#1. 42

#2. $24-\frac{2\sqrt{6}}{3}$

#3. $12x^2y^3$

#4. 7.02

#5. $3x+8y+6z=13$

#6. $\frac{1}{\sqrt{2}}\left(\frac{1}{2}-\sqrt{3}\pi\right)$

#7. $\frac{32}{3}$ $\quad\quad$ $*\frac{16}{3}$**은 3점**

#8. 2ln3

#9. (a) (5점) x축을 따라 극한값을 구하면 $\lim_{x\to 0}\frac{x^6}{x^4}=0$

$y=-x$를 따라 극한값을 구하면 $\lim_{x\to 0}\frac{x^4+x^4+x^6-x^5}{(2x^2)^2}=\lim_{x\to 0}\frac{2x^4+x^6-x^5}{4x^4}=\lim_{x\to 0}\frac{2+x^2-x}{4}=\frac{1}{2}$

$\Rightarrow$ 경로에 따라 극한값이 다르므로 f는 $(0,0)$에서 불연속이다.

(b) (5점) $f_x(0,0)=\lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h}=\lim_{h\to 0}\frac{1}{h}\times\frac{h^6}{h^4}=0$

$f_y(0,0)=\lim_{h\to 0}\frac{f(0,h)-f(0,0)}{h}=\lim_{h\to 0}\frac{1}{h}\times\frac{h^5}{h^4}=1$

#10. $u_x=\frac{x}{u},\ u_y=\frac{y}{u},\ v_x=-\frac{y}{u^2},\ v_y=\frac{x}{u^2},\ u_{xx}=\frac{1}{u}-\frac{x^2}{u^3}=\frac{y^2}{u^3},\ u_{yy}=\frac{x^2}{u^3},\ v_{xx}=\frac{2xy}{u^4},\ v_{yy}=-\frac{2xy}{u^4}$

$f_x=f_uu_x+f_vv_x,\ f_y=f_uu_y+f_vv_y$이므로

$$f_{xx}=\frac{\partial f_u}{\partial x}\times u_x+f_u\times u_{xx}+\frac{\partial f_v}{\partial x}\times v_x+f_v\times v_{xx}$$

$$=(f_{uu}u_x+f_{uv}v_x)\times u_x+f_uu_{xx}+(f_{vu}u_x+f_{vv}v_x)\times v_x+f_vv_{xx}$$

$$f_{yy}=\frac{\partial f_u}{\partial y}\times u_y+f_u\times u_{yy}+\frac{\partial f_v}{\partial y}\times v_y+f_v\times v_{yy}$$

$$=(f_{uu}u_y+f_{uv}v_y)\times u_y+f_uu_{yy}+(f_{vu}u_y+f_{vv}v_y)\times v_y+f_vv_{yy}$$

$$\Rightarrow f_{xx}+f_{yy}=f_{uu}\left((u_x)^2+(u_y)^2\right)+f_{uv}(u_xv_x+u_yv_y)+f_{vv}\left((v_x)^2+(v_y)^2\right)+f_u(u_{xx}+u_{yy})+f_v(v_{xx}+v_{yy})$$

$$=(6u+4v)\left(\frac{x^2}{u^2}+\frac{y^2}{u^2}\right)+(3u^2+4uv)\times\frac{x^2+y^2}{u^3}$$

$$=6u+4v+3u+4v$$

$$=9\sqrt{x^2+y^2}+8\arctan\left(\frac{y}{x}\right)$$

#11. $f_x(x,y)=3x^2+8xy+4y^2=(3x+2y)(x+2y)=0 \Leftrightarrow x=-\dfrac{2}{3}y$ 혹은 $x=-2y$

$f_y(x,y)=4x^2+6y^2+8xy-11=0 \Rightarrow x=-\dfrac{2}{3}y$이면 $\left(\dfrac{16}{9}+6-\dfrac{16}{3}\right)y^2=11$ 즉 $y=\pm\dfrac{3}{\sqrt{2}}$

$x=-2y$이면 $(16+6-16)y^2=11$ 즉 $y=\pm\dfrac{\sqrt{66}}{6}$

$\Rightarrow$ 임계점: $\left(\pm\dfrac{2}{\sqrt{2}},\mp\dfrac{3}{\sqrt{2}}\right)$, $\left(\pm\dfrac{\sqrt{66}}{3},\mp\dfrac{\sqrt{66}}{6}\right)$

$H=\begin{vmatrix} 6x+8y & 8x+8y \\ 8x+8y & 12y+8x \end{vmatrix}=8(3x+4y)(2x+3y)-64(x+y)^2$이므로

$H\left(\dfrac{2}{\sqrt{2}},-\dfrac{3}{\sqrt{2}}\right)=8\times15-64\times\dfrac{1}{2}>0$ & $f_{xx}\left(\dfrac{2}{\sqrt{2}},-\dfrac{3}{\sqrt{2}}\right)=8\times\left(-\dfrac{6}{\sqrt{2}}\right)<0 \Rightarrow \left(\dfrac{2}{\sqrt{2}},-\dfrac{3}{\sqrt{2}}\right)$는 극댓점

$H\left(-\dfrac{2}{\sqrt{2}},\dfrac{3}{\sqrt{2}}\right)=8\times15-64\times\dfrac{1}{2}>0$ & $f_{xx}\left(-\dfrac{2}{\sqrt{2}},\dfrac{3}{\sqrt{2}}\right)=8\times\dfrac{6}{\sqrt{2}}>0 \Rightarrow \left(-\dfrac{2}{\sqrt{2}},\dfrac{3}{\sqrt{2}}\right)$는 극솟점

$H\left(\pm\dfrac{\sqrt{66}}{3},\mp\dfrac{\sqrt{66}}{6}\right)=\dfrac{11}{6}\times(16-64)<0 \Rightarrow \left(\pm\dfrac{\sqrt{66}}{3},\mp\dfrac{\sqrt{66}}{6}\right)$는 안장점

#12. 1) $f_x(x,y)=\dfrac{3}{\sqrt{2}}y+3x(1-x^2-y^2)^{1/2}=0$, $f_y(x,y)=\dfrac{3}{\sqrt{2}}x+3y(1-x^2-y^2)^{1/2}=0$의 해를 구하기 위하여

위의 두 식을 더하면 $(x+y)\left(\dfrac{1}{\sqrt{2}}+\sqrt{1-x^2-y^2}\right)=0 \Rightarrow y=-x$

$\Rightarrow -\dfrac{3}{\sqrt{2}}x+3x\sqrt{1-2x^2}=0$ 즉 $x=0$ 혹은 $x=\pm\dfrac{1}{2}$

$\Rightarrow$ 임계점은 $(0,0)$, $\left(\dfrac{1}{2},-\dfrac{1}{2}\right)$ 혹은 $\left(-\dfrac{1}{2},\dfrac{1}{2}\right)$

$\therefore\ f(0,0)=-1$, $f\left(\pm\dfrac{1}{2},\mp\dfrac{1}{2}\right)=-\dfrac{3}{4\sqrt{2}}-\dfrac{1}{2\sqrt{2}}=-\dfrac{5}{4\sqrt{2}}$

2) $g(x,y)=x^2+y^2=1$에 대하여

$$\dfrac{3}{\sqrt{2}}y+3x(1-x^2-y^2)^{1/2}=2\lambda x$$

$$\dfrac{3}{\sqrt{2}}x+3y(1-x^2-y^2)^{1/2}=2\lambda y$$

$$x^2+y^2=1$$

세 번째 식을 위의 첫 번째 식과 두 번째 식을 대입하면

$$\dfrac{3}{\sqrt{2}}y=2\lambda x, \qquad \dfrac{3}{\sqrt{2}}x=2\lambda y$$

만약 $\lambda=0$이면 $x=y=0 \Rightarrow (0,0)$은 제약식을 만족하지 않으므로 $\lambda\neq0$

만약 $x=0$이면 위의 첫 번째 식에 의해 $y=0$이고 만약 $y=0$이면 두 번째 식에 의해 $x=0$

$\Rightarrow (0,0)$은 제약식을 만족하지 않으므로 $x\neq0$, $y\neq0$

따라서 위의 두 식을 나눌 수 있고, 이를 통해 $\dfrac{y}{x}=\dfrac{x}{y}$ 즉 $x^2=y^2$임을 알 수 있다.

$\Rightarrow$ 제약 조건을 만족하려면 $x^2=y^2=\dfrac{1}{2}$

$\therefore\ f\left(\pm\dfrac{1}{\sqrt{2}},\pm\dfrac{1}{\sqrt{2}}\right)=\dfrac{3}{2\sqrt{2}}$, $f\left(\pm\dfrac{1}{\sqrt{2}},\mp\dfrac{1}{\sqrt{2}}\right)=-\dfrac{3}{2\sqrt{2}}$

1)과 2)에 의하여 f는 $\left(\pm\dfrac{1}{\sqrt{2}},\mp\dfrac{1}{\sqrt{2}}\right)$에서 최솟값 $-\dfrac{3}{2\sqrt{2}}$을 갖고

$\left(\pm\dfrac{1}{\sqrt{2}},\pm\dfrac{1}{\sqrt{2}}\right)$에서 최댓값 $\dfrac{3}{2\sqrt{2}}$를 갖는다.