Engineering Circuits Analysis (ICE2002)

Chapter 8. Natural and Step Responses of RLC Circuits – Part 1/2/3/4

Contents

- Introduction to the Natural Response of a Parallel RLC Circuit
- The Forms of the Natural Response of a Parallel RLC Circuit
- The Step Response of a Parallel RLC Circuit
- The Natural and Step Response of a Series RLC Circuit

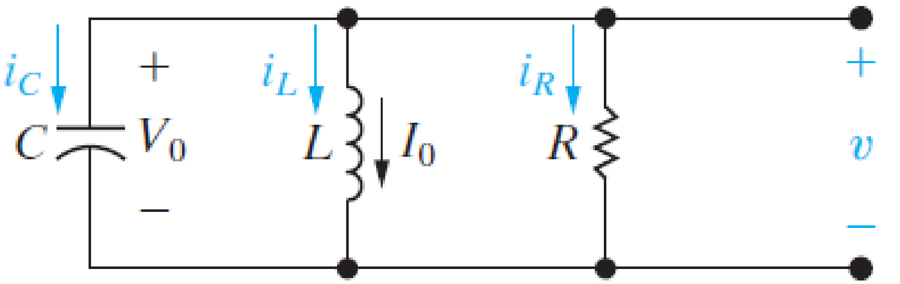
In Chapter 8

- We discuss the natural response and step response of circuits containing a resistor (R), an inductor (L), and a capacitor (C), known as RLC circuits.

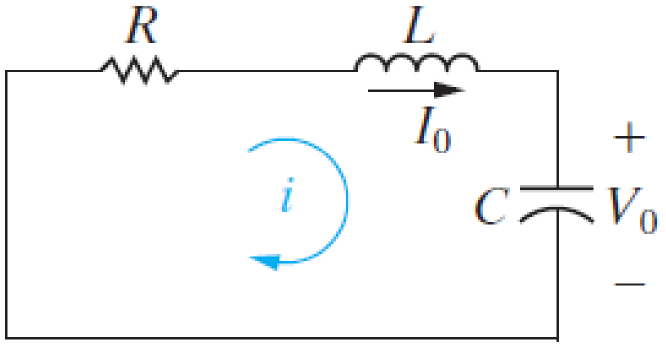
- Parallel RLC Circuits: find the voltage across the parallel branches created by the release of energy stored in the inductor or capacitor , or both.

- Series RLC Circuits: find the current generated in the series-connected elements by the release of stored energy in the inductor, capacitor, or both.

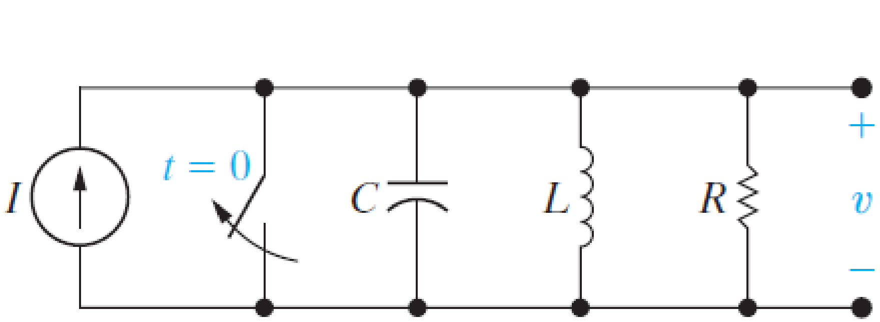
In Chapter 8



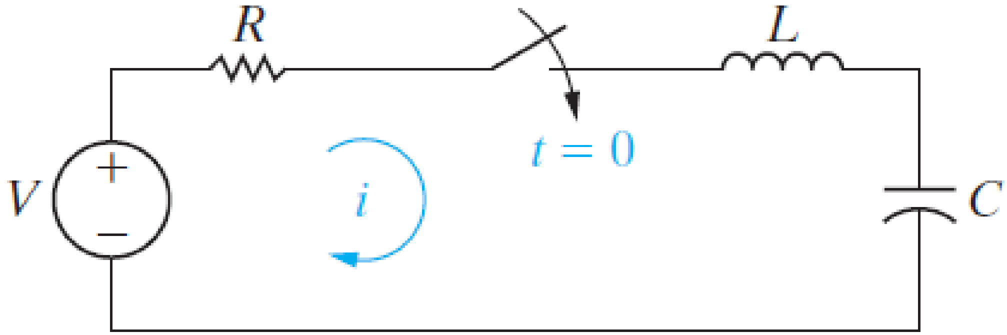
Natural response
of a parallel RLC circuit



Natural response
of a series RLC circuit



Step response
of a parallel RLC circuit



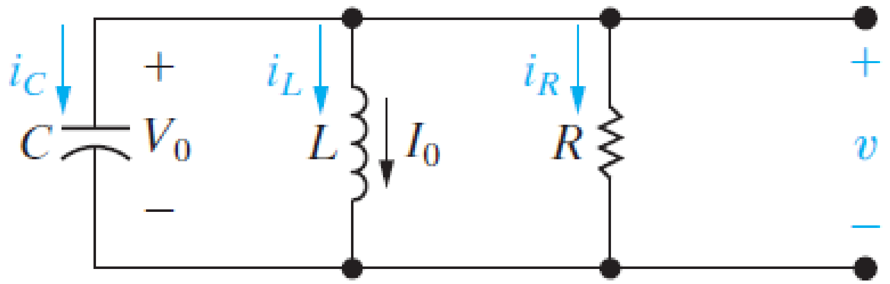
Step response
of a series RLC circuit

In Chapter 8

	Inductors	Capacitors
Primary v-i equation	$v(t) = L\frac{di(t)}{dt}$	$i(t) = C\frac{dv(t)}{dt}$
Alternate v-i equation	$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$	$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$
Initial condition	$i(t_0)$	$v(t_0)$
Behavior with a constant source	If $i(t) = I, v(t) = 0$ and the inductor behaves like a short circuit	If $v(t) = V, i(t) = 0$ and the capacitor behaves like an open circuit
Continuity requirement	$i(t)$ is continuous for all time so $v(t)$ is finite	$v(t)$ is continuous for all time so $i(t)$ is finite
Power equation	$p(t) = v(t)i(t) = Li(t)\frac{di(t)}{dt}$	$p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$
Energy equation	$w(t) = \frac{1}{2}Li(t)^2$	$w(t) = \frac{1}{2}Cv(t)^2$
Series-connected equivalent	$L_{eq} = \sum_{j=1}^{n} L_j$ $i_{eq}(t_0) = i_j(t_0)$ for all j	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ $v_{eq}(t_0) = \sum_{j=1}^{n} v_j(t_0)$
Parallel-connected equivalent	$\frac{1}{L_{eq}} = \sum_{j=1}^{n} \frac{1}{L_j}$ $i_{eq}(t_0) = \sum_{j=1}^{n} i_j(t_0)$	$C_{eq} = \sum_{j=1}^{n} C_j$ $v_{eq}(t_0) = v_j(t_0)$ for all j

Table 6.1

Introduction to Natural Response of Parallel RLC Circuit



Natural response
of a parallel RLC circuit

$$i_R + i_L + i_C = 0$$

$$\frac{v}{R} + \frac{1}{L}\int_0^t v\,d\tau + I_0 + C\frac{dv}{dt} = 0$$

$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} + C\frac{d^2v}{dt^2} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$

Ordinary second order differential equation
with constant coefficients

Introduction to Natural Response of Parallel RLC Circuit

General Solution of the Second Order Differential Equation (1)

$$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0} \longrightarrow v = v_1 + v_2 = A_1e^{s_1t} + A_2e^{s_2t}$$

$$v = Ae^{st}$$

$$As^2e^{st} + \frac{As}{RC}e^{st} + \frac{Ae^{st}}{LC} = 0$$

$$Ae^{st}\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right) = 0$$

$$\because e^{st} \neq 0, A \neq 0$$

$$\boxed{s^2 + \frac{s}{RC} + \frac{1}{LC} = 0}$$ Characteristic equation

$$\boxed{s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}\ ,\ s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}}$$

Introduction to Natural Response of Parallel RLC Circuit

General Solution of the Second Order Differential Equation (2)

$$\boxed{\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{v}{LC}=0} \longrightarrow v=v_1+v_2=A_1e^{s_1t}+A_2e^{s_2t}$$ (증명)

$$\frac{dv}{dt}=A_1s_1e^{s_1t}+A_2s_2e^{s_2t}$$

$$\frac{d^2v}{dt^2}=A_1s_1^2e^{s_1t}+A_2s_2^2e^{s_2t}$$

$$A_1e^{s_1t}\left(s_1^2+\frac{1}{RC}s_1+\frac{1}{LC}\right)+A_2e^{s_2t}\left(s_2^2+\frac{1}{RC}s_2+\frac{1}{LC}\right)=0$$

$$\boxed{\therefore v=A_1e^{s_1t}+A_2e^{s_2t}}$$ Solution

Introduction to Natural Response of Parallel RLC Circuit

General Solution of the Second Order Differential Equation (3)

$$\boxed{s^2 + \frac{s}{RC} + \frac{1}{LC} = 0}$$ Characteristic equation

$$\boxed{s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}\ ,\ s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2},\ \ s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Neper Frequency (감쇠 주파수)

$$\boxed{\alpha = \frac{1}{2RC}}$$

Resonant Radian Frequency (공진 주파수)

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

Unit [rad/s]

Introduction to Natural Response of Parallel RLC Circuit

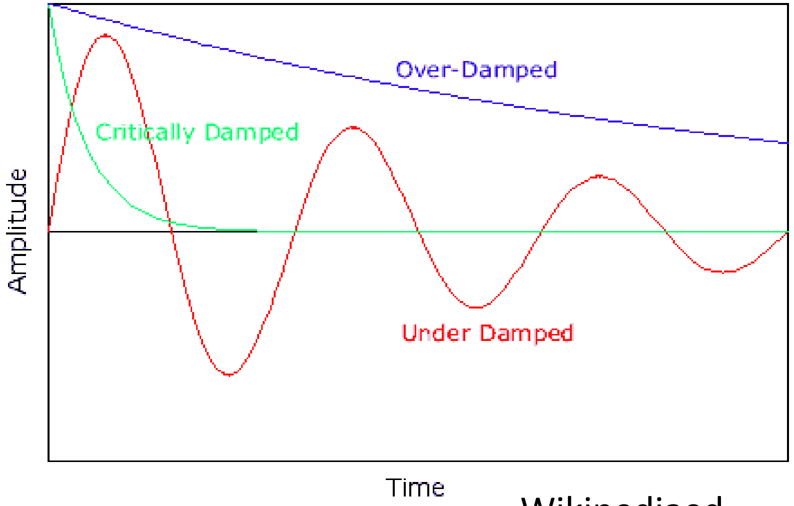
General Solution of the Second Order Differential Equation (4)

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \qquad \alpha = \frac{1}{2RC} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$\omega_0^2 < \alpha^2$ Voltage response overdamped (과 감쇠)

$\omega_0^2 > \alpha^2$ Voltage response underdamped (부족 감쇠)

$\omega_0^2 = \alpha^2$ Voltage response critically damped (임계 감쇠)

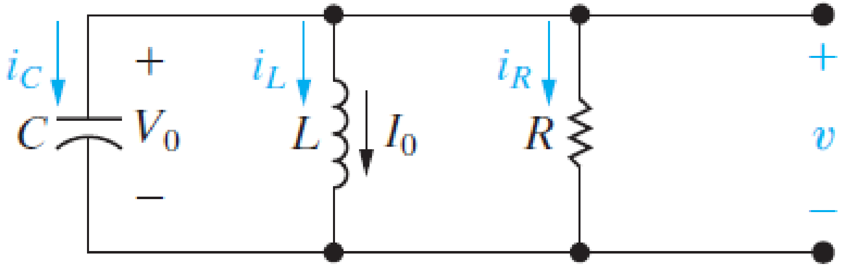


Wikipediaed

>> Damping affects the way the voltage response reaches its final (or steady-state) value

Example 8.1

(a) Find the roots of the characteristic equation, R=200ohm, L=50mH, C=0.2uH
(b) Will the response be overdamped, underdamped, or critically damped?
(c) Repeat (a) and (b) for R=312.5ohm
(d) What value of R causes the response to be critically damped?



Example 8.1

(a) Find the roots of the characteristic equation, R=200ohm, L=50mH, C=0.2uH
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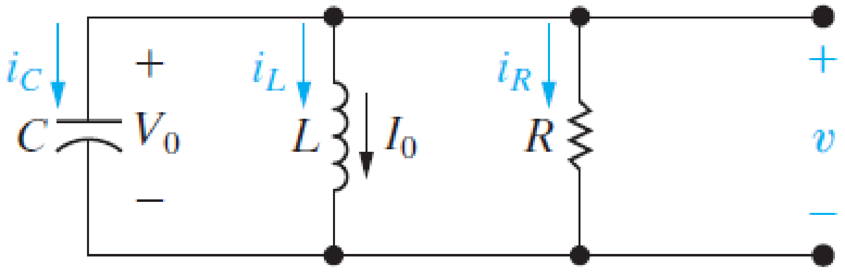
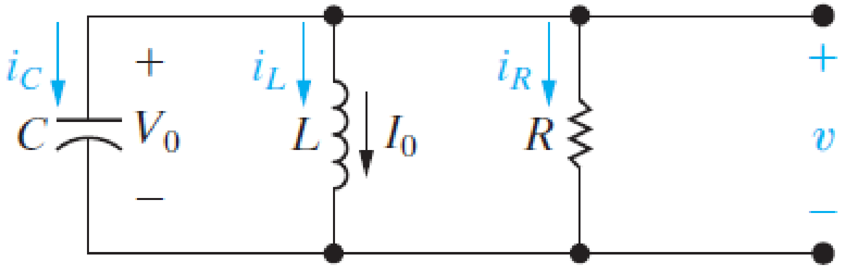


$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2},\quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Example 8.1

(c) Repeat (a) and (b) for R=312.5ohm

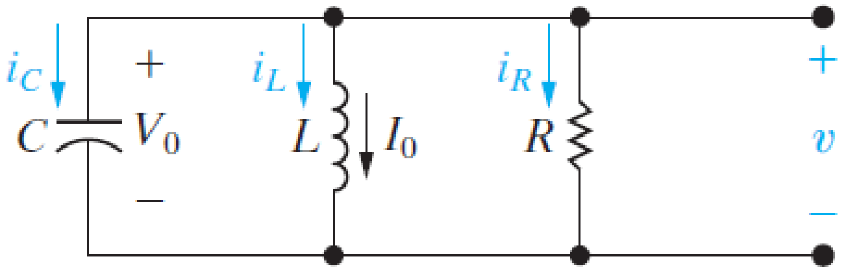


$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

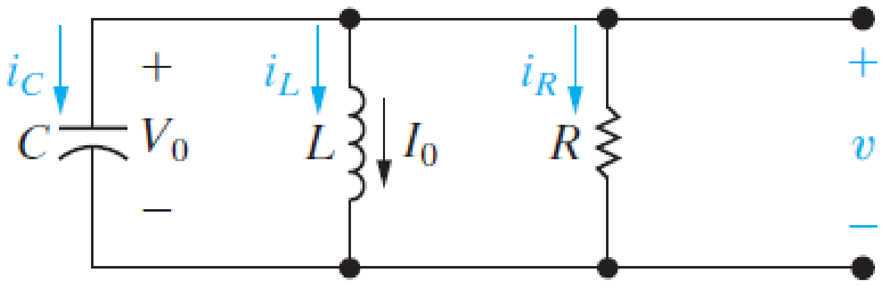
Example 8.1

(d) What value of R causes the response to be critically damped?



Summary (Part 1)

- Natural Response of a Parallel RLC Circuit



Natural response
of a parallel RLC circuit

Parameter	Terminology	Value in Natural Response
s_1, s_2	Characteristic roots	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
α	Neper frequency	$\alpha = \dfrac{1}{2RC}$
ω_0	Resonant radian frequency	$\omega_0 = \dfrac{1}{\sqrt{LC}}$

The Forms of the Natural Response of a Parallel RLC Circuits

(1) $\omega_0^2 < \alpha^2$ Overdamped voltage response (과 감쇠)

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\boxed{v = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$

Initial conditions

i) $v(0^+) = A_1 + A_2, \Rightarrow v(0^+) = V_0$

ii) $\dfrac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2, \Rightarrow \dfrac{dv(0^+)}{dt} = \dfrac{i_C(0^+)}{C}$

$\left(i_C(0^+) = -\dfrac{V_0}{R} - I_0 \right)$

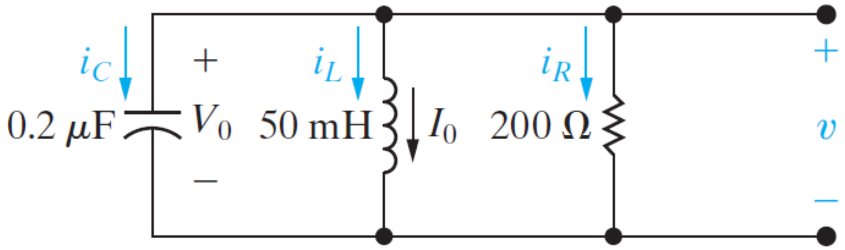
$$A_1 + A_2 = V_0$$

$$s_1 A_1 + s_2 A_2 + \frac{I_0}{C} + \frac{V_0}{RC} = 0$$

2 equations, 2 unknowns

Example 8.2

V(0+)=12V, i_L(0+)=30mA
Find the expression for v(t)



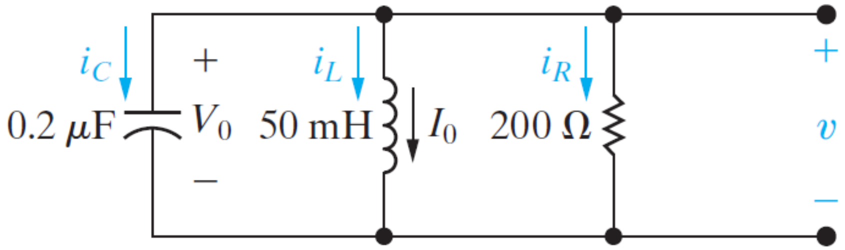
$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2},\ \ s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Example 8.2

V(0+)=12V, i_L(0+)=30mA
Find the expression for v(t)

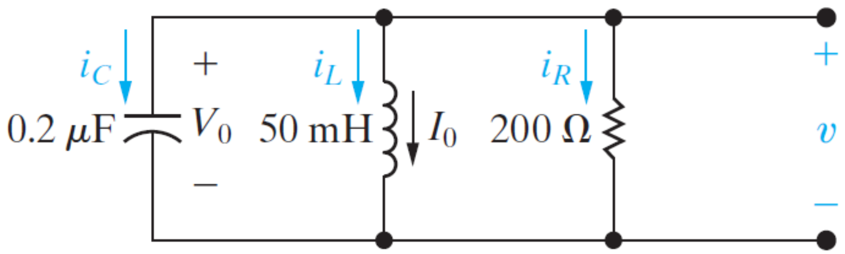


$$v(0^+) = A_1 + A_2, \qquad v(0^+) = V_0$$

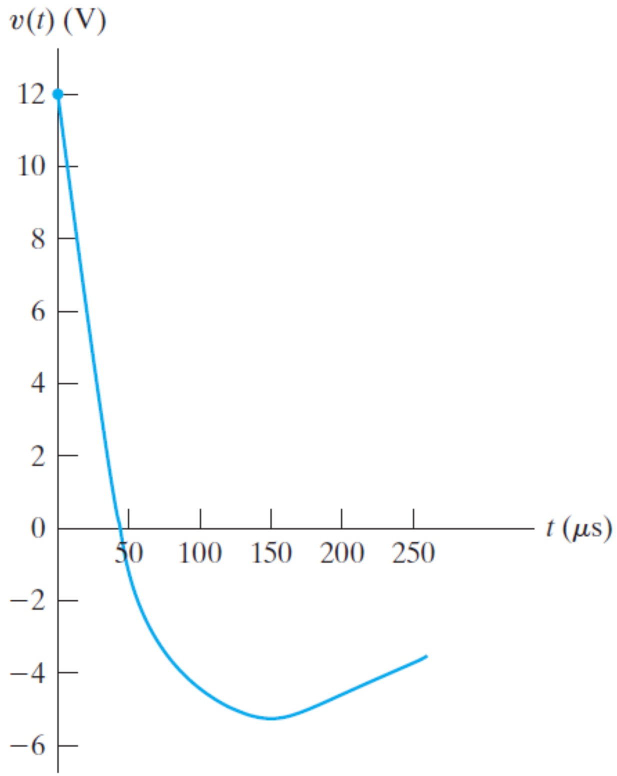
$$\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2, \quad \frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

Example 8.2/8.3

V(0+)=12V, i_L(0+)=30mA
Derive the expression for i_R, i_L, i_C



The Forms of the Natural Response of a Parallel RLC Circuits



Overdamped
Ex 8.2

The Forms of the Natural Response of a Parallel RLC Circuits

(2) $\omega_0^2 > \alpha^2$ Underdamped voltage response (부족 감쇠)

$$s_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha + j\underbrace{\sqrt{\omega_0^2 - \alpha^2}}_{w_d}$$

$$= -\alpha + j\omega_d$$

$$s_2 = -\alpha - j\omega_d$$

=Damped Radian Frequency

$$\boxed{v = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$

$$v(t) = A_1 e^{(-\alpha + j\omega_d)t} + A_2 e^{-(\alpha + j\omega_d)t}$$

$$= A_1 e^{-\alpha t} e^{j\omega_d t} + A_2 e^{-\alpha t} e^{-j\omega_d t}$$

$$= e^{-\alpha t}(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t})$$

Euler's identity: $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

$$= e^{-\alpha t}(A_1 \cos\omega_d t + jA_1 \sin\omega_d t$$
$$+ A_2 \cos\omega_d t - jA_2 \sin\omega_d t)$$

$$= e^{-\alpha t}[(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t]$$

The Forms of the Natural Response of a Parallel RLC Circuits

(2) $\omega_0^2 > \alpha^2$ Underdamped voltage response (부족 감쇠)

$$B_1 = A_1 + A_2, \quad B_2 = j(A_1 - A_2)$$

$$v(t) = e^{-\alpha t}[B_1 \cos \omega_d t + B_2 \sin \omega_d t]$$

$$\boxed{v(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t}$$

Damped Radian Frequency

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Initial conditions

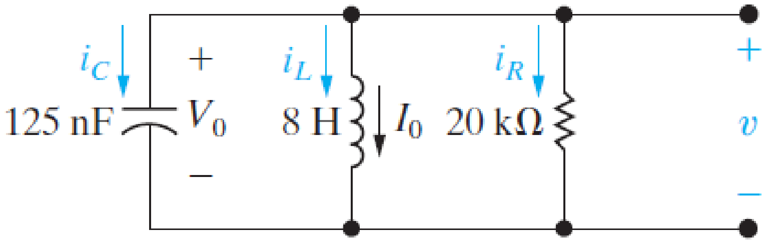
i) $v(0^+) = V_0 = B_1$

ii) $\dfrac{dv(0^+)}{dt} = \dfrac{i_C(0^+)}{C} = -\alpha B_1 + \omega_d B_2$

2 equations, 2 unknowns

Example 8.4

V_0=0, I_0=-12.25 mA
Calculate the voltage response t>=0



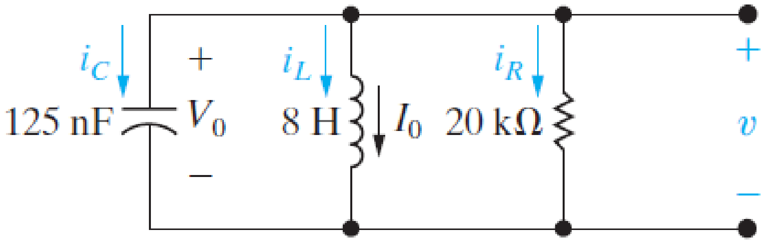
$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha + j\sqrt{\omega_0^2 - \alpha^2}$$
$$= -\alpha + j\omega_d$$
$$s_2 = -\alpha - j\omega_d$$
$$\alpha = \frac{1}{2RC} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$v(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$$

Example 8.4

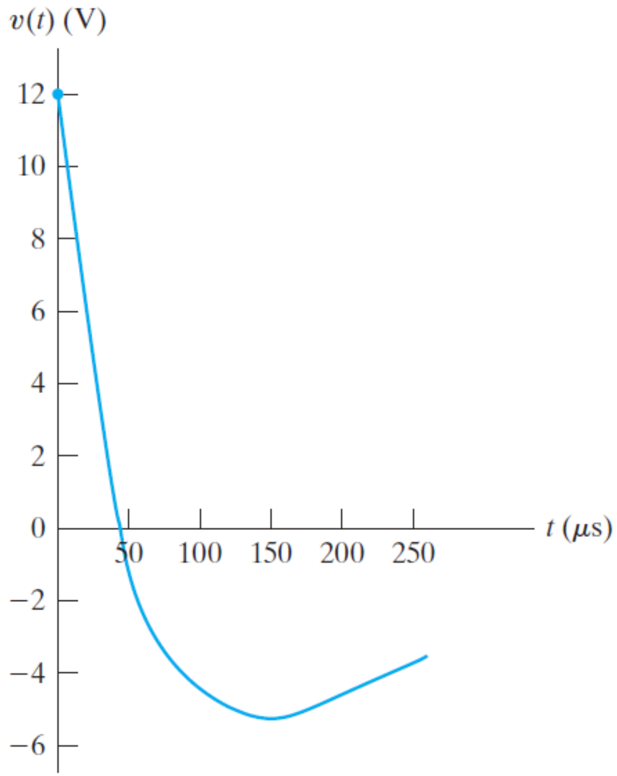
V_0=0, I_0=-12.25 mA
Calculate the voltage response t>=0

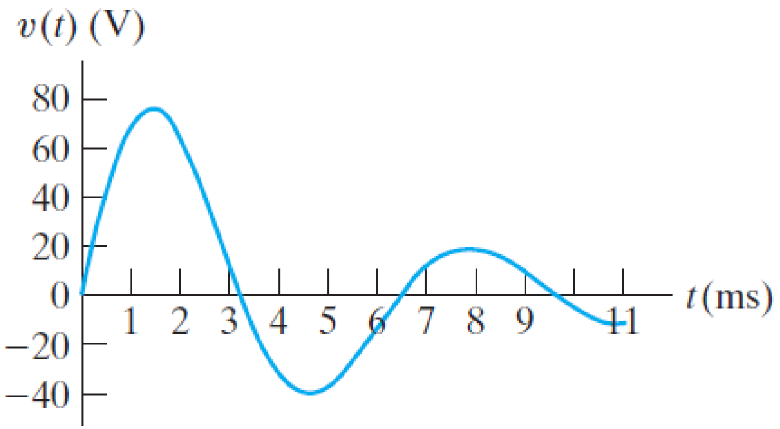


$$v(0^+) = V_0 = B_1$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = -\alpha B_1 + \omega_d B_2$$

The Forms of the Natural Response of a Parallel RLC Circuits





Overdamped
Ex 8.2

Underdamped
Ex 8.4

The Forms of the Natural Response of a Parallel RLC Circuits

(3) $\omega_0^2 = \alpha^2$ Critically damped voltage response (임계 감쇠)

$$s_1 = s_2 = -\alpha = -\frac{1}{2RC}$$

$$v_1 = D_1 t e^{st}, v_2 = D_2 e^{st}$$

$$\boxed{v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}}$$

Initial conditions

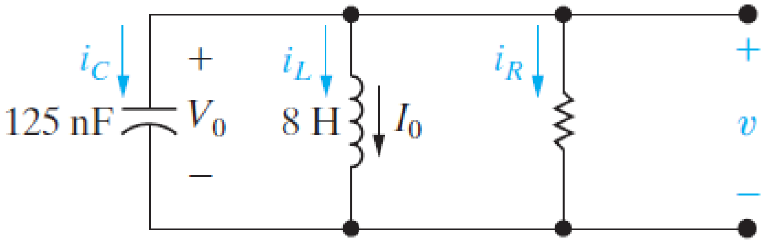
i) $v(0^+) = V_0 = D_2$

ii) $\dfrac{dv(0^+)}{dt} = \dfrac{i_C(0^+)}{C} = D_1 - \alpha D_2$

2 equations, 2 unknowns

Example 8.5

(a) Find the value of R that results in a critically damped voltage response
(b) Calculate v(t) for t>=0



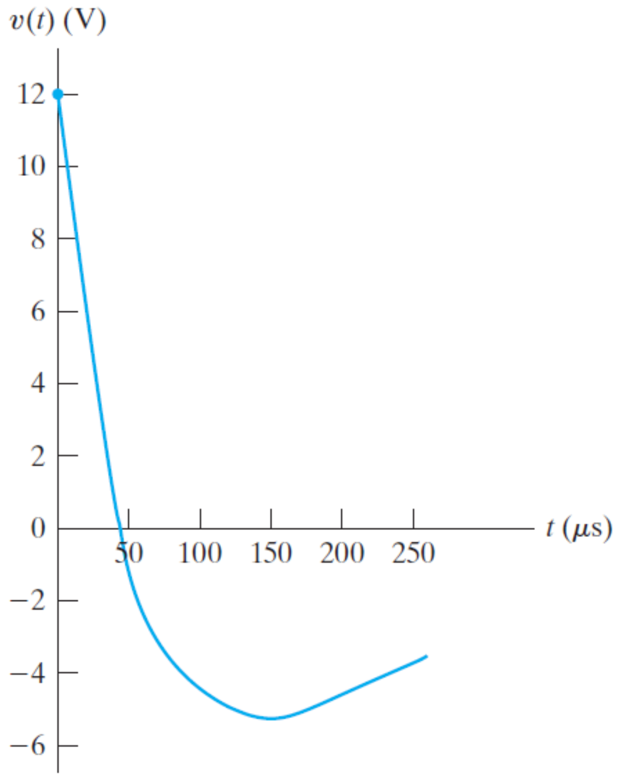
$$s_1 = s_2 = -\alpha = -\frac{1}{2RC}$$

$$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

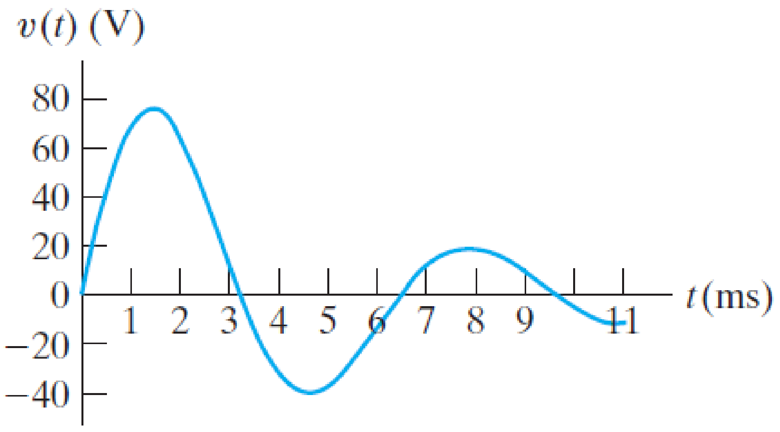
$$v(0^+) = V_0 = D_2$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = D_1 - \alpha D_2$$

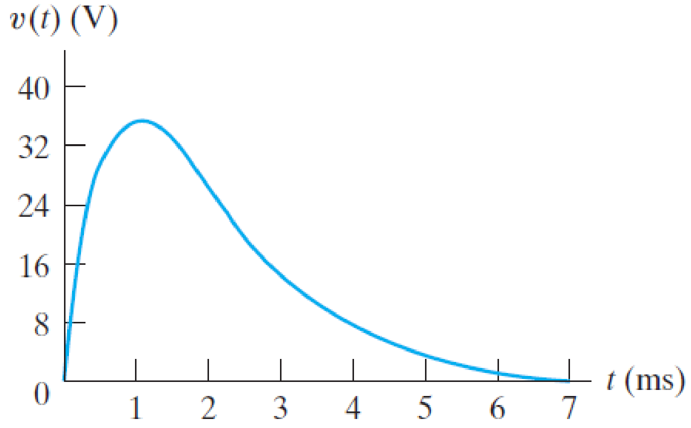
The Forms of the Natural Response of a Parallel RLC Circuits



Overdamped
Ex 8.2



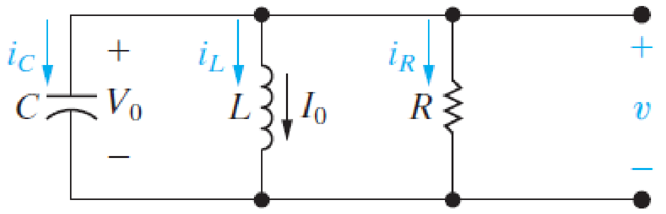
Underdamped
Ex 8.4



Critically damped
Ex 8.5

Summary (Part 2)

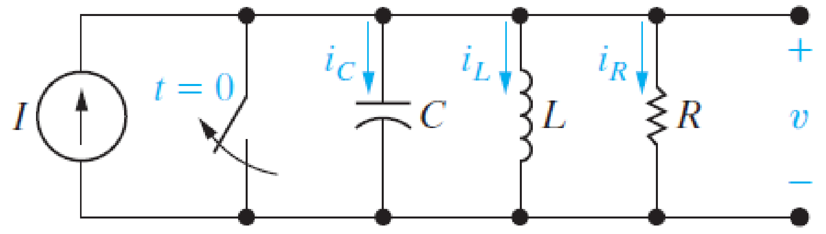
- Natural Response of a Parallel RLC Circuit



Characteristic equation	$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$
Neper, resonant, and damped frequencies	$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$v(t) = A_1e^{s_1t} + A_2e^{s_2t}$, t ≥ 0 $v(0^+) = A_1 + A_2 = V_0$ $\frac{dv(0^+)}{dt} = s_1A_1 + s_2A_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$
$\alpha^2 < \omega_0^2$: underdamped	$v(t) = B_1e^{-\alpha t}\cos\omega_d t + B_2e^{-\alpha t}\sin\omega_d t$, t ≥ 0 $v(0^+) = B_1 = V_0$ $\frac{dv(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$
$\alpha^2 = \omega_0^2$: critically damped	$v(t) = D_1te^{-\alpha t} + D_2e^{-\alpha t}$, t ≥ 0 $v(0^+) = D_2 = V_0$ $\frac{dv(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$

Table 8.2

Step Response of a Parallel RLC Circuit



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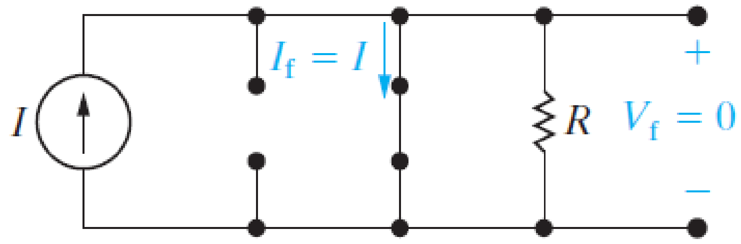


Step response
of a parallel RLC circuit

$$i_L + i_R + i_C = I$$

$$i_L + \frac{v}{R} + C\frac{dv}{dt} = I \qquad v = L\frac{di_L}{dt}, \frac{dv}{dt} = L\frac{d^2 i_L}{dt^2}$$

$$i_L + \frac{L}{R}\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} = I$$

$$\boxed{\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I}{LC}}$$

Ordinary second order differential equation
with constant coefficients

Step Response of a Parallel RLC Circuit

i) Indirect approach

$$i_L + \frac{v}{R} + C\frac{dv}{dt} = I$$

$$\frac{1}{L}\int_0^t v\,d\tau + \frac{v}{R} + C\frac{dv}{dt} = I$$

$$\frac{v}{L} + \frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} = 0$$

$$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0}$$

Note:

$$C\frac{dv}{dt} + i_L + \frac{v}{R} = I$$

$$i_L = I - C\frac{dv}{dt} - \frac{v}{R}$$

- $v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$
- $v = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$
- $v = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

Step Response of a Parallel RLC Circuit

i) Indirect approach

$$i_L + \frac{v}{R} + C\frac{dv}{dt} = I$$

$$\frac{1}{L}\int_0^t v\,d\tau + \frac{v}{R} + C\frac{dv}{dt} = I$$

$$\frac{v}{L} + \frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} = 0$$

$$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0}$$

Note:

$$C\frac{dv}{dt} + i_L + \frac{v}{R} = I$$

$$i_L = I - C\frac{dv}{dt} - \frac{v}{R}$$

- $v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$
- $v = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$
- $v = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

- $i_L = I + A_1' e^{s_1 t} + A_2' e^{s_2 t}$
- $i_L = I + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$
- $i_L = I + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$

Step Response of a Parallel RLC Circuit

ii) Direct approach

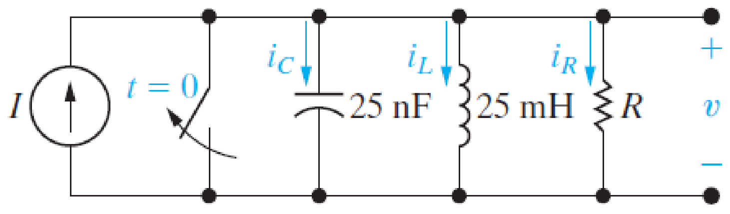
$$\boxed{\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I}{LC}}$$

$$I = I_f + (function\ of\ the\ same\ form\ as\ the\ natural\ response)$$

$$V = V_f + (function\ of\ the\ same\ form\ as\ the\ natural\ response)$$

Example 8.6

DC current source of 24 mA is applied. The value of the resistor is R=400ohm.
Find i_L for t>=0.

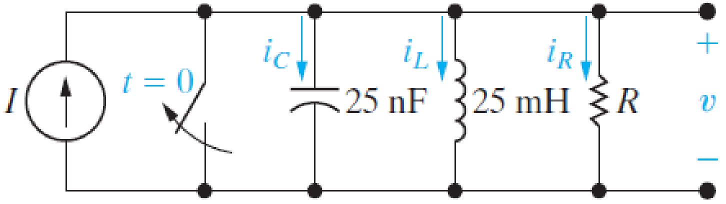


$$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

- $i_L = I + A_1' e^{s_1 t} + A_2' e^{s_2 t}$
- $i_L = I + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$
- $i_L = I + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$

Example 8.7

DC current source of 24 mA is applied. The value of the resistor is R=625ohm.
Find i_L for t>=0.

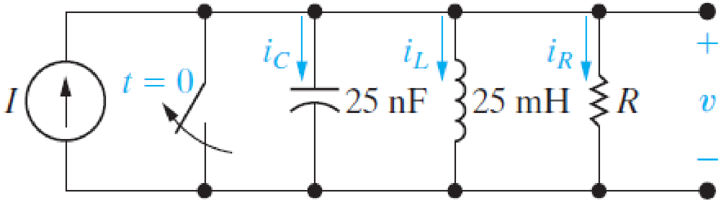


$$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

- $i_L = I + A_1' e^{s_1 t} + A_2' e^{s_2 t}$
- $i_L = I + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$
- $i_L = I + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$

Example 8.8

DC current source of 24 mA is applied. The value of the resistor is R=500ohm.
Find i_L for t>=0.

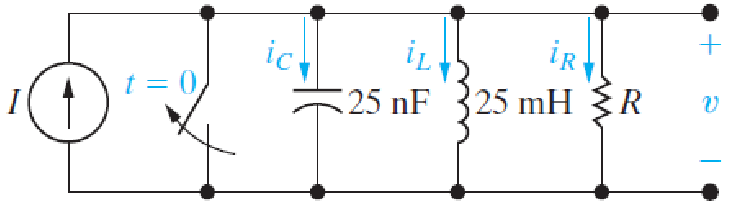


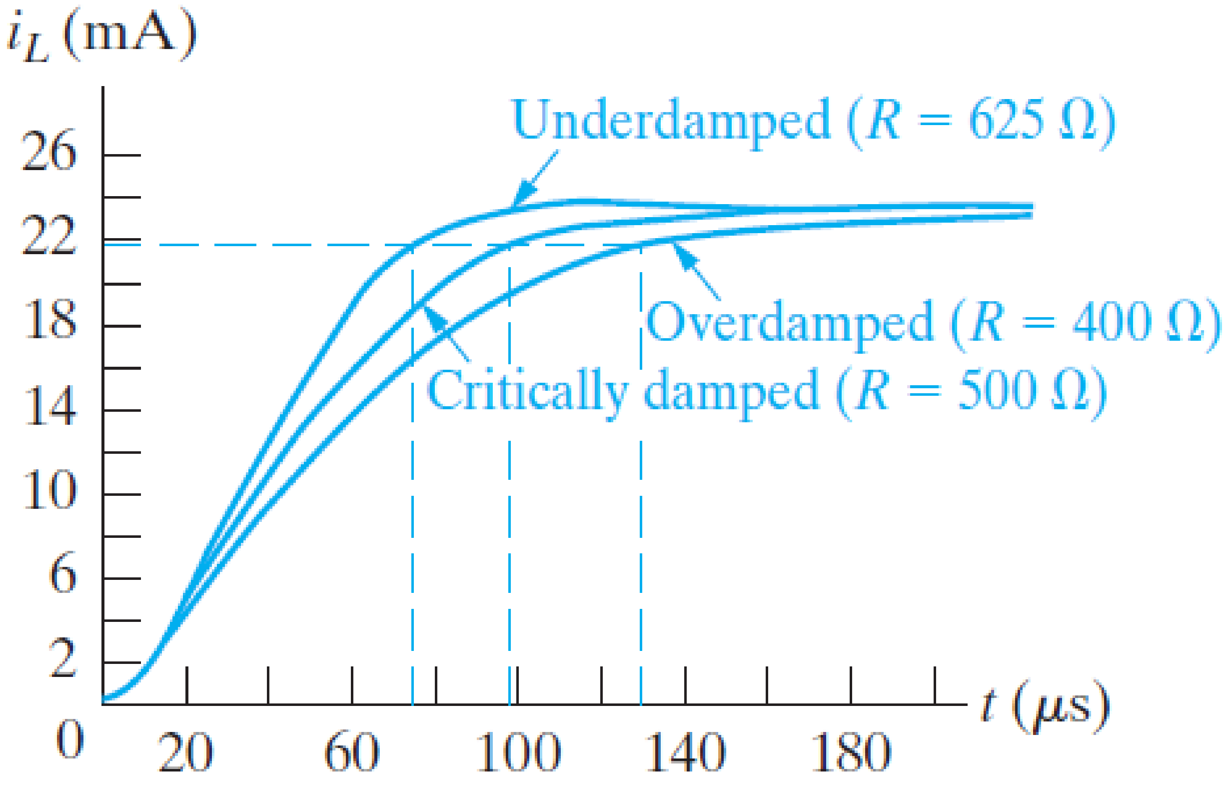
$$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

- $i_L = I + A_1' e^{s_1 t} + A_2' e^{s_2 t}$
- $i_L = I + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$
- $i_L = I + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$

Example 8.9

Example 8.6, 8.7, and 8.8





Summary

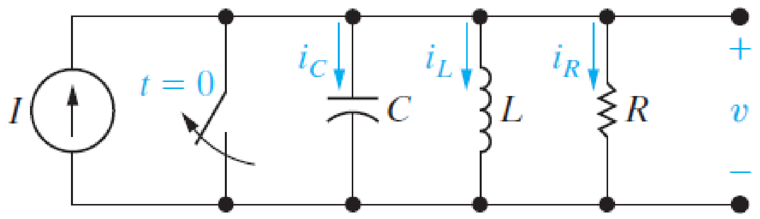
- Natural Response of a Parallel RLC Circuit
- Step Response of a Parallel RLC Circuit

$$I = I_f + (function\ of\ the\ same\ form\ as\ the\ natural\ response)$$

$$V = V_f + (function\ of\ the\ same\ form\ as\ the\ natural\ response)$$

Summary (Part 3)

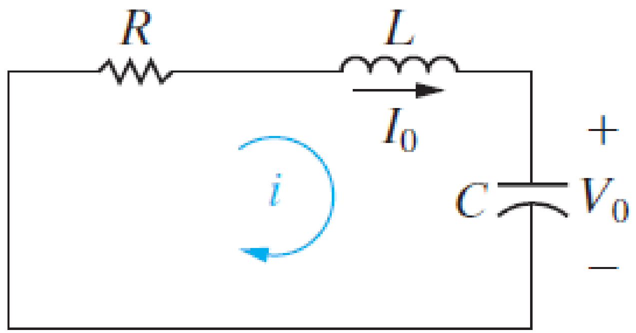
- Step Response of a Parallel RLC Circuit



Characteristic equation	$s^2 + \frac{1}{RC}s + \frac{1}{LC} = \frac{I}{LC}$
Neper, resonant, and damped frequencies	$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$i_L(t) = I_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}, \quad t \geq 0$ $i_L(0^+) = I_f + A_1' + A_2' = I_0$ $\frac{di_L(0^+)}{dt} = s_1 A_1' + s_2 A_2' = \frac{V_0}{L}$
$\alpha^2 < \omega_0^2$: underdamped	$i_L(t) = I_f + B_1' e^{-\alpha t}\cos \omega_d t + B_2' e^{-\alpha t}\sin \omega_d t, \quad t \geq 0$ $i_L(0^+) = I_f + B_1' = I_0$ $\frac{di_L(0^+)}{dt} = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$
$\alpha^2 = \omega_0^2$: critically damped	$i_L(t) = I_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}, \quad t \geq 0$ $i_L(0^+) = I_f + D_2' = I_0$ $\frac{di_L(0^+)}{dt} = D_1' - \alpha D_2' = \frac{V_0}{L}$

Table 8.3

Natural Response of a Series RLC Circuit



Natural response
of a series RLC circuit

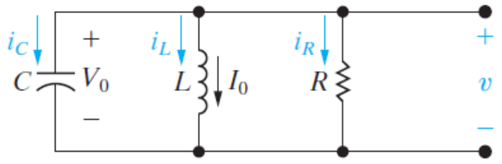
$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i d\tau + V_0 = 0$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0}$$

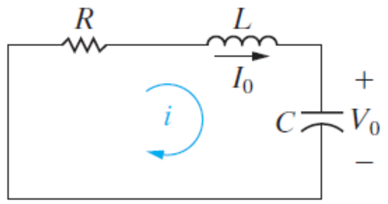
Ordinary second order differential equation
with constant coefficients

Natural Response of a Series RLC Circuit



Parallel RLC circuit $$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0 \rightarrow s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \qquad \alpha = \frac{1}{2RC} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$



Series RLC circuit $$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \qquad \boxed{\alpha = \frac{R}{2L} \quad \omega_0 = \frac{1}{\sqrt{LC}}}$$

Natural Response of a Series RLC Circuit

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \qquad \alpha = \frac{R}{2L} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$\omega_0^2 < \alpha^2$ Overdamped voltage response (과 감쇠)

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

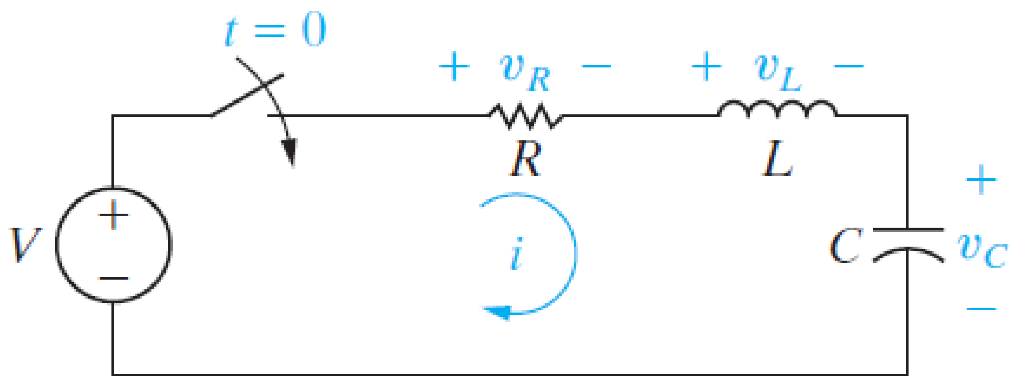
$\omega_0^2 > \alpha^2$ Underdamped voltage response (부족 감쇠)

$$i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$$

$\omega_0^2 = \alpha^2$ Critically damped voltage response (임계 감쇠)

$$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

Step Response of a Series RLC Circuit



Step response
of a series RLC circuit

$$V = v_R + v_L + v_C$$

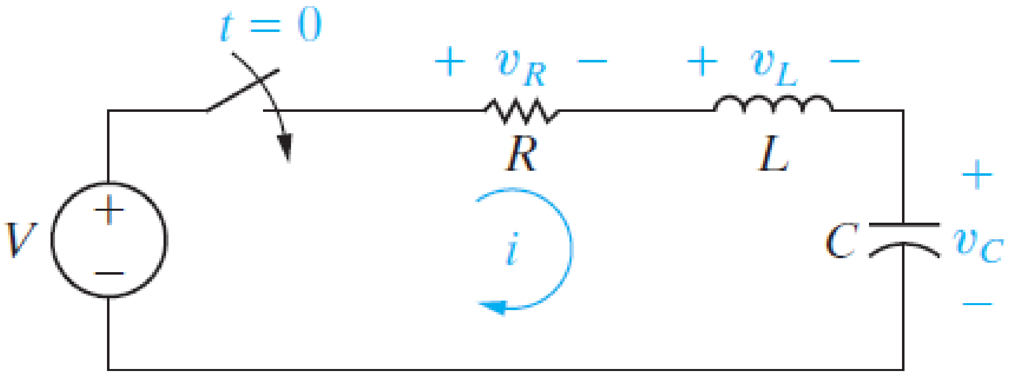
$$V = Ri + L\frac{di}{dt} + v_C \qquad i = C\frac{dv_C}{dt}, \frac{di}{dt} = C\frac{d^2 v_C}{dt^2}$$

$$v_C + RC\frac{dv_C}{dt} + LC\frac{d^2 v_C}{dt^2} = V$$

$$\boxed{\frac{d^2 v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{v_C}{LC} = \frac{V}{LC}}$$

Ordinary second order differential equation
with constant coefficients

Step Response of a Series RLC Circuit



Step response
of a series RLC circuit

$$\boxed{\frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{v_C}{LC} = \frac{V}{LC}}$$

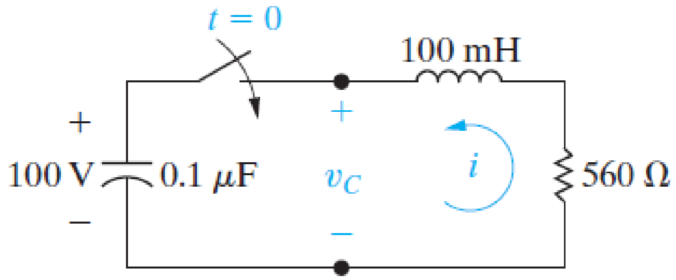
$$v_C = V_f + A_1'e^{s_1t} + A_2'e^{s_2t}$$

$$v_C = V_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$$

$$v_C = V_f + D_1'te^{-\alpha t} + D_2'e^{-\alpha t}$$

Example 8.11

(a) Find i(t) for t>=0 (b) Find $V_c(t)$ for t>=0.



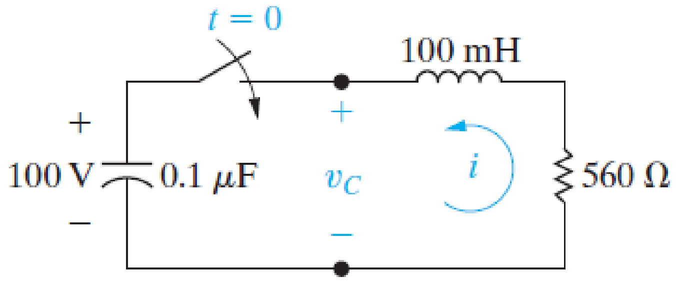
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$i(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$$
$$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

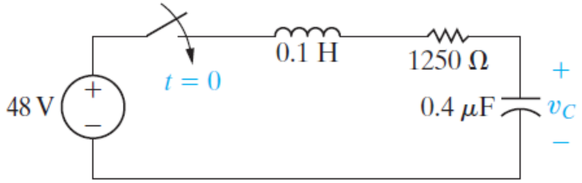
Example 8.11

(a) Find i(t) for t>=0 (b) Find $V_c(t)$ for t>=0.



Example 8.12

Find $V_c(t)$ for t>=0.



$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$v_C = V_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}$$
$$v_C = V_f + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$$
$$v_C = V_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$$

Summary (Part 4) Natural response of series RLC Circuit

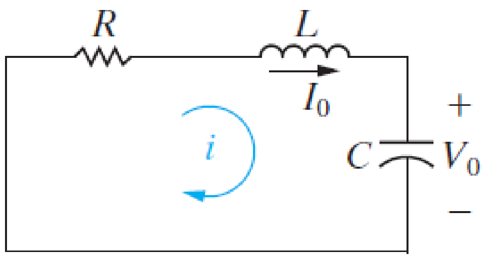
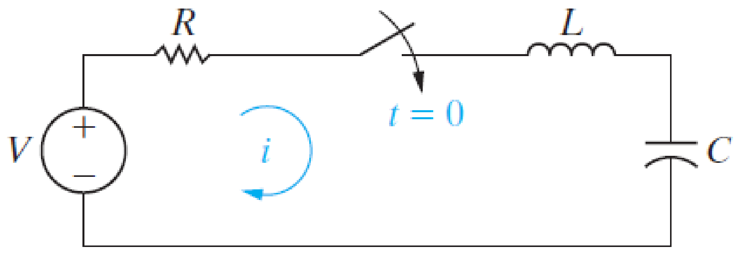


Table 8.4

Characteristic equation	$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$
Neper, resonant, and damped frequencies	$\alpha = \frac{R}{2L} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t \geq 0$ $i(0^+) = A_1 + A_2 = I_0$ $\frac{di(0^+)}{dt} = s_1 A_1 + s_2 A_2 = \frac{1}{L}(-RI_0 - V_0)$
$\alpha^2 < \omega_0^2$: underdamped	$i(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t, \quad t \geq 0$ $i(0^+) = B_1 = I_0$ $\frac{di(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{1}{L}(-RI_0 - V_0)$
$\alpha^2 = \omega_0^2$: critically damped	$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}, \quad t \geq 0$ $i(0^+) = D_2 = I_0$ $\frac{di(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{L}(-RI_0 - V_0)$

Summary (Part 4) Step response of series RLC Circuit



Characteristic equation	$s^2 + \frac{R}{L}s + \frac{1}{LC} = \frac{V}{LC}$
Neper, resonant, and damped frequencies	$\alpha = \frac{R}{2L} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$v_C(t) = V_f + A_1'e^{s_1 t} + A_2'e^{s_2 t}, t \geq 0$ $v_C(0^+) = V_f + A_1' + A_2' = V_0$ $\frac{dv_C(0^+)}{dt} = s_1A_1' + s_2A_2' = \frac{I_0}{C}$
$\alpha^2 < \omega_0^2$: underdamped	$v_C(t) = V_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t, t \geq 0$ $v_C(0^+) = V_f + B_1' = V_0$ $\frac{dv_C(0^+)}{dt} = -\alpha B_1' + \omega_d B_2' = \frac{I_0}{C}$
$\alpha^2 = \omega_0^2$: critically damped	$v_C(t) = V_f + D_1'te^{-\alpha t} + D_2'e^{-\alpha t}, t \geq 0$ $v_C(0^+) = V_f + D_2' = V_0$ $\frac{dv_C(0^+)}{dt} = D_1' - \alpha D_2' = \frac{I_0}{C}$

Table 8.5