01/27/2025

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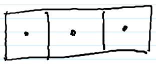
## < Example for 1D diffusivity equation with FDE >

$$\frac{P(x-\Delta x, t) - 2P(x,t) + P(x+\Delta x, t)}{\Delta x^2} = \frac{\phi \mu c_t}{0.00637k} \frac{P(x,t) - P(x, t-\Delta t)}{\Delta t}$$



$P(x-\Delta x,t)$ $P(x,t)$ $P(x+\Delta x,t)$ $P_{i-1}^{n-1}$ $P_i^{n-1}$ $P_{i+1}^{n-1}$ $t = t-\Delta t$

$P_{i-1}^n$ $P_i^n$ $P_{i+1}^n$ $t = t$

$P_{i-1}^{n+1}$ $P_i^{n+1}$ $P_{i+1}^{n+1}$ $t = t+\Delta t$

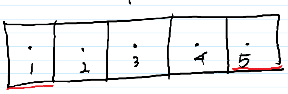
$$\frac{P_{i-1}^n - 2P_i^n + P_{i+1}^n}{\Delta x^2} = \frac{\phi \mu c_t}{0.00633k} \frac{P_i^n - P_i^{n-1}}{\Delta t}$$

&

unknowns. known

$$\frac{P_{i-1}^{n+1} - 2P_i^{n+1} + P_{i+1}^{n+1}}{\Delta x^2} = \frac{\phi \mu c_t}{0.00633k} \frac{P_i^{n+1} - P_i^n}{\Delta t}$$

## * Example problem



① Initial condition.

$P(x, t=0) = 1000$ psia.

② Boundary condition [ Open boundary condition. ⇒ Constant value b.c / Closed boundary condition. ⇒ Constant rate b.c.

$P(i=1, t) = 1000$ psia.
$P(i=5, t) = 500$ psia. ) ⇒ constant value boundary.

⇒ At $t=0$. (Initial time)

$P_1 = 1000$ psia, $P_2 = 1000$ psia, $P_3 = 1000$ psia. $P_4 = 1000$ psia. $P_5 = 500$ psia.

unknowns. known

$$\frac{P_{i-1}^{n+1} - 2P_i^{n+1} + P_{i+1}^{n+1}}{\Delta x^2} = \frac{\phi \mu c_t}{0.00633k} \frac{P_i^{n+1} - P_i^n}{\Delta t}$$

① Grid block 1.

$P_1^0 = 1000 = P_1^1 = P_1^2 = \cdots$ 1000 psia.

② Grid block 2

BC IC

$$\frac{\boxed{P_1^1} - 2P_2^1 + P_3^1}{\Delta x^2} = \frac{\phi \mu c_t}{0.00633k} \frac{P_2^1 - P_2^0}{\Delta t}$$

③ Grid block 3 IC

$$\frac{P_2^1 - 2P_3^1 + P_4^1}{\Delta x^2} = \frac{\phi \mu c_t}{0.00633k} \frac{P_3^1 - P_3^0}{\Delta t}$$

⇒ 3 equations
3 unknowns ($P_2^1, P_3^1, P_4^1$)

$$\frac{P_2 - 2P_3 + P_4}{\Delta x^2} = \frac{\phi \mu C_t}{0.00633k} \frac{P_3 - P_3}{\Delta t}$$

④ Grid block 4

$$\frac{P_3^1 - 2P_4^1 + \boxed{P_5^1}}{\Delta x^2} = \frac{\phi \mu C_t}{0.00633k} \frac{P_4^1 - P_4^0}{\Delta t}$$

BC IC

⑤ Grid block 5

$$P_5^1 = \cdots P_5^{100} \cdots = 500 \text{ psia.}$$

* Matrix equation. $Ax = b \Rightarrow x = A^{-1} \cdot b$

$A \Downarrow$ $N \times N$, $x \Downarrow$ $N \times 1$, $b \Downarrow$ $N \times 1$

$A \cdot P^{n+1} = b$. $A: 5 \times 5$, $P^{n+1} = 5 \times 1$, $b = 5 \times 1$.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{\Delta x^2} & A_{22} & \frac{1}{\Delta x^2} & 0 & 0 \\ 0 & \frac{1}{\Delta x^2} & A_{33} & \frac{1}{\Delta x^2} & 0 \\ 0 & 0 & \frac{1}{\Delta x^2} & A_{44} & \frac{1}{\Delta x^2} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_1^1 \\ P_2^1 \\ P_3^1 \\ P_4^1 \\ P_5^1 \end{pmatrix} = \begin{pmatrix} 1000 \\ \boxed{b_2} \\ \boxed{b_3} \\ \boxed{b_4} \\ 500 \end{pmatrix} \begin{array}{l} \Rightarrow P_1 \text{ equation} \\ \Rightarrow P_2 \text{ equation} \\ \Rightarrow P_3 \text{ equation} \\ \Rightarrow P_4 \text{ equation} \\ \Rightarrow P_5 \text{ equation.} \end{array}$$

$P_1$ equation $\Rightarrow P_1^1 = 1000$

$$\underbrace{A_{11}}_{1} P_1^1 + \cancel{A_{12}}^{0} P_2^1 + \cancel{A_{13}}^{0} P_3^1 + \cancel{A_{14}}^{0} P_4^1 + \cancel{A_{15}}^{0} P_5^1 = \underbrace{b_1}_{1000}.$$

$P_5$ equation $\Rightarrow P_5^1 = 500$

$$\cancel{A_{51}}^{0} P_1^1 + \cancel{A_{52}}^{0} P_2^1 + \cancel{A_{53}}^{0} P_3^1 + \cancel{A_{54}} P_4^1 + \underbrace{A_{55}}_{1} P_5^1 = \underbrace{b_5}_{500}.$$

$P_2$ equation.

$$\frac{P_1^1 - 2P_2^1 + P_3^1}{\Delta x^2} = \frac{\phi \mu C_t}{0.00633k} \frac{P_2^1 - P_2^0}{\Delta t}$$

$$A_{21} P_1^1 + A_{22} P_2^1 + A_{23} P_3^1 + \cancel{A_{24}}^{0} P_4^1 + \cancel{A_{25}}^{0} P_5^1 = b_2$$

$A_{21} \Downarrow \frac{1}{\Delta x^2}$, $A_{23} \Downarrow \frac{1}{\Delta x^2}$, $A_{22} \rightarrow -\frac{2}{\Delta x^2} - \frac{\phi \mu C_t}{0.00633k\, \Delta t}$, $b_2 \Downarrow -\frac{\phi \mu C_t}{0.00633k} \frac{P_2^0}{\Delta t}$

$P_3$ equation.

$$\frac{P_2^1 - 2P_3^1 + P_4^1}{\quad} = \frac{\phi \mu C_t}{\quad} \frac{P_3^1 - P_3^0}{\quad}$$

$P_3$ equation. $\dfrac{P_2^1 - 2P_3^1 + P_4^1}{\Delta x^2} = \dfrac{\phi \mu C_t}{0.00633 K} \dfrac{P_3^1 - P_3^0}{\Delta t}$

$$\cancel{A_{31}}^{\,0} P_1^1 + \underbrace{A_{32}}_{\frac{1}{\Delta x^2}} P_2^1 + \underbrace{A_{33}}_{-\frac{2}{\Delta x^2} - \frac{\phi \mu C_t}{0.00633 K \Delta t}} P_3^1 + \underbrace{A_{34}}_{\frac{1}{\Delta x^2}} P_4^1 + \cancel{A_{35}}^{\,0} P_5^1 = \underbrace{b_3}_{-\frac{\phi \mu C_t}{0.00633 K} \frac{P_3^0}{\Delta t}}$$

$P_4$ equation. $\dfrac{P_3^1 - 2P_4^1 + P_5^1}{\Delta x^2} = \dfrac{\phi \mu C_t}{0.00633 K} \dfrac{P_4^1 - P_4^0}{\Delta t}$

$$\cancel{A_{41}}^{\,0} P_1^1 + \cancel{A_{42}}^{\,0} P_2^1 + \underbrace{A_{43}}_{\frac{1}{\Delta x^2}} P_3^1 + \underbrace{A_{44}}_{-\frac{2}{\Delta x^2} - \frac{\phi \mu C_t}{0.00633 K \Delta t}} P_4^1 + \underbrace{A_{45}}_{\frac{1}{\Delta x^2}} P_5^1 = \underbrace{b}_{-\frac{\phi \mu C_t}{0.00633 K} \frac{P_4^0}{\Delta t}}$$

In summary,

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{\Delta x^2} & \left(-\frac{2}{\Delta x^2} - \boxed{\frac{\phi \mu C_t}{0.00633 K \Delta t}}\right) & \frac{1}{\Delta x^2} & 0 & 0 \\ 0 & \frac{1}{\Delta x^2} & \left(-\frac{2}{\Delta x^2} - \boxed{\frac{\phi \mu C_t}{0.00633 K \Delta t}}\right) & \frac{1}{\Delta x^2} & 0 \\ 0 & 0 & \frac{1}{\Delta x^2} & \left(-\frac{2}{\Delta x} - \boxed{\frac{\phi \mu C_t}{0.00633 K \Delta t}}\right) & \frac{1}{\Delta x^2} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_1^1 \\ P_2^1 \\ P_3^1 \\ P_4^1 \\ P_5^1 \end{pmatrix} = \begin{pmatrix} 1000 \\ -\boxed{\frac{\phi \mu C_t}{0.00633 K \Delta t}} P_2^0 \\ -\boxed{\frac{\phi \mu C_t}{0.00633 K \Delta t}} P_3^0 \\ -\boxed{\frac{\phi \mu C_t}{0.00633 K \Delta t}} P_4^0 \\ 500 \end{pmatrix}$$

$\phi = 0.2$, $\mu = 0.7$ cp, $C_t = 10^{-5} \frac{1}{psia}$, $K = 10$ md.

$\Delta x = 10$ ft, $\Delta t = 1$ day

$\Rightarrow \dfrac{1}{\Delta x^2} = 0.01$.

$$\frac{\phi \mu C_t}{0.00633 K \Delta t} = 2.2117 \times 10^{-5}.$$