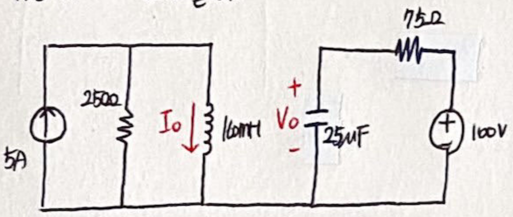
회로이론 HW #5.

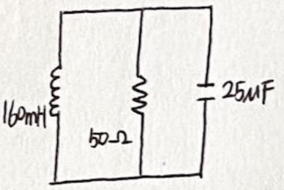
#8.11 $t<0$일 때



인덕터에 걸리는 전류 $I_0=5[A]$

축전기에 걸리는 전압 $V_0=100[V]$

$t>0$일 때, RLC회로에서의 Natural Response.



$R=50\Omega$

$L=160\cdot10^{-3}$ H

$C=25\cdot10^{-6}$ F.

$$\alpha=\frac{1}{2RC}=\frac{1}{2\cdot50\cdot25\cdot10^{-6}}=400$$

$$\omega_0^2=\frac{1}{LC}=\frac{1}{160\cdot10^{-3}\cdot25\cdot10^{-6}}=(500)^2.$$

$\alpha^2-\omega_0^2=-90000<0$. 즉, underdamped

$\omega_d=\sqrt{90000}=300.$

$$V(t)=B_1e^{-\alpha t}\cos\omega_d t+B_2e^{-\alpha t}\sin\omega_d t$$

$$=B_1e^{-400t}\cos300t+B_2e^{-400t}\sin300t$$

$V(0)=B_1=V_0=100[V]$ $\therefore B_1=100$

$$\frac{dV(0)}{dt}=\frac{i_C(0)}{C}=\frac{1}{C}(-i_R-i_L)=\frac{1}{C}\left(-\frac{100}{50}-5\right)$$

$$=\frac{1}{25\cdot10^{-6}}(-7)=-280000=B_1(-\alpha)+B_2\cdot\omega_d$$

$$=B_1(-400)+B_2\cdot300$$

$-400B_1+300B_2=-280000$

$300B_2=-280000+400\cdot100=-240000.$ $\therefore B_2=-800.$

$\therefore V(t)=100e^{-400t}\cos300t-800e^{-400t}\sin300t$ [V]

#8.12.

$I_0=5[A]$, $V_0=100[V]$로 동일.

$R=40\Omega$. $L=160\cdot10^{-3}$H. $C=25\cdot10^{-6}$F.

$$\alpha=\frac{1}{2RC}=\frac{1}{2\cdot40\cdot25\cdot10^{-6}}=500.$$

$\omega_0=500$ (#8.11에서와 동일).

$\alpha^2-\omega_0^2=0$. 즉. critically damped.

$$V(t)=D_1te^{-\alpha t}+D_2e^{-\alpha t}$$

$$=D_1te^{-500t}+D_2e^{-500t}$$

$V(0)=D_2=V_0=100$ $\therefore D_2=100$

$$\frac{dV(0)}{dt}=\frac{i_C(0)}{C}=D_1+D_2(-\alpha)=D_1-500D_2$$

$$=\frac{1}{C}\left(-\frac{V}{R}-i_L\right)=\frac{1}{25\cdot10^{-6}}\left(-\frac{100}{40}-5\right)=-300{,}000$$

$\therefore D_1=-300000+500D_2=-250000$

$\therefore V(t)=-250000te^{-500t}+100e^{-500t}$ [V]

#8.13

$I_0=5[A]$. $V_0=100[V]$로 동일.

$R=32[\Omega]$, $L=160\cdot10^{-3}$H, $C=25\cdot10^{6}$F

$$\alpha=\frac{1}{2RC}=\frac{1}{2\cdot32\cdot25\cdot10^{6}}=625.$$

$\omega_0=500.$ $\alpha^2-\omega_0^2>0$. 즉 overdamped!

$$V(t)=A_1e^{s_1t}+A_2e^{s_2t}$$

$$s=-625\pm\sqrt{625^2-500^2}=-625\pm375.$$

$s_1=-250$, $s_2=-1000$.

$$V(t)=A_1e^{-250t}+A_2e^{-1000t}$$

$V(0)=A_1+A_2=V_0=100.$

$$\frac{dV(0)}{dt}=\frac{di(0)}{C}=\frac{1}{C}\left(-\frac{100}{32}-5\right)=\frac{1}{25\cdot10^{6}}\left(-\frac{100}{32}-5\right)$$

$$=-325{,}000=s_1A_1+s_2A_2=-250A_1-1000A_2$$

$A_1+A_2=100$

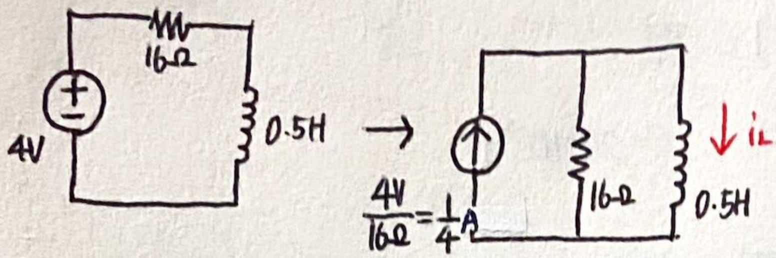
$A_1+4A_2=1300.$

$3A_2=1200$, $A_2=400.$

$A_1=-300$

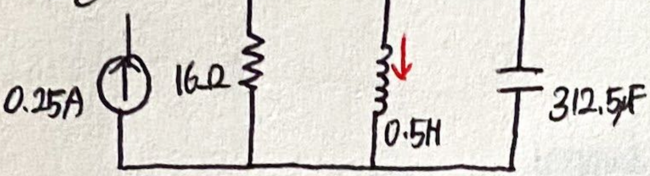
$V(t)=-300e^{-250t}+400e^{-1000t}$ [V]

#8.30 t<0일 때.



$I_{L0} = 0.25$ [A]

t≥0일 때.



$V_{C0} = 0$ [V].

$$\alpha = \frac{1}{2RC} = \frac{1}{2\cdot 16\cdot 312.5\cdot 10^{-6}} = 100. \quad \omega_0^2 = \frac{1}{LC} = 6400.$$

$\alpha^2 - \omega_0^2 > 0.$ $\alpha^2 > \omega_0^2$ 즉 overdamped response.

$$V(t) = A_1e^{S_1t} + A_2e^{S_2t}, \quad S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -100 \pm 60$$

$$= A_1e^{-40t} + A_2e^{-160t} \qquad = -40 \text{ or } -160.$$

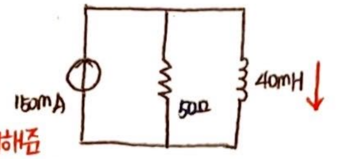
$$V(0) = A_1 + A_2 = V_{C0} = 0.$$

$$\frac{dV(0)}{dt} = \frac{i_C(0)}{C} = \frac{1}{312.5\cdot 10^{-6}}\cdot(-i_R - i_L + 0.25) = \frac{1}{312.5\cdot 10^{-6}}(-0 - 0.25 + 0.25)$$

$$= 0. = S_1A_1 + S_2A_2 = -40A_1 - 160A_2 = 0.$$

$A_1 = A_2 = 0.$ $\therefore V(t) = 0.$

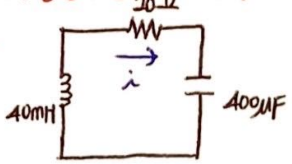
#8.46. $t<0$.



$I_0 = -150\cdot10^{-3}$ [A]

↳ 문제에서 제시해준 i의 방향과 반대이므로 (−)

$t>0$.



$R = 20\,\Omega$

$L = 40\cdot10^{-3}$ H

$C = 400\cdot10^{-6}$ F.

<Series, natural response>

$$\alpha = \frac{R}{2L} = \frac{20}{2\cdot40\cdot10^{-3}} = 250.\ [\text{rad/s}]$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{40\cdot10^{-3}\cdot400\cdot10^{-6}} = (250)^2.\ [\text{rad}^2/\text{s}^2]$$

$\alpha^2 = \omega_0^2$ 이므로 Critically damped!

$$I(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

$$I(0) = D_2 = I_0 = -150\cdot10^{-3}$$

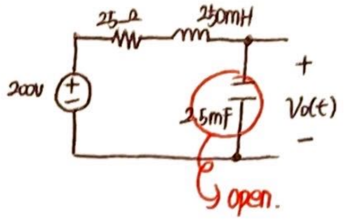
$$\frac{dI(0)}{dt} = \frac{1}{L}(-V_R - V_C) = \frac{1}{40\cdot10^{-3}}(-(20)\cdot(-150\cdot10^{-3}) - 0)$$

$$= \frac{1}{40\cdot10^{-3}}\cdot20\cdot150\cdot10^{-3} = 75 = D_1 - \alpha D_2$$

$$D_1 = 75 + \alpha D_2 = 75 + 250\cdot(-150\cdot10^{-3}) = 37.5$$

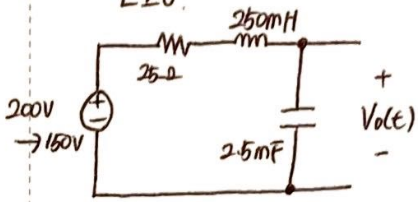
$$I(t) = 37500te^{-250t} - 150e^{-250t}\ [\text{mA}]$$

#8.53. $t<0$.



$V_0 = 200$ [V].

$t \geq 0$.



Series, Step response.

$R = 25\,\Omega$

$L = 250\cdot10^{-3}$ H

$C = 2.5\cdot10^{-3}$ F

$V_f = 150$ [V].

$$\alpha = \frac{R}{2L} = \frac{25}{2\cdot250\cdot10^{-3}} = 50\ [\text{rad/s}]$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{250\cdot10^{-3}\cdot2.5\cdot10^{-3}} = 1600\ [\text{rad}^2/\text{s}^2]$$

$\alpha^2 > \omega_0^2$, 즉, overdamped!

$$V_C(t) = V_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}.$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -50 \pm \sqrt{2500 - 1600} = -50 \pm 30.$$

$$s_1 = -20,\ s_2 = -80$$

$$V_C(0) = V_f + A_1' + A_2' = V_0 = 200$$

$$A_1' + A_2' = 200 - V_f = 200 - 150 = 50. \quad -①$$

capacitor의 초기 전류

$$\frac{dV_C(0)}{dt} = \frac{I_0}{C} = 0\ (\because I_0 = 0)$$

$$= A_1's_1 + A_2's_2 = -20A_1' - 80A_2' = 0. \quad -②$$

② → $20A_1' = -80A_2'$, $A_1' = -4A_2'$.

① 대입) $-3A_2' = 50$. $A_2' = -\frac{50}{3}$, $A_1' = -4\cdot A_2 = \frac{200}{3}$

$$A_1' = 66.67.\quad A_2' = -16.67$$

$$\therefore V_C(t) = 150 + 66.67e^{-20t} - 16.67e^{-80t}\ [\text{V}]$$