1. $z = r(\cos\theta + i\sin\theta) = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$

$* \; r = \sqrt{3+1} = 2 \qquad \mathrm{Arg}\,z = \mathrm{Arctan}\,\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

2. $z^n = r^n(\cos n\theta + i\sin n\theta)$

$(-\sqrt{3} - i)^{23} = 2^{23}(\cos\frac{23}{6}\pi + i\sin\frac{23}{6}\pi)$

$*$ $\cos\frac{23}{6}\pi = \cos(4\pi - \frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

$\sin\frac{23}{6}\pi = \sin(-\frac{\pi}{6}) = -\frac{1}{2}$

$2^{23}(\frac{\sqrt{3}}{2} + i(-\frac{1}{2})) = 2^{22}(\sqrt{3} - i)$

$\cancel{4}$. $u(x,y) = xy$

$u_x = v_y = y \rightarrow v = \frac{1}{2}y^2 + h(x)$

$u_y = -v_x = x \qquad v_x = h'(x) = -x$

$\rightarrow h(x) = -\frac{1}{2}x^2 + C$

$v(x,y) = \frac{1}{2}y^2 - \frac{1}{2}x^2 + C$

$f(z) = xy + \frac{1}{2}y^2 - \frac{1}{2}x^2 + C$

5. $f(z) = \underbrace{(ax^2 + ay^2 + c)}_{u(x,y)} + i\underbrace{bxy}_{v(x,y)}$

$u_x = 2ax = v_x = by$

$u_y = 2ay = -v_y = -bx$

$2ax = by, \quad 2ay = -bx$

$x = -\frac{2a}{b}y$

$2a(-\frac{2a}{b}y) = \frac{-4a^2}{b}y = by \Rightarrow 4a^2 + b^2 = 0$

모순.

6. $\ln z = (2 - \frac{1}{2i})\pi$의 해 z

$= (2 + \frac{i}{2})\pi$

$z = e^{2\pi} \cdot e^{\frac{\pi}{2}i} = ie^{2\pi}$

$(\because e^{\pi i/2} = i)$

7. $|z| = 2 \rightarrow L = 4\pi$

$|z^2 + 5| \geq ||z^2| - 5| = |4 - 5| = 1$

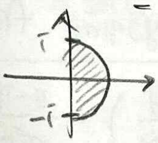
$\left|\frac{2z+1}{5+z^2}\right| \leq |2|z|+1| = 5$

$\left|\int_C \frac{2|z|+1}{5+z^2}\right| \leq 5 \cdot (4\pi) = 20\pi$

8. $z(t) = e^{it} \quad f(z(t)) = (1 + e^{it})/e^{it}$

$\int_{-\pi}^{\pi} (e^{-it} + 1)\, ie^{it}\, dt$

$= \int_{-\pi}^{\pi} (i + ie^{it})\, dt$



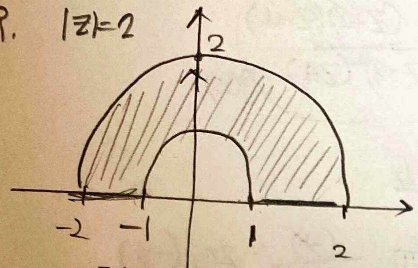
$= [it + e^{it}]_{-\pi}^{\pi}$

$= \pi i + \cancel{e^{i\pi}} - (-\pi i + \cancel{e^{-\pi i}})$

$= 2\pi i$

or $\int_C \frac{1+z}{z}\, dz = 2\pi i f(0) = 2\pi i$, $f(z) = 1+z$

9. $|z|=2$



* 구간에 따라 $z(t)$ 다르게 설정 $z(t)=2e^{it}$, $\overline{z(t)}=2e^{-it}$ $z(t)=e^{it}$

$$\int_c \frac{z}{\bar z}dz = \int_{-2}^{-1} 1dt + \int_1^2 1dt + \int_0^\pi \frac{\cancel{2}e^{it}}{\cancel{2}e^{-it}} 2ie^{it}dt - \int_0^\pi \frac{e^{it}}{e^{-it}}\cdot ie^{it}dt$$

$$= 1+1+2i\int_0^\pi e^{3it}dt - i\int_0^\pi e^{3it}dt = 2+i\int_0^\pi e^{3it}dt = \frac{4}{3}$$

PASS

서술형

1. ~~$\ln$~~ $n=0,\pm1,\pm2\cdots$ $\ln z = \operatorname{Ln} z \pm 2n\pi i$ $z=x+yi$로 가정

$\tan\theta=\frac{y}{x} \to \theta=\arctan\frac{y}{x}$

모든 n에 대해 0과 음의 실수축제외 해석함수.

$$\ln z = \ln r + i(\theta + c) = \ln\sqrt{x^2+y^2} + i\left(\arctan\frac{y}{x}+c\right) = \boxed{\underbrace{\frac{1}{2}\ln(x^2+y^2)}_{u(x,y)} + i\underbrace{\left(\arctan\frac{y}{x}+c\right)}_{v(x,y)}}$$

$u_x = v_y$
$u_y = -v_x$

$$\Rightarrow u_x = \frac{1}{2}\cdot\frac{2x}{x^2+y^2} = \frac{x}{x^2+y^2} = v_y \to \frac{1}{1+(y/x)^2}\cdot\frac{1}{x}$$

*$(\arctan x)' = \frac{1}{1+x^2}$

y미분 (y에 대해)

$$u_y = \frac{y}{x^2+y^2} = v_x(-1) = -\frac{1}{(1+(y/x)^2)}\left(-\frac{y}{x^2}\right)$$

$\Rightarrow$ Cauchy-Riemann이 성립

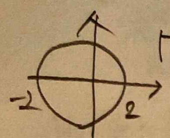
$\boxed{\text{* } f(z)=u(x,y)+iv(x,y) \text{일 때 } f'(z)=u_x+iv_x}$

$$f'(z) = \frac{x}{x^2+y^2} + i\left(\frac{-y}{x^2+y^2}\right) = \frac{x-iy}{x^2+y^2} = \frac{\bar z}{|z|^2} = \frac{1}{z}$$

∴ $(\ln z)' = \frac{1}{z}$ (z는 0 또는 음의 실수가 아니다.)

↳ $\ln z$에서 $z>0$, $\frac{1}{z}$에서 $z\neq0$인듯?

2. $\oint_{C:|z|=4} f(z)dz \rightarrow f(z) = \frac{1}{(z-1)\cancel{(z+2)}} \cdot \frac{\cancel{(z+2)}(z-4)}{(z-3)^2(z+5)}$ $g(z)$



$\mapsto z = 1, -2, 8,$ 에서
해석적X.

$z^2 - 2z - 8 = ($
-12
-4

$\oint_C f(z)dz = \int_0^{2\pi} \frac{g(z)}{z-1} dz$

$= 2\pi i\, g(1) = 2\pi i \cdot \frac{\cancel{(-3)}}{4 \cdot \cancel{6}\,2} = 2\pi i \left(-\frac{1}{8}\right)$

$\therefore -\frac{1}{4}\pi i$