인하대학교
INHA UNIVERSITY

Electromagnetics 1 (ICE2003)
-- Ch. 1. VECTOR ANALYSIS --

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Spring, 2021

Chapter Outline

- 전자기학을 배우기 위한 수학적 기초
- Scalar, Vector
  - 개념
  - 더하기, 빼기, 곱하기(내적, 외적)
- 3차원 좌표계
  - Rectangular, Cylindrical, Spherical

    직교 원기둥 구면
  - 각 좌표계에서의 위치 / vector 표현, 상호 변환
  - 각 좌표계에서의 미분, 적분을 위한 미소 길이, 면적, 부피

크기 크기+방향

Scalar, Vector

- Scalar: Temperature, Time, Distance, Mass, Density, Pressure, Voltage, ...
- Vector: Force, Velocity, Acceleration, ...

$\mathbb{E}$ $\mathbb{D}$

$\mathbb{H}$ $\mathbb{B}$

$\mathbb{J}$

ρ : 전하밀도

I

W

V

Q

Scalar, Vector

- Scalar: Temperature, Time, Distance, Mass, Density, Pressure, Voltage, …
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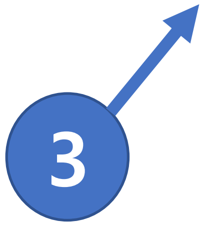
Scalar



Vector

Scalar, Vector

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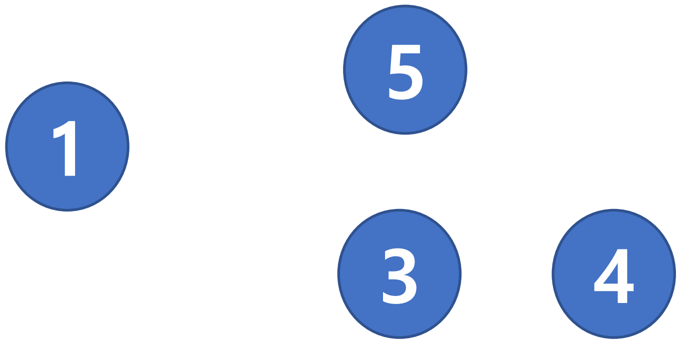
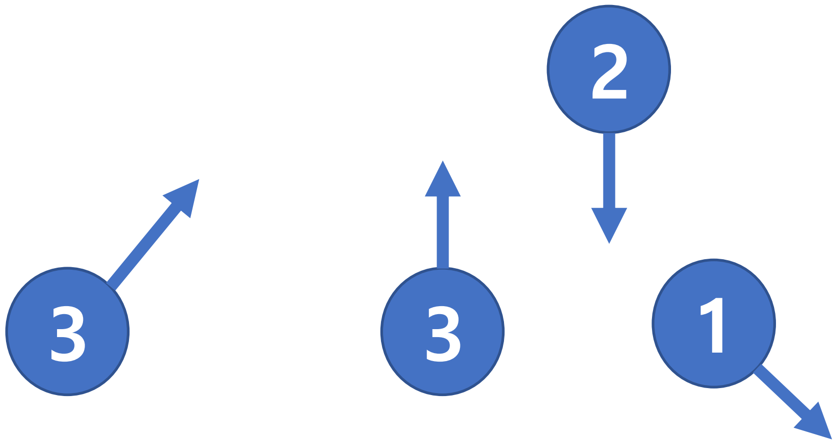


Scalar



Vector

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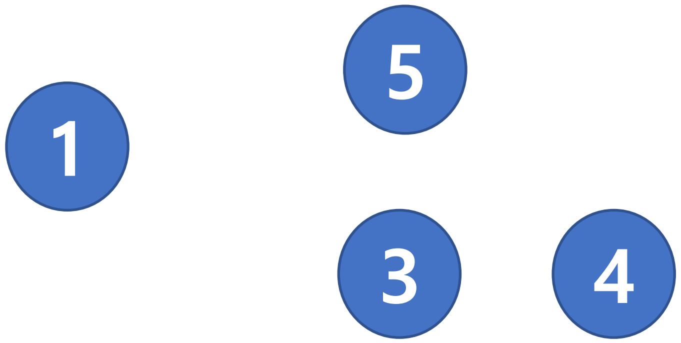
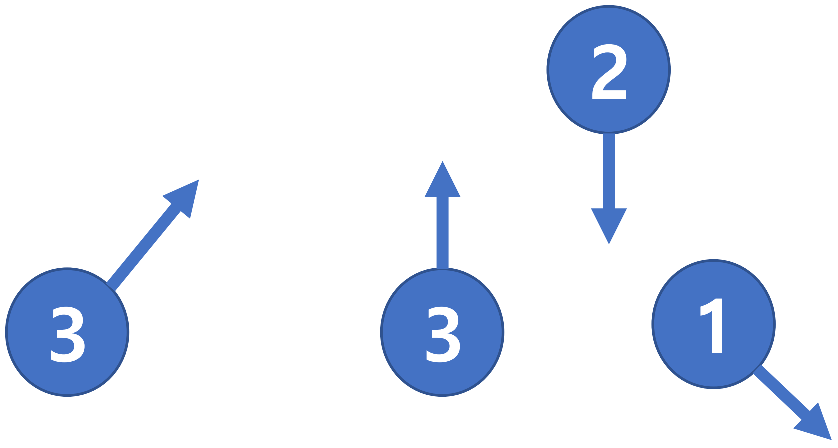


Scalar field



Vector field

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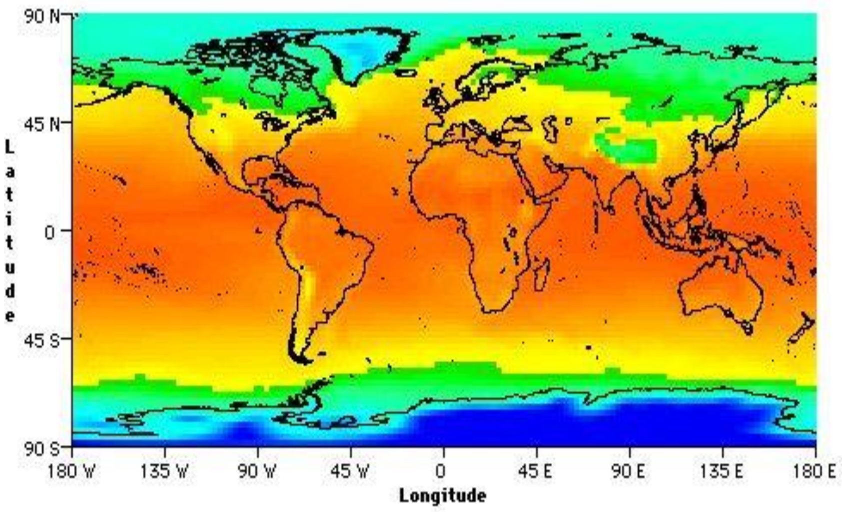
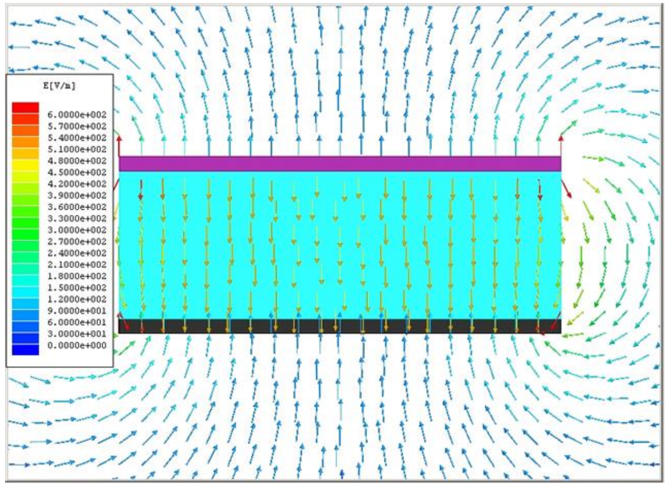


Scalar field



Vector field

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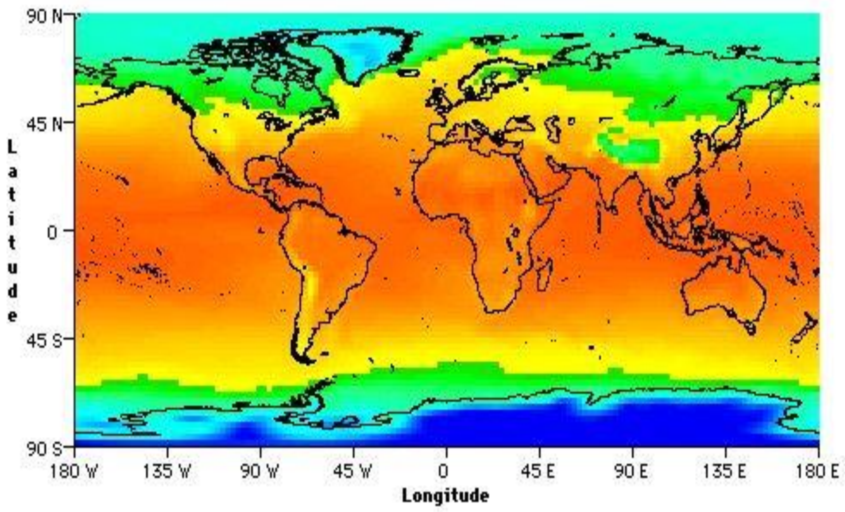
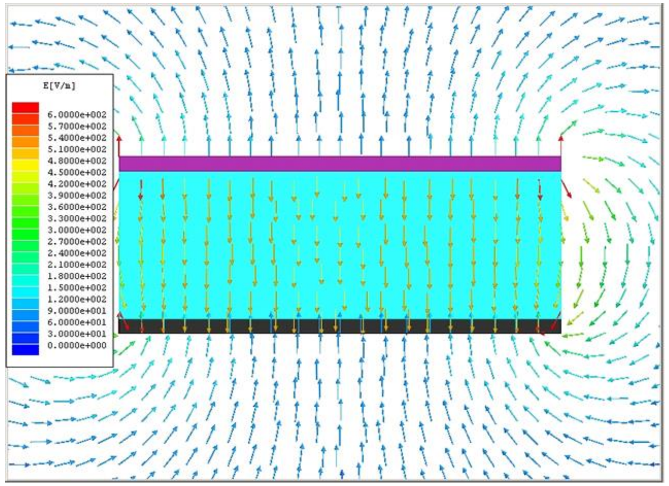


Scalar field $\rho(x, y, z)$



Vector field

$\mathbf{E}(x, y, z)$

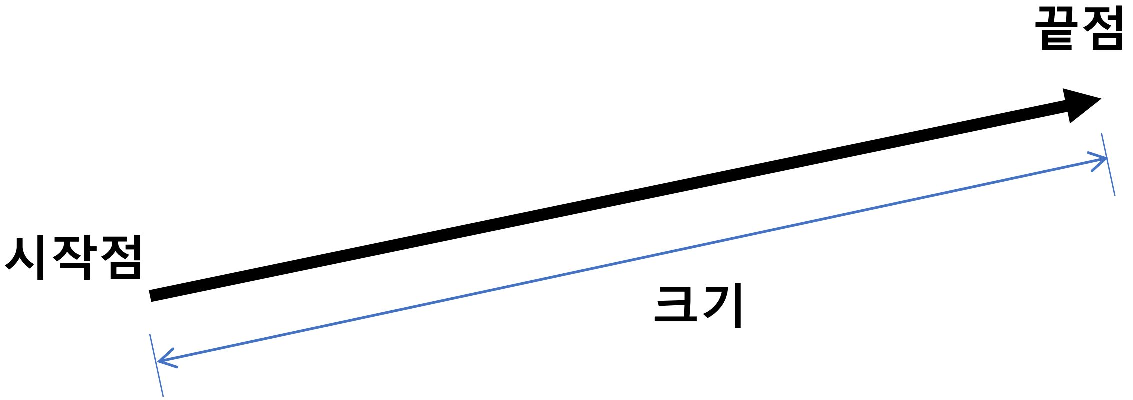
$\mathbf{D}(x, y, z)$

$\mathbf{H}(x, y, z)$

$\mathbf{B}(x, y, z)$

$\mathbf{J}(x, y, z)$

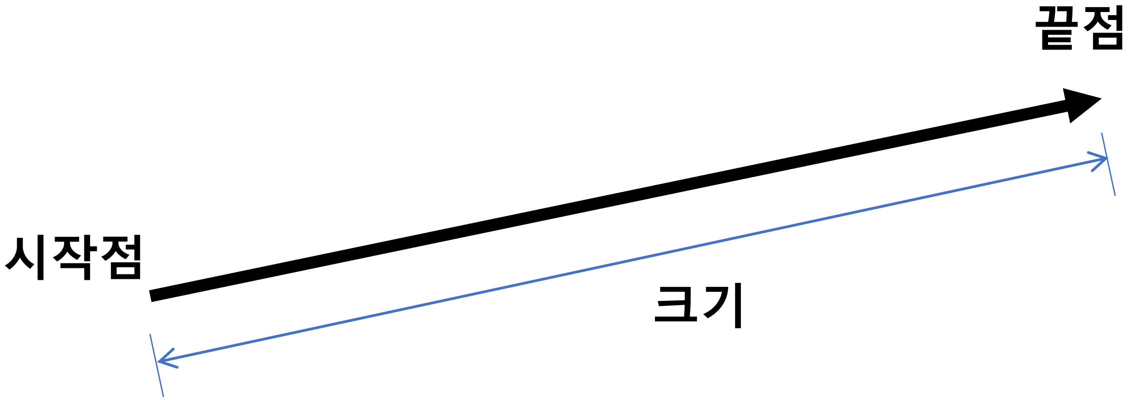
Vector



- 표시: $\mathbf{A}$ $\cancel{\vec{\mathbf{A}}}$ $\cancel{\vec{A}}$ $\cancel{\vec{a}}$ $\cancel{\vec{\mathbf{a}}}$ $\mathbf{a}$

나머지도 맞음

Vector

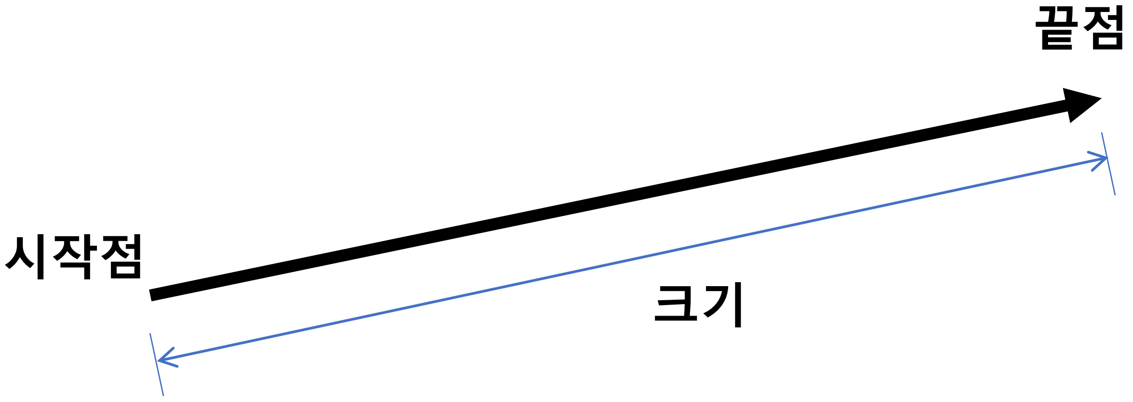


- 표시: $\mathbf{A}$ $\vec{\mathbf{A}}$ $\vec{A}$ $\vec{a}$ $\vec{\mathbf{a}}$ $\mathbf{a}$

벡터 벡터

Vector

A 벡터 A 스칼라



unit vector
단위

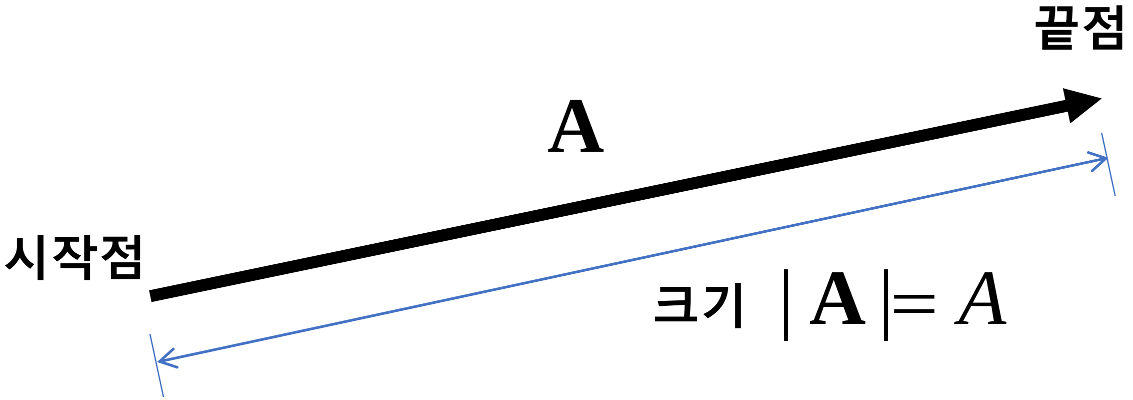
- 표시: $\mathbf{A}$ $\vec{\mathbf{A}}$ $\vec{A}$ $\vec{a}$ $\vec{\mathbf{a}}$ $\mathbf{a}$

벡터 벡터

A 스칼라

a 스칼라

Vector



- 표시: $\mathbf{A}$ $\vec{\mathbf{A}}$ $\vec{A}$ $\vec{a}$ $\vec{\mathbf{a}}$ $\mathbf{a}$

벡터 ($\mathbf{A}$), 벡터 ($\mathbf{a}$)

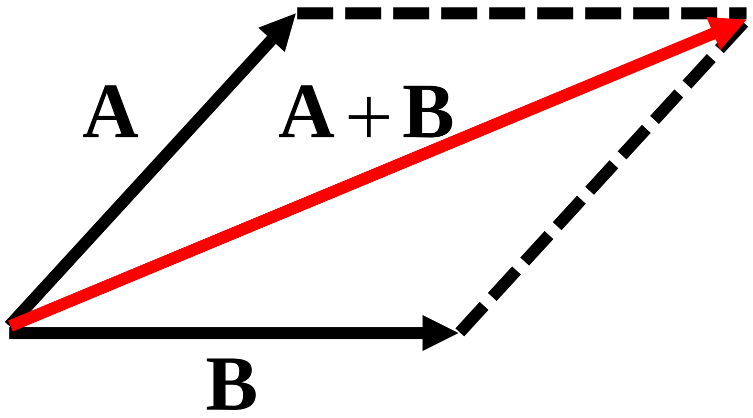
A 스칼라

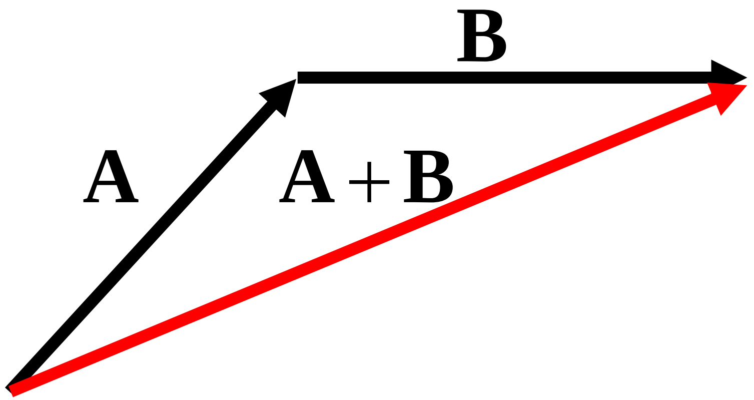
a 스칼라

Vector

Vector Addition

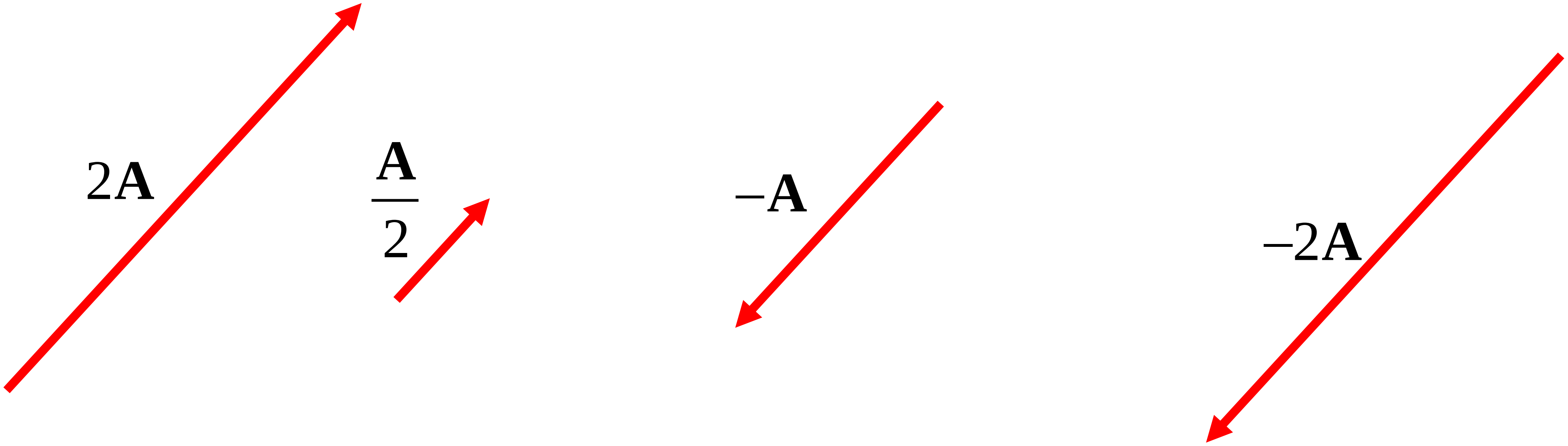
- A+B





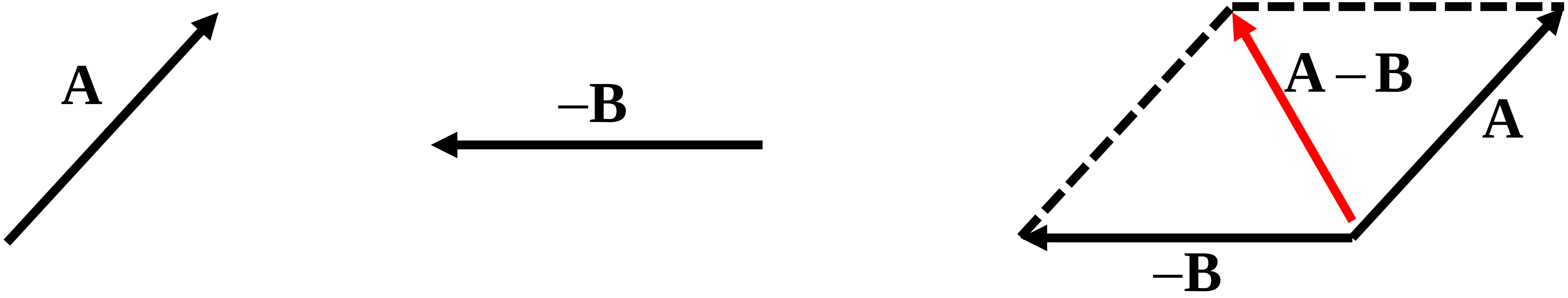
Vector scalar multiplication

- r**A**



Vector subtraction

- A-B



Vector addition / subtraction / scalar multiplication

Associative Law:

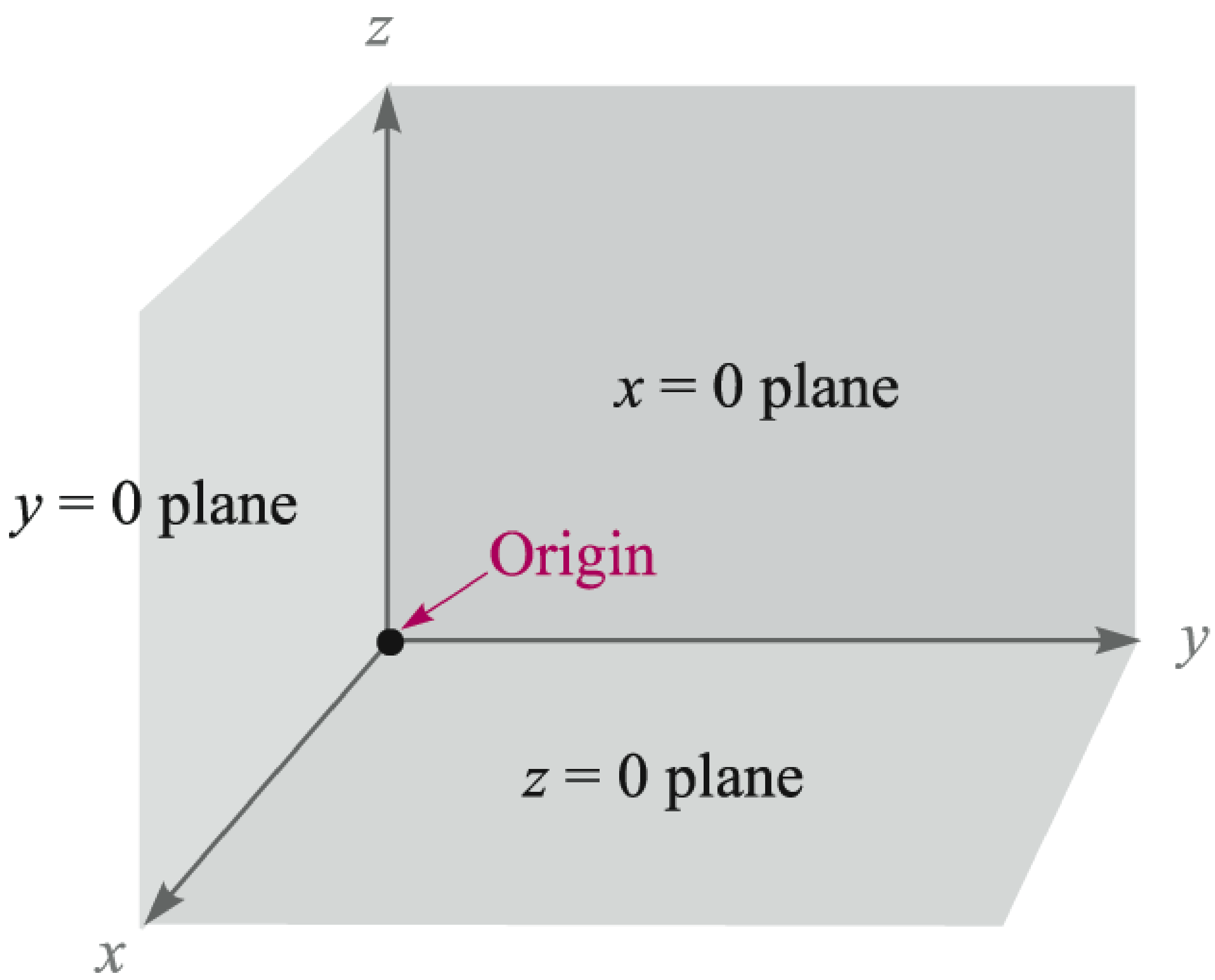
$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$$

결합

Distributive Law:

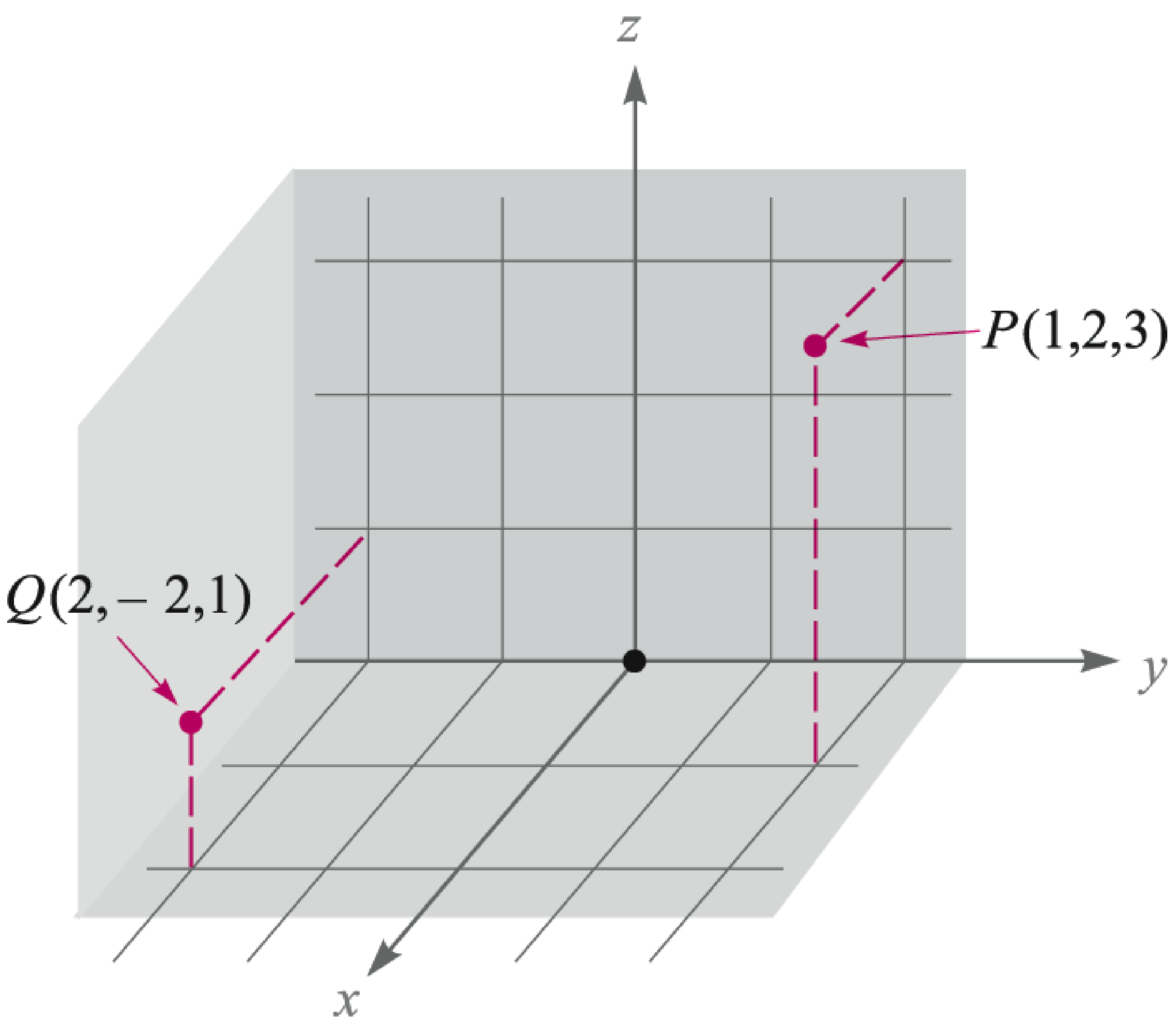
$$(r + s)(\mathbf{A} + \mathbf{B}) = r(\mathbf{A} + \mathbf{B}) + s(\mathbf{A} + \mathbf{B})$$

Rectangular Coordinate System



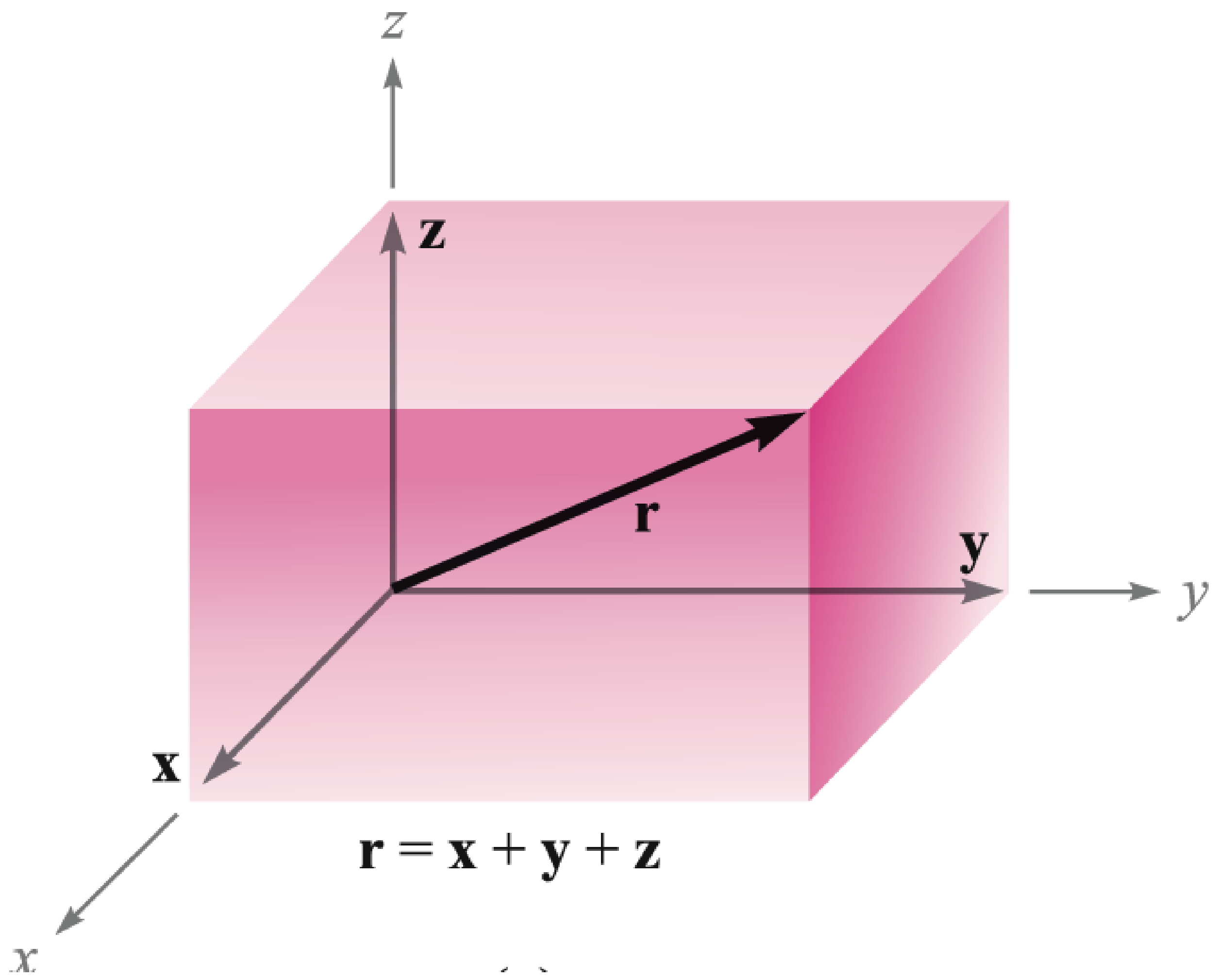
$$(x, y, z)$$

Point locations in Rectangular Coordinates



$$(x, y, z)$$

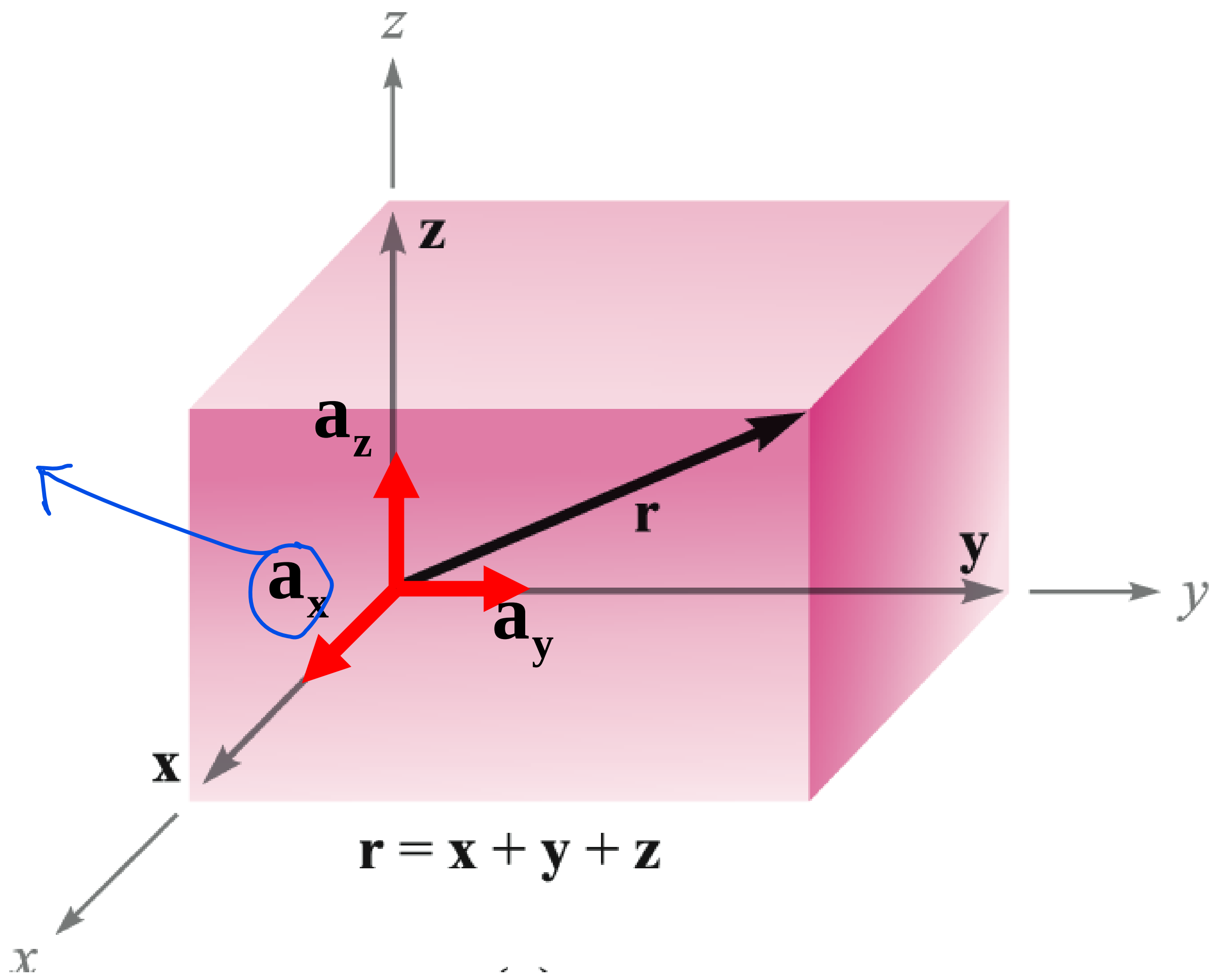
Orthogonal Vector Components



$$\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$$

Orthogonal Unit Vectors

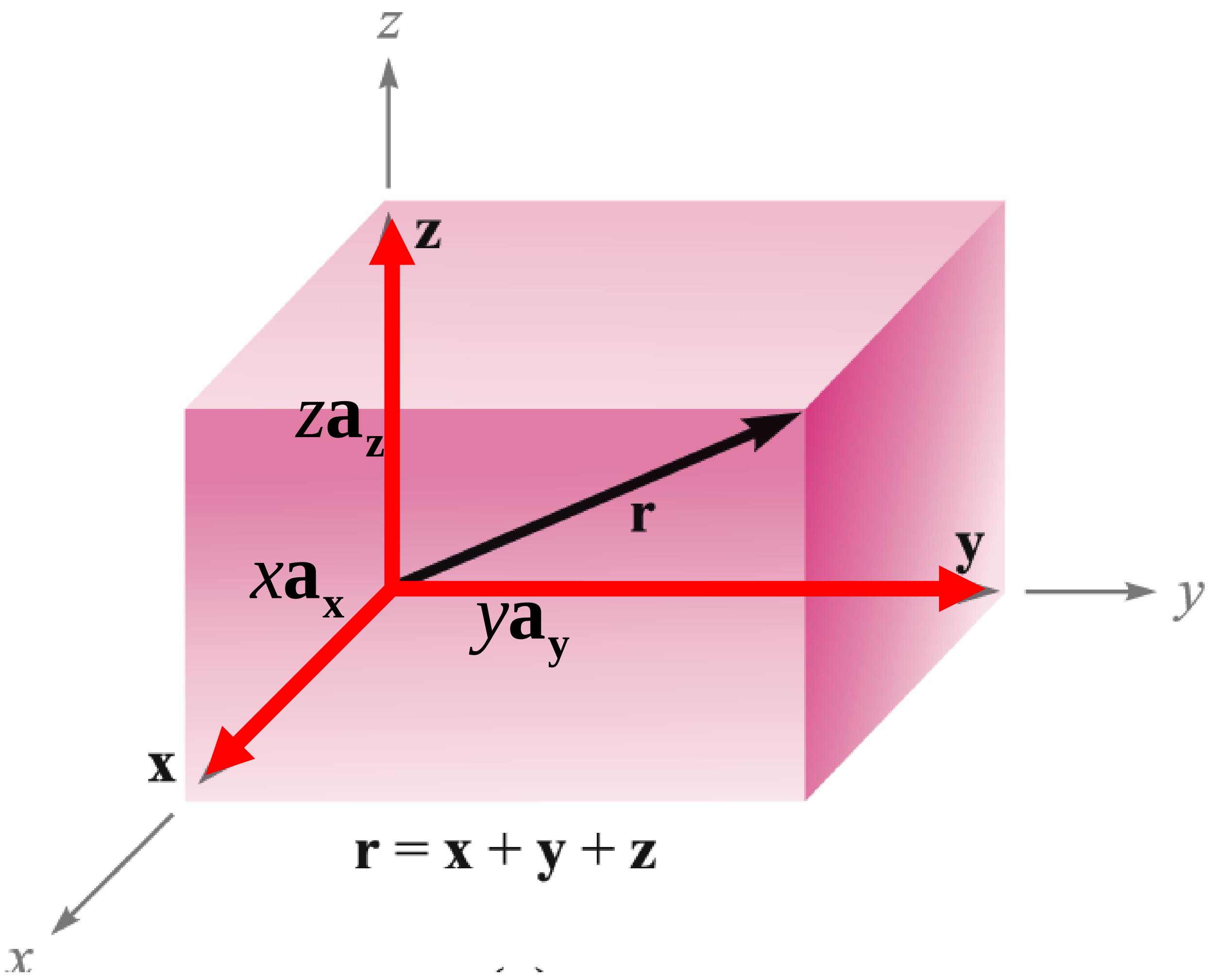
단위 벡터



$$\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$$

$$|\mathbf{a}_x| = |\mathbf{a}_y| = |\mathbf{a}_z| = 1$$

Orthogonal Vector Components

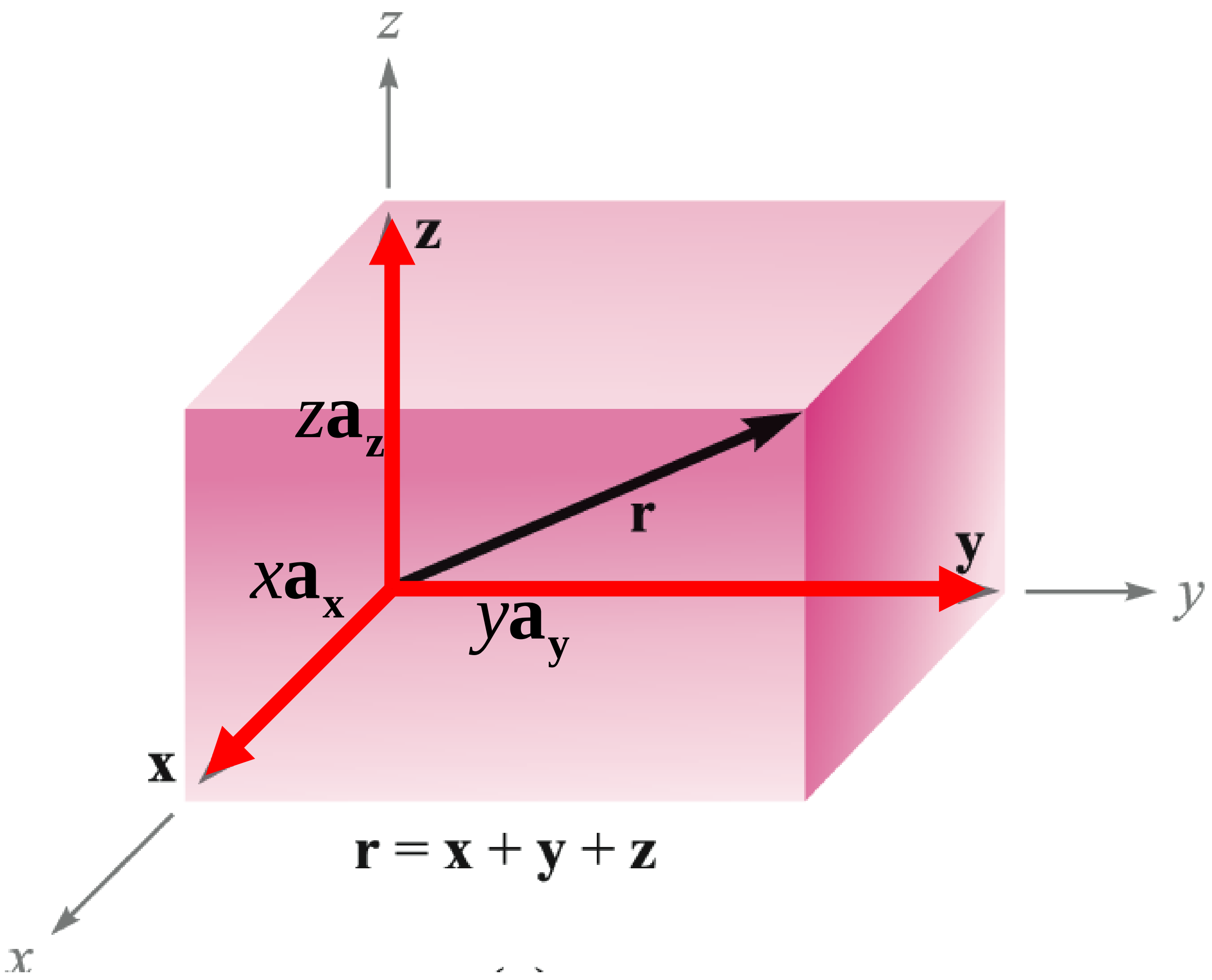


$$\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$$

$x\mathbf{a}_x \quad y\mathbf{a}_y \quad z\mathbf{a}_z$

스칼라 단위벡터

Vector Representation using Orthogonal Rectangular Unit Vectors

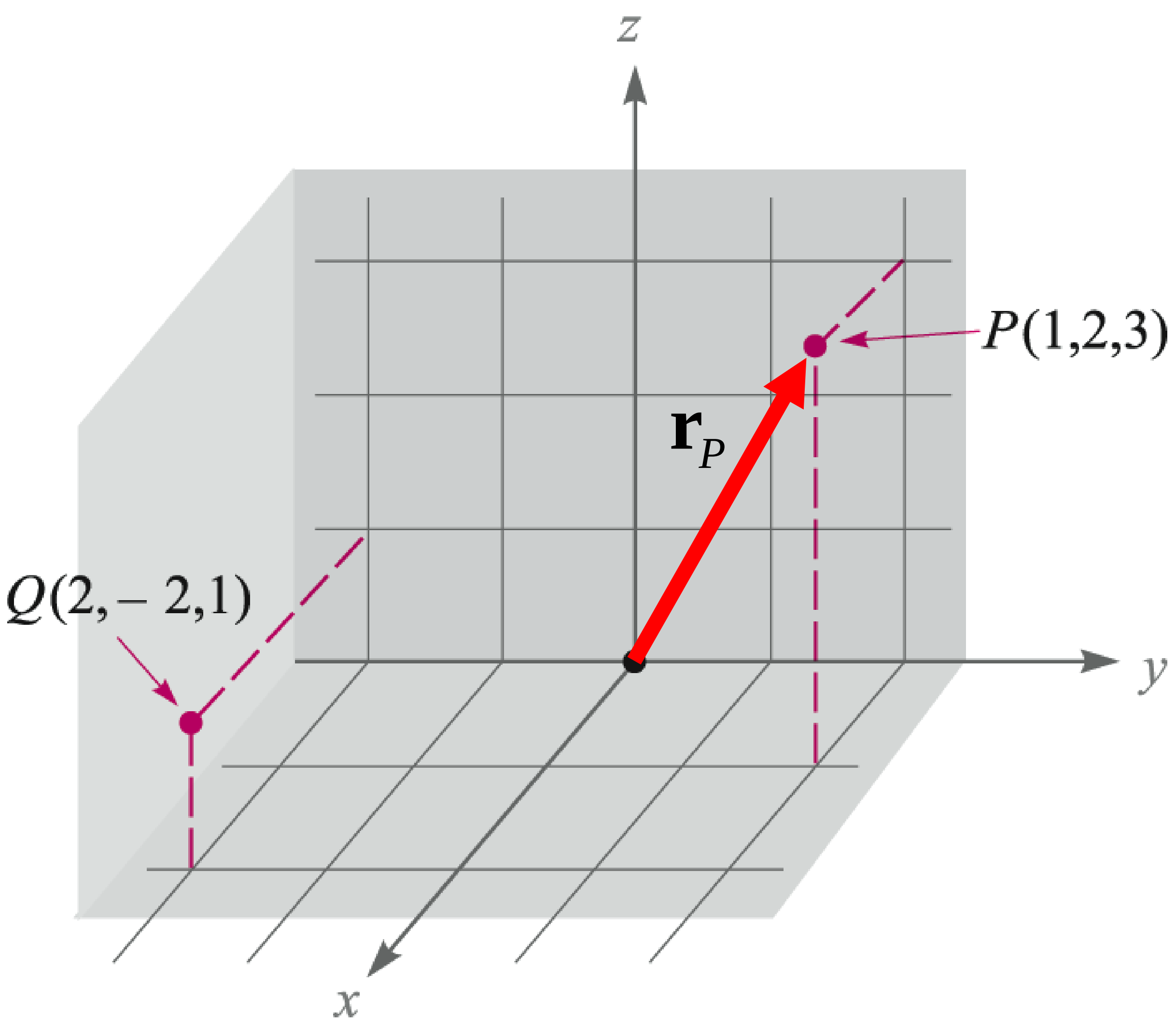


$$\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$$

$$x\mathbf{a}_x \qquad y\mathbf{a}_y \qquad z\mathbf{a}_z$$

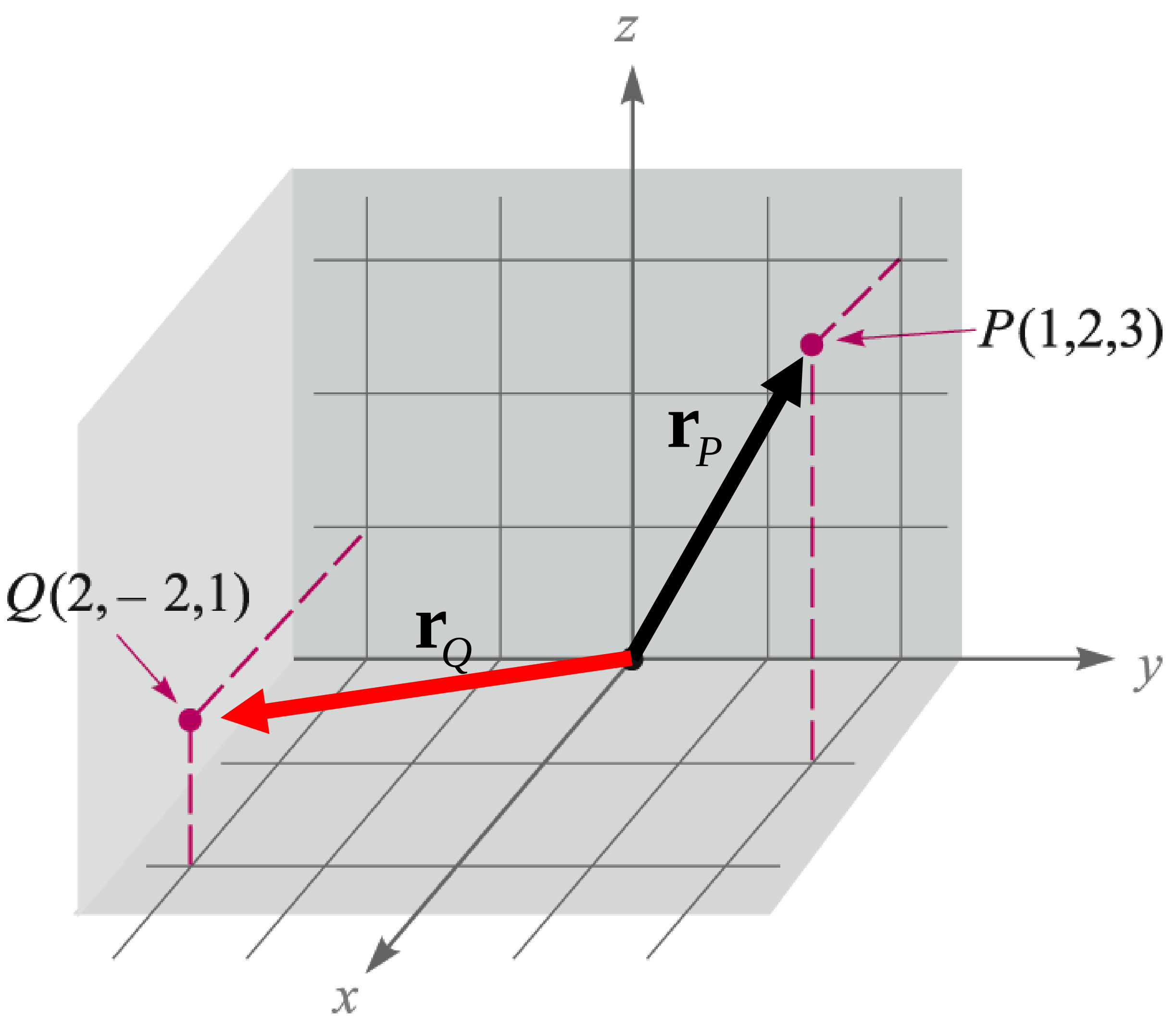
$$\boxed{\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z}$$

Vector Representation using Orthogonal Rectangular Unit Vectors



$$\mathbf{r}_P = 1\mathbf{a}_\mathbf{x} + 2\mathbf{a}_\mathbf{y} + 3\mathbf{a}_\mathbf{z}$$

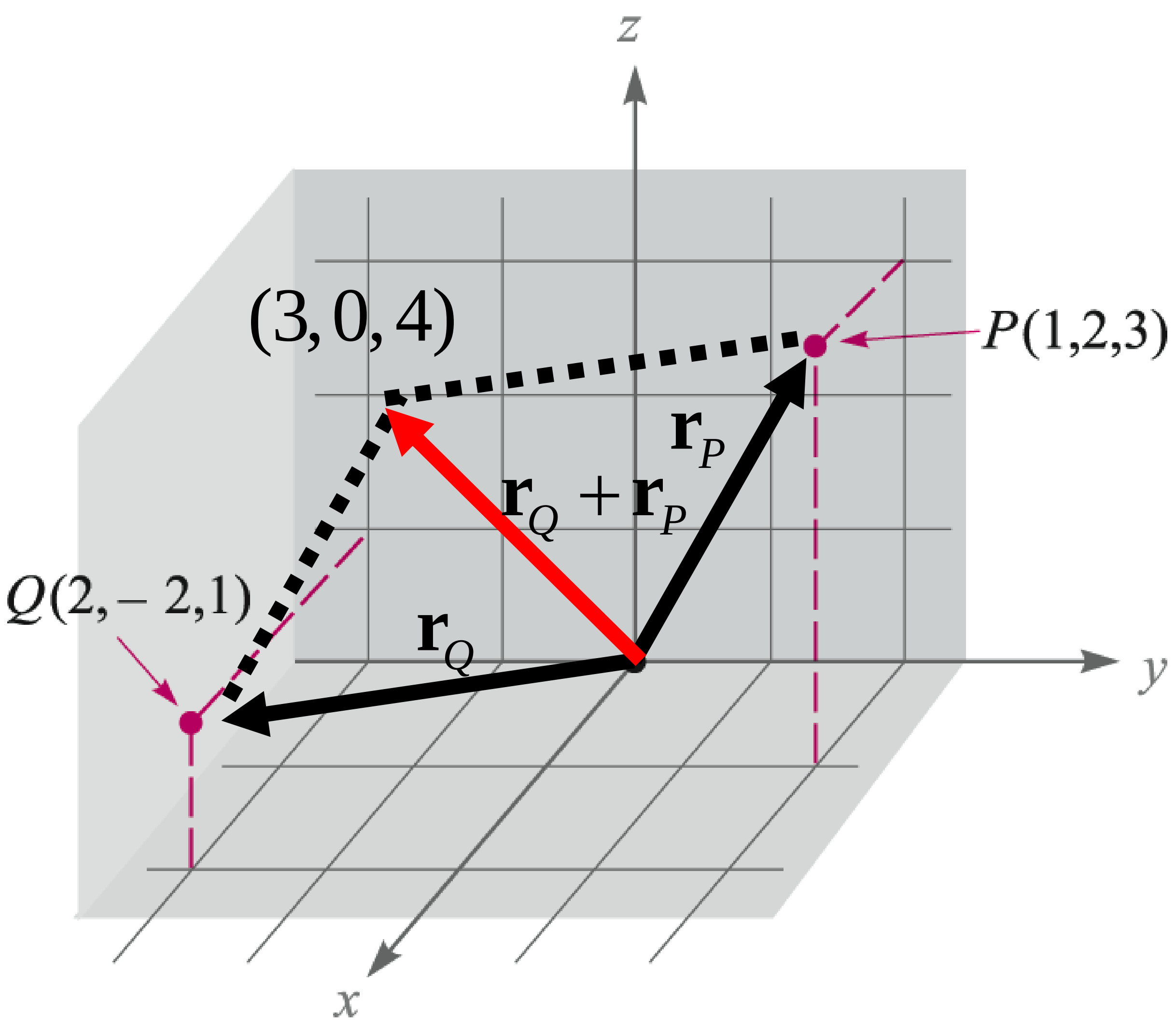
Vector Representation using Orthogonal Rectangular Unit Vectors



$$\mathbf{r}_P = 1\mathbf{a}_\mathbf{x} + 2\mathbf{a}_\mathbf{y} + 3\mathbf{a}_\mathbf{z}$$

$$\mathbf{r}_Q = 2\mathbf{a}_\mathbf{x} - 2\mathbf{a}_\mathbf{y} + 1\mathbf{a}_\mathbf{z}$$

Vector Representation using Orthogonal Rectangular Unit Vectors

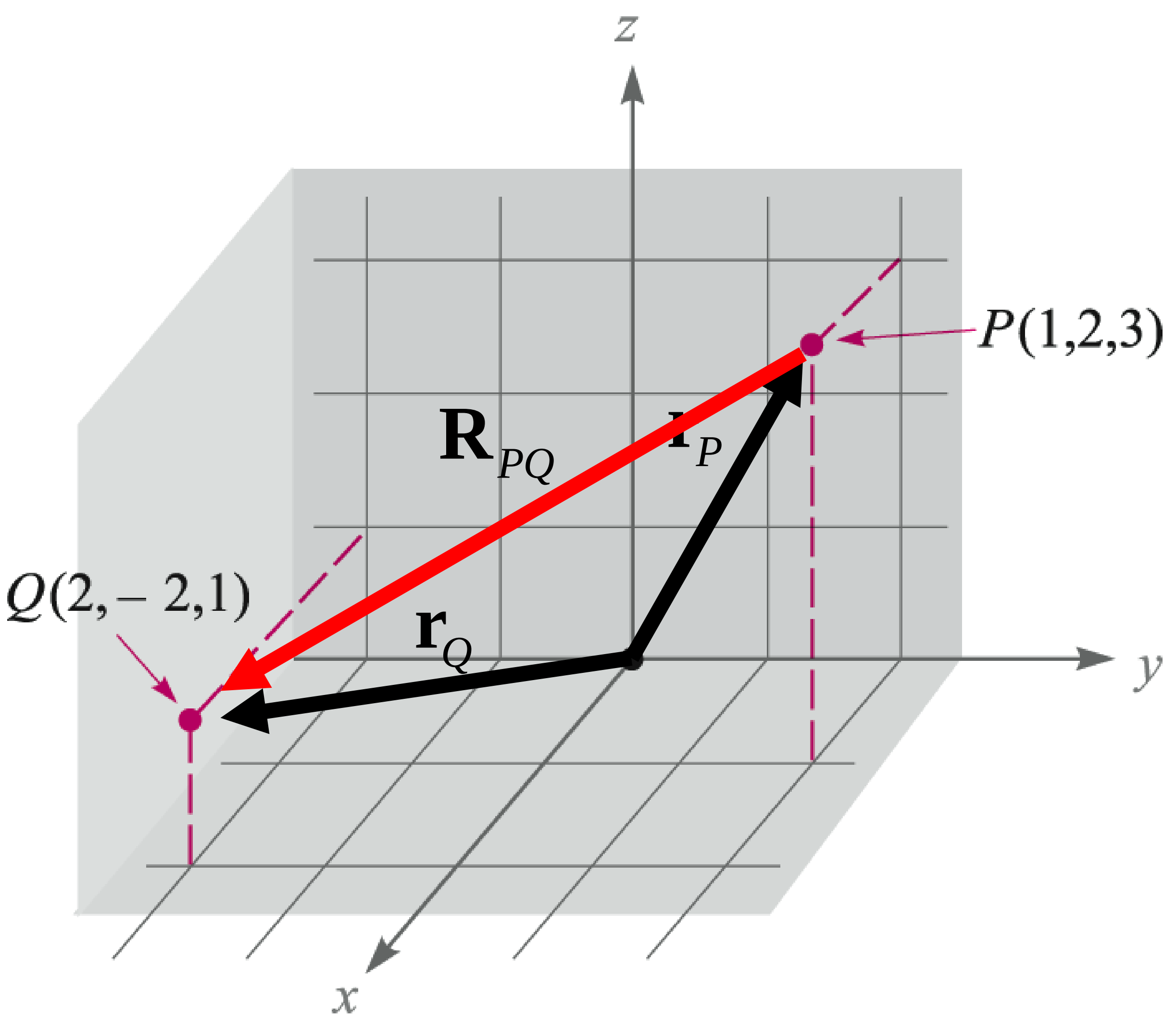


$$\mathbf{r}_P = 1\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$$

$$\mathbf{r}_Q = 2\mathbf{a}_x - 2\mathbf{a}_y + 1\mathbf{a}_z$$

$$\begin{aligned} &\mathbf{r}_Q + \mathbf{r}_P \\ &= (2+1)\mathbf{a}_x + (-2+2)\mathbf{a}_y + (1+3)\mathbf{a}_z \\ &= 3\mathbf{a}_x + 4\mathbf{a}_z \end{aligned}$$

Vector Representation using Orthogonal Rectangular Unit Vectors



$$\mathbf{r}_P = 1\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$$

$$\mathbf{r}_Q = 2\mathbf{a}_x - 2\mathbf{a}_y + 1\mathbf{a}_z$$

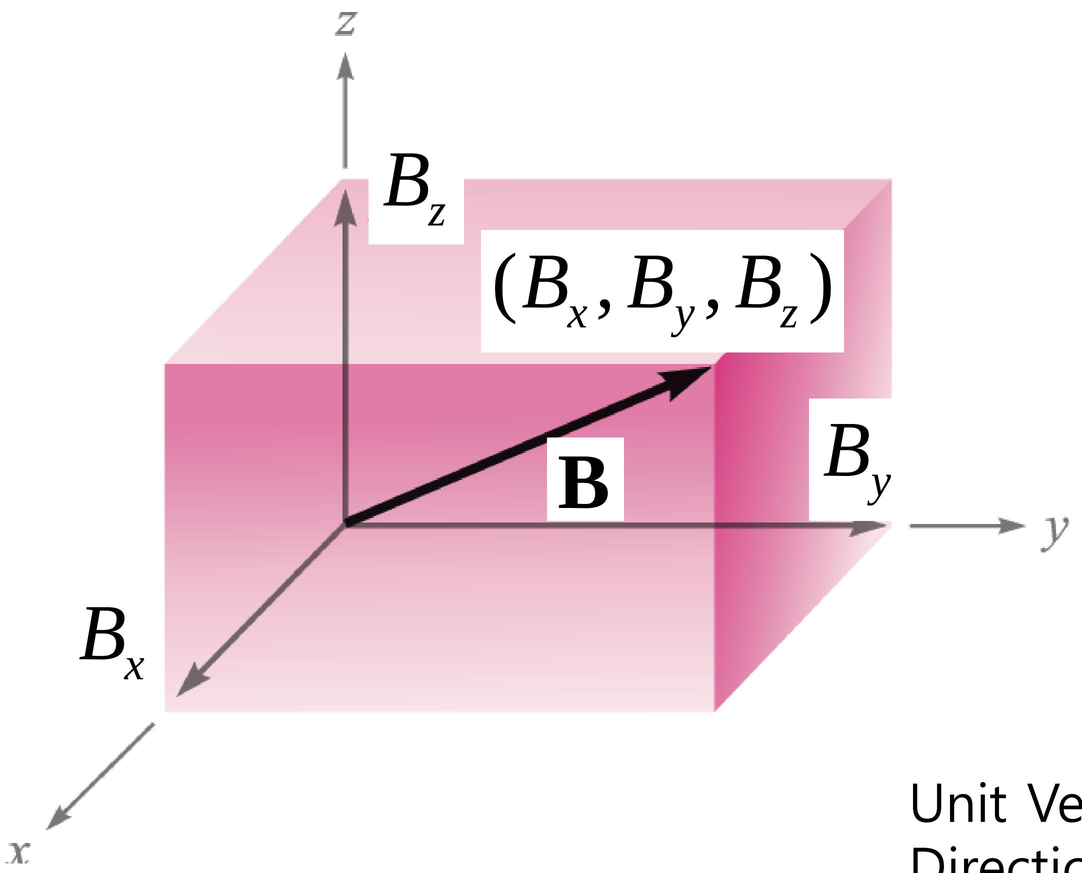
$$\mathbf{R}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P$$

$$= (2-1)\mathbf{a}_x + (-2-2)\mathbf{a}_y + (1-3)\mathbf{a}_z$$

$$= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z$$

(1, -4, -2) : 원점 기준

Vector Expressions in Rectangular Coordinates



General Vector, **B**:

$$\mathbf{B} = B_x \mathbf{a}_x + B_y \mathbf{a}_y + B_z \mathbf{a}_z$$

Magnitude of **B**:

$$|\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

Unit Vector in the Direction of **B**:

$$\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{B_x^2 + B_y^2 + B_z^2}} = \frac{\mathbf{B}}{|\mathbf{B}|}$$

Example

Specify the unit vector extending from the origin toward the point $G(2, -2, -1)$

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Specify the unit vector extending from the origin toward the point $G(2, -2, -1)$

$$\mathbf{G} = 2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z$$

$$|\mathbf{G}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2} = 3$$

$$\mathbf{a}_G = \frac{\mathbf{G}}{|\mathbf{G}|} = \tfrac{2}{3}\mathbf{a}_x - \tfrac{2}{3}\mathbf{a}_y - \tfrac{1}{3}\mathbf{a}_z = \underline{0.667\mathbf{a}_x - 0.667\mathbf{a}_y - 0.333\mathbf{a}_z}$$

Scalar, Vector

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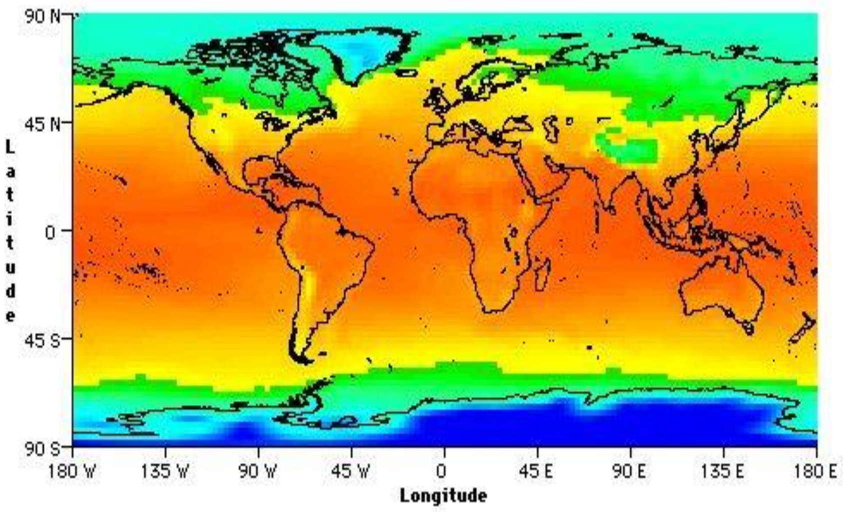
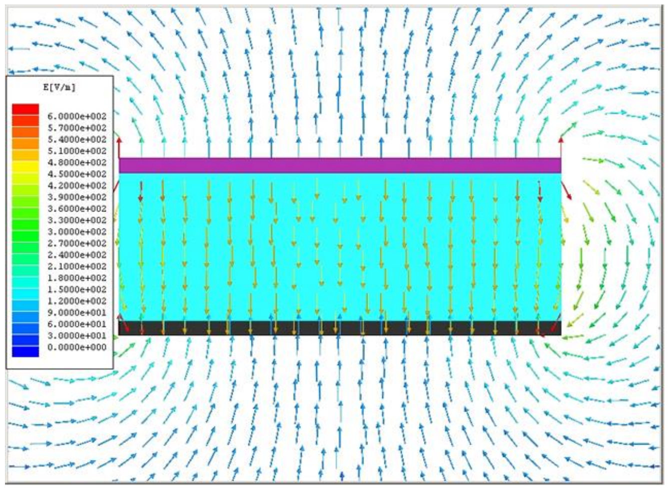
- Vector: Force, Velocity, Acceleration, …



Scalar field



Vector field

Vector Field

We are accustomed to thinking of a specific vector:

$$\mathbf{v} = v_x \mathbf{a}_x + v_y \mathbf{a}_y + v_z \mathbf{a}_z$$

A vector field is a *function* defined in space that has magnitude and direction at all points:

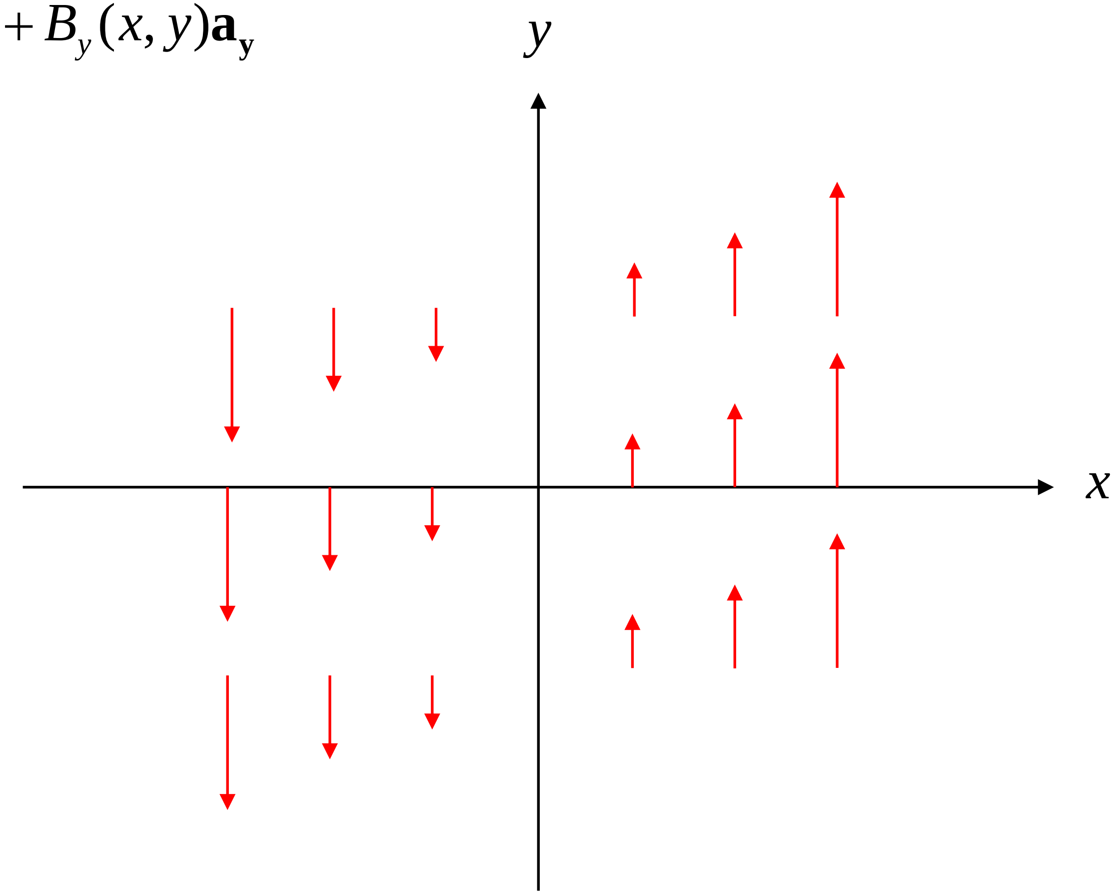
$$\mathbf{v}(\mathbf{r}) = v_x(\mathbf{r})\mathbf{a}_x + v_y(\mathbf{r})\mathbf{a}_y + v_z(\mathbf{r})\mathbf{a}_z$$

where r = (x,y,z)

Vector Field

$$\mathbf{B}(x, y) = B_x(x, y)\mathbf{a_x} + B_y(x, y)\mathbf{a_y}$$

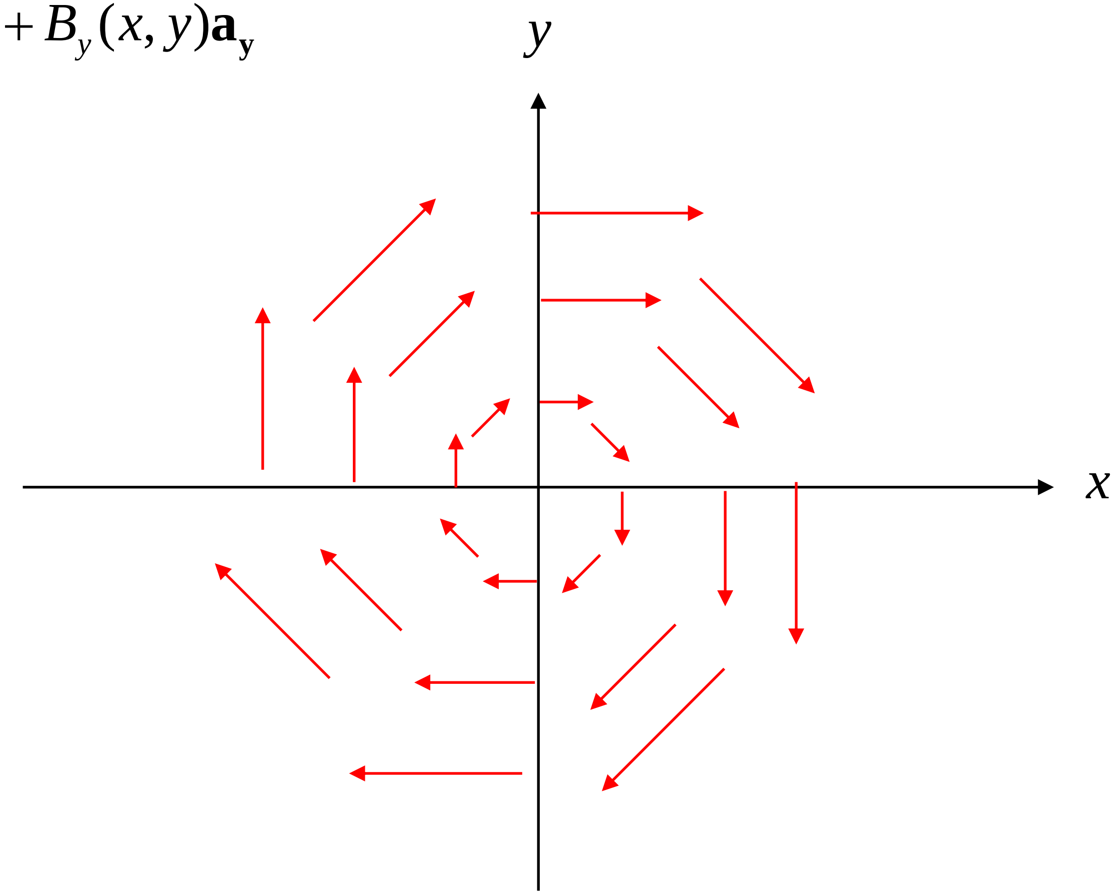
$$= 0\mathbf{a_x} + x\mathbf{a_y}$$



Vector Field

$$\mathbf{B}(x,y) = B_x(x,y)\mathbf{a_x} + B_y(x,y)\mathbf{a_y}$$

$$= y\mathbf{a_x} - x\mathbf{a_y}$$



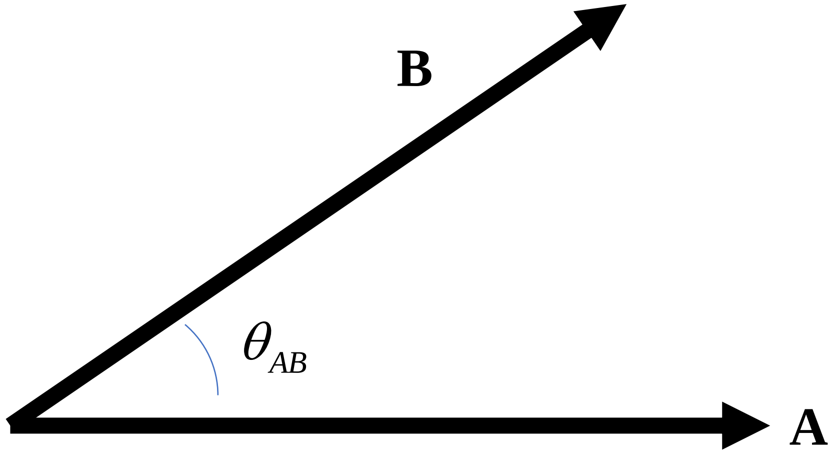
Product

$$\mathbf{A} \cdot \mathbf{B}$$

Dot product

$$\mathbf{A} \times \mathbf{B}$$

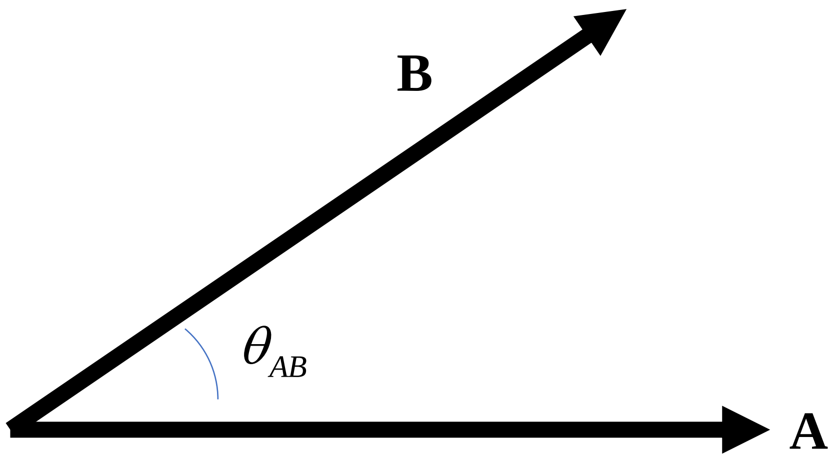
Cross product



Dot Product

Given two vectors **A** and **B**, the *dot product*, or *scalar product*, is defined as the product of the magnitude of **A**, the magnitude of **B**, and the cosine of the smaller angle between them,

$$\boxed{\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \cos \theta_{AB}}$$



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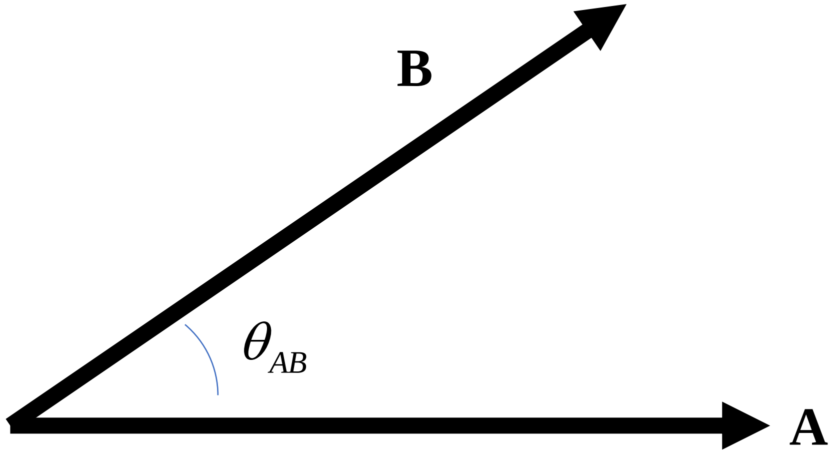
Vector Scalar



Dot Product = Scalar Product

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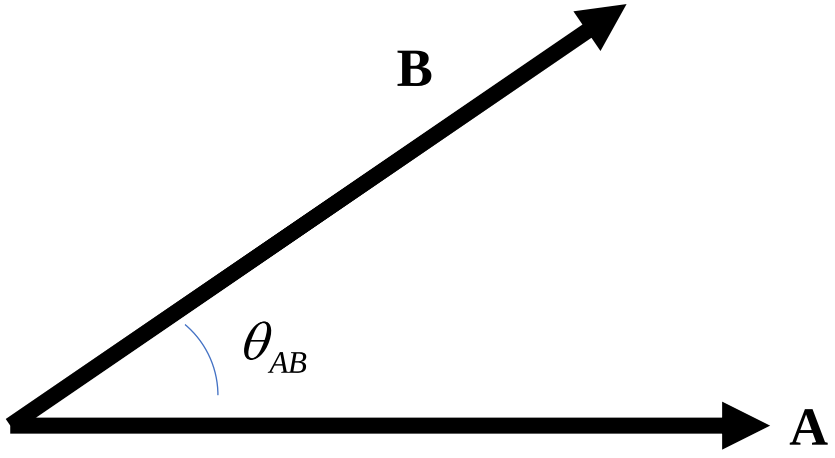
Vector Scalar



Dot Product = Scalar Product = Inner Product

Given two vectors **A** and **B**, the *dot product*, or *scalar product*, is defined as the product of the magnitude of **A**, the magnitude of **B**, and the cosine of the smaller angle between them,

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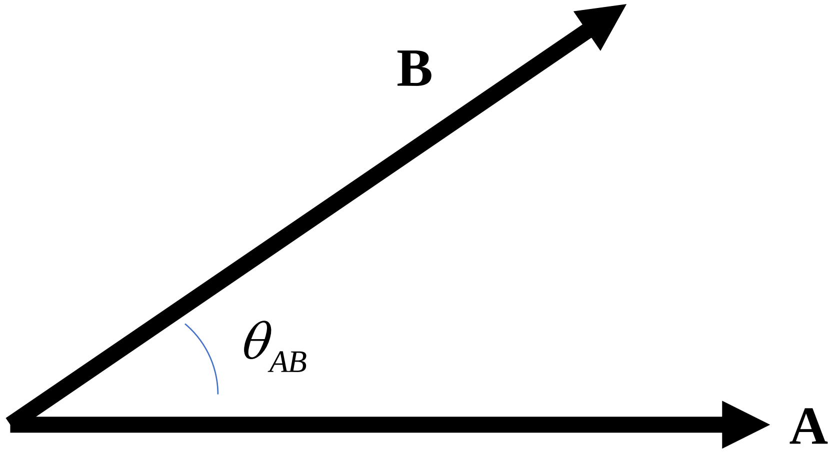
Vector Scalar



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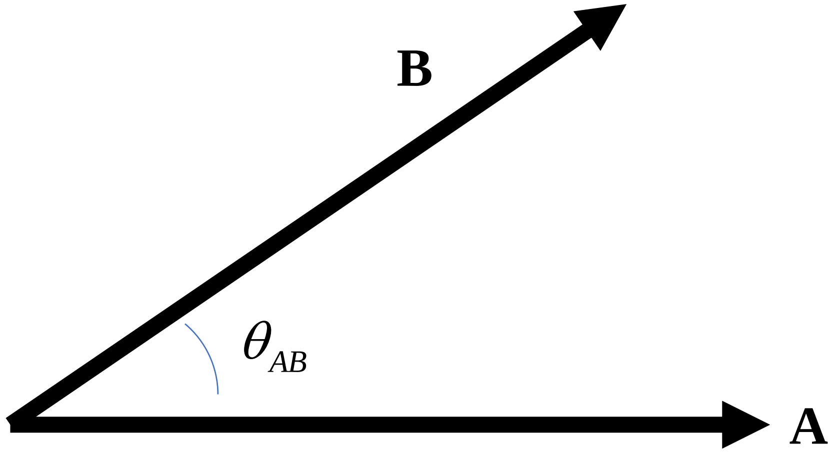

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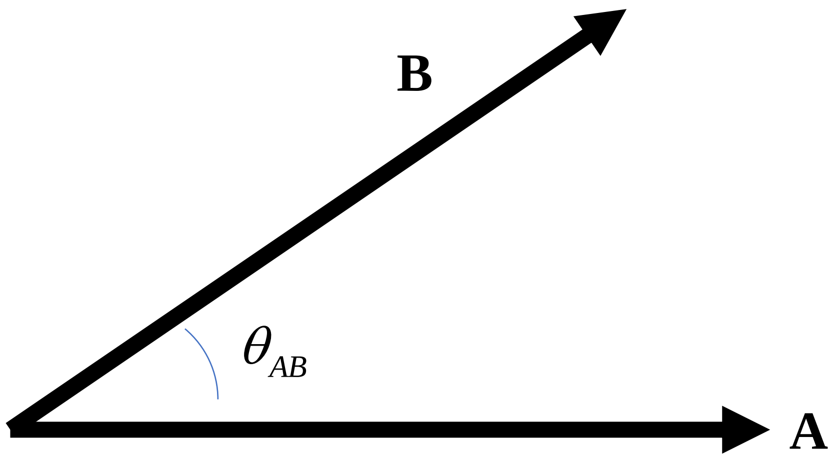

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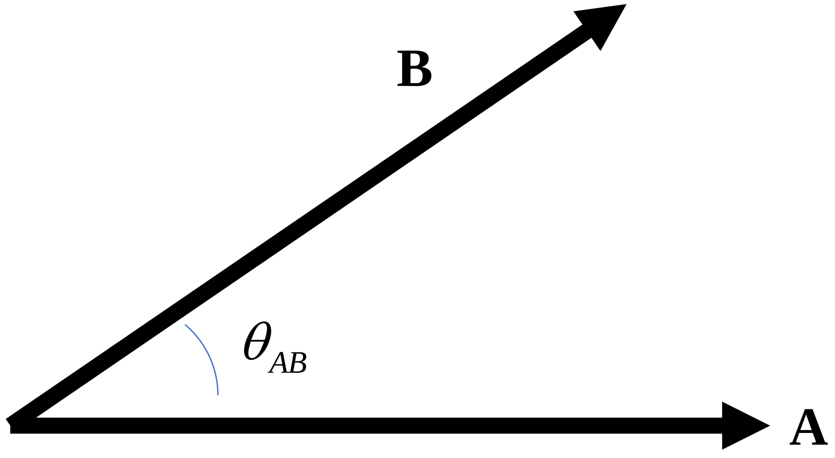
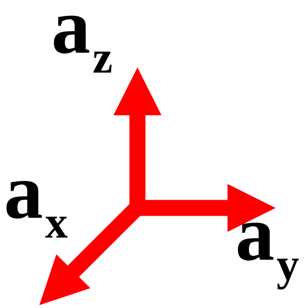
$$\boxed{\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}|\,|\mathbf{B}|\cos\theta_{AB}}$$

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$$

$$\mathbf{a}_x\cdot\mathbf{a}_x = \mathbf{a}_y\cdot\mathbf{a}_y = \mathbf{a}_z\cdot\mathbf{a}_z = 1$$





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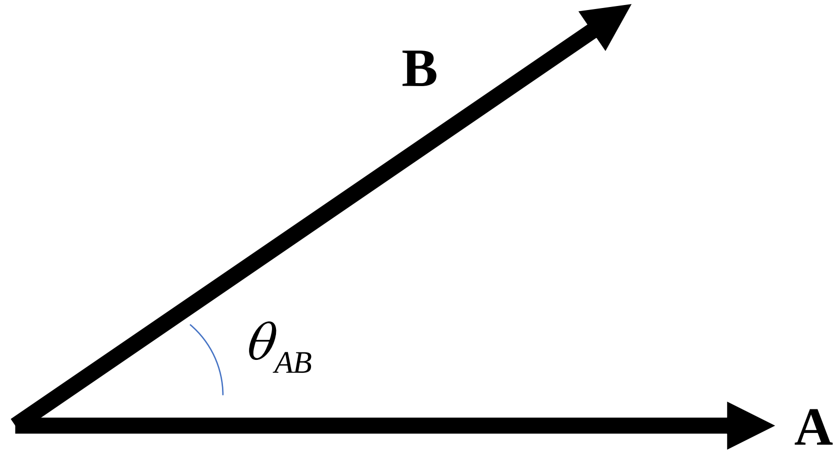
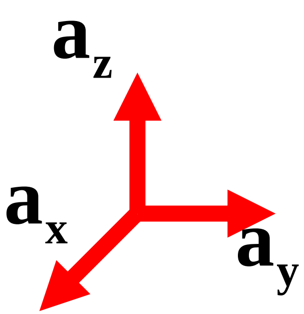
$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$$

$$\mathbf{a}_x\cdot\mathbf{a}_x = \mathbf{a}_y\cdot\mathbf{a}_y = \mathbf{a}_z\cdot\mathbf{a}_z = 1$$

$$\mathbf{a}_x\cdot\mathbf{a}_y = \mathbf{a}_y\cdot\mathbf{a}_z = \mathbf{a}_z\cdot\mathbf{a}_x = 0$$





Dot Product = Scalar Product = Inner Product

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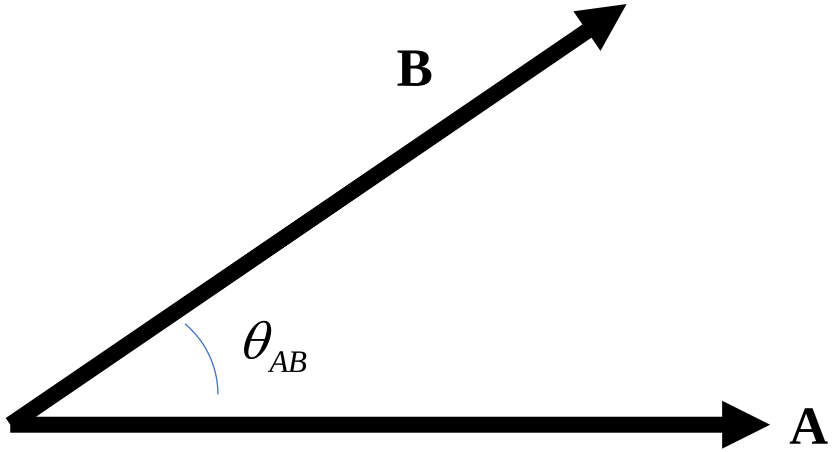

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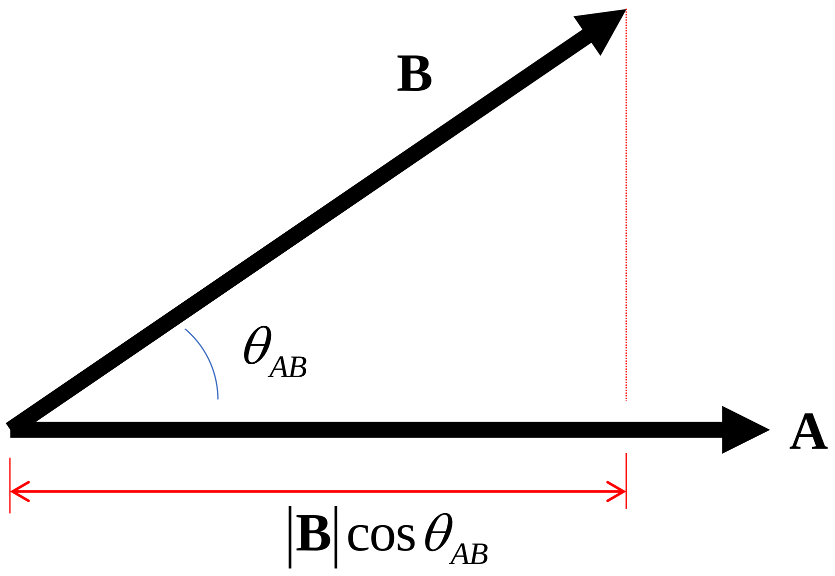

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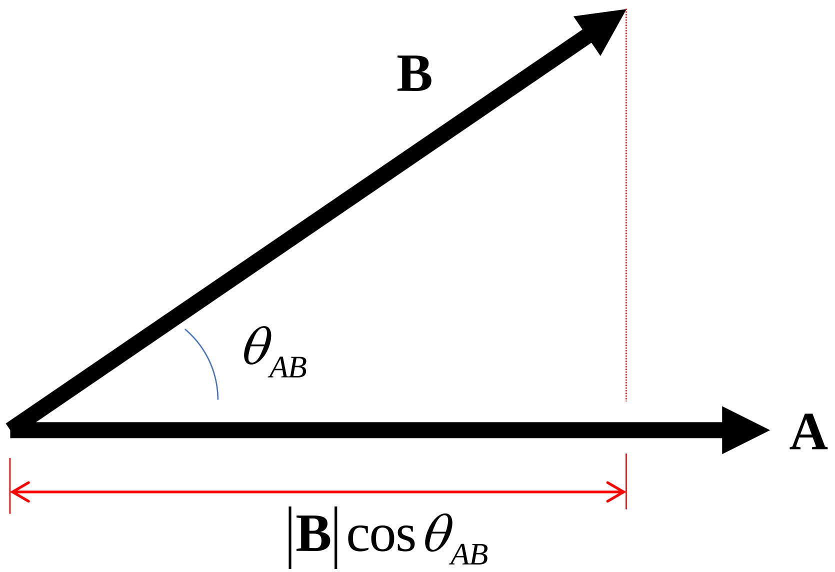

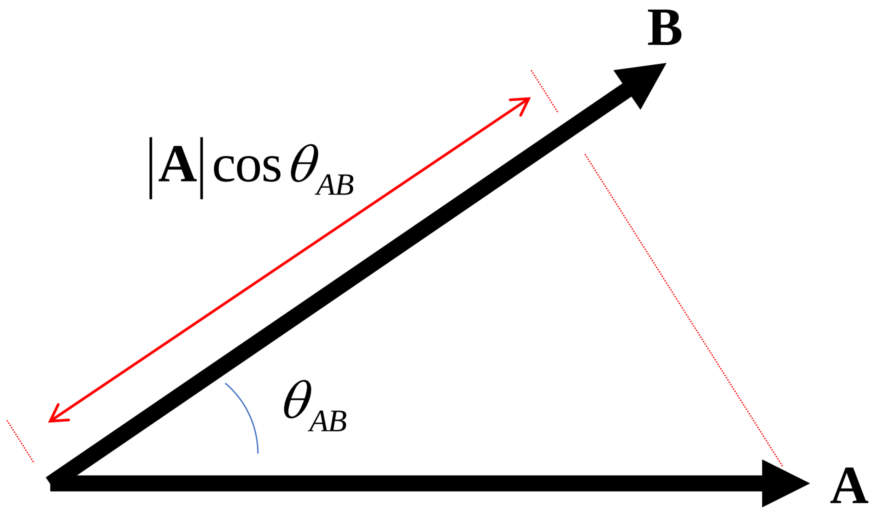

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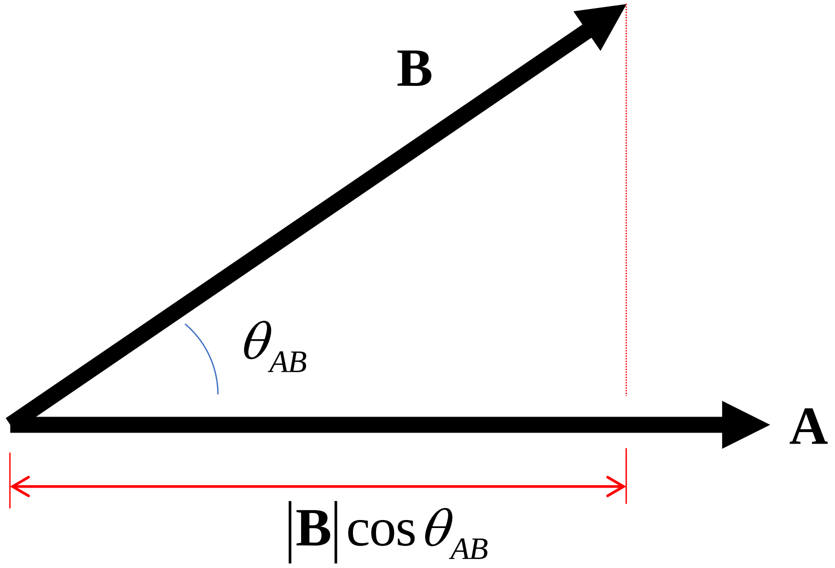

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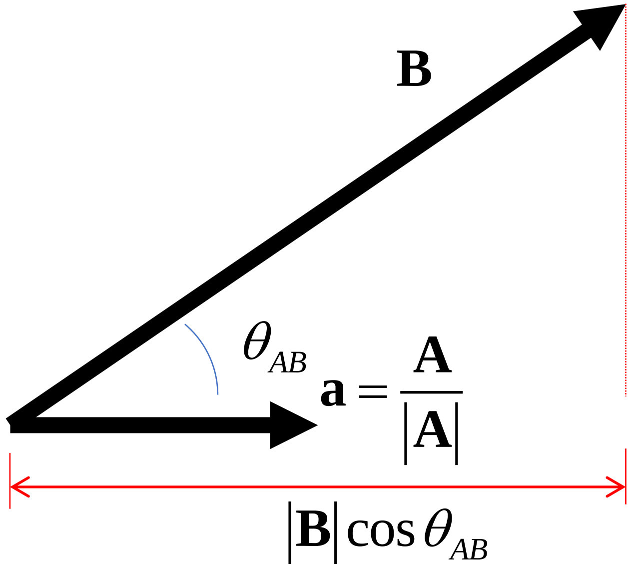

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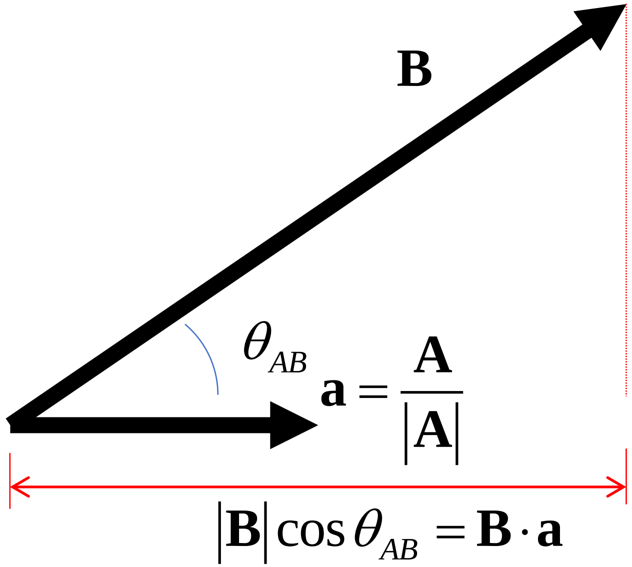

$$\mathbf{B}\cdot\mathbf{a} = |\mathbf{B}||\mathbf{a}|\cos\theta = |\mathbf{B}|\cos\theta$$

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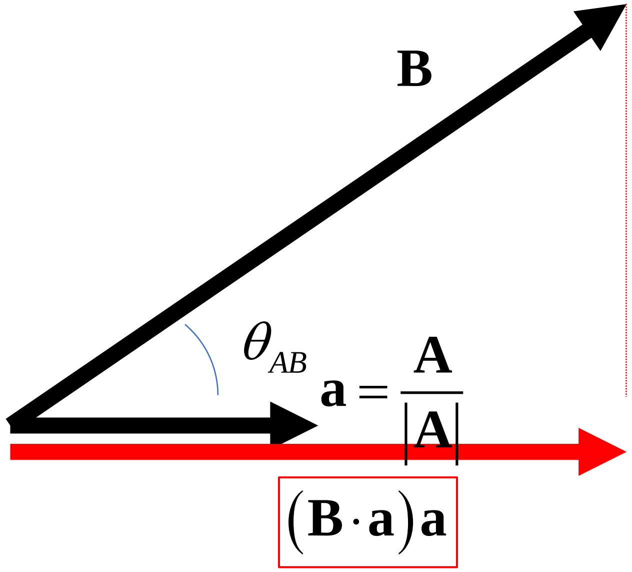

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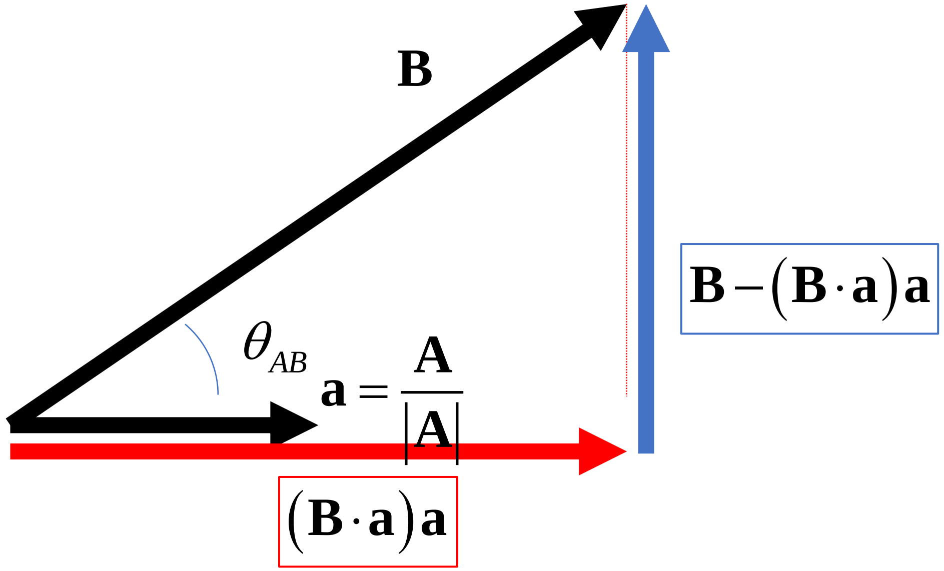

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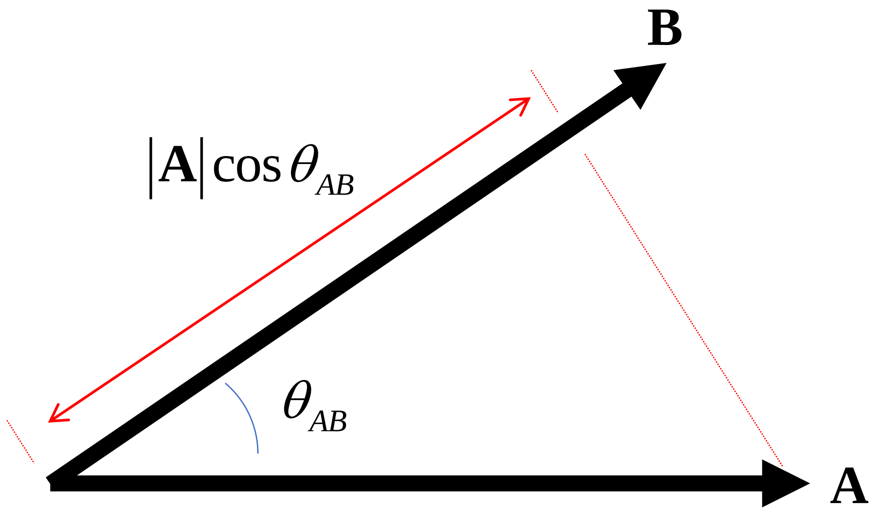

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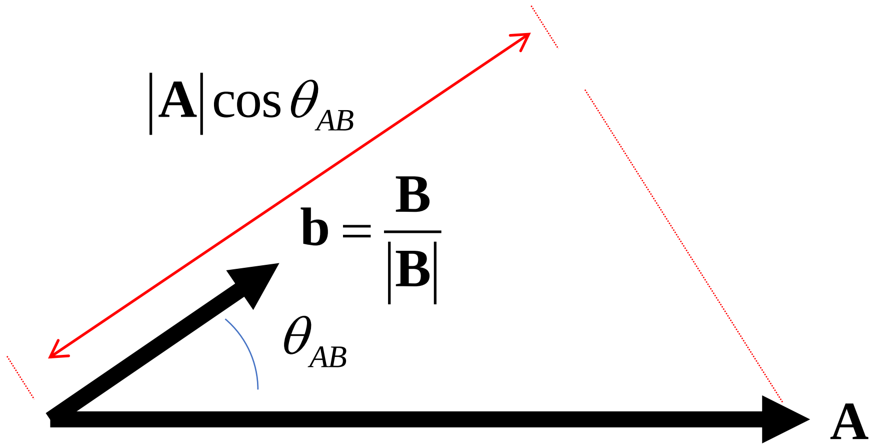

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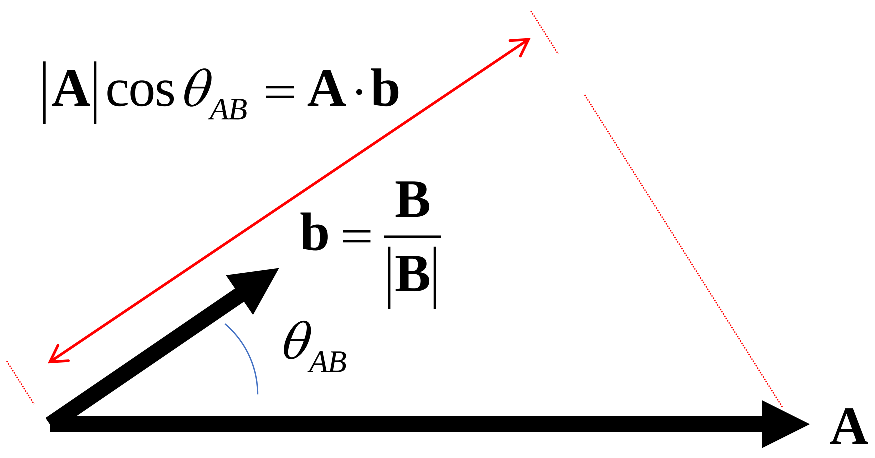

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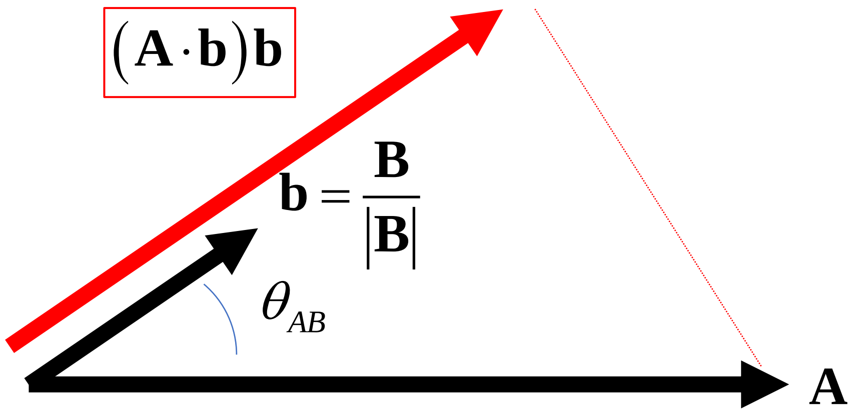

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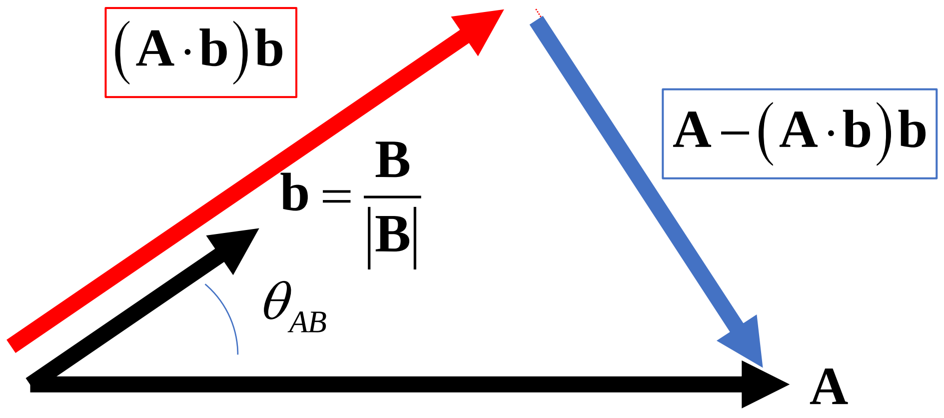

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$$\mathbf{A} = A_x \mathbf{a_x} + A_y \mathbf{a_y} + A_z \mathbf{a_z} \quad \Rightarrow$$

$$\mathbf{A} \cdot \mathbf{a_x} = \left(A_x \mathbf{a_x} + A_y \mathbf{a_y} + A_z \mathbf{a_z} \right) \cdot \mathbf{a_x} = A_x$$

$$\mathbf{A} \cdot \mathbf{a_y} = \left(A_x \mathbf{a_x} + A_y \mathbf{a_y} + A_z \mathbf{a_z} \right) \cdot \mathbf{a_y} = A_y$$

$$\mathbf{A} \cdot \mathbf{a_z} = \left(A_x \mathbf{a_x} + A_y \mathbf{a_y} + A_z \mathbf{a_z} \right) \cdot \mathbf{a_z} = A_z$$

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$$\left(\mathbf{A} \cdot \mathbf{a_y}\right)\mathbf{a_y} = A_y \mathbf{a_y}$$

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Vector   Scalar

$$\mathbf{A}\cdot\mathbf{B}=\mathbf{B}\cdot\mathbf{A}$$

$$\mathbf{a_x}\cdot\mathbf{a_x}=\mathbf{a_y}\cdot\mathbf{a_y}=\mathbf{a_z}\cdot\mathbf{a_z}=1$$

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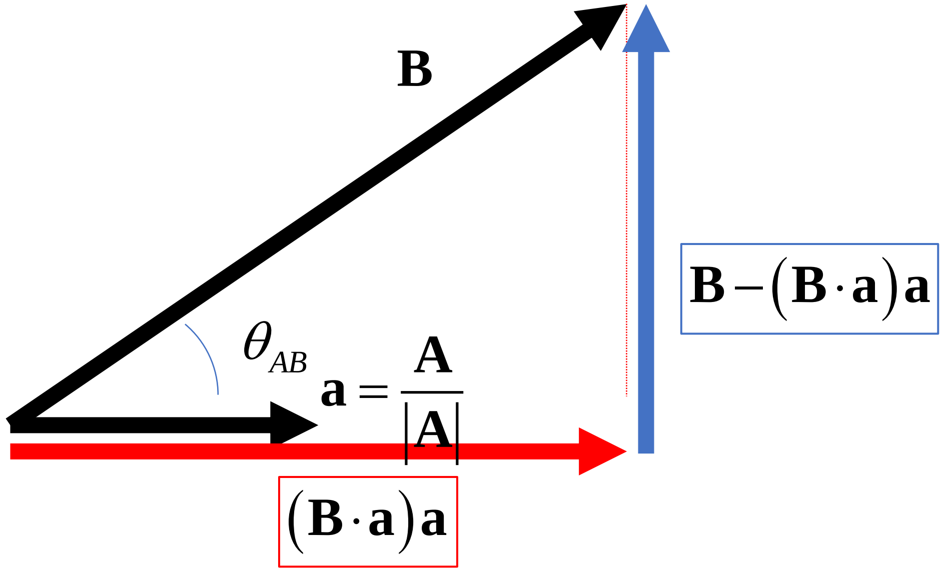


Dot Product = Scalar Product = Inner Product

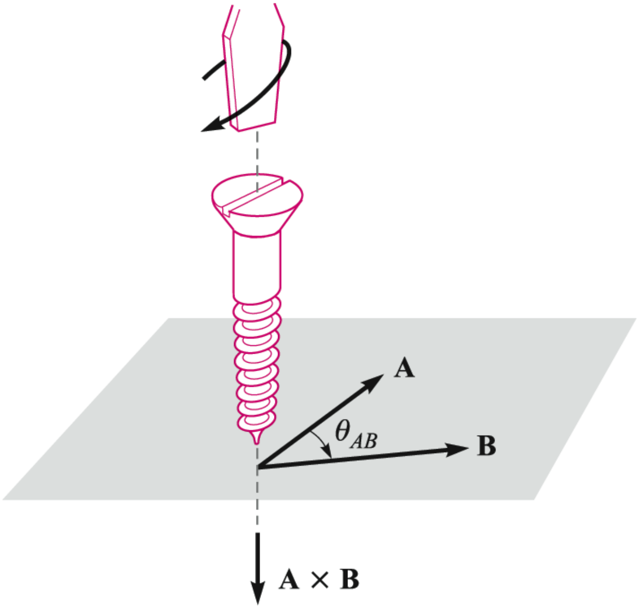
Consider a vector field $\mathbf{G} = y\mathbf{a}_x - 2.5x\mathbf{a}_y + 3\mathbf{a}_z$ and a point $Q(4,5,2)$

1. Find G at Q
2. Scalar component of G at Q in the direction of $\mathbf{a}_N = \frac{1}{3}\left(2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z\right)$
3. Vector component of G at Q in the direction of $\mathbf{a}_\mathbf{N}$

Cross Product

The cross product $\mathbf{A} \times \mathbf{B}$ is a vector; the magnitude of $\mathbf{A} \times \mathbf{B}$ is equal to the product of the magnitudes of $\mathbf{A}$, $\mathbf{B}$, and the sine of the smaller angle between $\mathbf{A}$ and $\mathbf{B}$; the direction of $\mathbf{A} \times \mathbf{B}$ is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$ and is along that one of the two possible perpendiculars which is in the direction of advance of a right-handed screw as $\mathbf{A}$ is turned into $\mathbf{B}$.

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$



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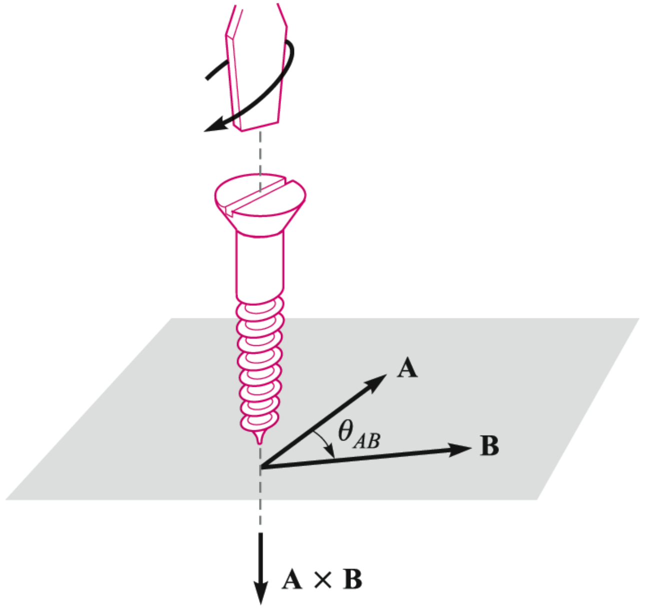
$$\boxed{\underbrace{\mathbf{A} \times \mathbf{B}}_{\text{Vector}} = \underbrace{\mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}_{\text{Vector}}}$$



Cross Product = Vector Product = Outer Product

$$\mathbf{B} \times \mathbf{A} =$$

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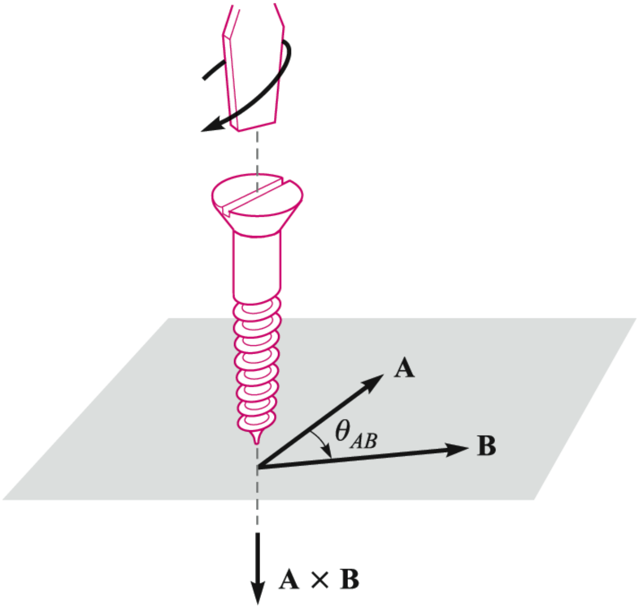
Vector



Cross Product = Vector Product = Outer Product

$$\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$$

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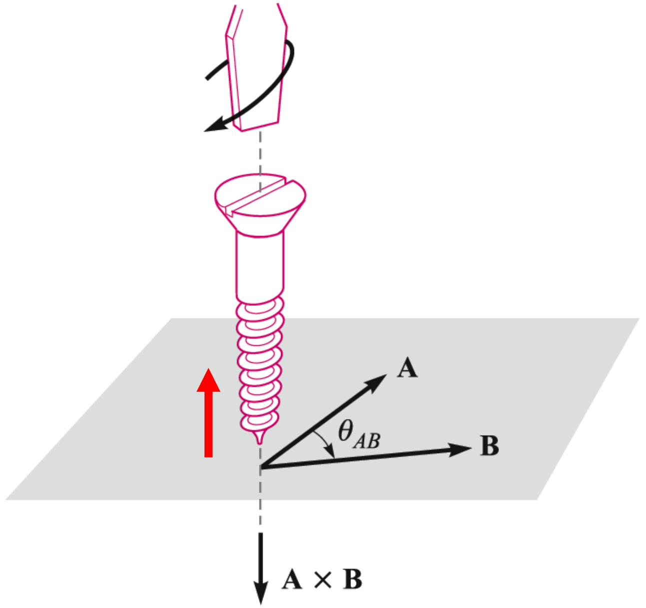
Vector Vector



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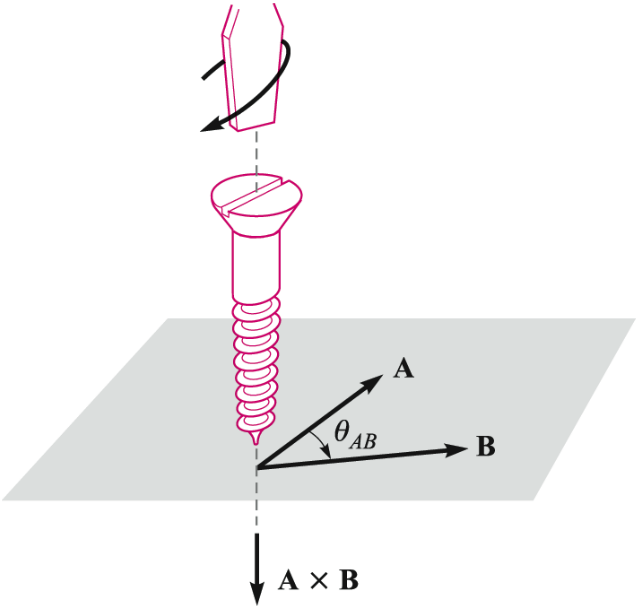

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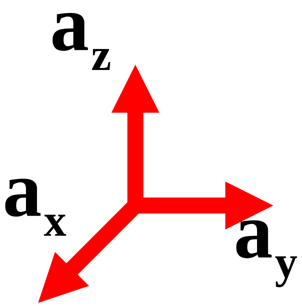

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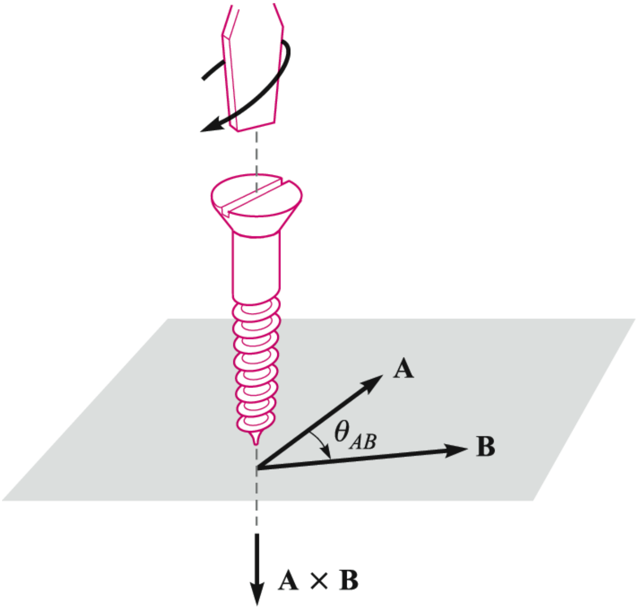

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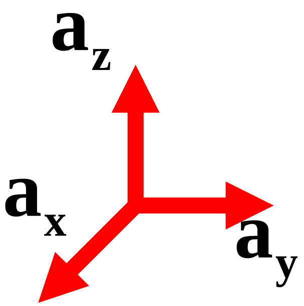
$$\mathbf{a}_x \times \mathbf{a}_x =$$



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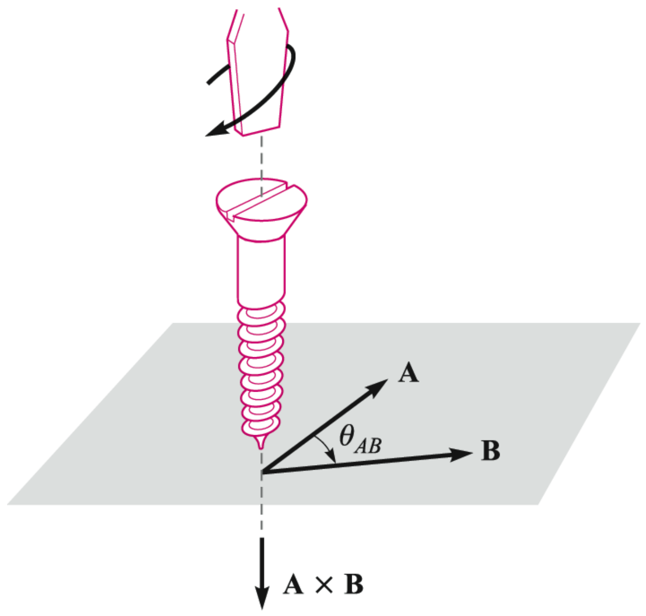
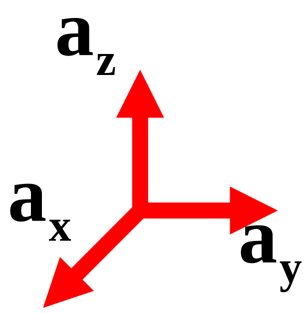
$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$



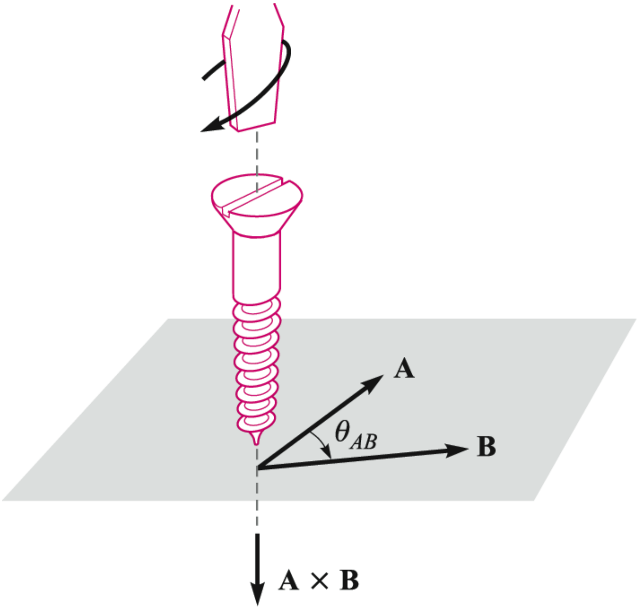
$$\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$$

$$\mathbf{a_x} \times \mathbf{a_x} = 0$$



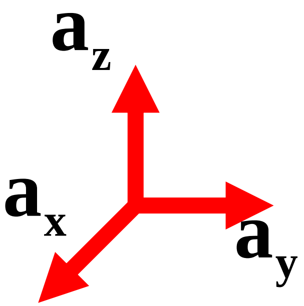
Cross Product = Vector Product = Outer Product

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$



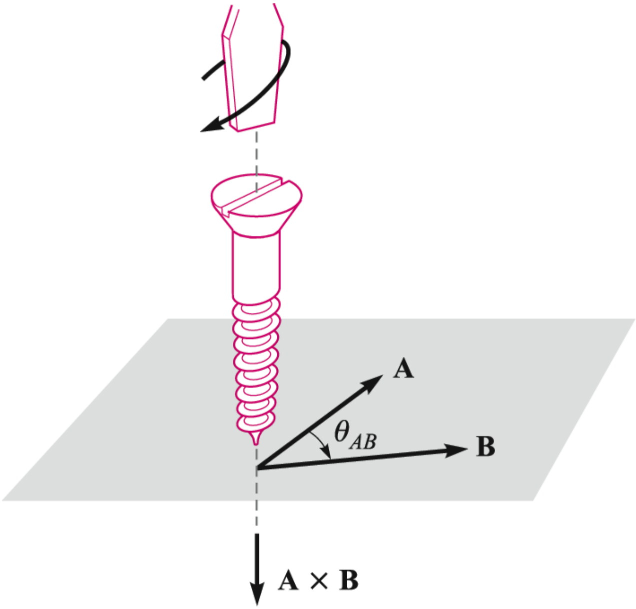
$$\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$$

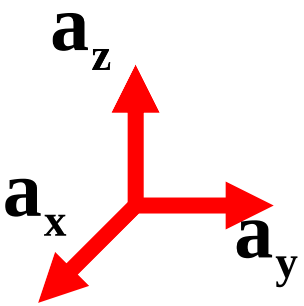
$$\left\{ \begin{aligned} \mathbf{a}_x \times \mathbf{a}_x &= 0 \\ \mathbf{a}_y \times \mathbf{a}_y &= 0 \\ \mathbf{a}_z \times \mathbf{a}_z &= 0 \end{aligned} \right.$$



Cross Product = Vector Product = Outer Product

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$





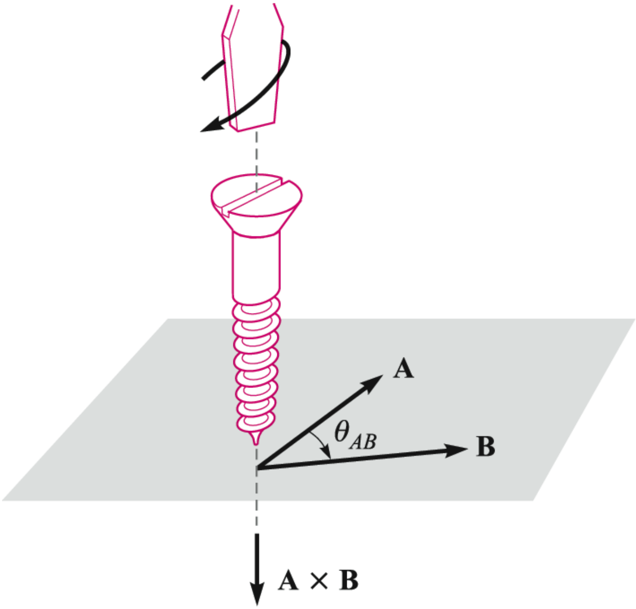
$$\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$$

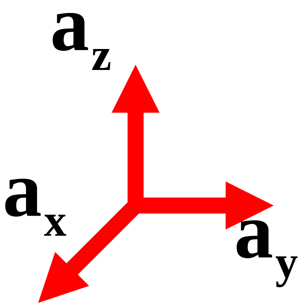
$$\left\{ \begin{aligned} \mathbf{a}_x \times \mathbf{a}_x &= 0 \\ \mathbf{a}_y \times \mathbf{a}_y &= 0 \\ \mathbf{a}_z \times \mathbf{a}_z &= 0 \end{aligned} \right.$$

$$\mathbf{a}_x \times \mathbf{a}_y =$$

Cross Product = Vector Product = Outer Product

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$





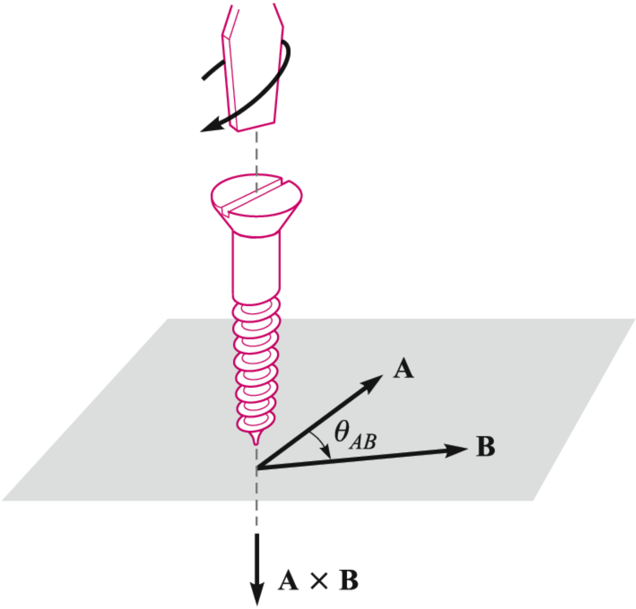
$$\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$$

$$\left\{ \begin{aligned} \mathbf{a}_x \times \mathbf{a}_x &= 0 \\ \mathbf{a}_y \times \mathbf{a}_y &= 0 \\ \mathbf{a}_z \times \mathbf{a}_z &= 0 \end{aligned} \right.$$

$$\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

Cross Product = Vector Product = Outer Product

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$

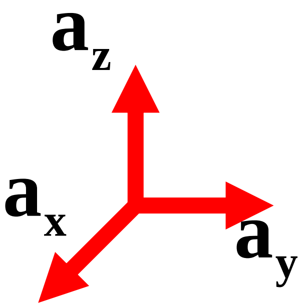


$$\mathbf{B}\times\mathbf{A} = -\mathbf{A}\times\mathbf{B}$$

$$\left.\begin{array}{l} \mathbf{a_x}\times\mathbf{a_x} = 0 \\ \mathbf{a_y}\times\mathbf{a_y} = 0 \\ \mathbf{a_z}\times\mathbf{a_z} = 0 \end{array}\right.$$

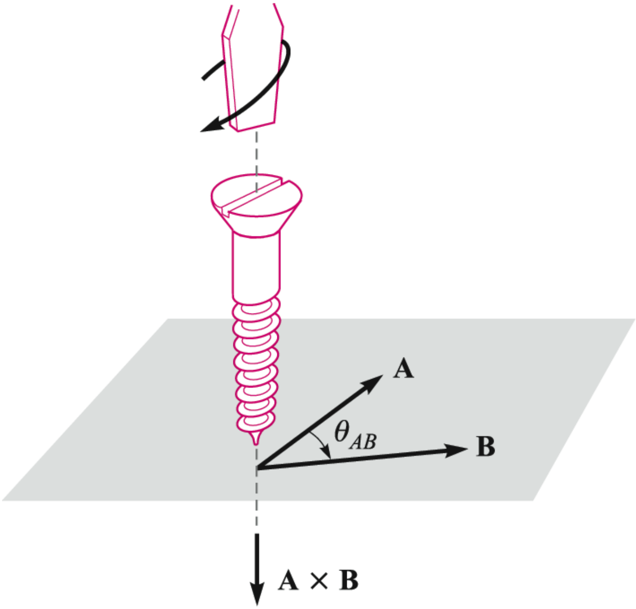
$$\mathbf{a_x}\times\mathbf{a_y} = \mathbf{a_z}$$

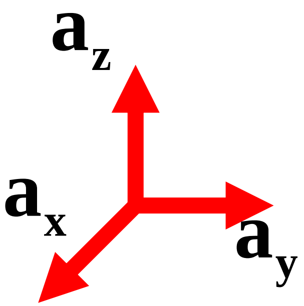
$$\mathbf{a_y}\times\mathbf{a_z} = \mathbf{a_x}$$



Cross Product = Vector Product = Outer Product

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\, |\mathbf{B}| \sin \theta_{AB}}$$





$$\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$$

$$\left\{ \begin{aligned} \mathbf{a}_x \times \mathbf{a}_x &= 0 \\ \mathbf{a}_y \times \mathbf{a}_y &= 0 \\ \mathbf{a}_z \times \mathbf{a}_z &= 0 \end{aligned} \right.$$

$$\left\{ \begin{aligned} \mathbf{a}_x \times \mathbf{a}_y &= \mathbf{a}_z \\ \mathbf{a}_y \times \mathbf{a}_z &= \mathbf{a}_x \\ \mathbf{a}_z \times \mathbf{a}_x &= \mathbf{a}_y \end{aligned} \right.$$

Cross Product = Vector Product = Outer Product

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$

$$\mathbf{B}\times\mathbf{A} = -\mathbf{A}\times\mathbf{B}$$

$$\mathbf{A} = A_x\mathbf{a_x} + A_y\mathbf{a_y} + A_z\mathbf{a_z}$$

$$\mathbf{B} = B_x\mathbf{a_x} + B_y\mathbf{a_y} + B_z\mathbf{a_z}$$

$$\left\{\begin{aligned} \mathbf{a_x}\times\mathbf{a_x} &= 0 \\ \mathbf{a_y}\times\mathbf{a_y} &= 0 \\ \mathbf{a_z}\times\mathbf{a_z} &= 0 \end{aligned}\right.$$

$$\mathbf{A}\times\mathbf{B} = \left(A_x\mathbf{a_x} + A_y\mathbf{a_y} + A_z\mathbf{a_z}\right)\times\left(B_x\mathbf{a_x} + B_y\mathbf{a_y} + B_z\mathbf{a_z}\right)$$

$$\left\{\begin{aligned} \mathbf{a_x}\times\mathbf{a_y} &= \mathbf{a_z} \\ \mathbf{a_y}\times\mathbf{a_z} &= \mathbf{a_x} \\ \mathbf{a_z}\times\mathbf{a_x} &= \mathbf{a_y} \end{aligned}\right.$$

Cross Product = Vector Product = Outer Product

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$

$$\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$$

$$\mathbf{A} = A_x\mathbf{a_x} + A_y\mathbf{a_y} + A_z\mathbf{a_z}$$

$$\mathbf{B} = B_x\mathbf{a_x} + B_y\mathbf{a_y} + B_z\mathbf{a_z}$$

$$\left\{\begin{aligned} \mathbf{a_x} \times \mathbf{a_x} &= 0 \\ \mathbf{a_y} \times \mathbf{a_y} &= 0 \\ \mathbf{a_z} \times \mathbf{a_z} &= 0 \end{aligned}\right.$$

$$\begin{aligned} \mathbf{A} \times \mathbf{B} &= \left(A_x\mathbf{a_x} + A_y\mathbf{a_y} + A_z\mathbf{a_z}\right) \times \left(B_x\mathbf{a_x} + B_y\mathbf{a_y} + B_z\mathbf{a_z}\right) \\ &= A_xB_y\mathbf{a_z} - A_xB_z\mathbf{a_y} + \ldots \end{aligned}$$

$$\left\{\begin{aligned} \mathbf{a_x} \times \mathbf{a_y} &= \mathbf{a_z} \\ \mathbf{a_y} \times \mathbf{a_z} &= \mathbf{a_x} \\ \mathbf{a_z} \times \mathbf{a_x} &= \mathbf{a_y} \end{aligned}\right.$$

Cross Product = Vector Product = Outer Product

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$

$$\mathbf{A} = A_x\mathbf{a_x} + A_y\mathbf{a_y} + A_z\mathbf{a_z}$$

$$\mathbf{B} = B_x\mathbf{a_x} + B_y\mathbf{a_y} + B_z\mathbf{a_z}$$

$$\begin{aligned}\mathbf{A}\times\mathbf{B} &= \left(A_x\mathbf{a_x} + A_y\mathbf{a_y} + A_z\mathbf{a_z}\right)\times\left(B_x\mathbf{a_x} + B_y\mathbf{a_y} + B_z\mathbf{a_z}\right)\\ &= \left(A_yB_z - A_zB_y\right)\mathbf{a_x} + \left(A_zB_x - A_xB_z\right)\mathbf{a_y} + \left(A_xB_y - A_yB_x\right)\mathbf{a_z}\end{aligned}$$

Cross Product = Vector Product = Outer Product

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$

$$\boxed{\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}}$$

$$\mathbf{A} = A_x\mathbf{a}_\mathbf{x} + A_y\mathbf{a}_\mathbf{y} + A_z\mathbf{a}_\mathbf{z}$$

$$\mathbf{B} = B_x\mathbf{a}_\mathbf{x} + B_y\mathbf{a}_\mathbf{y} + B_z\mathbf{a}_\mathbf{z}$$

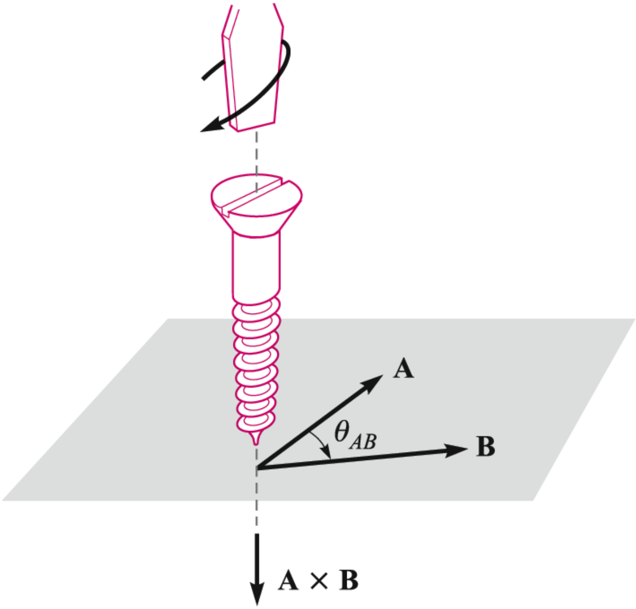
$$\begin{aligned}\mathbf{A}\times\mathbf{B} &= \left(A_x\mathbf{a}_\mathbf{x} + A_y\mathbf{a}_\mathbf{y} + A_z\mathbf{a}_\mathbf{z}\right)\times\left(B_x\mathbf{a}_\mathbf{x} + B_y\mathbf{a}_\mathbf{y} + B_z\mathbf{a}_\mathbf{z}\right) \\ &= \left(A_yB_z - A_zB_y\right)\mathbf{a}_\mathbf{x} + \left(A_zB_x - A_xB_z\right)\mathbf{a}_\mathbf{y} + \left(A_xB_y - A_yB_x\right)\mathbf{a}_\mathbf{z}\end{aligned}$$

Cross Product = Vector Product = Outer Product

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\, |\mathbf{B}| \sin\theta_{AB}}$$

Vector

Vector



$$\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$$

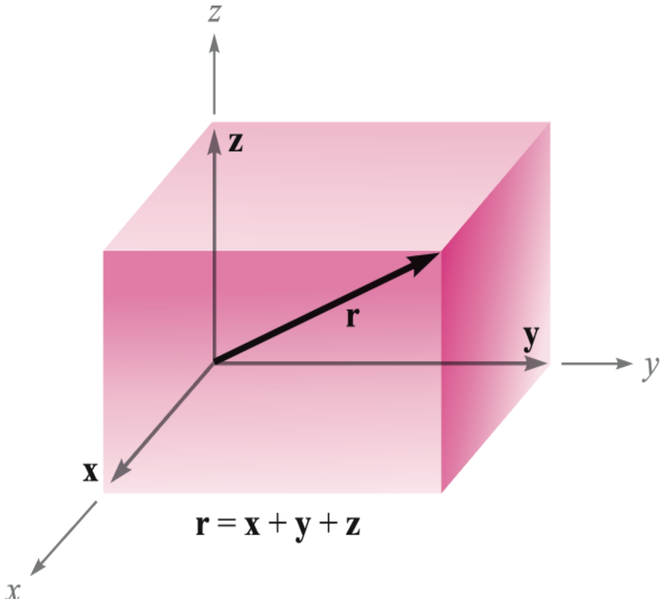
$$\begin{cases} \mathbf{a_x} \times \mathbf{a_x} = 0 \\ \mathbf{a_y} \times \mathbf{a_y} = 0 \\ \mathbf{a_z} \times \mathbf{a_z} = 0 \end{cases}$$

$$\begin{cases} \mathbf{a_x} \times \mathbf{a_y} = \mathbf{a_z} \\ \mathbf{a_y} \times \mathbf{a_z} = \mathbf{a_x} \\ \mathbf{a_z} \times \mathbf{a_x} = \mathbf{a_y} \end{cases}$$

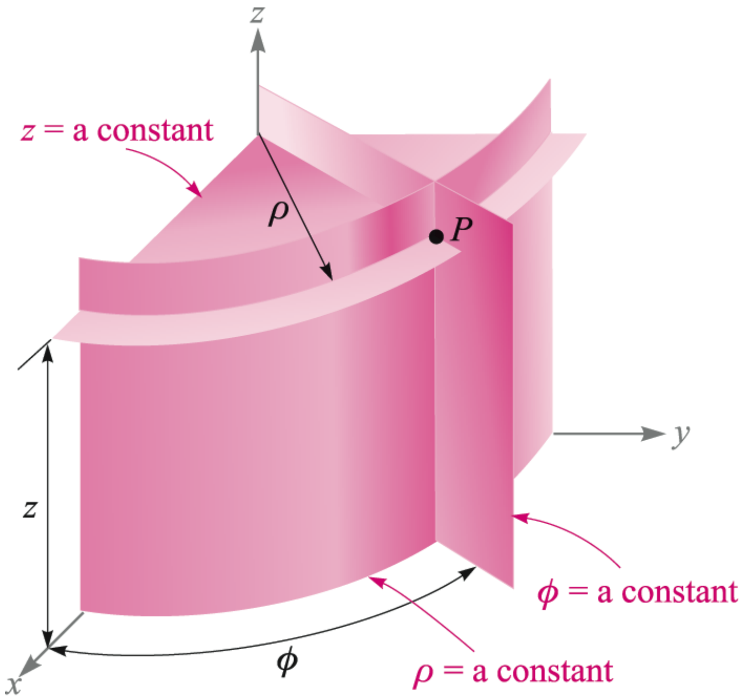
$$\boxed{\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}}$$

Coordinate System

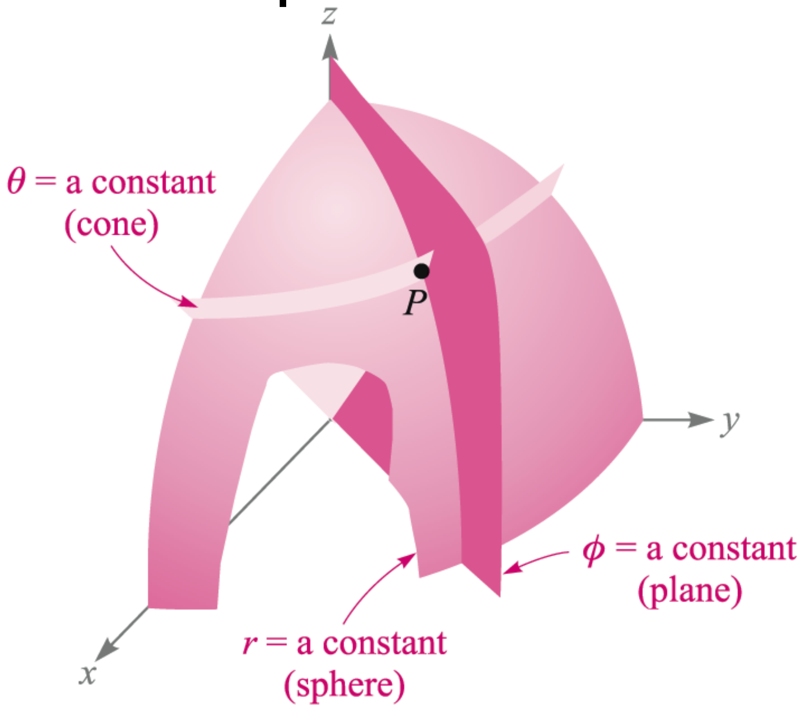
Rectangular



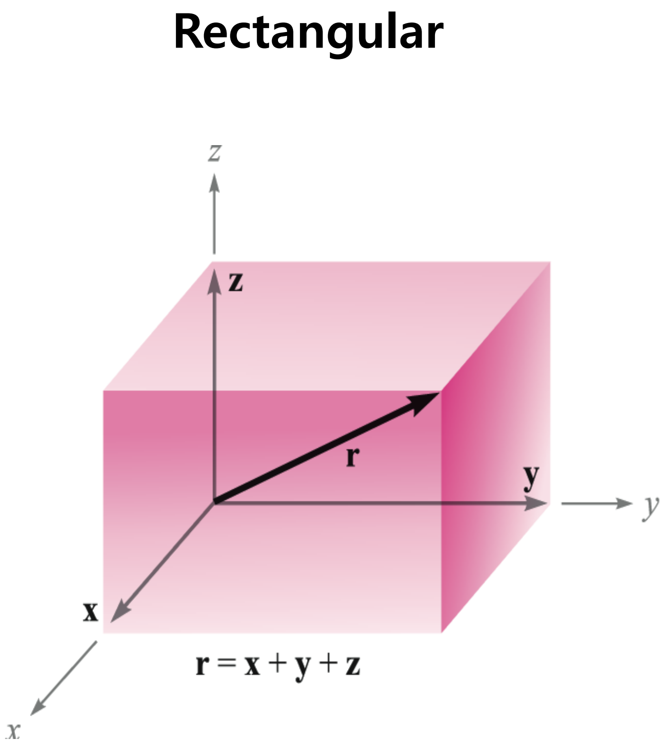
Cylindrical

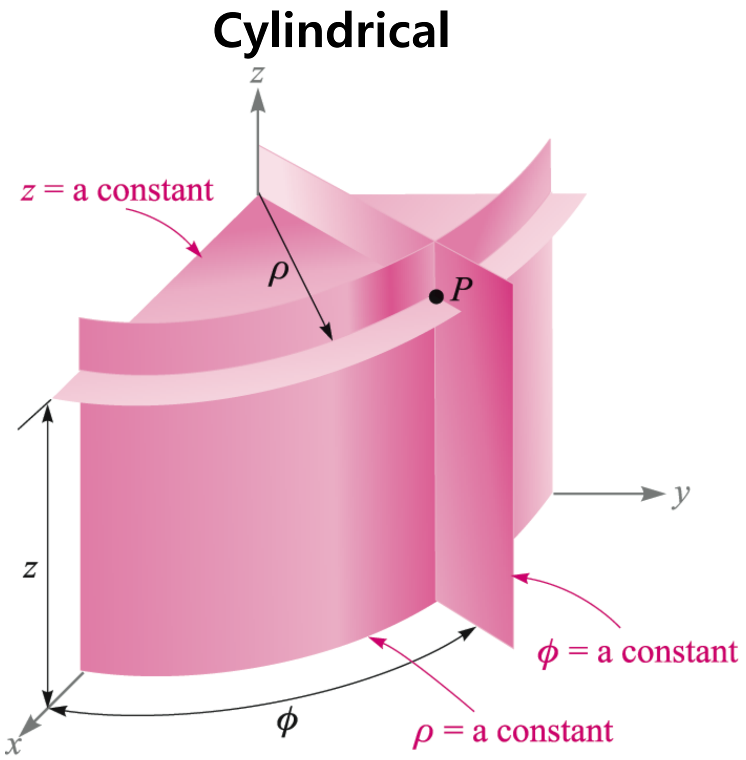


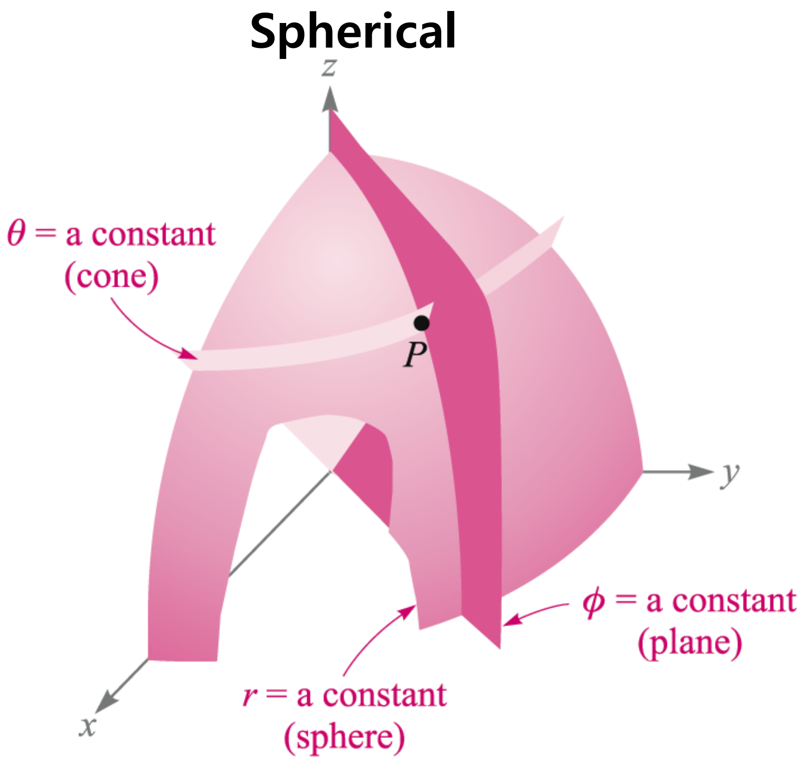
Spherical



Coordinate System

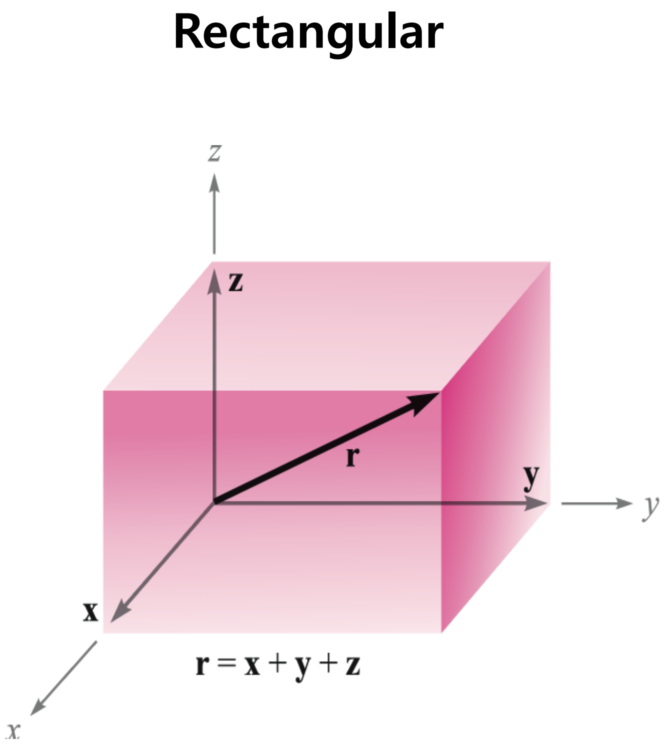


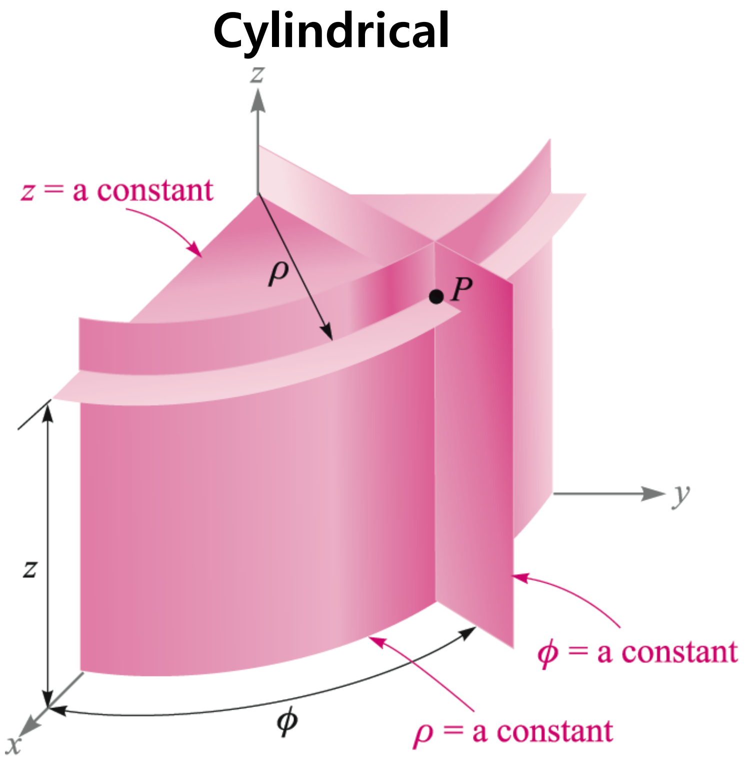


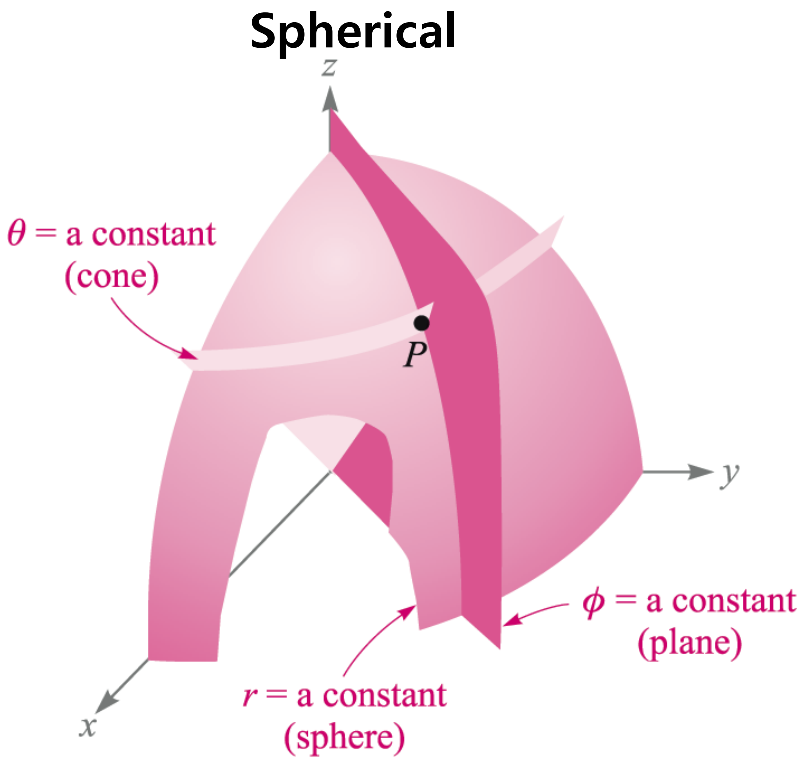


- 3차원 공간상에서의 위치를 각각의 좌표계로 표시하고 서로 변환하기

Coordinate System

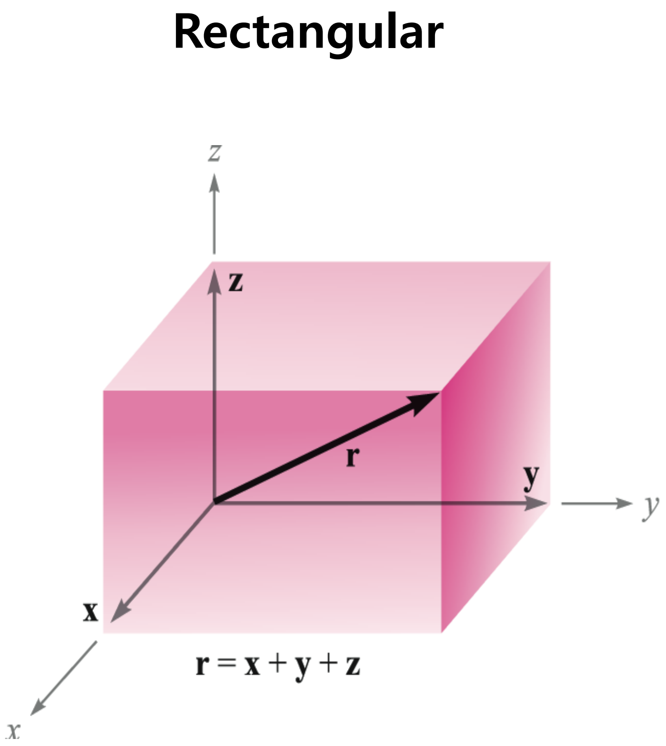


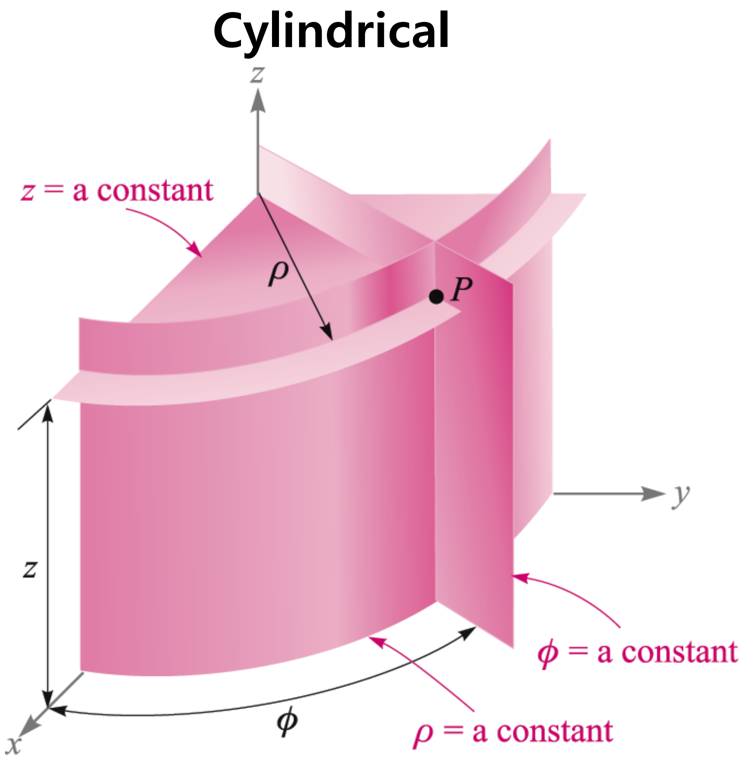


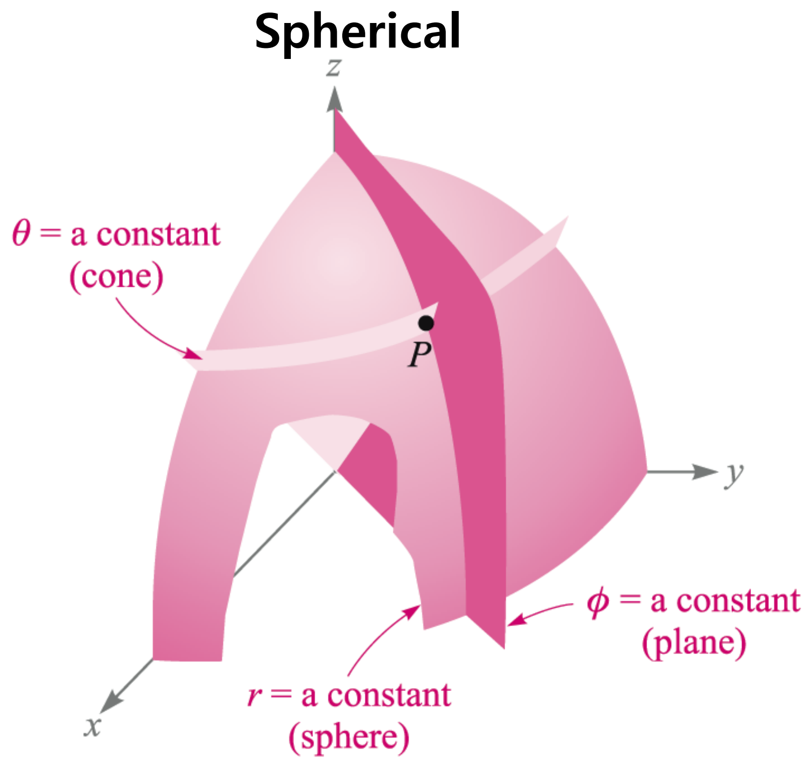


- 3차원 공간상에서의 위치를 각각의 좌표계로 표시하고 서로 변환하기
- Vector Field를 각각의 좌표계로 표시하고 서로 변환하기

Coordinate System



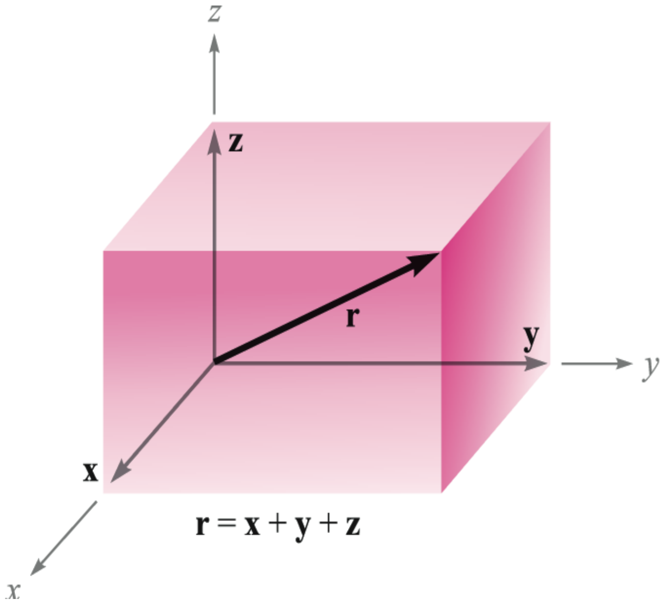




- 3차원 공간상에서의 위치를 각각의 좌표계로 표시하고 서로 변환하기
- Vector Field를 각각의 좌표계로 표시하고 서로 변환하기
- 미소 길이, 미소 면적, 미소 부피를 각각의 좌표계로 표시하고 계산하기

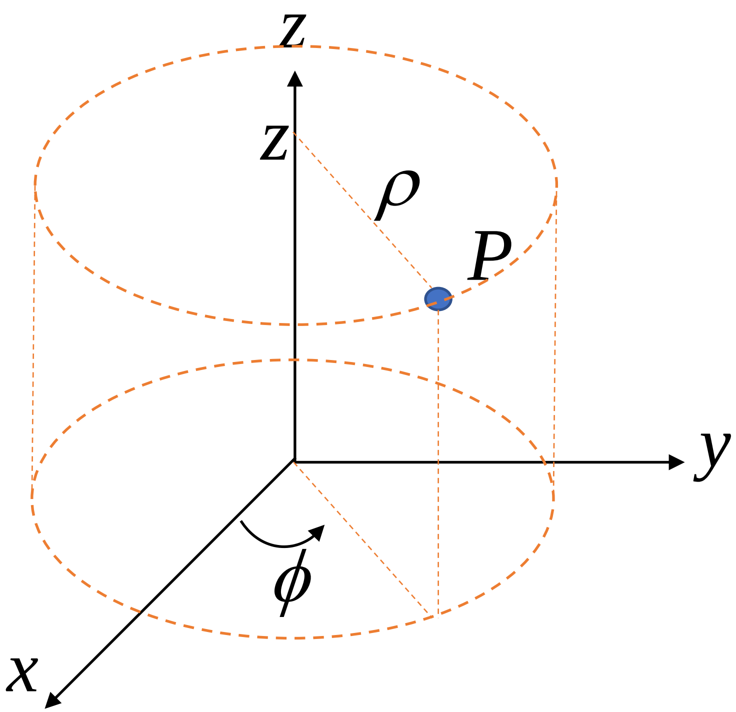
Coordinate System

Rectangular



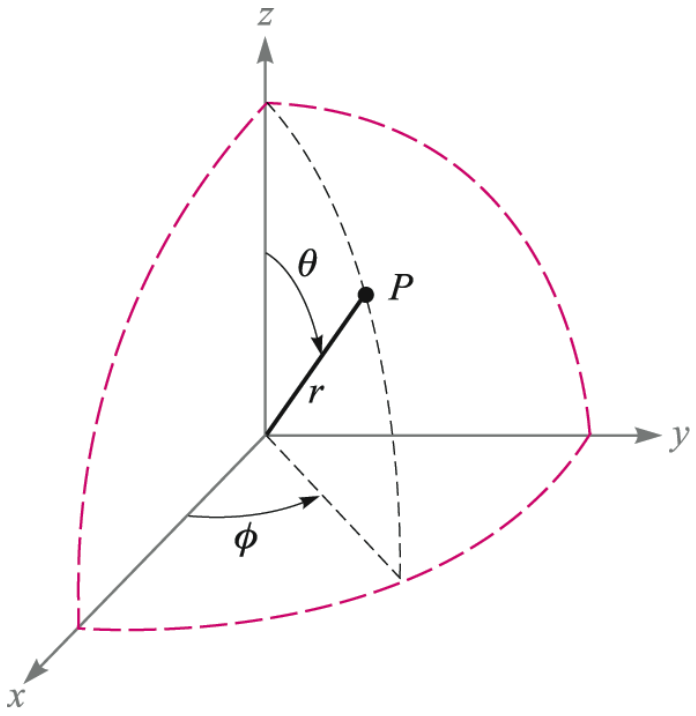
$P(x, y, z)$

Cylindrical



$P(\rho, \phi, z)$

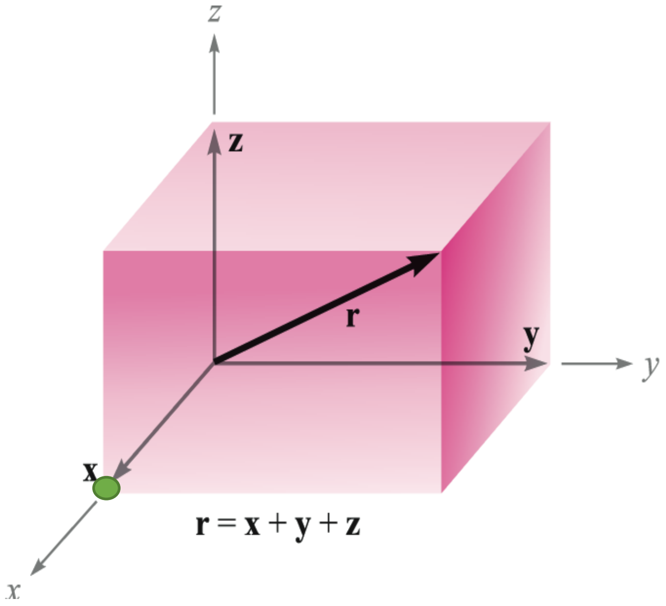
Spherical



$P(r, \theta, \phi)$

Coordinate System

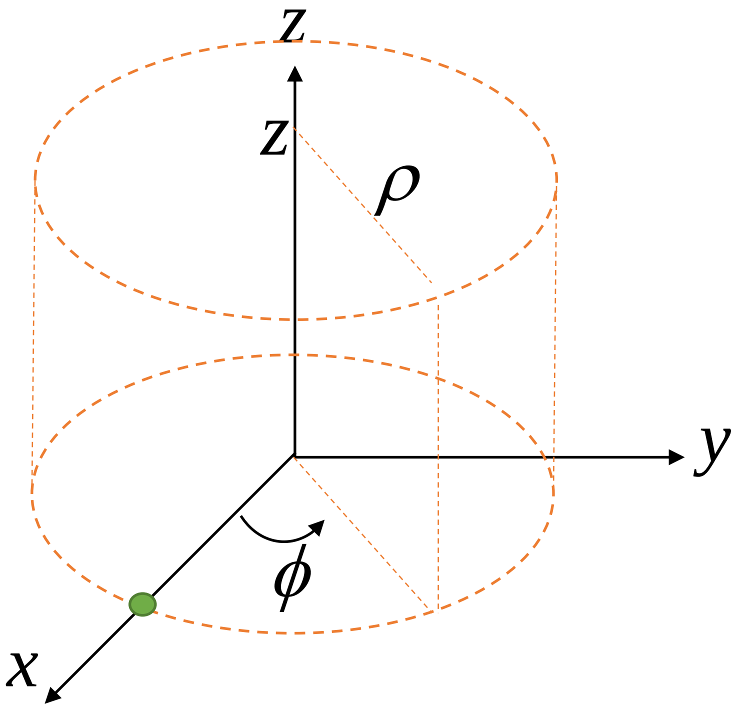
Rectangular



$P(x,y,z)$

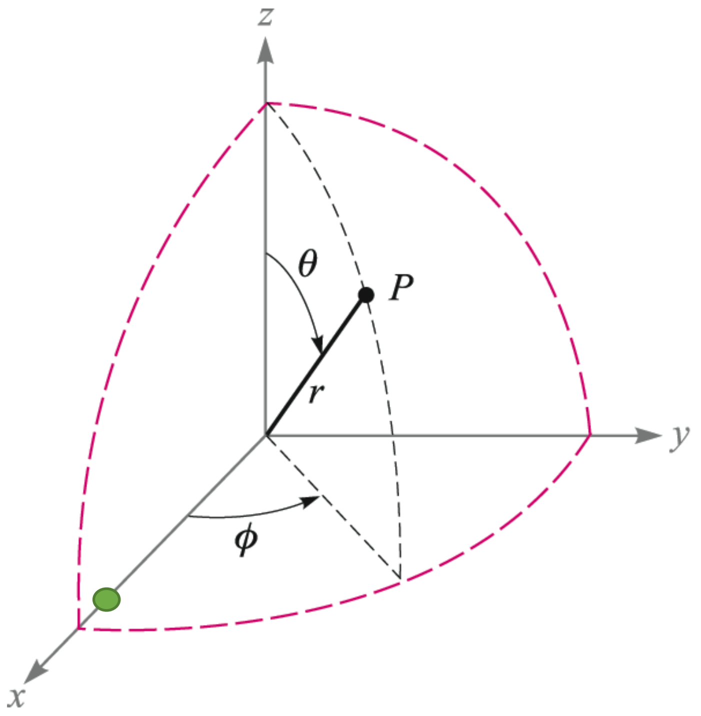
$(1,0,0)$

Cylindrical



$P(\rho,\phi,z)$

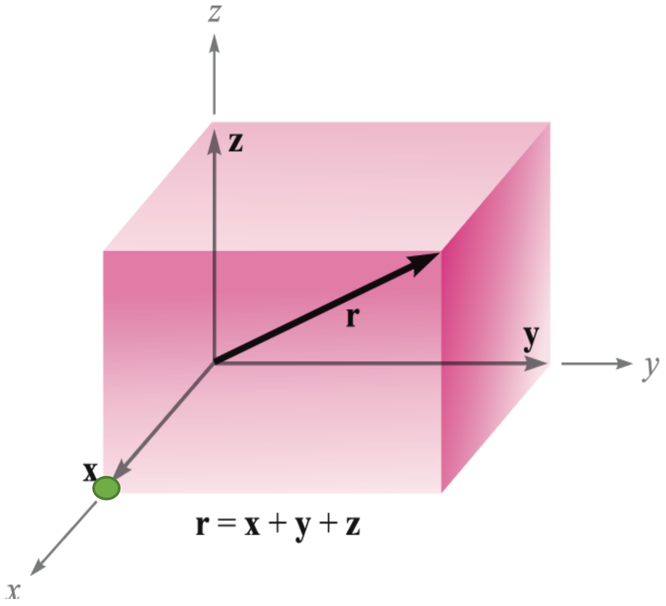
Spherical



$P(r,\theta,\phi)$

Coordinate System

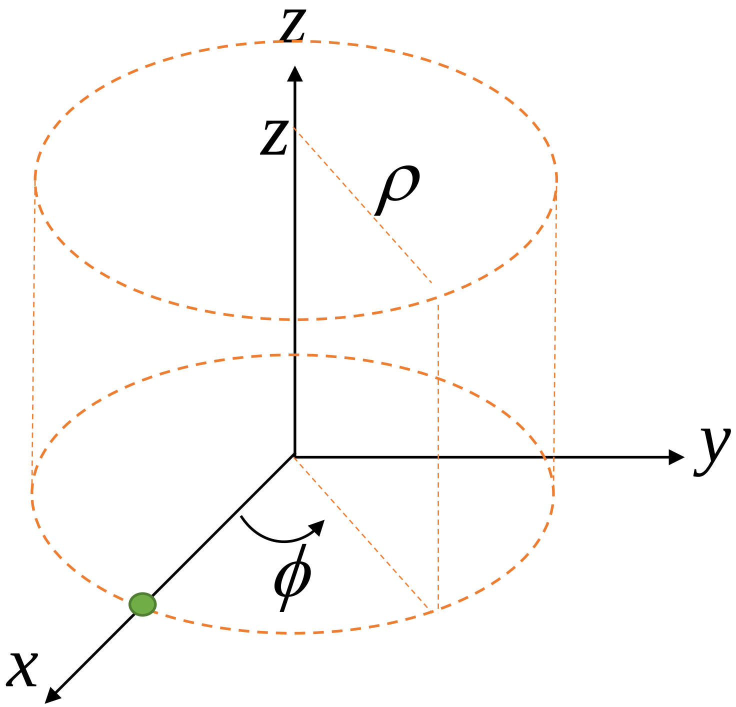
Rectangular



$P(x, y, z)$

$(1, 0, 0)$

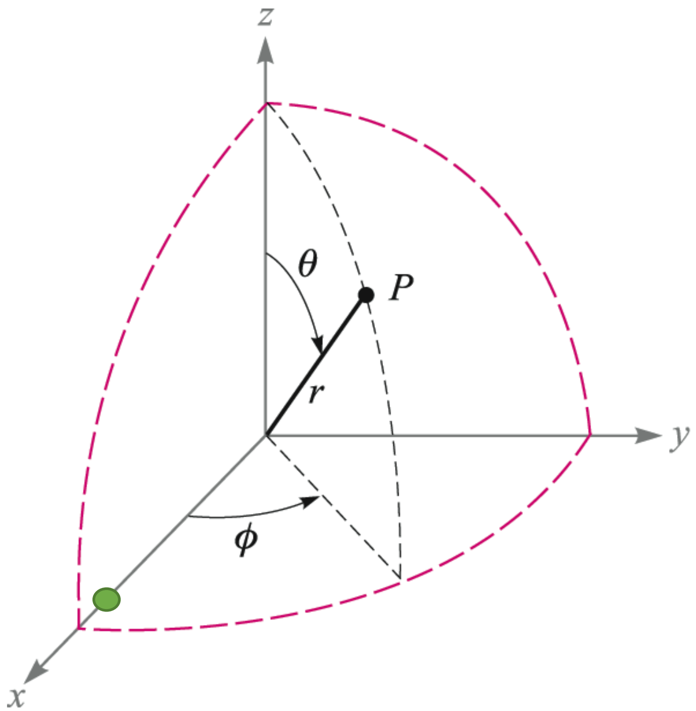
Cylindrical



$P(\rho, \phi, z)$

$(1, 0, 0)$

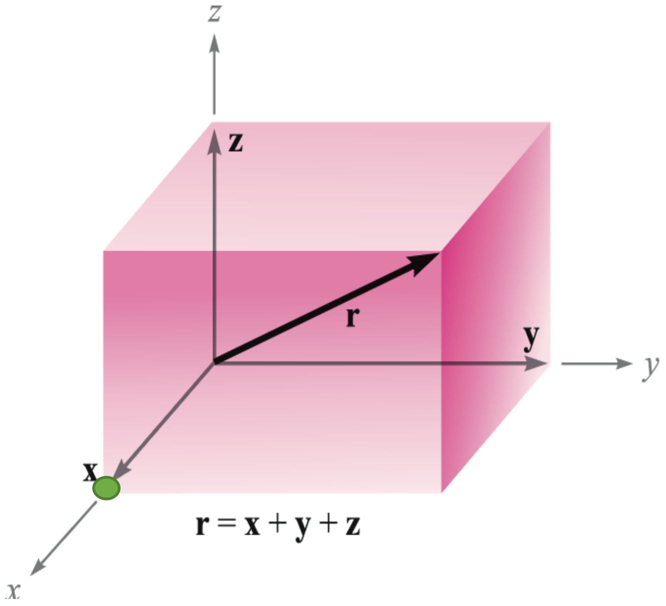
Spherical



$P(r, \theta, \phi)$

Coordinate System

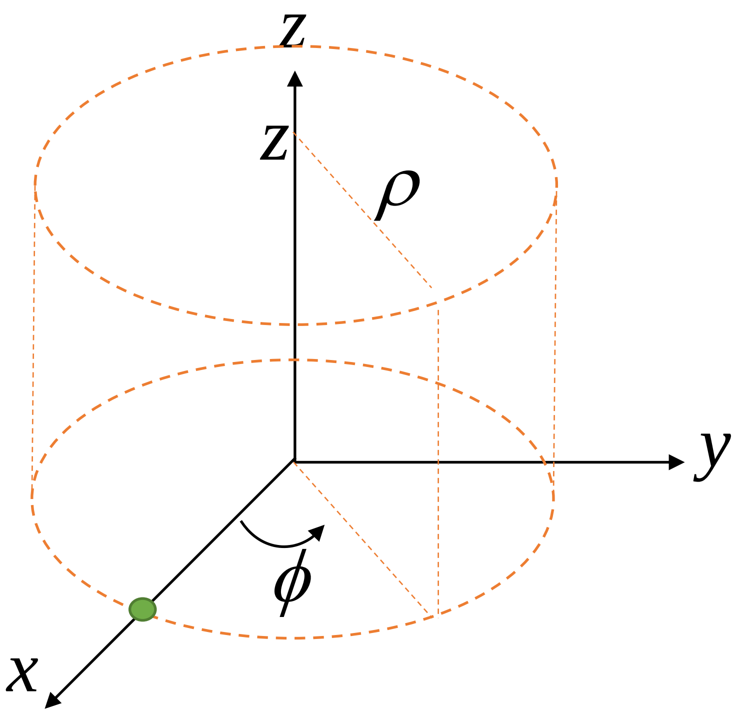
Rectangular



$$P(x, y, z)$$

$$(1, 0, 0)$$

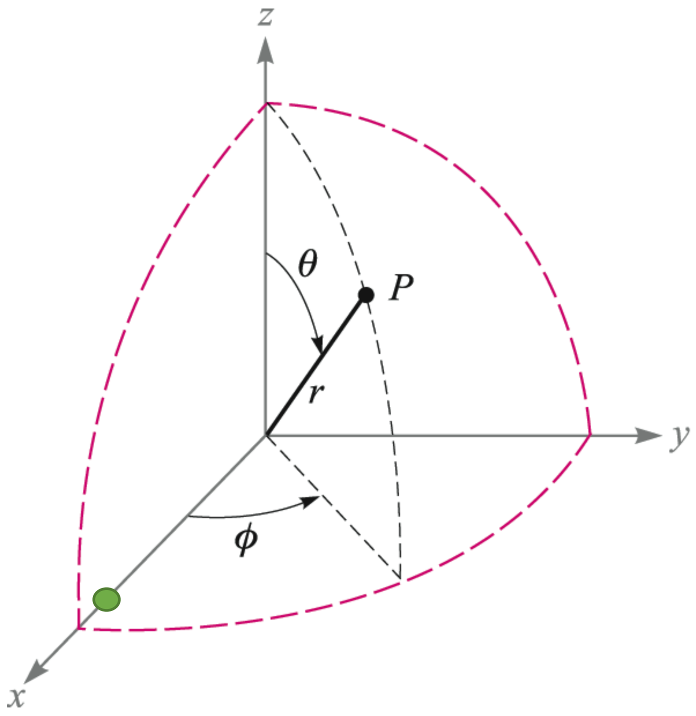
Cylindrical



$$P(\rho, \phi, z)$$

$$(1, 0, 0)$$

Spherical

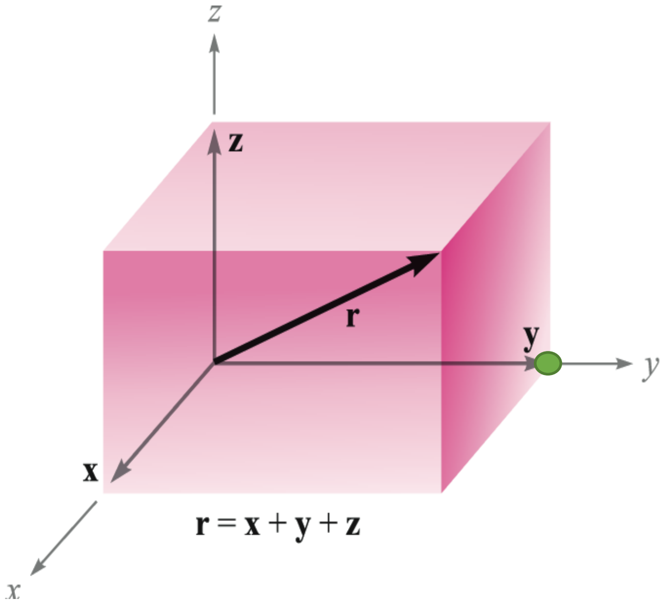


$$P(r, \theta, \phi)$$

$$\left(1, \frac{\pi}{2}, 0\right)$$

Coordinate System

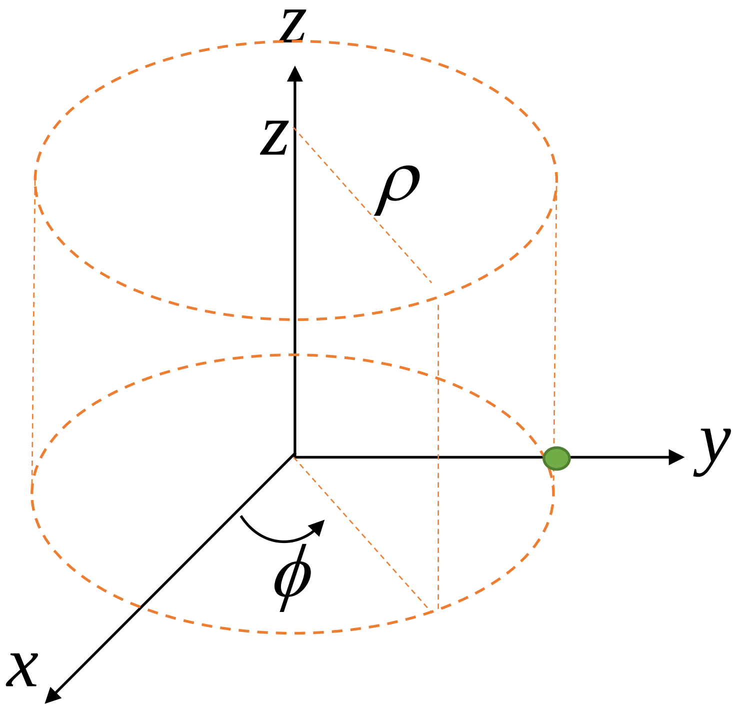
Rectangular



$P(x, y, z)$

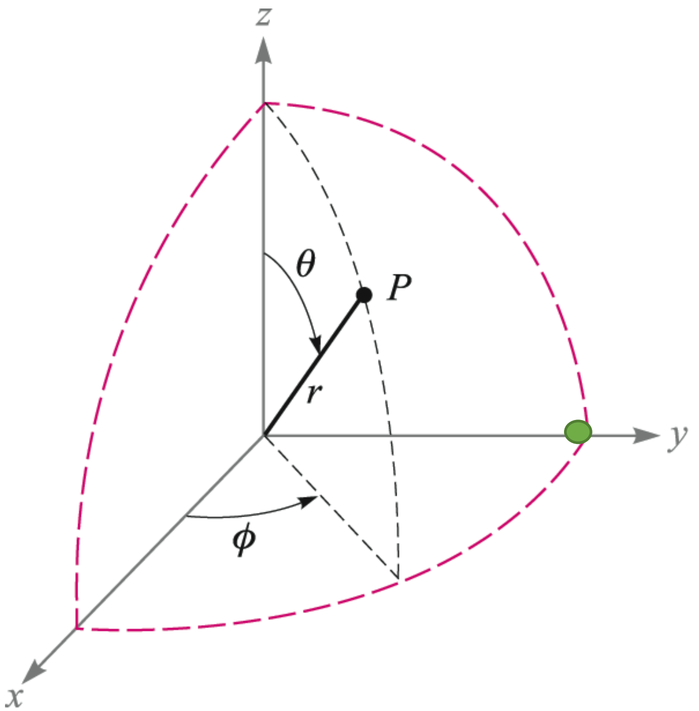
$(0,1,0)$

Cylindrical



$P(\rho, \phi, z)$

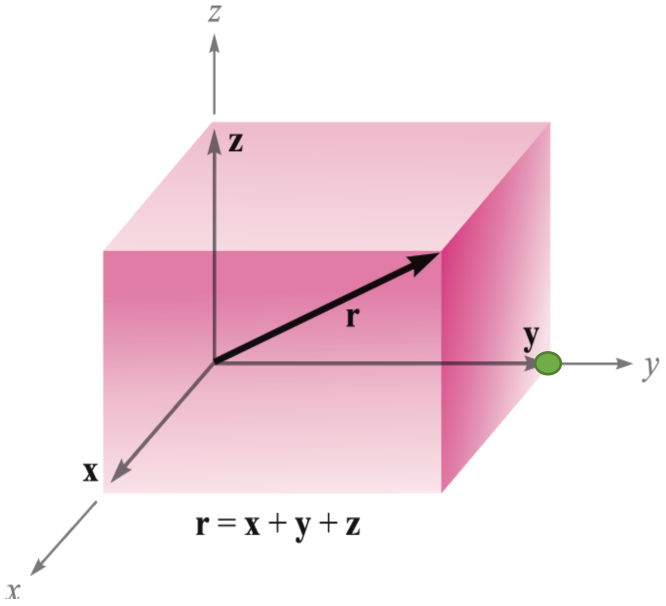
Spherical



$P(r, \theta, \phi)$

Coordinate System

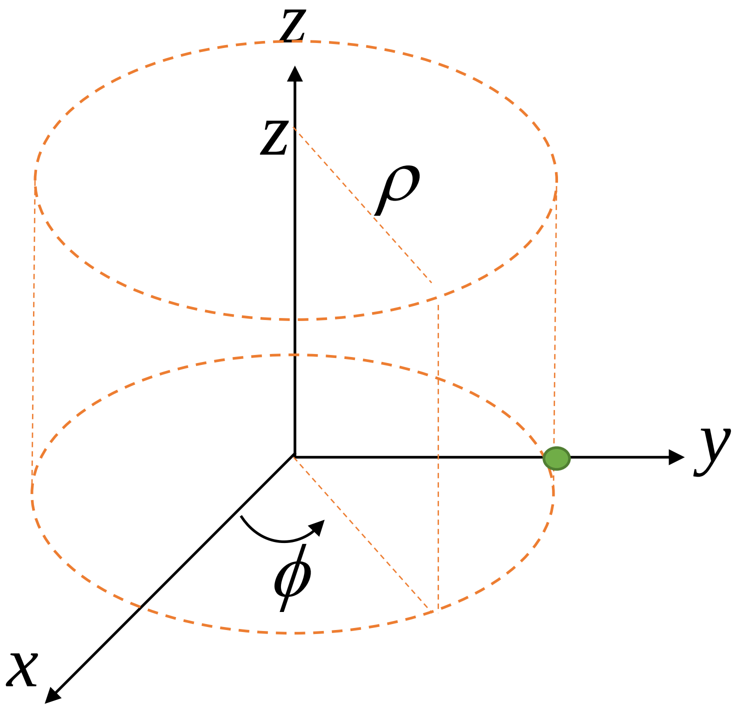
Rectangular



$P(x, y, z)$

$(0,1,0)$

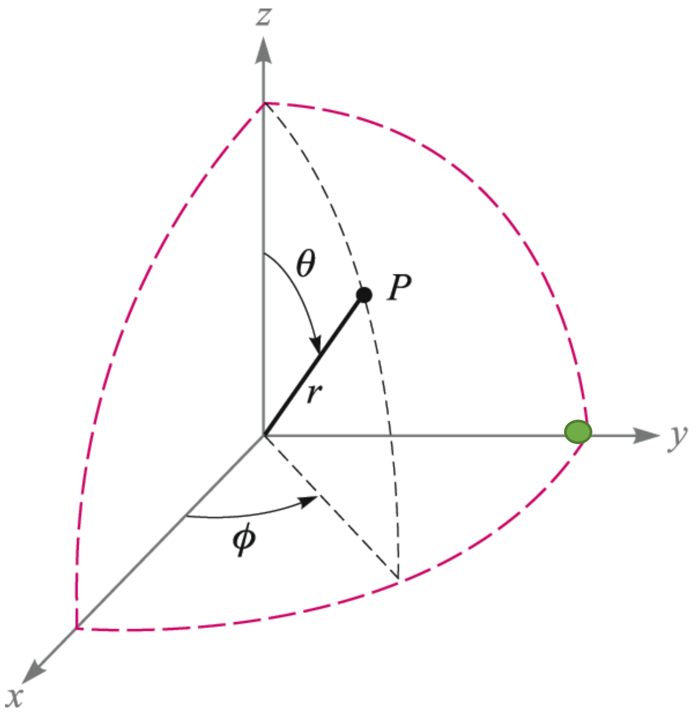
Cylindrical



$P(\rho, \phi, z)$

$\left(1, \frac{\pi}{2}, 0\right)$

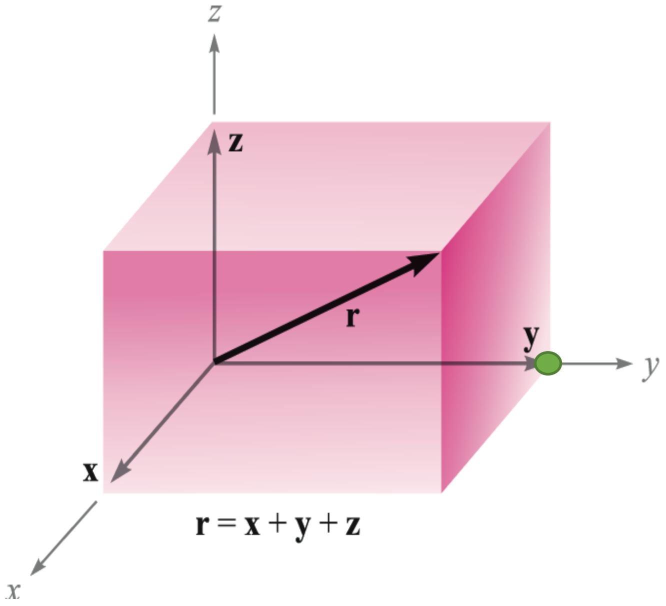
Spherical

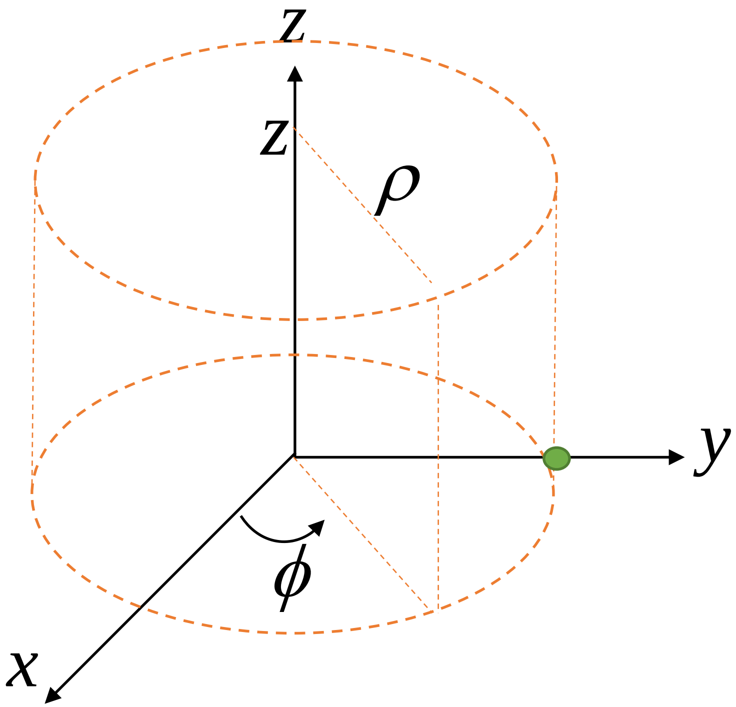


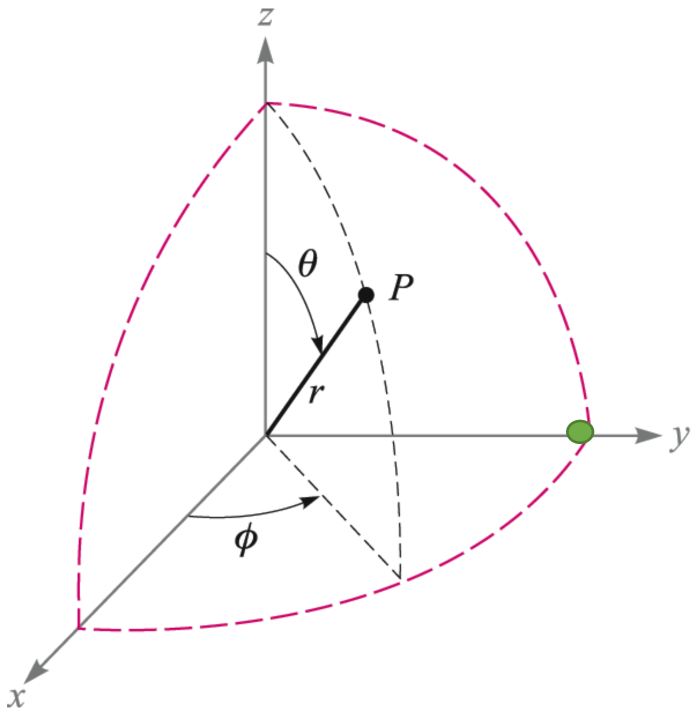
$P(r, \theta, \phi)$

Coordinate System

Rectangular	Cylindrical	Spherical





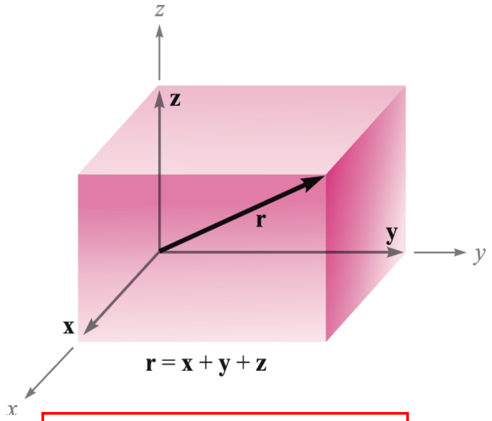


$P(x, y, z)$ $\quad$ $P(\rho, \phi, z)$ $\quad$ $P(r, \theta, \phi)$

$(0,1,0)$ $\quad$ $\left(1, \frac{\pi}{2}, 0\right)$ $\quad$ $\left(1, \frac{\pi}{2}, \frac{\pi}{2}\right)$

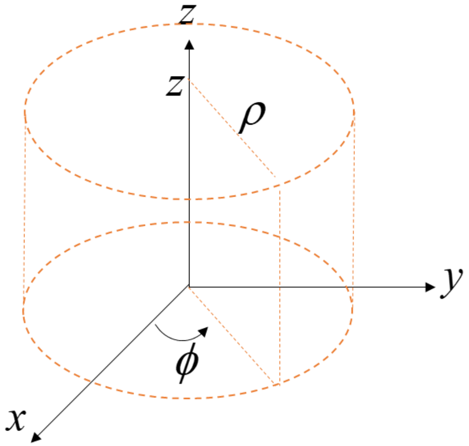
Coordinate System

Rectangular



$$\boxed{P(x, y, z)}$$

Cylindrical



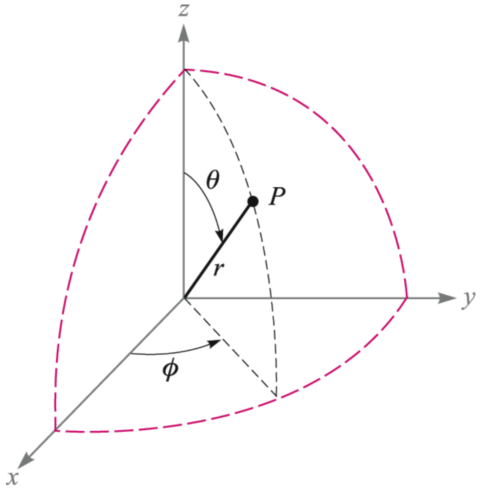
$$P(\rho, \phi, z)$$

$$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$

$$\phi = \tan^{-1} \frac{y}{x}$$

$$z = z$$

Spherical



$$P(r, \theta, \phi)$$

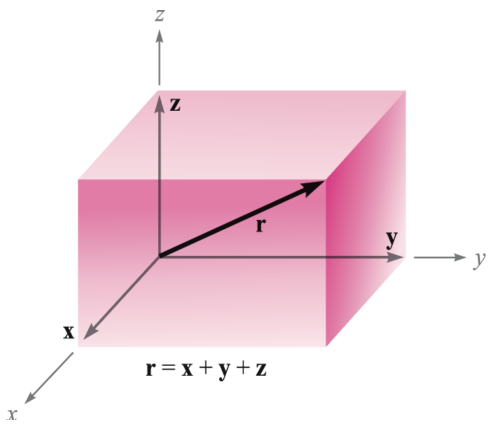
$$r = \sqrt{x^2 + y^2 + z^2} \qquad (r \geq 0)$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad (0^\circ \leq \theta \leq 180^\circ)$$

$$\phi = \tan^{-1} \frac{y}{x}$$

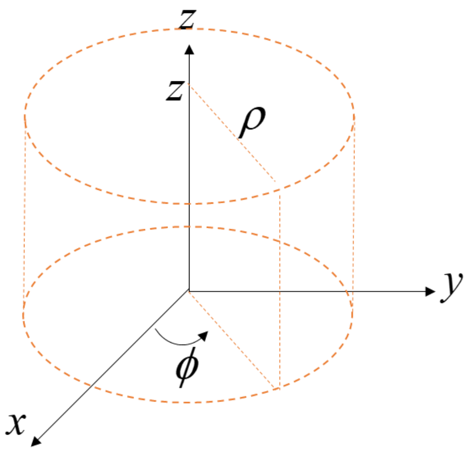
Coordinate System

Rectangular



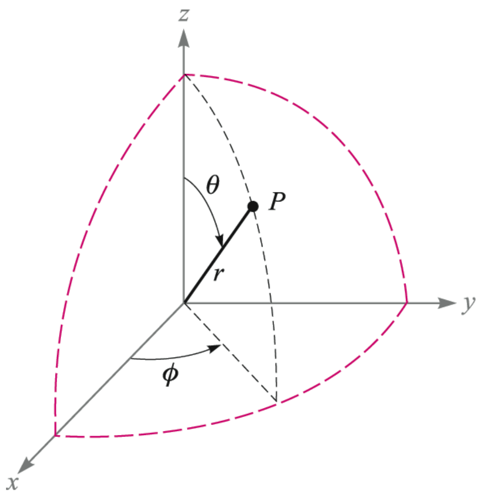
$$P(x, y, z)$$

Cylindrical



$$\boxed{P(\rho, \phi, z)}$$

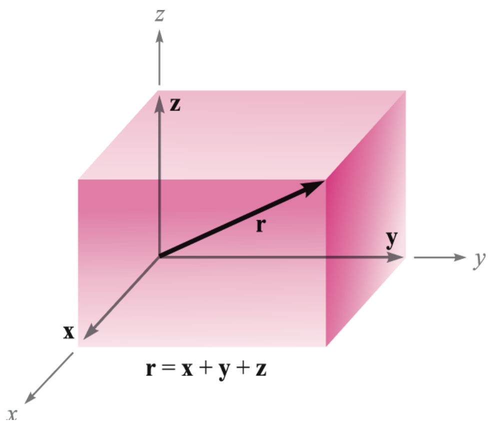
Spherical



$$P(r, \theta, \phi)$$

Coordinate System

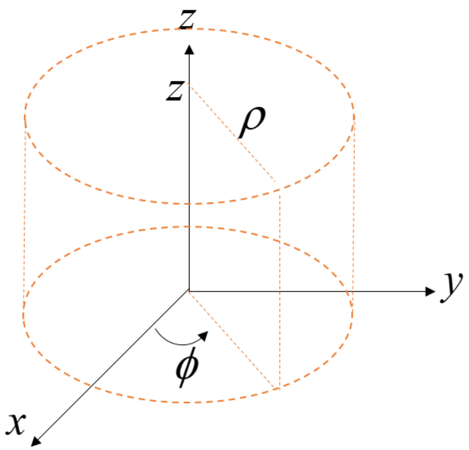
Rectangular



$$P(x, y, z)$$

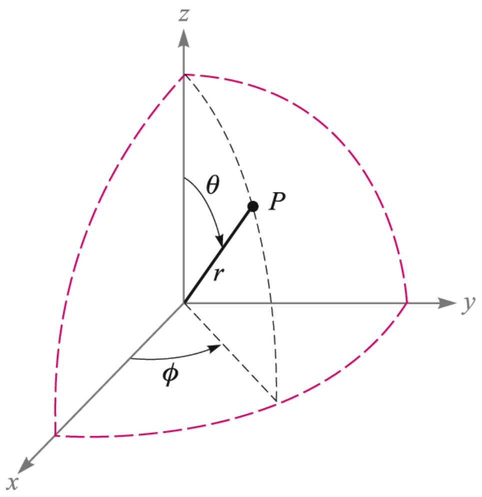
$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases}$$

Cylindrical



$$\boxed{P(\rho, \phi, z)}$$

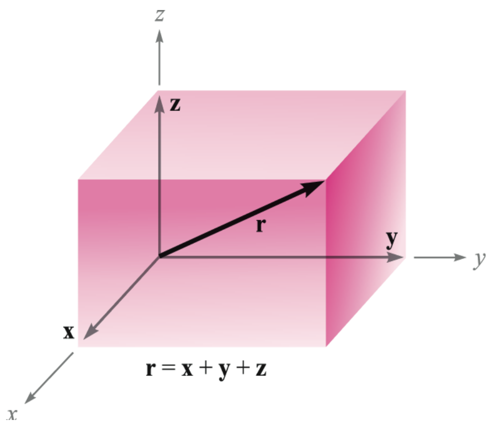
Spherical



$$P(r, \theta, \phi)$$

Coordinate System

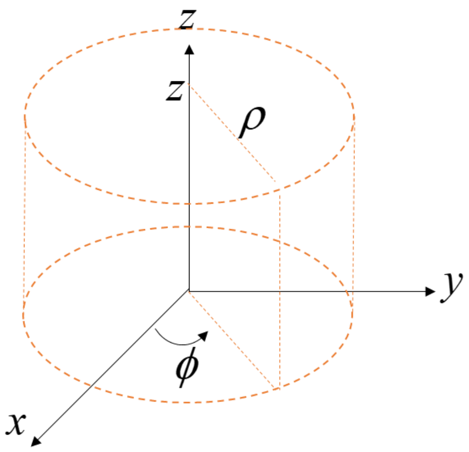
Rectangular



$$P(x, y, z)$$

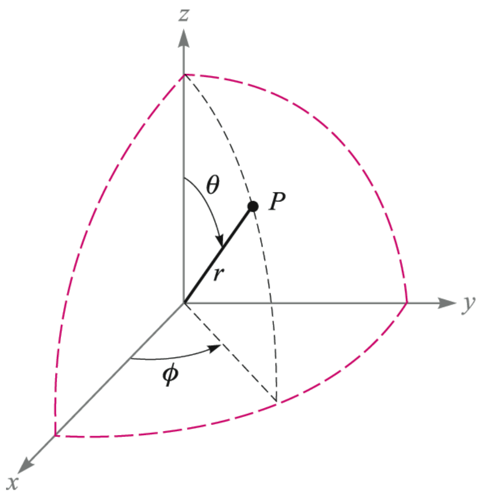
$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases}$$

Cylindrical



$$\boxed{P(\rho, \phi, z)}$$

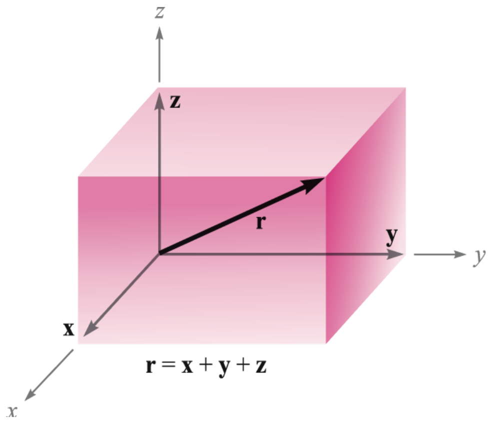
Spherical



$$P(r, \theta, \phi)$$

Coordinate System

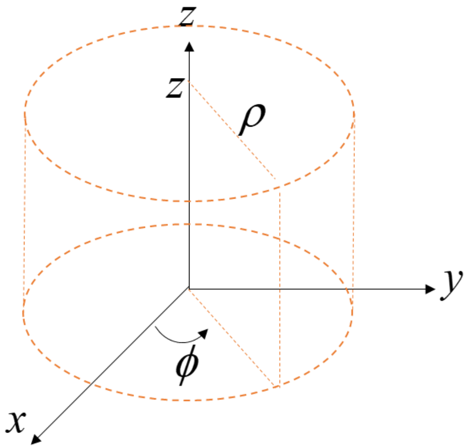
Rectangular



$$P(x, y, z)$$

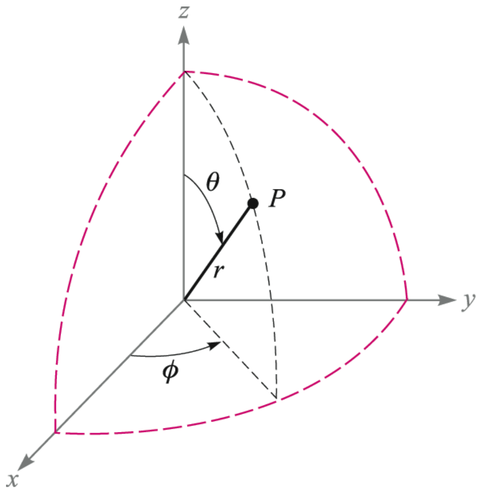
$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases}$$

Cylindrical



$$\boxed{P(\rho, \phi, z)}$$

Spherical



$$P(r, \theta, \phi)$$

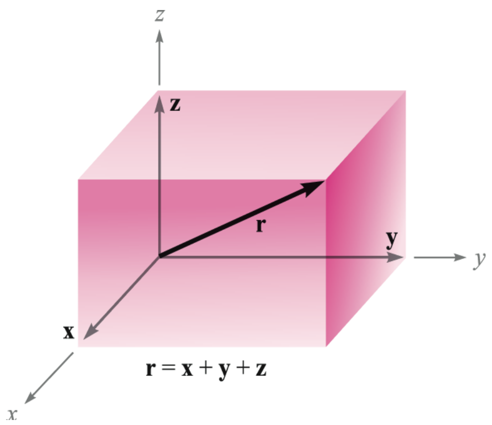
$$r = \sqrt{x^2 + y^2 + z^2} \qquad (r \geq 0)$$

$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad (0^\circ \leq \theta \leq 180^\circ)$$

$$\phi = \tan^{-1}\frac{y}{x}$$

Coordinate System

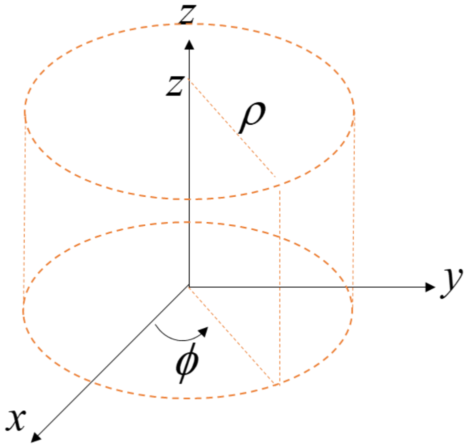
Rectangular



$$P(x, y, z)$$

$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases}$$

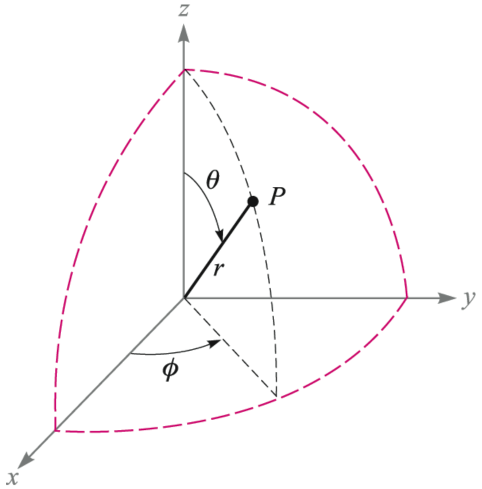
Cylindrical



$$\boxed{P(\rho, \phi, z)}$$

$$\left(1, \frac{\pi}{3}, 1\right)$$

Spherical



$$P(r, \theta, \phi)$$

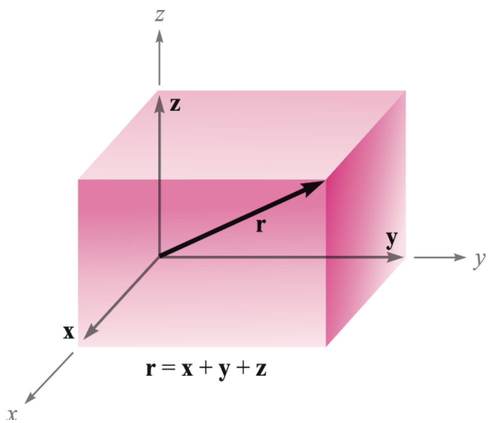
$$r = \sqrt{x^2 + y^2 + z^2} \qquad (r \geq 0)$$

$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad (0^\circ \leq \theta \leq 180^\circ)$$

$$\phi = \tan^{-1}\frac{y}{x}$$

Coordinate System

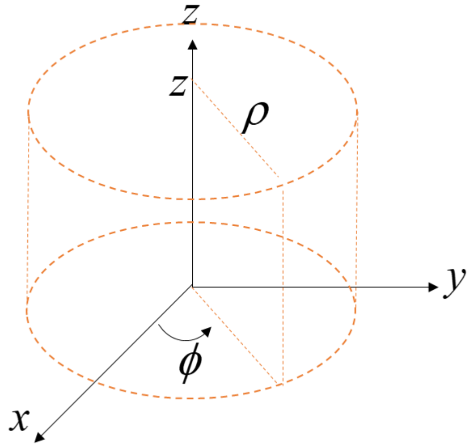
Rectangular



$$P(x, y, z)$$

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 1\right)$$

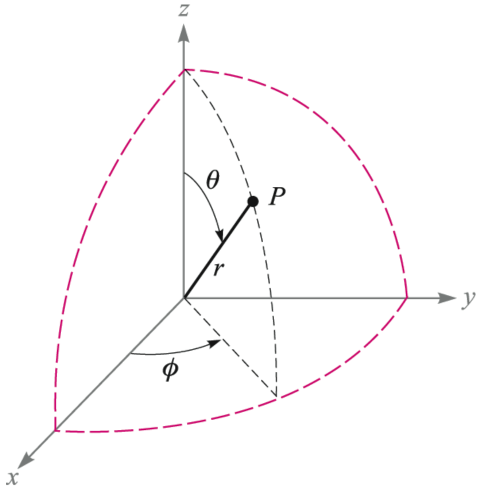
Cylindrical



$$\boxed{P(\rho, \phi, z)}$$

$$\left(1, \frac{\pi}{3}, 1\right)$$

Spherical



$$P(r, \theta, \phi)$$

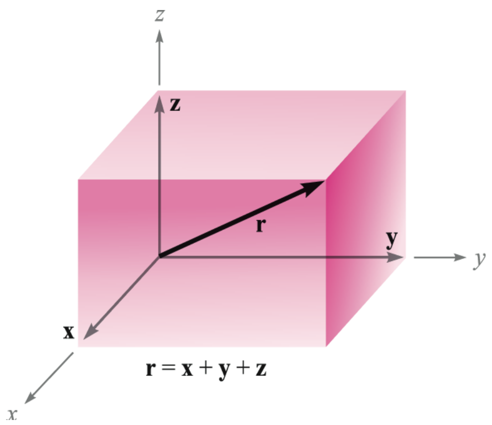
$$r = \sqrt{x^2 + y^2 + z^2} \qquad (r \geq 0)$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad (0^\circ \leq \theta \leq 180^\circ)$$

$$\phi = \tan^{-1} \frac{y}{x}$$

Coordinate System

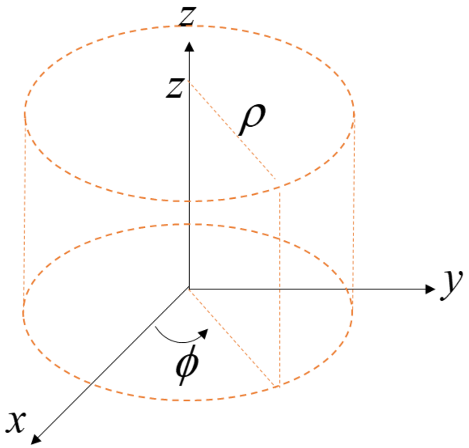
Rectangular



$$P(x, y, z)$$

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 1\right)$$

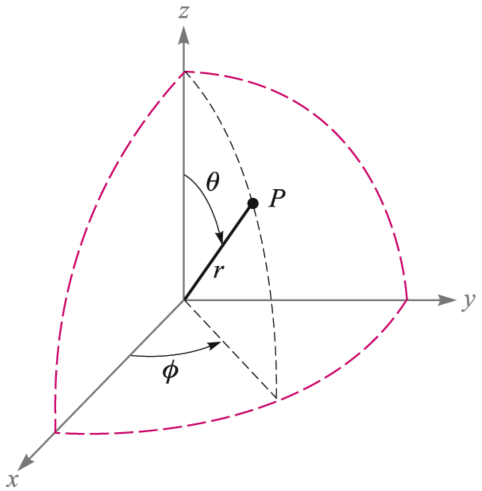
Cylindrical



$$\boxed{P(\rho, \phi, z)}$$

$$\left(1, \frac{\pi}{3}, 1\right)$$

Spherical

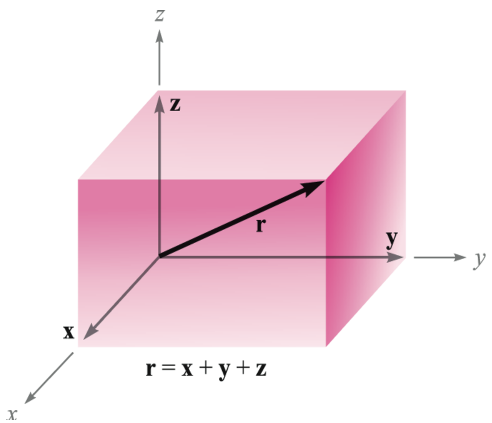


$$P(r, \theta, \phi)$$

$$\left(\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{3}\right)$$

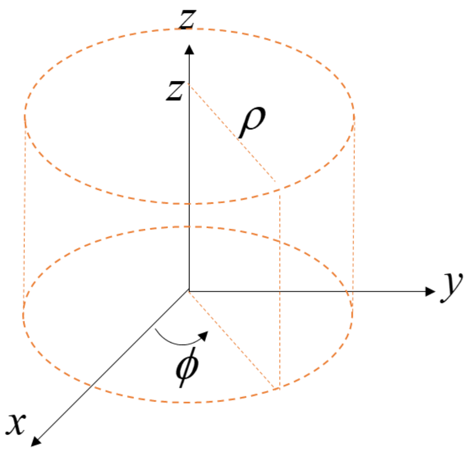
Coordinate System

Rectangular



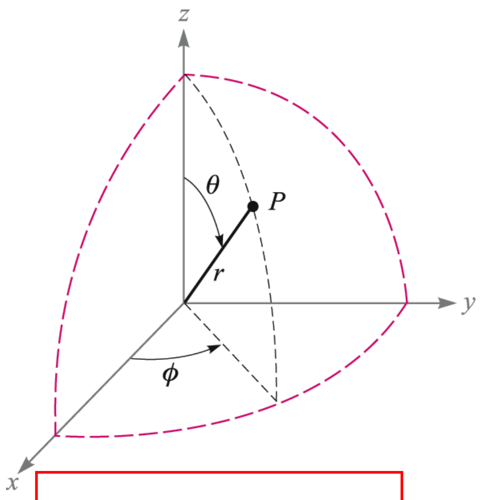
$$P(x, y, z)$$

Cylindrical



$$P(\rho, \phi, z)$$

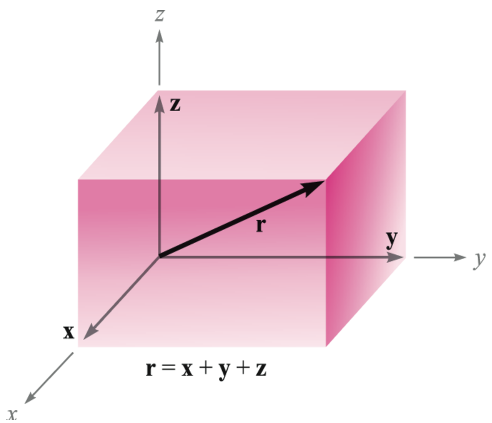
Spherical



$$\boxed{P(r, \theta, \phi)}$$

Coordinate System

Rectangular



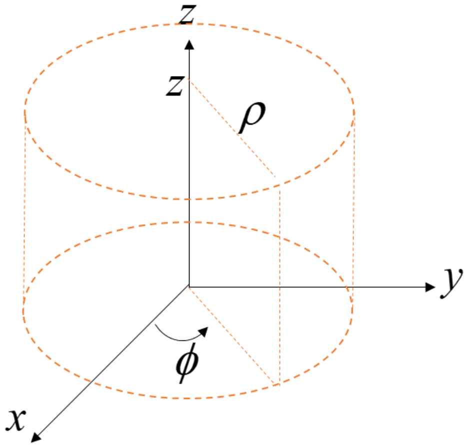
$$P(x, y, z)$$

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

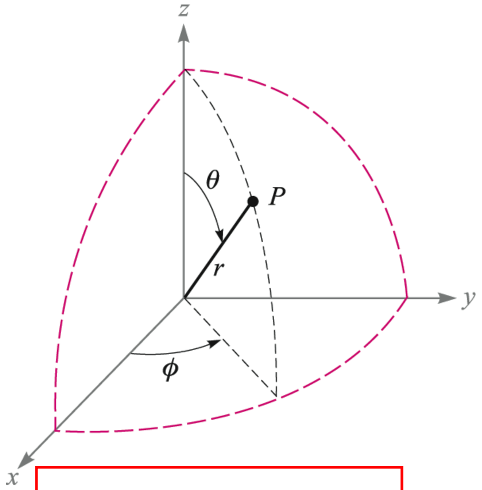
$$z = r \cos\theta$$

Cylindrical



$$P(\rho, \phi, z)$$

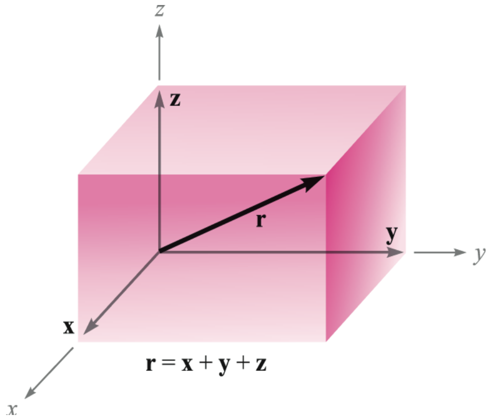
Spherical



$$\boxed{P(r, \theta, \phi)}$$

Coordinate System

Rectangular



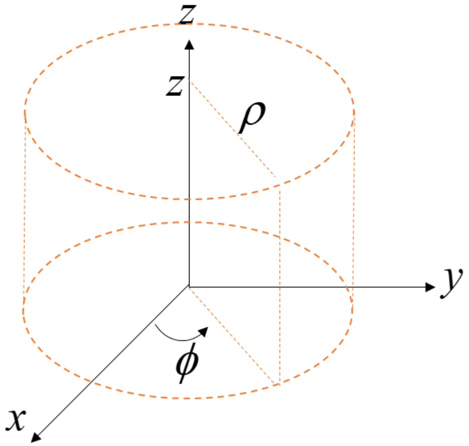
$$P(x, y, z)$$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

Cylindrical



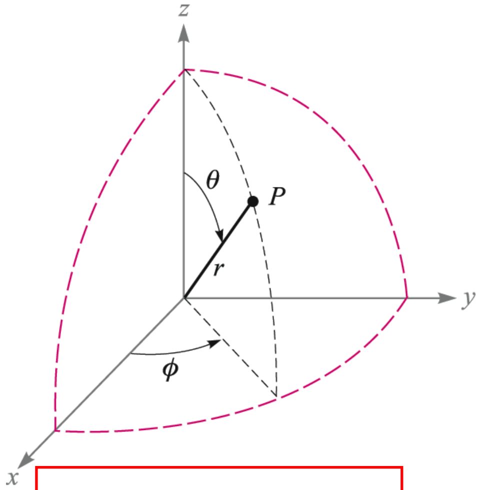
$$P(\rho, \phi, z)$$

$$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$

$$\phi = \tan^{-1}\frac{y}{x}$$

$$z = z$$

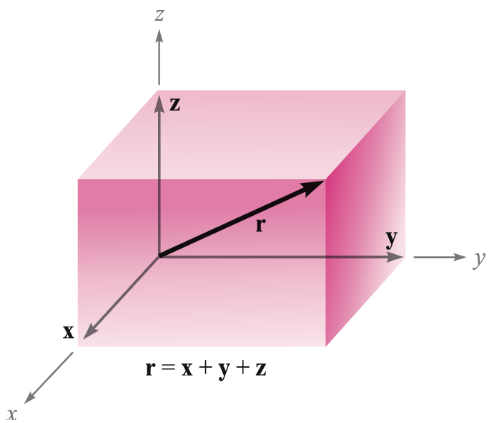
Spherical



$$\boxed{P(r, \theta, \phi)}$$

Coordinate System

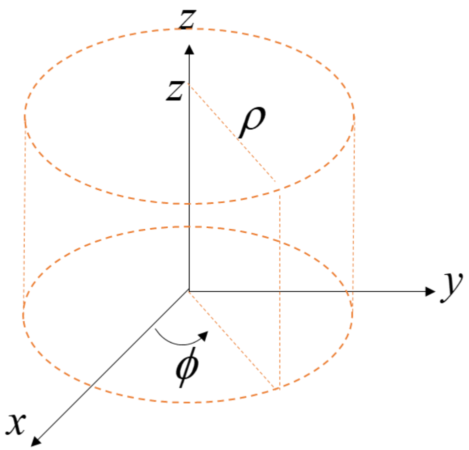
Rectangular



$$P(x, y, z)$$

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$

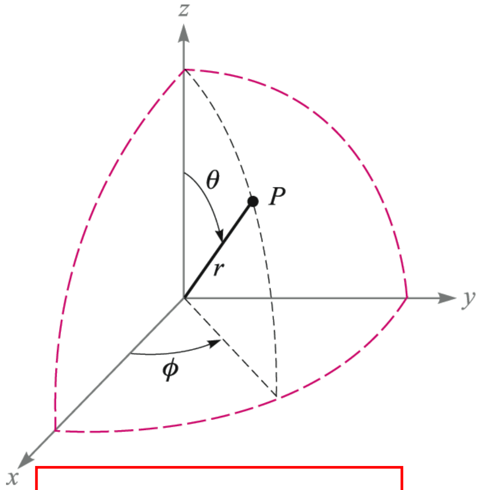
Cylindrical



$$P(\rho, \phi, z)$$

$$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$
$$\phi = \tan^{-1} \frac{y}{x}$$
$$z = z$$

Spherical

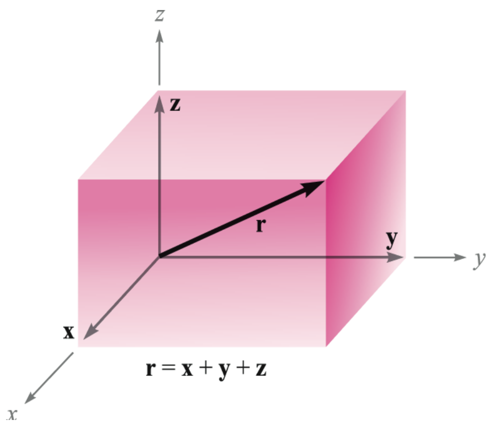


$$\boxed{P(r, \theta, \phi)}$$

$$\left(1, \frac{\pi}{3}, \pi\right)$$

Coordinate System

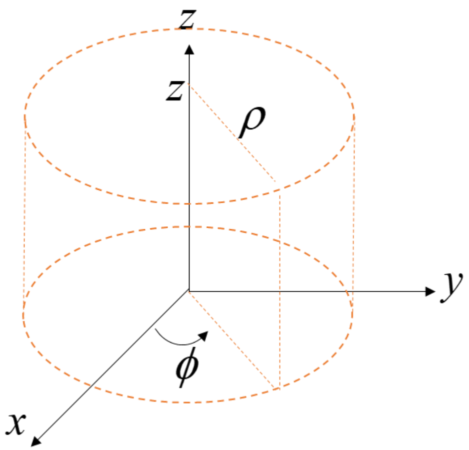
Rectangular



$$P(x, y, z)$$

$$\left(-\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)$$

Cylindrical



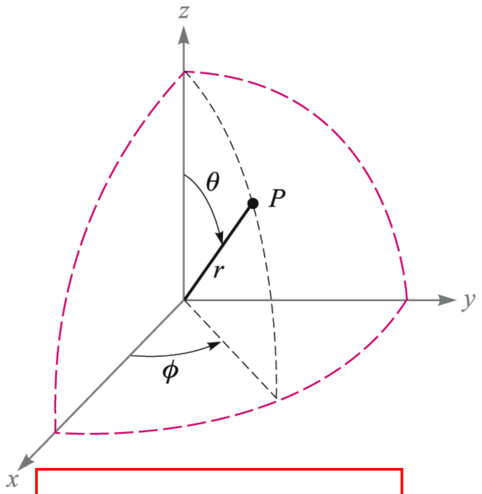
$$P(\rho, \phi, z)$$

$$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$

$$\phi = \tan^{-1} \frac{y}{x}$$

$$z = z$$

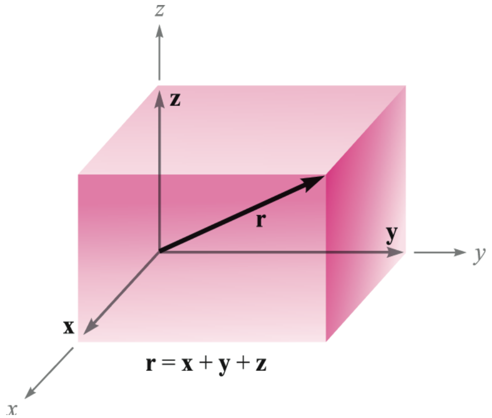
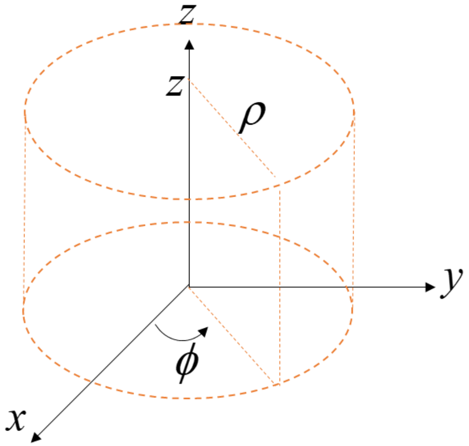
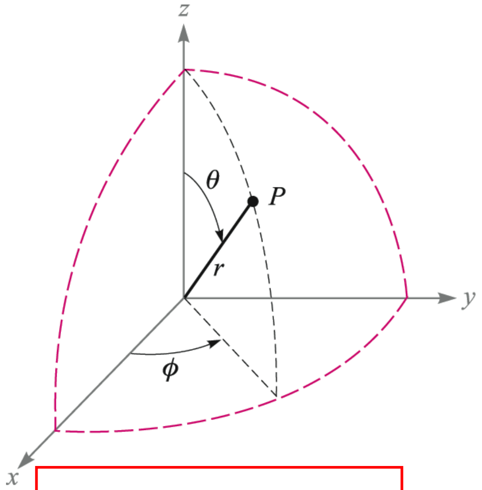
Spherical



$$\boxed{P(r, \theta, \phi)}$$

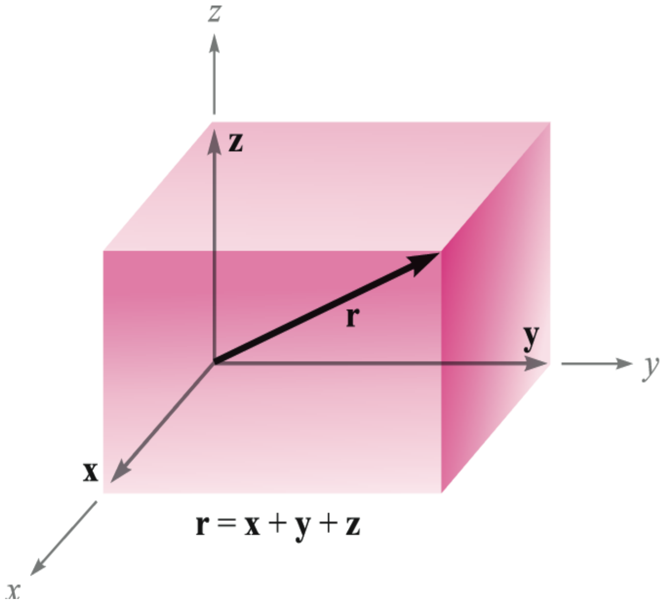
$$\left(1, \frac{\pi}{3}, \pi\right)$$

Coordinate System

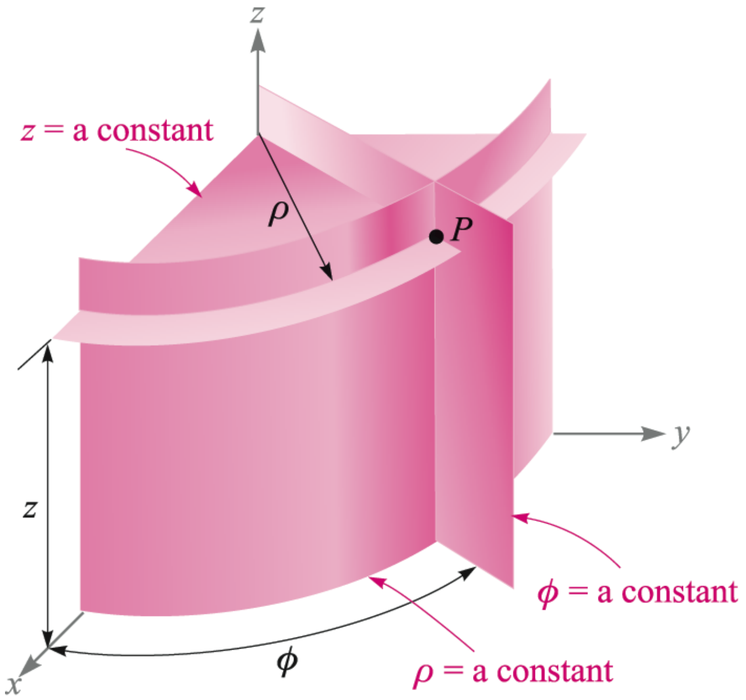
Rectangular	Cylindrical	Spherical
		
$P(x, y, z)$	$P(\rho, \phi, z)$	$\boxed{P(r, \theta, \phi)}$
$\left(-\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)$	$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}, \frac{1}{2}\right)$	$\left(1, \frac{\pi}{3}, \pi\right)$

Coordinate System

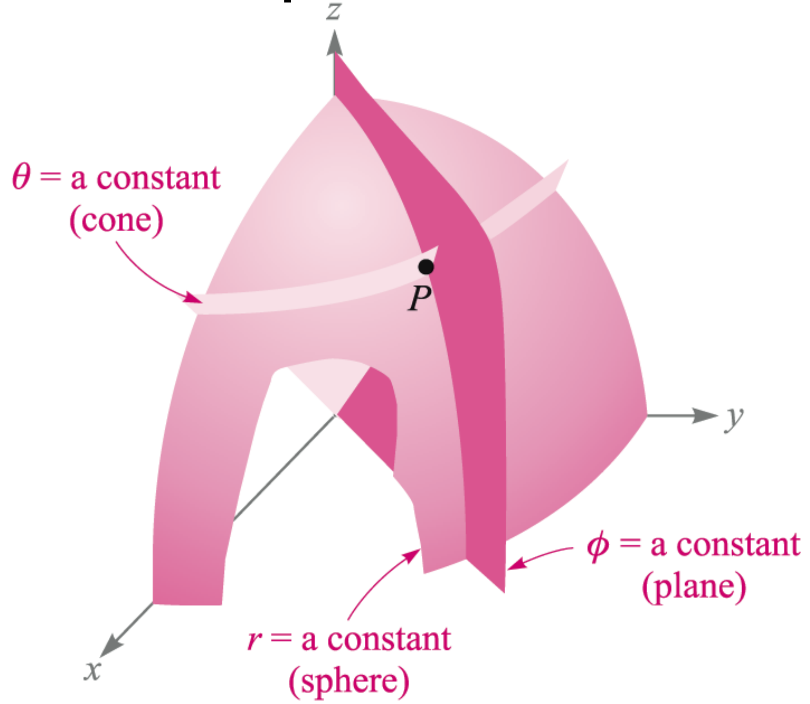
Rectangular



Cylindrical



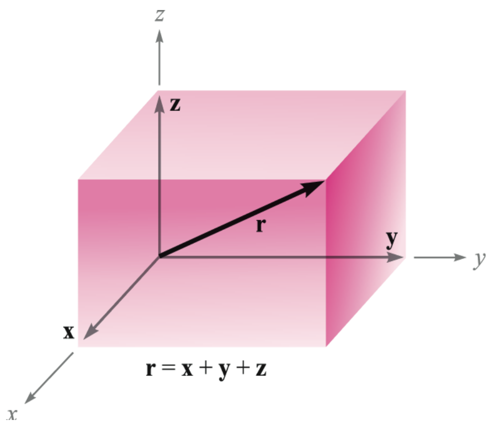
Spherical



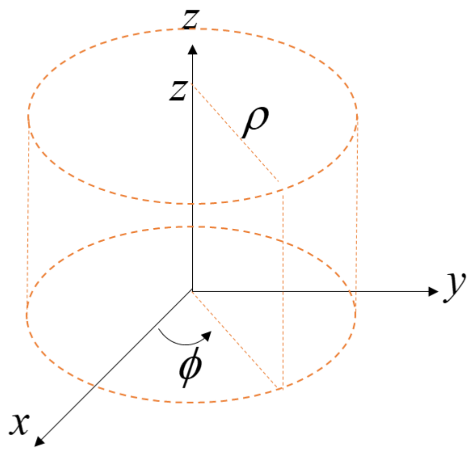
- 3차원 공간상에서의 위치를 각각의 좌표계로 표시하고 서로 변환하기
- Vector Field를 각각의 좌표계로 표시하고 서로 변환하기
- 미소 길이, 미소 면적, 미소 부피를 각각의 좌표계로 표시하고 계산하기

Coordinate System

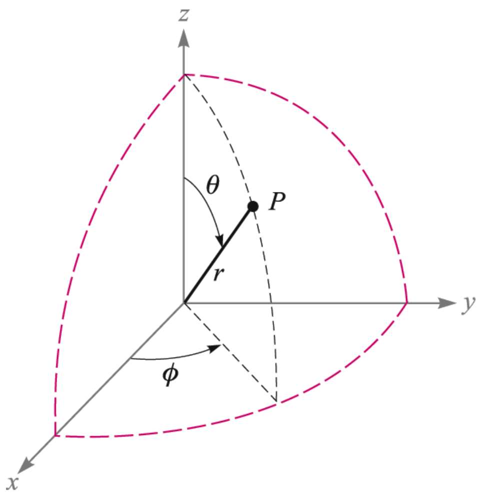
Rectangular



Cylindrical



Spherical



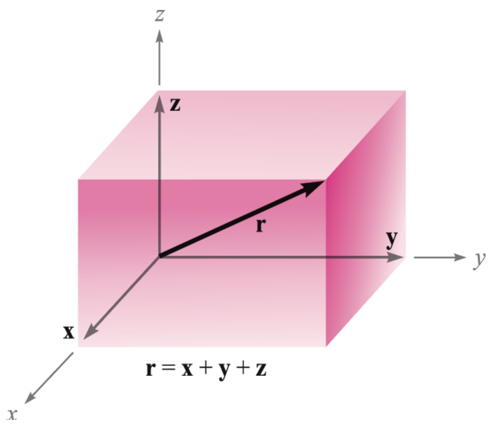
$\mathbf{a_x}$

$\mathbf{a_y}$

$\mathbf{a_z}$

Coordinate System

Rectangular

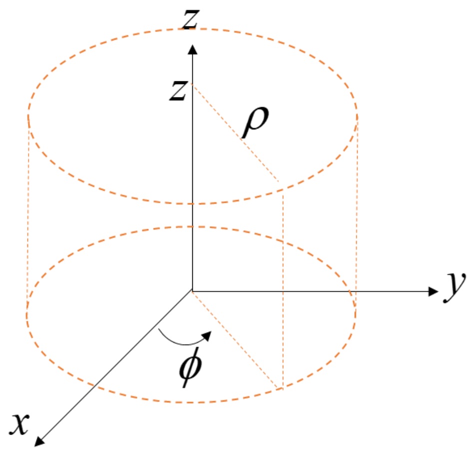


$\mathbf{a_x}$

$\mathbf{a_y}$

$\mathbf{a_z}$

Cylindrical

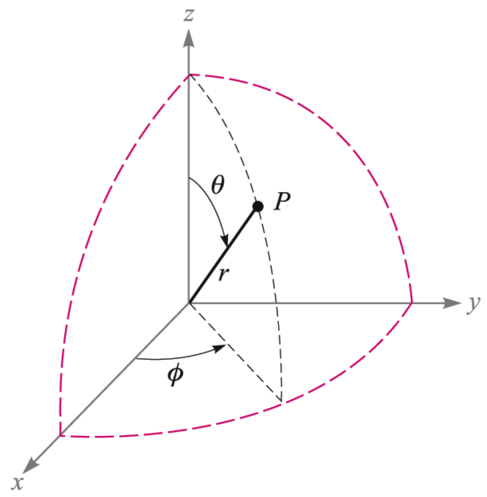


$\mathbf{a_\rho}$

$\mathbf{a_\phi}$

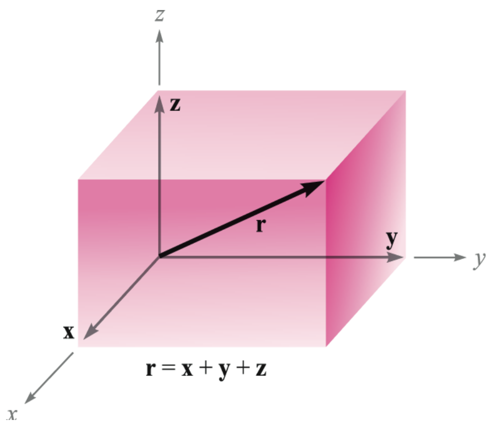
$\mathbf{a_z}$

Spherical



Coordinate System

Rectangular

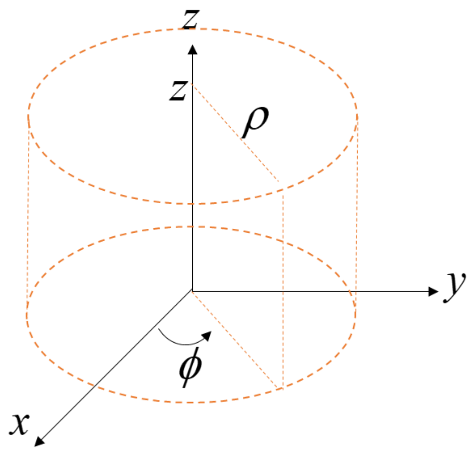


$\mathbf{a}_x$

$\mathbf{a}_y$

$\mathbf{a}_z$

Cylindrical

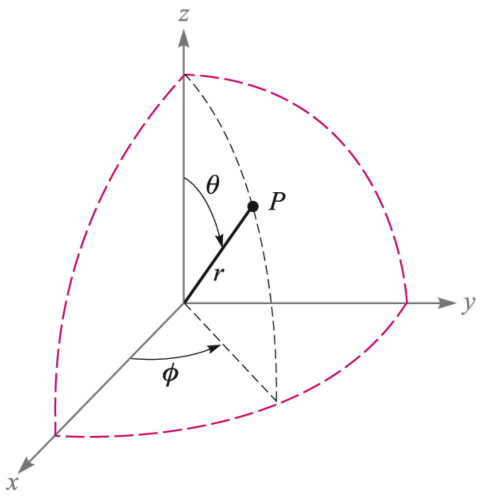


$\mathbf{a}_\rho$

$\mathbf{a}_\phi$

$\mathbf{a}_z$

Spherical



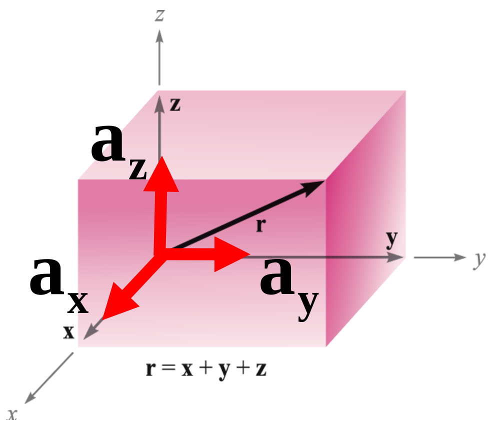
$\mathbf{a}_r$

$\mathbf{a}_\theta$

$\mathbf{a}_\phi$

Coordinate System

Rectangular

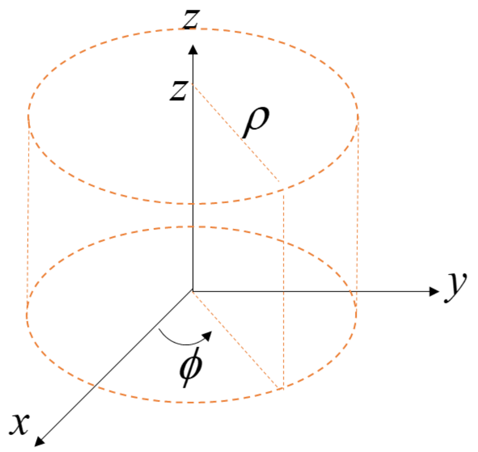


$\mathbf{a}_x$

$\mathbf{a}_y$

$\mathbf{a}_z$

Cylindrical

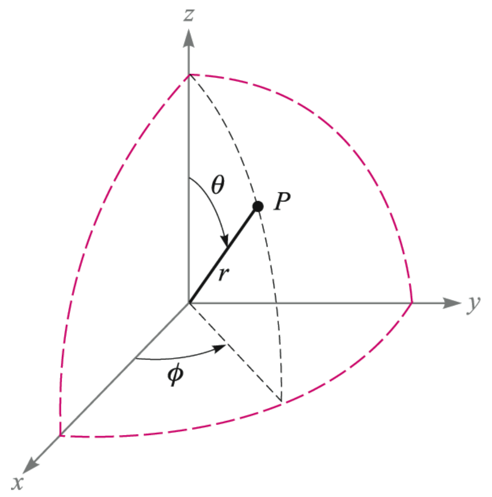


$\mathbf{a}_\rho$

$\mathbf{a}_\phi$

$\mathbf{a}_z$

Spherical



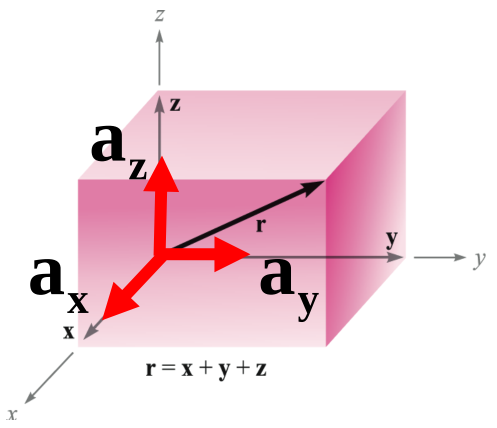
$\mathbf{a}_r$

$\mathbf{a}_\theta$

$\mathbf{a}_\phi$

Coordinate System

Rectangular

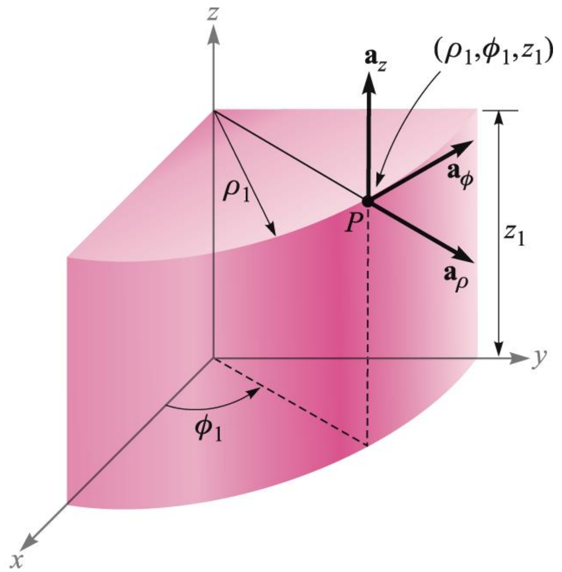


$\mathbf{a}_\mathrm{x}$

$\mathbf{a}_\mathrm{y}$

$\mathbf{a}_\mathrm{z}$

Cylindrical

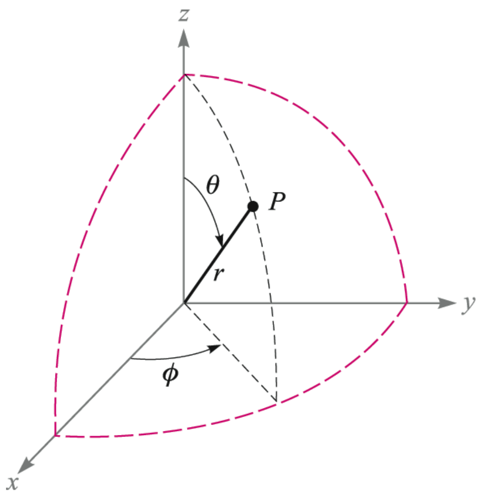


$\mathbf{a}_\rho$

$\mathbf{a}_\phi$

$\mathbf{a}_\mathrm{z}$

Spherical



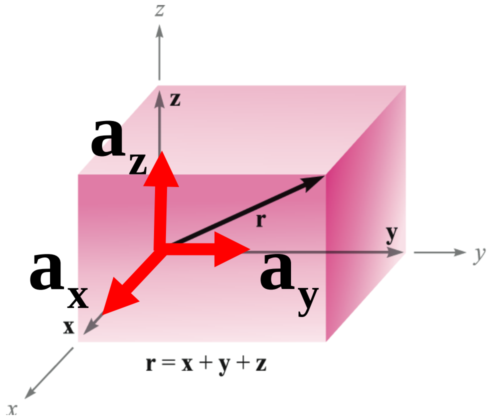
$\mathbf{a}_r$

$\mathbf{a}_\theta$

$\mathbf{a}_\phi$

Coordinate System

Rectangular

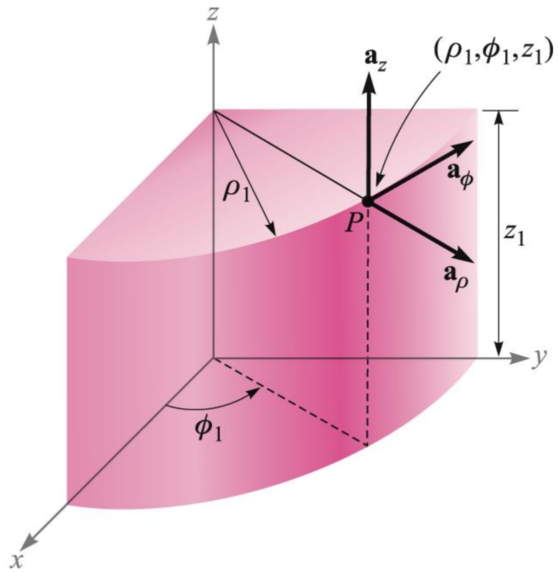


$\mathbf{a}_x$

$\mathbf{a}_y$

$\mathbf{a}_z$

Cylindrical

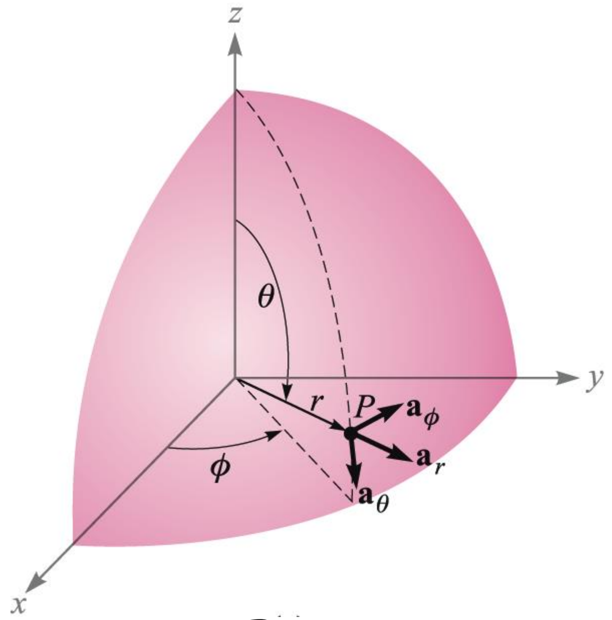


$\mathbf{a}_\rho$

$\mathbf{a}_\phi$

$\mathbf{a}_z$

Spherical



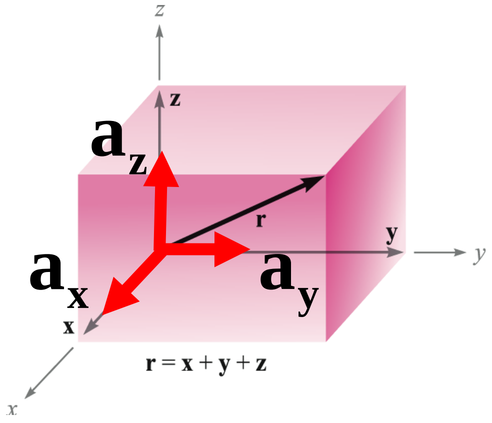
$\mathbf{a}_r$

$\mathbf{a}_\theta$

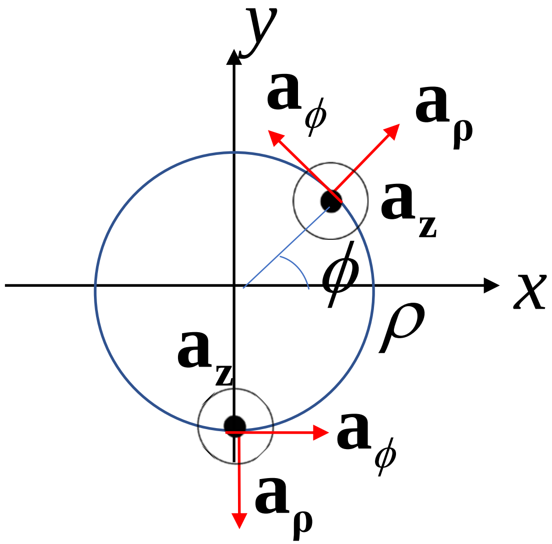
$\mathbf{a}_\phi$

Coordinate System

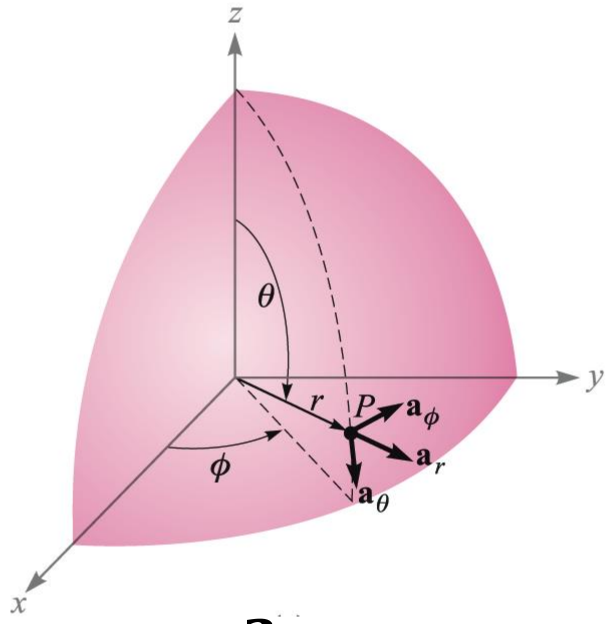
Rectangular



Cylindrical



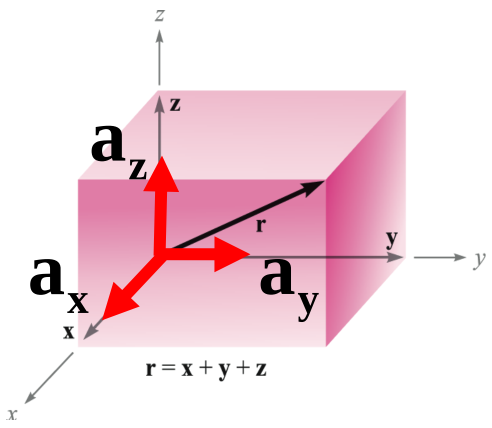
Spherical



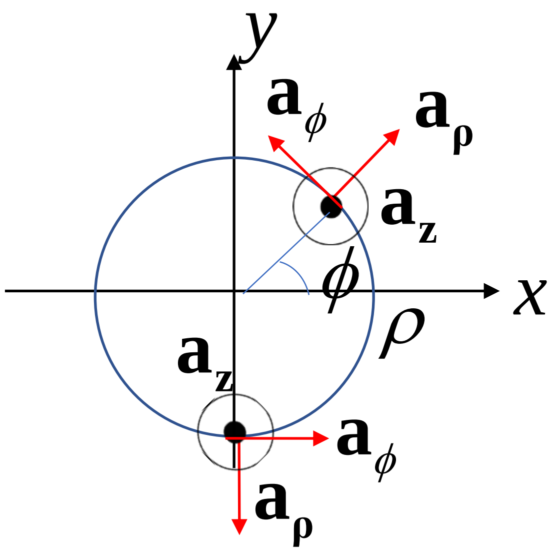
$\mathbf{a}_x$	$\mathbf{a}_\rho$	$\mathbf{a}_r$
$\mathbf{a}_y$	$\mathbf{a}_\phi$	$\mathbf{a}_\theta$
$\mathbf{a}_z$	$\mathbf{a}_z$	$\mathbf{a}_\phi$

Coordinate System

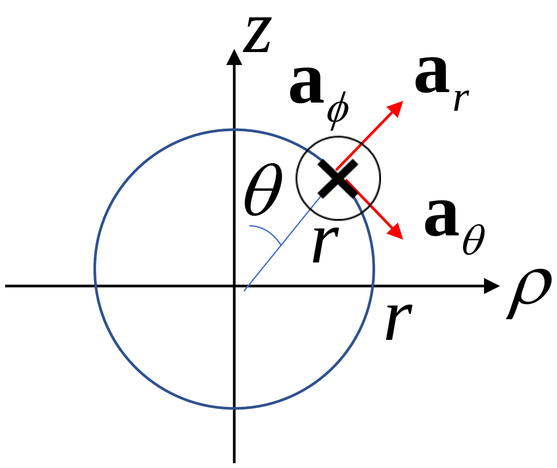
Rectangular



Cylindrical



Spherical



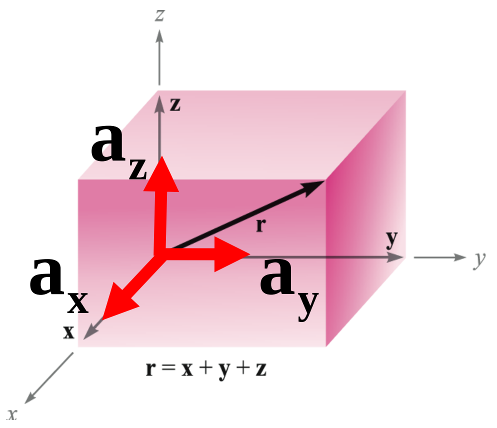
$$\mathbf{a}_\rho \cdot \mathbf{a}_x = \cos\phi$$

$$\mathbf{a}_\phi \cdot \mathbf{a}_x = -\sin\phi$$

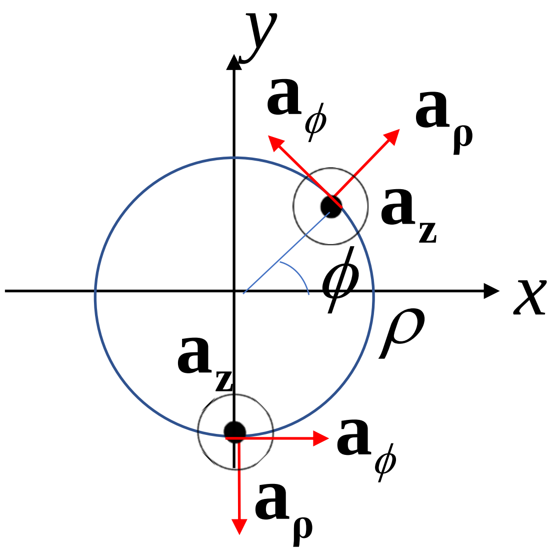
$$\mathbf{a}_z \cdot \mathbf{a}_x = 0$$

Coordinate System

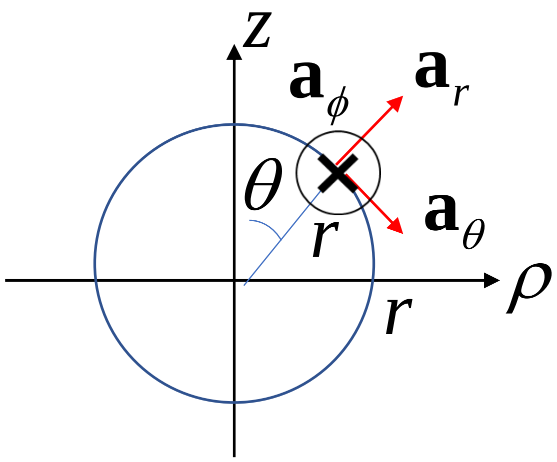
Rectangular



Cylindrical

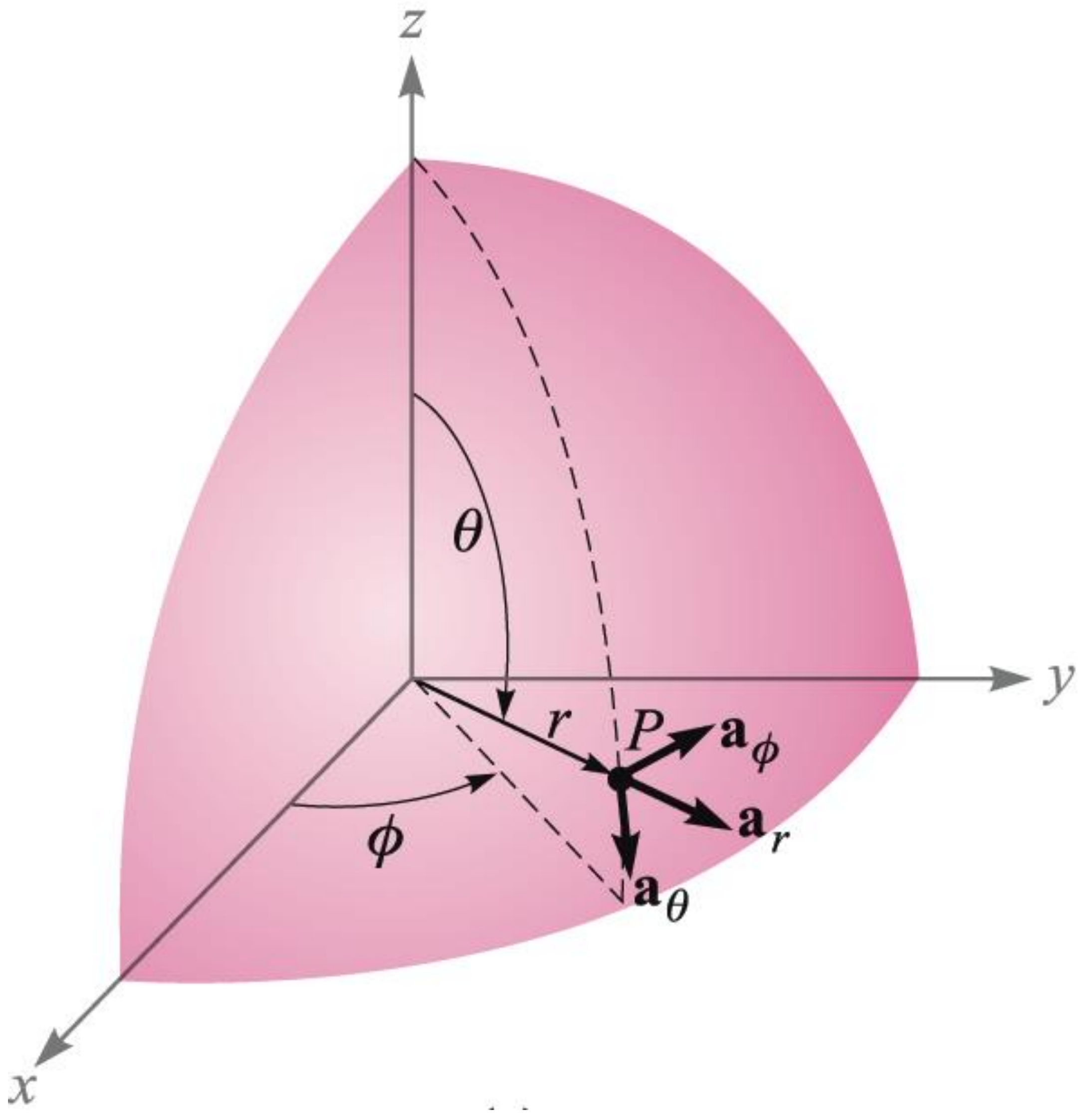


Spherical

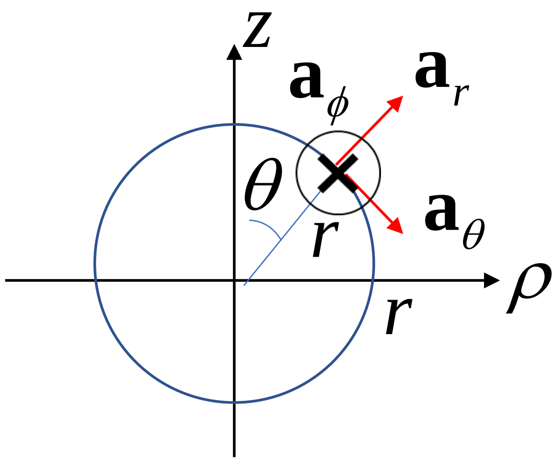


	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	0

Coordinate System



Spherical



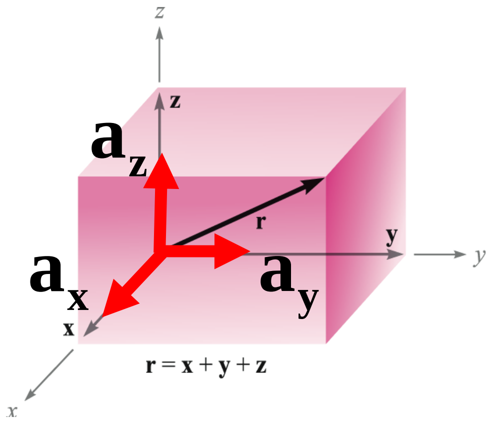
$$\mathbf{a}_r \cdot \mathbf{a}_\mathbf{x} = \sin\theta\cos\phi$$

$$\mathbf{a}_\theta \cdot \mathbf{a}_\mathbf{x} = \cos\theta\cos\phi$$

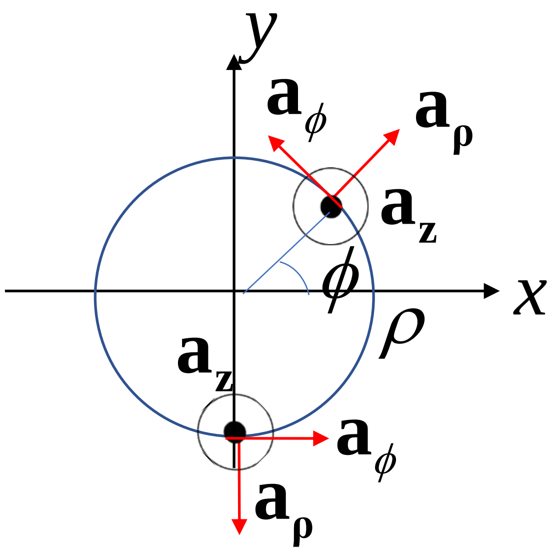
$$\mathbf{a}_\phi \cdot \mathbf{a}_\mathbf{x} = -\sin\phi$$

Coordinate System

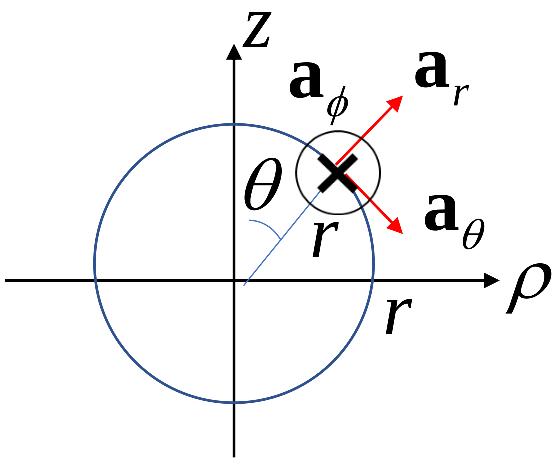
Rectangular



Cylindrical



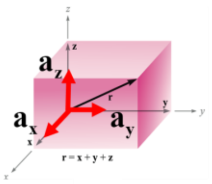
Spherical



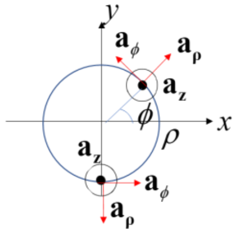
	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

Coordinate System

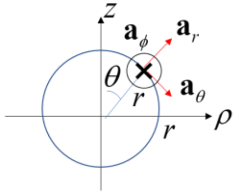
Rectangular



Cylindrical



Spherical



Rectangular	Cylindrical	Spherical
$\mathbf{a_x}$	$\mathbf{a_\rho}$	$\mathbf{a}_r$
$\mathbf{a_y}$	$\mathbf{a}_\phi$	$\mathbf{a}_\theta$
$\mathbf{a_z}$	$\mathbf{a_z}$	$\mathbf{a}_\phi$

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	0

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

Coordinate System

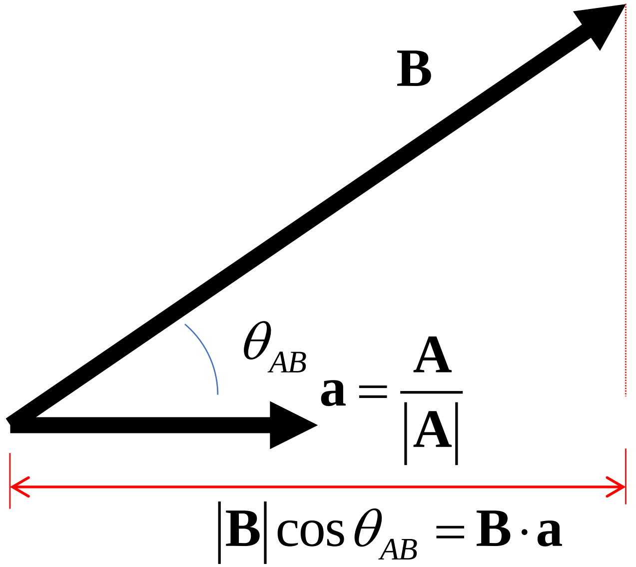
$$\mathbf{G}(x,y,z) = G_x(x,y,z)\mathbf{a_x} + G_y(x,y,z)\mathbf{a_y} + G_z(x,y,z)\mathbf{a_z}$$

$$= G_\rho(\rho,\phi,z)\mathbf{a}_\rho + G_\phi(\rho,\phi,z)\mathbf{a}_\phi + G_z(\rho,\phi,z)\mathbf{a_z}$$

Dot Product = Scalar Product = Inner Product

Given two vectors **A** and **B**, the *dot product*, or *scalar product*, is defined as the product of the magnitude of **A**, the magnitude of **B**, and the cosine of the smaller angle between them,

$$\boxed{\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}|\,\underline{|\mathbf{B}|\cos\theta_{AB}}}$$



$$\mathbf{B}\cdot\mathbf{a} = |\mathbf{B}||\mathbf{a}|\cos\theta = |\mathbf{B}|\cos\theta$$

Coordinate System

$$\mathbf{G}(x,y,z) = G_x(x,y,z)\mathbf{a}_\mathbf{x} + G_y(x,y,z)\mathbf{a}_\mathbf{y} + G_z(x,y,z)\mathbf{a}_\mathbf{z}$$

$$= G_\rho(\rho,\phi,z)\mathbf{a}_\rho + G_\phi(\rho,\phi,z)\mathbf{a}_\phi + G_z(\rho,\phi,z)\mathbf{a}_\mathbf{z}$$

$$\mathbf{G}(x,y,z)\cdot\mathbf{a}_\rho \qquad \mathbf{G}(x,y,z)\cdot\mathbf{a}_\phi \qquad \mathbf{G}(x,y,z)\cdot\mathbf{a}_\phi$$

Coordinate System

$$\mathbf{G}(x,y,z) = G_x(x,y,z)\mathbf{a_x} + G_y(x,y,z)\mathbf{a_y} + G_z(x,y,z)\mathbf{a_z}$$

$$= G_\rho(\rho,\phi,z)\mathbf{a}_\rho + G_\phi(\rho,\phi,z)\mathbf{a}_\phi + G_z(\rho,\phi,z)\mathbf{a_z}$$

$$\mathbf{G}(x,y,z)\cdot\mathbf{a}_\rho \qquad \mathbf{G}(x,y,z)\cdot\mathbf{a}_\phi \qquad \mathbf{G}(x,y,z)\cdot\mathbf{a}_\phi$$

$$= G_r(r,\theta,\phi)\mathbf{a}_r + G_\theta(r,\theta,\phi)\mathbf{a}_\theta + G_\phi(r,\theta,\phi)\mathbf{a}_\phi$$

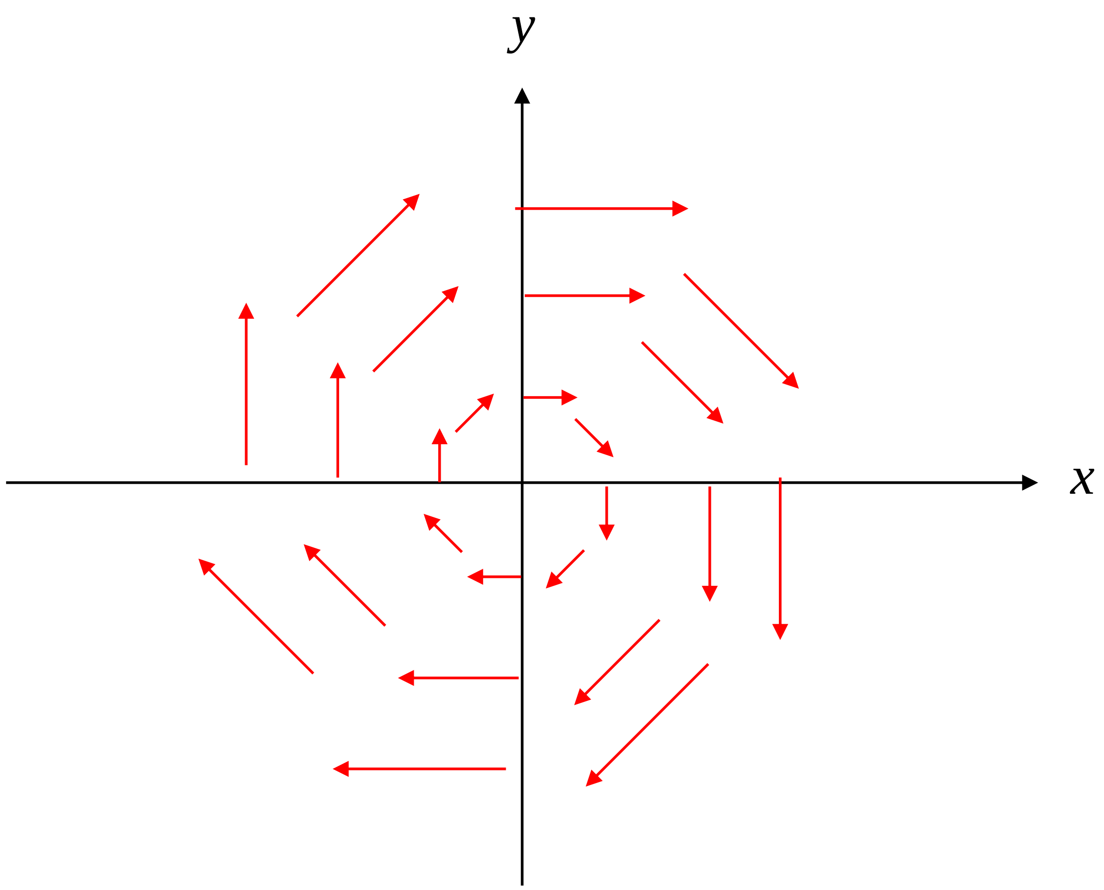
$$\mathbf{G}(x,y,z)\cdot\mathbf{a}_r \qquad \mathbf{G}(x,y,z)\cdot\mathbf{a}_\theta \qquad \mathbf{G}(x,y,z)\cdot\mathbf{a}_\phi$$

Coordinate system

$$\mathbf{B}(x,y,z) = B_x(x,y,z)\mathbf{a_x} + B_y(x,y,z)\mathbf{a_y} + B_z(x,y,z)\mathbf{a}_z$$

$$= y\mathbf{a_x} - x\mathbf{a_y}$$

$$= B_\rho(\rho,\phi,z)\mathbf{a}_\rho + B_\phi(\rho,\phi,z)\mathbf{a}_\phi + B_z(\rho,\phi,z)\mathbf{a_z}$$

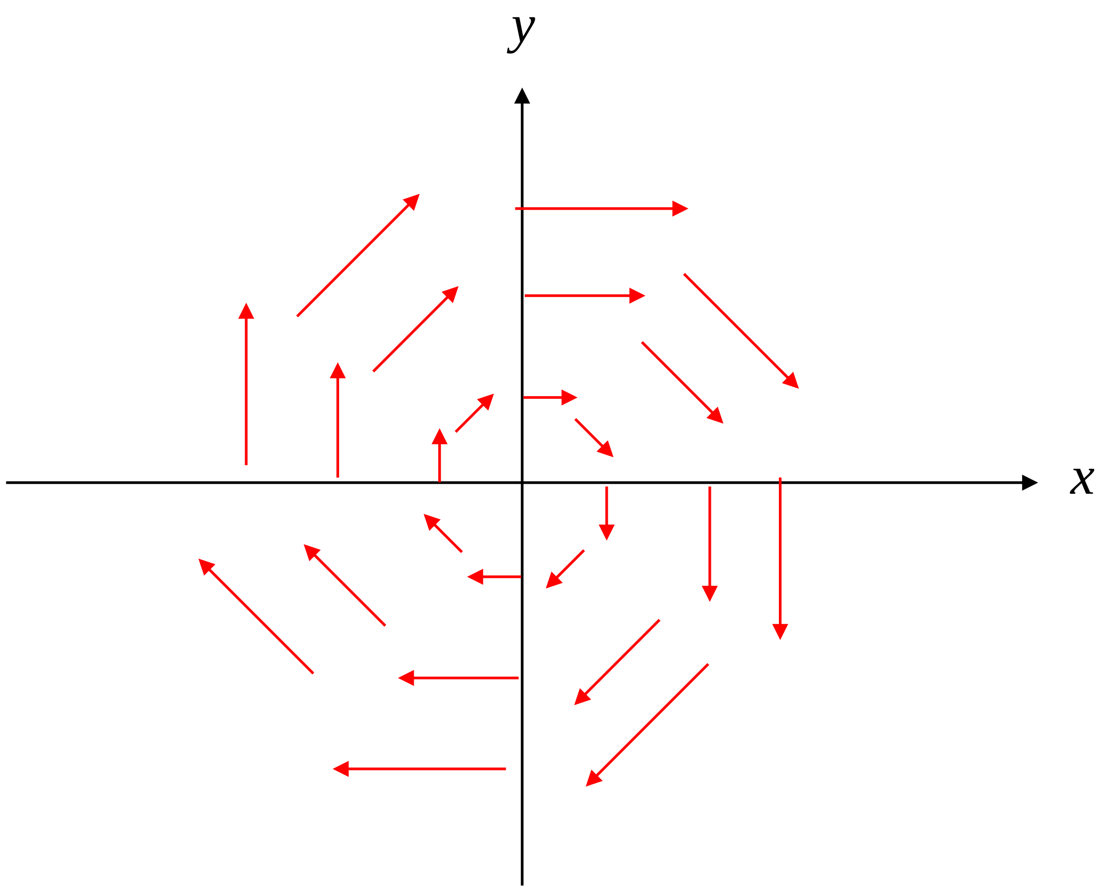


Coordinate system

$$\mathbf{B}(x,y,z) = B_x(x,y,z)\mathbf{a_x} + B_y(x,y,z)\mathbf{a_y} + B_z(x,y,z)\mathbf{a}_z$$

$$= y\mathbf{a_x} - x\mathbf{a_y}$$

$$= B_\rho(\rho,\phi,z)\mathbf{a}_\rho + B_\phi(\rho,\phi,z)\mathbf{a}_\phi + B_z(\rho,\phi,z)\mathbf{a_z}$$



$$B_\rho = \mathbf{B}\cdot\mathbf{a}_\rho = \left(y\mathbf{a_x} - x\mathbf{a_y}\right)\cdot\mathbf{a}_\rho$$

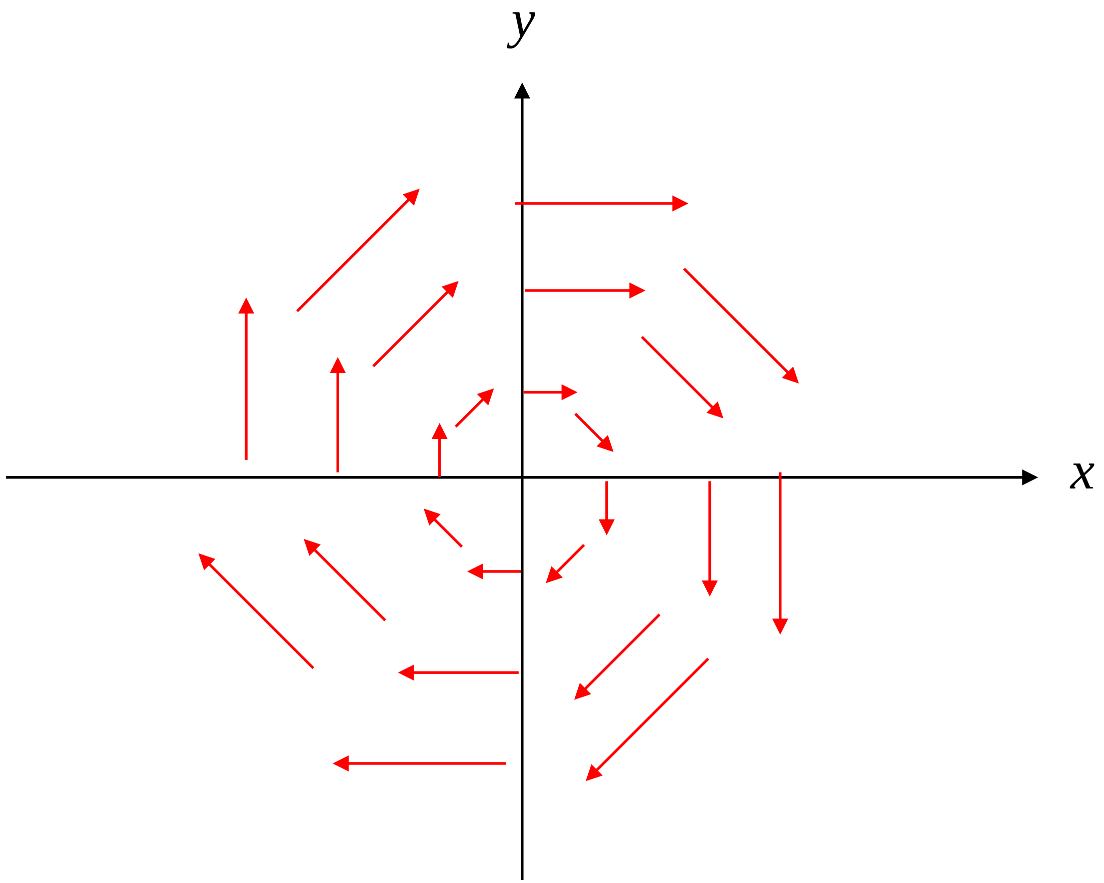
$$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$$

Coordinate system

$$\mathbf{B}(x,y,z) = B_x(x,y,z)\mathbf{a}_\mathbf{x} + B_y(x,y,z)\mathbf{a}_\mathbf{y} + B_z(x,y,z)\mathbf{a}_z$$

$$= y\mathbf{a}_\mathbf{x} - x\mathbf{a}_\mathbf{y}$$

$$= B_\rho(\rho,\phi,z)\mathbf{a}_\rho + B_\phi(\rho,\phi,z)\mathbf{a}_\phi + B_z(\rho,\phi,z)\mathbf{a}_\mathbf{z}$$



$$B_\rho = \mathbf{B}\cdot\mathbf{a}_\rho = \left(y\mathbf{a}_\mathbf{x} - x\mathbf{a}_\mathbf{y}\right)\cdot\mathbf{a}_\rho$$

$$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$$

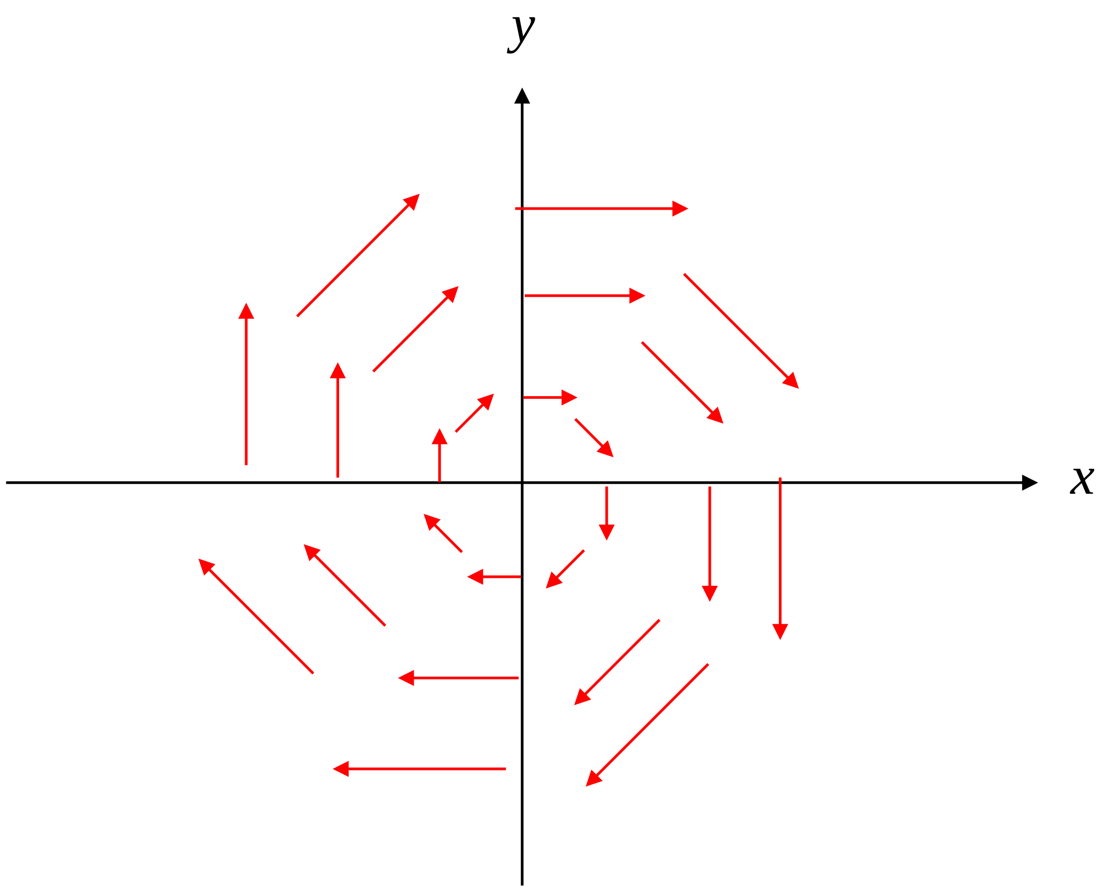
$$B_\phi = \mathbf{B}\cdot\mathbf{a}_\phi = \left(y\mathbf{a}_\mathbf{x} - x\mathbf{a}_\mathbf{y}\right)\cdot\mathbf{a}_\phi$$

$$= -y\sin\phi - x\cos\phi = -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$$

Coordinate system

$$\mathbf{B}(x,y,z) = B_x(x,y,z)\mathbf{a}_x + B_y(x,y,z)\mathbf{a}_y + B_z(x,y,z)\mathbf{a}_z$$
$$= y\mathbf{a}_x - x\mathbf{a}_y$$

$$\Rightarrow \quad = B_\rho(\rho,\phi,z)\mathbf{a}_\rho + B_\phi(\rho,\phi,z)\mathbf{a}_\phi + B_z(\rho,\phi,z)\mathbf{a}_z$$



$$B_\rho = \mathbf{B}\cdot\mathbf{a}_\rho = \left(y\mathbf{a}_x - x\mathbf{a}_y\right)\cdot\mathbf{a}_\rho$$
$$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$$

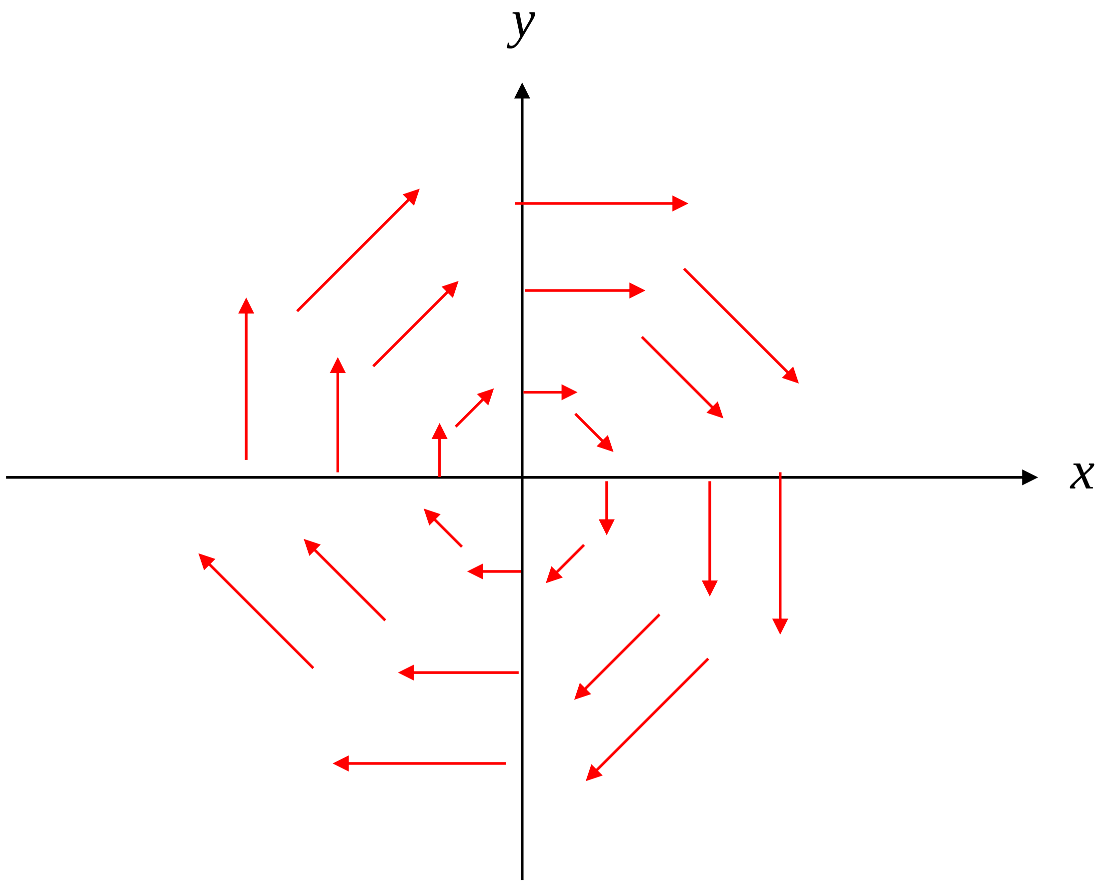
$$B_\phi = \mathbf{B}\cdot\mathbf{a}_\phi = \left(y\mathbf{a}_x - x\mathbf{a}_y\right)\cdot\mathbf{a}_\phi$$
$$= -y\sin\phi - x\cos\phi = -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$$

$$B_z = \mathbf{B}\cdot\mathbf{a}_z = \left(y\mathbf{a}_x - x\mathbf{a}_y\right)\cdot\mathbf{a}_z$$
$$= 0$$

Coordinate system

$$\mathbf{B}(x,y,z)=B_x(x,y,z)\mathbf{a_x}+B_y(x,y,z)\mathbf{a_y}+B_z(x,y,z)\mathbf{a_z}$$
$$=y\mathbf{a_x}-x\mathbf{a_y}$$

$$=B_\rho(\rho,\phi,z)\mathbf{a}_\rho+B_\phi(\rho,\phi,z)\mathbf{a}_\phi+B_z(\rho,\phi,z)\mathbf{a_z}$$
$$=-\rho\mathbf{a}_\phi$$



$$B_\rho=\mathbf{B}\cdot\mathbf{a}_\rho=\left(y\mathbf{a_x}-x\mathbf{a_y}\right)\cdot\mathbf{a}_\rho$$
$$=y\cos\phi-x\sin\phi=\rho\sin\phi\cos\phi-\rho\cos\phi\sin\phi=0$$

$$B_\phi=\mathbf{B}\cdot\mathbf{a}_\phi=\left(y\mathbf{a_x}-x\mathbf{a_y}\right)\cdot\mathbf{a}_\phi$$
$$=-y\sin\phi-x\cos\phi=-\rho\sin^2\phi-\rho\cos^2\phi=-\rho$$

$$B_z=\mathbf{B}\cdot\mathbf{a}_z=\left(y\mathbf{a_x}-x\mathbf{a_y}\right)\cdot\mathbf{a}_z$$
$$=0$$

Coordinate system

Transform the field, $\mathbf{G} = (xz/y)\mathbf{a}_x$, into spherical coordinates and components

$$G_r = \mathbf{G} \cdot \mathbf{a}_r = \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_r = \frac{xz}{y} \sin\theta \cos\phi$$

$$= r \sin\theta \cos\theta \frac{\cos^2\phi}{\sin\phi}$$

$$G_\theta = \mathbf{G} \cdot \mathbf{a}_\theta = \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_\theta = \frac{xz}{y} \cos\theta \cos\phi$$

$$= r \cos^2\theta \frac{\cos^2\phi}{\sin\phi}$$

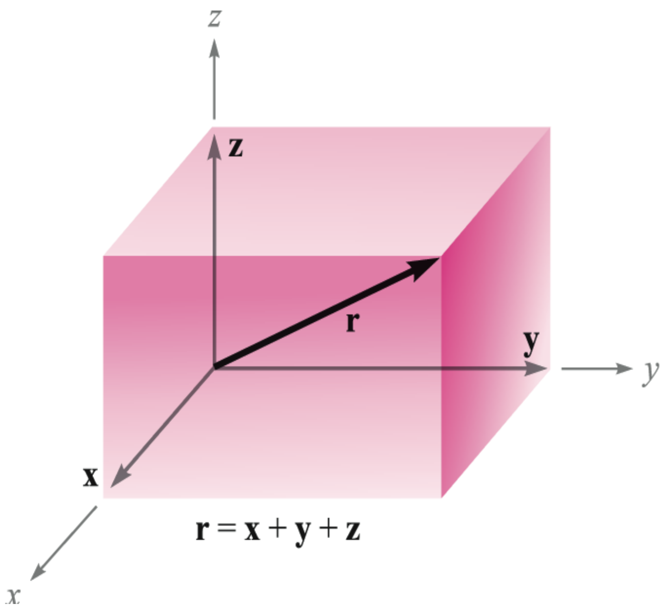
$$G\phi = \mathbf{G} \cdot \mathbf{a}_\phi = \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_\phi = \frac{xz}{y}(-\sin\phi)$$

$$= -r \cos\theta \cos\phi$$

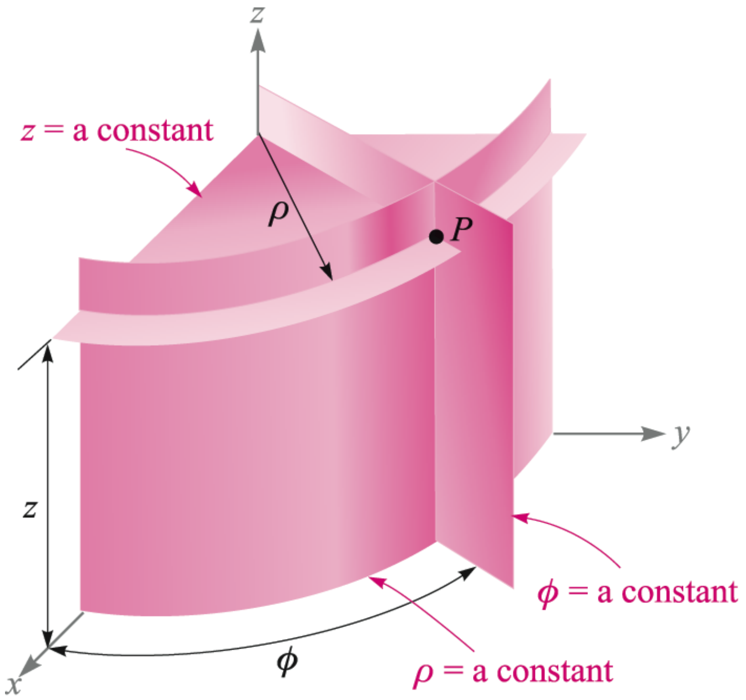
$$\mathbf{G} = r \cos\theta \cos\phi \, (\sin\theta \cot\phi \, \mathbf{a}_r + \cos\theta \cot\phi \, \mathbf{a}_\theta - \mathbf{a}_\phi)$$

Coordinate System

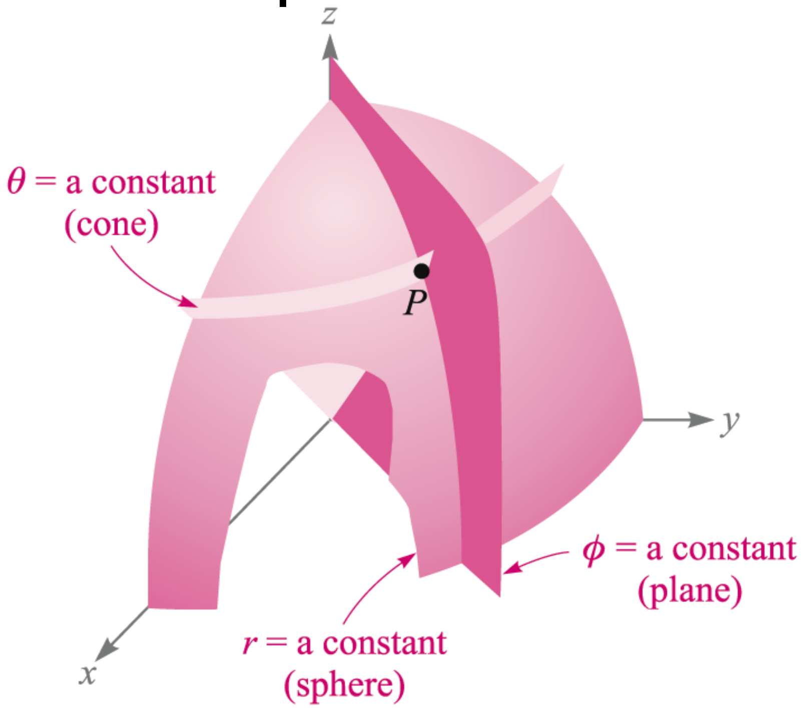
Rectangular



Cylindrical



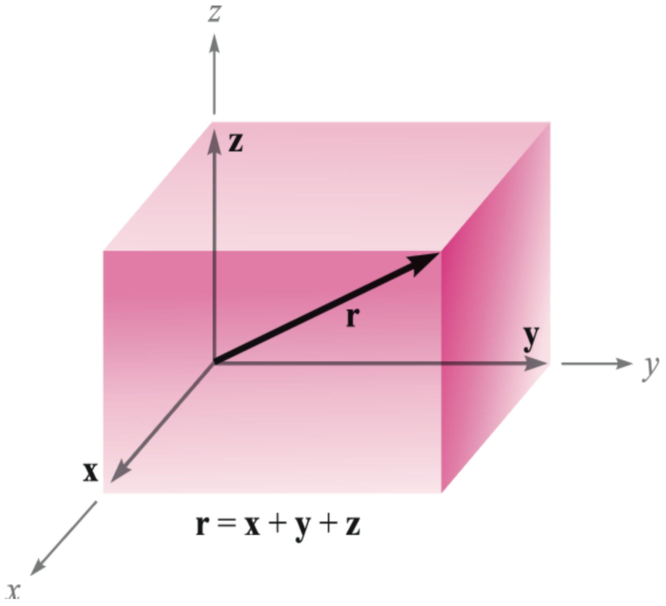
Spherical



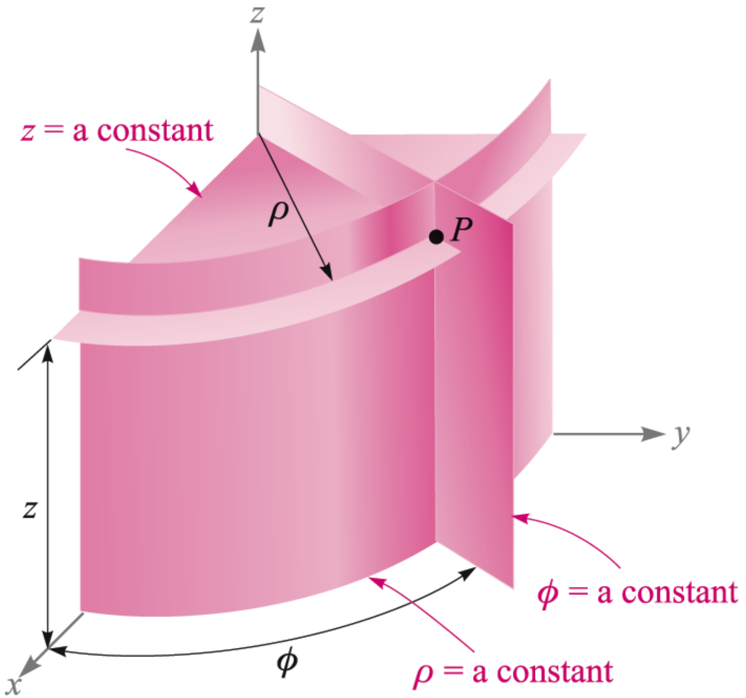
- 3차원 공간상에서의 위치를 각각의 좌표계로 표시하고 서로 변환하기
- Vector Field를 각각의 좌표계로 표시하고 서로 변환하기
- 미소 길이, 미소 면적, 미소 부피를 각각의 좌표계로 표시하고 계산하기

Coordinate System

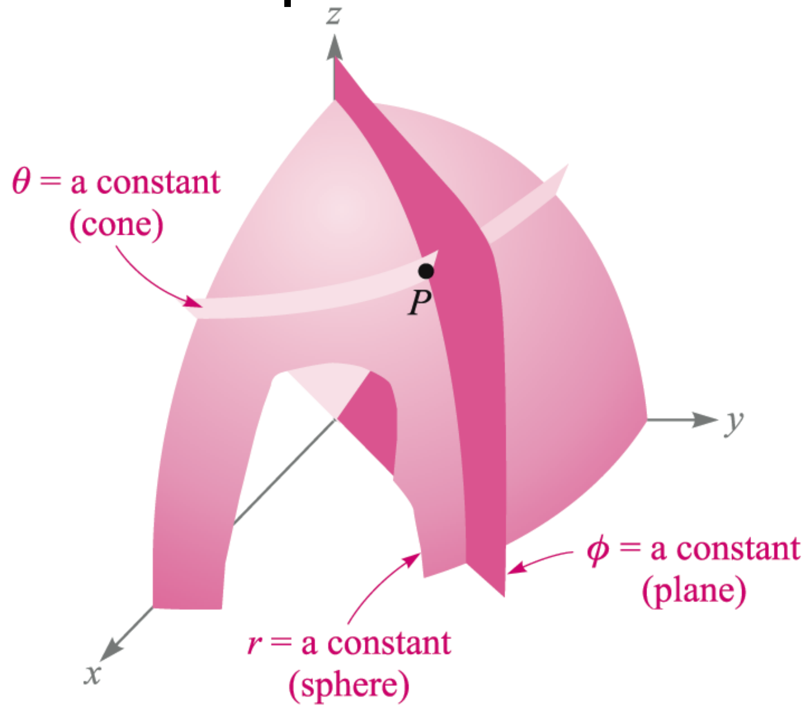
Rectangular



Cylindrical



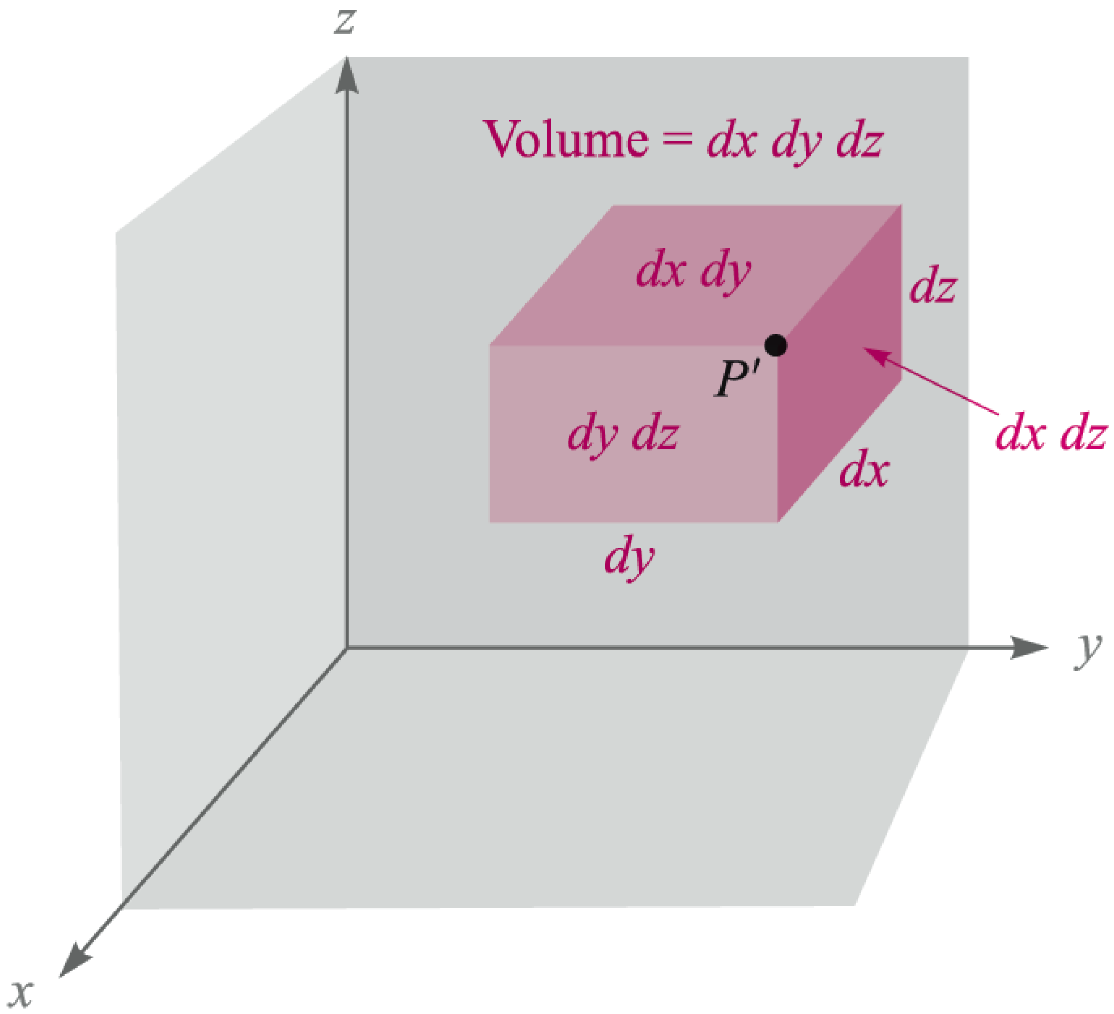
Spherical



- 3차원 공간상에서의 위치를 각각의 좌표계로 표시하고 서로 변환하기
- Vector Field를 각각의 좌표계로 표시하고 서로 변환하기
- 미소 길이, 미소 면적, 미소 부피를 각각의 좌표계로 표시하고 계산하기 dL dS dV

Coordinate System

Rectangular



$$P(x,y,z) \longrightarrow P'(x+dx, y+dy, z+dz)$$

$$dL = dx, \quad dy, \quad dz$$

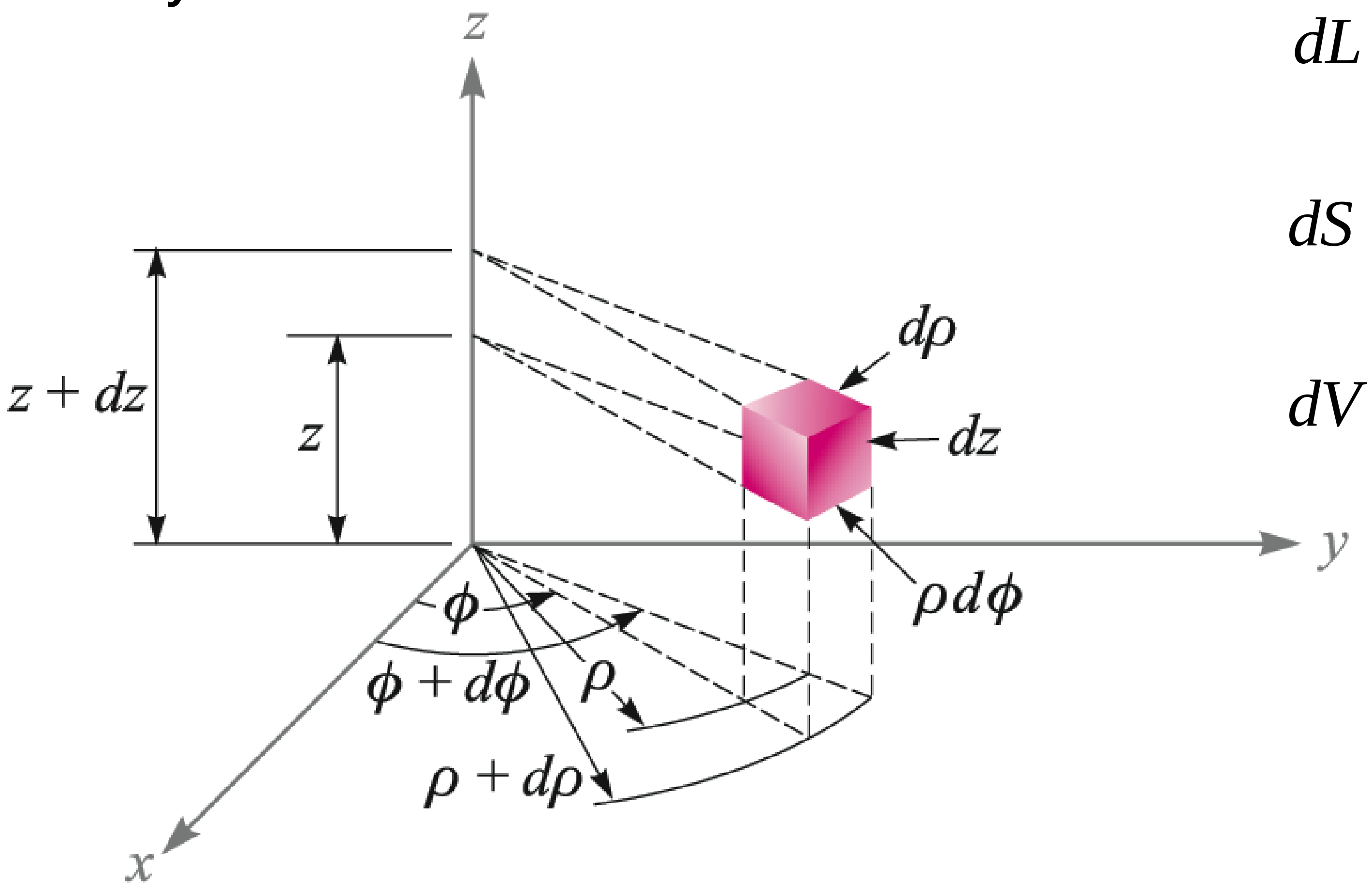
$$dS = dxdy, \quad dydz, \quad dzdx$$

$$dV = dxdydz$$

Coordinate System

$$P(\rho, \phi, z) \longrightarrow P'(\rho + d\rho, \phi + d\phi, z + dz)$$

Cylindrical



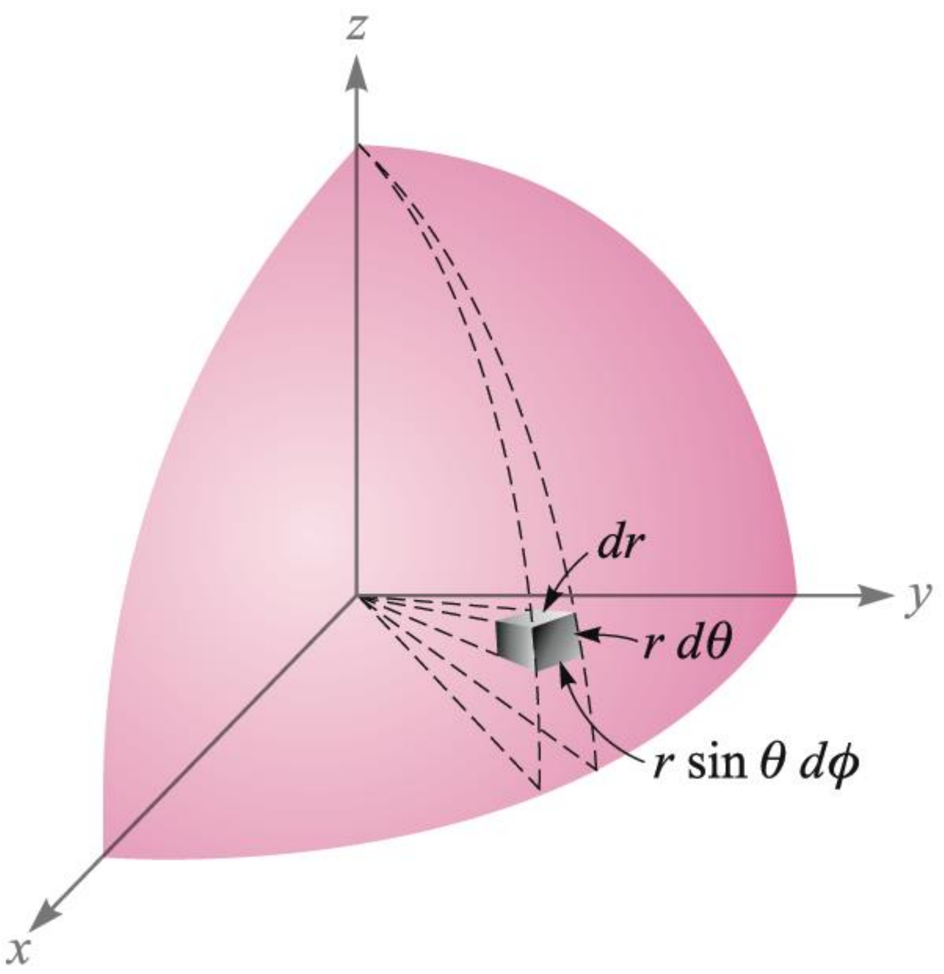
$$dL = d\rho, \quad \rho d\phi, \quad dz$$

$$dS = \rho d\rho d\phi, \quad \rho d\phi dz, \quad dz d\rho$$

$$dV = \rho d\rho d\phi dz$$

Coordinate System

Spherical



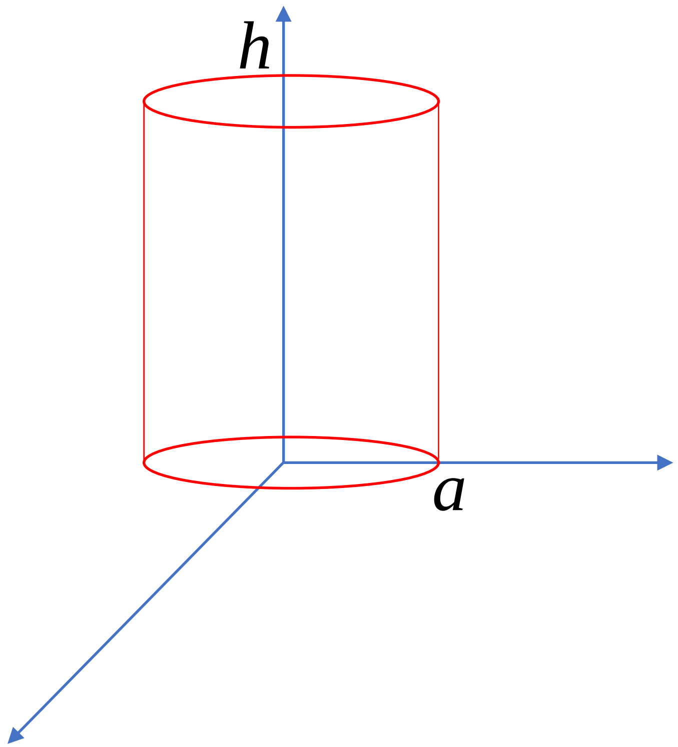
$$P(r,\theta,\phi) \longrightarrow P'(r+dr,\theta+d\theta,\phi+d\phi)$$

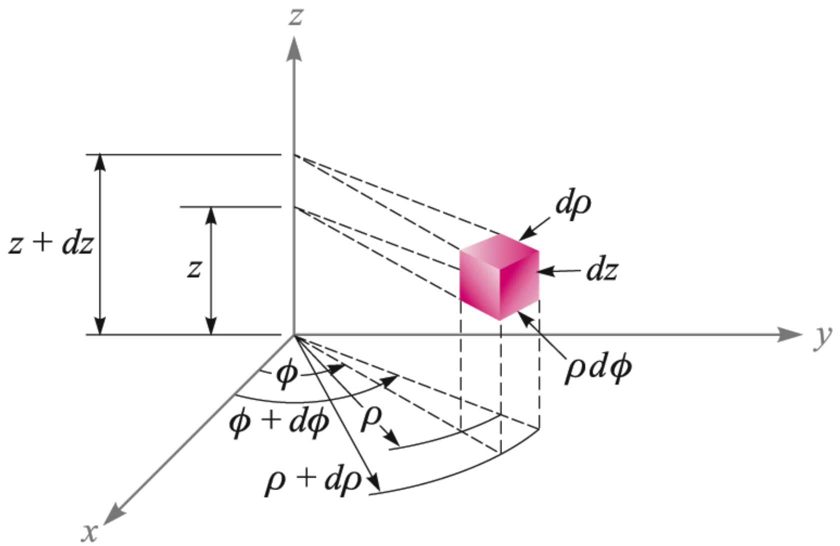
$$dL = dr, \quad rd\theta, \quad r\sin\theta d\phi$$

$$dS = rdrd\theta, \quad r^2\sin\theta d\theta d\phi, \quad r\sin\theta drd\phi$$

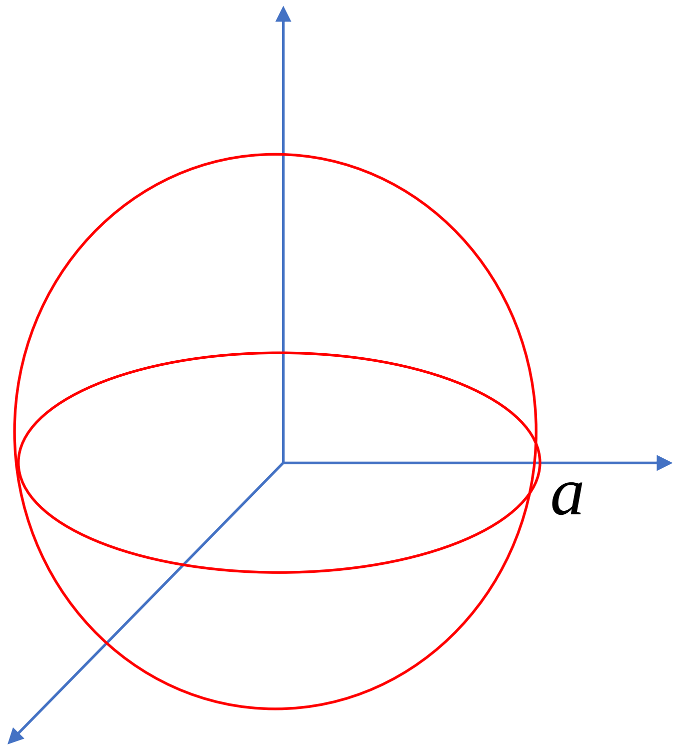
$$dV = r^2\sin\theta drd\theta d\phi$$

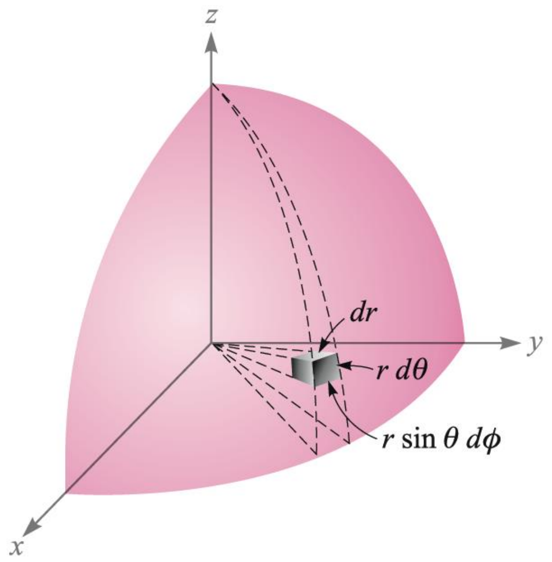
Coordinate System





Coordinate System





Chapter Summary

- 전자기학을 배우기 위한 수학적 기초
- Scalar, Vector
  - 개념
  - 더하기, 빼기, 곱하기(내적, 외적)
- 3차원 좌표계
  - Rectangular, Cylindrical, Spherical
  - 각 좌표계에서의 위치 / vector 표현, 상호 변환
  - 각 좌표계에서의 미분, 적분을 위한 미소 길이, 면적, 부피