[과제4]

$2L=2\pi$. $L=\pi$. $f(x)=x^3-\pi^2x$: 기함수.

$$f(x)=\sum_{n=1}^{\infty}B_n\cdot\sin\frac{n\pi}{L}x=\sum_{n=1}^{\infty}B_n\cdot\sin nx$$

$$B_n=\frac{2}{\pi}\int_0^{\pi}(x^3-\pi^2x)\sin nx\,dx$$

$$\int_0^{\pi}(x^3-\pi^2x)\sin nx\,dx$$

$$=\int_0^{\pi}x^3\sin nx-\pi^2x\sin nx\,dx$$

$$\int_0^{\pi}x^3\sin nx\,dx$$

$$=x^3\cdot\frac{-1}{n}\cos nx\Big|_0^{\pi}+\int_0^{\pi}3x^2\cdot\frac{1}{n}\cos nx\,dx$$

$$=x^3\cdot\frac{-1}{n}\cos nx\Big|_0^{\pi}+\overset{=0}{\cancel{3x^2\cdot\frac{1}{n^2}\sin nx\Big|_0^{\pi}}}-\int_0^{\pi}6x\cdot\frac{1}{n^2}\sin nx\,dx$$

$$=x^3\cdot\frac{-1}{n}\cos nx\Big|_0^{\pi}-\left(6x\cdot\frac{-1}{n^3}\cos nx\Big|_0^{\pi}+\overset{=0}{\cancel{\int_0^{\pi}6\cdot\frac{1}{n^3}\cos nx\,dx}}\right)$$

$$=-\frac{\pi^3}{n}\cos n\pi+\frac{6\pi}{n^3}\cos n\pi$$

$$\int_0^{\pi}\pi^2x\sin nx\,dx$$

$$=\pi^2x\cdot\frac{-1}{n}\cos nx\Big|_0^{\pi}+\overset{=0}{\cancel{\int_0^{\pi}\pi^2\cdot\frac{1}{n}\cos nx\,dx}}$$

$$=-\frac{\pi^3}{n}\cos n\pi$$

$$\Rightarrow\int_0^{\pi}x^3\sin nx-\pi^2x\sin nx\,dx$$

$$=\cancel{-\frac{\pi^3}{n}\cos n\pi}+\frac{6\pi}{n^3}\cos n\pi+\cancel{\frac{\pi^3}{n}\cos n\pi}=\frac{6\pi}{n^3}\cos n\pi$$

$$B_n=\frac{2}{\pi}\cdot\frac{6\pi}{n^3}\cos n\pi=\frac{12}{n^3}(-1)^n$$

(1) $f(x)=\sum_{n=1}^{\infty}\frac{12}{n^3}(-1)^n\cdot\sin nx$.

(2) $f(x)=-12\left(\frac{1}{1^3}\sin x-\frac{1}{2^3}\sin 2x+\frac{1}{3^3}\sin 3x-\frac{1}{4^3}\sin 4x+\frac{1}{5^3}\sin 5x-\cdots\right)$

$$f\left(\frac{\pi}{2}\right)=-12\left(1-\frac{1}{3^3}+\frac{1}{5^3}-\cdots\right)$$

$$\therefore\ 1-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\cdots=-\frac{1}{12}f\left(\frac{\pi}{2}\right)=-\frac{1}{12}\left(\frac{\pi^3}{8}-\frac{\pi^3}{2}\right)$$

$$=-\frac{1}{\cancel{12}_{4}}\cdot-\frac{\cancel{3}}{8}\pi^3=\frac{\pi^3}{32}$$

(3) by. Parseval's Identity

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\{f(x)\}^2dx=\sum_{n=1}^{\infty}b_n^2 \text{ 식 이용.}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}(x^3-\pi^2x)^2dx=\sum_{n=1}^{\infty}\left\{\frac{12}{n^3}(-1)^n\right\}^2$$

$$=\sum_{n=1}^{\infty}\frac{144}{n^6}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^6}=\frac{1}{144\pi}\int_{-\pi}^{\pi}(x^3-\pi^2x)^2dx$$

$$=\frac{1}{144\pi}\int_{-\pi}^{\pi}x^6-2\pi^2x^4+\pi^4x^2\,dx$$

$$=\frac{1}{72\pi}\int_0^{\pi}x^6-2\pi^2x^4+\pi^4x^2\,dx$$

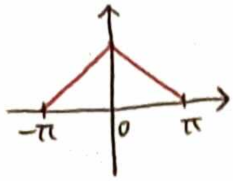
$$=\frac{1}{72\pi}\left[\frac{x^7}{7}-\frac{2\pi^2}{5}x^5+\frac{\pi^4}{3}x^3\right]_0^{\pi}$$

$$=\frac{1}{72\pi}\left(\frac{\pi^7}{7}-\frac{2\pi^7}{5}+\frac{\pi^7}{3}\right)$$

$$=\frac{1}{72\pi}\cdot\frac{8\pi^7}{105}=\frac{\pi^6}{945}$$

[과제5]

$$f(x) = \begin{cases} \pi + x & (-\pi < x < 0) \\ \pi - x & (0 < x < \pi) \end{cases}$$



∴ 우함수. $(B_n = 0)$

$2L = 2\pi$. $L = \pi$.

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cdot \cos\frac{n\pi}{L}x$$

$$= A_0 + \sum_{n=1}^{\infty} A_n \cdot \cos nx$$

$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{2}{2\pi}\int_0^{\pi} \pi - x\,dx$$

$$= \frac{1}{\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \frac{1}{\pi}\cdot\frac{\pi^2}{2} = \frac{\pi}{2}$$

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi}(\pi - x)\cos nx\,dx$$

$$\Rightarrow \int_0^{\pi} \pi\cos nx\,dx = \frac{\pi}{n}\sin nx\Big|_0^{\pi} = 0$$

$$\int_0^{\pi} x\cos nx\,dx = \overbrace{\cancel{\frac{x}{n}\sin nx\Big|_0^{\pi}}}^{=0} - \int_0^{\pi}\frac{1}{n}\sin nx\,dx$$

$$= -\left[\frac{-1}{n^2}\cos nx\right]_0^{\pi} = \frac{1}{n^2}(\cos n\pi - 1)$$

$$= \frac{1}{n^2}((-1)^n - 1)$$

$$A_n = \frac{2}{\pi}\cdot\frac{1}{n^2}(1 - (-1)^n)$$

n이 짝수면 $A_n = 0$.

홀수면 $A_n = \frac{2}{\pi}\cdot\frac{2}{n^2} = \frac{4}{n^2\pi}$

(1)

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} A_n \cdot \cos nx$$

$$= \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{4}{(2n-1)^2\pi}\cos((2n-1)x)$$

$$= \frac{\pi}{2} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos((2n-1)x)}{(2n-1)^2}$$

(2).

$$f(0) = \frac{\pi}{2} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

$$\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = f(0) - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{\pi}{2}\cdot\frac{\pi}{4} = \frac{\pi^2}{8}$$

(3).

by Parseval's Identity

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\{f(x)\}^2\,dx = 2a_0^2 + \sum_{n=1}^{\infty} a_n^2$$

$$= 2\cdot\left(\frac{\pi}{2}\right)^2 + \sum_{n=1}^{\infty}\frac{16}{(2n-1)^4\pi^2}$$

$$\frac{1}{L}\int_{-L}^{L}\{f(x)\}^2\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}\{f(x)\}^2\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}(\pi - x)^2\,dx = \frac{2}{\pi}\int_0^{\pi}\pi^2 - 2x\pi + x^2\,dx$$

$$= \frac{2}{\pi}\left[\pi^2 x - \pi x^2 + \frac{x^3}{3}\right]_0^{\pi} = \frac{2}{\pi}\left(\pi^3 - \pi^3 + \frac{\pi^3}{3}\right)$$

$$= \frac{2}{3}\pi^2$$

$$\sum_{n=1}^{\infty}\frac{16}{(2n-1)^4\pi^2} = \frac{2}{3}\pi^2 - \frac{\pi^2}{2} = \frac{\pi^2}{6}$$

$$\therefore \sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} = \frac{\pi^2}{6}\cdot\frac{\pi^2}{16} = \frac{\pi^4}{96}$$

(4)

$$\sum_{n=1}^{\infty}\frac{1}{n^4} = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} + \sum_{n=1}^{\infty}\frac{1}{(2n)^4}$$

$$= \sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} + \frac{1}{16}\sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$\frac{15}{16}\sum_{n=1}^{\infty}\frac{1}{n^4} = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$$

$$\therefore \sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{96}\cdot\frac{16}{15} = \frac{\pi^4}{90}$$