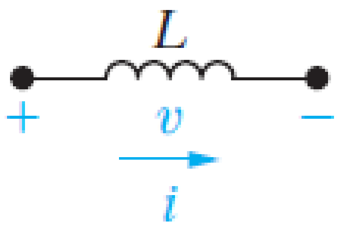
where
v is measured in volts [V], *L* in henrys [H],
i in amperes [A], and t in seconds [s].

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v dt + i(t_0)$$

where
$i(t_0)$ is the value of the inductor current at the time when we initiate the integration, namely, t_0.

V, I, P, W in an Inductor

- Power (P) and Energy (w) in an Inductor



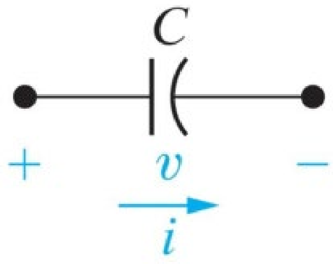
$$P = \left(L\frac{di}{dt}\right)i \quad [W]$$

$$P = v\left(\frac{1}{L}\int_{t0}^{t} v d\tau + i(t0)\right) \quad [W]$$

$$w = \frac{1}{2}Li^2 \qquad [J]$$

V, I, P, W in a Capacitor

- Capacitor I (current) - V (voltage) equation



$$i = C\frac{dv}{dt}$$

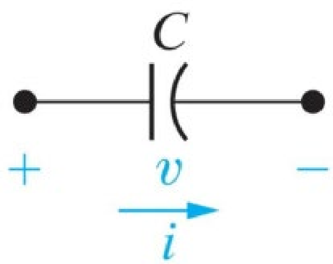
where
i in amperes [A], *C* in farads [F],
v is measured in volts [V],
and t in seconds [s].

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i dt + v(t_0)$$

where
$v(t_0)$ is the value of the capacitor voltage at the time when we initiate the integration, namely, t_0.

V, I, P, W in a Capacitor

- Power (P) and Energy (w) in a capacitor



P and w

$$P = v\left(C\frac{dv}{dt}\right) \quad [W]$$

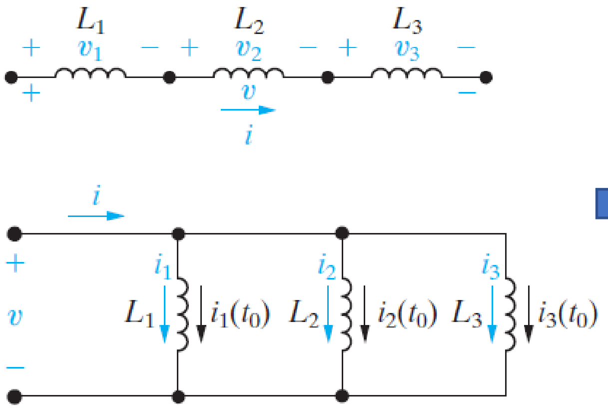
$$P = \left(\frac{1}{C}\int_{t0}^{t} i d\tau + v(t_0)\right) i \quad [W]$$

$$w = \frac{1}{2}Cv^2 \qquad [J]$$

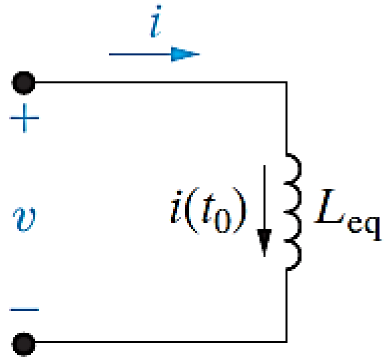
Series-Parallel Combinations of L and C

- Series-parallel combinations of inductors or capacitors can be redu ced to a single inductor or capacitor.

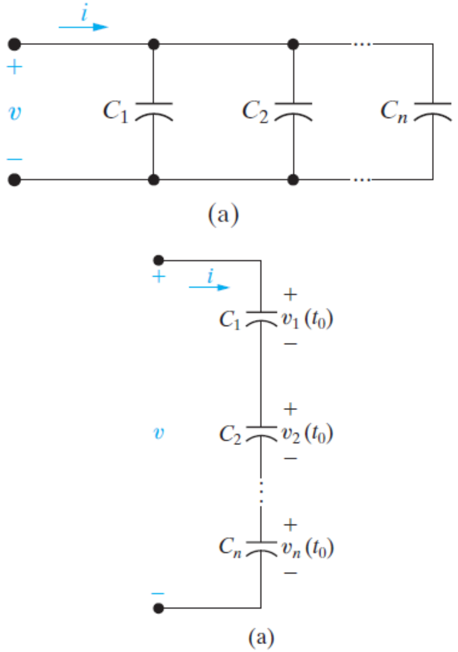
Inductors in
series and parallel



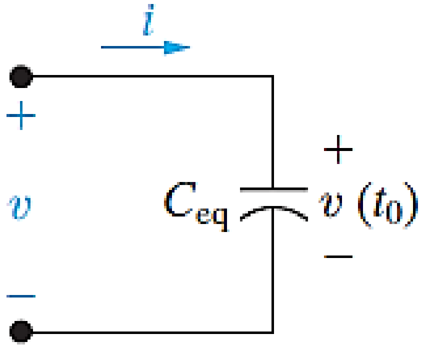
Equivalent inductor



Capacitors in
series and parallel

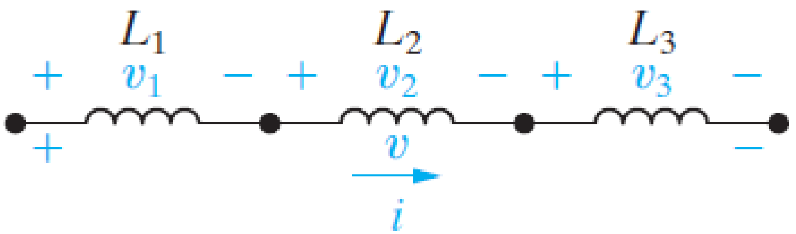


Equivalent capacitor



Inductors in Series and Parallel

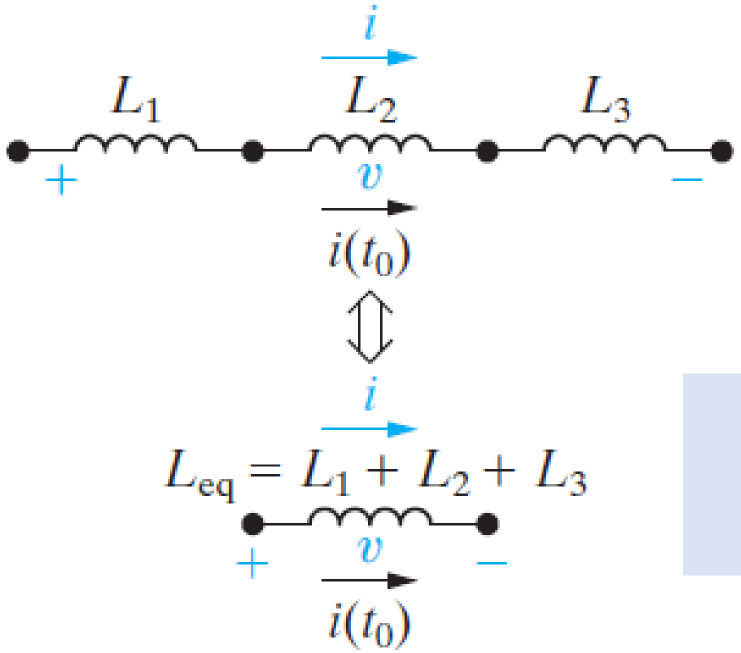
Inductors in series



$$v = v_1 + v_2 + v_3$$

$$v_1 = L_1 \frac{di}{dt} \qquad v_2 = L_2 \frac{di}{dt} \qquad v_3 = L_3 \frac{di}{dt}$$

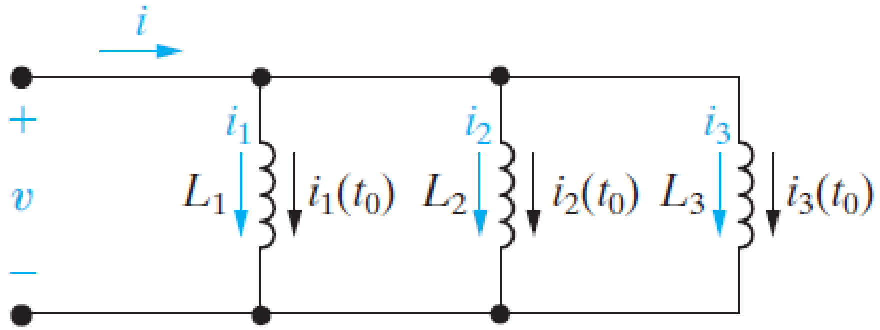
Combining inductors in series



$$L_{eq} = \sum_{i=1}^{n} L_i$$

Inductors in Series and Parallel

Inductors in parallel



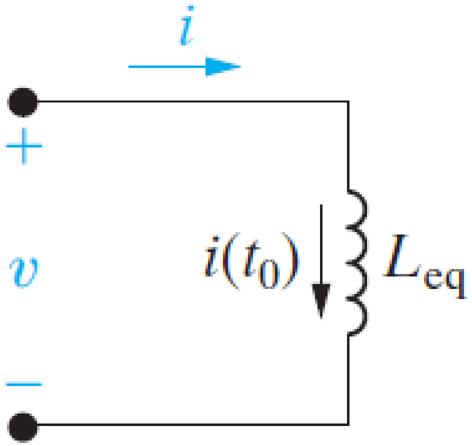
$$i = i_1 + i_2 + i_3$$

$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0)$$

$$i_2 = \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0)$$

$$i_3 = \frac{1}{L_3}\int_{t_0}^{t} v\,dt + i_3(t_0)$$

Combining inductors in parallel



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

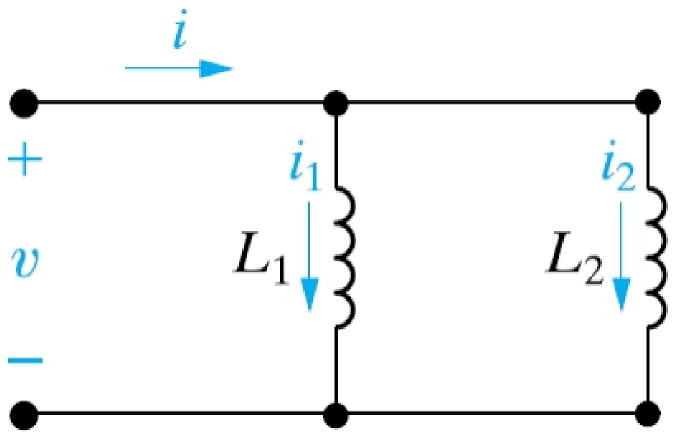
$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

$$\frac{1}{L_{eq}} = \sum_{i=1}^{n} \frac{1}{L_i}$$

$$i(t_0) = \sum_{j=1}^{n} i_j(t_0)$$

Inductors in Series and Parallel

Example



$$(1)\ L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

$$(2)\ i_1 = \frac{L_2}{L_1 + L_2} i$$

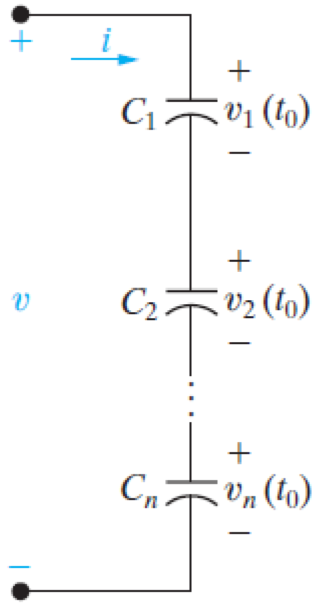
$$(3)\ L_1 i_1 = L_2 i_2$$

Assume that $L_1 = 3$ [mH], $L_2 = 2$ [mH], i=10[A], find i_1 in the given circuit.

$$i_1 = \frac{2}{3+2} \times 10 = 4\,[A]$$

Capacitors in Series and Parallel

Capacitors in series



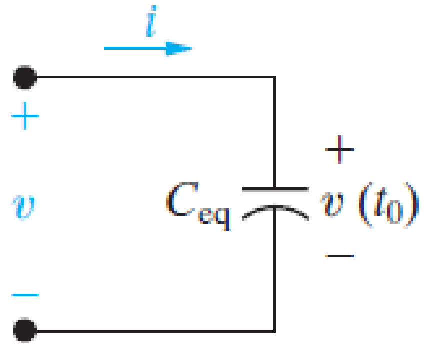
$$v = v_1 + v_2 + v_3$$

$$v_1(t) = \frac{1}{C_1}\int_{t_0}^{t} i dt + v_1(t_0)$$

$$v_2(t) = \frac{1}{C_2}\int_{t_0}^{t} i dt + v_2(t_0)$$

$$v_3(t) = \frac{1}{C_3}\int_{t_0}^{t} i dt + v_3(t_0)$$

Combining Capacitors in series



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

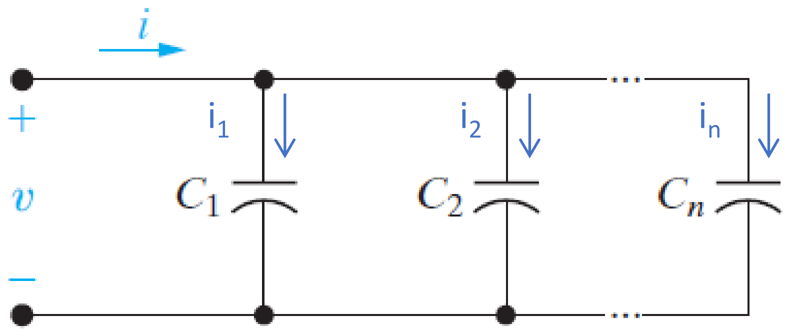
$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_n(t_0)$$

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

$$v(t_0) = \sum_{j=1}^{n} v_j(t_0)$$

Capacitors in Series and Parallel

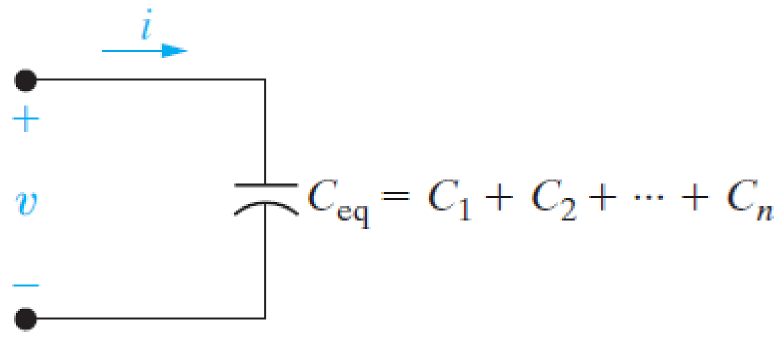
Capacitors in parallel



$i = i_1 + \ i_2 + \ i_3$

$i_1 = C_1 \dfrac{dv}{dt} \qquad i_2 = C_2 \dfrac{dv}{dt} \qquad i_3 = C_3 \dfrac{dv}{dt}$

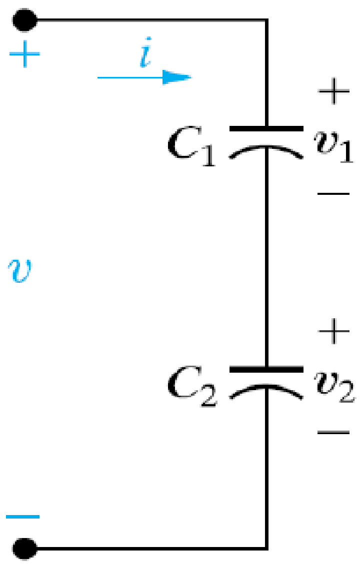
Combining Capacitors in parallel



$$C_{eq} = \sum_{i=1}^{n} C_i$$

Capacitors in Series and Parallel

Example



$$(1)\ C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$(2)\ v_1 = \frac{C_2}{C_1 + C_2} v$$

$$(3)\ C_1 v_1 = C_2 v_2$$

$$(4)\ Q_1 = Q_2$$

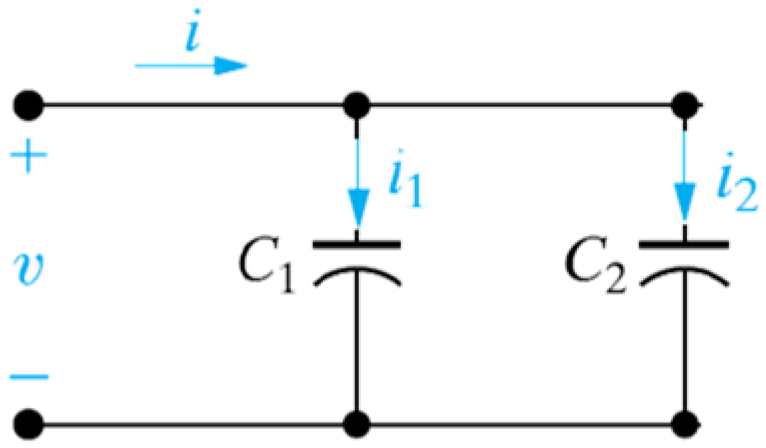
Assume that C_1 = 3 [μF], C_2 = 2 [μF], V_1 = 10 [V], find V_2 and V in the given circuit.

$$v_2 = \frac{C_1}{C_2} v_1 = \frac{3}{2} \times 10 = 15\,[V]$$

$$v = v_1 + v_2 = 10 + 15 = 25\,[V]$$

Capacitors in Series and Parallel

Example



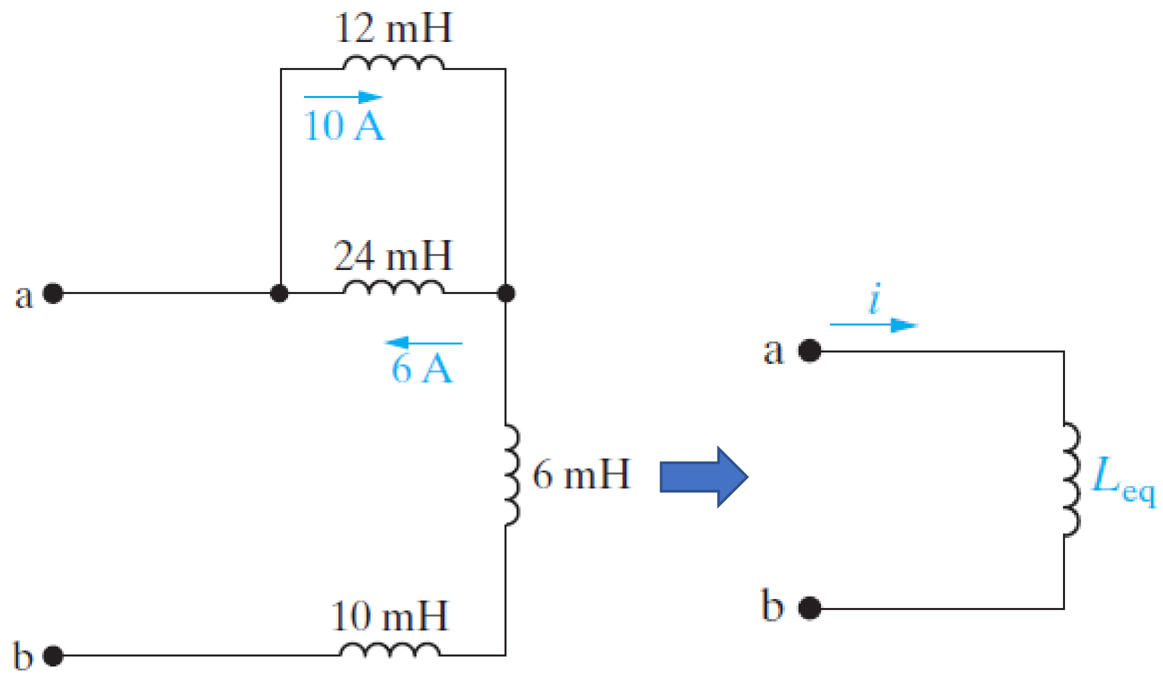
$$\boxed{\begin{aligned} &(1)\ C_{eq} = C_1 + C_2 \\ &(2)\ i_1 = \frac{C_1}{C_1 + C_2} i \\ &(3)\ C_1 i_2 = C_2 i_1 \end{aligned}}$$

Assume that $C_1 = 3$ [μF], $C_2 = 2$ [μF], $i_1 = 6$[A], find i in the given circuit.

$$i = \frac{C_1 + C_2}{C_1} i_1 = \frac{3+2}{3} \times 6 = 10\,[A]$$

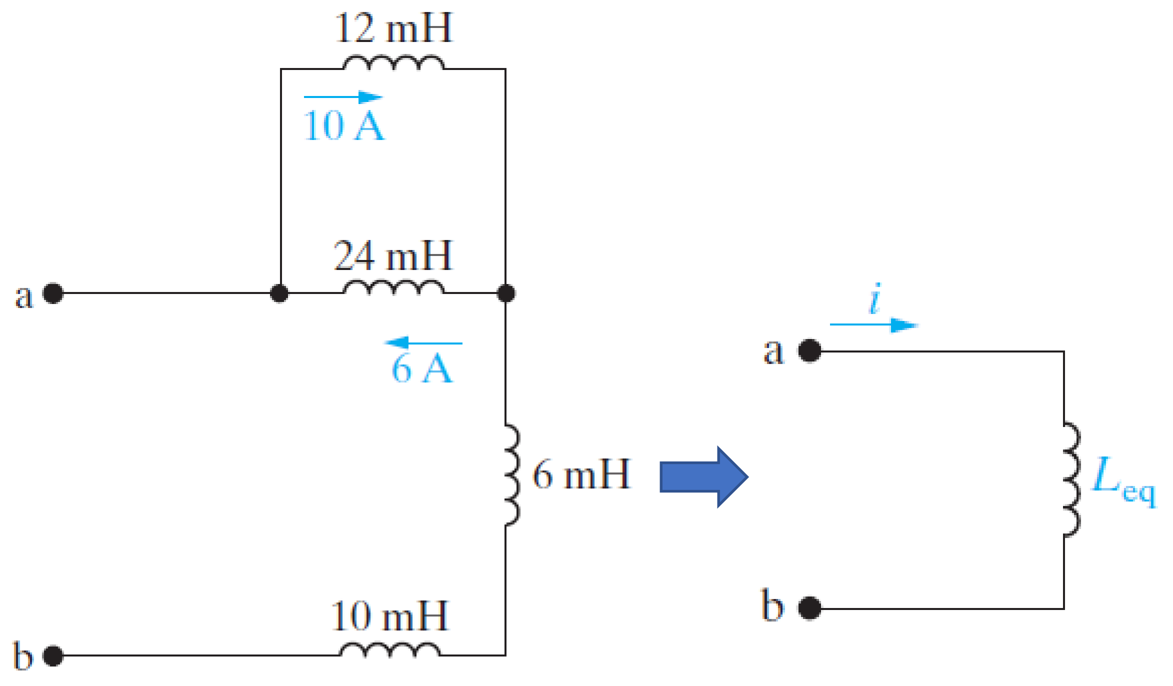
Example 6.6

Q. Find the equivalent inductance, L_{eq}.
Find the initial current in the equivalent inductor.



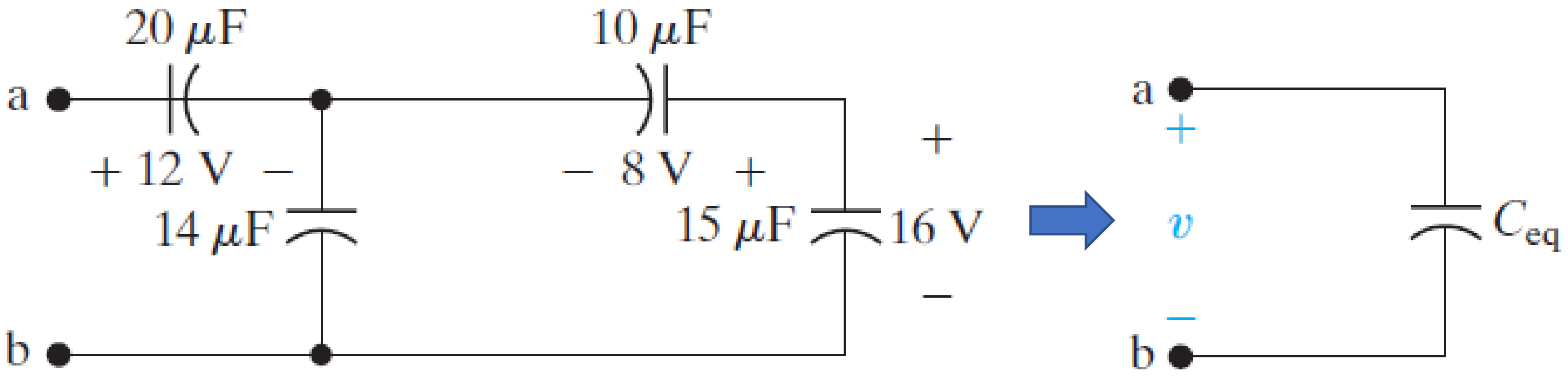
Example 6.6

Q. Find the equivalent inductance, L_{eq}.
Find the initial current in the equivalent inductor.



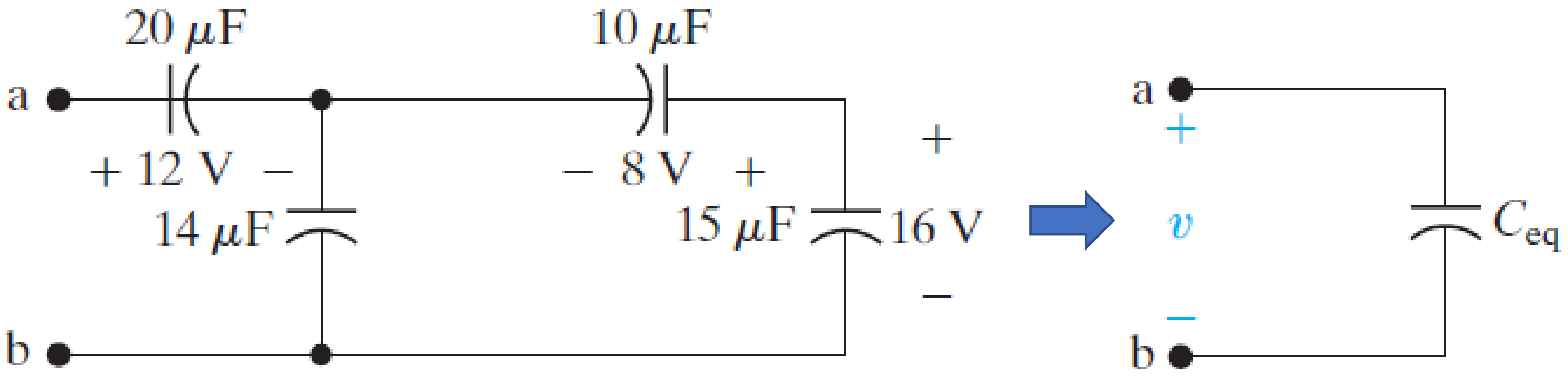
Example 6.7

Q. Find the equivalent Capacitance, C_{eq}.
Find the initial voltage across the equivalent capacitor.



Example 6.7

Q. Find the equivalent Capacitance, C_{eq}.
Find the initial voltage across the equivalent capacitor.



Summary

- Inductor & Capacitor
  - IV equation
  - power and energy
- Series-Parallel Combinations of Inductance and Capacitance

>> Summarized in Table 6.1

Table 6.1

	Inductors	Capacitors
Primary v-i equation	$v(t) = L\frac{di(t)}{dt}$	$i(t) = C\frac{dv(t)}{dt}$
Alternate v-i equation	$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$	$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$
Initial condition	$i(t_0)$	$v(t_0)$
Behavior with a constant source	If $i(t) = I, v(t) = 0$ and the inductor behaves like a short circuit	If $v(t) = V, i(t) = 0$ and the capacitor behaves like an open circuit
Continuity requirement	$i(t)$ is continuous for all time so $v(t)$ is finite	$v(t)$ is continuous for all time so $i(t)$ is finite
Power equation	$p(t) = v(t)i(t) = Li(t)\frac{di(t)}{dt}$	$p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$
Energy equation	$w(t) = \frac{1}{2}Li(t)^2$	$w(t) = \frac{1}{2}Cv(t)^2$
Series-connected equivalent	$L_{eq} = \sum_{j=1}^{n} L_j$ $i_{eq}(t_0) = i_j(t_0)$ for all j	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ $v_{eq}(t_0) = \sum_{j=1}^{n} v_j(t_0)$
Parallel-connected equivalent	$\frac{1}{L_{eq}} = \sum_{j=1}^{n} \frac{1}{L_j}$ $i_{eq}(t_0) = \sum_{j=1}^{n} i_j(t_0)$	$C_{eq} = \sum_{j=1}^{n} C_j$ $v_{eq}(t_0) = v_j(t_0)$ for all j