반	겨울	이름	임서경	학번	12201944	1

문제 1 $y' = e^{2x-1}y^2$

답 $y = -\frac{2e}{e^{2x}+2e-1}$

풀이 $\frac{dy}{dx} = e^{2x-1}\cdot y^2$

$\int \frac{1}{y^2}dy = \int e^{2x-1}dx$, $y(0)=1$

$-\frac{1}{y} = \frac{1}{2}e^{2x-1}+C$

$1 = \frac{1}{2e}+C \quad C = 1-\frac{1}{2e}$

$-\frac{1}{y} = \frac{1}{2}e^{2x-1}+1-\frac{1}{2e}$

$\frac{1}{y} = -\frac{1}{2}e^{2x-1}-1+\frac{1}{2e}$ $\frac{e^{2x}}{2e}$

$\frac{1}{y} = \frac{-e^{2x}-2e+1}{2e}$

문제 2

답 $y = x\ln|x|+x$

풀이 $xy' = x+y$, $y(1)=1$

$y' = 1+\frac{y}{x}$

$\frac{y}{x} = u \quad y = ux$, $y' = u'x+u$

$y' = 1+u$

$u' = \frac{1}{x}$

$u = \ln|x|+C$

$\frac{y}{x} = \ln|x|+C \leftarrow (1,1)$

$1 = C$

$y = x\ln|x|+x$

문제 3 $y''-3y'-4y=0 \quad y(0)=2 \quad y'(0)=1$

답 $y = \frac{3}{5}e^{4x}+\frac{7}{5}e^{-x}$

풀이 $y = e^{\lambda x}$ $\left(\begin{array}{l} y' = \lambda\cdot e^{\lambda x} \\ y'' = \lambda^2\cdot e^{\lambda x}\end{array}\right)$

$\rightarrow \lambda^2-3\lambda-4=0 \quad (\lambda-4)(\lambda+1)=0$

$y = c_1e^{4x}+c_2e^{-x}$

$y' = 4c_1e^{4x}-c_2e^{-x}$

$c_1+c_2=2$

$4c_1-c_2=1$

$c_1 = \frac{3}{5}, \ c_2 = \frac{7}{5}$

문제 4 $y''-6y'+9y=0$

답 $y = c_1e^{3x}+c_2x\cdot e^{3x}$

풀이 $\left(\begin{array}{l} y' = \lambda e^{\lambda x} \\ y'' = \lambda^2 e^{\lambda x}\end{array}\right)$

$y = e^{\lambda x}$

$\rightarrow \lambda^2-6\lambda+9=0 \quad (\lambda-3)^2=0$

$y_1 = e^{3x} \rightarrow y_2 = x\cdot e^{3x}$

$y = c_1e^{3x}+c_2x\cdot e^{3x}$

$y_2 = e^{3x}u(x)$

$y_2' = 3e^{3x}u(x)+e^{3x}u'(x)$

$y_2'' = 9e^{3x}u(x)+3e^{3x}u'(x)$

$+3e^{3x}u'(x)+e^{3x}u''(x)$

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문제 5 $y''+4y=0$

답 $y=C_1\cos 2x+C_2\sin 2x$

풀이 $y=e^{\lambda x} \to \lambda^2+4=0 \to \lambda=\pm 2i$

$y=Ae^{2ix}+Be^{-2ix}$

$(e^{ix}=\cos x+i\sin x)$

$=C_1\cos 2x+C_2\sin 2x$

$\left(\begin{array}{l} y'=\lambda e^{\lambda x} \\ y''=\lambda^2 e^{\lambda x}\end{array}\right)$

문제 7 $x^3y''-5x^2y'+9xy=0$

답 $y=C_1x^3+C_2x^3\ln x$.

풀이 주어진식을 x로 나눔. (오일러-코시)

$x^2y''-5xy'+9y=0$

$y'=mx^{m-1}$

$y''=m(m-1)x^{m-2}$

$y'''=m(m-1)(m-2)x^{m-3}$

$y=x^m \to m^2-6m+9=0$

$(m-3)^2=0 \to y_1=x^3$

$y=C_1x^3+C_2x^3\ln x$

$y_2=x^3\int\frac{1}{x^6}\exp\int\frac{5}{x}dx\, dx$

$=x^3\int\frac{1}{x^6}x^5\, dx$

$=x^3\ln x$

문제 6 $x^3y'''+2x^2y''-xy'+y=0$.

답 $y=C_1x+C_2x\ln x+C_3x^{-1}$

풀이 $y=x^m$

$y'=mx^{m-1}$

$y''=m(m-1)x^{m-2}$

$y'''=m(m-1)(m-2)x^{m-3}$

(오일러-코시)

$m(m-1)(m-2)+2m(m-1)-m+1=0$

$(m-1)(m^2-2m+2m-1)=0$

$(m-1)^2(m+1)=0$.

$y=C_1x+C_2x\ln x+C_3x^{-1}$

문제 8 $x^2y''-4xy'+6y=0$

답 $y=C_1x^2+C_2x^3$

풀이 (오일러-코시)

$y'=mx^{m-1}$

$y''=m(m-1)x^{m-2}$

$y'''=m(m-1)(m-2)x^{m-3}$

$y=x^m \to m^2-5m+6=0$

$(m-2)(m-3)=0$

$y_1=x^2,\ y_2=x^3$

$y=C_1x^2+C_2x^3$

분반	겨울	이름	임서경	학번	12201944	3

문제 9 $y' = \frac{4e^x - e^4}{xe^4 - e^x}$

답 $ye^x - xe^y = c$

풀이 $\frac{dy}{dx} = \frac{ye^x - e^y}{xe^y - e^x}$

$dy(xe^y - e^x) = dx(ye^x - e^y)$

$dx(ye^x - e^y) + dy(e^x - xe^y) = 0$

$(e^x - e^y) = (e^x - e^y$ 이므로 완전상미분.

$u(x,y) = \int (ye^x - e^y)dx$

$= ye^x - xe^y + k(y) = c$

$\frac{\partial u}{\partial y} = e^x - xe^y + k'(y) = e^x - xe^y$

$u(x,y) = ye^x - xe^y = c$

문제 11 풀토벤

답 2500마리

풀이 비감염자수 $y(t)$

$y'(t) = 10000\{0.4\frac{y(t)}{10000}(1 - \frac{y(t)}{10000})\}$

$- 0.1(10000 - y(t))$

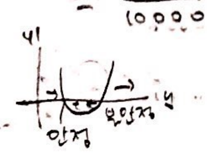
$y' = 0.4y(t)(-1 + \frac{y(t)}{10000})$

$+ 1000 - 0.1y(t)$

$= -0.5y + \frac{0.4}{10000}y^2 + 1000$

$= \frac{1}{10000}(0.4y^2 - 5000y + 10000000)$

$= \frac{1}{10000}(y - 10000)(0.4y - 1000)$



안정해 $y = \frac{1000}{0.4}$

$= 2500$

문제 10 $y' = (x+y)^2 + 3,\ y(0) = 2$

답 $y = 2\tan(2x + \frac{\pi}{4}) - x$

풀이 $u = x + y,\quad y = u - x$

$\begin{cases} y' = u' - 1 \\ y' = u^2 + 3 \end{cases}$

$u' = u^2 + 4$

$\int \frac{1}{u^2+4}du = \int dx$

$u = 2\tan t$
$du = 2\sec^2 t\,dt$

$\int \frac{1}{2}dt = x + C$

$\frac{1}{2}\tan^{-1}(\frac{u}{2}) = x + C$

$\frac{u}{2} = \tan(2x + C_1)$

$y = 2\tan(2x + C_1)$ $\quad u = 2\tan(2x + C_1)$

$- x$ ← $(0, 2)$ 대입

$2 = 2\tan C_1,\ C_1 = \frac{\pi}{4}$

문제 12 $x^3y''' - 5x^2y'' + 12xy' - 12y = x^{-2}$

답 $y = C_1x + C_2x^3 + C_3x^4$
$- \frac{1}{90}x^{-2}$

풀이 $y = x^m \rightarrow m(m-1)(m-2) - 5m(m-1) + 12m$
$- 12 = 0$

(오일러-코시)

$(m-1)(m^2 - 2m - 5m + 12) = 0$

$(m-1)(m-3)(m-4) = 0$

$y_h = C_1x + C_2x^3 + C_3x^4$

$W = \begin{vmatrix} x & x^3 & x^4 \\ 1 & 3x^2 & 4x^3 \\ 0 & 6x & 12x^2 \end{vmatrix} = x \cdot 12x^4 - x^3(12x^2)$
$+ x^4(6x)$

$= 6x^5$

$W_1 = x^6$

$W_2 = -3x^4$ $\quad h(x) = x^{-5}$ $\quad -\frac{1}{18}x^{-6}$

$W_3 = 2x^3$ $\quad \frac{1}{6}x^{-4}$ $\quad -\frac{1}{18}x^{-3}$ $\quad -\frac{1}{2}x^{-6}$ $\quad \frac{1}{10}x^{-5}$

$y_p = x\int \frac{x^6 \cdot x^{-5}}{6x^5}dx + x^3\int \frac{-3x^4 \cdot x^{-5}}{6x^5}dx + x^4 \int \frac{2x^3 \cdot x^{-5}}{6x^5}dx$ $\quad \frac{x^{-2}}{?}$

$= -\frac{1}{10}x^{-2} + \frac{1}{10}x^{-2} - \frac{1}{18}x^{-2} = -\frac{x^{-2}}{90}$

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문제 13 $xyy' + (x-1)y^2 = 2x^2 \quad (y^2 = z)$

답 $y^2 = \dfrac{2x^2 - x}{x-1}$

풀이 $y' + (1 - \frac{1}{x})y = 2x\, y^{-1}$

$z = y^2$ $\quad z' + (2 - \frac{2}{x})z = 4x$

$z' = 2yy'$ $\quad F = \exp\int (2 - \frac{2}{x})dx$ $\quad 2x - 2\ln x$

$= 2y((\frac{1}{x} - 1)y + 2xy^{-1})$ $\quad = \frac{e^{2x}}{x^2}$

$= (\frac{2}{x} - 2)z + 4x$

$z = \frac{x^2}{e^{2x}} \int \frac{e^{2x}}{x^2} \cdot 4x\, dx$ $\quad \frac{1}{2}e^{2x} \cdot x^{-1} + \frac{1}{2}\int e^{2x} \cdot x^{-2} dx$

$= 4x^2 \cdot e^{-2x} \left(\int e^{2x} \cdot x^{-1} dx\right)$ $\quad \frac{1}{2}e^{2x} \cdot x^{-2} + \int e^{2x} \cdot x^{-3} dx$

$y^2 = 2x + 2x^2 e^{-2x} \int e^{2x} \cdot x^{-2} dx$ $\quad \frac{1}{2}e^{2x} x^{-3} + \frac{3}{2}\int e^{2x} x^{-4}$

$= 2x + 1 + 2x^2 e^{-2x} \int e^{2x} \cdot x^{-3} dx$

$= 2x + 1 + \frac{1}{x} + \frac{1}{x^2} + \cdots = 2x + \frac{1}{1 - \frac{1}{x}} = 2x + \frac{x}{x-1}$

문제 14

답 $y = c_1 e^x + c_2 x \cdot e^x + x^2 + 4x + 6$

풀이 $y = e^{\lambda x} \to \lambda^2 - 2\lambda + 1 = 0$

$(\lambda - 1)^2 = 0$

$y_h = c_1 e^x + c_2 x \cdot e^x$

$y_p = ax^2 + bx + c$

$y_p' = 2ax + b. \quad y_p'' = 2a$

$2a - 2(2ax + b) + ax^2 + bx + c = x^2$

$\cancel{x^2} + 2 - 4x - 2b + bx + c = \cancel{x^2}$

$a = 1, \ b = 4, \ c = 6$

$y_p = x^2 + 4x + 6$

문제 15

답 $y = c_1 x + c_2 x^{-1} + 2x^3$

풀이 $y = x^m \ + \quad m^2 - 1 = 0$

$(c_0 \cdot 2x - 2x(1))$ $\quad y_h = c_1 x + c_2 x^{-1}$

$w = \begin{vmatrix} x & x^{-1} \\ 1 & -x^{-2} \end{vmatrix} = -\frac{2}{x}$ $\quad f(x) = 16x$

$y_p = -x \int \frac{x^{-1} \cdot 16x}{-\frac{2}{x}} dx + \frac{1}{x} \int \frac{x \cdot 16x}{-\frac{2}{x}} dx$

$= -x \int -8x\, dx + \frac{1}{x} \int (-8x^3) dx$

$-4x^2 \qquad -2x^4$

$= 4x^3 - 2x^3 = 2x^3$

$y = y_h + y_p.$

문제 16

답 $y = c_1 \cos x + c_2 \sin x + 4e^x - \frac{x}{2}\cos x$

풀이 $y = e^{\lambda x} \to \lambda^2 + 1 = 0 \to \lambda = \pm i$

$y_h = c_1 \cos x + c_2 \sin x$

$y_p = ke^x + x(A\cos x + B\sin x)$

$y_p' = ke^x + A\cos x + B\sin x + x(-A\sin x + B\cos x)$

$y_p'' = ke^x - 2A\sin x + 2B\cos x + x(-A\cos x - B\sin x)$

$y_p'' + y_p = 2ke^x - 2A\sin x + 2B\cos x = 8e^x + \sin x$

$k = 4, \ A = -\frac{1}{2}, \ B = 0$

$y_p = 4e^x + x(-\frac{1}{2}\cos x)$

$y = y_h + y_p$

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문제 17

답

12m

풀이

$m=2,\ y(0)=4,\ y'(0)=0,\ 20=8k,\ k=2.5$

$2y''+2.5y=0 \quad \omega=\sqrt{\frac{k}{m}}=\frac{\sqrt5}{2}$

$y=A\cos\frac{\sqrt5}{2}x+B\sin\frac{\sqrt5}{2}x$

$y(0)=A=4 \quad y=4\cos\frac{\sqrt5}{2}x$

$y'(0)=B=0 \quad -4\le\cos\frac{\sqrt5}{2}x\le4$

$f=10=5y$ 최고점 - 최저점

$y=2.$ $=(4+2)-(-4-2)$

$=12$

문제 19

답

2

풀이

$\frac{2y}{x}\ne0$ (완전상미분X)

1) F_x : $F_x\cdot\frac{2y}{x}=-F_x'\cdot 2y$

$\frac{F_x'}{F_x}=-\frac{1}{x}$ $\ln|F_x|=-\ln x+C$

$F_x=\frac{1}{x}$

$(1+\frac{y^2}{x^2})dx-\frac{2y}{x}dy=0$

$u(x,y)=x-\frac{y^2}{x}+\cancel{k(y)}=C$ $\frac{\partial u}{\partial y}=-\frac{2y}{x}$

$\hookrightarrow(1,0)$

$C=1$

$x=2\rightarrow 2-\frac{y^2}{2}=1 \quad y^2=2$

문제 18

답

$y=e^{-x}(c_1\cos\frac{\sqrt2}{2}x+c_2\sin\frac{\sqrt2}{2}x)$
$-\frac{1}{5}\cos2t+\frac{1}{8}\sin2t$

풀이

$2I''+4I'+3I=2\cos2t$

$I''+2I'+1.5I=\cos2t$

$y=e^{\lambda x}\rightarrow \lambda^2+2\lambda+1.5=0$ $\frac{-4\pm\sqrt{16-24}}{4}$ $\frac{2\sqrt{2}i}{4}$

$2\lambda^2+4\lambda+3=0.$ $-1\pm\frac{\sqrt2 i}{2}$

$y_h=a\,e^{(-1+\frac{\sqrt2}{2}i)x}+b\,e^{(-1+\frac{\sqrt2}{2}i)x}$

$=e^{-x}(c_1\cos\frac{\sqrt2}{2}x+c_2\sin\frac{\sqrt2}{2}x)$

$y_p=A\cos2t+B\sin2t$

$y_p'=-2A\sin2t+2B\cos2t$ $A=\frac{1}{5}$ $B=\frac{1}{8}$

$y_p''=-4A\cos2t-4B\sin2t$ $-2.5A+4B=1$ $-2.5A-4B=0$

$I''+2I'+1.5I$

$=-4A\cos2t-4B\sin2t-4A\sin2t$

$+4B\cos2t+1.5A\cos2t+1.5B\sin2t$

$=\cos2t$

문제 20

답

$y=-x+x\ln|x|+1$

풀이

$y_1=x \quad y''-\frac{1}{x-1}y'+\frac{1}{x^2-x}y=0$

$y_2=x\int\frac{1}{x^2}\left(\exp\int(+\frac{1}{x-1})dx\right)dx$

$=x\int\frac{1}{x^2}\cdot(x-1)\,dx$

$=x\int(\frac{1}{x}-\frac{1}{x^2})dx$

$=x\ln|x|+1$

$y=c_1x+c_2(x\ln x+1)$

$e(c_1+c_2)+c_2=1$ $c_1+c_2+\frac{c_2}{e}=\frac{1}{e}$

$c_1+c_2\times2=1$ $c_1+2c_2=1$

$\rightarrow c_2=1,\ c_1=-1$

$y=-x+x\ln|x|+1$

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