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문제 1

[질문 1] 답 $\mathcal{L}(2t+1)$

$F(3) = \frac{5}{9}$

$F(s) = \frac{2}{s^2} + \frac{1}{s} \Rightarrow \frac{2}{9} + \frac{1}{3} = \frac{5}{9}$

[질문 2] 답 $\mathcal{L}(\sin 3t + 1)$

$F(1) = \frac{3}{10}$

$F(s) = \frac{3}{s^2+9}$

[질문 3] 답 $\mathcal{L}(e^{2t})$

$F(4) = \frac{1}{2}$

$F(s) = \frac{1}{s-2}$

[질문 4] 답 $\mathcal{L}(\frac{1}{3}t^3)$

$F(2) = \frac{1}{8}$

$F(s) = \frac{1}{3} \cdot \frac{3!}{s^4} = \frac{2}{s^4}$

[질문 5] 답 $\mathcal{L}(e^{3t}\cos t)$

$F(4) = \frac{1}{2}$

$\frac{s-3}{(s-3)^2+1}$

문제 3

[질문 1] 답 $\mathcal{L}(\frac{1}{4!}t^4)$

$f(2) = \frac{2}{3}$

$f(t) = \frac{1}{4!}t^4 \quad f(2) = \frac{16}{24}$

[질문 2] 답

$f(\ln 2) = 25$

$\mathcal{L}^{-1}\left(10\frac{s}{s^2-4} + 5\frac{4}{s^2-4}\right) = 10\cosh 2t + 2.5\sinh 2t$

$= 10 \cdot \frac{17}{8} + 2 \cdot \frac{15}{8} = \frac{85+15}{4} = \frac{100}{4} = 25$

[질문 3] 답

$f(0) = -1$

$\mathcal{L}^{-1}\left(\frac{2}{s-3} - \frac{3}{s+1}\right) \quad f(t) = 2e^{3t} - 3e^{-t}$

[질문 4] 답

$f(\frac{\pi}{3}) = +e^{-\frac{\pi}{3}}(\sqrt{3}-1)$

$\mathcal{L}^{-1}\left(\frac{2(s+1)-4}{(s+1)^2+4}\right) \quad 2e^{-t}\cos 2t + 2.5\sin 2t\, e^{-t}$

[질문 5] 답

$e^{4(t-2)}u(t-2)$

$\mathcal{L}^{-1}\left(\frac{e^{-2s}}{s-4}\right) = e^{4(t-2)}u(t-2)$

문제 2

[질문 1] 답

$F(2) = \frac{1}{2}$

$\frac{5}{2} - 2$

$F(s) = \mathcal{L}(5 - 5\cos 3t) = \mathcal{L}(5 - 5\cos t) = \frac{5}{s} - \frac{5s}{s^2+1}$

[질문 2] 답

$F(1) = -\frac{12}{24}$

$\mathcal{L}(4)\,\mathcal{L}(s+2) = \frac{-4s}{s^2+4} \quad F(1) = \frac{-4}{5} \cdot \frac{3}{5}$

[질문 3] 답

$F(1) = \frac{1}{2}$

$\mathcal{L}^{-1}\mathcal{L}(4)(s^2+9) = \frac{10}{s+1} \quad F(1) \cdot 10 = 5$

[질문 4] 답

$F(4) = 4e^{-\frac{3}{2}}$

$\mathcal{L}((t-3)u(t-3)) = \frac{e^{-3s}}{s^2}$

[질문 5] 답

$\frac{1}{2}e^{4(t-2)}(t-2)^2u(t-2)$

$\mathcal{L}^{-1}\left(\frac{e^{-2s}}{(s-4)^3}\right) = e^{-(t-2)} \times \frac{1}{2}(t-2)^2 \times u(t-2)$

문제 4

답

$-\ln 2$

풀이

$f(t) = 1 - e^t \quad \mathcal{L}(f(t)) = \frac{1}{s} - \frac{1}{s-1}$

$\int_s^\infty \left(\frac{1}{x} - \frac{1}{x-1}\right)dx = [\ln|x| - \ln|x+1|]_s^\infty$

$= \ln\frac{s-1}{s}$

$F(2) = \ln\frac{1}{2} = -\ln 2$

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문제 5 $f(t)=\int_0^\infty \tau e^{(1-t)\tau}\sin\tau\, d\tau$.

답

$\frac{1}{2}$

풀이

$f(t)=\int_0^\infty t e^{t\tau}\sin\tau\cdot e^{-t\tau}\,d\tau$

$=\mathcal{L}(te^t\sin t)=-\left(\frac{1}{(s-1)^2+1}\right)'$

$\mathcal{L}(e^t\sin t)=\frac{1}{(s-1)^2+1}$ $=\frac{2t-2}{(t^2-2t+2)^2}$

$f(2)=\frac{2}{(4-4+2)^2}=\frac{1}{2}$

문제 6

답

$\sqrt{3}-\frac{\pi}{3}$

풀이

$h(t)=\mathcal{L}^{-1}\left(\frac{2}{s^2+1}\right)\times\mathcal{L}^{-1}\left(\frac{2}{s^2+1}\right)$

$=2\sin t * 2\sin t$

$=4\int_0^t \sin\tau\times\sin(t-\tau)\,d\tau$

$=-2\int_0^t(\cos t-\cos(2\tau-t))\,d\tau$

$=-2\left[\tau\cos t-\frac{1}{2}\sin(2\tau-t)\right]_0^t$

$=-2(t\cos t-\sin t)$

$h\left(\frac{\pi}{3}\right)=-2\left(\frac{\pi}{6}-\frac{\sqrt{3}}{2}\right)=-\frac{\pi}{3}+\sqrt{3}$

문제 7 $f(t)=\mathcal{L}^{-1}\left(\ln\left(\frac{s+1}{s}\right)\right)$ $f(\pi)$?

답

$\frac{3}{\pi}$

풀이

$F(s)=\ln(s^2+1)-\ln s$

$F'(s)=\frac{2s}{s^2+1}-\frac{1}{s}$

$\mathcal{L}^{-1}(F'(s))=2\cos t-1=-tf(t)$

$f(t)=\frac{(-2\cos t}{t}$

$f(\pi)=\frac{3}{\pi}$

문제 8 $ty''+(1-t)y'+y=0$. $F(2)=4+\frac{a}{2}$, $F(s)=\mathcal{L}(y)$

답

$F(4)=\frac{3}{16}$

풀이

$-2sF-s^2F'+sF+F+sF'+F=0$

$(-s+s^2)F'+(-2+s)F=0$

$\ln F=+\ln|s-1|+2\ln s$

$F(s)=\frac{s-1}{s^2}$, $F(4)=\frac{3}{16}$

$\frac{F'}{F}=\frac{s-2}{s(1-s)}=\frac{-s+2}{s(s-1)}=\frac{a}{s-1}-\frac{b}{s}$

$a-b=-1$ $as-b(s-1)=-s+2$

$b=+2$

$a=+1$

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문제 9

답

$\frac{1}{\pi}$

풀이

$\int \frac{1}{y^2}\,dy = \int (1-2x)\,dx$

$-\frac{1}{y} = x - x^2 + C \rightarrow (0,1)$

$-1 = C$,

$x = 2 \rightarrow -\frac{1}{y} = 2 - 4 - 1$

$y(2) = \frac{1}{\pi}$

문제 10

[질문 1] 답

(0, 2),
(4, -2)

[질문 2] 답

① (0,2) → 마디점

② (4,-2) → 안장점

풀이

$y_1' = y_1(4 - y_1)$
$y_2' = y_2 + y_1 - 2$

y_1 y_2
(0, 2)
(4, -2)

① (0,2) → $y_1' = 4y_1 - y_1^2 \approx 4y_1$
$y_2' = y_2 + y_1 - 2$

$\begin{pmatrix} 4 & 0 \\ 1 & 1 \end{pmatrix}$

$p > 0 \quad q > 0 \quad \Delta > 0$

마디점

② (4,-2) → $y_1' = 4(\tilde{y}_1 + 4) - (\tilde{y}_1 + 4)^2 = -\tilde{y}_1^2 - 4\tilde{y}_1$
$\approx -4\tilde{y}_1$
$y_2' = y_2 + y_1 - 2$

$\tilde{y}_1 = y_1 - 4$

$\begin{pmatrix} -4 & 0 \\ 1 & 1 \end{pmatrix}$

안장점 $p < 0 \quad q < 0$

문제 11

$y'' + 9y = \delta(t - \frac{\pi}{3})$

답

$-\frac{1}{3}u(-\frac{\pi}{3}) - \frac{1}{3}u(\frac{\pi}{3}) - 4$

풀이

$s^2 \mathcal{L}(y) - 2s + 9\mathcal{L}(y) = e^{-\frac{\pi}{3}s}$

$(s^2 + 9)\mathcal{L}(y) = e^{-\frac{\pi}{3}s} + 2s$

$\mathcal{L}(y) = \frac{e^{-\frac{\pi}{3}s} + 2s}{s^2 + 9}$

$y = \frac{1}{3}\sin 3(t - \frac{\pi}{3})\,u(t - \frac{\pi}{3}) + 2\cos 3t$

$y(\frac{\pi}{3}) = -\frac{1}{3}u(-\frac{\pi}{3}) - 2$

$y(\pi) = -\frac{1}{3}u(\frac{\pi}{3}) - 2$

$y(\frac{\pi}{3}) + y(\pi) = -\frac{1}{3}u(-\frac{\pi}{3}) - \frac{1}{3}u(\frac{\pi}{3}) - 4$

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문제 12 $xy'' + (1-2x)y' + (x-1)y = 0.$ $y(1)=2e$, $y'(1)=3e$

답

$e^2(2+\ln 2)$

풀이

$b(x) = 1-2x$. $c(x) = x(x-1)$

$b(0) = 1$, $c(0) = 0$.

$r(r-1)+r = 0$. $r = 0$

$y = \sum_{m=0}^{\infty} a_m x^m$. $y' = \sum_{m=1}^{\infty} m a_m x^{m-1}$ $y'' = \sum_{m=2}^{\infty} m(m-1) a_m x^{m-2}$

$\sum_{m=2}^{\infty} m(m-1) a_m x^{m-1} + \sum_{m=1}^{\infty} m a_m x^{m-1} - 2\sum_{m=1}^{\infty} m a_m x^m$

$+ \sum_{m=0}^{\infty} a_m x^{m+1} - \sum_{m=0}^{\infty} a_m x^m = 0$

$\sum_{m=1}^{\infty} (m+1) m a_{m+1} x^m + \sum_{m=0}^{\infty} a_{m+1} x^m (m+1) - 2\sum_{m=1}^{\infty} m a_m x^m$

$+ \sum_{m=1}^{\infty} a_{m-1} x^m - \sum_{m=0}^{\infty} a_m x^m = 0$

$(-a_0 + a_1) + \sum_{m=1}^{\infty} x^m ((m+1)^2 a_{m+1} - (2m+1) a_m + a_{m-1}) = 0$

$(a_0 = a_1)$

$(m+1)^2 (a_{m+1} - \frac{a_m}{m+1}) - m(a_m - \frac{a_{m-1}}{m}) = 0$

$(m \geq 1)$

$\rightarrow (m+1)^2 b_{m+1} - m b_m = 0$

$b_{m+1} = \frac{m}{(m+1)^2} b_m$

$b_{m+1} = a_{m+1} - \frac{a_m}{m+1} = 0$

$a_{m+1} = \frac{a_m}{m+1} \rightarrow a_m = \frac{a_0}{m!}$

$y_1 = e^x$

따라서 $y_2 = e^x \int e^{-2x} \cdot e^{2x} \cdot \frac{1}{x} dx$

$= e^x \ln x$

$y = c_1 e^x + c_2 e^x \ln x$ $y' = c_1 e^x$ $+ c_2(\ln x + 1)$

$\rightarrow y = 2e^x + e^x \ln x$ $c_1 = 2$

$c_2 = 1$

$y(2) = 2e^2 + e^2 \ln 2$

문제 13

답

6

풀이

#1) $x\,dy = (y+x^2)dx$

$(y+x^2)dx - x\,dy = 0$ $(\neq (-1))$

완전상미분방정식 X

() $f_x = f_x'(-x) - f_x$, $2f_x = f_x'(-x)$

$\frac{f_x'}{f_x} = -\frac{2}{x}$ $f_x = \frac{1}{x^2}$ $(\frac{y}{x^2}+1)dx - \frac{1}{x}dy = 0$

#2) $y' = \frac{y}{x} + x$ $y = ux$ $-\frac{y}{x}+x = 0$

$y' = u + x$ $y' = u'x + u$ $y(1,2)$

$u' = 1$, $u = x + C$

$y = x^2 + Cx$ $\rightarrow C(1,2) \rightarrow C = 1$

$(x=2)$ $y = 4+2 = 6$

나는 정직하게 시험에 응할 것을 서약합니