HW#1 Solution

Chapter 1.

[Problem 1] Text page 48, Problem 1.9

P 1.9 [a] First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 40te^{-500t}.$$

Therefore, $dq = 40te^{-500t}\,dt$.

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 40\int_0^t ye^{-500y}\,dy.$$

We solve the integral and make the substitutions for the limits of the integral:

$$q(t) - q(0) = 40\frac{e^{-500y}}{(-500)^2}(-500y-1)\Bigg|_0^t$$

$$= 160\times 10^{-6}e^{-500t}(-500t-1) + 160\times 10^{-6}$$

$$= 160\times 10^{-6}(1 - 500te^{-500t} - e^{-500t}).$$

But $q(0) = 0$ by hypothesis, so

$$q(t) = 160(1 - 500te^{-500t} - e^{-500t})\,\mu\text{C}.$$

[b] $q(0.001) = (160)[1 - 500(0.001)e^{-500(0.001)} - e^{-500(0.001)} = 14.4\,\mu\text{C}.$

[Problem 2] Text page 49, Problem 1.14

P 1.14 Assume we are standing at box A looking toward box B. Use the passive sign convention to get $p = vi$, since the current i is flowing into the + terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

[a] $p = (40)(8) = 320$ W   320 W from A to B;

[b] $p = (-10)(-2) = 20$ W   20 W from A to B;

[c] $p = (-50)(2) = -100$ W   100 W from B to A;

[d] $p = (20)(-10) = -200$ W   200 W from B to A.

[Problem 3] Text page 49, Problem 1.18

P 1.18 [a] $p = vi = (3e^{-50t})(0.005e^{-50t}) = 0.015e^{-100t}$ W

$$p(0.005) = 0.015e^{-100(0.005)} = 0.015e^{-0.5} = 9.1 \text{ mW}.$$

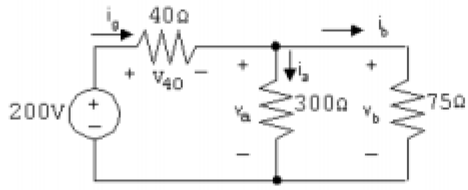
[b] $$w_{\text{total}} = \int_0^\infty p(x)\,dx = \int_0^\infty 0.015e^{-100x}\,dx = 0.015\frac{e^{-100x}}{e^{-100}}\bigg|_0^\infty$$

$$= -0.00015(e^{-\infty} - e^0) = 0.00015 = 0.15 \text{ mJ}.$$

Chapter 2.

[Problem 4] Text page 81, Problem 2.18

P 2.18



[a] Write a KVL equation clockwise aroud the right loop, starting below the $300\,\Omega$ resistor:

$-v_a + v_b = -0$ so $v_a = v_b$.

Using Ohm's law,

$v_a = 300i_a$ and $v_b = 75i_b$.

Substituting,

$300i_a = 75i_b$ so $i_b = 4i_a$.

Write a KCL equation at the top middle node, summing the currents leaving:

$-i_g + i_a + i_b = 0$ so $i_g = i_a + i_b = i_a + 4i_a = 5i_a$.

Write a KVL equation clockwise around the left loop, starting below the voltage source:

$-200\,\text{V} + v_{40} + v_a = 0.$

From Ohm's law,

$v_{40} = 40i_g \qquad \text{and} \qquad v_a = 300i_a.$

Substituting,

$-200\,\text{V} + 40i_g + 300i_a = 0$

Substituting for i_g:

$-200\,\text{V} + 40(5i_a) + 300i_a = -200\,\text{V} + 200i_a + 300i_a = -200\,\text{V} + 500i_a = 0.$

Thus,

$$500i_a = 200\,\text{V} \qquad \text{so} \qquad i_a = \frac{200\,\text{V}}{500} = 0.4\,\text{A}.$$

[b] From part (a), $i_b = 4i_a = 4(0.4\,\text{A}) = 1.6\,\text{A}$.

[c] From the circuit, $v_o = 75\,\Omega(i_b) = 75\,\Omega(1.6\,\text{A}) = 120\,\text{V}$.

[d] Use the formula $p_R = Ri_R^2$ to calculate the power absorbed by each resistor:

$p_{40\Omega} = i_g^2(40\,\Omega) = (5i_a)^2(40\,\Omega) = [5(0.4)]^2(40\,\Omega) = (2)^2(40\,\Omega) = 160\,\text{W};$

$p_{300\Omega} = i_\text{a}^2(300\,\Omega) = (0.4)^2(300\,\Omega) = 48\,\text{W};$

$p_{75\Omega} = i_\text{b}^2(75\,\Omega) = (4i_a)^2(75\,\Omega) = [4(0.4)]^2(75\,\Omega) = (1.6)^2(75\,\Omega) = 192\,\text{W}.$

[e] Using the passive sign convention,

$$\begin{aligned} p_\text{source} &= -(200\,\text{V})i_g = -(200\,\text{V})(5i_a) = -(200\,\text{V})[5(0.4\,\text{A})] \\ &= -(200\,\text{V})(2\,\text{A}) = -400\,\text{W}. \end{aligned}$$

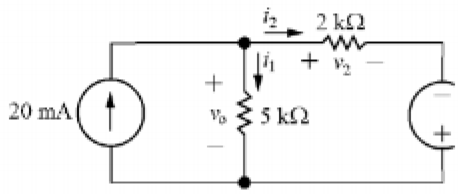
Thus the voltage source delivers 400 W of power to the circuit. Check:

$\sum P_\text{dis} = 160 + 48 + 192 = 400\,\text{W};$

$\sum P_\text{del} = 400\,\text{W}.$

[Problem 5] Text page 81, Problem 2.20

P 2.20 Label the unknown resistor currents and voltages:



[a] KCL at the top node: $\quad 0.02 = i_1 + i_2;$

KVL around the right loop: $\quad -v_o + v_2 - 5 = 0.$

Use Ohm's law to write the resistor voltages in the previous equation in terms of the resistor currents:

$$-5000i_1 + 2000i_2 - 5 = 0 \qquad \rightarrow \qquad -5000i_1 + 2000i_2 = 5.$$

Multiply the KCL equation by -2000 and add it to the KVL equation to eliminate i_2:

$$-2000(i_1 + i_2) + (-5000i_1 + 2000i_2) = -2000(0.02) + 5 \quad \rightarrow \quad -7000i_1 = -35.$$

Solving,

$$i_1 = \frac{-35}{-7000} = 0.005 = 5 \text{ mA}.$$

Therefore,

$$v_o = Ri_1 = (5000)(0.005) = 25 \text{ V}.$$

[b] $p_{20\text{mA}} = -(0.02)v_o = -(0.02)(25) = -0.5 \text{ W};$

$$i_2 = 0.02 - i_1 = 0.02 - 0.005 = 0.015 \text{ A};$$

$$p_{5\text{V}} = -(5)i_2 = -(5)(0.015) = -0.075 \text{ W};$$

$$p_{5\text{k}} = 5000i_1^2 = 5000(0.005)^2 = 0.125 \text{ W};$$

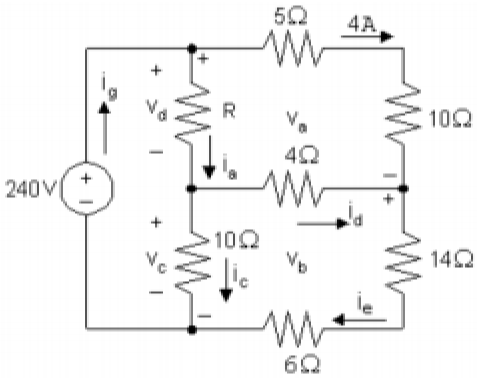
$$p_{2\text{k}} = 2000i_2^2 = 2000(0.015)^2 = 0.45 \text{ W};$$

$$p_{\text{total}} = p_{20\text{mA}} + p_{5\text{V}} + p_{5\text{k}} + p_{2\text{k}} = -0.5 - 0.075 + 0.125 + 0.45 = 0.$$

Thus the power in the circuit balances.

[Problem 6] Text page 82, Problem 2.26

P 2.26 [a]



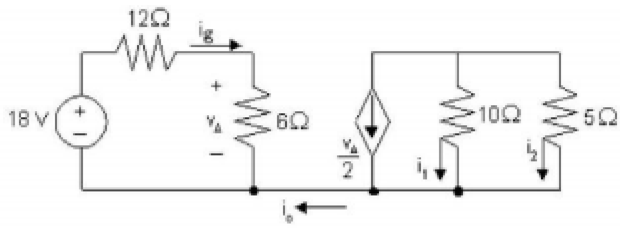
$v_a = (5 + 10)(4) = 60\,\text{V};$
$-240 + v_a + v_b = 0 \qquad \text{so} \qquad v_b = 240 - v_a = 240 - 60 = 180\,\text{V};$
$i_e = v_b/(14 + 6) = 180/20 = 9\,\text{A};$
$i_d = i_e - 4 = 9 - 4 = 5\,\text{A};$
$v_c = 4i_d + v_b = 4(5) + 180 = 200\,\text{V};$
$i_c = v_c/10 = 200/10 = 20\,\text{A};$
$v_d = 240 - v_c = 240 - 200 = 40\,\text{V};$
$i_a = i_d + i_c = 5 + 20 = 25\,\text{A};$
$R = v_d/i_a = 40/25 = 1.6\,\Omega.$

[b] $i_g = i_a + 4 = 25 + 4 = 29$ A;
p_g (supplied) $= (240)(29) = 6960$ W.

[Problem 7] Text page 83, Problem 2.35

P 2.35 [a] $i_o = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[b]



$18 = (12 + 6)i_g \qquad i_g = 1$ A
$v_\Delta = 6i_g = 6\text{V} \qquad v_\Delta/2 = 3$ A
$10i_1 = 5i_2 \qquad \text{so} \qquad i_1 + 2i_1 = -3$ A; therefore $i_1 = -1$ A.

[c] $i_2 = 2i_1 = -2$ A.