## I. Test 4: Complicated Distribution

After these first three tests are done we can move onto more complicated distributions. Radially symmetric distributions would be interesting since the particles tend to travel in circles and therefore cause significant distortions to particles that are initially distributed on a rectangular grid. There is lots of literature that use these types of distributions. Namely, Perlman and Beale have some interesting articles on simulations of this type at long times.

a) For each point $\boldsymbol{i} \in [0, \cdots, N_p]^2$, with $h_p = 1/N_p$ and $N_p = 256$ (this can also be $N_p = 1/(\Delta x/2)$. We want our particles to be on a finer grid than the grid we use for deposition and interpolation). Particles that satisfy,

$$\|\boldsymbol{i}h_p - (0.5, 0.375)\|_{\mathrm{L2}} \leq 0.12 \text{ or } \|\boldsymbol{i}h_p - (0.5, 0.625)\|_{\mathrm{L2}} \leq 0.12$$

are given a strength of $h_p^2/(\Delta x \Delta y)$. This should be run to a time $T = 12.5$.
b) From [1], we can sample the radially symmetric distribution on a uniform grid

$$\omega(x, y) = \begin{cases} (r_{\mathrm{bdry}} - \|(x, y) - (0.5, 0.5)\|_{\mathrm{L2}}^2)^7 \text{ if } \|(x, y) - (0.5, 0.5)\|_{\mathrm{L2}} \leq r_{\mathrm{bdry}} \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

with $r_{\mathrm{bdry}} = 1/4$.

[1] Mirta Perlman. On the accuracy of vortex methods. *Jourdnal of Computational Physics*, 59(2):200-223, 1985.