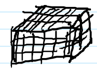
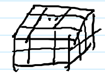
01/13/2025

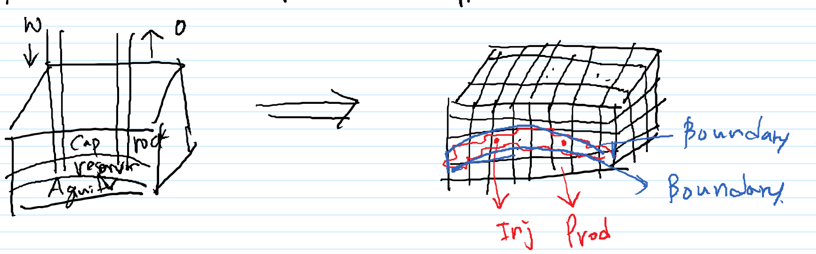
Monday, January 13, 2025 1:04 PM

1. Subsurface Reservoir System.

* Reservoir mangement workflow.

- Geological model build – fine scale. 
- Upscale to computational model 
- Establish boundary conditions – Aquifer
  - Prod / Inj.
  - Faults
- Define & solve equation describing fluid flow.
  - P, S, T
- History matching & Uncertainty quantification.

* Reservoir Simulation model



Single-phase flow : $P = f(x, y, z, t)$

Two-phase flow ; $P, S = f(x, y, z, t)$ → By solving a series of PDEs.

Fluid flow & heat : $P, T = f(x, y, z, t)$.

<Mathematical Concepts>

- Vector notation.

Scalar : u

Vector : u_1, u_2, u_3. $\qquad \vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$

① operator ∇. ⇒

$$\nabla a = \begin{pmatrix} \frac{\partial a}{\partial x} \\ \frac{\partial a}{\partial y} \\ \frac{\partial a}{\partial z} \end{pmatrix} = \frac{\partial a}{\partial x} i + \frac{\partial a}{\partial y} j + \frac{\partial a}{\partial z} k.$$

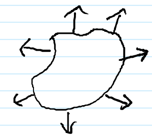
$$\begin{pmatrix} \frac{\partial a}{\partial y} \\ \frac{\partial a}{\partial z} \end{pmatrix}$$

② Inner product $(\cdot)$

$$\vec{u}\cdot\vec{v} = \sum_{i=1}^{n} u_i v_i \Rightarrow u_1v_1 + u_2v_2 + u_3v_3$$

③ Divergence. ⇒ ① & ② $\quad \nabla\cdot$

$$\nabla\cdot\vec{u} = \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z}.$$



Net flux out of the system.

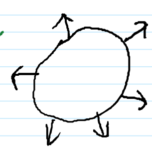
<Diffusivity Equation>

① Continuity equation : $\nabla\cdot(\rho\vec{u}) = -\frac{\partial(\phi\rho)}{\partial t}$ ... ①

② Darcy equation : $\vec{u} = -\frac{k}{\mu}\nabla P$ ... ②

③ Equation of State : $C_f\rho = \frac{d\rho}{\partial P}$, $C_R\phi = \frac{d\phi}{dP}$, $C_t = C_f + C_R$

$$\left[C = -\frac{1}{V}\frac{\partial V}{\partial P} \right]$$



$\boxed{L}$ Total mass flow rate out of the system

= Total mass loss in the system

$\boxed{L} = 30.$

$\boxed{R} = -30$

$70 = 100 - 30.$

* Derivation of diffusivity equation

(1) Substite Darcy's law to continuity equation. ② → ①

$$\nabla\cdot(\rho\,\vec{u}) = -\frac{\partial(\phi\rho)}{\partial t}.$$

$\uparrow$ ②

$$\Rightarrow \nabla\cdot\left(\rho\left(\cancel{-}\frac{k}{\mu}\nabla P\right)\right) = \cancel{-}\frac{\partial(\phi\rho)}{\partial t}$$

Assume k, μ = constant.

$$\Rightarrow \frac{k}{\mu}\nabla\cdot(\rho\nabla P) = \frac{\partial(\phi\rho)}{\partial t}.$$

$$\Rightarrow \nabla\cdot(\rho\nabla P) = \mu\,\partial(\phi\rho)$$

$\nabla\cdot\nabla P = \nabla^2 P$

$$\Rightarrow \nabla \cdot (\rho \nabla p) = \frac{\mu}{k} \frac{\partial (\phi \rho)}{\partial t}$$

$\nabla \cdot \nabla p = \nabla^2 p$

$$\Rightarrow \underbrace{\nabla \rho \cdot \nabla p + \rho \nabla^2 p}_{LHS} = \underbrace{\frac{\mu}{k} \frac{\partial (\phi \rho)}{\partial t}}_{RHS}.$$

$\boxed{LHS}$ $\nabla \rho = \frac{\partial \rho}{\partial p} \nabla p.$

Applying equation of state.

$\nabla \rho = C_f \rho \nabla p.$

$$C_f \rho \cancel{\nabla p \cdot \nabla p} + \rho \nabla^2 p.$$

Slightly compressible fluid. eg.) water, oil

$\nabla p \approx 10^{-5}.$

$$\rho \nabla^2 p.$$

LHS = RHS.

$$\rho \nabla^2 p = \frac{\mu}{k} \frac{\partial (\phi \rho)}{\partial t}$$

$\boxed{RHS}$ $\frac{\mu}{\rho k} \frac{\partial (\phi \rho)}{\partial t}.$

$L(a+b) \Rightarrow a L(b) + b L(a)$

$$= \frac{\mu}{k \rho} \left[\rho \frac{\partial \phi}{\partial t} + \phi \frac{\partial \rho}{\partial t} \right]$$

by EOS.

$$= \frac{\mu}{k \rho} \left[\rho \frac{\partial \phi}{\partial p} \frac{\partial p}{\partial t} + \phi \frac{\partial \rho}{\partial p} \frac{\partial p}{\partial t} \right]$$

$C_f \rho = \frac{\partial \rho}{\partial p}, \quad C_R \phi = \frac{\partial \phi}{\partial p}.$

$$= \frac{\mu}{k \cancel{\rho}} \left[\cancel{\rho} C_R \phi \frac{\partial p}{\partial t} + \phi C_f \cancel{\rho} \frac{\partial p}{\partial t} \right]$$

$$= \frac{\mu \phi}{k} \left[C_R + C_f \right] \frac{\partial p}{\partial t}.$$

$C_t = C_R + C_f.$

$$= \frac{\mu \phi C_t}{k} \frac{\partial p}{\partial t} \Rightarrow \text{RHS}.$$

LHS = RHS

$$\boxed{\nabla^2 p = \frac{\phi \mu C_t}{K} \frac{\partial p}{\partial t}}$$ Diffusivity equation.

$\boxed{1D}$ $\nabla^2 p = \frac{\partial^2 p}{\partial x^2}$

$\boxed{2D}$ $\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}$

$\boxed{3D}$ $\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}$

$\boxed{\text{Radial}}$ $\nabla^2 p = \frac{1}{r} \frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial p}{\partial r}\right)$, (In θ, z direction there is no change)

∇ = gradient. $\nabla P \Rightarrow \frac{\partial P}{\partial x} i + \frac{\partial P}{\partial y} j + \frac{\partial P}{\partial z} k$.

∇^2 = Laplacian. $= \nabla \cdot \nabla \Rightarrow \nabla \cdot \left(\frac{\partial P}{\partial x} i, \frac{\partial P}{\partial y} j, \frac{\partial P}{\partial z} k\right)$

$$= \left(\frac{\partial}{\partial x} i, \frac{\partial}{\partial y} j, \frac{\partial}{\partial z} k\right) \cdot \left(\frac{\partial P}{\partial x} i, \frac{\partial P}{\partial y} j, \frac{\partial P}{\partial z} k\right).$$

$$= \frac{\partial}{\partial x} i \cdot \frac{\partial P}{\partial x} i + \frac{\partial}{\partial y} j \cdot \frac{\partial P}{\partial y} j + \frac{\partial k}{\partial z} \cdot \frac{\partial p}{\partial z} K$$

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}$$

$\nabla^2 p = \nabla \cdot (\nabla p)$