# Chapter 5

# *Converting Between Analog & Digital Signals*

## 5.1 Problems

**Problem 5.1 (Sampling audio waveforms)** *A talking book generates a digital audio waveform that was generated by sampling the original audio waveform* $2,000$ *times a s. What is the maximum allowed frequency for proper operation?*

*(ans:*

$$f_s = 2,000\ Hz \quad \rightarrow \quad f_{max} < \frac{f_s}{2} = 1,000\ Hz$$

*)*

**Problem 5.2 (Cell phone audio waveforms)** *The audio signals on your cell phone occupy the frequency range from 300Hz to 3,400Hz. What sampling frequency would you specify for digitizing the voice waveform?*

*(ans:*

$$f_{max} = 3,400\ Hz \quad \rightarrow \quad f_s > 2f_{max} = 6,800\ Hz$$

*Many systems use* $f_s = 8,000\ Hz$ *as the standard.*
*)*

**Problem 5.3 (Cell phone HD Voice waveforms)** *The HD Voice systems for cell phone systems occupy the frequency range from 50Hz to 7,000Hz and permit high-quality speech to be transmitted. What sampling frequency would you specify for digitizing the voice waveform?*

*(ans:*

$$f_{max} = 7,000\ Hz \quad \rightarrow \quad f_s > 2f_{max} = 14,000\ Hz$$

*A reasonable value falls in the range* $15,000 < f_s < 20,000$ *Hz.*
*)*



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**Problem 5.4 (Aliasing)** *Let the sampling frequency $f_s$ = 10,000 Hz with the result that the alias frequency $f_a$ = 1,000 Hz is observed. What two values of the original frequency $f_o$ could have produced this alias frequency?*

*(ans:*

$$f_a = |f_s - f_o| = |10\,kHz - f_o| = 1\,kHz \quad \to 10\,kHz - f_o = \pm 1\,kHz$$

*or*

$$f_o = (10 \pm 1)\,kHz$$

*Hence, the possible values are $f_o = 9$ kHz and $f_o = 1.1$ kHz.*
*)*

**Problem 5.5 (Maximum spacial frequencies)** *Your old cellphone display has a 180 rows and 320 columns. What are the maximum horizontal and vertical spacial frequencies present in an icon image that would produce an un-aliased image on that display?*

*(ans: In the horizontal (row) direction there are 320 columns. The maximum frequency is then*

$$f_{H,max} < 160 \text{ cycles/row}$$

*In the vertical (column) direction there are 180 rows. The maximum frequency is then*

$$f_{V,max} < 90 \text{ cycles/column}$$

*)*

**Problem 5.6 ($2014_{10}$ in binary)** *What is the binary representation of $2014_{10}$?*

*(ans:*
*Starting with the smallest power of 2 value that does not exceed 2014, we begin subtracting powers of 2. If subtraction produces positive result, the corresponding bit = 1; if not, we do not perform subtraction and make the corresponding bit = 0.*

| | | |
|---|---|---|
| $2014 - 1024$ | $= 990$ | 1 *(MSB)* |
| $990 - 512$ | $= 478$ | 1 |
| $478 - 256$ | $= 222$ | 1 |
| $222 - 128$ | $= 94$ | 1 |
| $94 - 64$ | $= 30$ | 1 |
| $30 - 32$ | $= -2$ | 0 |
| $30 - 16$ | $= 14$ | 1 |
| $14 - 8$ | $= 6$ | 1 |
| $6 - 4$ | $= 2$ | 1 |
| $2 - 2$ | $= 0$ | 1 |
| $0 - 1$ | $= -1$ | 0 *(LSB)* |

*Hence,*

$$2014_{10} = 11111011110_2$$

*)*

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**Problem 5.7** ($01011010_2$ **in decimal)** *What is the decimal representation of* $010101010_2$*?*

*(ans: Starting from the LSB and including the powers of 2 that correspond to the 1-bit values gives*

$$01011010_2 = 2 + 8 + 16 + 64 = 90_{10}$$

*)*

**Problem 5.8 (Magic cards)** Magic cards *allow you to guess a number between 0 and 15 that a person was thinking of by answering whether the number appeared on a set of four cards. These cards appear in Figure 5.1 with one number replaced by dashes in each card. What are the missing numbers?*
*(ans: The missing numbers have a binary representation with a 1 in the position corresponding to the*

| | | |
|---|---|---|
| 1 | 3 | 5 |
| 7 | __ | 11 |
| 13 | 15 | |

| | | |
|---|---|---|
| 2 | 3 | 6 |
| 7 | 10 | __ |
| 14 | 15 | |

| | | |
|---|---|---|
| 4 | 5 | __ |
| 7 | 12 | 13 |
| 14 | 15 | |

| | | |
|---|---|---|
| 8 | 9 | 10 |
| 11 | 12 | 13 |
| __ | 15 | |

Figure 5.1: Magic cards used in Problem 5.8.

*number in the shaded square. For example, in the first box (1 box) all numbers have binary numbers that end in 1 (xxx1), in the 2 box (xx1x), in the 4 box (x1xx), and in the 8 box (1xxx).*

*The missing number is the value obtained by adding the smallest number that results in a 1 in the corresponding bit location. For example, in the 1 box (xxx1)* $710 = 0111_2$*, adding 2 gives the next value with a 1 on the LSB, or* $1001_2 = 9$*.*

| | | |
|---|---|---|
| 1 | 3 | 5 |
| 7 | 9 | 11 |
| 13 | 15 | |

| | | |
|---|---|---|
| 2 | 3 | 6 |
| 7 | 10 | 11 |
| 14 | 15 | |

| | | |
|---|---|---|
| 4 | 5 | 6 |
| 7 | 12 | 13 |
| 14 | 15 | |

| | | |
|---|---|---|
| 8 | 9 | 10 |
| 11 | 12 | 13 |
| 14 | 15 | |

*)*

**Problem 5.9 (Video game audio)** *Your computer game uses digitized audio stored as 10 bits per sample. How many voltage levels does this represent? If the audio is reproduced over a range of 0 V to 5 V, what is step size* $\Delta$*? If the audio samples were generated 44,000 times per s in each of two stereo channels, how many bits per s does the system produce to give your game sound?*

*(ans:*
*Ten bits can represent up to* $2^{10} = 1024$ *levels.*

$$\Delta = \frac{5\,V}{1024} = 0.0049\,V = 4.9\,mV$$

*A commonly used approximation is* $2^{10} = 1,000$*, giving* $\Delta = 5$ *mV.*

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*For sampling rate $f_s = 44$ kHz per channel, two 10-bit values are acquired 44 thousand times per s, or*

$$2 \times 10 \times 44,000 = 880,000 \textit{ bps}$$

*)*

**Problem 5.10 (Binary addresses on the Internet)** *Current binary addresses of Websites use 4 bytes. What is the number of possible unique addresses?*

*(ans: Four bytes corresponds to 32 bits, giving*

$$2^{32} = 4 \times \left(2^{10}\right)^3 = 4 \times \left(10^3\right)^3 = 4 \times 10^9$$

*or 4 billion addresses.*
*)*

**Problem 5.11 (CD audio duration)** *An audio CD stores 650 megabytes ($650\ MB$) of data, where $1\ MB = 2^{20}\ bytes = 1,048,576\ bytes$, and 1 byte is an 8-bit data unit. The sampling rate $f_s = 44$ kHz is used with 16-bit quantization. What duration of stereo music (two separate waveforms) can be stored on a CD? Give answer in minutes.*

*(ans: The bit rate of stereo (2 channel) music is*

$$2 \times 16 \textit{ bits/sample } \times 44,000 \textit{ samples/s } = 1,408,000 \textit{ bps}$$

*giving a byte rate equal to*

$$1,408,000 \textit{ bits/s} \times \frac{1 \textit{ byte}}{8 \textit{ bits}} = 176,000 \textit{ Bps}$$

*The music duration is*

$$\frac{650 \times 1,048,576 \textit{ bytes}}{176,000 \textit{ bytes/s}} = 3,872.6 \textit{ s}$$

*Dividing this duration by 60 s/min gives 64.5 minutes.*
*)*

**Problem 5.12 (ADC with staircase quantizer)** *An 8-bit ADC performs a conversion every sampling period $T_s$ = 0.1 ms. What is the period of the staircase $T_\Delta$?*

*(ans: Eight bits gives $2^8 = 256$ levels. A staircase that produces 256 levels in $T_s$ ss requires*

$$T_\Delta = \frac{T_s}{256} = \frac{0.1 \textit{ ms}}{256} = 3.9 \times 10^{-4} \textit{ ms} = 0.39 \,\mu s$$

*)*

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**Problem 5.13 (Quantizing with successive approximation)** *An 8-bit ADC performs a conversion using successive approximation with sampling period $T_s = 0.1\,ms$. What is the conversion time per bit $T_\Delta$?*

*(ans: Successive approximation using $b$ bits requires $b$ comparisons, giving*

$$T_\Delta = \frac{T_s}{8} = \frac{0.1\ ms}{8} = 0.0125\ ms = 12.5\,\mu s$$

*)*

**Problem 5.14 (Boxcar DAC output waveform)** *A digital memory produces the following bit sequence that goes to a 8-bit boxcar DAC that ranges between 0 V and 5 V, and produces a reconstruction every $T_s$ = 0.1 ms. Sketch the reconstructed analog waveform produced by the following bits, providing amplitude and time values in the sketch.*

101000000110011100000111

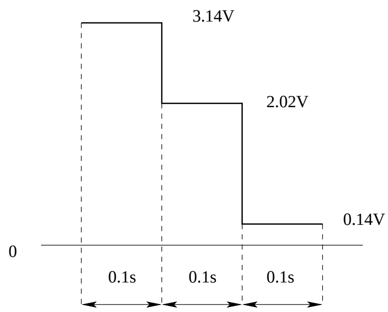
*(ans: The step size for an 8-bit DAC extending from 0 to 5V is*

$$\Delta = \frac{V_{max}}{2^8 - 1} = \frac{5V}{255} = 0.0196V$$

*Dividing the bit sequence into 8-bit units and multiplying by $\Delta$ gives*

$$\overbrace{10100000}^{160 \to 3.14V}\ \overbrace{01100111}^{103 \to 2.02V}\ \overbrace{00000111}^{7 \to 0.14V}$$

*These voltages occur every 0.1 ms to produce the waveform shown.*




*)*

**Problem 5.15 (Musical greeting card with boxcar DAC)** *A musical greeting card plays a 10-s segment of a song when it is opened. If the original audio was processed to remove all frequencies at and above 3 kHz and 5 bits quantize the samples, what is the minimum number of bits stored on the card digital memory?*

*(ans: Using the minimum sampling rate $f_s = 6,000$ Hz gives*

$$6,000\ samples/s \times 5\ bits/sample \times 10\ s = 300,000\ bits$$

*)*

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**Problem 5.16 (PWM duty cycle)** *A PWM waveform extends from 0 V to 5 V and has a period $T_{PWM}$ = 10 ms. What is the duty cycle of the PWM waveform that produces a 1 V average?*

*(ans: To produce 1V from a 5V supply requires a 20% duty cycle. The period $T_{PWM}$ does not matter.*
*)*

**Problem 5.17 (PWM DAC on-time)** *Let $V_{max}$ = 5 V and $T_{PWM}$ = 2 ms. What value of $T_{ON}$ produces $V_{ave}$ = 3.4 V?*

*(ans:*

$$T_{ON} = \frac{V_{ave}}{V_{max}} T_{PWM} = \frac{3.4V}{5V} 2\ ms = 1.36\ ms$$

*)*

**Problem 5.18 (PWM DAC value)** *Let $V_{max}$ = 5 V and $T_{PWM}$ = 2 ms in a PWM DAC that uses 8-bits. What $V_q$ is closest to $V_{ave}$ = 3.4 V?*

*(ans:*

$$T_{ON} = \frac{V_{ave}}{V_{max}} T_{PWM} = \frac{3.4V}{5V} 2\ ms = 1.36\ ms$$

*An 8-bit quantizer requires*

$$T_{\Delta} = \frac{T_{PWM}}{2^8 - 1} = \frac{2\ ms}{255} = 0.0078\ ms$$

*Rounding to the closest integer gives the number of ON intervals as*

$$\frac{T_{ON}}{T_{\Delta}} = \frac{1.36\ ms}{0.0078\ ms} = 174.4 \to 174$$

*This gives*

$$V_q = \frac{174}{255} 5V = 3.41V$$

*)*

**Problem 5.19 (SNR of quantized signals)** *The signal variance $\sigma_s^2 = 4V_{rms}^2$ and the quantizer has step size $\Delta$ = 0.01 V. What is the SNR expressed as a ratio of powers and in dB units?*

*(ans: A quantizer having step size $\Delta$ gives equivalent noise variance equal to*

$$\sigma_n^2 = \frac{\Delta^2}{12} V_{rms}^2 = \frac{10^{-4} V^2}{12} = 8.33 \times 10^{-6} V_{rms}^2$$

*The $\sigma_s^2 = 4V_{rms}^2$ gives*

$$SNR = \frac{\sigma_s^2}{\sigma_n^2} = \frac{4}{8.33 \times 10^{-6}} = 0.48 \times 10^6$$

*and*

$$SNR_{dB} = 10 \log_{10}\left(0.48 \times 10^6\right) = 60 + \overbrace{10 \log_{10}(0.48)}^{-3.2} = 56.8\ dB$$

*)*

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**Problem 5.20 (Maximum SNR of a sinusoidal signal)** *A sinusoidal signal with amplitude $A$ is offset and applied to a 10-bit quantizer with range from 0 to 5 V. What value of $A$ maximizes the SNR of the samples? What is the SNR?*

*(ans: To obtain a very good match of the sinusoid to the quantizer we apply an offset of 2.5 V and make $A = 2.5$ V to give a waveform swing from 0 to 5 V. In this case the signal power equals*

$$\sigma_s^2 = \frac{A^2}{2} = 3.125\, V_{rms}^2$$

*The 10-bit quantizer has step size*

$$\Delta = \frac{5V}{1023} = 0.00489\, V$$

*and produces*

$$\sigma_n^2 = \frac{\Delta^2}{12} V_{rms}^2 = \frac{0.00489V^2}{12} = 1.99 \times 10^{-6} V_{rms}^2$$

*The SNR is*

$$SNR = \frac{\sigma_s^2}{\sigma_n^2} = \frac{3.125}{1.99 \times 10^{-6}} = 1.57 \times 10^6$$

*and*

$$SNR_{dB} = 10 \log_{10}\left(1.57 \times 10^6\right) = 60 + \overbrace{10 \log_{10}(1.57)}^{1.96} = 61.96\; dB$$

*The best match is made by making the waveform extend from $-\Delta/2$ V to $5 + \Delta/2$ V, so the extreme values have the same quantization error as the rest of the samples. In this case,*

$$A = \frac{5\,V + \Delta}{2} = \frac{5.00489\,V}{2} = 2.502\,V \rightarrow \quad \sigma_s^2 = 3.131\, V_{rms}^2$$

*Doing this gives the same values for the SNR as above.*
*)*

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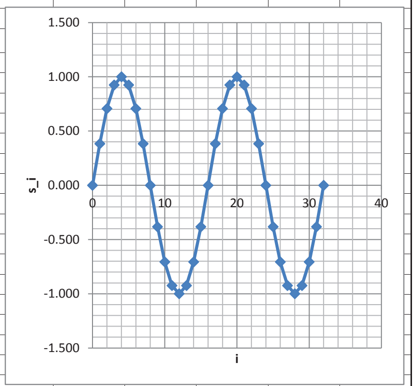
## 5.2 Excel Projects

**Project 5.1 (Sampling 2 periods of a sinusoid)** *Using Example 13.23 as a guide, compose a worksheet to plot 2 periods of a sinusoidal waveform that are sampled 16 times per period.*

*(ans:*

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | $f_o$= | **0.0625** | | | | | | |
| 2 | | | | | | | | |
| 3 | i | sin($2\pi f_o i$) | | | | | | |
| 4 | 0 | 0.000 | | | | | | |
| 5 | 1 | 0.383 | | | | | | |
| 6 | 2 | 0.707 | | | | | | |
| 7 | 3 | 0.924 | | | | | | |
| 8 | 4 | 1.000 | | | | | | |
| 9 | 5 | 0.924 | | | | | | |
| 10 | 6 | 0.707 | | | | | | |
| 11 | 7 | 0.383 | | | | | | |
| 12 | 8 | 0.000 | | | | | | |
| 13 | 9 | -0.383 | | | | | | |
| 14 | 10 | -0.707 | | | | | | |
| 15 | 11 | -0.924 | | | | | | |
| 16 | 12 | -1.000 | | | | | | |
| 17 | 13 | -0.924 | | | | | | |
| 18 | 14 | -0.707 | | | | | | |
| 19 | 15 | -0.383 | | | | | | |
| 20 | 16 | 0.000 | | | | | | |
| 21 | 17 | 0.383 | | | | | | |
| 22 | 18 | 0.707 | | | | | | |
| 23 | 19 | 0.924 | | | | | | |
| 24 | 20 | 1.000 | | | | | | |
| 25 | 21 | 0.924 | | | | | | |
| 26 | 22 | 0.707 | | | | | | |
| 27 | 23 | 0.383 | | | | | | |
| 28 | 24 | 0.000 | | | | | | |
| 29 | 25 | -0.383 | | | | | | |
| 30 | 26 | -0.707 | | | | | | |
| 31 | 27 | -0.924 | | | | | | |
| 32 | 28 | -1.000 | | | | | | |
| 33 | 29 | -0.924 | | | | | | |
| 34 | 30 | -0.707 | | | | | | |
| 35 | 31 | -0.383 | | | | | | |
| 36 | 32 | 0.000 | | | | | | |




| | A | B |
|---|---|---|
| 1 | $f_o$= | **=1/16** |
| 2 | | |
| 3 | i | sin($2\pi f_o i$) |
| 4 | 0 | = SIN(2*PI()*$B$1*A4) |
| 5 | =1+A4 | = SIN(2*PI()*$B$1*A5) |
| 6 | =1+A5 | = SIN(2*PI()*$B$1*A6) |
| 7 | =1+A6 | = SIN(2*PI()*$B$1*A7) |
| 8 | =1+A7 | = SIN(2*PI()*$B$1*A8) |
| 9 | =1+A8 | = SIN(2*PI()*$B$1*A9) |
| 10 | =1+A9 | = SIN(2*PI()*$B$1*A10) |

*)*

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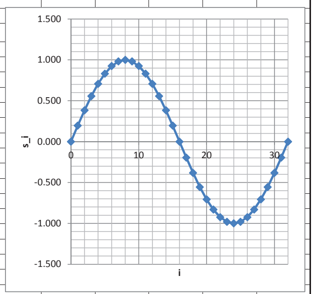
**Project 5.2 (Sampling a sinusoid at 32 points)** *Using Example 13.23 as a guide, compose a worksheet to plot one period of a sinusoidal waveform that is sampled 32 times per period.*

*(ans: The $x$-axis has been re-formatted to set the minimum value to zero and the maximum value to 32.*

| | A | B |
|---|---|---|
| 1 | $f_o$= | 0.03125 |
| 2 | | |
| 3 | i | sin(2π$f_o$i) |
| 4 | 0 | 0.000 |
| 5 | 1 | 0.195 |
| 6 | 2 | 0.383 |
| 7 | 3 | 0.556 |
| 8 | 4 | 0.707 |
| 9 | 5 | 0.831 |
| 10 | 6 | 0.924 |
| 11 | 7 | 0.981 |
| 12 | 8 | 1.000 |
| 13 | 9 | 0.981 |
| 14 | 10 | 0.924 |
| 15 | 11 | 0.831 |
| 16 | 12 | 0.707 |
| 17 | 13 | 0.556 |
| 18 | 14 | 0.383 |
| 19 | 15 | 0.195 |
| 20 | 16 | 0.000 |
| 21 | 17 | -0.195 |
| 22 | 18 | -0.383 |
| 23 | 19 | -0.556 |
| 24 | 20 | -0.707 |
| 25 | 21 | -0.831 |
| 26 | 22 | -0.924 |
| 27 | 23 | -0.981 |
| 28 | 24 | -1.000 |
| 29 | 25 | -0.981 |
| 30 | 26 | -0.924 |
| 31 | 27 | -0.831 |
| 32 | 28 | -0.707 |
| 33 | 29 | -0.556 |
| 34 | 30 | -0.383 |
| 35 | 31 | -0.195 |
| 36 | 32 | 0.000 |



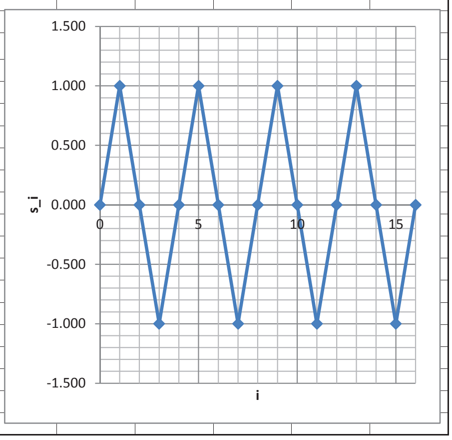

| | A | B |
|---|---|---|
| 1 | $f_o$= | =1/32 |
| 2 | | |
| 3 | i | sin(2π$f_o$i) |
| 4 | 0 | = SIN(2*PI()*$B$1*A4) |
| 5 | =1+A4 | = SIN(2*PI()*$B$1*A5) |
| 6 | =1+A5 | = SIN(2*PI()*$B$1*A6) |
| 7 | =1+A6 | = SIN(2*PI()*$B$1*A7) |
| 8 | =1+A7 | = SIN(2*PI()*$B$1*A8) |
| 9 | =1+A8 | = SIN(2*PI()*$B$1*A9) |
| 10 | =1+A9 | = SIN(2*PI()*$B$1*A10) |
| 11 | =1+A10 | = SIN(2*PI()*$B$1*A11) |
| 12 | =1+A11 | = SIN(2*PI()*$B$1*A12) |

*)*

**Project 5.3 (Sampling a sinusoid at 4 points)** *Using Example 13.23 as a guide, compose a worksheet to plot four periods of a sinusoidal waveform that is sampled 4 times per period.*

*(ans: The $x$-axis has been re-formatted to set the minimum value to zero and the maximum value to 16.*

| | A | B |
|---|---|---|
| 1 | $f_o$= | 0.25 |
| 2 | | |
| 3 | i | sin(2π$f_o$i) |
| 4 | 0 | 0.000 |
| 5 | 1 | 1.000 |
| 6 | 2 | 0.000 |
| 7 | 3 | -1.000 |
| 8 | 4 | 0.000 |
| 9 | 5 | 1.000 |
| 10 | 6 | 0.000 |
| 11 | 7 | -1.000 |
| 12 | 8 | 0.000 |
| 13 | 9 | 1.000 |
| 14 | 10 | 0.000 |
| 15 | 11 | -1.000 |
| 16 | 12 | 0.000 |
| 17 | 13 | 1.000 |
| 18 | 14 | 0.000 |
| 19 | 15 | -1.000 |
| 20 | 16 | 0.000 |




| | A | B |
|---|---|---|
| 1 | $f_o$= | =1/4 |
| 2 | | |
| 3 | i | sin(2π$f_o$i) |
| 4 | 0 | = SIN(2*PI()*$B$1*A4) |
| 5 | =1+A4 | = SIN(2*PI()*$B$1*A5) |
| 6 | =1+A5 | = SIN(2*PI()*$B$1*A6) |
| 7 | =1+A6 | = SIN(2*PI()*$B$1*A7) |
| 8 | =1+A7 | = SIN(2*PI()*$B$1*A8) |
| 9 | =1+A8 | = SIN(2*PI()*$B$1*A9) |
| 10 | =1+A9 | = SIN(2*PI()*$B$1*A10) |

*)*

**Project 5.4 (Demonstration of aliasing)** *Using Example 13.24 as a guide, demonstrate the aliasing that occurs when we make $f_o = 0.9$ when $f_s = 1$. Explain the difference in the observed aliased*

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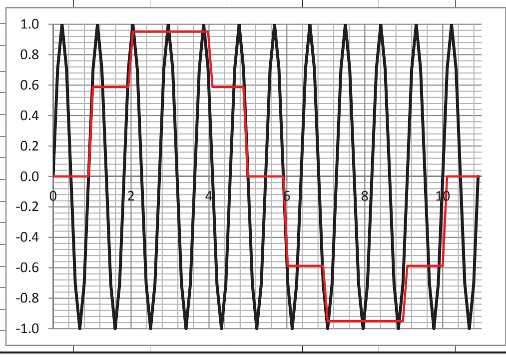
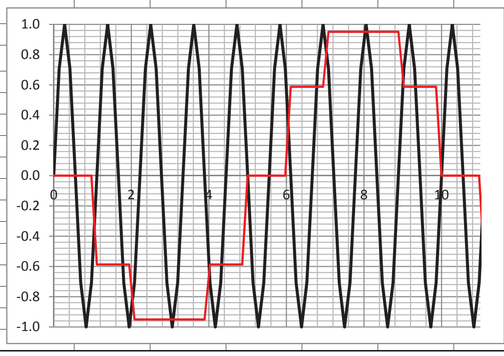
*waveforms between $f_o = 0.9$ and $f_o = 1.1$.*

*ans: When $f_o = 1.1 > f_s = 1$ a positive aliased sinusoid occurs, while when $f_o = 0.9 < f_s = 1$ a negative aliased sinusoid occurs.*

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | $f_o$= | 1.1 | | $T_o$ = | 0.11 |
| 2 | | | | | |
| 3 | $T_j$ | $s_j$ | i | $sa_i$ | |
| 4 | 0.000 | 0.000 | 0 | 0.000 | |
| 5 | 0.114 | 0.707 | 0 | 0.000 | |
| 6 | 0.227 | 1.000 | 0 | 0.000 | |
| 7 | 0.341 | 0.707 | 0 | 0.000 | |
| 8 | 0.455 | 0.000 | 0 | 0.000 | |
| 9 | 0.568 | -0.707 | 0 | 0.000 | |
| 10 | 0.682 | -1.000 | 0 | 0.000 | |
| 11 | 0.795 | -0.707 | 0 | 0.000 | |
| 12 | 0.909 | 0.000 | 0 | 0.000 | |
| 13 | 1.023 | 0.707 | 1 | 0.588 | |
| 14 | 1.136 | 1.000 | 1 | 0.588 | |
| 15 | 1.250 | 0.707 | 1 | 0.588 | |
| 16 | 1.364 | 0.000 | 1 | 0.588 | |




| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | $f_o$= | 0.9 | | $T_o$ = | 0.14 |
| 2 | | | | | |
| 3 | $T_j$ | $s_j$ | i | $sa_i$ | |
| 4 | 0.000 | 0.000 | 0 | 0.000 | |
| 5 | 0.139 | 0.707 | 0 | 0.000 | |
| 6 | 0.278 | 1.000 | 0 | 0.000 | |
| 7 | 0.417 | 0.707 | 0 | 0.000 | |
| 8 | 0.556 | 0.000 | 0 | 0.000 | |
| 9 | 0.694 | -0.707 | 0 | 0.000 | |
| 10 | 0.833 | -1.000 | 0 | 0.000 | |
| 11 | 0.972 | -0.707 | 0 | 0.000 | |
| 12 | 1.111 | 0.000 | 1 | -0.588 | |
| 13 | 1.250 | 0.707 | 1 | -0.588 | |
| 14 | 1.389 | 1.000 | 1 | -0.588 | |
| 15 | 1.528 | 0.707 | 1 | -0.588 | |
| 16 | 1.667 | 0.000 | 1 | -0.588 | |




| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | $f_o$= | 1.1 | | $T_o$ = | =1/(8*B1) |
| 2 | | | | | |
| 3 | $T_j$ | $s_j$ | i | $sa_i$ | |
| 4 | 0 | =SIN(2*PI()*$B$1*A4) | =FLOOR(A4,1) | =SIN(2*PI()*$B$1*C4) | |
| 5 | =A4+$E$1 | =SIN(2*PI()*$B$1*A5) | =FLOOR(A5,1) | =SIN(2*PI()*$B$1*C5) | |
| 6 | =A5+$E$1 | =SIN(2*PI()*$B$1*A6) | =FLOOR(A6,1) | =SIN(2*PI()*$B$1*C6) | |
| 7 | =A6+$E$1 | =SIN(2*PI()*$B$1*A7) | =FLOOR(A7,1) | =SIN(2*PI()*$B$1*C7) | |
| 8 | =A7+$E$1 | =SIN(2*PI()*$B$1*A8) | =FLOOR(A8,1) | =SIN(2*PI()*$B$1*C8) | |
| 9 | =A8+$E$1 | =SIN(2*PI()*$B$1*A9) | =FLOOR(A9,1) | =SIN(2*PI()*$B$1*C9) | |
| 10 | =A9+$E$1 | =SIN(2*PI()*$B$1*A10) | =FLOOR(A10,1) | =SIN(2*PI()*$B$1*C10) | |

*)*

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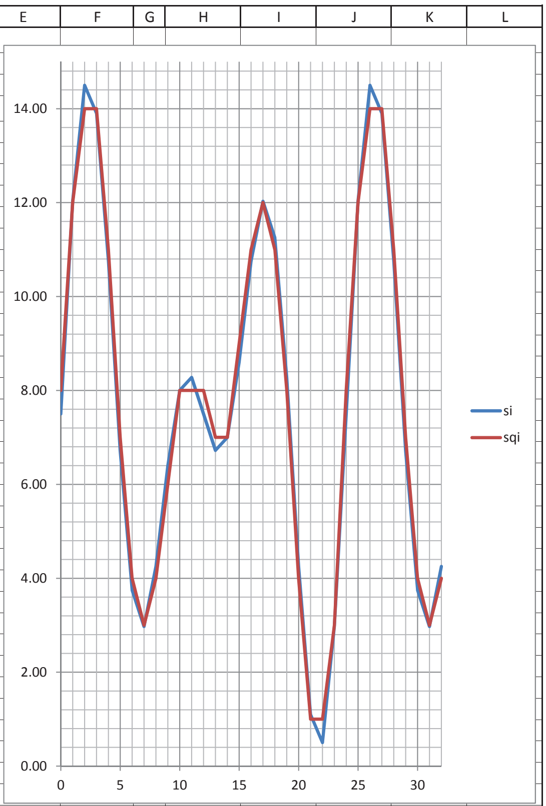
**Project 5.5 (Quantizing waveforms)** *Using Example 13.25 as a guide, compose a worksheet to quantize the waveform*

$$(V_{max}/4)(2 + \sin(2\pi i/8) + \sin(2\pi i/12)) \text{ for } 0 \leq i \leq 32$$

*using code words with $n_b = 4$ bits.*

*(ans:*

| | A | B | C | D |
|---|---|---|---|---|
| 1 | $f_1$= | $f_2$= | $n_b$= | $V_{max}$= |
| 2 | 0.125 | 0.08 | 4 | 15 |
| 3 | | | | |
| 4 | i | $s_i$ | $sq_i$ | bit code |
| 5 | 0 | 7.50 | 8 | 1000 |
| 6 | 1 | 12.03 | 12 | 1100 |
| 7 | 2 | 14.50 | 14 | 1110 |
| 8 | 3 | 13.90 | 14 | 1110 |
| 9 | 4 | 10.75 | 11 | 1011 |
| 10 | 5 | 6.72 | 7 | 0111 |
| 11 | 6 | 3.75 | 4 | 0100 |
| 12 | 7 | 2.97 | 3 | 0011 |
| 13 | 8 | 4.25 | 4 | 0100 |
| 14 | 9 | 6.40 | 6 | 0110 |
| 15 | 10 | 8.00 | 8 | 1000 |
| 16 | 11 | 8.28 | 8 | 1000 |
| 17 | 12 | 7.50 | 8 | 1000 |
| 18 | 13 | 6.72 | 7 | 0111 |
| 19 | 14 | 7.00 | 7 | 0111 |
| 20 | 15 | 8.60 | 9 | 1001 |
| 21 | 16 | 10.75 | 11 | 1011 |
| 22 | 17 | 12.03 | 12 | 1100 |
| 23 | 18 | 11.25 | 11 | 1011 |
| 24 | 19 | 8.28 | 8 | 1000 |
| 25 | 20 | 4.25 | 4 | 0100 |
| 26 | 21 | 1.10 | 1 | 0001 |
| 27 | 22 | 0.50 | 1 | 0001 |
| 28 | 23 | 2.97 | 3 | 0011 |
| 29 | 24 | 7.50 | 8 | 1000 |
| 30 | 25 | 12.03 | 12 | 1100 |
| 31 | 26 | 14.50 | 14 | 1110 |
| 32 | 27 | 13.90 | 14 | 1110 |
| 33 | 28 | 10.75 | 11 | 1011 |
| 34 | 29 | 6.72 | 7 | 0111 |
| 35 | 30 | 3.75 | 4 | 0100 |
| 36 | 31 | 2.97 | 3 | 0011 |
| 37 | 32 | 4.25 | 4 | 0100 |
| 38 | | | | |




| | A | B | C | D |
|---|---|---|---|---|
| 1 | $f_1$= | $f_2$= | $n_b$= | $V_{max}$= |
| 2 | =1/8 | =1/12 | 4 | =2^C2-1 |
| 3 | | | | |
| 4 | i | $s_i$ | $sq_i$ | bit code |
| 5 | 0 | = ($D$2/4)*(2+SIN(2*PI()*$A$2*A5) + SIN(2*PI()*$B$2*A5)) | =ROUND(B5,0) | =DEC2BIN(C5,$C$2) |
| 6 | =1+A5 | = ($D$2/4)*(2+SIN(2*PI()*$A$2*A6) + SIN(2*PI()*$B$2*A6)) | =ROUND(B6,0) | =DEC2BIN(C6,$C$2) |
| 7 | =1+A6 | = ($D$2/4)*(2+SIN(2*PI()*$A$2*A7) + SIN(2*PI()*$B$2*A7)) | =ROUND(B7,0) | =DEC2BIN(C7,$C$2) |
| 8 | =1+A7 | = ($D$2/4)*(2+SIN(2*PI()*$A$2*A8) + SIN(2*PI()*$B$2*A8)) | =ROUND(B8,0) | =DEC2BIN(C8,$C$2) |
| 9 | =1+A8 | = ($D$2/4)*(2+SIN(2*PI()*$A$2*A9) + SIN(2*PI()*$B$2*A9)) | =ROUND(B9,0) | =DEC2BIN(C9,$C$2) |
| 10 | =1+A9 | = ($D$2/4)*(2+SIN(2*PI()*$A$2*A10) + SIN(2*PI()*$B$2*A10)) | =ROUND(B10,0) | =DEC2BIN(C10,$C$2) |

*)*

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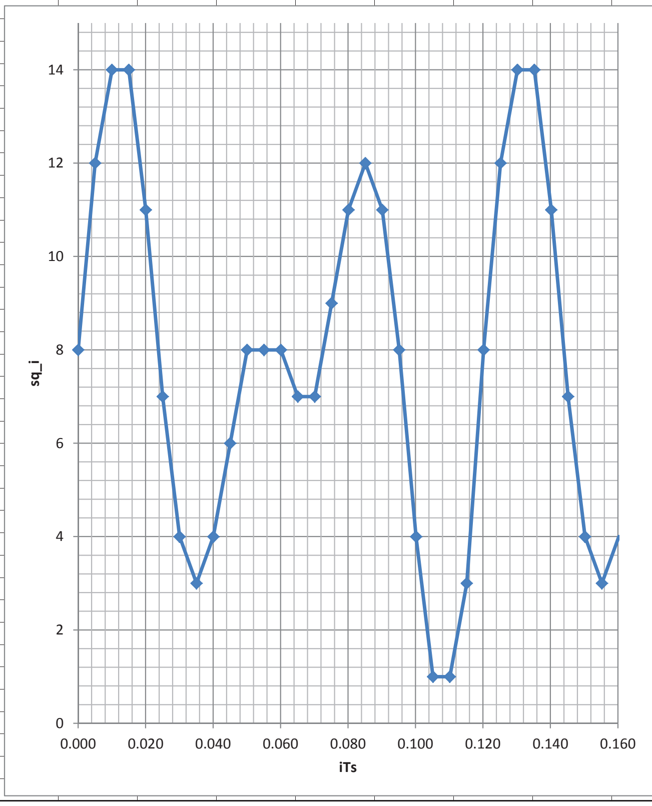
**Project 5.6 (Boxcar DAC)** *Using the results of Project 5.5 and Example 13.26 as a guide, reconstruct the quantized waveform $sq_i$ from the codewords having $n_b = 4$ bits.*

*(ans:*

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | T_s= | #bits= | Δ= | | |
| 2 | 0.0050 | 4 | 1 | | |
| 3 | | | | | |
| 4 | i | bit code | | iT_s | sq_i |
| 5 | 0 | 1000 | | 0.000 | 8 |
| 6 | 1 | 1100 | | 0.005 | 12 |
| 7 | 2 | 1110 | | 0.010 | 14 |
| 8 | 3 | 1110 | | 0.015 | 14 |
| 9 | 4 | 1011 | | 0.020 | 11 |
| 10 | 5 | 0111 | | 0.025 | 7 |
| 11 | 6 | 0100 | | 0.030 | 4 |
| 12 | 7 | 0011 | | 0.035 | 3 |
| 13 | 8 | 0100 | | 0.040 | 4 |
| 14 | 9 | 0110 | | 0.045 | 6 |
| 15 | 10 | 1000 | | 0.050 | 8 |
| 16 | 11 | 1000 | | 0.055 | 8 |
| 17 | 12 | 1000 | | 0.060 | 8 |
| 18 | 13 | 0111 | | 0.065 | 7 |
| 19 | 14 | 0111 | | 0.070 | 7 |
| 20 | 15 | 1001 | | 0.075 | 9 |
| 21 | 16 | 1011 | | 0.080 | 11 |
| 22 | 17 | 1100 | | 0.085 | 12 |
| 23 | 18 | 1011 | | 0.090 | 11 |
| 24 | 19 | 1000 | | 0.095 | 8 |
| 25 | 20 | 0100 | | 0.100 | 4 |
| 26 | 21 | 0001 | | 0.105 | 1 |
| 27 | 22 | 0001 | | 0.110 | 1 |
| 28 | 23 | 0011 | | 0.115 | 3 |
| 29 | 24 | 1000 | | 0.120 | 8 |
| 30 | 25 | 1100 | | 0.125 | 12 |
| 31 | 26 | 1110 | | 0.130 | 14 |
| 32 | 27 | 1110 | | 0.135 | 14 |
| 33 | 28 | 1011 | | 0.140 | 11 |
| 34 | 29 | 0111 | | 0.145 | 7 |
| 35 | 30 | 0100 | | 0.150 | 4 |
| 36 | 31 | 0011 | | 0.155 | 3 |
| 37 | 32 | 0100 | | 0.160 | 4 |




| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | T_s= | #bits= | Δ= | | |
| 2 | 0.005 | 4 | 1 | | |
| 3 | | | | | |
| 4 | i | bit code | | iT_s | sq_i |
| 5 | 0 | 1000 | | =A5*$A$2 | =$C$2*BIN2DEC(B5) |
| 6 | =1+A5 | 1100 | | =A6*$A$2 | =$C$2*BIN2DEC(B6) |
| 7 | =1+A6 | 1110 | | =A7*$A$2 | =$C$2*BIN2DEC(B7) |
| 8 | =1+A7 | 1110 | | =A8*$A$2 | =$C$2*BIN2DEC(B8) |
| 9 | =1+A8 | 1011 | | =A9*$A$2 | =$C$2*BIN2DEC(B9) |
| 10 | =1+A9 | 0111 | | =A10*$A$2 | =$C$2*BIN2DEC(B10) |

*)*

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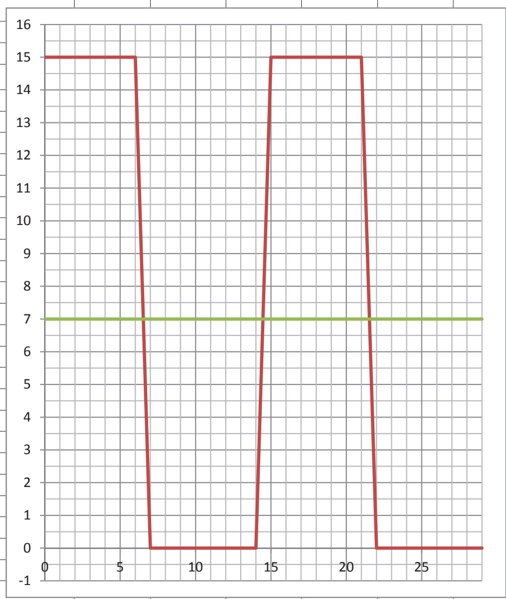
**Project 5.7 (PWM DAC)** *Using Example 13.27 as a guide, compose a worksheet to form a PWM reconstruction using the code words formed in Project 5.5.*

*(ans: First, the worksheet and VBA code show an animation as $sq_i$ increases from 0 to $2^b - 1$ for $b = 4$ bits. Two PWM periods are shown, and the average is computed over these two periods.*

| | A | B | C | D |
|---|---|---|---|---|
| 1 | $sq_i$= | $n_b$= | $V_{max}$= | PWM Period |
| 2 | 7 | 4 | 15 | 15 |
| 3 | | | | |
| 4 | i | PWM interval | interval val | $sq_i$ |
| 5 | 0 | 0 | 15 | 7 |
| 6 | 1 | 1 | 15 | 7 |
| 7 | 2 | 2 | 15 | 7 |
| 8 | 3 | 3 | 15 | 7 |
| 9 | 4 | 4 | 15 | 7 |
| 10 | 5 | 5 | 15 | 7 |
| 11 | 6 | 6 | 15 | 7 |
| 12 | 7 | 7 | 0 | 7 |
| 13 | 8 | 8 | 0 | 7 |
| 14 | 9 | 9 | 0 | 7 |
| 15 | 10 | 10 | 0 | 7 |
| 16 | 11 | 11 | 0 | 7 |
| 17 | 12 | 12 | 0 | 7 |
| 18 | 13 | 13 | 0 | 7 |
| 19 | 14 | 14 | 0 | 7 |
| 20 | 15 | 0 | 15 | 7 |
| 21 | 16 | 1 | 15 | 7 |
| 22 | 17 | 2 | 15 | 7 |
| 23 | 18 | 3 | 15 | 7 |
| 24 | 19 | 4 | 15 | 7 |
| 25 | 20 | 5 | 15 | 7 |
| 26 | 21 | 6 | 15 | 7 |
| 27 | 22 | 7 | 0 | 7 |
| 28 | 23 | 8 | 0 | 7 |
| 29 | 24 | 9 | 0 | 7 |
| 30 | 25 | 10 | 0 | 7 |
| 31 | 26 | 11 | 0 | 7 |
| 32 | 27 | 12 | 0 | 7 |
| 33 | 28 | 13 | 0 | 7 |
| 34 | 29 | 14 | 0 | 7 |




PWM Demo

| | A | B | C | D |
|---|---|---|---|---|
| 1 | $sq_i$= | $n_b$= | $V_{max}$= | PWM Period |
| 2 | 7 | 4 | =2^B2 - 1 | =C2 |
| 3 | | | | |
| 4 | i | PWM interval | interval val | $sq_i$ |
| 5 | 0 | =MOD(A5,$D$2) | =IF($A$2>B5,$C$2,0) | =AVERAGE($C$5:$C$34) |
| 6 | =1+A5 | =MOD(A6,$D$2) | =IF($A$2>B6,$C$2,0) | =AVERAGE($C$5:$C$34) |
| 7 | =1+A6 | =MOD(A7,$D$2) | =IF($A$2>B7,$C$2,0) | =AVERAGE($C$5:$C$34) |
| 8 | =1+A7 | =MOD(A8,$D$2) | =IF($A$2>B8,$C$2,0) | =AVERAGE($C$5:$C$34) |
| 9 | =1+A8 | =MOD(A9,$D$2) | =IF($A$2>B9,$C$2,0) | =AVERAGE($C$5:$C$34) |
| 10 | =1+A9 | =MOD(A10,$D$2) | =IF($A$2>B10,$C$2,0) | =AVERAGE($C$5:$C$34) |

```
Sub PWM()
Do While Range("a2") < 100           ' endless loop
    Range("a2") = Range("a2") + 1    ' increment sq_i
    If Range("a2") > Range("d2") Then
        Range("a2") = 0              ' mod (sq_i, V_max)
    End If
    Application.Wait Now + TimeSerial(0, 0, 1)    ' 1 s delay
Loop
End Sub
```

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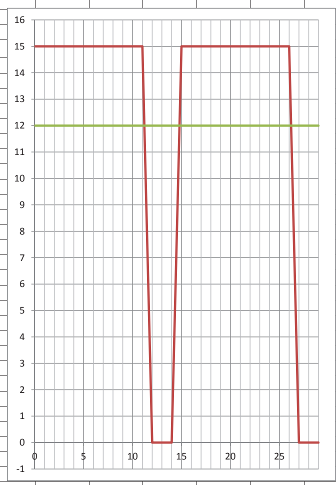
*The second worksheet applies the binary sequence from Project 5.5 and shows an animation of the average over two periods that produces the quantized value $sq_i$. The binary sequence from Project 5.5 is inserted into columns* F−G *changing the index to $k$. The binary number is converted into a decimal number in* H *and then inserted into* A2 *sequentially with the VBA Macro. The Macros also changes the color of the $sq_k$ value being applied to $sq_i$ for clarity.*

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | $sq_i$= | $n_b$= | $V_{max}$= | PWM Period | | | | |
| 2 | 12 | 4 | 15 | 15 | | | | |
| 3 | | | | | | | | |
| 4 | i | PWM interval | interval val | $sq_i$ | | k | bit code | $sq_k$= |
| 5 | 0 | 0 | 15 | 12 | | 0 | 1000 | 8 |
| 6 | 1 | 1 | 15 | 12 | | 1 | 1100 | 12 |
| 7 | 2 | 2 | 15 | 12 | | 2 | 1110 | 14 |
| 8 | 3 | 3 | 15 | 12 | | 3 | 1110 | 14 |
| 9 | 4 | 4 | 15 | 12 | | 4 | 1011 | 11 |
| 10 | 5 | 5 | 15 | 12 | | 5 | 0111 | 7 |
| 11 | 6 | 6 | 15 | 12 | | 6 | 0100 | 4 |
| 12 | 7 | 7 | 15 | 12 | | 7 | 0011 | 3 |
| 13 | 8 | 8 | 15 | 12 | | 8 | 0100 | 4 |
| 14 | 9 | 9 | 15 | 12 | | 9 | 0110 | 6 |
| 15 | 10 | 10 | 15 | 12 | | 10 | 1000 | 8 |
| 16 | 11 | 11 | 15 | 12 | | 11 | 1000 | 8 |
| 17 | 12 | 12 | 0 | 12 | | 12 | 1000 | 8 |
| 18 | 13 | 13 | 0 | 12 | | 13 | 0111 | 7 |
| 19 | 14 | 14 | 0 | 12 | | 14 | 0111 | 7 |
| 20 | 15 | 0 | 15 | 12 | | 15 | 1001 | 9 |
| 21 | 16 | 1 | 15 | 12 | | 16 | 1011 | 11 |
| 22 | 17 | 2 | 15 | 12 | | 17 | 1100 | 12 |
| 23 | 18 | 3 | 15 | 12 | | 18 | 1011 | 11 |
| 24 | 19 | 4 | 15 | 12 | | 19 | 1000 | 8 |
| 25 | 20 | 5 | 15 | 12 | | 20 | 0100 | 4 |
| 26 | 21 | 6 | 15 | 12 | | 21 | 0001 | 1 |
| 27 | 22 | 7 | 15 | 12 | | 22 | 0001 | 1 |
| 28 | 23 | 8 | 15 | 12 | | 23 | 0011 | 3 |
| 29 | 24 | 9 | 15 | 12 | | 24 | 1000 | 8 |
| 30 | 25 | 10 | 15 | 12 | | 25 | 1100 | 12 |
| 31 | 26 | 11 | 15 | 12 | | 26 | 1110 | 14 |
| 32 | 27 | 12 | 0 | 12 | | 27 | 1110 | 14 |
| 33 | 28 | 13 | 0 | 12 | | 28 | 1011 | 11 |
| 34 | 29 | 14 | 0 | 12 | | 29 | 0111 | 7 |
| 35 | | | | | | 30 | 0100 | 4 |
| 36 | | | | | | 31 | 0011 | 3 |
| 37 | | | | | | 32 | 0100 | 4 |




PWM Demo

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | $sq_i$= | $n_b$= | $V_{max}$= | PWM Period | | | | |
| 2 | 12 | 4 | =2^B2 - 1 | =C2 | | | | |
| 3 | | | | | | | | |
| 4 | i | PWM interval | interval val | $sq_i$ | | k | bit code | $sq_k$= |
| 5 | 0 | =MOD(A5,$D$2) | =IF($A$2>B5,$C$2,0) | =AVERAGE($C$5:$C$34) | | 0 | 1000 | =BIN2DEC(G5) |
| 6 | =1+A5 | =MOD(A6,$D$2) | =IF($A$2>B6,$C$2,0) | =AVERAGE($C$5:$C$34) | | =1+F5 | 1100 | =BIN2DEC(G6) |
| 7 | =1+A6 | =MOD(A7,$D$2) | =IF($A$2>B7,$C$2,0) | =AVERAGE($C$5:$C$34) | | =1+F6 | 1110 | =BIN2DEC(G7) |
| 8 | =1+A7 | =MOD(A8,$D$2) | =IF($A$2>B8,$C$2,0) | =AVERAGE($C$5:$C$34) | | =1+F7 | 1110 | =BIN2DEC(G8) |
| 9 | =1+A8 | =MOD(A9,$D$2) | =IF($A$2>B9,$C$2,0) | =AVERAGE($C$5:$C$34) | | =1+F8 | 1011 | =BIN2DEC(G9) |
| 10 | =1+A9 | =MOD(A10,$D$2) | =IF($A$2>B10,$C$2,0) | =AVERAGE($C$5:$C$34) | | =1+F9 | 0111 | =BIN2DEC(G10) |

```
Sub PWM()
Dim K As Integer                   ' Define sq_k pointer
K = 5                              ' first row of data
Do While Range("a2") < 100         ' endless loop
    Range("H" & K).Font.Color = RGB(255, 0, 0)  ' color sq_k red
    Range("a2").Value = Range("H" & K).Value    ' set sq_i = sq_k
    Application.Wait Now + TimeSerial(0, 0, 1)  ' 1 s delay
    Range("H" & K).Font.Color = RGB(0, 0, 0)    'restore color sq_k to black
    K = K + 1
    If K > 37 Then                 ' last row of data
        K = 5                      ' return to first row of data
    End If
Loop                               ' End While loop
End Sub
```

*)*

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