[과제6]

$f(x)$: 우함수 $(b_n=0)$ $\quad 2L=4, \ L=2$

$$a_0=\frac{1}{2L}\int_{-L}^{L} f(x)\,dx=\frac{1}{2}\int_0^2 f(x)\,dx=\frac{1}{2}\int_0^2 x\,dx=\frac{1}{2}\cdot\frac{1}{2}\cdot x^2\Big|_0^2$$

$$=1.$$

$$a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cdot\cos\frac{n\pi}{L}x\,dx=\frac{\cancel{2}}{\cancel{2}}\int_0^2 x\cdot\cos\frac{n\pi}{2}x\,dx$$

$$=\underbrace{\cancel{x\cdot\frac{2}{n\pi}\sin\frac{n\pi}{2}x\Big|_0^2}}_{=0}-\int_0^2\frac{2}{n\pi}\sin\frac{n\pi}{2}x\,dx$$

$$=\left(\frac{2}{n\pi}\right)^2\cdot\cos\frac{n\pi}{2}x\Big|_0^2=\frac{4}{n^2\pi^2}\{(-1)^n-1\}$$

$$a_n=\begin{cases}0 & (n\text{이 짝수})\\ \dfrac{-8}{n^2\pi^2} & (n\text{이 홀수})\end{cases}$$

$$f(x)=a_0+\sum_{n=1}^{\infty}a_n\cdot\cos\frac{n\pi}{2}x$$

$$=1+\sum_{n=1}^{\infty}\frac{-8}{(2n-1)^2\pi^2}\cos\frac{n\pi}{2}x$$

$$=1-\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\frac{n\pi}{2}x$$

$$f(0)=1-\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

$=$이$=0$

$$\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=1$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$$

[과제7]

$$f(x) = \begin{cases} \pi^2 & -\pi < x < 0 \\ \pi^2 - x^2 & 0 \le x < \pi \end{cases}$$

$2L = 2\pi$. $L = \pi$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi}\left\{\int_{-\pi}^{0} \pi^2 dx + \int_{0}^{\pi} \pi^2 - x^2\,dx\right\}$$

$$= \frac{1}{2\pi}\left\{\int_0^{\pi} 2\pi^2 - x^2\,dx\right\} = \frac{1}{2\pi}\left[2\pi^2 x - \frac{1}{3}x^3\right]_0^{\pi}$$

$$= \frac{1}{2\pi}\left(2\pi^3 - \frac{\pi^3}{3}\right) = \frac{1}{2\pi}\cdot\frac{5}{3}\pi^3 = \frac{5}{6}\pi^2$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos nx\,dx$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} \pi^2\cos nx\,dx + \int_0^{\pi}(\pi^2 - x^2)\cos nx\,dx\right\}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}(2\pi^2\cos nx - x^2\cos nx)\,dx\right\}$$

$$\int_0^{\pi} 2\pi^2\cos nx\,dx = 2\pi^2\left[\frac{1}{n}\sin nx\right]_0^{\pi} = 0$$

$$\int_0^{\pi} x^2\cos nx\,dx = \underbrace{\frac{x^2}{n}\cancel{\sin nx}\Big|_0^{\pi}}_{=0} - \int_0^{\pi}\frac{2x}{n}\sin nx\,dx$$

$$= -\left\{-\frac{2x}{n^2}\cos nx\Big|_0^{\pi} + \underbrace{\int_0^{\pi}\frac{2}{n^2}\cancel{\cos nx}\,dx}_{=0}\right\} = \frac{2\pi}{n^2}\cos n\pi$$

$$= \frac{2\pi}{n^2}\cos n\pi = \frac{2\pi}{n^2}(-1)^n$$

$$a_n = \frac{1}{\pi}\left\{-\frac{2\pi}{n^2}(-1)^n\right\} = \frac{1}{\pi}\cdot\frac{2\pi}{n^2}(-1)^{n+1}$$

$$= \frac{2}{n^2}(-1)^{n+1}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0}\pi^2\sin nx\,dx + \int_0^{\pi}(\pi^2\sin nx - x^2\sin nx)\,dx\right\}$$

$$= \frac{1}{\pi}\left\{-\int_0^{\pi} x^2\sin nx\,dx\right\}$$

$$\int_0^{\pi} x^2\sin nx\,dx = -\frac{x^2}{n}\cos nx\Big|_0^{\pi} + \int_0^{\pi}\frac{2x}{n}\cos nx$$

$$= -\frac{\pi^2}{n}\cos n\pi + \left\{\underbrace{\frac{2x}{n^2}\cancel{\sin nx}\Big|_0^{\pi}}_{=0} - \int_0^{\pi}\frac{2}{n^2}\sin nx\,dx\right.$$

$$= -\frac{\pi^2}{n}(-1)^n + \frac{2}{n^3}\left[\cos nx\right]_0^{\pi} = \frac{\pi^2}{n}(-1)^{n+1} + \frac{2}{n^3}(\cos n\pi - 1)$$

$$= \frac{\pi^2}{n}(-1)^{n+1} + \frac{2}{n^3}\left\{(-1)^n - 1\right\}$$

$$b_n = \frac{\pi}{n}(-1)^n + \frac{2}{n^3\pi}\left\{1 - (-1)^n\right\}$$

과제7 이어서...

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$= \frac{5}{6}\pi^2 + \sum_{n=1}^{\infty} \frac{2}{n^2}(-1)^{n+1} \cos nx + \left(\frac{\pi}{n}(-1)^n + \frac{2}{n^3\pi}\{1-(-1)^n\} \right) \sin nx$$

$$f(0) = \frac{5}{6}\pi^2 + \sum_{n=1}^{\infty} \frac{2}{n^2}(-1)^{n+1} + 0 = \pi^2$$

$$\sum_{n=1}^{\infty} \frac{2}{n^2}(-1)^{n+1} = \frac{\pi^2}{6}, \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{6} \cdot \frac{1}{2} = \frac{\pi^2}{12}$$

[과제8]

$$f(x) = \begin{cases} -1 & (-2 \le x \le 0) \\ 2 & (0 < x \le 2) \end{cases} \qquad 2L = 4. \ L = 2$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{2}x + b_n \sin \frac{n\pi}{2}x$$

$$a_0 = \frac{1}{4}\int_{-2}^{2} f(x)\,dx = \frac{1}{4}\left\{ \int_{-2}^{0} \underbrace{(-1)\,dx}_{\text{우함수} = \int_0^2 -1\,dx} + \int_0^2 2\,dx \right\}$$

$$= \frac{1}{4}\int_0^2 1\,dx = \frac{1}{4}\cdot 2 = \frac{1}{2}$$

$$a_n = \frac{1}{2}\int_{-2}^{2} f(x) \cos\frac{n\pi}{2}x\,dx$$

우함수 $= \int_0^2 -\cos\frac{n\pi}{2}x\,dx$

$$= \frac{1}{2}\left\{ \int_{-2}^{0} -\cos\frac{n\pi}{2}x\,dx + \int_0^2 2\cos\frac{n\pi}{2}x\,dx \right\}$$

$$= \frac{1}{2}\int_0^2 \cos\frac{n\pi}{2}x\,dx = 0$$

$$b_n = \frac{1}{2}\int_{-2}^{2} f(x)\cdot \sin\frac{n\pi}{2}x\,dx$$

$$= \frac{1}{2}\left\{ \int_{-2}^{0} -\sin\frac{n\pi}{2}x\,dx + \int_0^2 2\sin\frac{n\pi}{2}x\,dx \right\}$$

기함수 $= \int_0^2 \sin\frac{n\pi}{2}x\,dx$

$$= \frac{1}{2}\left\{ 3\int_0^2 \sin\frac{n\pi}{2}x\,dx \right\} = \frac{3}{2}\left[\frac{-2}{n\pi}\cos\frac{n\pi}{2}x \right]_0^2$$

$$= \frac{3}{2}\left(-\frac{2}{n\pi}\right)(\cos n\pi - 1) = -\frac{3}{n\pi}\cdot((-1)^n - 1) = \frac{3}{n\pi}(1-(-1)^n)$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{3}{n\pi}(1-(-1)^n) \sin\frac{n\pi}{2}x.$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} \frac{3}{(2n-1)\pi} \cdot 2 \cdot \sin\frac{(2n-1)\pi}{2}x$$

$$= \frac{1}{2} + \frac{6}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \cdot \sin\frac{(2n-1)\pi}{2}x.$$

$$\frac{6}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \cdot \sin\frac{(2n-1)\pi}{2}x = f(x) - \frac{1}{2}.$$

$x=1$ 대입.

$$\frac{6}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \cdot \sin\frac{(2n-1)}{2}\pi = f(1) - \frac{1}{2} = \frac{3}{2}$$

$$\sum_{n=1}^{\infty} \frac{1}{2n-1} \underbrace{\boxed{\sin\frac{2n-1}{2}\pi}}_{=(-1)^{n-1}} = \frac{3}{2}\cdot\frac{\pi}{6} = \frac{\pi}{4}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} = \frac{\pi}{4}$$

[과제9]

$$f(x)=\begin{cases}0 & (-\pi<x<0)\\ \sin x & (0\le x<\pi)\end{cases} \qquad 2L=2\pi,\ L=\pi$$

$$f(x)=a_0+\sum_{n=1}^{\infty} a_n\cos nx+b_n\sin nx$$

$$a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx=\frac{1}{2\pi}\int_0^{\pi}\sin x\,dx$$

$$=\frac{1}{2\pi}\cdot 2=\frac{1}{\pi}$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos nx\,dx$$

$$=\frac{1}{\pi}\int_0^{\pi}\sin x\cdot\cos nx\,dx$$

$$=\frac{1}{\pi}\int_0^{\pi}\frac{1}{2}\{\sin(x+nx)+\sin(x-nx)\}\,dx$$

$$=\frac{1}{2\pi}\int_0^{\pi}\sin((1+n)x)+\sin((1-n)x)\,dx$$

$$=\frac{-1}{2\pi}\left[\frac{1}{1+n}\cos((1+n)x)+\frac{1}{1-n}\cos((1-n)x)\right]_0^{\pi}$$

$$=\frac{-1}{2\pi}\left(\frac{1}{1+n}\cos(1+n)\pi-\frac{1}{1+n}+\frac{1}{1-n}\cos(1-n)\pi-\frac{1}{1-n}\right)$$

$$=\frac{-1}{2\pi}\left(\frac{1}{1+n}\cdot(-1)^{n+1}-\frac{1}{1+n}+\frac{1}{1-n}\cdot(-1)^{n+1}-\frac{1}{1-n}\right)$$

$$a_n=\begin{cases}0 & (n\text{이 홀수})\\ \frac{-1}{2\pi}\cdot(-2)\left(\frac{1}{1+n}+\frac{1}{1-n}\right)=\frac{1}{\pi}\cdot\frac{2}{1-n^2} & (n\text{이 짝수})\end{cases}$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\sin nx\,dx$$

$$=\frac{1}{\pi}\int_0^{\pi}\sin x\cdot\sin nx\,dx$$

$$=\frac{1}{\pi}\int_0^{\pi}-\frac{1}{2}\{\cos\underbrace{(x+nx)}_{(1+n)x}-\cos\underbrace{(x-nx)}_{(1-n)x}\}\,dx$$

$$=-\frac{1}{2\pi}\left[\frac{1}{1+n}\sin(1+n)x-\frac{1}{1-n}\sin(1-n)x\right]_0^{\pi}=0$$

$$f(x)=a_0+\sum_{n=1}^{\infty}a_n\cos nx$$

$$=\frac{1}{\pi}+\sum_{n=1}^{\infty}\frac{2}{\pi}\cdot\frac{1}{(1-2n)(1+2n)}\cos 2nx$$

$$f(0)=\frac{1}{\pi}+\sum_{n=1}^{\infty}\frac{2}{\pi}\cdot\frac{1}{(1-2n)(1+2n)}=0$$

$$\frac{1}{\pi}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{(1-2n)(1+2n)}=0$$

$$\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{(1-2n)(1+2n)}=-\frac{1}{\pi},\quad \sum_{n=1}^{\infty}\frac{1}{(1-2n)(1+2n)}=-\frac{1}{2}$$

$$\sum_{n=1}^{\infty}\frac{1}{(1-2n)(1+2n)}$$

$$=\frac{1}{-1\cdot 3}+\frac{1}{-3\cdot 5}+\frac{1}{-5\cdot 7}+\cdots$$

$$-\sum_{n=1}^{\infty}\frac{1}{(1-2n)(1+2n)}=\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots$$

$$=\frac{1}{2}.$$

$$\frac{1}{2}+\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots=1.$$

$$\therefore 1$$

[과제10]

$$f(x)=\begin{cases}0 & (x<-1)\\ -1 & (-1<x<0)\\ 2 & (0<x<1)\\ 0 & (x>1)\end{cases}$$

$$f(x)=\int_0^{\infty}A(\omega)\cos\omega x+B(\omega)\sin\omega x\,d\omega$$

$$A(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}f(x)\cdot\cos\omega x\,dx$$

$$=\frac{1}{\pi}\left\{\int_{-1}^{0}\underbrace{-\cos\omega x}_{\text{우함수}=\int_0^1-\cos\omega x\,dx}\,dx+\int_0^1 2\cos\omega x\,dx\right\}$$

$$=\frac{1}{\pi}\left\{\int_0^1\cos\omega x\,dx\right\}$$

$$=\frac{1}{\pi}\left[\frac{1}{\omega}\sin\omega x\right]_0^1=\frac{1}{\omega\pi}\sin\omega$$

$$B(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}f(x)\cdot\sin\omega x\,dx$$

$$=\frac{1}{\pi}\left\{\int_{-1}^{0}\underbrace{-\sin\omega x}_{\text{기함수}=\int_0^1\sin\omega x\,dx}\,dx+\int_0^1 2\sin\omega x\,dx\right\}$$

$$=\frac{1}{\pi}\cdot 3\int_0^1\sin\omega x\,dx$$

$$=\frac{3}{\pi}\left[\frac{-1}{\omega}\cdot\cos\omega x\right]_0^1$$

$$=\frac{-3}{\omega\pi}(\cos\omega-1)=\frac{3}{\omega\pi}(1-\cos\omega)$$

$$\therefore f(x)=\int_0^{\infty}\left\{\frac{1}{\omega\pi}\sin\omega\cdot\cos\omega x+\frac{3}{\omega\pi}(1-\cos\omega)\sin\omega x\right\}d\omega$$

[과제 11]

$f(x) = e^{-|x|}\sin x$: 기함수. $(\because A(\omega)=0)$

$f(x) = \int_0^\infty A(\omega)\cos\omega x + B(\omega)\sin\omega x \, d\omega$

$A(\omega)=0,$

$B(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cdot\sin\omega x \, dx$

$= \frac{1}{\pi}\int_{-\infty}^{\infty} e^{-|x|}\sin x\cdot\sin\omega x \, dx$

$= \frac{2}{\pi}\int_0^\infty e^{-x}\sin x\cdot\sin\omega x \, dx$

$= \frac{2}{\pi}\int_0^\infty e^{-x}\cdot\left(-\frac{1}{2}\right)\left(\cos((1+\omega)x) - \cos((1-\omega)x)\right) dx$

$= -\frac{1}{\pi}\int_0^\infty e^{-x}\left(\cos((1+\omega)x) - \cos((1-\omega)x)\right) dx$

$= -\frac{1}{\pi}\left\{\int_0^\infty e^{-x}\cdot\cos((1+\omega)x)\,dx - \int_0^\infty e^{-x}\cdot\cos((1-\omega)x)\,dx\right\}$

$= -\frac{1}{\pi}\left(\mathcal{L}(\cos((1+\omega)x))\Big|_{s=1} - \mathcal{L}(\cos(1-\omega)x)\Big|_{s=1}\right)$

$= -\frac{1}{\pi}\left(\frac{1}{1^2+(1+\omega)^2} - \frac{1}{1^2+(1-\omega)^2}\right)$

$= \frac{1}{\pi}\left(\frac{-1}{\omega^2+2\omega+2} + \frac{1}{\omega^2-2\omega+2}\right)$

$f(x) = \int_0^\infty \frac{1}{\pi}\left(\frac{1}{\omega^2-2\omega+2} - \frac{1}{\omega^2+2\omega+2}\right)\cdot\sin\omega x \, d\omega$

[과제 12]

$f(x) = \begin{cases} |x|, & |x|<\pi \\ 0, & |x|>\pi \end{cases}$: 우함수 $(B(\omega)=0)$

$f(x) = \int_0^\infty A(\omega)\cos\omega x + B(\omega)\sin\omega x \, d\omega$

$A(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cdot\cos\omega x \, dx$

$= \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{|x|}_{우}\cdot\underbrace{\cos\omega x}_{우} \, dx$

$= \frac{2}{\pi}\int_0^\pi x\cdot\cos\omega x \, dx$

$= \frac{2}{\pi}\left\{\frac{x}{\omega}\cdot\overset{=0}{\cancel{\sin\omega x}}\Big|_0^\pi - \frac{1}{\omega}\int_0^\pi \sin\omega x \, dx\right\}$

$= \frac{2}{\pi}\left(-\frac{1}{\omega}\right)\left(\left[\frac{-1}{\omega}\cos\omega x\right]_0^\pi\right)$

$= \frac{2}{\pi\omega^2}\left(\cos\omega\pi - 1\right)$

$f(x) = \int_0^\infty \frac{2}{\pi\omega^2}(\cos\omega\pi - 1)\cdot\cos\omega x \, d\omega$

[과제 13]

$f(x) = xe^{-2x} \ (x>0)$

1) 푸리에 사인 적분

$f(x) = \int_0^\infty B(\omega)\sin\omega x \, d\omega$

$A(\omega) = 0$

$B(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin\omega x \, dx = \frac{2}{\pi}\int_0^\infty f(x)\sin\omega x \, dx$

$= \frac{2}{\pi}\int_0^\infty e^{-2x} \cdot x\sin\omega x \, dx = \frac{2}{\pi}\mathcal{L}(x\sin\omega x)\Big|_{s=2}$

$g(x) = x\sin\omega x. \quad g(0)=0$

$g'(x) = \sin\omega x + \omega x\cos\omega x, \quad g'(0)=0$

$g''(x) = \omega\cos\omega x + \omega\cos\omega x - \omega^2 x\sin\omega x$

$= 2\omega\cos\omega x - \omega^2 x\sin\omega x$

$\mathcal{L}(g'') = 4\mathcal{L}(g)$

$\int_0^\infty (2\omega\cos\omega x - \omega^2 x\sin\omega x)e^{-2x}dx$

$= 4\int_0^\infty x\sin\omega x \, e^{-2x} dx$

$(4+\omega^2)\int_0^\infty e^{-2x} \cdot x\sin\omega x \, dx = \int_0^\infty 2\omega\cos\omega x \, e^{-2x} dx$

$(4+\omega^2)\mathcal{L}(x\sin\omega x)\Big|_{s=2} = 2\omega\mathcal{L}(\cos\omega x)\Big|_{s=2}$

$= 2\omega \cdot \frac{2}{2^2+\omega^2} = \frac{4\omega}{\omega^2+4}$

$\mathcal{L}(x\sin\omega x) = \frac{4\omega}{(\omega^2+4)^2}$

$B(\omega) = \frac{2}{\pi} \cdot \frac{4\omega}{(\omega^2+4)^2} = \frac{8}{\pi} \cdot \frac{\omega}{(\omega^2+4)^2}$

$\therefore f(x) = \int_0^\infty \frac{8}{\pi} \cdot \frac{\omega}{(\omega^2+4)^2}\sin\omega x \, d\omega$

2) 푸리에 코사인 적분

$f(x) = \int_0^\infty A(\omega)\cos\omega x \, d\omega$

$B(\omega) = 0$

$A(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos\omega x \, dx = \frac{2}{\pi}\int_0^\infty f(x)\cos\omega x \, dx$

$= \frac{2}{\pi}\int_0^\infty e^{-2x} \cdot x\cos\omega x \, dx$

$= \frac{2}{\pi}\mathcal{L}(x\cos\omega x)\Big|_{s=2}$

$g(x) = x\cos\omega x, \quad g(0)=0$

$g'(x) = \cos\omega x - \omega x\sin\omega x, \quad g'(0)=1$

$g''(x) = -\omega\sin\omega x - \omega\sin\omega x - \omega^2 x\cos\omega x$

$= -2\omega\sin\omega x - \omega^2 x\cos\omega x$

$\mathcal{L}(g'') = 4\mathcal{L}(g) - 2g(0) - g'(0)$

$= 4\mathcal{L}(g) - 1$

$\int_0^\infty (-2\omega\sin\omega x - \omega^2 x\cos\omega x)e^{-2x}dx$

$= 4\int_0^\infty x\cos\omega x \cdot e^{-2x}dx - 1$

$\int_0^\infty (4+\omega^2)x\cos\omega x \cdot e^{-2x}dx = (4+\omega^2)\mathcal{L}(x\cos\omega x)\Big|_{s=2}$

$= \int_0^\infty -2\omega\sin\omega x \, e^{-2x}dx + 1$

$= -2\omega \cdot \mathcal{L}(\sin\omega x)\Big|_{s=2} + 1$

$= -2\omega \cdot \frac{\omega}{2^2+\omega^2} + 1 = \frac{-2\omega^2}{\omega^2+4} + 1$

$(4+\omega^2)\mathcal{L}(x\cos\omega x)\Big|_{s=2}$

$= \frac{-2\omega^2}{\omega^2+4} + 1 \Rightarrow \mathcal{L}(x\cos\omega x)\Big|_{s=2} = \frac{-2\omega^2}{(\omega^2+4)^2} + \frac{1}{\omega^2+4}$

$A(\omega) = \frac{2}{\pi}\left\{\frac{-2\omega^2}{(\omega^2+4)^2} + \frac{1}{\omega^2+4}\right\}$

$f(x) = \int_0^\infty \frac{2}{\pi}\left\{\frac{-2\omega^2}{(\omega^2+4)^2} + \frac{1}{\omega^2+4}\right\}\cos\omega x \, d\omega$

[과제14]

$$\int_0^\infty f(x)\cos\omega x\,dx = e^{-\omega}.$$

$$\frac{2}{\pi}\int_0^\infty f(x)\cdot\cos\omega x\,dx = \frac{2}{\pi}e^{-\omega} = A(\omega)$$

$$f(x) = \int_0^\infty A(\omega)\cdot\cos\omega x\,d\omega = \int_0^\infty \frac{2}{\pi}e^{-\omega}\cdot\cos\omega x\,d\omega$$

$$= \frac{2}{\pi}\int_0^\infty e^{-\omega}\cdot\cos\omega x\,d\omega$$

$$= \frac{2}{\pi}\mathcal{L}(\cos(x\omega))\Big|_{s=1}$$

$$= \frac{2}{\pi}\cdot\frac{1}{1^2+x^2} = \frac{2}{\pi}\left(\frac{1}{x^2+1}\right) \quad (x>0)$$

[과제15]

$f(x)=\dfrac{1}{1+x^2}$: 우함수.

$$\hat{f}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{1}{1+x^2}\cdot e^{-i\omega x}\,dx$$

$$=\sqrt{\frac{2}{\pi}}\int_{0}^{\infty}\frac{1}{1+x^2}\cdot\cos\omega x\,dx$$

$g(x)=e^{-|x|}$: 우함수.

$$\hat{g}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-|x|}\cdot e^{-i\omega x}\,dx$$

$$=\sqrt{\frac{2}{\pi}}\int_{0}^{\infty}e^{-x}\cdot\cos\omega x\,dx=\sqrt{\frac{2}{\pi}}\mathcal{L}(\cos\omega x)\big|_{s=1}$$

$$=\sqrt{\frac{2}{\pi}}\cdot\frac{1}{1^2+\omega^2}$$

$$g(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\hat{g}(\omega)\,e^{i\omega x}\,d\omega$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\sqrt{\frac{2}{\pi}}\cdot\frac{1}{1+\omega^2}\cdot e^{i\omega x}\,d\omega=e^{-|x|}$$

$$=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+\omega^2}\cdot e^{i\omega x}\,d\omega$$

$$\int_{-\infty}^{\infty}\frac{1}{1+\omega^2}e^{i\omega x}\,d\omega=\pi\cdot e^{-|x|}.$$ x대신 $-x$ 대입.

$$\int_{-\infty}^{\infty}\frac{1}{1+\omega^2}e^{-i\omega x}\,d\omega=\pi\cdot e^{-|x|}$$

$\Rightarrow$ 적분변수 변경 $(\omega\to x)$

$$\int_{-\infty}^{\infty}\frac{1}{1+x^2}e^{-i\omega x}\,dx=\pi e^{-|\omega|}$$

$$\hat{f}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{1}{1+x^2}\cdot e^{-i\omega x}\,dx$$

$$=\frac{1}{\sqrt{2\pi}}\cdot\pi e^{-|\omega|}=\sqrt{\frac{\pi}{2}}e^{-|\omega|}$$

[과제 16]

$$f(x)=\begin{cases}1-\dfrac{|x|}{a} & (|x|\le a)\\ 0 & (|x|>a)\end{cases}$$

f : 우함수

$$\hat{f}_c(\omega)=\sqrt{\frac{2}{\pi}}\int_0^a\left(1-\frac{x}{a}\right)\cos\omega x\,dx$$

$$=\sqrt{\frac{2}{\pi}}\int_0^a \cos\omega x-\frac{x}{a}\cos\omega x\,dx$$

$$=\sqrt{\frac{2}{\pi}}\left\{\frac{1}{\omega}\sin\omega x\Big|_0^a-\left(\frac{x}{a\omega}\sin\omega x\Big|_0^a-\int_0^a\frac{1}{a\omega}\sin\omega x\,dx\right)\right\}$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{1}{\omega}\cancel{\sin a\omega}-\frac{a}{a\omega}\cancel{\sin a\omega}+\int_0^a\frac{1}{a\omega}\sin\omega x\,dx\right)$$

$$=\sqrt{\frac{2}{\pi}}\cdot\left[\frac{-1}{a\omega^2}\cos\omega x\right]_0^a=\sqrt{\frac{2}{\pi}}\cdot\frac{1}{a\omega^2}(1-\cos a\omega)$$

$$\lim_{\omega\to0}\hat{f}(\omega)=\lim_{\omega\to0}\sqrt{\frac{2}{\pi}}\cdot\frac{1}{a\omega^2}(1-\cos a\omega)$$

$$=\sqrt{\frac{2}{\pi}}\lim_{\omega\to0}\frac{(1-\cos a\omega)(1+\cos a\omega)}{a^2\omega^2(1+\cos a\omega)}\cdot a$$

$$=\sqrt{\frac{2}{\pi}}\lim_{\omega\to0}\frac{\sin^2 a\omega}{a^2\omega^2(1+\cos a\omega)}\cdot a$$

$$=\sqrt{\frac{2}{\pi}}\cdot\frac{a}{2}=\frac{a}{2}\sqrt{\frac{2}{\pi}}=a\sqrt{\frac{1}{2\pi}}$$

[과제 17]

$$\int_0^\infty f(x)\cos wx\,dx = \begin{cases} 1-w & (0\le w\le 1) \\ 0 & (w>1) \end{cases}$$

$$\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos wx\,dx = \begin{cases} \sqrt{\frac{2}{\pi}}(1-w) & (0\le w\le 1) \\ 0 & (w>1) \end{cases}$$

$$F_c(f)=\hat{f}_c(w)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos wx\,dx$$

$$= \begin{cases} \sqrt{\frac{2}{\pi}}(1-w) & (0\le w\le 1) \\ 0 & (w>1) \end{cases}$$

$$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty \hat{f}(w)e^{iwx}\,dw.$$

$$=\sqrt{\frac{2}{\pi}}\int_0^\infty \hat{f}(w)\cdot\cos wx\,dw$$

$$=\sqrt{\frac{2}{\pi}}\int_0^1 \sqrt{\frac{2}{\pi}}(1-w)\cos wx\,dw$$

$$=\frac{2}{\pi}\int_0^1 \cos wx - w\cos wx\,dw$$

$$=\frac{2}{\pi}\left\{\frac{1}{x}\sin wx\Big|_0^1-\left(w\cdot\frac{1}{x}\sin wx\Big|_0^1-\int_0^1\frac{1}{x}\sin wx\,dw\right)\right\}$$

$$=\frac{2}{\pi}\left(\frac{1}{x}\sin x-\frac{1}{x}\sin x+\left[\frac{-1}{x^2}\cos wx\right]_0^1\right)$$

$$=\frac{2}{\pi}\left(\frac{1}{x^2}-\frac{1}{x^2}\cos x\right)=\frac{2}{\pi x^2}(1-\cos x)$$

[과제 18]

$$f(x)=\begin{cases} e^{-x} & (x\ge 0) \\ e^{x} & (x<0) \end{cases}$$

f : 우함수.

$$\hat{f}_c(w)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos wx\,dx$$

$$=\sqrt{\frac{2}{\pi}}\int_0^\infty e^{-x}\cdot\cos wx\,dx$$

$$=\sqrt{\frac{2}{\pi}}\mathcal{L}(\cos wx)\Big|_{s=1}$$

$$=\sqrt{\frac{2}{\pi}}\cdot\frac{1}{1^2+w^2}$$

$$\therefore \sqrt{\frac{2}{\pi}}\cdot\frac{1}{1+w^2}$$