12201856 김다영 1

(10)

1. (1) $\mathbb{A}(x,y,z) = x\,a_x$

$\mathbb{A}(x,y,z)\cdot a_\rho = x\,a_x\cdot a_\rho = x\cos\phi$

$\mathbb{A}(x,y,z)\cdot a_\phi = x a_x\cdot a_\phi = -x\cdot\sin\phi$

$\mathbb{A}(x,y,z)\cdot a_z = 0.$

$(\cancel{\mathbb{A}(x,y,z)\cdot a_\rho)a_\rho +})$

$\therefore \mathbb{A}(\rho,\phi,z)$
$= x\cos\phi\, a_\rho + (-x)\sin\phi\, a_\phi$

(2) $\mathbb{B}(r,\theta,\phi) = 2a_r.$

$\mathbb{B}(r,\theta,\phi)\cdot a_x = 2a_r\cdot a_x = 2\cdot\sin\theta\cdot\cos\phi$

$\mathbb{B}(r,\theta,\phi)\cdot a_y = 2a_r\cdot a_y = 2\cdot\sin\theta\,\sin\phi.$

$\mathbb{B}(r,\theta,\phi)\cdot a_z = 2a_r\cdot a_z = 2\cdot\cos\theta$

$\therefore \mathbb{B}(x,y,z) = 2\sin\theta\cos\phi\, a_x + 2\sin\theta\sin\phi\, a_y + 2\cos\theta\, a_z.$

(10).

2. (1). $d\mathbb{E} = \dfrac{dQ\,(\mathbb{r}-\mathbb{r}')}{4\pi\varepsilon_0|\mathbb{r}-\mathbb{r}'|^{3/2}}$

$\mathbb{r} = z a_z,\quad \mathbb{r}' = \rho a_\rho.\quad \cancel{dS}\ dS = \rho\, d\rho\, d\phi$

$\mathbb{r}-\mathbb{r}' = z a_z - \rho a_\rho.$

$$\mathbb{E} = \int \frac{dQ(\mathbb{r}-\mathbb{r}')}{4\pi\varepsilon_0(\mathbb{r}-\mathbb{r}')^{3/2}} = \int \frac{\rho_s dS\,(\mathbb{r}-\mathbb{r}')}{4\pi\varepsilon_0(\mathbb{r}-\mathbb{r}')^{3/2}} = \int_0^b\int_0^{2\pi}\frac{\rho\,\rho_s\,(z a_z-\rho a_\rho)}{4\pi\varepsilon_0\,(z^2+\rho^2)^{3/2}}\,d\phi\,d\rho$$

$$= \int_0^b\int_0^{2\pi}\frac{\rho\,\rho_s\cdot z a_z.}{4\pi\varepsilon_0(z^2+\rho^2)^{3/2}}\,d\phi\,d\rho. = \frac{\rho_s z}{4\pi\varepsilon_0}a_z.\int_0^b\int_0^{2\pi}\frac{\rho}{(z^2+\rho^2)^{3/2}}\,d\phi\,d\rho$$

$$= \frac{\rho_s z}{2\varepsilon_0}a_z.\int_0^b\frac{\rho}{(z^2+\rho^2)^{3/2}}\,d\rho$$

$$\int_0^b \frac{\rho}{(z^2+\rho^2)^{3/2}}d\rho = \left[-(z^2+\rho^2)^{-\frac{1}{2}}\right]_0^b = \left[-\frac{1}{\sqrt{z^2+\rho^2}}\right]_0^b = \frac{1}{z}-\frac{1}{\sqrt{z^2+b^2}}$$

$$\therefore \mathbb{E} = \frac{\rho_s z}{2\varepsilon_0}\left(\frac{1}{z}-\frac{1}{\sqrt{z^2+b^2}}\right)a_z.$$

$= \left(1-\left(1+\left(\frac{b}{z}\right)^2\right)^{-\frac{1}{2}}\right).$

(2). $\mathbb{E} = \dfrac{\rho_s}{2\varepsilon_0}\left(1-\dfrac{z}{\sqrt{z^2+b^2}}\right)a_z = \dfrac{\rho_s}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{1+(\frac{b}{z})^2}}\right)a_z.$ $z \gg b$ 이므로.

$\mathbb{E} \approx \dfrac{\rho_s}{2\varepsilon_0}\left(1-\left(1-\frac{1}{2}\left(\frac{b}{z}\right)^2\right)\right)a_z = \dfrac{\rho_s}{2\varepsilon_0}\cdot\dfrac{1}{2}\cdot\dfrac{b^2}{z^2}a_z = \dfrac{\rho_s b^2}{4\varepsilon_0 z^2}a_z$ 로 근사시킬수 있다.

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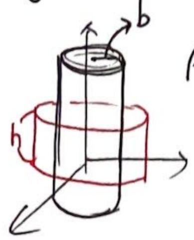
3.



$\rho_v = \rho_0$.

$V = \pi b^2 h$.

(1).

① $\rho > b$ 일때 (외부).

$\oint \mathbf{D} \cdot d\mathbf{S} = Q = \rho_v V = \rho_0 V$. $\mathbf{D} = D_\rho(\rho)\, a_\rho$. $d\mathbf{S} = 2\pi\rho h \cdot a_\rho$.

$D_\rho(\rho)\, a_\rho \cdot 2\pi\rho h \cdot a_\rho = D_\rho(\rho) \cdot 2\pi\rho h = \rho_0 V = \rho_0 \cdot \pi b^2 h$.

$D_\rho(\rho) = \frac{\rho_0 \pi b^2 \cancel{h}}{2\pi\rho \cancel{h}} = \frac{\rho_0 b^2}{2\rho}$ $\mathbf{D} = \frac{\rho_0 b^2}{2\rho} a_\rho$, $\Rightarrow$ $\underline{\mathbf{E} = \frac{\rho_0 b^2}{2\rho\varepsilon_0} a_\rho}$.

② $0 < \rho < b$ 일때 (내부).

$\oint \mathbf{D} \cdot d\mathbf{S} = \rho_0 V$

$D_\rho(\rho)\, a_\rho \cdot 2\pi\rho h \cdot a_\rho = D_\rho(\rho) \cdot 2\pi\rho h = \rho_0 V = \rho_0 \pi \rho^2 \cdot h$.

$D_\rho(\rho) = \frac{\rho_0 \pi \rho^2 \cancel{h}}{2\pi\rho \cancel{h}} = \frac{\rho_0 \rho}{2}$ $\Rightarrow$ $\underline{\mathbf{E} = \frac{\rho_0 \rho}{2\varepsilon_0} a_\rho}$.

(2).

① 외부 $\nabla \cdot \mathbf{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho D_\rho) = \frac{1}{\rho} \cdot \frac{\partial}{\partial \rho}\left(\rho \cdot \frac{\rho_0 b^2}{2\rho}\right) = \underline{0}$. $\left(\because \frac{\partial(\frac{\rho_0 b^2}{2})}{\partial \rho} = 0\right)$.

뒷부분은 0이므로 생략.

② 내부. $\nabla \cdot \mathbf{D} = \frac{1}{\rho} \cdot \frac{\partial}{\partial \rho}\left(\rho \cdot \frac{\rho_0 \rho}{2}\right) = \frac{1}{\rho} \cdot \frac{\partial}{\partial \rho}\left(\frac{\rho_0 \rho^2}{2}\right) = \frac{1}{\rho} \cdot \rho_0 \cdot \frac{2\rho}{2} = \underline{\rho_0}$. 이다.

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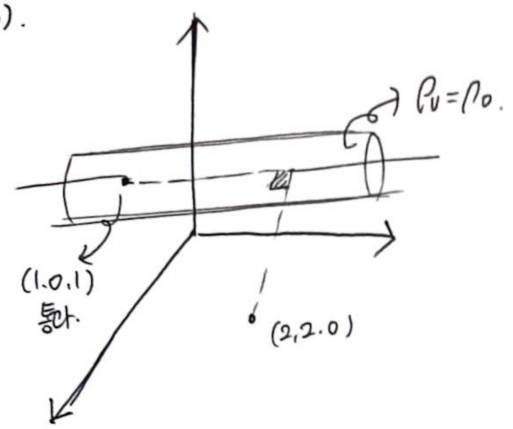
(3).



$\mathbf{r} = 2\mathbf{a}_x + 2\mathbf{a}_y$

$\mathbf{r}' = \mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$.

$\mathbf{r} - \mathbf{r}' = \mathbf{a}_x - \mathbf{a}_z$, $|\mathbf{r} - \mathbf{r}'| = \sqrt{2}$.

점. (2,2,0)은 원통 외부의 점이고. y축에 평행하므로.

$\mathbf{E} = \dfrac{\rho_0 b^2}{2\rho \varepsilon_0} \mathbf{a}_\rho$ 라 동일하다.

ρ은 (2,2,0)에서 원통의 중심에 내린 수선의 발과의 거리이므로.

(2,2,0)과 (1,2,1)과의 거리 즉 $\rho = \sqrt{1+1} = \sqrt{2}$이다.

$\therefore \mathbf{E} = \dfrac{\rho_0 b^2}{2\sqrt{2}\varepsilon_0} \cdot \dfrac{\mathbf{a}_x - \mathbf{a}_z}{\sqrt{2}} = \dfrac{\rho_0 b^2}{4\varepsilon_0}(\mathbf{a}_x - \mathbf{a}_z)$ 이다.

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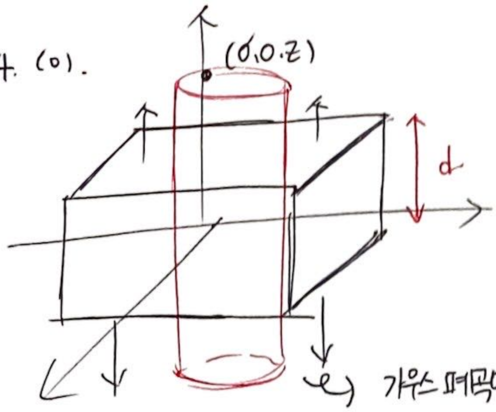
4. (이).



$\mathbf{D} = D_z(z)\mathbf{a}_z$.

↳ 가우스 폐곡면 = 원기둥 (옆면 넓이 = S)

① $z > \frac{d}{2}$일때..

$\oint_S \mathbf{D} \cdot d\mathbf{S} = \rho_0 V$.

$= D_z(z) \cdot 2S = \rho_0 \cdot 2S \cdot d$.

$D_z(z) = \rho_0 d. \Rightarrow \mathbf{D} = \rho_0 d\, \mathbf{a}_z$. $\mathbf{E} = \frac{\rho_0 d}{\varepsilon_0}\mathbf{a}_z$.

② $z < \frac{d}{2}$일때.

$z > \frac{d}{2}$일때와. 크기동일, 방향만 반대. $\therefore \mathbf{E} = \frac{-\rho_0 d}{\varepsilon_0}\mathbf{a}_z$.

③ $0 < z < \frac{d}{2}$

$\oint_S \mathbf{D} \cdot d\mathbf{S} = D_z(z) \cdot 2S = \rho_0 V = \rho_0 \cdot 2S \cdot 2z$.

$D_z(z) = 2\rho_0 z \Rightarrow \mathbf{D} = 2\rho_0 z\, \mathbf{a}_z$. $\therefore \mathbf{E} = \frac{2\rho_0 z}{\varepsilon_0}\mathbf{a}_z$.

④ $-\frac{d}{2} < z < 0$.

$0 < z < \frac{d}{2}$일 때와 크기 동일, 방향만 반대. $\therefore \mathbf{E} = -\frac{2\rho_0 z}{\varepsilon_0}\mathbf{a}_z$.

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~~(5)~~ #5·(0)

$dS = r^2 \sin\theta \, d\theta d\phi$

$r_1 = 2. \quad \rho_{s1} = 20. \rightarrow Q_{r_1} = \int \rho_{s1} \cdot dS = \int_0^{\pi}\int_0^{2\pi} \rho_{s1} \cdot r^2 \sin\theta \, d\phi d\theta$

$= \rho_{s1} \cdot r^2 \int_0^{\pi}\int_0^{2\pi} \sin\theta \, d\phi d\theta = \rho_{s1} \cdot r^2 \cdot 2 \cdot 2\pi$

$= 20 \cdot 4 \cdot 4\pi = \underline{320\pi} \; C.$

$r_2 = 4. \quad \rho_{s2} = -4.$

$r_3 = 6. \quad \rho_{s3}.$

$\rightarrow Q_{r3} = \rho_{s3} \cdot 36 \cdot 4\pi$

$= 144\pi \cdot \rho_{s3}$

$\rightarrow Q_{r2} = \int \rho_{s2} \cdot dS = \int_0^{\pi}\int_0^{2\pi} \rho_{s2} \cdot r^2 \sin\theta \, d\phi d\theta \cdot = \rho_{s2} \cdot r^2 \cdot 2 \cdot 2\pi.$

$= -4 \cdot 16 \cdot 2 \cdot 2\pi = \underline{-256\pi \; C}.$

(1). $r=1$ 위치에서의. 전속밀도 D.

$\oint D \cdot dS = Q$ 인데. $r=1$ 내부에는 전하가 존재하지 않는다. 따라서 $Q=0$. 이고 $\underline{D=0}$ 이다.

$r=3$ 위치에서의 $D. = D_r(r)\, a_r.$ (가우스 폐곡면 : 구)

$\oint D \cdot dS = D_r(r) \cdot a_r \cdot 4\pi r^2_{a_r} = 320\pi C.$ ($r=3$인 구 내부에 전하량 $= 320\pi$ C)

$D_r(r) = \dfrac{320\pi}{4\pi \cdot 9} = \dfrac{80}{9}$ $\quad \therefore \underline{D = \dfrac{80}{9} a_r.}$

$r=5$ 위치에서의 $D = D_r(r) a_r$

$\oint D \cdot dS = D_r(r) \cdot a_r \cdot 4\pi \cdot 25 \cdot a_r = 320\pi - 256\pi = 64\pi.$

$D_r(r) = \dfrac{64}{4\pi \cdot 25} = \dfrac{16}{25\pi}$ $\quad \therefore \underline{D = \dfrac{16}{25\pi} a_r.}$

$$\begin{array}{r} 320 \\ -256 \\ \hline 64 \end{array}$$

(2). $r=7$ 에서의 $D=0$ 이라는 것은.

~~r=7인 구 내부~~ $r=7$인 가우스폐곡면인 구 내부의 전하량이 0인 것이다

따라서 $320\pi - 256\pi + 144\pi \rho_{s3} = 0.$

$64 + 144\rho_{s3} = 0. \quad \rho_{s3} = -\dfrac{64}{144} = -\dfrac{16}{36} = \underline{\dfrac{-4}{9}}$

$$\begin{array}{r} 16 \\ \times 4 \\ \hline 64 \end{array} \qquad \begin{array}{r} 9 \\ \cdot 16 \\ \hline 144 \end{array}$$