Chapter 7

Detecting Data Signals in Noise

7.1 Problems

Problem 7.1 (Linear processor) *A linear processor operates on $n_x = 5$ samples with coefficients*

$$c_0 = 1, c_1 = 2, c_2 = 3, c_3 = 2, c_4 = 1$$

The observed signal sequence X_i equals

$$X_0 = 2, X_1 = -3, X_2 = 1, X_3 = 0, X_4 = -1$$

Compute output V produced by X_i.

(ans:

$$V = \sum_{i=0}^{4} c_i (X_i) = 1(2) + 2(-3) + 3(1) + 2(0) + 1(-1) = -2$$

)

Problem 7.2 (Linear processor with $c_i = 0$) *A linear processor operates on $n_x = 5$ samples with coefficients*

$$c_0 = 0.5, c_1 = 2, c_2 = 0, c_3 = -3, c_4 = -1$$

Rank the importance of each of the input signal sequence values X_i for $0 \le i \le 4$, as determined by the coefficient magnitudes.

(ans: The rule is that the importance of the signal value is proportional to the coefficient magnitude.

$$\begin{array}{lcll}
|c_3| = 3 & \rightarrow & X_3 & \textit{most important} \\
|c_1| = 2 & \rightarrow & X_1 & \\
|c_4| = 1 & \rightarrow & X_4 & \\
|c_0| = 0.5 & \rightarrow & X_0 & \\
|c_2| = 0 & \rightarrow & X_2 & \textit{irrelevant}
\end{array}$$

)

Problem 7.3 (Matched processor) *Consider the signal sequence*

$$s_0 = 1, s_1 = 2, s_2 = 3, s_3 = 4, s_4 = 5$$

1. *Compute the signal energy $\mathcal{E}_s$.*
2. *Design the matched processor.*
3. *Compute the matched processor output V when $X_i = s_i$ for $0 \le i \le 4$.*
4. *Specify a different signal sequence s'_i for $0 \le i \le 4$ having the same $\mathcal{E}_s$.*
5. *Compute the matched processor output V' when $X_i = s'_i$.*

(ans:

1.

$$\mathcal{E}_s = \sum_{i=0}^{4} s_i^2 = 1 + 4 + 9 + 16 + 25 = 45$$

2.

$$c_i = s_i \quad \rightarrow \; c_0 = 1 \;\; c_1 = 2 \;\; c_2 = 3 \;\; c_3 = 4 \;\; c_4 = 5$$

3. *Compute the matched processor output V when $X_i = s_i$ for $0 \le i \le 4$.*

$$V = \sum_{i=0}^{4} s_i \overbrace{X_i}^{=s_i} = \sum_{i=0}^{4} s_i^2 = 1 + 4 + 9 + 16 + 25 = 45 \quad (= \mathcal{E}_s)$$

4. *Let $s'_i = s_{4-i}$ for $0 \le i \le 4$. That is, it is the time-reversed version given by*

$$s\prime_0 = s_4 = 5, s'_1 = s_3 = 4, s'_2 = s_2 = 3, s'_3 = s_1 = 2, s'_4 = s_0 = 1$$

$$\mathcal{E}_s = \sum_{i=0}^{4} (s'_i)^2 = 25 + 16 + 9 + 4 + 1 = 45$$

   Hence, the energies are equal. Any permutation of s_i produces the same result.

5. *Compute the matched processor output V' when $X_i = s'_i$.*

$$V' = \sum_{i=0}^{4} s_i \overbrace{(X_i)}^{=s'_i} = 1(5) + 2(4) + 3(3) + 4(2) + 5(1) = 35 \quad (< \mathcal{E}_s)$$

)

Problem 7.4 (Designing a maximum energy signal) *A transmission channel is constrained by allowing signals that have magnitudes $|s_i| \le 2$ V.*

1. *Design a valid signal sequence* s_i *for* $0 \le i \le 4$ *that has the maximum* $\mathcal{E}_s$.
2. *Compute the signal energy* $\mathcal{E}_s$.
3. *Design the matched processor.*
4. *Compute the matched processor output* V *when* $X_i = s_i$.

(ans:

1. *The simplest signal is* $s_i = 2\ (= V_{max})$ *for* $0 \le i \le 4$.
2. $$\mathcal{E}_s = \sum_{i=0}^{4} s_i^2 = 4 + 4 + 4 + 4 + 4 = 20$$
3. $$c_i = s_i \quad \rightarrow c_0 = 2 \ \ c_1 = 2 \ \ c_2 = 2 \ \ c_3 = 2 \ \ c_4 = 2$$
4. $$V = \sum_{i=0}^{4} s_i \overbrace{X_i}^{=s_i} = \sum_{i=0}^{4} s_i^2 = 4 + 4 + 4 + 4 + 4 = 20 \quad (= \mathcal{E}_s)$$

)

Problem 7.5 (Complementary signals) *Let* $s_i = i + 1$ *for* $0 \le i \le 4$.

1. *Form the complementary pair* $s1_i$ *and* $s0_i$*. Compute the matched processor output* V *when each signal is observed.*
2. *Specify a different signal sequence* s'_i *for* $0 \le i \le 4$ *having the same* $\mathcal{E}_s$.
3. *Compute the matched processor output* V' *when* $X_i = s'_i$.

(ans:

1. $$s1_i = s_i : \quad s1_0 = 1, \quad s1_1 = 2, \quad s1_2 = 3, \quad s1_3 = 4, \quad s1_4 = 5$$
$$s0_i = -s_i : \quad s0_0 = -1, \quad s0_1 = -2, \quad s0_2 = -3, \quad s0_3 = -4, \quad s0_4 = -5$$
$$V_{|X=s1} = \sum_{i=0}^{4} s_i \overbrace{s1_i}^{=s_i} = \sum_{i=0}^{4} s_i^2 1 + 4 + 9 + 16 + 25 = 45$$
$$V_{|X=s0} = \sum_{i=0}^{4} s_i \overbrace{s0_i}^{=-s_i} = -\sum_{i=0}^{4} s_i^2 - 1 - 4 - 9 - 16 - 25 = -45$$

2. *Let* $s'_i = s_{4-i}$ *for* $0 \le i \le 4$. *That is, it is the time-reversed version given by*

$$s'_0 = s_4 = 5, s'_1 = s_3 = 4, s'_2 = s_2 = 3, s'_3 = s_1 = 2, s'_4 = s_0 = 1$$

$$\mathcal{E}_s = \sum_{i=0}^{4} (s'_i)^2 = 25 + 16 + 9 + 4 + 1 = 45$$

   Hence, the energies are equal. Any permutation of s_i *produces the same result.*

3. *Compute the matched processor output* V' *when* $X_i = s'_i$.

$$V' = \sum_{i=0}^{4} s_i \overbrace{(X_i)}^{=s'_i} = 1(5) + 2(4) + 3(3) + 4(2) + 5(1) = 35 \quad (< \mathcal{E}_s)$$

)

Problem 7.6 (Sinusoidal signal energy) *Compute the energy of the sinusoidal signal*

$$s_i = 2\sin(2\pi i/8) \textit{ for } 0 \le i \le 31$$

(ans:

$$\mathcal{E}_s = \sum_{i=0}^{31} s_i^2 = \sum_{i=0}^{31} [2\sin(2\pi i/8)]^2 = 4\sum_{i=0}^{31} \sin^2(2\pi i/8)$$

Note that there are four complete periods of the sinusoid in the 32 samples (from 0 to 31). Hence,

$$\sum_{i=0}^{31} \sin^2(2\pi i/8) = \frac{n_x}{2} = 16$$

This gives

$$\mathcal{E}_s = 4(16) = 64$$

)

Problem 7.7 (Determining σ_N^2 from observing a time sequence) *You observe a sequence containing a large number of Gaussian noise samples and notice that almost all samples fall within the range* $[-120, 120]$ *mV. What are reasonable values for* σ_N *and* σ_N^2*?*

(ans: If almost all, but not all, values fall within a particular interval, and the number of random values are large, then it is reasonable to assume that the interval contains 95% of the values. For Gaussian random numbers, the interval $[-2\sigma, 2\sigma]$ *contain approximately 95% of the numbers. Hence, a reasonable SD estimate is*

$$\sigma_N \approx \frac{240\,mV}{4} = 60\,mV_{rms}$$

and the associated variance

$$\sigma_N^2 \approx 3600\,mV_{rms}^2$$

This provides a ball park value that can be determined simply.
)

Problem 7.8 (Signal-to-noise ratio of sinusoidal signal) *Let the sinusoidal signal be given by*

$$s_i = 2\sin(2\pi i/8) \textit{ for } 0 \le i \le 31$$

The detected signal is

$$X_i = s_i + N_i \textit{ for } 0 \le i \le 31$$

where N_i is Gaussian noise having $\sigma_N = 0.1$. What are the values of the power signal-to-noise ratio and SNR_{dB}?

(ans:

$$SNR = \frac{\mathcal{E}_s}{\sigma_N^2}$$

The signal energy equals

$$\mathcal{E}_s = \sum_{i=0}^{31} s_i^2 = \sum_{i=0}^{31} [2\sin(2\pi i/8)]^2 = 4(16) = 64 \quad (= n_x A^2/2)$$

The noise variance equals $\sigma_N^2 = (0.1)^2 = 0.01$. These values give

$$SNR = \frac{64}{0.01} = 6400$$

Using decibel units gives

$$SNR_{dB} = 10\overbrace{\log_{10} 6400}^{=3.81} = 38.1 \; dB$$

)

Problem 7.9 (Simulating random bits) *If Y_i is a random number produced by the uniform PRNG and* floor *is a function that rounds down (truncates) a number value down to its integer value, explain why $D = floor(2 * Y_i)$ produces values 0 and 1 that are equally likely.*

(ans: The random number Y_i lies in the interval [0,1). Multiplying Y_i by two, produces random number $X_i = 2Y_i$ that lies in the interval [0,2). The floor function produces the digit value D_i to the left of the decimal point, which gives

$$X_i = [0,1) \rightarrow D_i = 0 \quad \textit{and} \quad x_i = [1,2) \rightarrow D_i = 1$$

)

7.2 Excel Projects

Project 7.1 (Linear processor) *Extend Example 13.36 to implement a linear processor having coefficient set*

$$c_i = 2i + 1 \textit{ for } \; i = 0, 1, 2, \ldots, n_x - 1 \;\; (n_X = 10)$$

Enter a Gaussian random input sequence X_i for $0 \le i \le n_X - 1$, and compute output V.

(ans:

	A	B	C	D
1	i	c_i	X_i	$c_i\,X_i$
2	0	1	-1.11	-1.11
3	1	3	0.12	0.37
4	2	5	-0.72	-3.58
5	3	7	-0.07	-0.47
6	4	9	0.22	2.00
7	5	11	1.02	11.25
8	6	13	0.58	7.49
9	7	15	-0.44	-6.58
10	8	17	0.03	0.50
11	9	19	-0.85	-16.09
12				
13			V=	-6.23

	A	B	C	D
1	i	c_i	X_i	$c_i\,X_i$
2	0	=2*A2+1	=NORM.S.INV(RAND())	=B2*C2
3	=1+A2	=2*A3+1	=NORM.S.INV(RAND())	=B3*C3
4	=1+A3	=2*A4+1	=NORM.S.INV(RAND())	=B4*C4
5	=1+A4	=2*A5+1	=NORM.S.INV(RAND())	=B5*C5
6	=1+A5	=2*A6+1	=NORM.S.INV(RAND())	=B6*C6
7	=1+A6	=2*A7+1	=NORM.S.INV(RAND())	=B7*C7
8	=1+A7	=2*A8+1	=NORM.S.INV(RAND())	=B8*C8
9	=1+A8	=2*A9+1	=NORM.S.INV(RAND())	=B9*C9
10	=1+A9	=2*A10+1	=NORM.S.INV(RAND())	=B10*C10
11	=1+A10	=2*A11+1	=NORM.S.INV(RAND())	=B11*C11
12				
13			V=	=SUM(D2:D11)

)

Project 7.2 (Matched processor) *Extend Example 13.37 to implement a matched processor having coefficient set that is matched to input signal*

$$s_i = \sin(2\pi i/16)\, \textit{for}\;\; i = 0, 1, 2, \ldots, n_x - 1 \;\; (n_X = 16)$$

and compute output V *when* $X_i = s_i$ *for* $i = 0, 1, 2, \ldots, n_x - 1.$

(ans:

	A	B	C	D	E	F
1	i	s_i		$c_i = s_i$	X_i	$c_i\, X_i$
2	0	0.00		0.00	0.00	0.00
3	1	0.38		0.38	0.38	0.15
4	2	0.71		0.71	0.71	0.50
5	3	0.92		0.92	0.92	0.85
6	4	1.00		1.00	1.00	1.00
7	5	0.92		0.92	0.92	0.85
8	6	0.71		0.71	0.71	0.50
9	7	0.38		0.38	0.38	0.15
10	8	0.00		0.00	0.00	0.00
11	9	-0.38		-0.38	-0.38	0.15
12	10	-0.71		-0.71	-0.71	0.50
13	11	-0.92		-0.92	-0.92	0.85
14	12	-1.00		-1.00	-1.00	1.00
15	13	-0.92		-0.92	-0.92	0.85
16	14	-0.71		-0.71	-0.71	0.50
17	15	-0.38		-0.38	-0.38	0.15
18						
19					V=	8

	A	B	C	D	E	F
1	i	s_i		$c_i = s_i$	X_i	$c_i\, X_i$
2	0	=SIN(2*PI()*A2/16)		=B2	=B2	=D2*E2
3	=1+A2	=SIN(2*PI()*A3/16)		=B3	=B3	=D3*E3
4	=1+A3	=SIN(2*PI()*A4/16)		=B4	=B4	=D4*E4
5	=1+A4	=SIN(2*PI()*A5/16)		=B5	=B5	=D5*E5
6	=1+A5	=SIN(2*PI()*A6/16)		=B6	=B6	=D6*E6
7	=1+A6	=SIN(2*PI()*A7/16)		=B7	=B7	=D7*E7
8	=1+A7	=SIN(2*PI()*A8/16)		=B8	=B8	=D8*E8
9	=1+A8	=SIN(2*PI()*A9/16)		=B9	=B9	=D9*E9
10	=1+A9	=SIN(2*PI()*A10/16)		=B10	=B10	=D10*E10
11	=1+A10	=SIN(2*PI()*A11/16)		=B11	=B11	=D11*E11
12	=1+A11	=SIN(2*PI()*A12/16)		=B12	=B12	=D12*E12
13	=1+A12	=SIN(2*PI()*A13/16)		=B13	=B13	=D13*E13
14	=1+A13	=SIN(2*PI()*A14/16)		=B14	=B14	=D14*E14
15	=1+A14	=SIN(2*PI()*A15/16)		=B15	=B15	=D15*E15
16	=1+A15	=SIN(2*PI()*A16/16)		=B16	=B16	=D16*E16
17	=1+A16	=SIN(2*PI()*A17/16)		=B17	=B17	=D17*E17
18						
19					V=	=SUM(F2:F17)

)

Project 7.3 (Complementary signals) *Extend Example 13.38 to implement a matched processor having coefficients matched to input signal*

$$s_i = \sin(2\pi i/16) \textit{ for } \; i = 0, 1, 2, \ldots, n_x - 1 \;\; (n_X = 16)$$

Generate the pair of complementary signals $s1_i$ and $s0_i$, and compute outputs $V|_{X=s1}$ and $V|_{X=s0}$.

(ans:

	A	B	C	D	E	F	G	H	I	J
1				$X_i = s1_i = s_i$				$X_i = s0_i = -s_i$		
2	i	s_i		c_i	X_i	$c_i X_i$		c_i	X_i	$c_i X_i$
3	0	0.00		0.00	0.00	0.00		0.00	0.00	0.00
4	1	0.38		0.38	0.38	0.15		0.38	-0.38	-0.15
5	2	0.71		0.71	0.71	0.50		0.71	-0.71	-0.50
6	3	0.92		0.92	0.92	0.85		0.92	-0.92	-0.85
7	4	1.00		1.00	1.00	1.00		1.00	-1.00	-1.00
8	5	0.92		0.92	0.92	0.85		0.92	-0.92	-0.85
9	6	0.71		0.71	0.71	0.50		0.71	-0.71	-0.50
10	7	0.38		0.38	0.38	0.15		0.38	-0.38	-0.15
11	8	0.00		0.00	0.00	0.00		0.00	0.00	0.00
12	9	-0.38		-0.38	-0.38	0.15		-0.38	0.38	-0.15
13	10	-0.71		-0.71	-0.71	0.50		-0.71	0.71	-0.50
14	11	-0.92		-0.92	-0.92	0.85		-0.92	0.92	-0.85
15	12	-1.00		-1.00	-1.00	1.00		-1.00	1.00	-1.00
16	13	-0.92		-0.92	-0.92	0.85		-0.92	0.92	-0.85
17	14	-0.71		-0.71	-0.71	0.50		-0.71	0.71	-0.50
18	15	-0.38		-0.38	-0.38	0.15		-0.38	0.38	-0.15
19										
20					$V\|_{X=s1}=$	8.00			$V\|_{X=s0}=$	-8.00

	A	B	C	D	E	F	G	H	I	J
1				$X_i = s1_i = s_i$				$X_i = s0_i = -s_i$		
2	i	s_i		c_i	X_i	$c_i X_i$		c_i	X_i	$c_i X_i$
3	0	=SIN(2*PI()*A3/16)		=B3	=B3	=D3*E3		=B3	=-B3	=H3*I3
4	=1+A3	=SIN(2*PI()*A4/16)		=B4	=B4	=D4*E4		=B4	=-B4	=H4*I4
5	=1+A4	=SIN(2*PI()*A5/16)		=B5	=B5	=D5*E5		=B5	=-B5	=H5*I5
6	=1+A5	=SIN(2*PI()*A6/16)		=B6	=B6	=D6*E6		=B6	=-B6	=H6*I6
7	=1+A6	=SIN(2*PI()*A7/16)		=B7	=B7	=D7*E7		=B7	=-B7	=H7*I7
8	=1+A7	=SIN(2*PI()*A8/16)		=B8	=B8	=D8*E8		=B8	=-B8	=H8*I8
9	=1+A8	=SIN(2*PI()*A9/16)		=B9	=B9	=D9*E9		=B9	=-B9	=H9*I9
10	=1+A9	=SIN(2*PI()*A10/16)		=B10	=B10	=D10*E10		=B10	=-B10	=H10*I10
11	=1+A10	=SIN(2*PI()*A11/16)		=B11	=B11	=D11*E11		=B11	=-B11	=H11*I11
12	=1+A11	=SIN(2*PI()*A12/16)		=B12	=B12	=D12*E12		=B12	=-B12	=H12*I12
13	=1+A12	=SIN(2*PI()*A13/16)		=B13	=B13	=D13*E13		=B13	=-B13	=H13*I13
14	=1+A13	=SIN(2*PI()*A14/16)		=B14	=B14	=D14*E14		=B14	=-B14	=H14*I14
15	=1+A14	=SIN(2*PI()*A15/16)		=B15	=B15	=D15*E15		=B15	=-B15	=H15*I15
16	=1+A15	=SIN(2*PI()*A16/16)		=B16	=B16	=D16*E16		=B16	=-B16	=H16*I16
17	=1+A16	=SIN(2*PI()*A17/16)		=B17	=B17	=D17*E17		=B17	=-B17	=H17*I17
18	=1+A17	=SIN(2*PI()*A18/16)		=B18	=B18	=D18*E18		=B18	=-B18	=H18*I18
19										
20					$V\|_{X=s1}=$	=SUM(F3:F18)			$V\|_{X=s0}=$	=SUM(J3:J18)

)

Project 7.4 (Matched processor output when observing signals in noise) *Modify Example 13.39 to implement a matched processor having coefficients matched to input signal*

$$s_i = \sin(2\pi i/16) \, \textit{for} \;\; i = 0, 1, 2, \ldots, n_x - 1 \;\; (n_X = 16)$$

To form the observed signal X_i add Gaussian noise having a specified σ_N to s_i. Determine the value of σ_N for which errors errors begin to occur (an error is observed 10% of the time).

(ans: The value $\sigma_N = 2$ causes errors to occur about every ten transmissions. Conditional formatting was applied to make errors evident. In `G22 <0.5` *condition makes cell red when error occurs. In* `K22 >0.5` *condition makes cell red when error occurs.*

	A	B	C	D	E	F	G	H	I	J	K
1	σ_N=	2			$X_i = s1_i+N_i = s_i+N_i$				$X_i = s0_i+N_i = -s_i+N_i$		
2	i	s_i	N_i		c_i	X_i	$c_i\,X_i$		c_i	$X_i = s0_i$	$c_i\,X_i$
3	0	0.00	0.15		0	0.15	0.00		0	0.15	0.00
4	1	0.38	1.11		0.38	1.49	0.57		0.38	0.73	0.28
5	2	0.71	-2.17		0.71	-1.46	-1.04		0.71	-2.88	-2.04
6	3	0.92	-3.36		0.92	-2.44	-2.25		0.92	-4.28	-3.96
7	4	1.00	-3.61		1	-2.61	-2.61		1	-4.61	-4.61
8	5	0.92	-0.53		0.92	0.39	0.36		0.92	-1.46	-1.35
9	6	0.71	0.68		0.71	1.39	0.98		0.71	-0.02	-0.02
10	7	0.38	2.49		0.38	2.88	1.10		0.38	2.11	0.81
11	8	0.00	-3.09		0	-3.09	0.00		0	-3.09	0.00
12	9	-0.38	0.57		-0.4	0.19	-0.07		-0.38	0.95	-0.36
13	10	-0.71	-1.63		-0.7	-2.34	1.65		-0.71	-0.92	0.65
14	11	-0.92	-0.07		-0.9	-0.99	0.92		-0.92	0.85	-0.79
15	12	-1.00	4.09		-1	3.09	-3.09		-1	5.09	-5.09
16	13	-0.92	1.29		-0.9	0.37	-0.34		-0.92	2.21	-2.05
17	14	-0.71	-2.26		-0.7	-2.97	2.10		-0.71	-1.56	1.10
18	15	-0.38	1.82		-0.4	1.44	-0.55		-0.38	2.21	-0.84
19											
20						$V\|_{X=s1+N}$=	-2.26			$V\|_{X=s0+N}$=	-18.26
21											
22						~B=	0			~B=	0

	A	B	C	D	E	F	G	H	I	J	K
1	σ_N=	2			$X_i = s1_i+N_i = s_i+N_i$				$X_i = s0_i+N_i = -s_i+N_i$		
2	i	s_i	N_i		c_i	X_i	$c_i\,X_i$		c_i	$X_i = s0_i$	$c_i\,X_i$
3	0	=SIN(2*PI()*A3/16)	=$B$1*NORM.S.INV(RAND())		=B3	=B3+C3	=E3*F3		=B3	=-B3+C3	=I3*J3
4	=1+A3	=SIN(2*PI()*A4/16)	=$B$1*NORM.S.INV(RAND())		=B4	=B4+C4	=E4*F4		=B4	=-B4+C4	=I4*J4
5	=1+A4	=SIN(2*PI()*A5/16)	=$B$1*NORM.S.INV(RAND())		=B5	=B5+C5	=E5*F5		=B5	=-B5+C5	=I5*J5
6	=1+A5	=SIN(2*PI()*A6/16)	=$B$1*NORM.S.INV(RAND())		=B6	=B6+C6	=E6*F6		=B6	=-B6+C6	=I6*J6
7	=1+A6	=SIN(2*PI()*A7/16)	=$B$1*NORM.S.INV(RAND())		=B7	=B7+C7	=E7*F7		=B7	=-B7+C7	=I7*J7
8	=1+A7	=SIN(2*PI()*A8/16)	=$B$1*NORM.S.INV(RAND())		=B8	=B8+C8	=E8*F8		=B8	=-B8+C8	=I8*J8
9	=1+A8	=SIN(2*PI()*A9/16)	=$B$1*NORM.S.INV(RAND())		=B9	=B9+C9	=E9*F9		=B9	=-B9+C9	=I9*J9
10	=1+A9	=SIN(2*PI()*A10/16)	=$B$1*NORM.S.INV(RAND())		=B10	=B10+C10	=E10*F10		=B10	=-B10+C10	=I10*J10
11	=1+A10	=SIN(2*PI()*A11/16)	=$B$1*NORM.S.INV(RAND())		=B11	=B11+C11	=E11*F11		=B11	=-B11+C11	=I11*J11
12	=1+A11	=SIN(2*PI()*A12/16)	=$B$1*NORM.S.INV(RAND())		=B12	=B12+C12	=E12*F12		=B12	=-B12+C12	=I12*J12
13	=1+A12	=SIN(2*PI()*A13/16)	=$B$1*NORM.S.INV(RAND())		=B13	=B13+C13	=E13*F13		=B13	=-B13+C13	=I13*J13
14	=1+A13	=SIN(2*PI()*A14/16)	=$B$1*NORM.S.INV(RAND())		=B14	=B14+C14	=E14*F14		=B14	=-B14+C14	=I14*J14
15	=1+A14	=SIN(2*PI()*A15/16)	=$B$1*NORM.S.INV(RAND())		=B15	=B15+C15	=E15*F15		=B15	=-B15+C15	=I15*J15
16	=1+A15	=SIN(2*PI()*A16/16)	=$B$1*NORM.S.INV(RAND())		=B16	=B16+C16	=E16*F16		=B16	=-B16+C16	=I16*J16
17	=1+A16	=SIN(2*PI()*A17/16)	=$B$1*NORM.S.INV(RAND())		=B17	=B17+C17	=E17*F17		=B17	=-B17+C17	=I17*J17
18	=1+A17	=SIN(2*PI()*A18/16)	=$B$1*NORM.S.INV(RAND())		=B18	=B18+C18	=E18*F18		=B18	=-B18+C18	=I18*J18
19											
20						$V\|_{X=s1+N}$=	=SUM(G3:G18)			$V\|_{X=s0+N}$=	=SUM(K3:K18)
21											
22						~B=	=IF(G20<0,0,1)			~B=	=IF(K20<0,0,1)

)

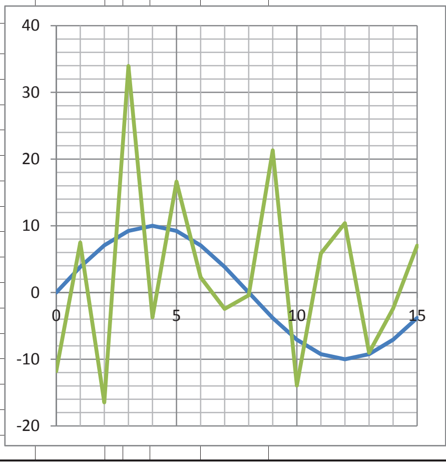
Project 7.5 (Observing signals in noise) *Modify Example 13.40 to implement a matched processor having coefficients matched to input signal*

$$s_i = \sin(2\pi i/16)\ \textit{for}\ \ i = 0, 1, 2, \ldots, n_x - 1 \ \ (n_X = 16)$$

To form the observed signal X_i add Gaussian noise having a specified σ_N to s_i. Determine the value of σ_N for which it is difficult to notice the signal component.

(ans:

	A	B	C	D
1	σ_N=	10		
2	i	s_i	N_i	X_i
3	0	0.00	-11.80	-11.80
4	1	3.83	3.65	7.47
5	2	7.07	-23.53	-16.46
6	3	9.24	24.71	33.95
7	4	10.00	-13.73	-3.73
8	5	9.24	7.37	16.61
9	6	7.07	-4.80	2.27
10	7	3.83	-6.28	-2.46
11	8	0.00	-0.42	-0.42
12	9	-3.83	25.13	21.30
13	10	-7.07	-6.87	-13.94
14	11	-9.24	15.08	5.85
15	12	-10.00	20.38	10.38
16	13	-9.24	0.23	-9.01
17	14	-7.07	4.67	-2.40
18	15	-3.83	10.84	7.02



	A	B	C	D
1	σ_N=	10		
2	i	s_i	N_i	X_i
3	0	=10*SIN(2*PI()*A3/16)	=$B$1*NORM.S.INV(RAND())	=B3+C3
4	=1+A3	=10*SIN(2*PI()*A4/16)	=$B$1*NORM.S.INV(RAND())	=B4+C4
5	=1+A4	=10*SIN(2*PI()*A5/16)	=$B$1*NORM.S.INV(RAND())	=B5+C5
6	=1+A5	=10*SIN(2*PI()*A6/16)	=$B$1*NORM.S.INV(RAND())	=B6+C6
7	=1+A6	=10*SIN(2*PI()*A7/16)	=$B$1*NORM.S.INV(RAND())	=B7+C7
8	=1+A7	=10*SIN(2*PI()*A8/16)	=$B$1*NORM.S.INV(RAND())	=B8+C8
9	=1+A8	=10*SIN(2*PI()*A9/16)	=$B$1*NORM.S.INV(RAND())	=B9+C9
10	=1+A9	=10*SIN(2*PI()*A10/16)	=$B$1*NORM.S.INV(RAND())	=B10+C10
11	=1+A10	=10*SIN(2*PI()*A11/16)	=$B$1*NORM.S.INV(RAND())	=B11+C11
12	=1+A11	=10*SIN(2*PI()*A12/16)	=$B$1*NORM.S.INV(RAND())	=B12+C12
13	=1+A12	=10*SIN(2*PI()*A13/16)	=$B$1*NORM.S.INV(RAND())	=B13+C13
14	=1+A13	=10*SIN(2*PI()*A14/16)	=$B$1*NORM.S.INV(RAND())	=B14+C14
15	=1+A14	=10*SIN(2*PI()*A15/16)	=$B$1*NORM.S.INV(RAND())	=B15+C15
16	=1+A15	=10*SIN(2*PI()*A16/16)	=$B$1*NORM.S.INV(RAND())	=B16+C16
17	=1+A16	=10*SIN(2*PI()*A17/16)	=$B$1*NORM.S.INV(RAND())	=B17+C17
18	=1+A17	=10*SIN(2*PI()*A18/16)	=$B$1*NORM.S.INV(RAND())	=B18+C18

)

Project 7.6 (Histogram of complementary signals in noise) *Using Example 13.32 as a guide, generate a histogram of one thousand matched processor output values V when processing signals in the presence of noise. Choose your favorite complementary signal pair with $n_X = 16$ and a signal-to-noise ratio $\mathcal{E}_S/\sigma_N^2 = 2$. Repeat for $\mathcal{E}_S/\sigma_N^2 = 4$.*

(ans: The signal parameters are specified in the worksheet in terms of the signal duration n_x, the maximum allowable amplitude A_{max}, and the noise variance σ_N^2. The signal sequence is specified in `K3:K18`*. For the signal sequence we compute the signal energy E_s, the matched processor coefficients in* `C3:C18`*. The filter output variance*

$$\sigma_V^2 = \sigma_N^2 \sum_{i=0}^{n_x-1} c_i^2$$

is computed and its square root in computed in `E2`*.*

The histogram extends from hist min *to* hist max *$(= -$* hist min*) and its parameters are computed in the worksheet. The* hist min *value in* `A8` *is computed from $\mathcal{E}_s$ and σ_V as* hist min $= -(\mathcal{E}_s - 4\sigma_V))$ *to effectively guarantee that it will be less than the minimum V value encountered. The number of bins is specified by the user in* `B8`*. The bin width is computed in* `C8` *as $-2($ hist min)/(# bins-1). The bin number of an observed value is the row number is the histogram display and is computed as rounded((value minus* hist min*)/bin width) +2 in* `C5` *as*

```
=ROUND((P20-A8)/C8,0)+2
```

This allows the verification of the bin number for each manual observation (produce by `F9`*).*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	n_x=	A_{max} =	E_s=	σ^2_N=	σ_V=		bin	count		D_t=	1			$X_i = sT_i + N_i$		
2	16	10	800	80	253		-1812	0		i	s_i	sT_i=		c_i	X_i	$c_i\ X_i$
3							-1611	1		0	0.00	0.00		0.00	9.87	0.00
4	n_T=	i=	bin#=		SNR=		-1410	8		1	3.83	3.83		3.83	-13.22	-50.58
5	1000	1000	15		10		-1209	48		2	7.07	7.07		7.07	18.43	130.31
6							-1008	101		3	9.24	9.24		9.24	-5.56	-51.38
7	hist min =	# bins=	width=				-807	159		4	10.00	10.00		10.00	19.85	198.55
8	-1812	19	201				-606	112		5	9.24	9.24		9.24	13.97	129.11
9							-405	63		6	7.07	7.07		7.07	9.91	70.10
10							-204	12		7	3.83	3.83		3.83	2.70	10.33
11							-3	4		8	0.00	0.00		0.00	3.08	0.00
12	Hist						198	16		9	-3.83	-3.83		-3.83	-3.84	14.70
13							399	37		10	-7.07	-7.07		-7.07	-2.32	16.40
14							600	116		11	-9.24	-9.24		-9.24	-0.16	1.52
15							801	158		12	-10.00	-10.00		-10.00	-4.17	41.72
16							1002	125		13	-9.24	-9.24		-9.24	-13.60	125.62
17							1203	30		14	-7.07	-7.07		-7.07	-8.92	63.07
18							1404	9		15	-3.83	-3.83		-3.83	-5.71	21.84
19							1605	1								
20							1806	0							$V\|_{X=sT+N}$ =	721.31

	A	B	C	D	E
1	n_x=	A_{max} =	E_s=	σ^2_N=	σ_V=
2	16	10	=SUMSQ(K3:K18)	80	=SQRT(D2*SUMSQ(N3:N18))
3					
4	n_T=	i=	bin#=		SNR=
5	1000	1000	=ROUND((P20-A8)/C8,0)+2		=C2/D2
6					
7	hist min =	# bins=	width=		
8	= -ROUND(C2+4*E2,0)	19	=-ROUND(2*A8/(B8-1),0)		

	J	K	L	M	N	O	P
1	D_t=	=ROUND(RAND(),0)			$X_i = sT_i + N_i$		
2	i	s_i	sT_i=		c_i	X_i	c_i X_i
3	0	=$B$2*SIN(2*PI()*J3/16)	=IF($K$1=1,K3,-K3)		=K3	=L3+SQRT($D$2)*NORM.S.INV(RAND())	=N3*O3
4	=1+J3	=$B$2*SIN(2*PI()*J4/16)	=IF($K$1=1,K4,-K4)		=K4	=L4+SQRT($D$2)*NORM.S.INV(RAND())	=N4*O4
5	=1+J4	=$B$2*SIN(2*PI()*J5/16)	=IF($K$1=1,K5,-K5)		=K5	=L5+SQRT($D$2)*NORM.S.INV(RAND())	=N5*O5
6	=1+J5	=$B$2*SIN(2*PI()*J6/16)	=IF($K$1=1,K6,-K6)		=K6	=L6+SQRT($D$2)*NORM.S.INV(RAND())	=N6*O6
7	=1+J6	=$B$2*SIN(2*PI()*J7/16)	=IF($K$1=1,K7,-K7)		=K7	=L7+SQRT($D$2)*NORM.S.INV(RAND())	=N7*O7
8	=1+J7	=$B$2*SIN(2*PI()*J8/16)	=IF($K$1=1,K8,-K8)		=K8	=L8+SQRT($D$2)*NORM.S.INV(RAND())	=N8*O8
9	=1+J8	=$B$2*SIN(2*PI()*J9/16)	=IF($K$1=1,K9,-K9)		=K9	=L9+SQRT($D$2)*NORM.S.INV(RAND())	=N9*O9
10	=1+J9	=$B$2*SIN(2*PI()*J10/16)	=IF($K$1=1,K10,-K10)		=K10	=L10+SQRT($D$2)*NORM.S.INV(RAND())	=N10*O10
11	=1+J10	=$B$2*SIN(2*PI()*J11/16)	=IF($K$1=1,K11,-K11)		=K11	=L11+SQRT($D$2)*NORM.S.INV(RAND())	=N11*O11
12	=1+J11	=$B$2*SIN(2*PI()*J12/16)	=IF($K$1=1,K12,-K12)		=K12	=L12+SQRT($D$2)*NORM.S.INV(RAND())	=N12*O12
13	=1+J12	=$B$2*SIN(2*PI()*J13/16)	=IF($K$1=1,K13,-K13)		=K13	=L13+SQRT($D$2)*NORM.S.INV(RAND())	=N13*O13
14	=1+J13	=$B$2*SIN(2*PI()*J14/16)	=IF($K$1=1,K14,-K14)		=K14	=L14+SQRT($D$2)*NORM.S.INV(RAND())	=N14*O14
15	=1+J14	=$B$2*SIN(2*PI()*J15/16)	=IF($K$1=1,K15,-K15)		=K15	=L15+SQRT($D$2)*NORM.S.INV(RAND())	=N15*O15
16	=1+J15	=$B$2*SIN(2*PI()*J16/16)	=IF($K$1=1,K16,-K16)		=K16	=L16+SQRT($D$2)*NORM.S.INV(RAND())	=N16*O16
17	=1+J16	=$B$2*SIN(2*PI()*J17/16)	=IF($K$1=1,K17,-K17)		=K17	=L17+SQRT($D$2)*NORM.S.INV(RAND())	=N17*O17
18	=1+J17	=$B$2*SIN(2*PI()*J18/16)	=IF($K$1=1,K18,-K18)		=K18	=L18+SQRT($D$2)*NORM.S.INV(RAND())	=N18*O18
19							
20						$V\vert_{x=sT+N}$ =	=SUM(P3:P18)

Performing most of the computations in the worksheet allows the following VBA code to be relatively simple.

```
Sub hist()
Dim Val As Integer
Range("B5").Value = 0                ' Reset counter i
For Val = 1 To Range("B8").Value     ' Set Hist Bin Vals
    Range("G" & 1 + Val).Value = Range("A8").Value + (Val - 1) * Range("C8").Value
    Range("H" & 1 + Val).Value = 0  ' Initialize counts
Next
Do While Range("B5").Value < Range("A5").Value  ' loop for nT times
    Val = (Range("P20").Value - Range("A8").Value) / Range("C8").Value + 2  'bin value
    If Val > 1 Then                                ' if valid bin value
        Range("H" & Val).Value = Range("H" & Val).Value + 1    ' incr bin count
    End If
    Range("B5").Value = Range("B5").Value + 1   ' increment counter i
Loop
End Sub
```

)

Project 7.7 (Estimating probability of error for a matched processor) *Modify Examples 13.41, 13.42 and 13.43 to compute the probability of error as σ_N^2 increases between $0.1\mathcal{E}_s$ and $10\mathcal{E}_s$ by factor $\sqrt{2}$, using signal*

$$s_i = \sin(2\pi i/16) \textit{ for } \; i = 0, 1, 2, \ldots, n_x - 1 \;\; (n_X = 16)$$

(ans: The worksheets are shown for extending each example.

1. *From Example 13.41, the VBA program suspends automatic calculations, which occur when any cell value changes and several changes occur with each transmission. VBA Macro* `restore` *restores the automatic cell updates. If not restored, the SNR value does not update when the σ_N value is changed.*

	A	B	C	D	E	F	G	H
1	E_s=	8		D_t=	0		#Transmits=	12
2	σ_N=	2		~D_r=	0		#Errors=	2
3	SNR=	2.0		Err	0		P[err]=	0.167
4								
5	i	s_i	sT_i	N_i		c_i	X_i	c_i X_i
6	0	0.00	0.00	2.08		0.00	2.08	0.00
7	1	0.38	-0.38	0.01		0.38	-0.38	-0.14
8	2	0.71	-0.71	1.82		0.71	1.11	0.79
9	3	0.92	-0.92	2.07		0.92	1.15	1.06
10	4	1.00	-1.00	1.01		1.00	0.01	0.01
11	5	0.92	-0.92	-0.51		0.92	-1.44	-1.33
12	6	0.71	-0.71	1.78		0.71	1.07	0.76
13	7	0.38	-0.38	4.23		0.38	3.85	1.47
14	8	0.00	0.00	3.46		0.00	3.46	0.00
15	9	-0.38	0.38	1.39		-0.38	1.77	-0.68
16	10	-0.71	0.71	1.57		-0.71	2.28	-1.61
17	11	-0.92	0.92	-0.23		-0.92	0.70	-0.64
18	12	-1.00	1.00	-1.13		-1.00	-0.13	0.13
19	13	-0.92	0.92	-2.10		-0.92	-1.18	1.09
20	14	-0.71	0.71	-0.34		-0.71	0.37	-0.26
21	15	-0.38	0.38	3.83		-0.38	4.21	-1.61
22								
23	Start		Xmit	Restore			$V\vert_{X=sT+N}$=	-0.97

	A	B	C	D	E	F	G	H
1	E_s=	=SUMSQ(B6:B22)		D_t=	=ROUND(RAND(),0)		#Transmits=	12
2	σ_N=	2		~D_r=	=IF(H23<0,0,1)		#Errors=	2
3	SNR=	=B1/(B2^2)		Err	=IF(E1=E2,0,1)		P[err]=	=IF(H1>0, H2/H1,0)
4								
5	i	s_i	sT_i	N_i		c_i	X_i	c_i X_i
6	0	=SIN(2*PI()*A6/16)	=IF($E$1=1,B6,-B6)	=$B$2*NORM.S.INV(RAND())		=B6	=C6+D6	=F6*G6
7	=1+A6	=SIN(2*PI()*A7/16)	=IF($E$1=1,B7,-B7)	=$B$2*NORM.S.INV(RAND())		=B7	=C7+D7	=F7*G7
8	=1+A7	=SIN(2*PI()*A8/16)	=IF($E$1=1,B8,-B8)	=$B$2*NORM.S.INV(RAND())		=B8	=C8+D8	=F8*G8
9	=1+A8	=SIN(2*PI()*A9/16)	=IF($E$1=1,B9,-B9)	=$B$2*NORM.S.INV(RAND())		=B9	=C9+D9	=F9*G9
10	=1+A9	=SIN(2*PI()*A10/16)	=IF($E$1=1,B10,-B10)	=$B$2*NORM.S.INV(RAND())		=B10	=C10+D10	=F10*G10
11	=1+A10	=SIN(2*PI()*A11/16)	=IF($E$1=1,B11,-B11)	=$B$2*NORM.S.INV(RAND())		=B11	=C11+D11	=F11*G11
12	=1+A11	=SIN(2*PI()*A12/16)	=IF($E$1=1,B12,-B12)	=$B$2*NORM.S.INV(RAND())		=B12	=C12+D12	=F12*G12
13	=1+A12	=SIN(2*PI()*A13/16)	=IF($E$1=1,B13,-B13)	=$B$2*NORM.S.INV(RAND())		=B13	=C13+D13	=F13*G13
14	=1+A13	=SIN(2*PI()*A14/16)	=IF($E$1=1,B14,-B14)	=$B$2*NORM.S.INV(RAND())		=B14	=C14+D14	=F14*G14
15	=1+A14	=SIN(2*PI()*A15/16)	=IF($E$1=1,B15,-B15)	=$B$2*NORM.S.INV(RAND())		=B15	=C15+D15	=F15*G15
16	=1+A15	=SIN(2*PI()*A16/16)	=IF($E$1=1,B16,-B16)	=$B$2*NORM.S.INV(RAND())		=B16	=C16+D16	=F16*G16
17	=1+A16	=SIN(2*PI()*A17/16)	=IF($E$1=1,B17,-B17)	=$B$2*NORM.S.INV(RAND())		=B17	=C17+D17	=F17*G17
18	=1+A17	=SIN(2*PI()*A18/16)	=IF($E$1=1,B18,-B18)	=$B$2*NORM.S.INV(RAND())		=B18	=C18+D18	=F18*G18
19	=1+A18	=SIN(2*PI()*A19/16)	=IF($E$1=1,B19,-B19)	=$B$2*NORM.S.INV(RAND())		=B19	=C19+D19	=F19*G19
20	=1+A19	=SIN(2*PI()*A20/16)	=IF($E$1=1,B20,-B20)	=$B$2*NORM.S.INV(RAND())		=B20	=C20+D20	=F20*G20
21	=1+A20	=SIN(2*PI()*A21/16)	=IF($E$1=1,B21,-B21)	=$B$2*NORM.S.INV(RAND())		=B21	=C21+D21	=F21*G21
22								
23	Start		Xmit	Restore			$V\vert_{X=sT+N}$=	=SUM(H6:H21)

```
Sub start()     ' resets counters to zero
Application.Calculation = xlCalculationManual ' Stop Auto Calculations
Range("H1").Value = 0 ' Reset # transmits
Range("H2").Value = 0 ' Reset # errors
```

```
End Sub
'---
Sub xmit()        ' transmits another data signal
Calculate         ' Force one recalculation, New D
Range("H1").Value = Range("H1").Value + 1 ' Increment # transmits
' Calculate         ' Force one recalculation, New D
If Range("E1").Value <> Range("E2").Value Then ' ~B not equal D -> error
    Range("H2") = Range("H2") + 1              ' Increment # errors
End If
End Sub
'---
Sub restore()       ' resets counters to zero
Application.Calculation = xlCalculationAutomatic ' restore Auto Calculations
End Sub
```

2. *Extending Example 13.42, the VBA Macro* `auto` *has been added to the program in Example 13.41.*

	A	B	C	D	E	F	G	H
1	E_s=	8		D_t=	1		#Transmits=	1000
2	σ_N=	2		~Dr=	1		#Errors=	67
3	SNR=	2		Err	0		P[err]=	0.07
4								
5	i	s_i	sT_i	N_i		c_i	X_i	$c_i X_i$
6	0	0.00	0.00	1.64		0.00	1.64	0.00
7	1	0.38	0.38	-2.23		0.38	-1.85	-0.71
8	2	0.71	0.71	5.11		0.71	5.81	4.11
9	3	0.92	0.92	2.74		0.92	3.67	3.39
10	4	1.00	1.00	-0.42		1.00	0.58	0.58
11	5	0.92	0.92	-0.68		0.92	0.24	0.22
12	6	0.71	0.71	0.42		0.71	1.13	0.80
13	7	0.38	0.38	0.01		0.38	0.39	0.15
14	8	0.00	0.00	0.16		0.00	0.16	0.00
15	9	-0.38	-0.38	0.52		-0.38	0.14	-0.05
16	10	-0.71	-0.71	-0.41		-0.71	-1.12	0.79
17	11	-0.92	-0.92	-3.99		-0.92	-4.92	4.54
18	12	-1.00	-1.00	-2.16		-1.00	-3.16	3.16
19	13	-0.92	-0.92	-1.12		-0.92	-2.04	1.89
20	14	-0.71	-0.71	-2.23		-0.71	-2.94	2.08
21	15	-0.38	-0.38	2.02		-0.38	1.63	-0.63
22								
23	Start	Xmit	Restore	Auto			$V\|_{X=sT+N}$=	20.32

	A	B	C	D	E	F	G	H
1	E_s=	=SUMSQ(B6:B21)		D_t=	=IF(RAND()<0.5,0,1)		#Transmits=	1000
2	σ_N=	2		~Dr=	=IF(H23<0,0,1)		#Errors=	67
3	SNR=	=B1/B2^2		Err	=IF(E1=E2,0,1)		P[err]=	=IF(H1>0,H2/H1,0)
4								
5	i	s_i	sT_i	N_i		c_i	X_i	$c_i X_i$
6	0	=SIN(2*PI()*A6/16)	=IF($E$1=1,B6,-B6)	=$B$2*NORM.S.INV(RAND())		=B6	=C6+D6	=F6*G6
7	=1+A6	=SIN(2*PI()*A7/16)	=IF($E$1=1,B7,-B7)	=$B$2*NORM.S.INV(RAND())		=B7	=C7+D7	=F7*G7
8	=1+A7	=SIN(2*PI()*A8/16)	=IF($E$1=1,B8,-B8)	=$B$2*NORM.S.INV(RAND())		=B8	=C8+D8	=F8*G8
9	=1+A8	=SIN(2*PI()*A9/16)	=IF($E$1=1,B9,-B9)	=$B$2*NORM.S.INV(RAND())		=B9	=C9+D9	=F9*G9
10	=1+A9	=SIN(2*PI()*A10/16)	=IF($E$1=1,B10,-B10)	=$B$2*NORM.S.INV(RAND())		=B10	=C10+D10	=F10*G10
11	=1+A10	=SIN(2*PI()*A11/16)	=IF($E$1=1,B11,-B11)	=$B$2*NORM.S.INV(RAND())		=B11	=C11+D11	=F11*G11
12	=1+A11	=SIN(2*PI()*A12/16)	=IF($E$1=1,B12,-B12)	=$B$2*NORM.S.INV(RAND())		=B12	=C12+D12	=F12*G12
13	=1+A12	=SIN(2*PI()*A13/16)	=IF($E$1=1,B13,-B13)	=$B$2*NORM.S.INV(RAND())		=B13	=C13+D13	=F13*G13
14	=1+A13	=SIN(2*PI()*A14/16)	=IF($E$1=1,B14,-B14)	=$B$2*NORM.S.INV(RAND())		=B14	=C14+D14	=F14*G14
15	=1+A14	=SIN(2*PI()*A15/16)	=IF($E$1=1,B15,-B15)	=$B$2*NORM.S.INV(RAND())		=B15	=C15+D15	=F15*G15
16	=1+A15	=SIN(2*PI()*A16/16)	=IF($E$1=1,B16,-B16)	=$B$2*NORM.S.INV(RAND())		=B16	=C16+D16	=F16*G16
17	=1+A16	=SIN(2*PI()*A17/16)	=IF($E$1=1,B17,-B17)	=$B$2*NORM.S.INV(RAND())		=B17	=C17+D17	=F17*G17
18	=1+A17	=SIN(2*PI()*A18/16)	=IF($E$1=1,B18,-B18)	=$B$2*NORM.S.INV(RAND())		=B18	=C18+D18	=F18*G18
19	=1+A18	=SIN(2*PI()*A19/16)	=IF($E$1=1,B19,-B19)	=$B$2*NORM.S.INV(RAND())		=B19	=C19+D19	=F19*G19
20	=1+A19	=SIN(2*PI()*A20/16)	=IF($E$1=1,B20,-B20)	=$B$2*NORM.S.INV(RAND())		=B20	=C20+D20	=F20*G20
21	=1+A20	=SIN(2*PI()*A21/16)	=IF($E$1=1,B21,-B21)	=$B$2*NORM.S.INV(RAND())		=B21	=C21+D21	=F21*G21
22								
23	Start	Xmit	Restore	Auto			$V\|_{X=sT+N}$=	=SUM(H6:H21)

```
Sub start()     ' resets counters to zero
Application.Calculation = xlCalculationManual ' Stop Auto Calculations
Range("H1").Value = 0 ' Reset # transmits
Range("H2").Value = 0 ' Reset # errors
End Sub
'---
Sub xmit()       ' transmits another data signal
Calculate        ' Force one recalculation, New D
Range("H1").Value = Range("H1").Value + 1 ' Increment # transmits
' Calculate        ' Force one recalculation, New D
If Range("E1").Value <> Range("E2").Value Then  ' ~B not equal D -> error
    Range("H2") = Range("H2") + 1               ' Increment # errors
End If
End Sub
'---
Sub auto()
start
Do While Range("H1").Value < 1000   ' 1000 transmissions
    xmit
Loop
Application.Calculation = xlCalculationAutomatic ' restore Auto Calculations
End Sub
'---
Sub restore()
Application.Calculation = xlCalculationAutomatic ' restore Auto Calculations
End Sub
```

3. Extending Example 13.43, the VBA Macro `SNR` *has been added to the program in Example 13.42.*

	A	B	C	D	E	F	G	H	I	J	K
1	E_s=	8		D_t=	1		#Transmits=	1000		SNR	P[err]
2	σ_N=	102.4		~D_r=	0		#Errors=	498		12.5000	0.0000
3	SNR=	0.0008		err	1		P[err]=	0.498		6.2500	0.0040
4										3.1250	0.0380
5	i	s_i	sT_i	N_i		c_i	X_i	$c_i\ X_i$		1.5625	0.1110
6	0	0.00	0.00	126.99		0.00	126.99	0.00		0.7813	0.1970
7	1	0.38	0.38	12.55		0.38	12.93	4.95		0.3906	0.2950
8	2	0.71	0.71	14.14		0.71	14.85	10.50		0.1953	0.3100
9	3	0.92	0.92	-4.41		0.92	-3.49	-3.22		0.0977	0.3730
10	4	1.00	1.00	-182.25		1.00	-181.25	-181.25		0.0488	0.4120
11	5	0.92	0.92	12.37		0.92	13.30	12.28		0.0244	0.4240
12	6	0.71	0.71	-16.20		0.71	-15.49	-10.96		0.0122	0.4610
13	7	0.38	0.38	-138.91		0.38	-138.53	-53.01		0.0061	0.4800
14	8	0.00	0.00	-80.21		0.00	-80.21	0.00		0.0031	0.4780
15	9	-0.38	-0.38	-17.58		-0.38	-17.97	6.88		0.0015	0.4980
16	10	-0.71	-0.71	37.31		-0.71	36.60	-25.88			
17	11	-0.92	-0.92	-2.67		-0.92	-3.60	3.32			
18	12	-1.00	-1.00	-3.02		-1.00	-4.02	4.02			
19	13	-0.92	-0.92	30.22		-0.92	29.29	-27.06			
20	14	-0.71	-0.71	-6.66		-0.71	-7.37	5.21			
21	15	-0.38	-0.38	-49.05		-0.38	-49.44	18.92			
22											
23	Start	Xmit	Restore	Auto			$V\|_{X=sT+N}$=	-235.30			
24	P[err] as SNR										
25											

	A	B	C	D	E	F	G	H
1	E_s=	=SUMSQ(B6:B21)		D_t=	=IF(RAND()<0.5,0,1)		#Transmits=	1000
2	σ_N=	102.4		~D_r=	=IF(H23<0,0,1)		#Errors=	498
3	SNR=	=B1/B2^2		err	=IF(E1=E2,0,1)		P[err]=	=IF(H1>0,H2/H1,0)
4								
5	i	s_i	sT_i	N_i		c_i	X_i	c_i X_i
6	0	=SIN(2*PI()*A6/16)	=IF($E$1=1,B6,-B6)	=$B$2*NORM.S.INV(RAND())		=B6	=C6+D6	=F6*G6
7	=1+A6	=SIN(2*PI()*A7/16)	=IF($E$1=1,B7,-B7)	=$B$2*NORM.S.INV(RAND())		=B7	=C7+D7	=F7*G7
8	=1+A7	=SIN(2*PI()*A8/16)	=IF($E$1=1,B8,-B8)	=$B$2*NORM.S.INV(RAND())		=B8	=C8+D8	=F8*G8
9	=1+A8	=SIN(2*PI()*A9/16)	=IF($E$1=1,B9,-B9)	=$B$2*NORM.S.INV(RAND())		=B9	=C9+D9	=F9*G9
10	=1+A9	=SIN(2*PI()*A10/16)	=IF($E$1=1,B10,-B10)	=$B$2*NORM.S.INV(RAND())		=B10	=C10+D10	=F10*G10
11	=1+A10	=SIN(2*PI()*A11/16)	=IF($E$1=1,B11,-B11)	=$B$2*NORM.S.INV(RAND())		=B11	=C11+D11	=F11*G11
12	=1+A11	=SIN(2*PI()*A12/16)	=IF($E$1=1,B12,-B12)	=$B$2*NORM.S.INV(RAND())		=B12	=C12+D12	=F12*G12
13	=1+A12	=SIN(2*PI()*A13/16)	=IF($E$1=1,B13,-B13)	=$B$2*NORM.S.INV(RAND())		=B13	=C13+D13	=F13*G13
14	=1+A13	=SIN(2*PI()*A14/16)	=IF($E$1=1,B14,-B14)	=$B$2*NORM.S.INV(RAND())		=B14	=C14+D14	=F14*G14
15	=1+A14	=SIN(2*PI()*A15/16)	=IF($E$1=1,B15,-B15)	=$B$2*NORM.S.INV(RAND())		=B15	=C15+D15	=F15*G15
16	=1+A15	=SIN(2*PI()*A16/16)	=IF($E$1=1,B16,-B16)	=$B$2*NORM.S.INV(RAND())		=B16	=C16+D16	=F16*G16
17	=1+A16	=SIN(2*PI()*A17/16)	=IF($E$1=1,B17,-B17)	=$B$2*NORM.S.INV(RAND())		=B17	=C17+D17	=F17*G17
18	=1+A17	=SIN(2*PI()*A18/16)	=IF($E$1=1,B18,-B18)	=$B$2*NORM.S.INV(RAND())		=B18	=C18+D18	=F18*G18
19	=1+A18	=SIN(2*PI()*A19/16)	=IF($E$1=1,B19,-B19)	=$B$2*NORM.S.INV(RAND())		=B19	=C19+D19	=F19*G19
20	=1+A19	=SIN(2*PI()*A20/16)	=IF($E$1=1,B20,-B20)	=$B$2*NORM.S.INV(RAND())		=B20	=C20+D20	=F20*G20
21	=1+A20	=SIN(2*PI()*A21/16)	=IF($E$1=1,B21,-B21)	=$B$2*NORM.S.INV(RAND())		=B21	=C21+D21	=F21*G21
22								
23	Start	Xmit	Restore	Auto			$V\|_{X=sT+N}$=	=SUM(H6:H21)

```
Sub start()      ’ resets counters to zero
Application.Calculation = xlCalculationManual ’ Stop Auto Calculations
Range("H1").Value = 0 ’ Reset # transmits
Range("H2").Value = 0 ’ Reset # errors
End Sub
’---
Sub xmit()       ’ transmits another data signal
Calculate        ’ Force one recalculation, New D
Range("H1").Value = Range("H1").Value + 1 ’ Increment # transmits
’ Calculate       ’ Force one recalculation, New D
If Range("E1").Value <> Range("E2").Value Then  ’ ~B not equal D -> error
    Range("H2") = Range("H2") + 1               ’ Increment # errors
End If
End Sub
’---
Sub auto()
start                   ’ call start Macro
Do While Range("H1").Value < 1000   ’ 1000 transmissions
    xmit                   ’ call xmit Macro
Loop
restore
End Sub
’---
Sub restore()
Application.Calculation = xlCalculationAutomatic ’ restore Auto Calculations
End Sub
’---
Sub SNR()
Dim RowNum As Integer                ’ specify an integer variable
RowNum = 2                           ’ starting row of data compilation
Range("B2").Value = Range("B1").Value / 10       ’ initial noise SD value
Do While Range("B2").Value < 10 * Range("B1").Value ’ end noise SD value
    auto
    Range("J" & RowNum).Value = Range("B3").Value    ’ enter SNR
    Range("K" & RowNum).Value = Range("H3").Value    ’ enter Prob[err]
    RowNum = RowNum + 1              ’ increment row
    Range("B2").Value = Range("B2").Value * Sqr(2)   ’ increase noise SD
Loop
End Sub
```

)

Project 7.8 (Designing a signal that has minimum probability of error) *Assume a channel constrains signals to have a limited amplitude*

$$|s_i| \leq 1V \textit{ for } i = 0, 1, 2, \ldots, n_x - 1$$

Design a signal with $n_X = 16$ *that has a maximum* $\mathcal{E}_s$. *Use two worksheets, each modeled after Example 13.43 that compares your signal performance with that of the sinusoidal signal*

$$s_i = \sin(2\pi i/16) \textit{ for } i = 0, 1, 2, \ldots, n_x - 1 \;\; (n_X = 16)$$

(ans: The simplest signal that satisfies this constraint and has the maximum energy is

$$s_i = 1 \textit{ for } i = 0, 1, 2, \ldots, n_x - 1$$

and is shown below. Starting with the same σ_N *value as with the sinusoidal waveform, the SNR increases by a factor of two. The P[err] is only dependent on the SNR, but the designed signal has a larger SNR.*

The only changes to this program from the previous project are the following.

1. *Copy and paste the sinusoidal results three columns to the right.*
2. *Change* s_i *to have values that meet the allowed limits. Many signals are possible, including the random binary signal sequence.*

	B	C	D	E	F	G	H	I	J	K	L	M	N
1	16		D_t=	1		#Transmits=	1000		SNR	P[err]		SNR	P[err]
2	204.8		~D_r=	0		#Errors=	478		25.0000	0.0000		12.5000	0.0000
3	0.00038147		err	1		P[err]=	0.478		12.5000	0.0010		6.2500	0.0040
4									6.2500	0.0060		3.1250	0.0380
5	s_i	sT_i	N_i		c_i	X_i	$c_i X_i$		3.1250	0.0480		1.5625	0.1110
6	1.00	1.00	-65.44		1.00	-64.44	-64.44		1.5625	0.0970		0.7813	0.1970
7	1.00	1.00	-113.72		1.00	-112.72	-112.72		0.7813	0.2220		0.3906	0.2950
8	1.00	1.00	428.28		1.00	429.28	429.28		0.3906	0.2420		0.1953	0.3100
9	1.00	1.00	-408.62		1.00	-407.62	-407.62		0.1953	0.3200		0.0977	0.3730
10	1.00	1.00	-173.42		1.00	-172.42	-172.42		0.0977	0.3750		0.0488	0.4120
11	1.00	1.00	4.19		1.00	5.19	5.19		0.0488	0.4060		0.0244	0.4240
12	1.00	1.00	-54.58		1.00	-53.58	-53.58		0.0244	0.4460		0.0122	0.4610
13	1.00	1.00	-1.38		1.00	-0.38	-0.38		0.0122	0.4370		0.0061	0.4800
14	1.00	1.00	171.51		1.00	172.51	172.51		0.0061	0.4720		0.0031	0.4780
15	1.00	1.00	101.84		1.00	102.84	102.84		0.0031	0.4840		0.0015	0.4980
16	1.00	1.00	161.98		1.00	162.98	162.98		0.0015	0.5090			
17	1.00	1.00	3.16		1.00	4.16	4.16		0.0008	0.4780			
18	1.00	1.00	-76.21		1.00	-75.21	-75.21						
19	1.00	1.00	-77.59		1.00	-76.59	-76.59		Max amplitude signal results			Sinusoidal signal results	
20	1.00	1.00	-103.68		1.00	-102.68	-102.68						
21	1.00	1.00	174.06		1.00	175.06	175.06						
22													
23	Xmit	Restore	Auto			$V\|_{x=sT+N}$=	-13.62						
24	P[err] as SNR												
25													

)