* Diffusivity Equation (PDE)

$$\nabla^2 P = \frac{\phi \mu C_t}{k} \frac{\partial P}{\partial t} \quad \text{(SI unit, cgs unit)}$$

$$\nabla^2 P = \frac{\phi \mu C_t}{0.00633k} \frac{\partial P}{\partial t} \quad \text{(Field unit)}$$

$\Downarrow$

P [psia]. ϕ [-]. μ [cp]. C_t [$\frac{1}{psia}$]. k [md]. t [days]

(In case of t in hours,

$$\nabla^2 P = \frac{\phi \mu C_t}{0.0002637k} \frac{\partial P}{\partial t} \;)$$

In 1 D.

$$\boxed{\frac{\partial^2 P}{\partial x^2} = \frac{\phi \mu C_t}{0.00633k} \frac{\partial P}{\partial t}}$$

$\Rightarrow$ 1 D diffusivity equation (PDE) in field unit.

$\Rightarrow$ To convert PDE into FDE.

Finite Difference Equation.

<Taylor Series for FDE>

$$f(x+\Delta x) = a_0 + a_1 \Delta x + a_2 \Delta x^2 + a_3 \Delta x^3 + \dots + a_n \Delta x^n + \dots$$

$$\lim_{\Delta x \to 0} f(x+\Delta x) = a_0 = f(x)$$

$$\lim_{\Delta x \to 0} f'(x+\Delta x) = a_1 + 2a_2\Delta x + 3a_3 \Delta x^2 + \dots$$

$$= a_1 = f'(x)$$

$$\lim_{\Delta x \to 0} f''(x+\Delta x) = 2a_2 + 2\times 3\, a_3 \Delta x + \dots$$

$$= 2a_2 = f''(x) \Rightarrow a_2 = \frac{f''(x)}{2}$$

$\vdots$

$$\lim_{\Delta x \to 0} f^{(n)}(x+\Delta x) = n!\, a_n \Rightarrow a_n = \frac{f^{(n)}(x)}{n!}$$

$$\Rightarrow f(x+\Delta x) = f(x) + f'(x)\Delta x + \frac{f''(x)}{2!}\Delta x^2 + \frac{f'''(x)}{3!}\Delta x^3 + \dots$$

$\boxed{\text{LHS}}$ $\frac{\partial^2 P}{\partial x^2} = P''(x)$

$$P(x+\Delta x) = P(x) + P'(x)\Delta x + \frac{P''(x)}{2!}\Delta x^2 + \frac{f'''(x)}{3!}\Delta x^3 + \dots \quad ①$$

$$P(x-\Delta x) = P(x) - P'(x)\Delta x + \frac{P''(x)}{2!}\Delta x^2 - \frac{f'''(x)}{3!}\Delta x^3 + \dots \quad ②$$

① + ②

$$P(x+\Delta x) + P(x-\Delta x) = 2P(x) + 2\cdot\frac{P''(x)}{2!}\Delta x^2 + 2\cdot\frac{P^{(4)}(x)}{4!}\Delta x^4 + \dots$$

$$P''(x)\Delta x^2 = P(x+\Delta x) + P(x-\Delta x) - 2P(x) - \frac{2P^{(4)}(x)}{4!}\Delta x^4 + \dots$$

$$P''(x) = \frac{P(x-\Delta x) - 2P(x) + P(x+\Delta x)}{\Delta x^2} \Bigg| - \frac{2P^{(4)}(x)}{4!}\Delta x^2 + \dots$$

$$P''(x) = \frac{P(x-\Delta x) - 2P(x) + P(x+\Delta x)}{\Delta x^2} - \frac{2P^{(4)}(x)}{4!}\Delta x^2 + \cdots$$

Error term

$$P''(x) = \frac{P(x-\Delta x) - 2P(x) + P(x+\Delta x)}{\Delta x^2} + E(\Delta x^2)$$

RHS $\frac{\phi \mu C_t}{0.00633K} \frac{\partial P}{\partial t}$

$$\frac{\partial P}{\partial t} = P'(t)$$

$$P(t-\Delta t) = P(t) - P'(t)\Delta t + \frac{P''(t)}{2!}\Delta t^2 - \frac{P'''(t)}{3!}\Delta t^3 + \cdots$$

Error.

$$P'(t) = \frac{P(t) - P(t-\Delta t)}{\Delta t} + \frac{P''(t)}{2!}\Delta t - \cdots$$

$$P'(t) = \frac{P(t) - P(t-\Delta t)}{\Delta t} + E(\Delta t)$$

Our FDE for 1D FDE is

$$\frac{P(x-\Delta x) - 2P(x) + P(x+\Delta x)}{\Delta x^2} + E(\Delta x^2) = \frac{\phi \mu C_t}{0.00633K} \frac{P(t) - P(t-\Delta t)}{\Delta t} + E(\Delta t)$$

$$\Rightarrow \boxed{\frac{P(x-\Delta x) - 2P(x) + P(x+\Delta x)}{\Delta x^2} = \frac{\phi \mu C_t}{0.00633K} \frac{P(t) - P(t-\Delta t)}{\Delta t}}$$

With Error $\propto \Delta x^2, \Delta t$.

1. LHS at $t-\Delta t$.

$$\frac{P(x-\Delta x, t-\Delta t) - 2P(x, t-\Delta t) + P(x+\Delta x, t-\Delta t)}{\Delta x^2}$$

Explicit method

only unknown. Known

$$= \frac{\phi \mu C_t}{0.00633K} \frac{P(x,t) - P(x, t-\Delta t)}{\Delta t}$$

2. LHS at t.

$$\frac{P(x-\Delta x, t) - 2P(x) + P(x+\Delta x, t)}{\Delta x^2}$$

unknown

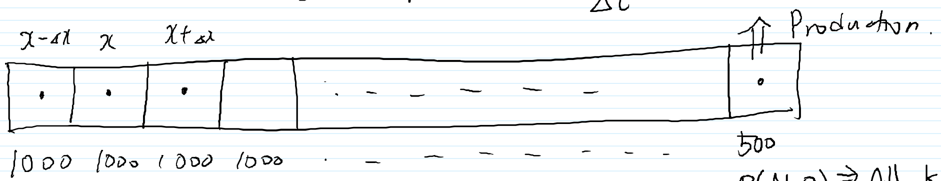
Implicit method

unknown Known.

$$= \frac{\phi \mu C_t}{0.00633K} \frac{P(x,t) - P(x, t-\Delta t)}{\Delta t}$$

Production

$$= \frac{\phi \mu c_t}{0.00633K} \frac{p(x,t) - p(x,t-\Delta t)}{\Delta t}$$



$t=0.$ $P(1,0)$ $P(2,0)$ $P(3,0)$ . . . $P(N,0) \Rightarrow$ All known.

$t=\Delta t$ $P(1,\Delta t)$ $P(2,\Delta t)$ $P(3,\Delta t)$ . . . $P(N,\Delta t) \Rightarrow$ All Unknown.

?

For explicit method.

$$\frac{P(1,0) - 2P(2,0) + P(3,0)}{\Delta x^2} = \frac{\phi \mu C_t}{0.00633K} \frac{P(2,\Delta t) - P(2,0)}{\Delta t}$$

For implicit method

$$\frac{P(1,\Delta t) - 2P(1,\Delta t) + P(3,\Delta t)}{\Delta x^2} = \frac{\phi \mu C_t}{0.00633K} \frac{P(2,\Delta t) - P(2,0)}{\Delta t}$$

Explicit method doesn't need matrix solution.

$$\frac{0.00633K}{\phi \mu C_t}\left[\frac{\Delta t}{\Delta x^2}\right] < 0.5$$

Implicit method, we will need matrix solution.

We don't need convergence criteria.

IMPES (Mutiphase flow)

$\Rightarrow$ Implicit Pressure Explicit Saturation.