Engineering Circuits Analysis (ICE2002)

Chapter 3. Simple Resistive Circuits – Part 1/2

Contents

Review Chapter 2: Ohm's Law

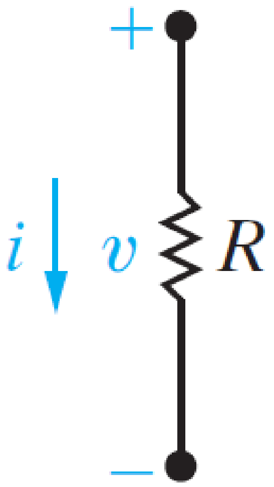
- Ohm's law establishes the proportionality of voltage and current in a resistor. It states that the voltage across a resistor is directly proportional to the current I flowing through the resistor.
- Two extreme possible values of R:
  zero (R = 0) – short circuit and infinite – open circuit

$$v = Ri$$

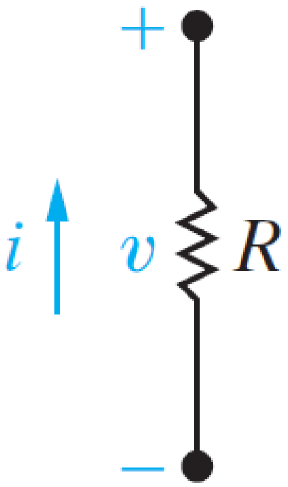
where
v = the voltage in volts,
i = the current in amperes,
R = the resistance in ohms.

Review Chapter 2: Ohm's Law

- If the current flow in the resistor is in the direction of the voltage drop across it >> (a)
- If the current flow in the resistor is in the direction of the voltage rise across it >> (b)



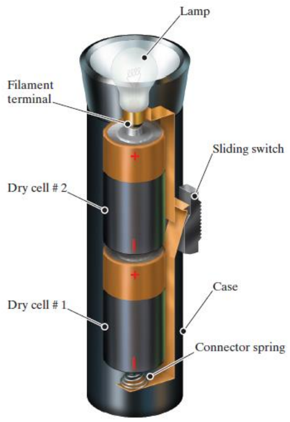
(a) $v = iR$

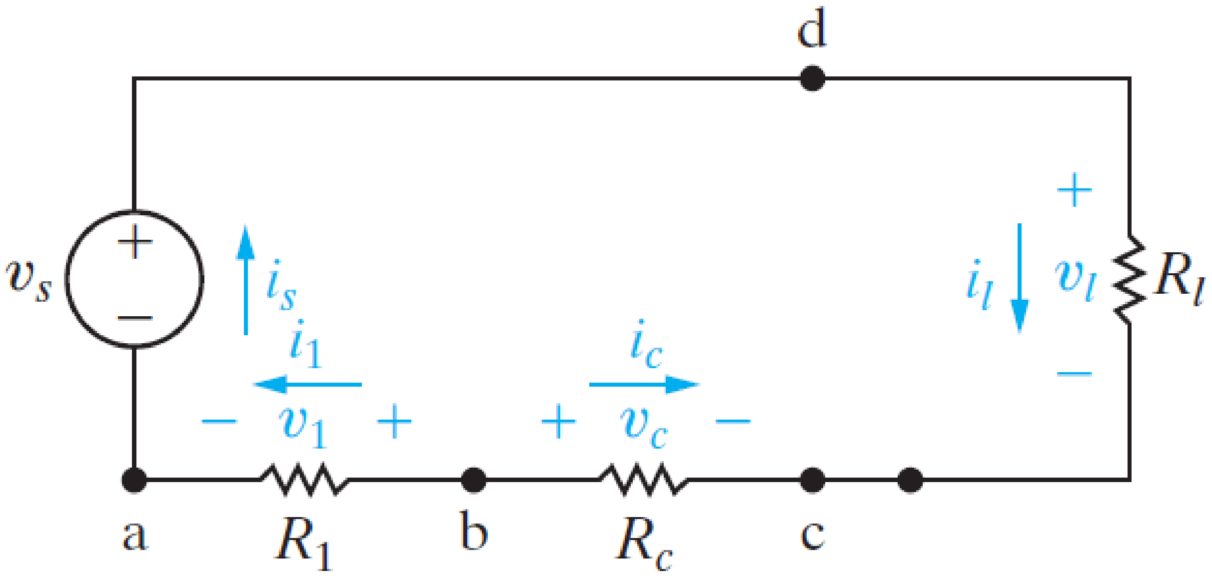


(b) $v = -iR$

Review Chapter 2: Kirchhoff's Law

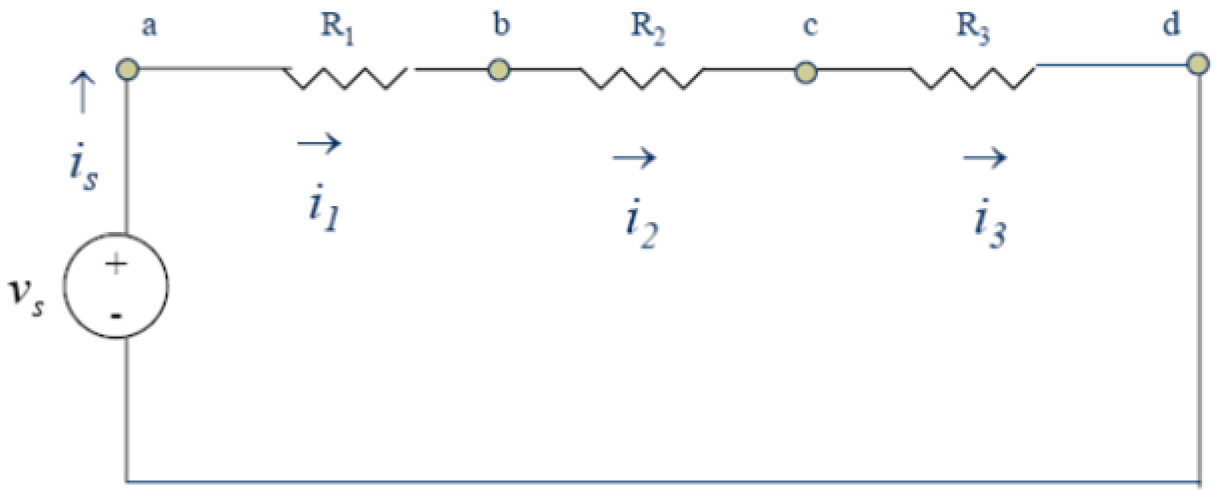
- KCL: the algebraic sum of all the currents at any node in a circuit equals zero
- KVL: the algebraic sum of all the voltage around any closed path in a circuit equals zero





Resistors in Series

- KCL: circuit elements that are connected in series carry the same current.



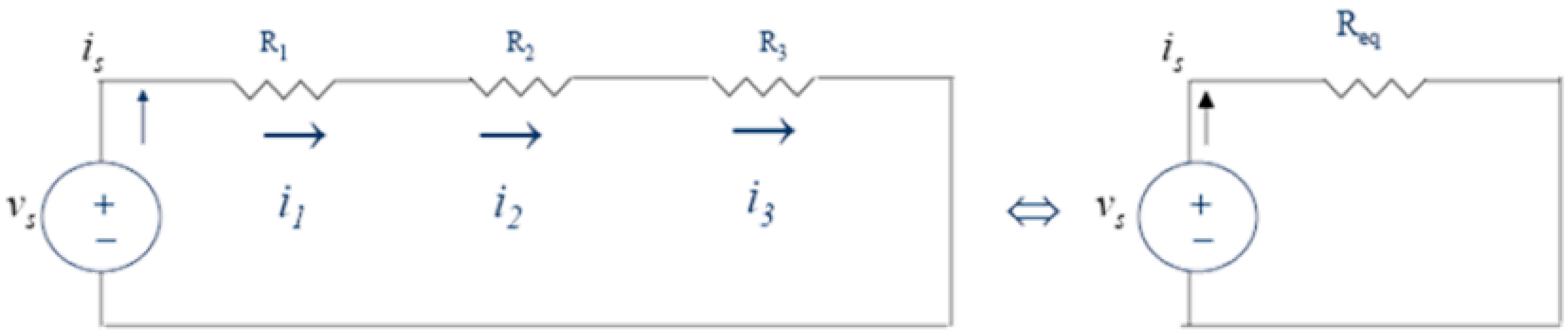
By KCL : at node a node b node c

$i_s = i_1$ $i_1 = i_2$ $i_2 = i_3$

$\rightarrow \quad i_s = i_1 = i_2 = i_3$

Resistors in Series

We want to show that the resistors in series add up to form a single equivalent resistor.



Can we find a value for R_{eq}, such that when used in place of R_1, R_2, R_3, the circuit draws the same i_s for the same v_s?

By KVL :

$$v_s = i_1 R_1 + i_2 R_2 + i_3 R_3$$

$$i_s = i_1 = i_2 = i_3$$

$$v_s = i_s (R_1 + R_2 + R_3)$$

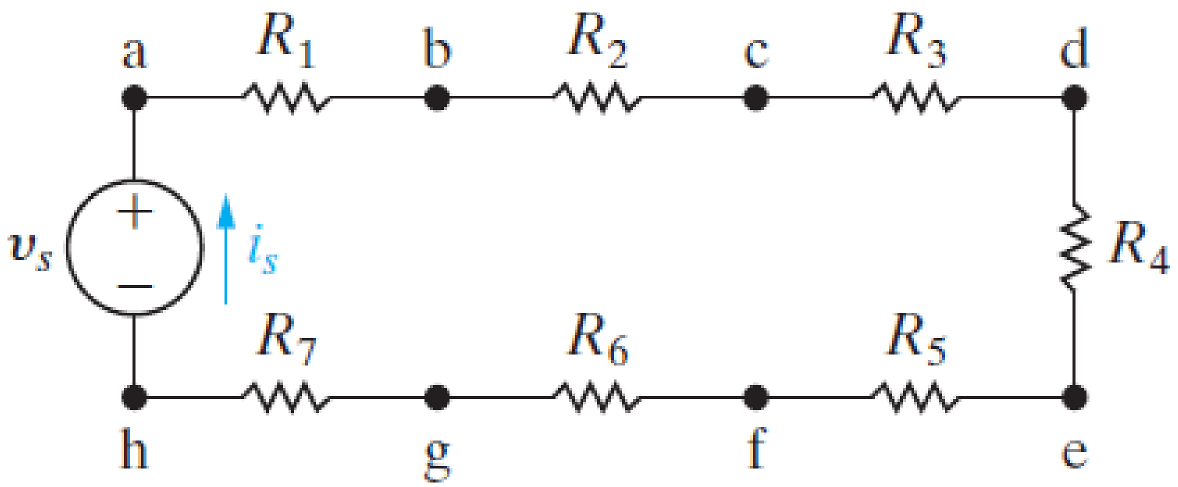
$$v_s = i_s R_{eq}$$

$$\boxed{R_1 + R_2 + R_3 = R_{eq}}$$

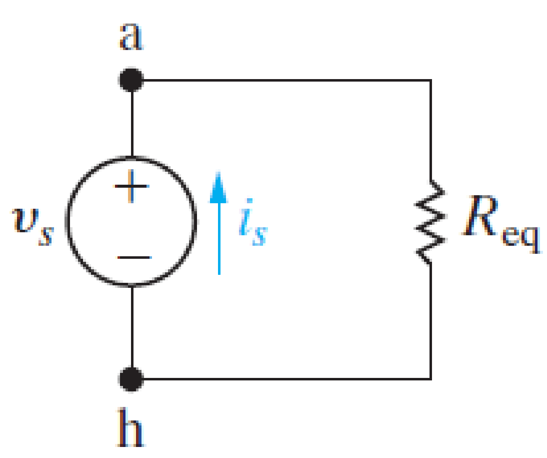
Resistors in Series

- Resistors in series
  - These resistors carry the same current by applying Kirchhoff's current law to each node in the circuit.

Resistors connected in series



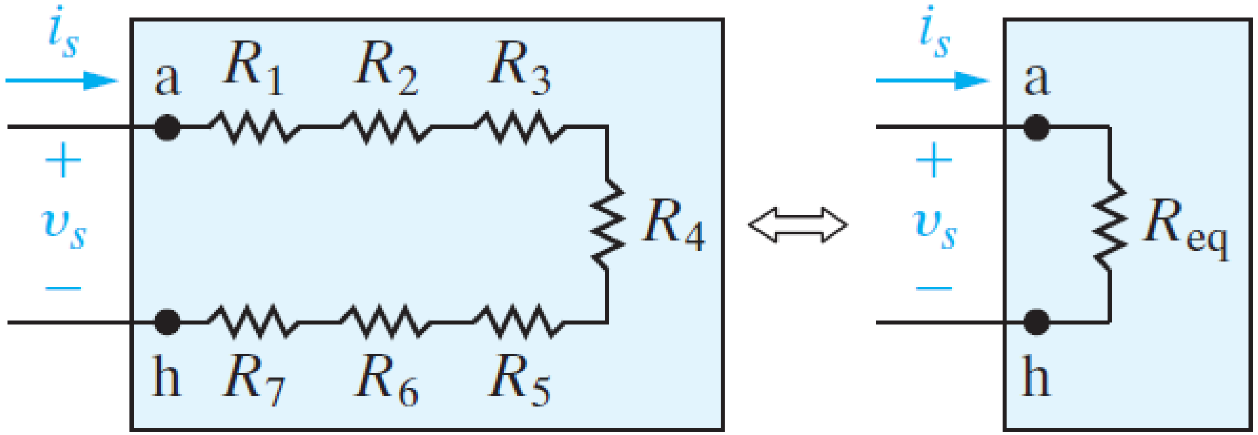
Simplified version of the circuit



Resistors in Series

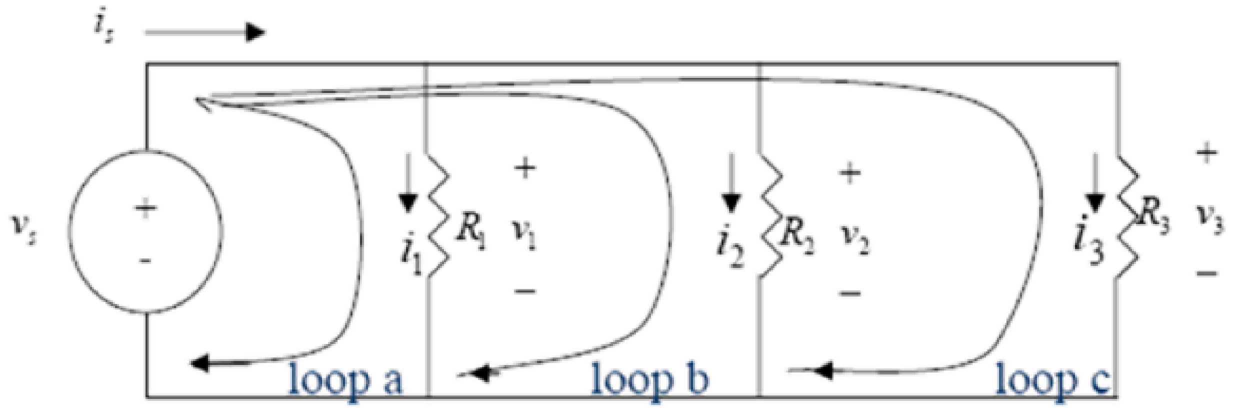
- Resistors in series
  - If k resistors are connected in series, the equivalent single resistor has a resistance equal t the sum of the k resistance, or

$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \ldots + R_k$$



Resistors in Parallel

- KVL: circuit elements that are connected in parallel have the same voltage across their terminals



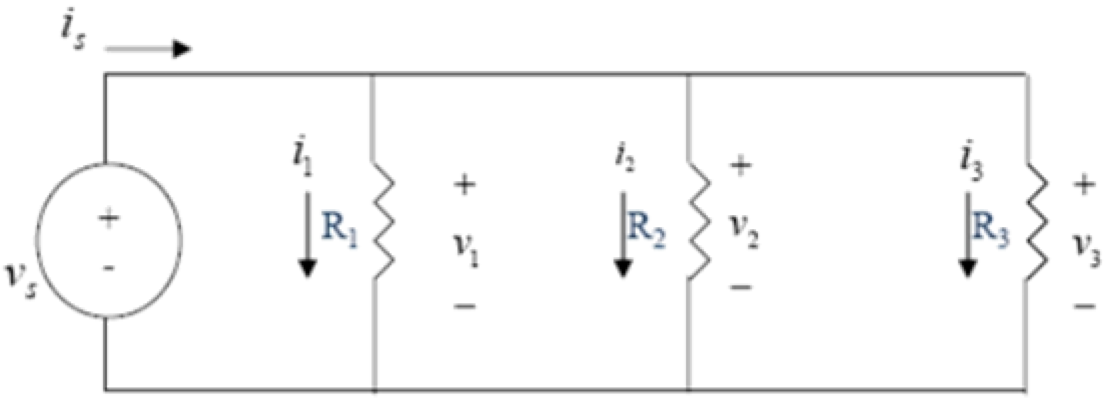
* By KVL :

loop a	loop b	loop c
$v_s - v_1 = 0$	$v_s - v_2 = 0$	$v_s - v_3 = 0$
$v_s = v_1$	$v_s = v_2$	$v_s = v_3$

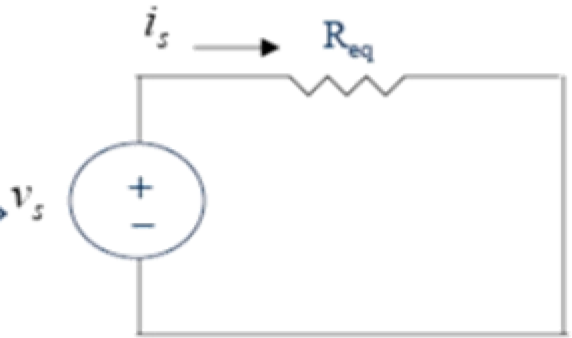
$$\boxed{v_s = v_1 = v_2 = v_3}$$

Resistors in Parallel

- KVL: circuit elements that are connected in parallel have the same voltage across their terminals



$\Leftrightarrow$



* By KCL :

$$i_s = i_1 + i_2 + i_3$$

$$i_s = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

$$i_s = v_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

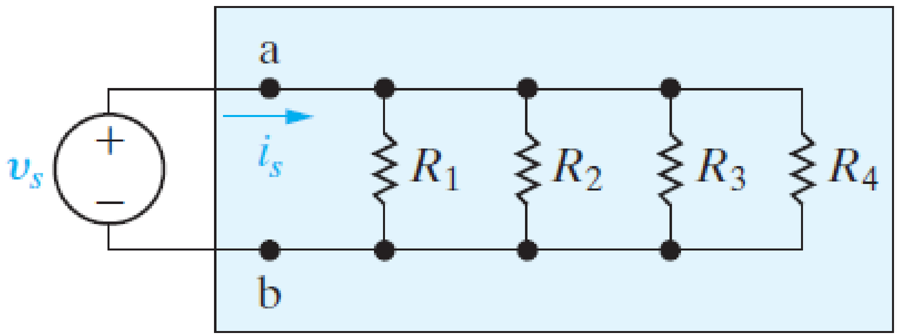
$$i_s = \frac{v_s}{R_{eq}}$$

$$\boxed{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_{eq}}}$$

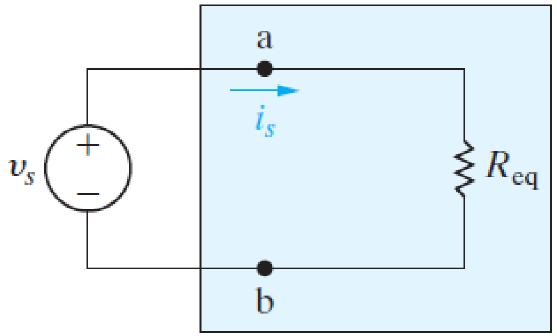
Resistors in Parallel

- Resistors in parallel
  - Parallel resistors can be combined to obtain a single equivalent resistance.

Resistors connected in parallel



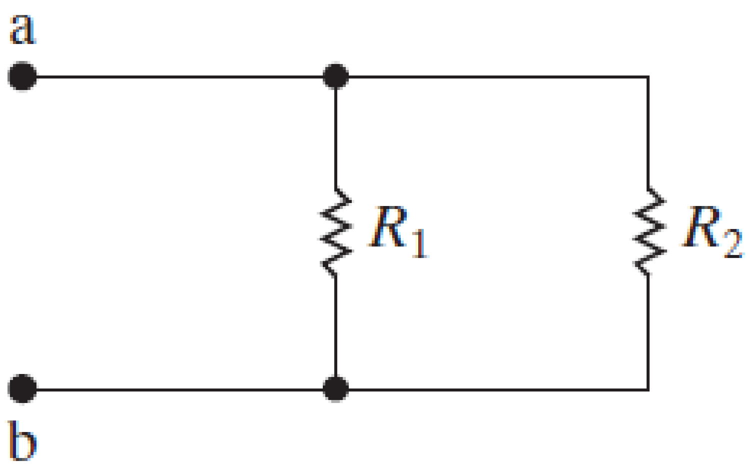
Simplified version of the circuit



$$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_k}$$

Resistors in Parallel

- Resistors in parallel
  - Parallel resistors can be combined to obtain a single equivalent resistance.

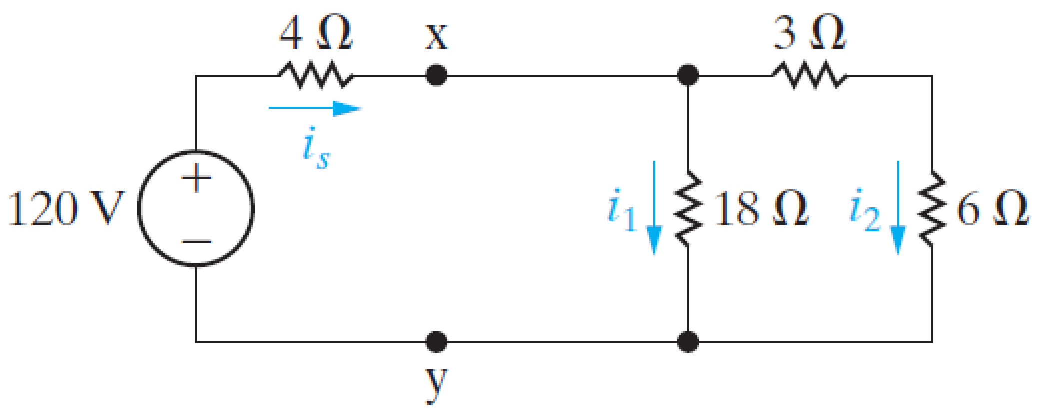


Representation of parallel

$$R_{eq} = R_1 \| R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

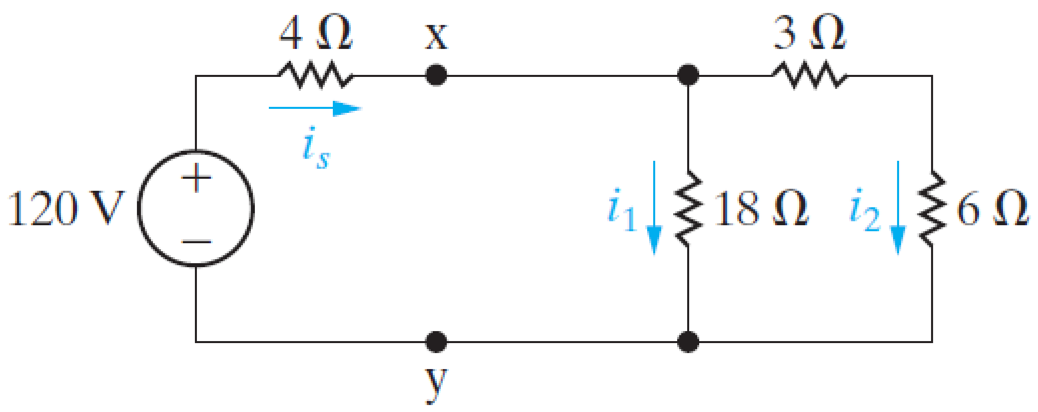
Example 3.2

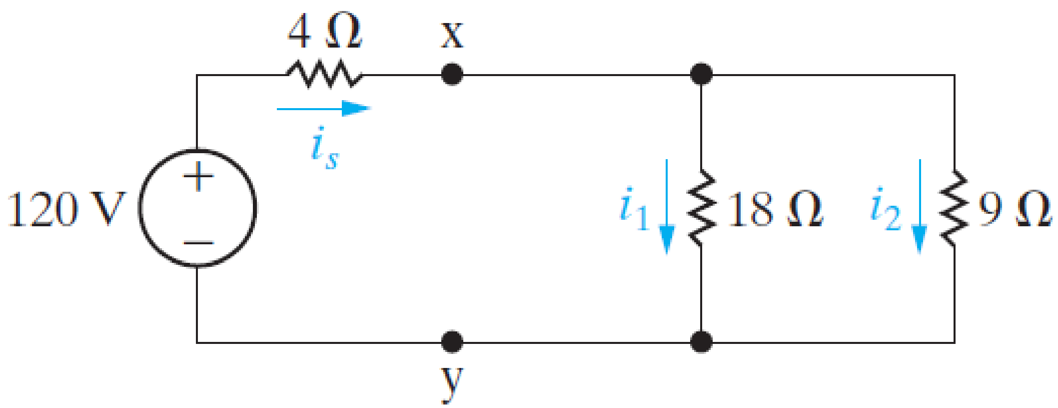
Q. Find i_s, i_1, and i_2



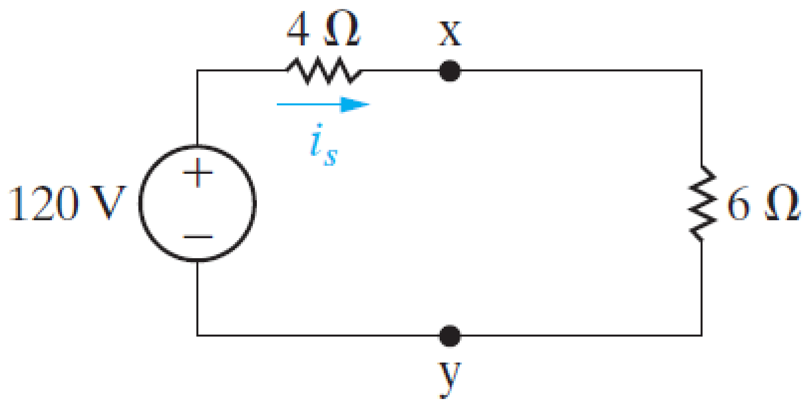
Example 3.2

Q. Find i_s, i_1, and i_2





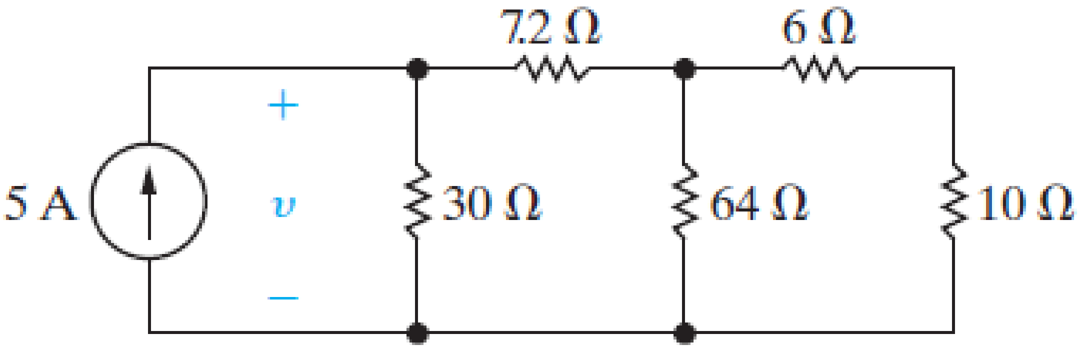
(a)



(b)

Assessment Problem

Q. Find v, power delivered to the circuit by the current source, and power dissipated in the 10-ohm resistor



Assessment Problem

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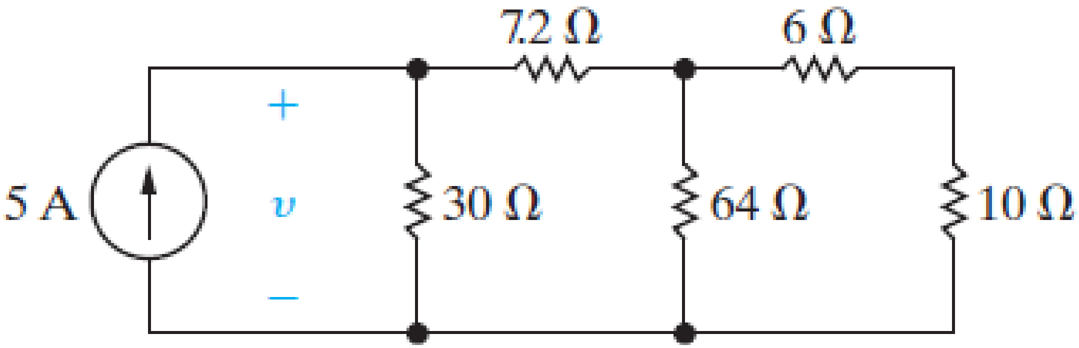

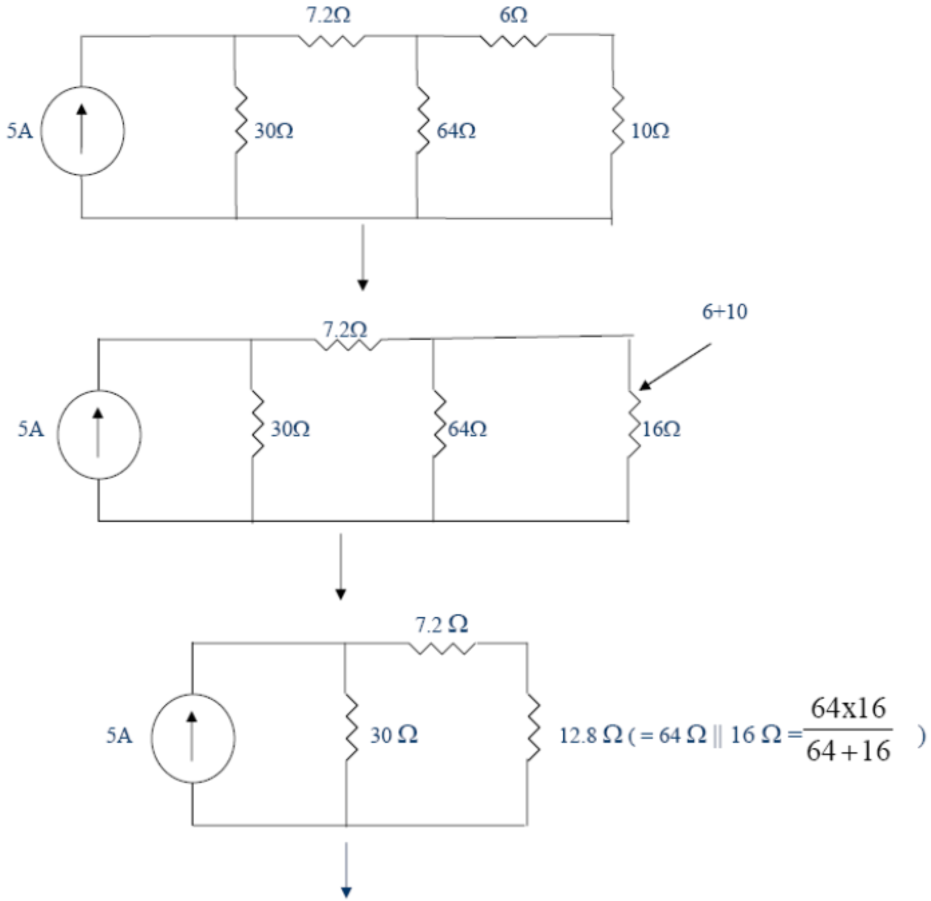

Assessment Problem

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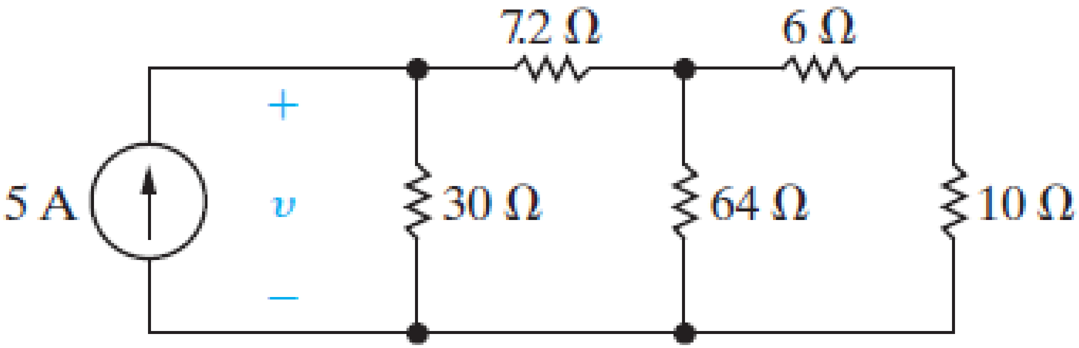
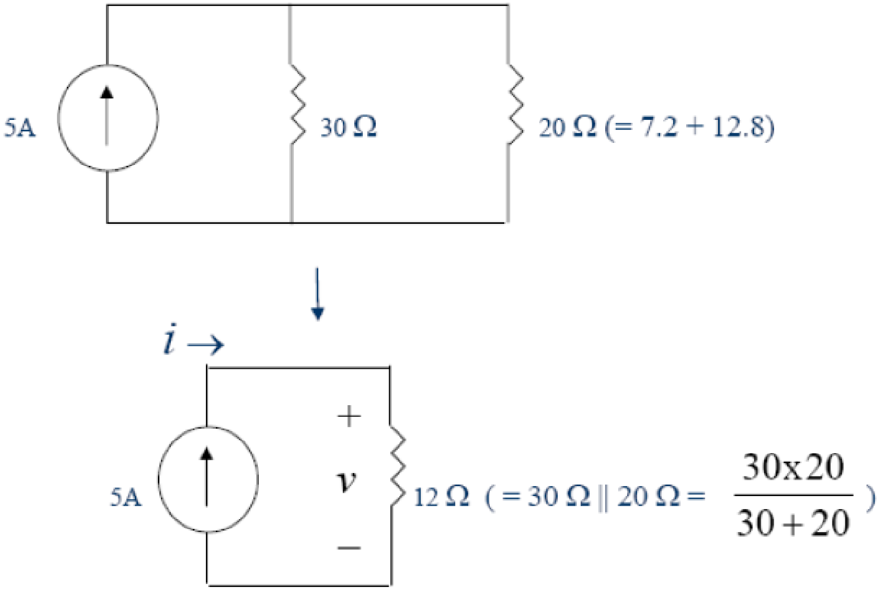




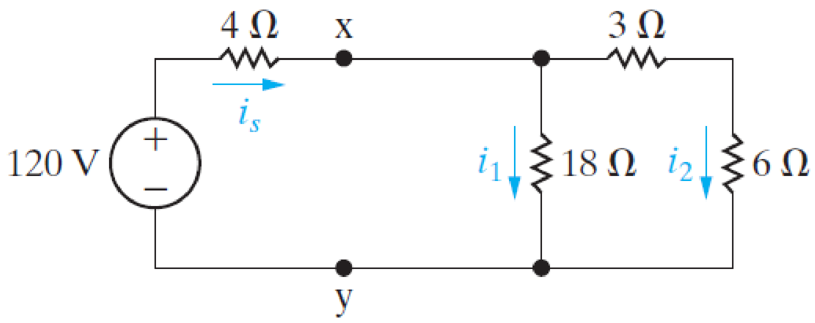
$$i = 5 \text{ amps}$$

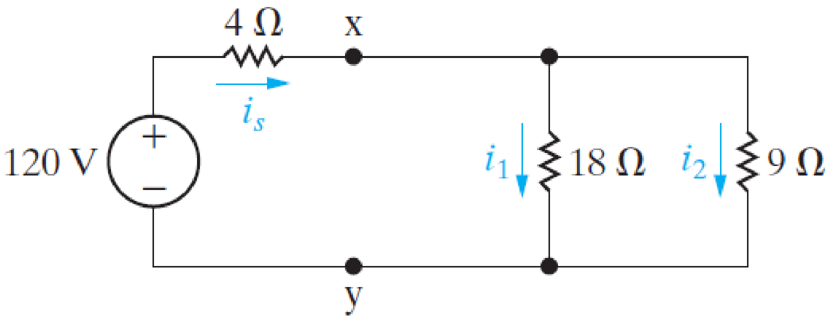
$$v = i \cdot R = 5 \cdot 12 = 60 \; volts$$

$$p = v \cdot i = 60 \cdot 5 = 300 \; watts$$

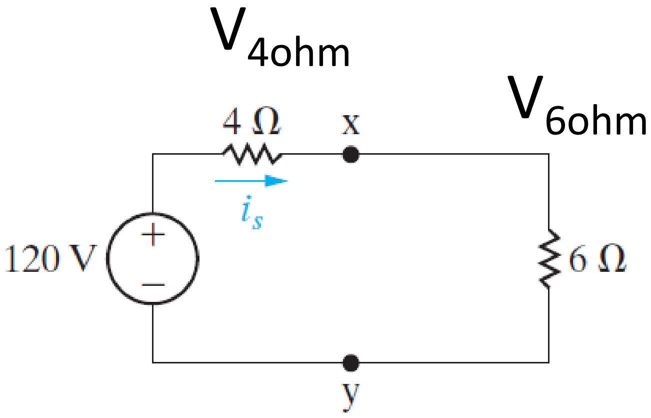
Voltage-divider & Current-Divider Circuits

In Example 3.2, we already used
voltage-divider and current-divider circuits





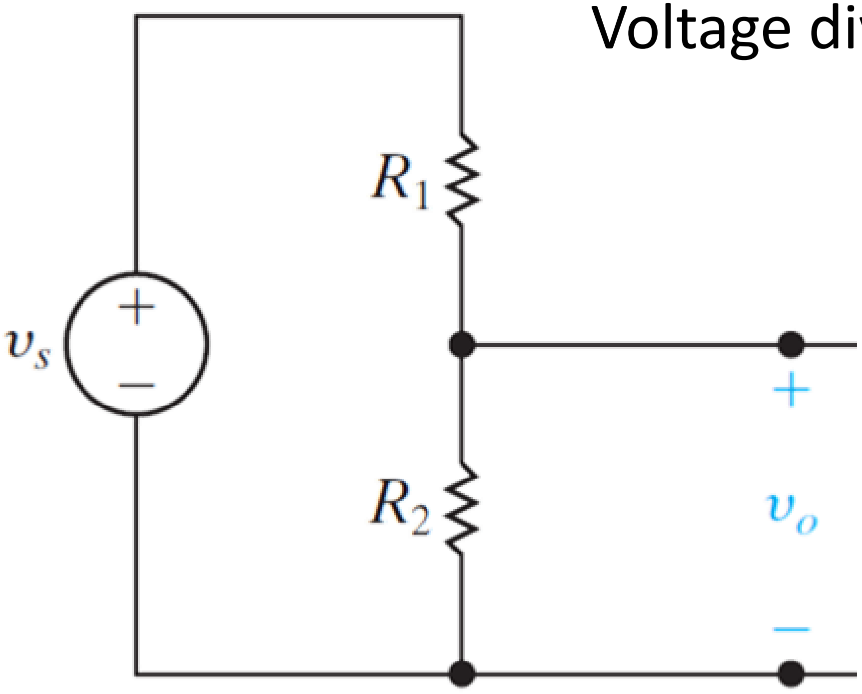
(a)



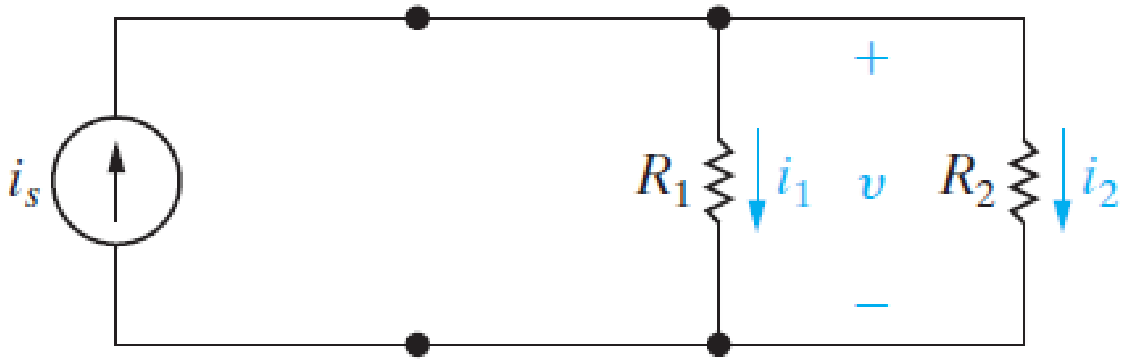
(b)

Voltage-divider & Current-Divider Circuits

Voltage divider circuit

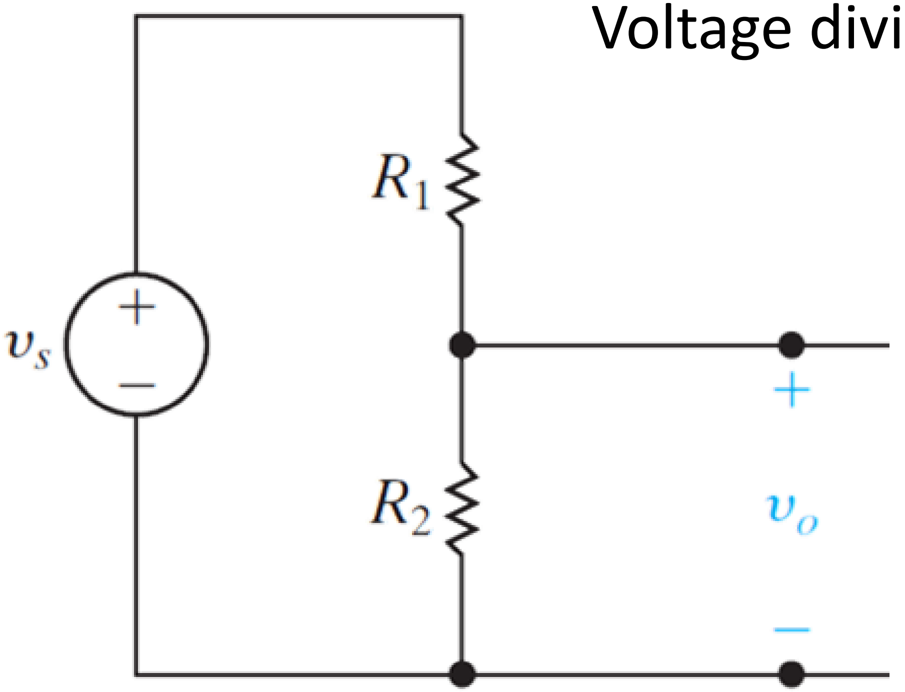


Current divider circuit



Voltage-divider & Current-Divider Circuits

Voltage divider circuit



$$v_s = i(R_1 + R_2)$$

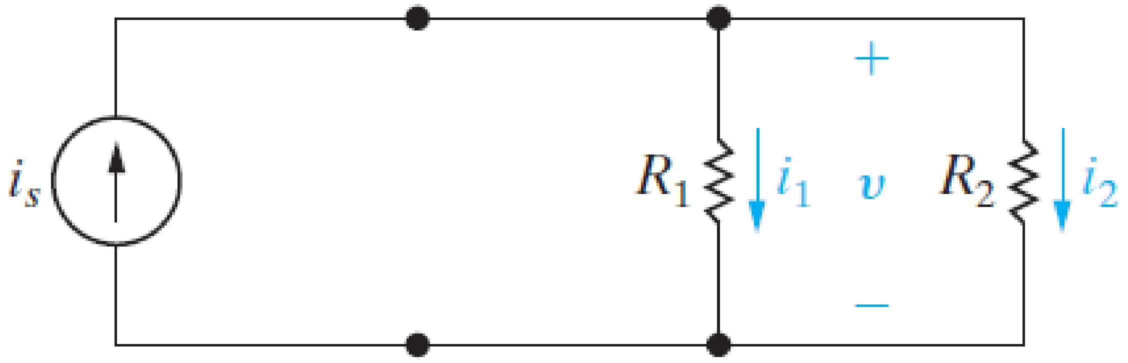
$$i = \frac{v_s}{R_1 + R_2}$$

$$v_o = iR_2$$

$$\boxed{v_0 = \frac{R_2}{R_1 + R_2} v_s}$$

Voltage-divider & Current-Divider Circuits

Current divider circuit



$$i_1 R_1 = i_2 R_2 \Rightarrow i_1 = \frac{i_2 R_2}{R_1}$$

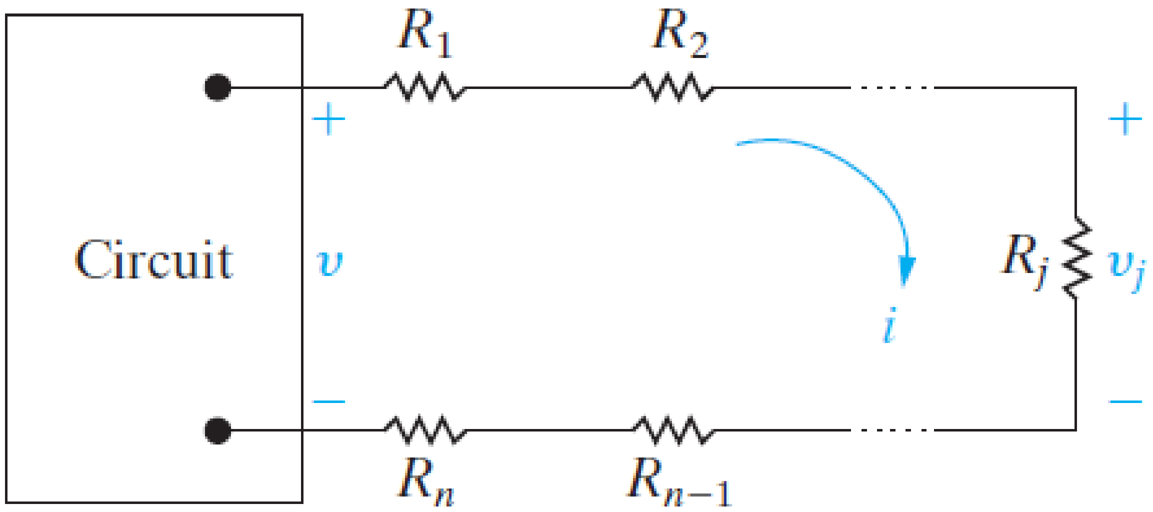
$$i_s = i_1 + i_2 = \frac{i_2 R_2}{R_1} + i_2 = \frac{i_2 R_2 + i_2 R_1}{R_1} = \frac{i_2 (R_2 + R_1)}{R_1}$$

$$\boxed{i_2 = \frac{R_1}{R_1 + R_2} i_s}$$

$$i_1 = \frac{R_2}{R_1 + R_2} i_s,$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_s.$$

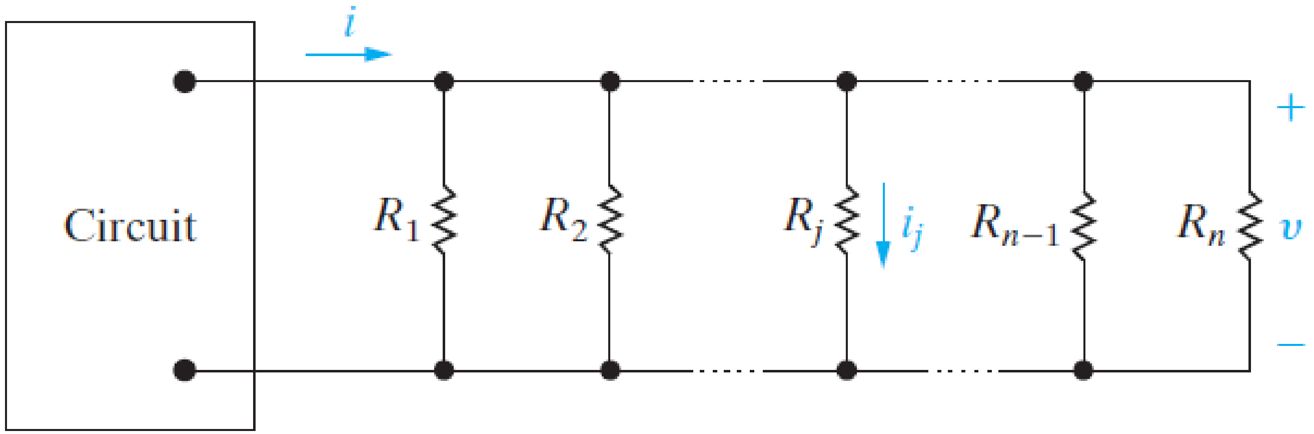
Voltage Division



$$i = \frac{v}{R_1 + R_2 + \ldots + R_n} = \frac{v}{R_{eq}},$$

$$v_j = iR_j = \left(\frac{R_j}{R_{eq}}\right)v$$

Current Division



Circuit used to illustrate current division.

$$v = i(R_1 \| R_2 \| \ldots \| R_n) = i R_{\mathrm{eq}}.$$

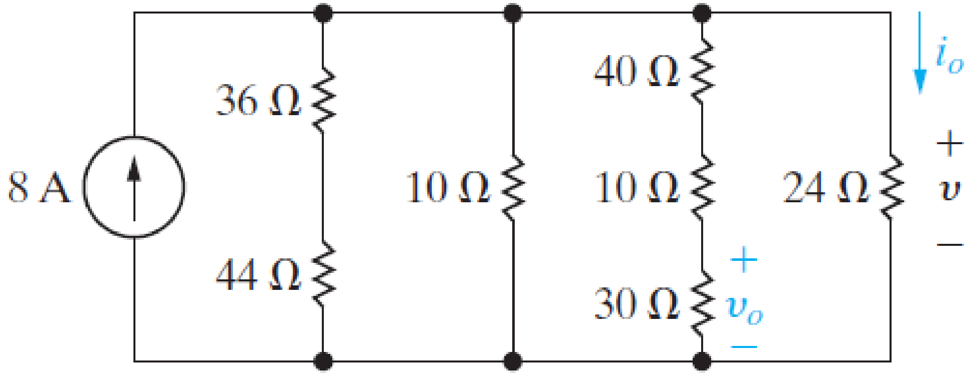
$$i_j = \frac{v}{R_j} = \frac{R_{eq}}{R_j} i$$

i_j = *current through the resistance* R_j

i = *current into the parallel-connected resistances whose*

Example 3.7

Q. Find i_o, and v_o



Measuring Voltage and Current

Analog multimeter

DC Voltage

DC Current

Resistance

Digital multimeter

Resistance

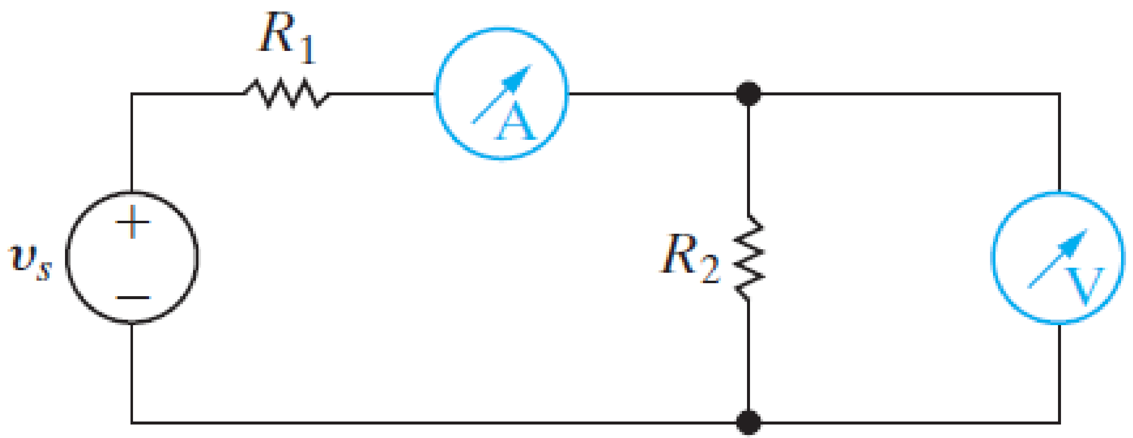
DC Voltage

DC Current

Ref) https://en.wikipedia.org/wiki/Multimeter

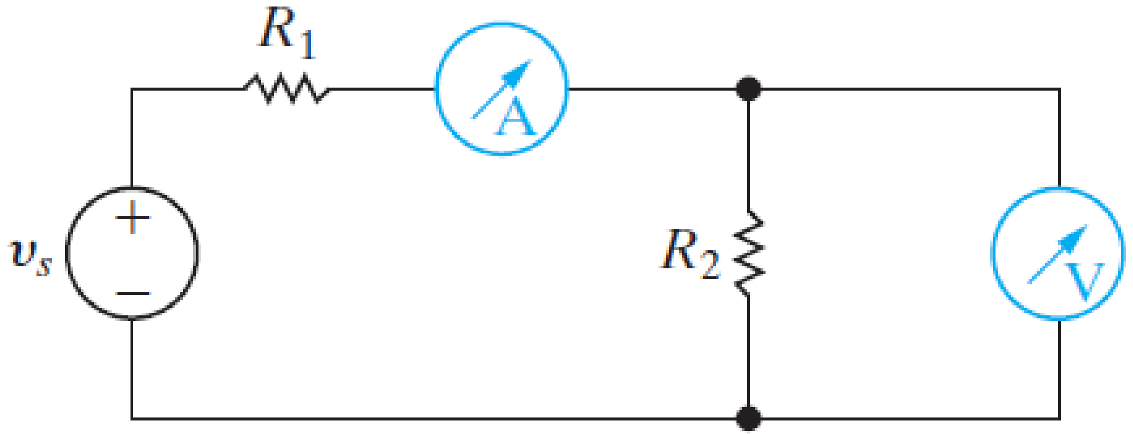
Measuring Voltage and Current

- Voltage (Voltmeter)
  - Voltmeter measures voltage and must be placed in parallel with the voltage being measured.
  - An ideal voltmeter has infinite internal resistance and thus does not alter the voltage being measured.



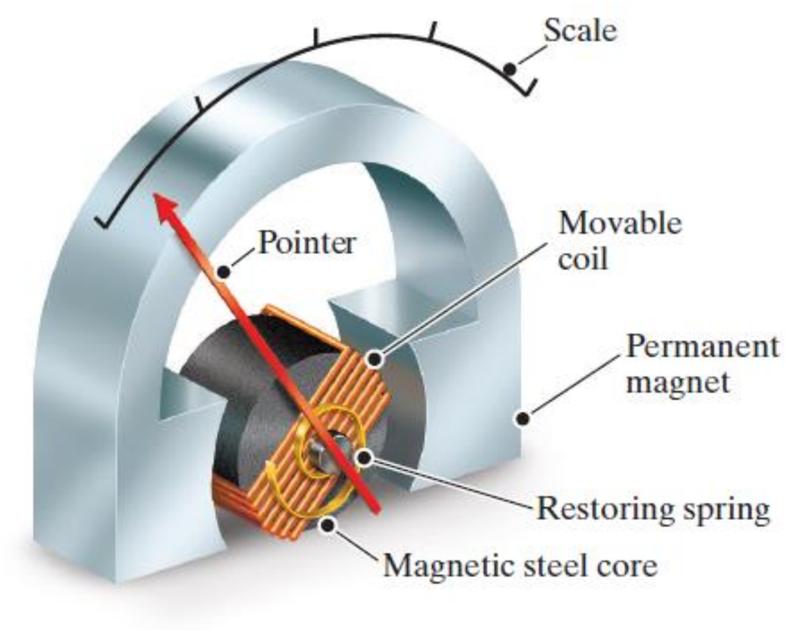
Measuring Voltage and Current

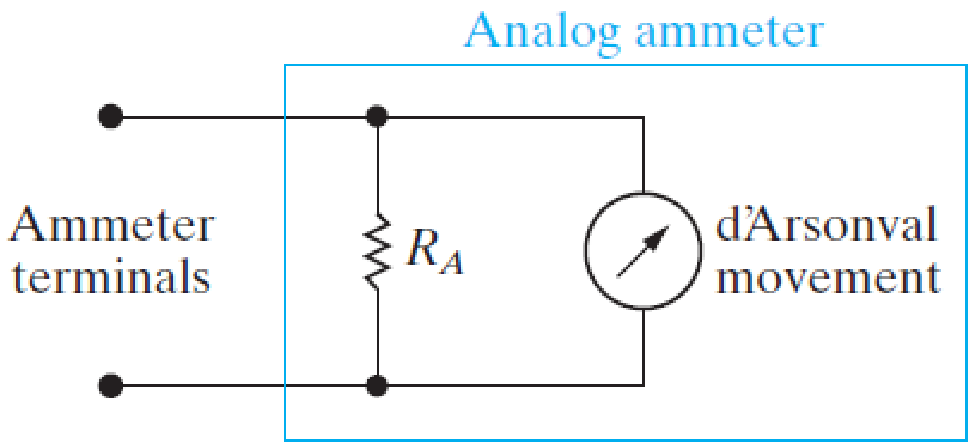
- Current (Ammeter)
  - Ammeter measures current and must be placed in series with the current being measured.
  - An ideal ammeter has zero internal resistance and thus does not alter the current being measured.



Measuring Voltage and Current

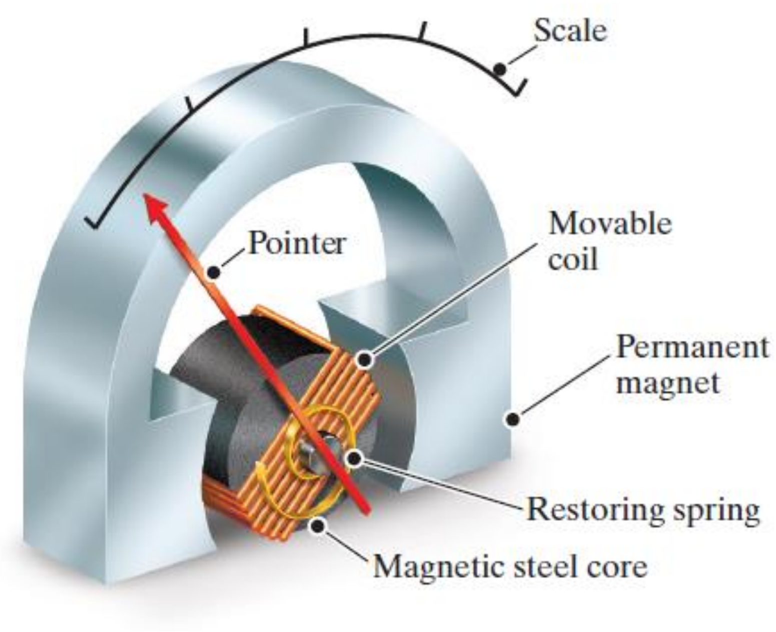
- d'Arsonval meter movement
  - The purpose of the parallel resistor is to limit the amount of current in the movement's coil by shunting some of it through R_A.

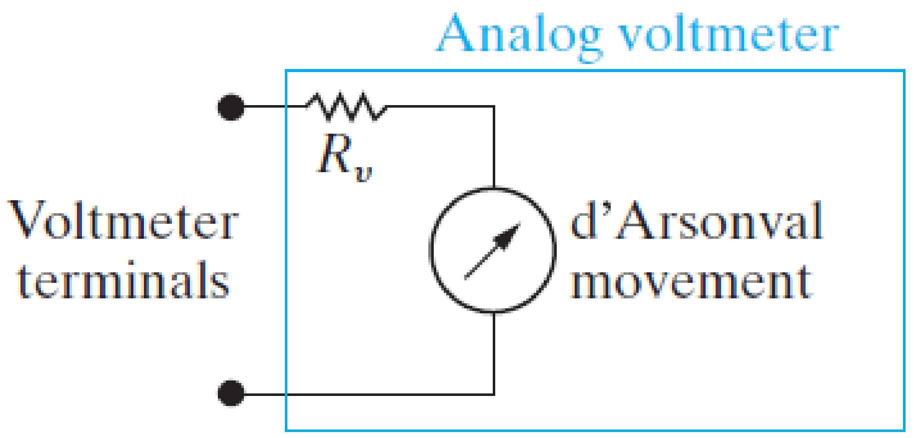




Measuring Voltage and Current

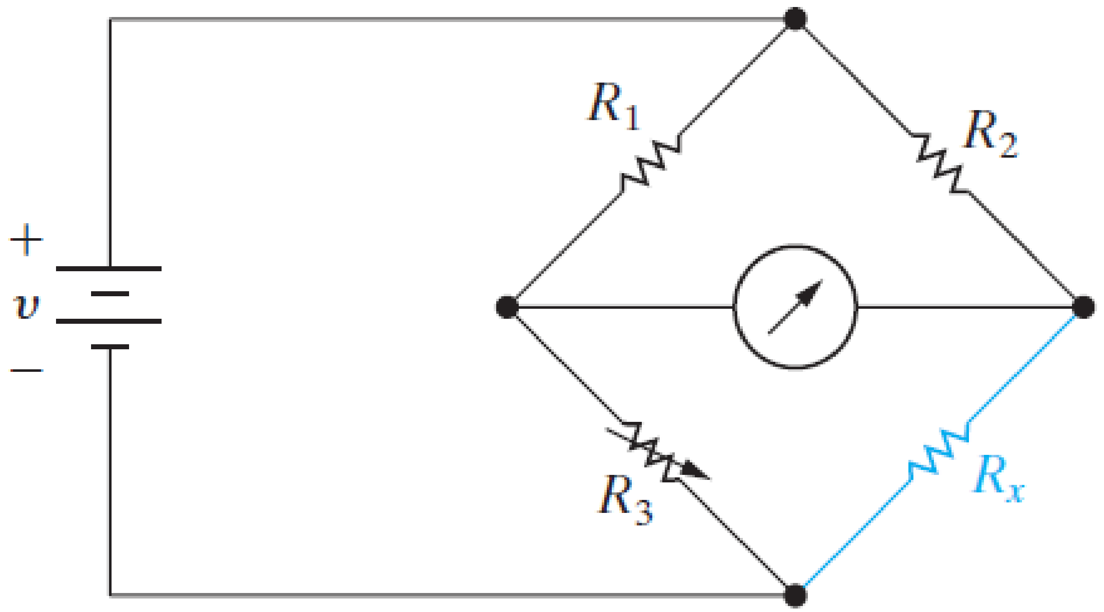
- d'Arsonval meter movement
  - The resistor is used to limit the voltage drop across the meter 's coil.





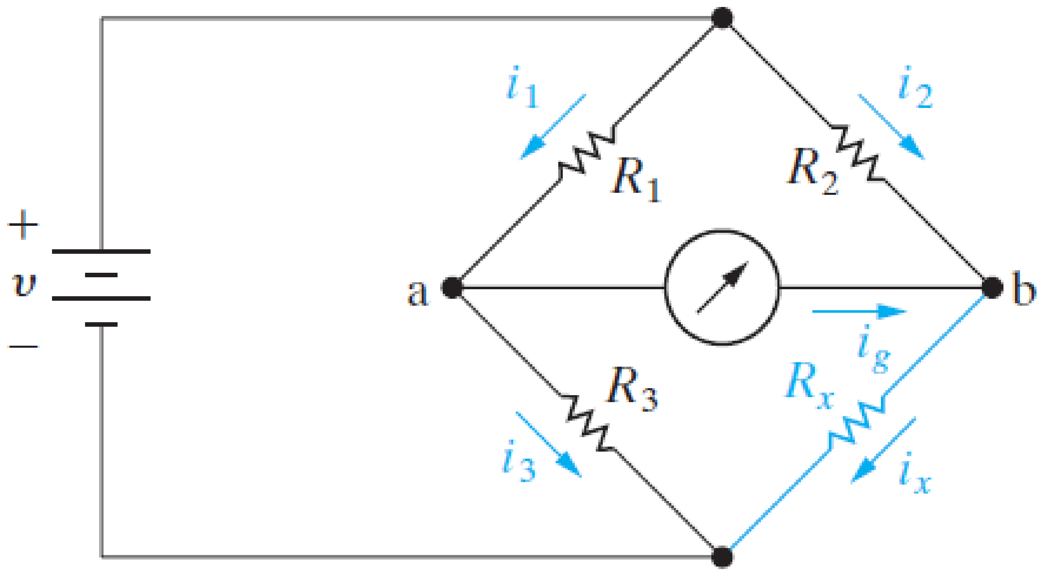
Measuring Resistance

- Wheatstone bridge circuit
  - The Wheatstone bridge circuit is used to make precise measurements of a resistor's value using
    four resistors, a DC voltage source, and a galvanometer.



Measuring Resistance

- Wheatstone bridge circuit



Balanced Wheatstone bridge circuit

i_g=0 [A]

KCL $i_1=i_3$ and $i_2=i_x$

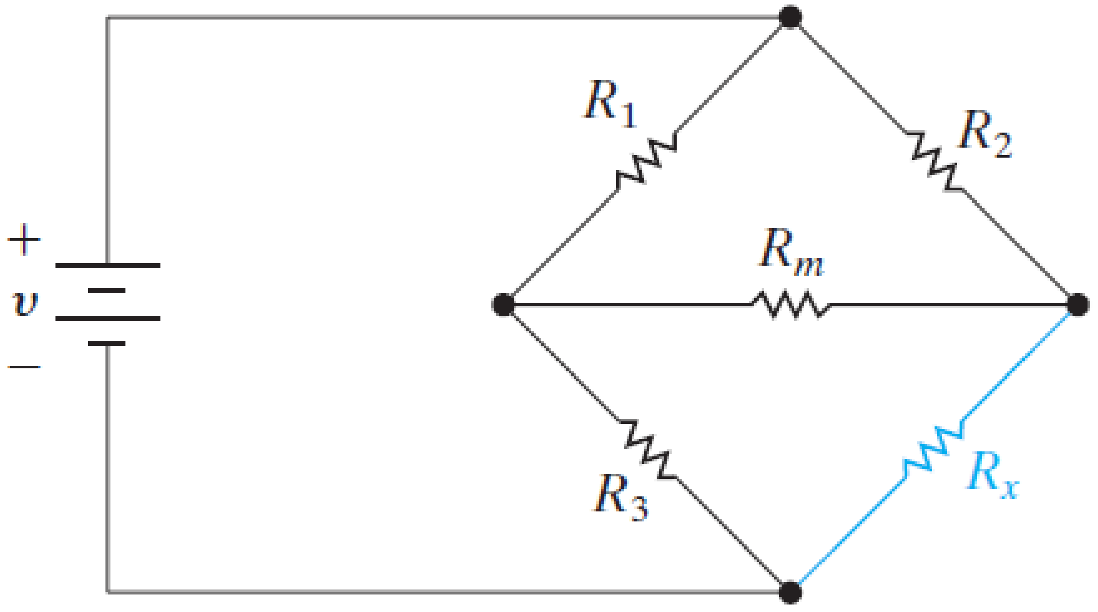
KVL $i_1R_1 = i_2R_2$

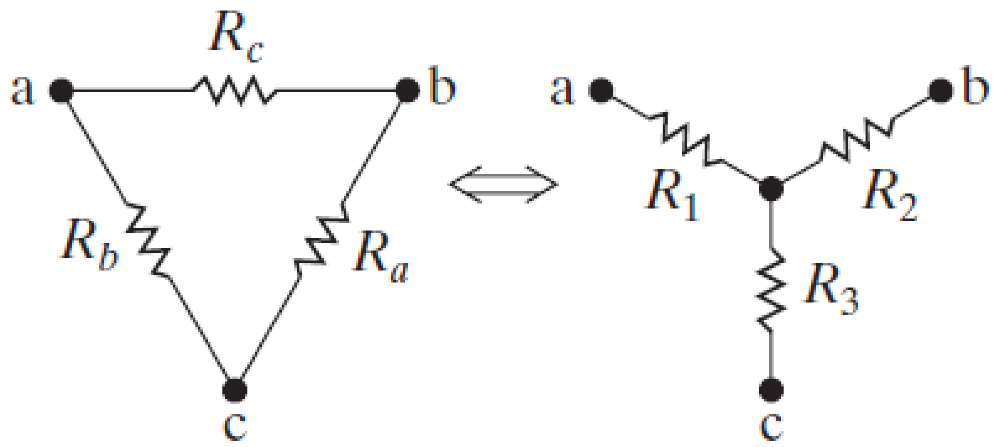
$i_3R_3 = i_xR_x$

$$R_x = \left(\frac{R_2}{R_1}\right)R_3$$

Delta-to-Wye (Pi-to-Tee) Equivalent Circuits

- Delta-to-Wye (or called Pi-to-Tee)
  - Cannot reduce the interconnected resistors of this circuit to a single equivalent resistance because of the R_m.
    >> delta-to-wye or pi-to-tee equivalent circuit

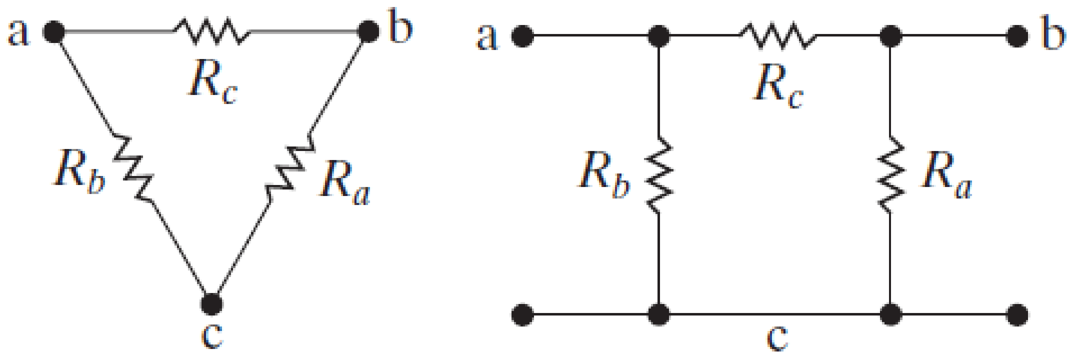




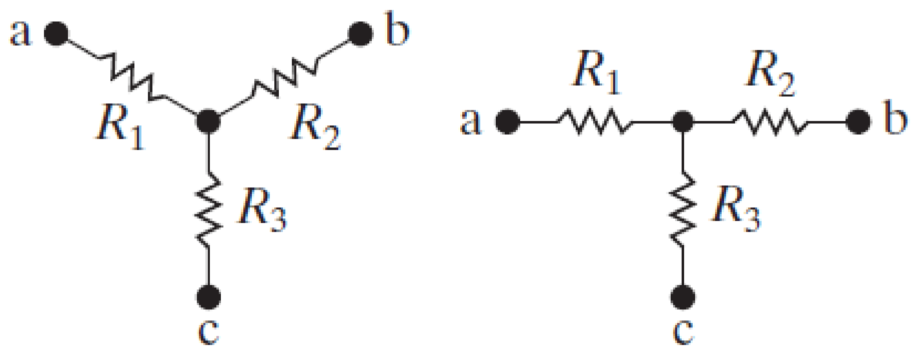
Delta-to-Wye (Pi-to-Tee) Equivalent Circuits

- Delta-to-Wye (or called Pi-to-Tee)

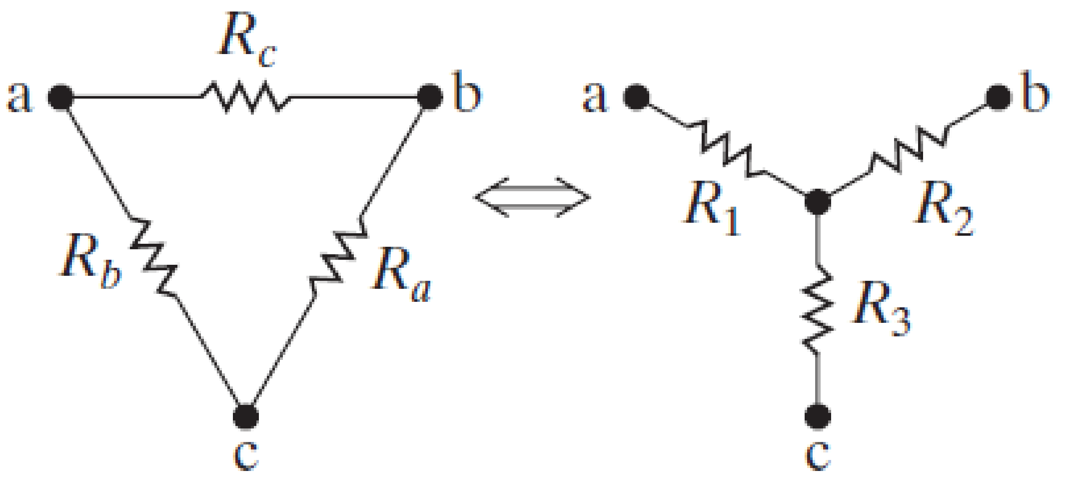
Delta or Pi



Wye or Tee



Delta-to-Wye (Pi-to-Tee) Equivalent Circuits



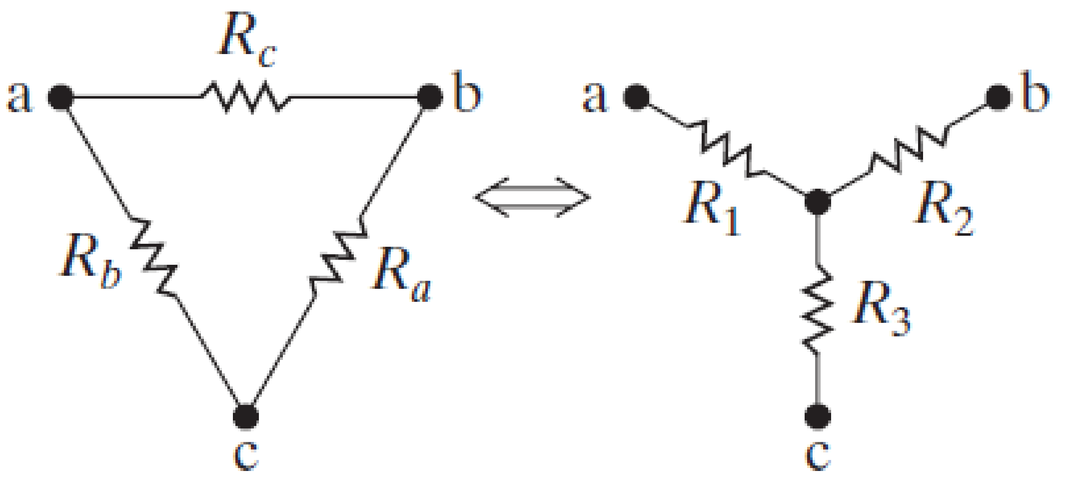
$$R_{ab} = R_{ab}$$

$$R_c \parallel (R_a + R_b) = R_1 + R_2$$

$$\frac{R_c(R_a + R_b)}{R_c + R_a + R_b} = R_1 + R_2$$

$$\boxed{\frac{R_a R_c + R_b R_c}{R_a + R_b + R_c} = R_1 + R_2 \quad \text{(Eq 1)}}$$

Delta-to-Wye (Pi-to-Tee) Equivalent Circuits



$$R_{ac} = R_{ac}$$

$$\frac{R_b(R_a+R_c)}{R_a+R_b+Rc} = R_1+R_3$$

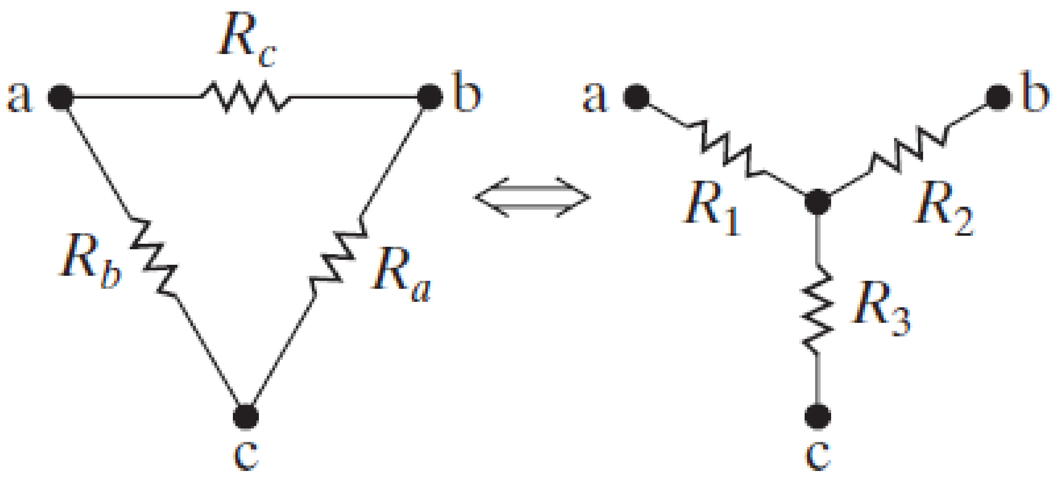
$$\frac{R_aR_b+R_bR_c}{R_a+R_b+R_c} = R_1+R_3 \text{ (Eq 2)}$$

$$R_{cb} = R_{cb}$$

$$\frac{R_a(R_b+R_c)}{R_a+R_b+Rc} = R_2+R_3$$

$$\frac{R_aR_b+R_aR_c}{R_a+R_b+R_c} = R_2+R_3 \text{ (Eq 3)}$$

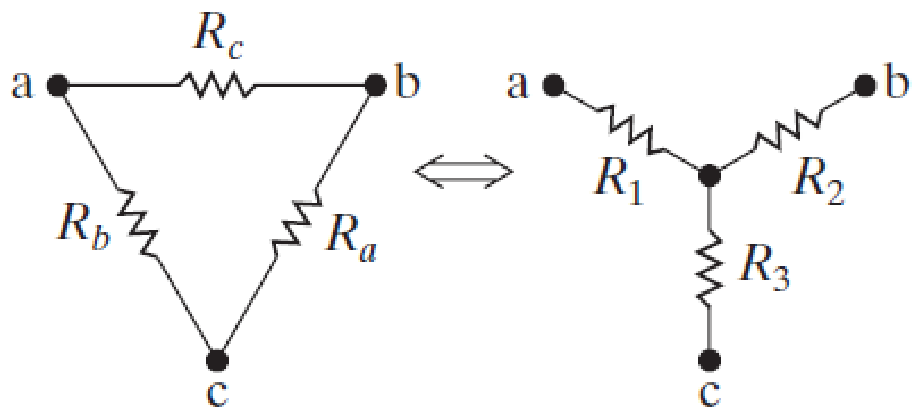
Delta-to-Wye (Pi-to-Tee) Equivalent Circuits



Solve for R_1, R_2, and R_3 from Eqs 1 – 3, in terms of R_a, R_b, and R_c.

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}}$$

Delta-to-Wye (Pi-to-Tee) Equivalent Circuits



Delta-to-Wye

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Wye-to-Delta

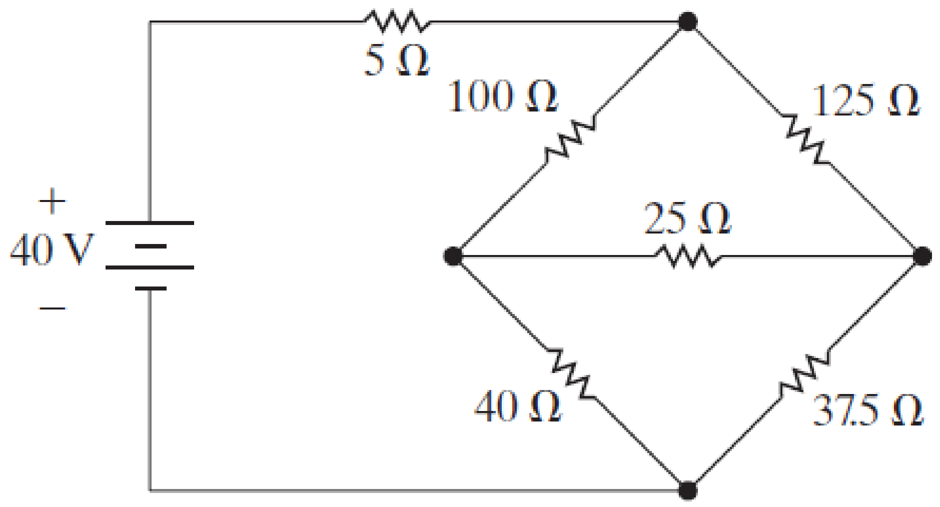
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

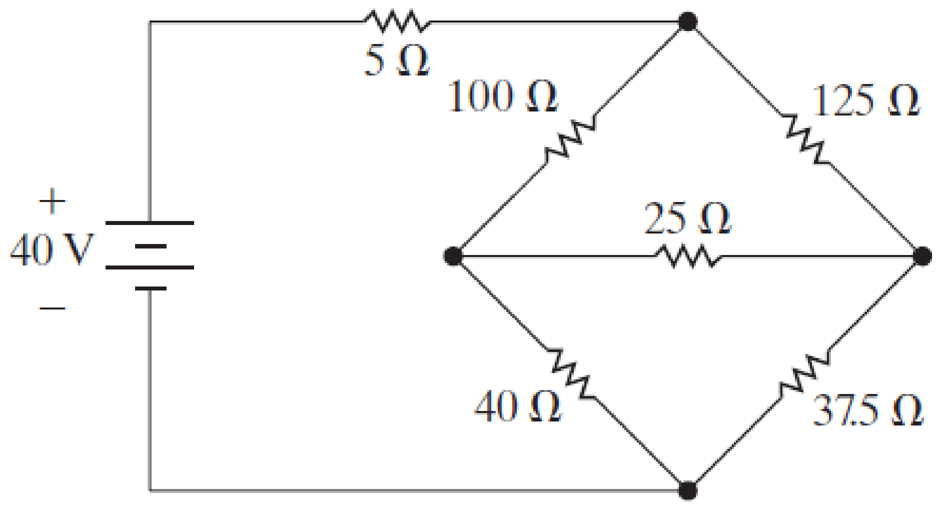
Example 3.11

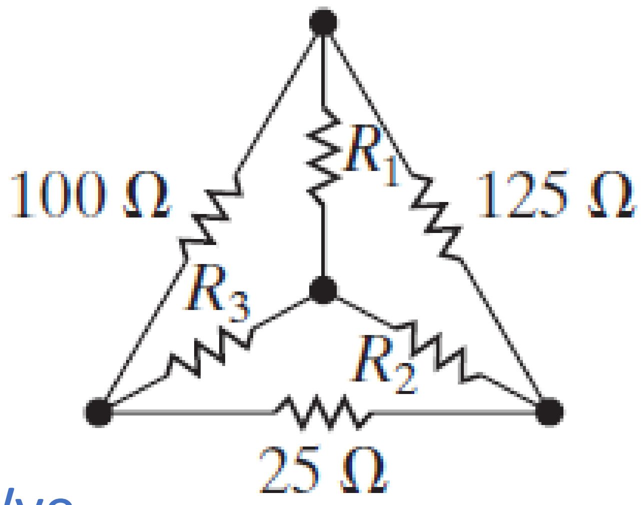
Q. Find current and power



Example 3.11

Q. Find current and power





Delta to Wye

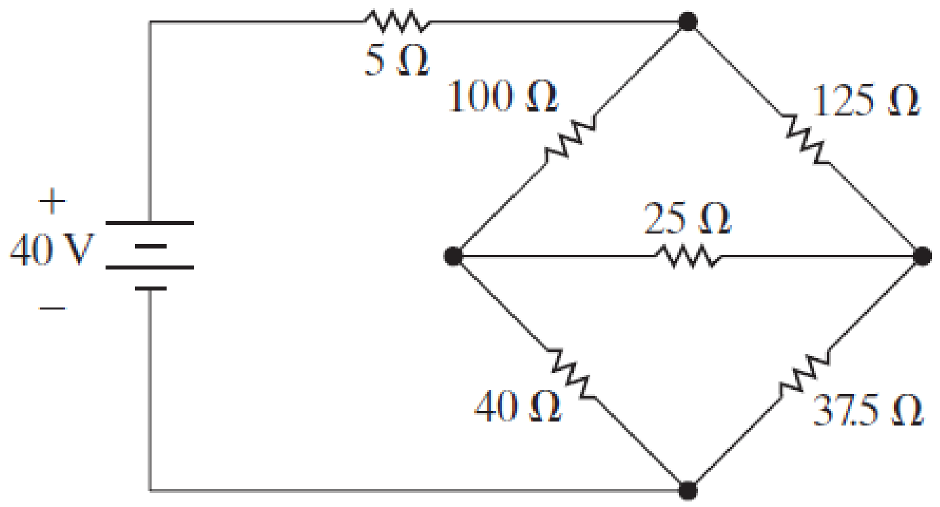
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

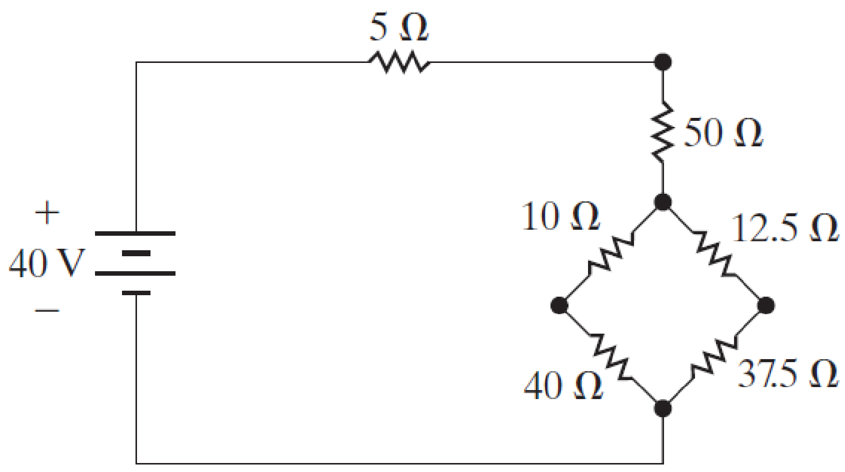
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

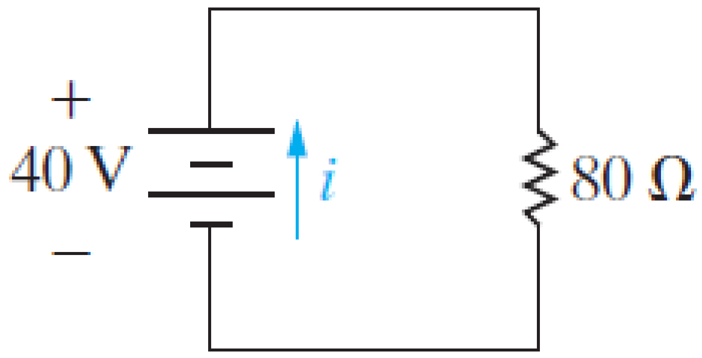
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Example 3.11

Q. Find current and power

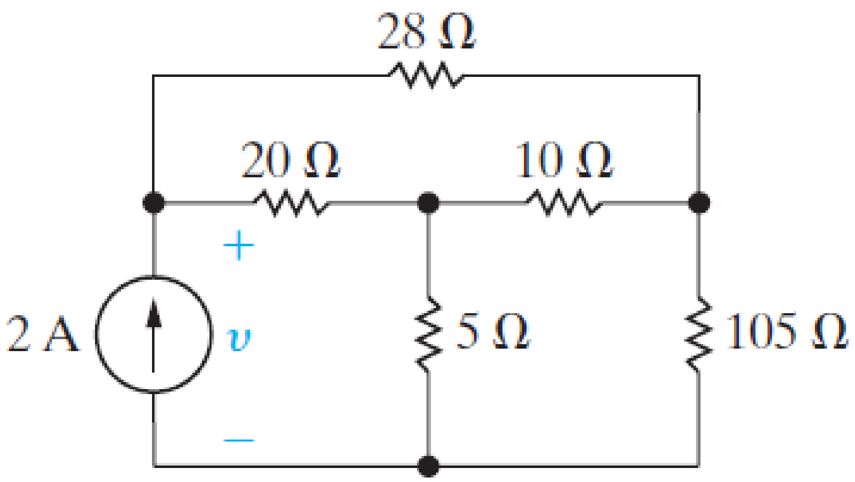


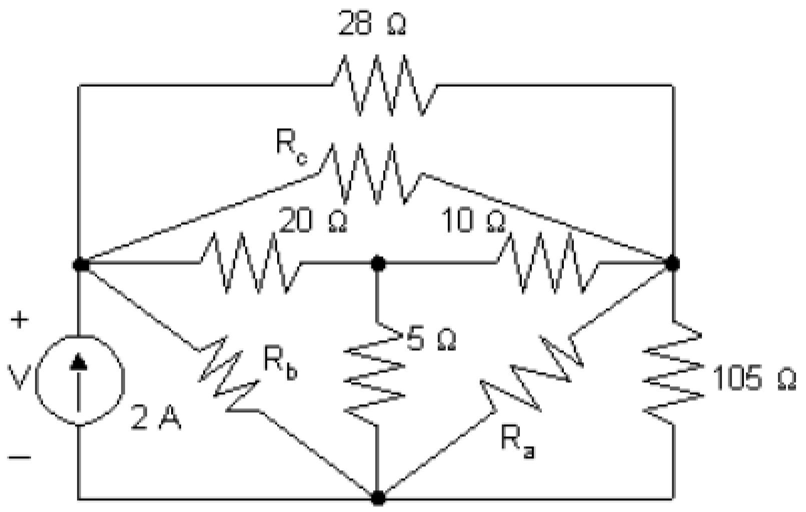




Assessment Problem

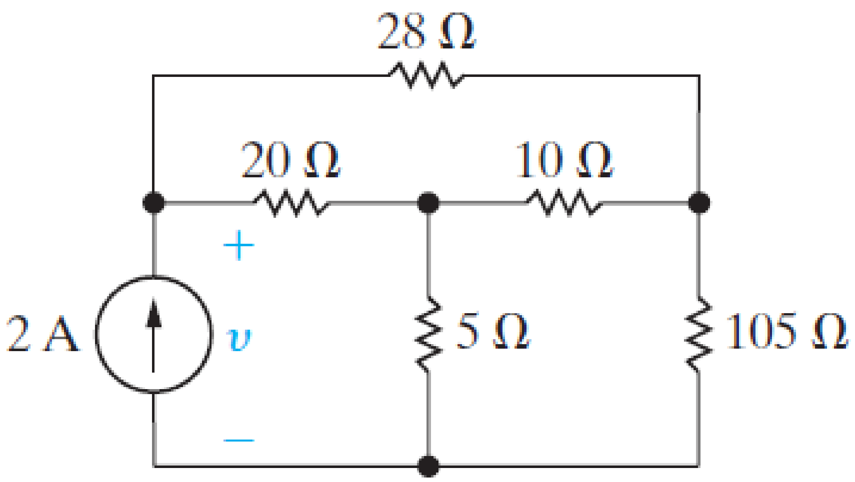
Q. Find voltage using Wye-to-Delta transformation

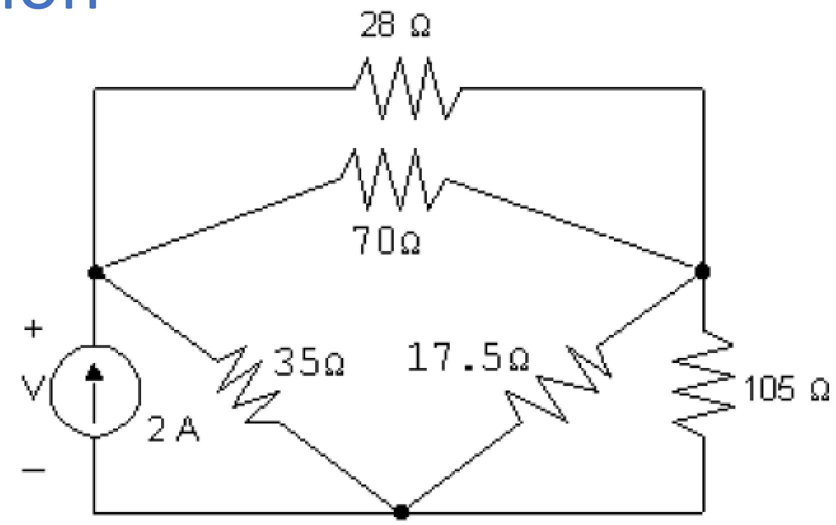




Assessment Problem

Q. Find voltage using Wye-to-Delta transformation





Wye to Delta

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

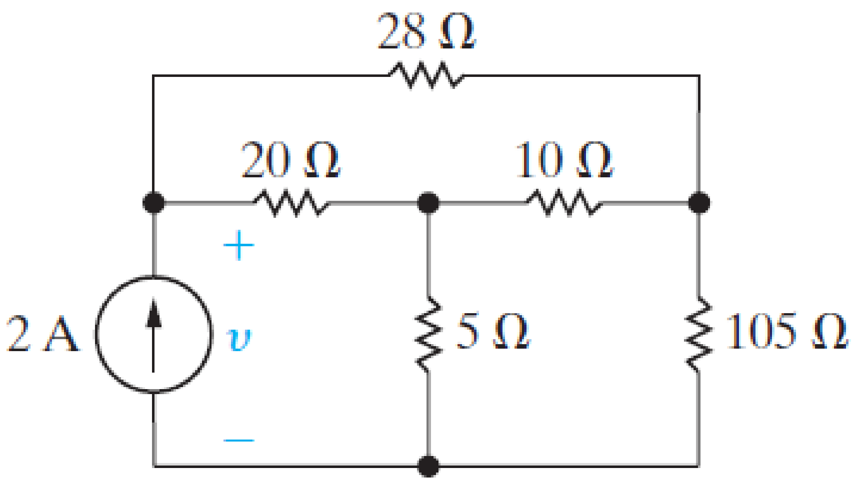
$$R_a = \frac{(5)(10) + (5)(20) + (10)(20)}{20} = 17.5\,\Omega;$$

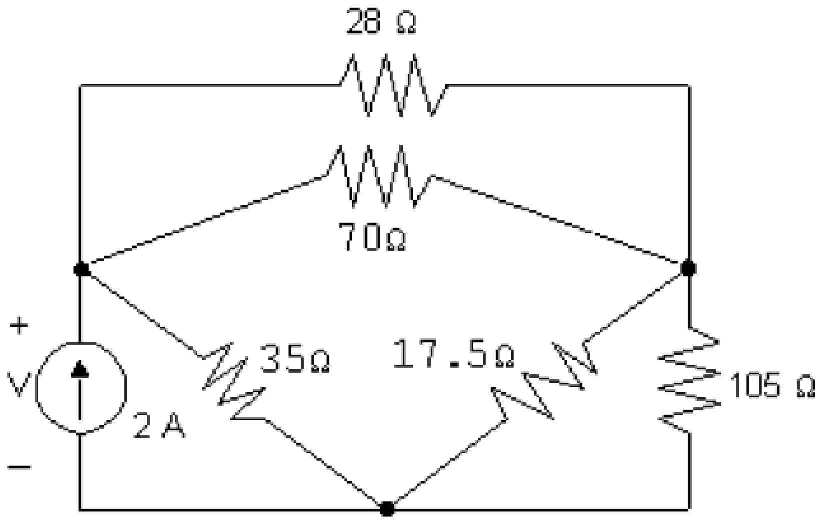
$$R_b = \frac{(5)(10) + (5)(20) + (10)(20)}{10} = 35\,\Omega;$$

$$R_c = \frac{(5)(10) + (5)(20) + (10)(20)}{5} = 70\,\Omega.$$

Assessment Problem

Q. Find voltage using Wye-to-Delta transformation





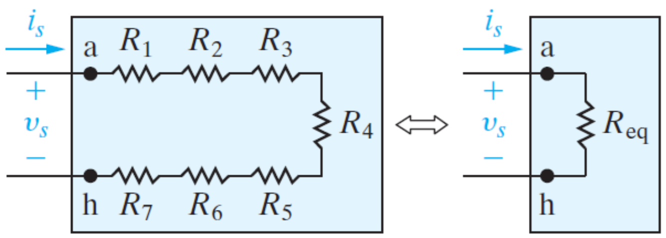
$$70\,\Omega \| 28\,\Omega = \frac{(70)(28)}{70+28} = 20\,\Omega.$$

$$17.5\,\Omega \| 105\,\Omega = \frac{(17.5)(105)}{17.5+105} = 15\,\Omega.$$

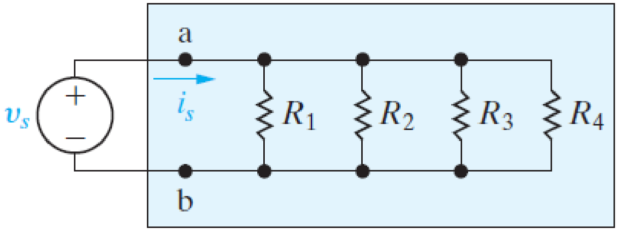
$$35\,\Omega \| 35\,\Omega = \frac{(35)(35)}{35+35} = 17.5\,\Omega.$$

Summary

- Resistors in series
- Resistors in parallel
- Voltage division and current division
- Measuring voltage, current, and resistance
- Delta-to-Wye and Wye-to-Delta Transformation



Series resistors



Parallel resistors

$$v_j = iR_j = \left(\frac{R_j}{R_{eq}}\right)v$$

Voltage division

$$i_j = \frac{v}{R_j} = \frac{R_{eq}}{R_j} i$$

Current division