1. 8

2. $\frac{\ln 2}{10}$

3. $\frac{10}{3}$

4. $\frac{1}{24}$

5. $9\sqrt{2}\pi^3$

6. $\frac{1}{32}\sinh\left(4\sinh^{-1}(2\sqrt{2})\right)-\frac{1}{8}\sinh^{-1}(2\sqrt{2})+\frac{13\pi}{6}$

 * 전원 5점처리 합니다.

7. $x-y+\sqrt{2}z=2\sqrt{2}-2$

8. $\frac{\ln 5}{2}+5$

9. $\frac{6}{7}$

10. $\frac{11}{3}$

11. E: $x^2+y^2 \leqq 1$, $0 \leqq z \leqq x^2+y^2+1$

$$\Rightarrow V = \iiint_E 1\,dV = \int_0^{2\pi}\int_0^1\int_0^{r^2+1} r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 (r^3+r)\,dr\,d\theta$$

$$= 2\pi\left(\frac{1}{4}+\frac{1}{2}\right) = \frac{3}{2}\pi$$

$\partial E = S_1 + S_2 + S_3$, S_1: $z = x^2+y^2+1$, $x^2+y^2 \leqq 1$

S_2: $x^2+y^2=1$, $0 \leqq z \leqq 2$

S_3: $x^2+y^2 \leqq 1$, $z=0$

$$\Rightarrow A(\partial E) = A(S_1)+A(S_2)+A(S_3)$$

$$= \iint_{S_1} 1\,dS + 4\pi + \pi$$

$$= \iint_{x^2+y^2 \leqq 1} \sqrt{1+4x^2+4y^2}\,dx\,dy + 5\pi = \int_0^{2\pi}\int_0^1 r\sqrt{1+4r^2}\,dr\,d\theta + 5\pi$$

$$= \frac{\pi}{6}(5\sqrt{5}-1)+5\pi = \frac{\pi}{6}(5\sqrt{5}+29)$$

- 부피 4점, $A(S_1)$ 3점, $A(S_2)$ 2점, $A(S_3)$ 1점
- 부피 식 세우면 2점

12. $A(D) = \int_C x\,dy = \int_0^{2\pi} (5+2\cos t)\cos t \times 5\cos t\,dt$

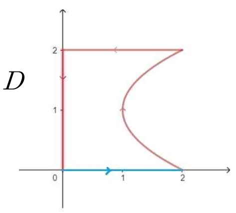
$$= \int_0^{2\pi} (25\cos^2 t + 10\cos^3 t)\,dt = \int_0^{2\pi}\left(\frac{25}{2}+\frac{25}{2}\cos 2t + 10\cos t - 10\cos t\sin^2 t\right)dt$$

$$= 25\pi$$

- $dy = 5\cos t\,dt$ 틀리면 -3점
- 선적분을 생각하긴 하면 2점

13. 아래의 그림과 같이 $C^* = C + L$, L: $C(t) = (t, 0)$, $0 \leq t \leq 2$라 하자.

D



$$\int_{C^*} P\,dx + Q\,dy = \int_C P\,dx + Q\,dy + \int_L P\,dx + Q\,dy \text{이므로}$$

$$\int_C P\,dx + Q\,dy = \int_{C^*} P\,dx + Q\,dy - \int_L P\,dx + Q\,dy$$

$$= \iint_D (Q_x - P_y)\,dA - \int_L P\,dx + Q\,dy$$

$$= \int_0^2\int_0^{(y-1)^2+1} (\sec^2 x - \sec^2 x + 2y - 1)\,dx\,dy - \int_0^2 1\,dx$$

$$= \int_0^2 (2y-1)(y^2-2y+2)\,dy - 2$$

$$= \frac{8}{3} - 2 = \frac{2}{3}$$

- $\iint_D (2y-1)\,dA$까지 쓰면 3점, 영역 D까지 표기하면 5점
- L에서의 선적분 계산 3점

14. D: $0 \leqq \rho \leqq 1$, $0 \leqq \phi \leqq \dfrac{\pi}{4}$, $0 \leqq \theta \leqq 2\pi$

$$\Rightarrow \iint_S \mathrm{F} \cdot \mathrm{n}\, dS = \iiint_D \operatorname{div} \mathrm{F}\, dV = \iiint_D (-y^2 + 3y^2 + 2x^2 + 3z^2)\, dV$$

$$= \int_0^{2\pi} \int_0^{\pi/4} \int_0^1 \rho^2 \sin\phi (2\rho^2 + \rho^2 \cos^2\phi)\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi} 1 d\theta \int_0^{\pi/4} (2\sin\phi + \sin\phi\cos^2\phi) d\phi \int_0^1 \rho^4 d\rho$$

$$= \frac{2\pi}{5}\left[-2\cos\phi - \frac{1}{3}\cos^3\phi\right]_0^{\pi/4} = \frac{2\pi}{5}\left(2 + \frac{1}{3} - \sqrt{2} - \frac{1}{6\sqrt{2}}\right)$$

$$= \frac{2\pi}{5}\left(\frac{7}{3} - \frac{13\sqrt{2}}{12}\right)$$

- 발산 구하면 3점
- 구면좌표계로 변환할 때 피적분함수 정리 틀리면 -2점
- 구면좌표계로 식 세우면 7점, 세우다 말면 정도에 따라 -2점

15. C는 $z = x^2 - y^2$, $x^2 + y^2 = 1$의 교선이므로 S: $z = x^2 - y^2$, $x^2 + y^2 \leqq 1$의 경계곡선

$\mathrm{F}(x, y, z) = (x^2 + z, x + e^y, z) \;\Rightarrow\; \operatorname{curl}\mathrm{F} = (0, 1, 1)$

$$\int_C \mathrm{F} \cdot d\mathrm{s} = \iint_S \operatorname{curl}\mathrm{F} \cdot \mathrm{n}\, dS = \iint_{x^2+y^2 \leqq 1} (0, 1, 1) \cdot (-2x, 2y, 1)\, dx\, dy$$

$$= \iint_{x^2+y^2 \leqq 1} (2y + 1) dx\, dy$$

$$= \int_0^{2\pi} \int_0^1 r(2r\sin\theta + 1) dr\, d\theta = \int_0^1 \int_0^{2\pi} (r + 2r^2 \sin\theta) d\theta\, dr$$

$$= \pi$$

- Stokes 정리 기술하면 2점
- curl F 구하면 3점
- 식 세우면 7점