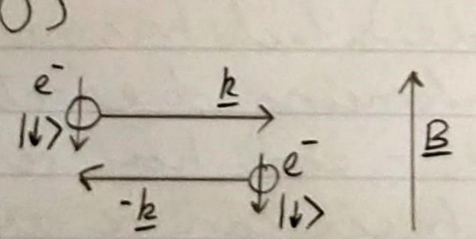
16/11/20 SS08 - Superconductivity

Objective:

In this experiment, we will aim to demonstrate that superconductivity occurs in metals at low temperatures, and we will aim to determine the behaviour of these type 1 superconductors in response to external perturbations (e.g. an applied magnetic field).

Due to current world events, we are doing this lab remotely. Thus, we are using precollected data, but will still be following the data analysis procedures described in the lab script, "SS08, Superconductivity" 2013.

- We can attempt to describe why an applied magnetic field may suppress superconductivity. If the electrons in the bound pair are both anti-alligned with the field, they gain an energy each,



$: \Delta E = g\mu B = \Delta$

If the increase in energy per electron is $\sim\Delta$, the electrons will have sufficient energy to become unbound, and scatter, suppressing the superconductivity.

Under high current densities, we might expect superconductivity to fail due to the creation of its own magnetic field. We would also see the Fermi energy increase as $\epsilon_F \sim j^{2/3}$, which would increase the energy of electrons at the Fermi level, meaning they require less thermal energy to overcome their binding energy.

3.1.1 - We are also asked to calculate the conversion between revolutions of the cryostat's exhaust gas meter to the volume of liquid ^4He lost:

$$1 \text{ revolution} \equiv 50L \text{ of } {}^4He \text{ gas.}$$

$$740L \ {}^4He \text{ gas} \equiv 1L \text{ of liquid } {}^4He$$

$$\therefore 1 \text{ revolution} \equiv \frac{50}{740} L \text{ of liquide } {}^4He$$

$$1 \text{ revolution} \equiv 0.0676L \text{ of liquid } {}^4He \text{ loss} \quad (3sf)$$

3.4 - The data recording program records the integral of the voltage readings. It is important any DC offset is removed, but high frequency AC noise shouldn't be an issue as over the measurement time, it should integrate to an average of 0.

- Having completed the preliminary precautions we can begin with the experiment proper, starting with a measurement of the superconducting phase transition for tin.

4.1 <u>Observation of the normal - superconducting transition using the resistivity of tin wire.</u>

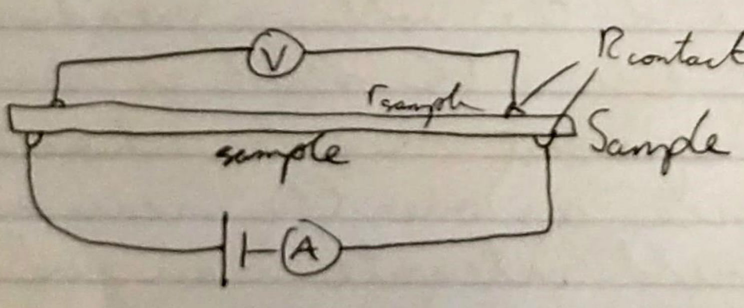
- The 4 wire method is preferable to a 2 wire measurement method, as the contact resistance of the leads to the sample won't affect the measurement of its resistance. In a 2 wire method, the in-line measurement of the current / voltage will change based on the contact resistance.

e.g: $R_{measured} = r_{sample} + R_{contact}$.

For the 4-wire method, as the voltage measurement is in parallel solely with the sample, we get

$$\frac{1}{R_{measured}} = \frac{1}{r_{sample}} + \frac{1}{R_{contact}}$$

$$\simeq \frac{1}{r_{sample}} \quad \text{for } r_{sample} << R_{contact}$$



- It is necessary to reverse the current through the sample and measure the potential difference both times, as if the voltage contacts aren't alligned with the current, the measured voltage will take the odd function

form $V = \text{const.} + f(B \cdot I)$, where B is the ambient magnetic field. Thus, reversing the current allows us to take 2 measurements and cancel out the affects of the magnetic field:

$$\bar{V} = \frac{V(I) + V(-I)}{2}$$

- We can check to see if the current is heating the sample by taking many voltage readings a fixed current. If there is no sample heating, Ohm's law will apply and the resistance will be constant.

<u>Preliminary Measurements</u>:

- There were two resistance measurements made at high temperature for the tin sample. The data was given as

T[K]	V+ [V]	V- [V]
300	5.86	-5.85
77	1.102	-1.095

$\Big\} I = 7.687 mA$

$$\therefore R(300K) = \frac{V_+ - (V_-)}{2I}$$

$$= 762 \Omega \text{ (3sf)}$$

$$R(77K) = 143 \Omega \text{ (3sf)}$$

- We also want the resistance at the highest temperature we are recording actual data from. The temperature has to be inferred from a pressure measurement, using an empirical formula,

$$x = \log_{10}(P/1\text{mbar}),$$

$$\frac{T}{1K} = 1.24177 + 0.23793x + 0.36207x^2$$

$$- 0.33188x^3 + 0.20738x^4 - 0.05294x^5$$

$$+ 0.00552x^6.$$

- Using this gave

	$T[K]$	$V+ [\times 10^{-3} V]$	$V- [\times 10^{-3} V]$
$I = 1.002A$:	4.23	0.0913	-0.0832

$$R(4.23K) = 8.70 \times 10^{-5} \Omega.$$

- If we look at the ratio $R(300K)/R(4.23K)$, we expect that for a purer sample, the ratio will be large. This is because there will be fewer lattice imperfections for electrons to scatter off at low temperatures, leading to a decrease in $R(4.23K)$.

$$\frac{R(300K)}{R(4.23K)} = 8.76 \times 10^6 \Omega \text{ (3sf)}$$

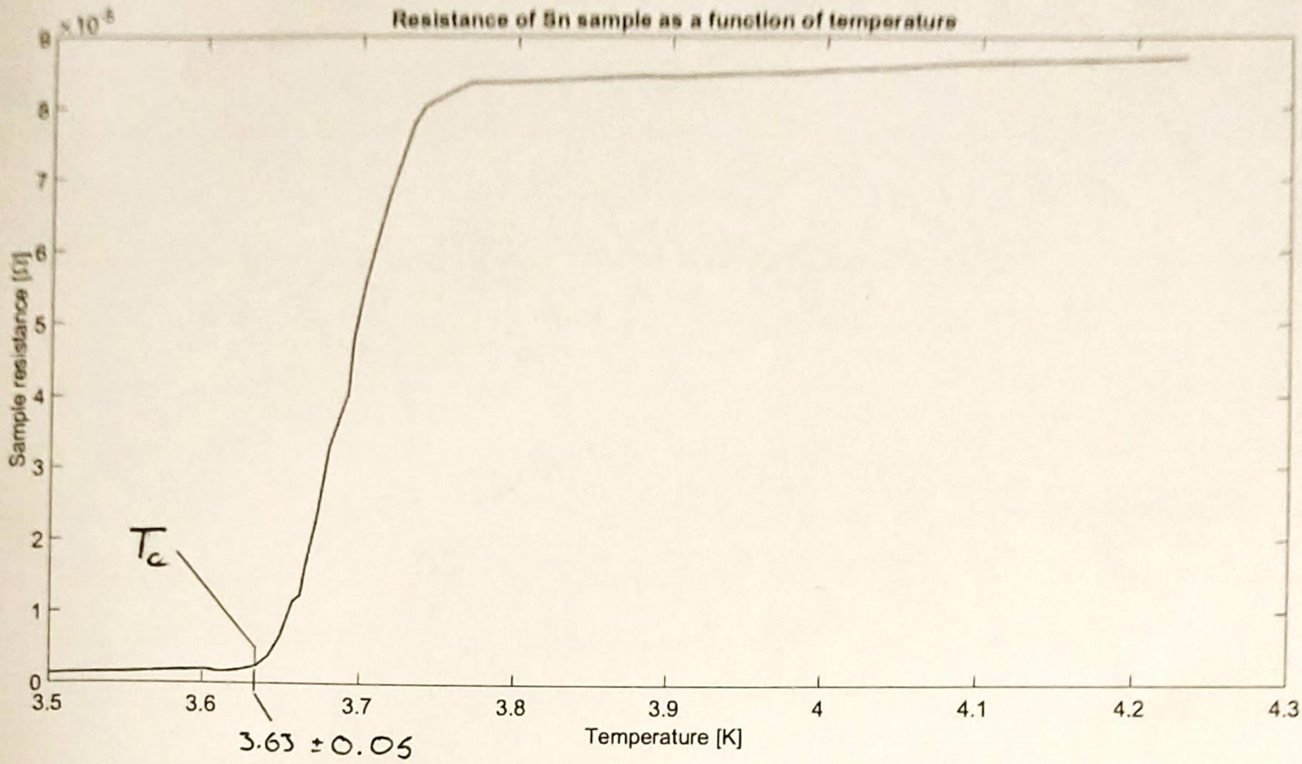
The large ratio implies our sample is very pure, and has few lattice imperfections from impurities or internal stress.

Plot of resistance vs. temperature for the superconducting phase transition of tin (and code used).

- The finite width of the phase transition is due to the finite width of the Fermi distribution's decrease at low temperature. Because of this, as T decreases, fewer electrons have the energy to thermally excite and overcome their pair-binding energy, so the resistance rapidly drops over a finite width until negligeable numbers of electrons are ~~conduct~~ scattering.

- We can measure the transition temperature as the temperature at which we begin to leave the superconducting regime.

$$\therefore T_c = 3.63 \pm 0.05 \text{ K}$$




```matlab
%% ResistivityDataRemote analysis
% This section requires us to determine the resistance of our sample at
% different temperatures and constant current. The file has the inputs of
% pressure, v+ and v- (voltages for diffferent current bias)

% SPECIFICALLY, this section will read in the data and convert it into an
% appropriate format

% The below data was taken at current I = 1.002A
I = 1.002;
%gets the helium vapour pressure in mbar
p = ResistivityDataRemote.Heliumpressure;
% gets the positive voltage measurement in millivolts, which we will
% convert into volts
vmax = (1/1000)*ResistivityDataRemote.V;
% gets the reversed voltage measurement in millivolts, which we will
% convert into volts
vmin = (1/1000)*ResistivityDataRemote.V1;

% we calculate the average voltage value to better determine the resistance
vbar = (vmax - vmin)/2;

% here we use the empirical formula to change the pressures into a
% temperature based on trhe measurements made by Duriex and Rusby:
x = log10(p);
T = 1.24177 + 0.23793*x + 0.36207*x.^2 -0.33188*x.^3 +0.20738*x.^4 -0.05294*x.^5 +0.00552*x.^6;

%% In this section we will compute the resistances and plot them against temperature
R = vbar./I;

p = plot(T,R);

xlabel("Temperature [K]");
ylabel("Sample resistance [\Omega]");
title("Resistance of Sn sample as a function of temperature");
set(findall(gcf,'-property','FontSize'),'FontSize',20);
p.LineWidth = 2;
```

- After that, we started looking at data recorded at a fixed pressure that demonstrated the loss of superconductivity for large B fields.

- The data was included in files that recorded the shunt voltage and sample voltage. The shunt voltage could be turned into a magnetic field via the relation

$$B = \frac{0.018\ \mathrm{TA^{-1}}}{0.5\ \mathrm{VA^{-1}}} V_{shunt} .$$

Doing this, we were able to plot the sample voltage against the magnetic field strength for a variety of temperatures.

Sample voltage, V_s, vs. applied magnetic field, B, at fixed temperatures.

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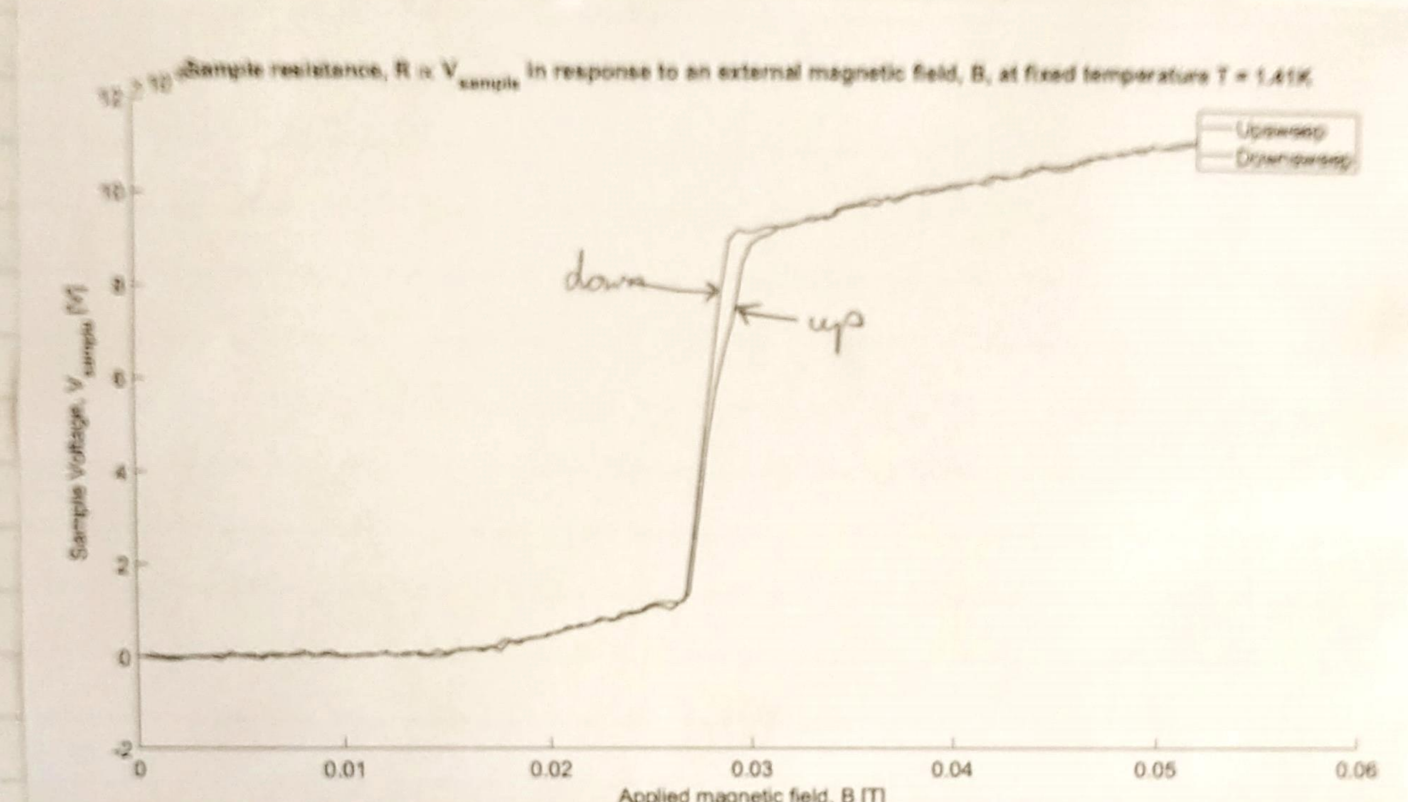
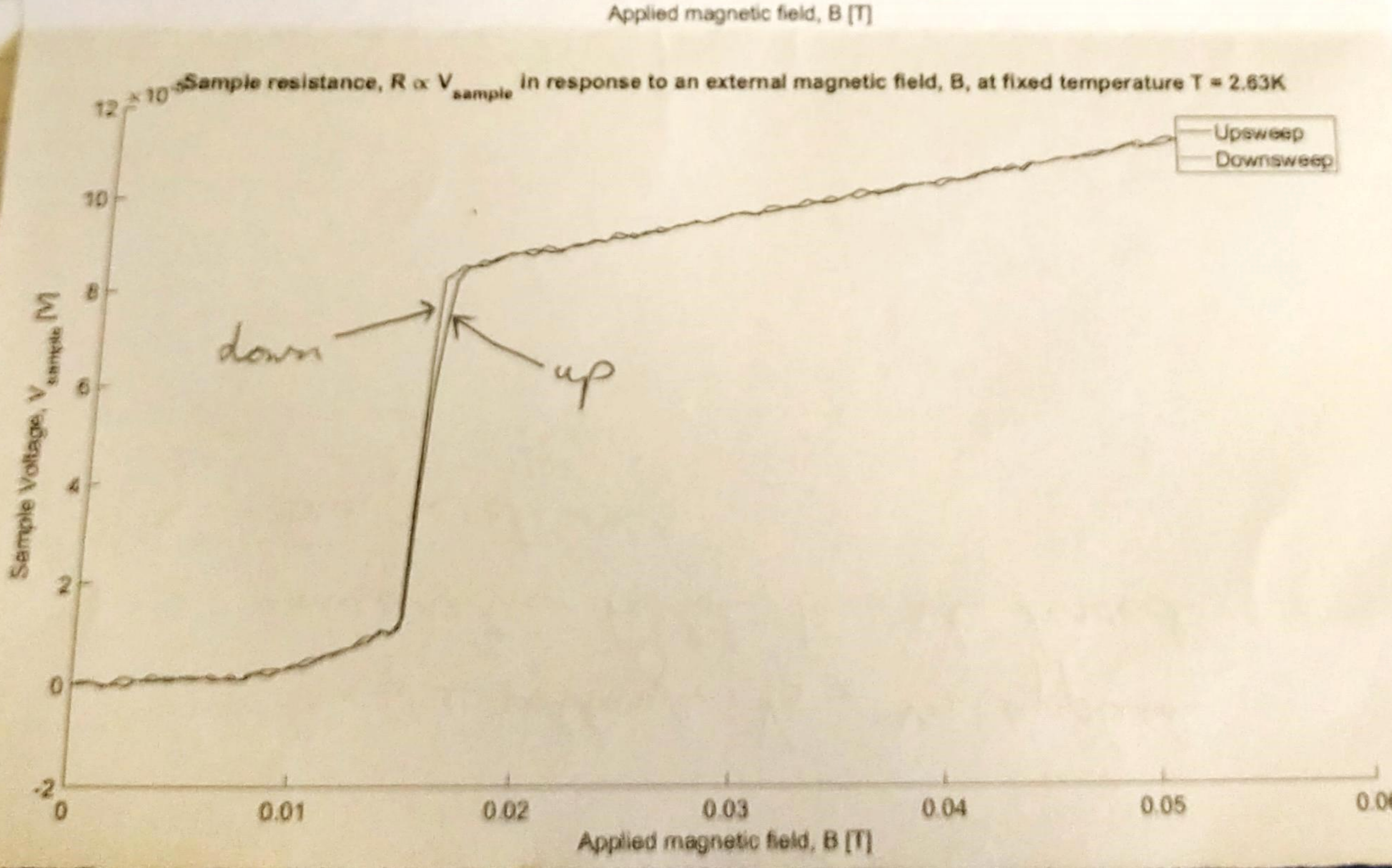




59.

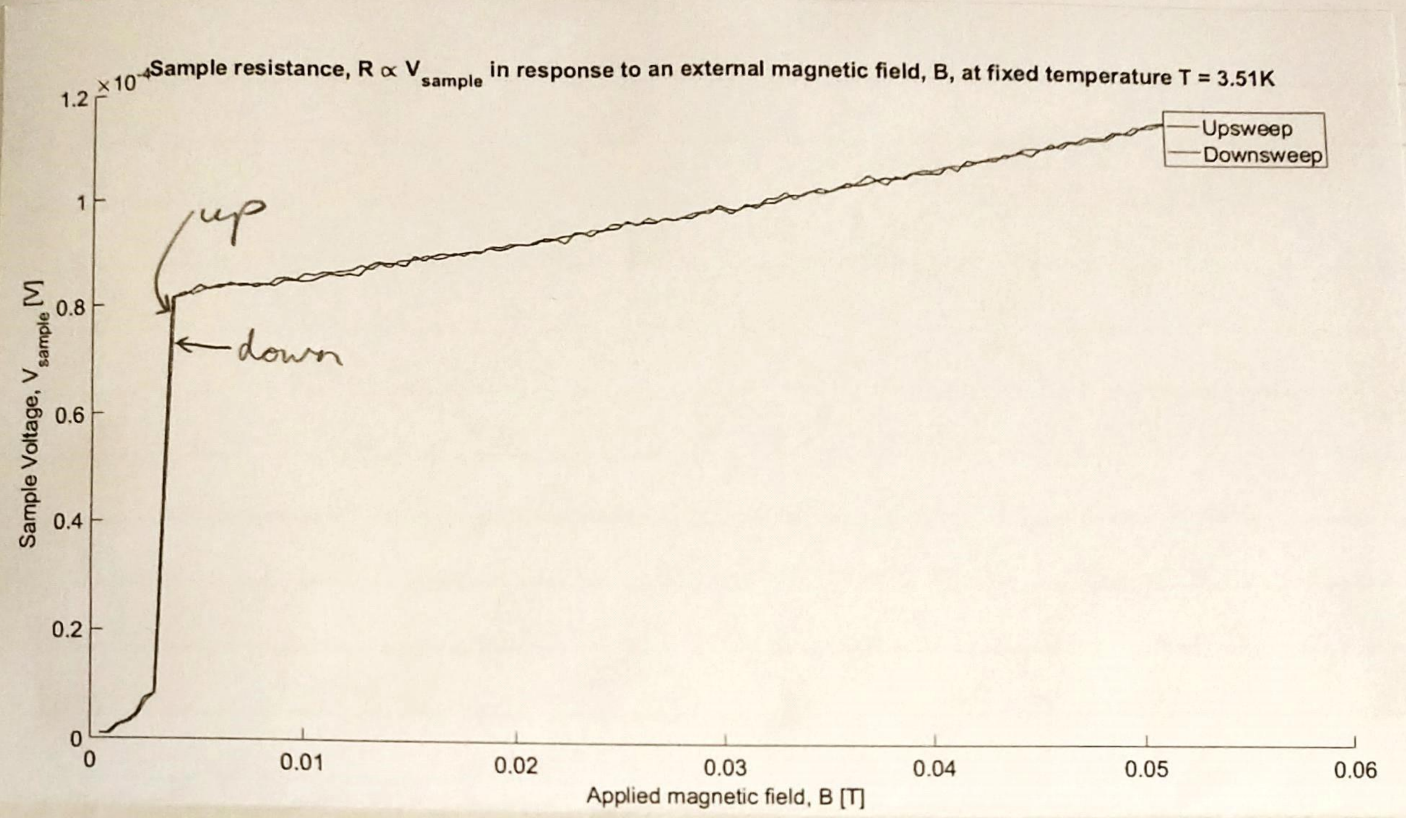
Sample voltage, V_s, vs. applied magnetic field, B, at fixed ~~te~~ temperatures.

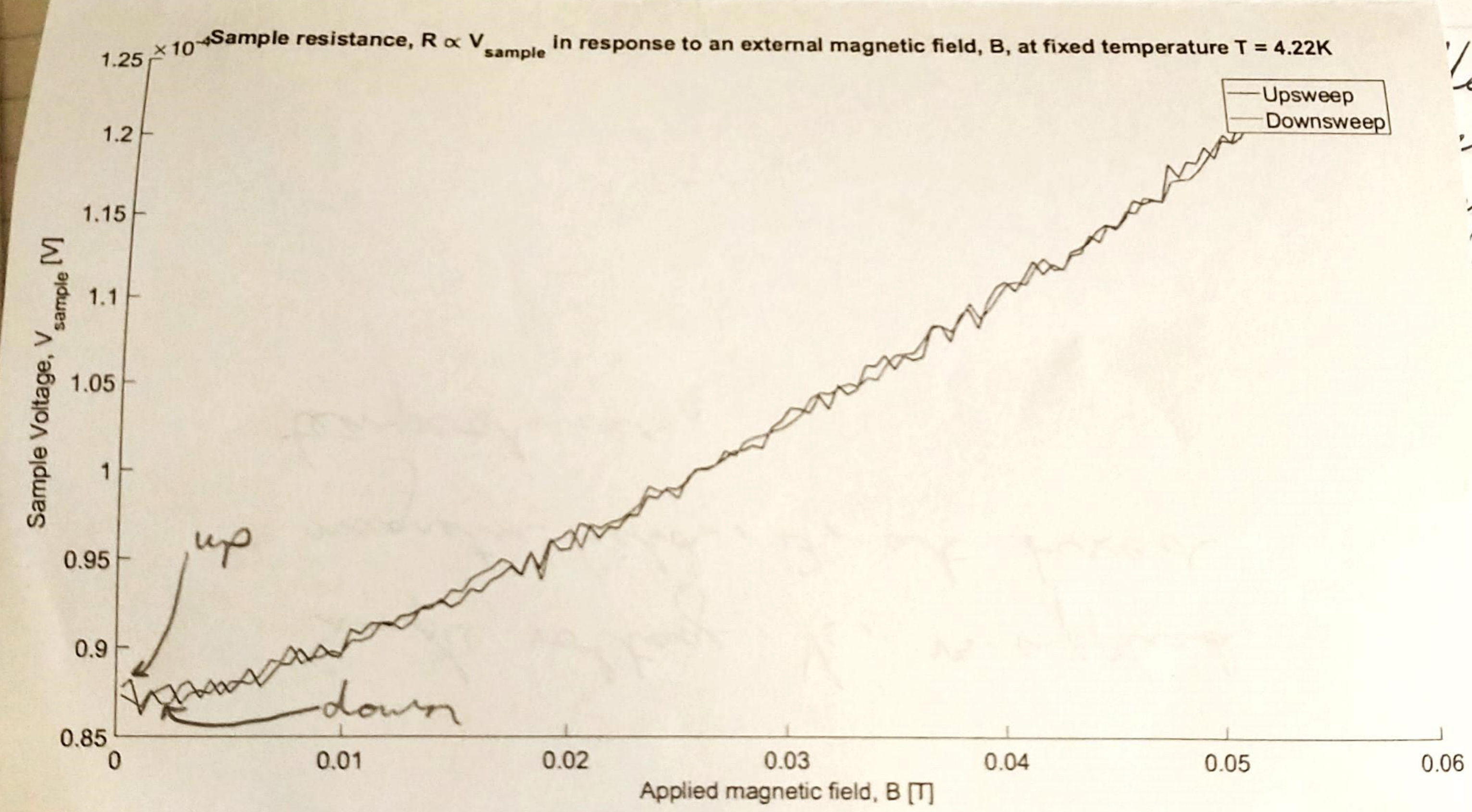
- The shape of the transition is very simillar to that of ~~V~~ R vs. T, as the ~~chan~~ increase ~~in~~ in energy due to the increased field strength B, for anti-alligned (wrt the field) parallel pairs of electrons, allowing them to overcome their binding energy and begin to scatter. This doesn't happen all at once, but rather electrons further from the Fermi surface require stronger fields to overcome the Δ energy cost.

- The downsweep occuring at lower magnetic fields to the upsweep could be due to the rotation of atoms in the lattice, in ~~order~~ response to the magnetic field, heating the system. This would lead to more thermal energy being available to the electrons, so they can remain unbound at lower B fields.

$$\cancel{4}\,\mu_B B_{up} \simeq \Delta \,,$$

$$\mu_B B_{down} + k_B \Delta T \simeq \Delta \,.$$





fields to the upsweep could be due to

- B_c being defined as a 50% of the extrapolated "normal-state" value is a poor definition as this value could be poorly defined and it isn't related to any predictable quantity.
If instead we define B_c as when the sample voltage / resistance initially begins to increase significantly, this can be estimated by considering the energy requirements of electrons at the fermi level at finite temperature.

- For the provided temperatures, we then attempted to determine the critical field strength from each plot:

T[K]	B_c[T]
1.41	0.027 ± 0.0005
1.87	0.023 ± 0.001
2.12	0.0205 ± 0.001
2.24	0.019 ± 0.001
2.34	0.0178 ± 0.0005
2.45	0.0165 ± 0.0005
2.63	0.0145 ± 0.0005
2.78	0.0125 ± 0.001
2.91	0.0114 ± 0.0005
3.03	0.0095 ± 0.001
3.13	0.0082 ± 0.0005
3.22	0.0073 ± 0.0005
3.31	0.0062 ± 0.0005
3.37	0.0050 ± 0.0004
3.43	0.0042 ± 0.0005
3.46	0.0036 ± 0.0005

T [K]	B_c [T]
3.51	0.0031 ± 0.0003
3.55	0.0024 ± 0.0010
3.59	0.0018 ± 0.0005
3.62	0.0013 ± 0.0005

All $V_{sample} \sim B$ plots
on same axes,
and code used to generate
plots.

Plot of $B_c \sim T^2$

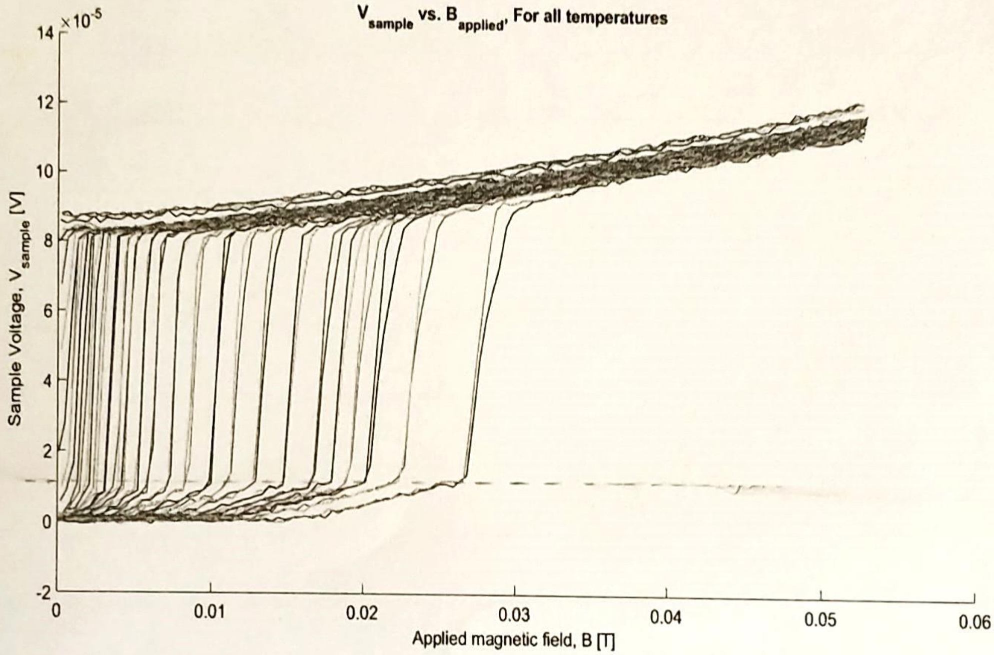
$B(0) = 30.7 \pm 0.5$ mT

$B(13.7 \pm 0.3\ K^2) = 0$

- If we try and fit our data to the curve

$$B_c = B_0\left(1 - \frac{T^2}{T_c^2}\right),$$

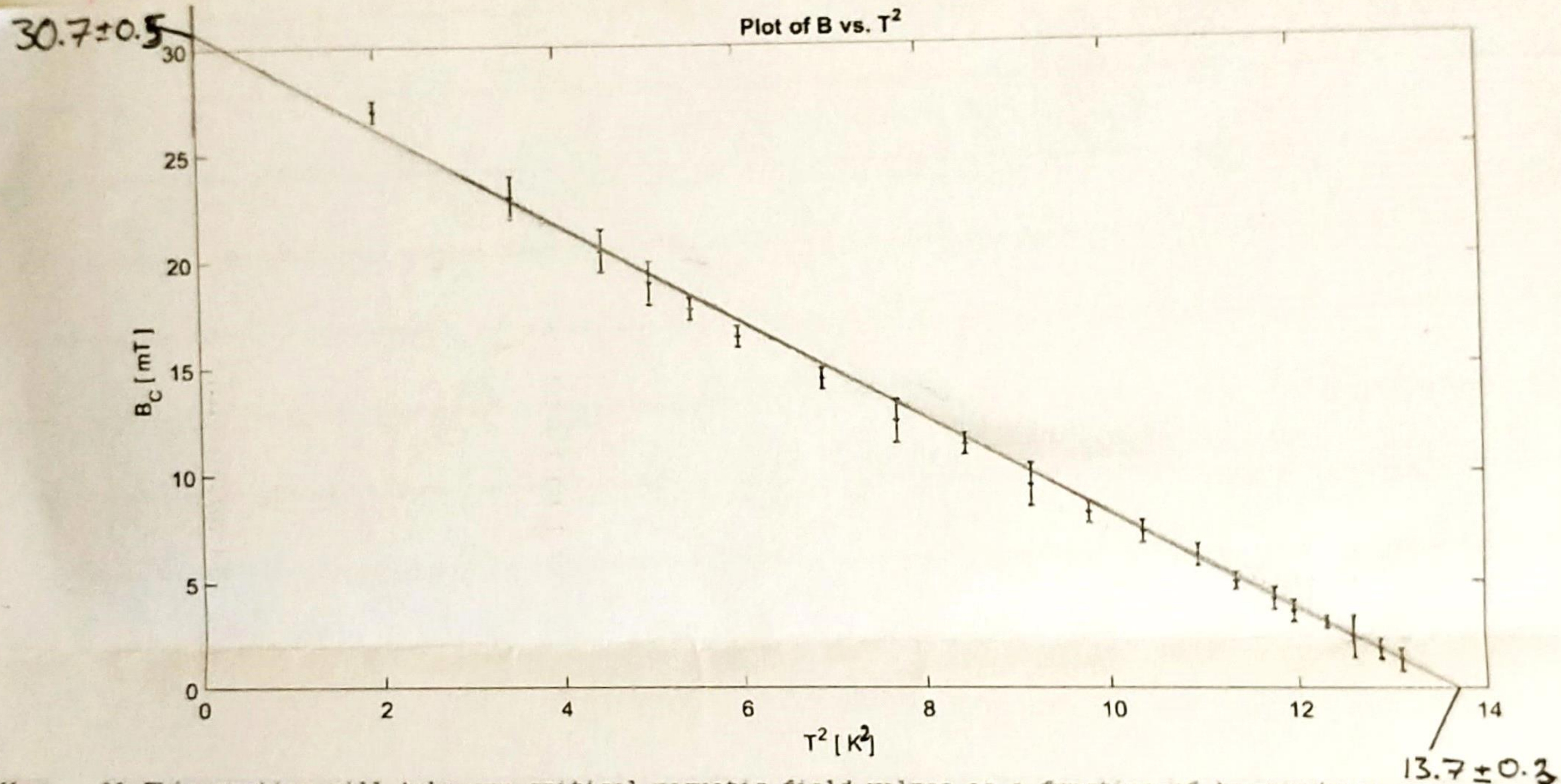
we get the values:




```matlab
%% This script is for the critical magnetic field strnegth vs T analysis.
% We will need to read in each of the given files (recording time, voltage
% and integrated voltage at fixed pressure) and determine the critical
% magnetic field strength from each visually via a plot

%Here we take the pressures in mBar that the data was recorded at
pressures = [1010,890,630,610,595,570,560,540,520,500,473,450,430,400,370,330,290,250,210,170,130,92,74,58,43,21,3];
% next, we'll determine the tempereatures these occur at using the formula
% from Duriex and Rusby
x = log10(pressures);
T = 1.24177 + 0.23793*x + 0.36207*x.^2 -0.33188*x.^3 +0.20738*x.^4 -0.05294*x.^5 +0.00552*x.^6;

for i = 1:size(pressures,2)
    filename = strcat('ProbeA',int2str(pressures(i)),'mB');
    data = importfile(filename, 3, 242);

    t = data.t;
    shuntV = data.shuntV;
    sampleV = data.sampleV;
    integratedV = data.integratedV;

    % having read in the data, the shunt voltage can be converted into a
    % magnetic field measurement via its relation to the current, and that
    % to the flux density
    B = (0.018/0.5)*shuntV;

    %this creates a plot of the sample voltage against the shunt voltage (which is proportional to the applied electric field)
    f = figure();
    changeIndex = floor(size(shuntV,1)/2);
    hold on
    plot(B(1:changeIndex),sampleV(1:changeIndex),'LineWidth',1.5);
    plot(B(changeIndex+1:end),sampleV(changeIndex+1:end),'LineWidth',1.5);
    legend('Upsweep','Downsweep');
    xlabel("Applied magnetic field, B [T]");
    ylabel("Sample Voltage, V_(sample) [V]");
    title("Sample resistance, R \propto V_(sample) in response to an external magnetic field, B, at fixed temperature T = " + num2str(round(T(i),2)) + "K");
    set(findall(gcf,'-property','FontSize'),'FontSize',20);
    hold off
end
```

curve

$$B_c = B_0\left(1 - \frac{T^2}{T_c^2}\right),$$




```
%% This section will take our critical magnetic field values as a function of temperature
% The temperatures at which we could measure the critical magnetic field [Kelvin]
T = [1.41 ,1.87,2.12,2.24,2.34,2.45,2.63,2.78,2.91,3.03,3.13,3.22,3.31,3.37,3.43,3.46,3.51,3.55,3.59,3.62];

% The observed magnetic fields, and their uncertanties [milli Tesla]
Bc = [27,23,20.5,19,17.8,16.5,14.5,12.5,11.4,9.5,8.2,7.3,6.2,5.0,4.2,3.6,3.1,2.4,1.8,1.3];
uncert = [.5,1,1,1,.5,.5,.5,1,.5,1,.5,.5,.5,.4,.5,.5,.3,1,.5,.5];

% We want to plot B vs T^2 to get a relation of the form Bc = B0(1 - (T/Tc))
T2 = T.^2;
p = errorbar(T2,Bc,uncert,'+');
xlabel("T^2 [ K ]");
ylabel("B_C [ mT ]");
title("Plot of B vs. T^2");
set(findall(gcf,'-property','FontSize'),'FontSize',20);
p.LineWidth = 2;
%plot(T2,Bc,'+');
```

$$B_0 = (30.7 \pm 0.5) \times 10^{-3}\,T,$$
$$T_c^2 = 13.7 \pm 0.3 \ K^2$$

$$\therefore T_c = 3.70 \pm 0.06 \ K$$

4.2 The Meissner Effect in Tin:

· In this section of the lab, the probe being used would have been swapped out for one with a central tin core wrapped with a copper wire, designed to measure the magnetic flux internal to the tin.

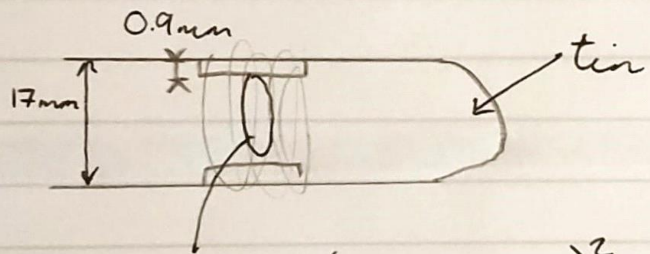


figure 9 in lab script

$$A = \frac{\pi(17 - 2\times0.9)^2}{4} \ mm^2$$

$$= 181.46 \ mm^2 \ (5sf)$$
$$A = 1.8146 \times 10^{-4} \ m^2 \ (5sf).$$

- We can measure the internal magnetic field by considering that a change in internal magnetic flux induces a voltage around the coil.

$$\therefore V_{coil} = nA \frac{\partial B_{int}}{\partial t}.$$

$$\therefore \int_{t_0}^{t_1} V_{coil}\, dt = nA\left(B(t_1) - B(t_0)\right).$$

- At the start of each measurement run, the shunt voltage starts from zero, so there is no initially applied magnetic field.

$$\therefore B(t_0) \simeq 0.$$

- The data program records the integrated voltage, $V_{int} = \int_{t_0}^{t_1} V_{coil}\, dt$, so we can say

$$B_{int}(t) = \frac{1}{nA} V_{int}(t).$$

- A quick note on procedure, half of the files for this section were .dat files, and half ~~were~~ didn't have any file extension. I converted all of them to .dat files for convenience with the coding.

- We wrote some code to plot the internal magnetic field against the applied magnetic field. Some of the plots for different temperatures are overleaf.

Plots of $B_{internal}$ vs. $B_{applied}$

for $T = 1.34K$,

$T = 2.85K$.

Plots of $B_{internal}$ vs. $B_{applied}$

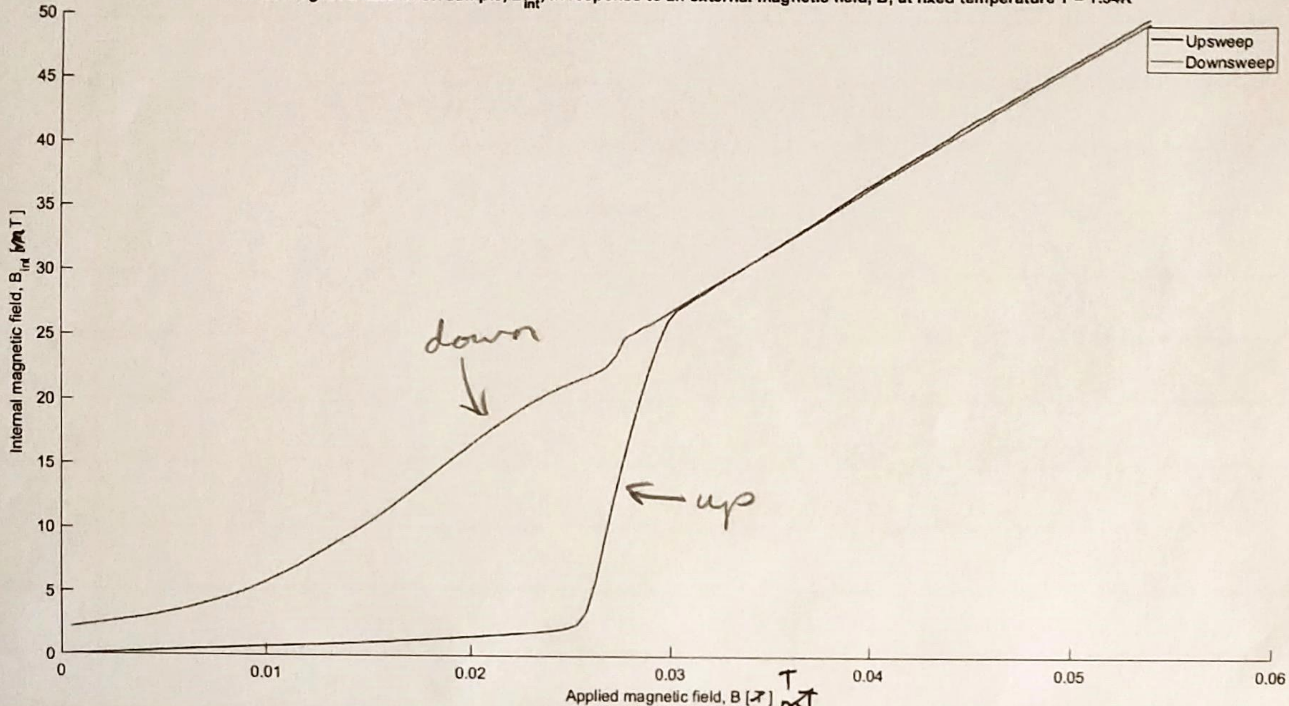
for $T = 3.45K$,

$T = 4.22K$.

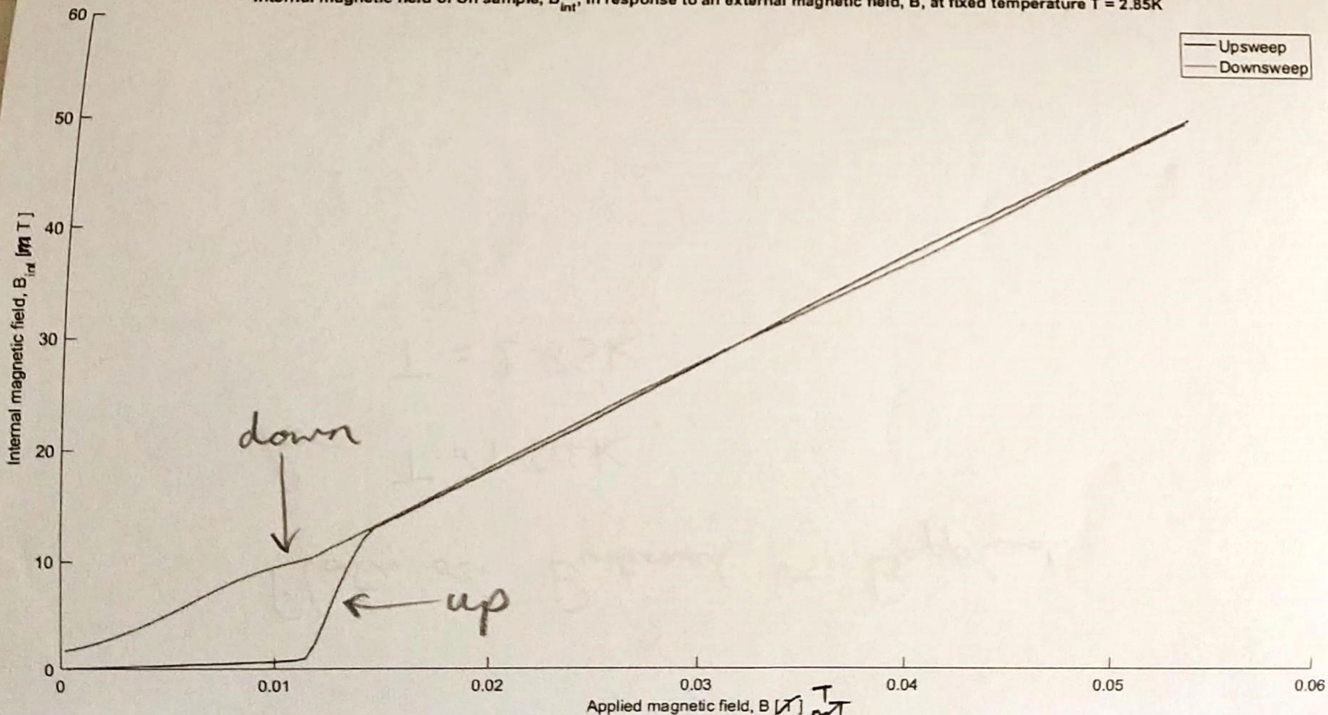
- On the upsweep, the tin expells the applied field almost entirely, until the applied field is sufficient to suppress the superconductivity. At this point, the internal flux rapidly increases towards the line where ~~B_{int} is proportional to $B_{applied}$~~. $B_{int} \simeq B_{applied}$.

This is because the tin can no longer

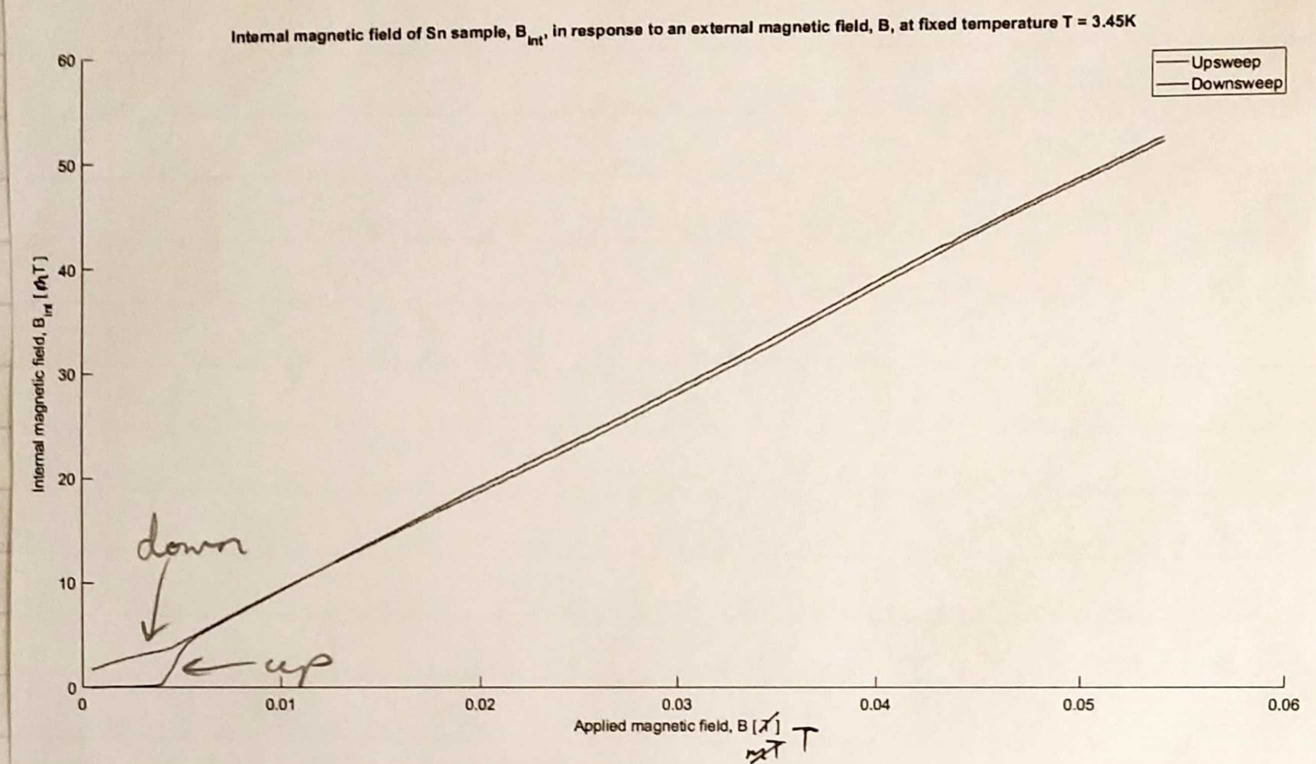
Internal magnetic field of Sn sample, B_{int}, in response to an external magnetic field, B, at fixed temperature T = 1.34K



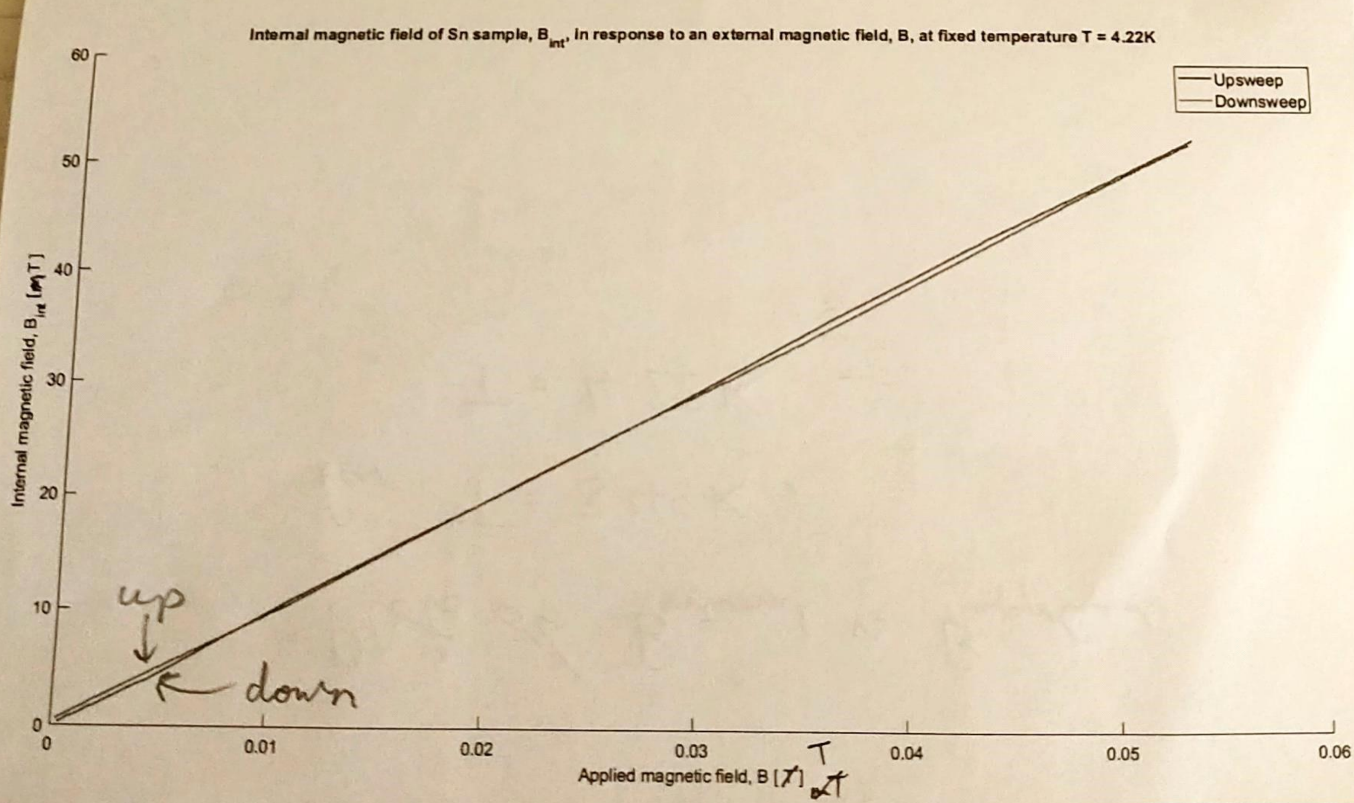
Internal magnetic field of Sn sample, B_{int}, in response to an external magnetic field, B, at fixed temperature T = 2.85K



$T = 2.85K.$



Internal magnetic field of Sn sample, B_{int}, in response to an external magnetic field, B, at fixed temperature T = 3.45K
Upsweep
Downsweep
Internal magnetic field, B_{int} [mT]
Applied magnetic field, B [~~T~~] ~~mT~~ T
0
10
20
30
40
50
60
0
0.01
0.02
0.03
0.04
0.05
0.06
down
up



Internal magnetic field of Sn sample, B_{int}, in response to an external magnetic field, B, at fixed temperature T = 4.22K
Upsweep
Downsweep
Internal magnetic field, B_{int} [mT]
Applied magnetic field, B [~~T~~] ~~mT~~ T
0
10
20
30
40
50
60
0
0.01
0.02
0.03
0.04
0.05
0.06
up
down

have arbitrarily large currents to oppose the external field, and the currents within it rapidly dissipate. This reduces the amount of expulsion, and at high magnetic fields, hardly any field density is expelled (hence $B_{int.} \Rightarrow B_{applied}$, which we see).

- If the probe's diameter was larger, we would expect to observe the same critical B_c value, but the increase of $B_{int.} \rightarrow B_{applied}$ would occur much quicker, as the larger sample's surface would have a large resistance, so the electrons generating the expelling field would scatter at a faster rate.

- We can determine the critical flux density, B_c, for each temperature by measuring where the upsweep curve begins to rapidly increase:

T[K]	B_c [μT]
1.34	25.3 ± 0.5
1.86	21.4 ± 0.5
2.1	19.0 ± 0.5
2.24	18.0 ± 0.2
2.48	15.3 ± 0.5
2.64	13.6 ± 0.2
2.76	12.4 ± 0.3
2.85	11.5 ± 0.3
2.94	10.4 ± 0.3
2.99	9.7 ± 0.3

T[K]	B_c [μT]
3.08	8.6 ± 0.3
3.2	7.1 ± 0.3
3.29	6.1 ± 0.5
3.35	5.4 ± 0.2
3.39	4.6 ± 0.2
3.45	~~4.05 ± 0.2~~ 4.0 ± 0.3
3.5	3.0 ± 0.2
3.52	3.0 ± 0.3
3.54	2.9 ± 0.2
3.59	1.8 ± 0.5
3.62	1.55 ± 0.2
3.7	———
3.84	———
3.88	———
4.22	———

} Didn't display any $\underline{B}$ expulsion.

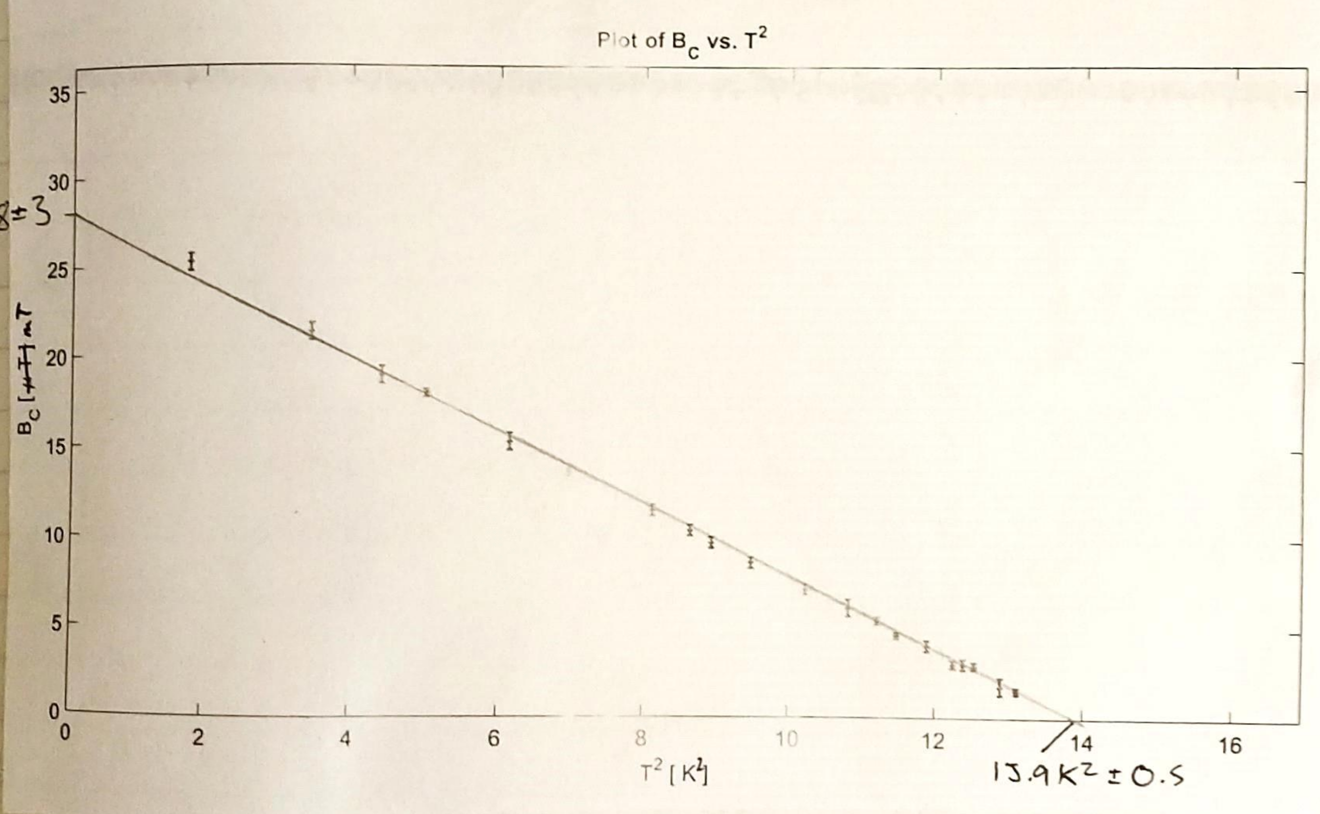
~~· A quick note on procedure, we ended up with μT for B internal because the data in the files for V integrated has units [Vms], not [V] as is stated in the files. This also gives the expected order of magnitude for our measurements, which is a good sanity check.~~

- Using our plot for $B_c \sim T^2$, we can see it is linear. Using the form

$$B_c = B_0\left(1 - \left(\frac{T}{T_c}\right)^2\right),$$

we get $B_0 = 28 \pm 3$ ~~μT~~ mT.

Code used to generate B_{int} vs. $B_{applied}$ plots, and to create B_c vs. T^2 plot.



$$B_0 = 28 \pm 3 \text{ mT},$$
$$T_c = 3.73 \pm 0.07 \text{ K}$$

- As we can see, our values for B_0 and T_c agree with the earlier experiment within error bounds.

```
%We will start by creating a list of pressures at which measurements were
%taken.
pressures = [1010,720,690,590,540,525,490,480,470,440,410,390,360,320,270,230,220,190,165,132,98,80,58,41,20,2];
% again, we'll determine the tempereatures these occur at using the formula
% from Duriex and Rusby
x = log10(pressures);
T = 1.24177 + 0.23793*x + 0.36207*x.^2 -0.33188*x.^3 + 0.20738*x.^4 -0.05294*x.^5 +0.00552*x.^6;
% These variables are the turning number and area of the coil [mm^2]
n = 676;
A = 181.46;
% goes through the file for each of the pressures and gets the data for it.
for i = 1:size(pressures,2)
    filename = strcat('ProbeB',int2str(pressures(i)),'bar.dat');
    data = importfile(filename, 3, 242);
    t = data.t;
    shuntV = data.shuntV;
    sampleV = data.sampleV;
    integratedV = data.integratedV;
    % having read in the data, the shunt voltage can be converted into a
    % magnetic field measurement via its relation to the current, and that
    % to the flux density
    Bapplied = (0.018/0.5)*shuntV;
    % we can also convert the integrated voltage into an internal flux
    % density in the tin probe. Using formula 3 in the lab script, letting
    % t1 be the start time, and saying that B1 ~= 0, we get
    Binternal = 1/(n*A) * integratedV;
    %this creates a plot of the sample voltage against the shunt voltage (which is proportional to the applied electric field)
    f = figure();
    % roughly cuts the data in half so we can plot the up and down sweep
    % seperately.
    changeIndex = floor(size(shuntV,1)/2);
    hold on
    plot(Bapplied(1:changeIndex)*10^3,Binternal(1:changeIndex)*10^9,'LineWidth',1.5);
    plot(Bapplied(changeIndex+1:end)*10^3,Binternal(changeIndex+1:end)*10^9,'LineWidth',1.5);
    legend('Upsweep','Downsweep');
    xlabel("Applied magnetic field, B [ \mu T ]");
    ylabel("Internal magnetic field, B_{int} [ \mu T ]");
    title("Internal magnetic field of Sn sample, B_{int}, in response to an external magnetic field, B, at fixed temperature T
    set(findall(gcf,'-property','FontSize'),'FontSize',15);
    hold off
end
%% This section is to test the Bc ~ T^2 relation as described in equation 1 in the lab script
% The temperatures at which we could measure the critical magnetic field [Kelvin]
T = [1.34,1.86,2.10,2.24,2.48,2.64,2.76,2.85,2.94,2.99,3.08,3.2,3.29,3.35,3.39,3.45,3.5,3.52,3.54,3.59,3.62];
% The observed magnetic fields, and their uncertanties [micro Tesla]
Bc = [25.3,21.4,19.0,18,15.3,13.6,12.4,11.5,10.4,9.7,8.6,7.1,6.1,5.4,4.6,4.0,3,3,2.9,1.8,1.55];
uncert = [.5,.5,.5,.2,.5,.2,.3,.3,.3,.3,.3,.3,.5,.2,.2,.3,.2,.3,.2,.5,.2];
% We want to plot B vs T^2 to get a relation of the form Bc = B0(1 - (T/Tc))
T2 = T.^2;
p = errorbar(T2,Bc,uncert,'+');
xlabel("T^2 [ K ]");
ylabel("B_C [ \mu T ]");
title("Plot of B_C vs. T^2");
set(findall(gcf,'-property','FontSize'),'FontSize',20);
p.LineWidth = 2;
%plot(T2,Bc,'+');
```

- If the specimen was instead a classical perfect conductor, we wouldn't observe the Meissner effect. If we consider Ohm's law,

$$\underline{j} = \sigma \underline{E}, \quad \sigma \to \infty,$$

the current density must remain finite to be physical. Thus, as $\sigma \to \infty$, $\underline{E}$ must be zero uniformly.

$$\therefore \underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} = \underline{0}.$$

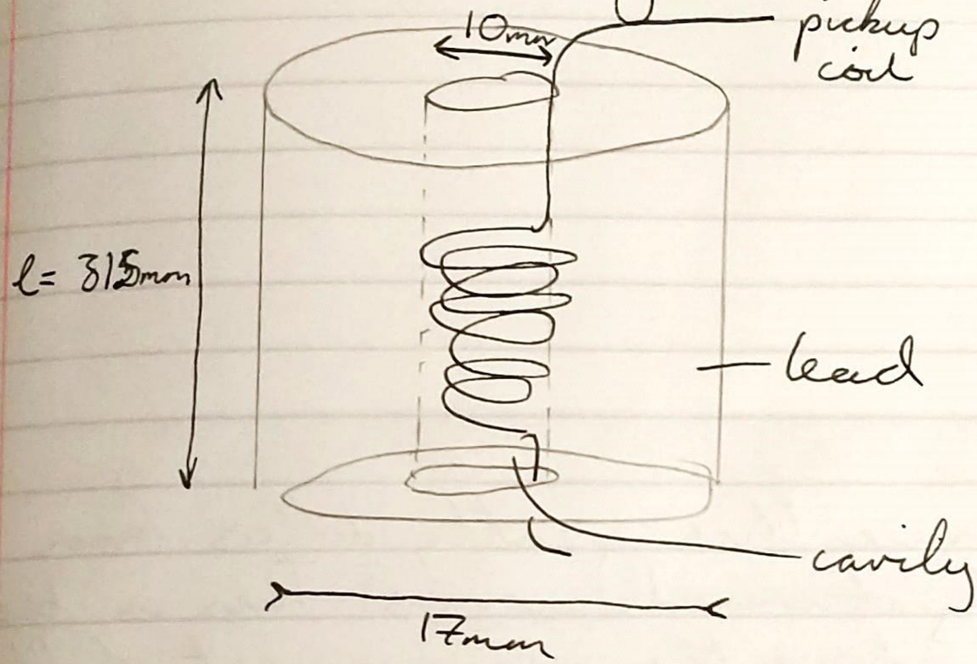
$$\therefore \frac{\partial \underline{B}}{\partial t} = \underline{0},$$

$$\underline{B} = \underline{B}_0 \quad : \text{constant}.$$

- In a perfect conductor, the internal magnetic field would remain constant for all time, regardless of the specimen's temperature or the ambient magnetic field, etc. Thus, there can be no breakdown of this state, which categorizes the superconductive phase transition.

4.3 Persistant currents in Lead:

- In this part of the experiment, we've swapped the tin cores for a lead core with a central cylinder bored out of it:



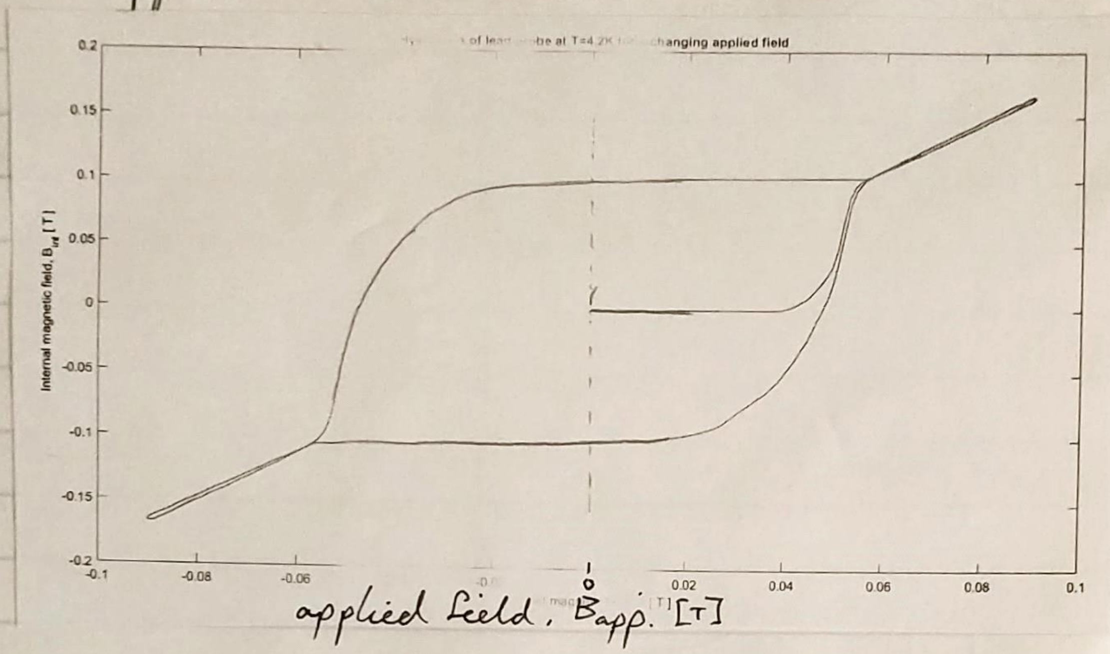
- Taking measurements of the voltage through the search coil, we can convert them into magnetic flux densities again by

$$B_{int} = \frac{1}{nA} V_{coil\,(integrated)},$$

where nA is given in the datafile as $nA = 0.0755012$ [presumably m^2].

Plotting this against the applied field (calculated from the shunt voltage) we got a hysteresis curve that is shown overleaf.

Hysteresis of lead probe at $T = 4.2K$ for a changing applied field:



As we can see, this greatly differs from previously observed curves. This is because we are taking the measurement of flux within a cavity/hole in the lead probe. As this isn't part of the superconductor, we don't see the total expulsion of fields.

Initially, that is what we see, but upon decreasing B beyond B_c, the currents within the superconductor's inner surface become persistant, remaining roughly constant even as the applied field is reduced. This is why the magnetic field we observe in the hole (generated by these persistant currents) remains roughly constant across the superconducting regime. This process occurs again for negative fields too, and the process of creating and destroying the core's magnetic field creates a hysteresis loop.

· In the absence of applied field the magnetic flux density measured in the core is

$$B_{core} = 0.099 \pm \cancel{0.02}\ 0.005 \text{ T}.$$

· Treating the inner surface of the core as a solenoid, and using Ampere's law, we can say that

$$lB = \mu_0 I_{persistant}$$

$$I_{persistant} = \frac{31.5\times10^{-3}\text{ m} \times 0.099\text{ T}}{4\pi\times10^{-7}\text{ Hm}^{-1}}$$

$$I_{persistant} = 2481.6 \pm 125 \text{ A}$$

- This would conventionally be a very large current, however the material is in its superconducting phase so a large current is to be expected due to the negligeable resistance.

- This can be related to the current density by considering the thickness of the layer in which the persistant current is present. This is given as $t_{sheath} = 5\times10^{-8}$ m

$$\therefore V_{\substack{current \\ region}} = l \times \left(\frac{\pi(d_{core} + 2t_{sheath})^2}{4} - \frac{\pi d_{core}^2}{4} \right)$$

$$= 4.948\times10^{-11} \text{ m}^3 \text{ (4sf)}.$$

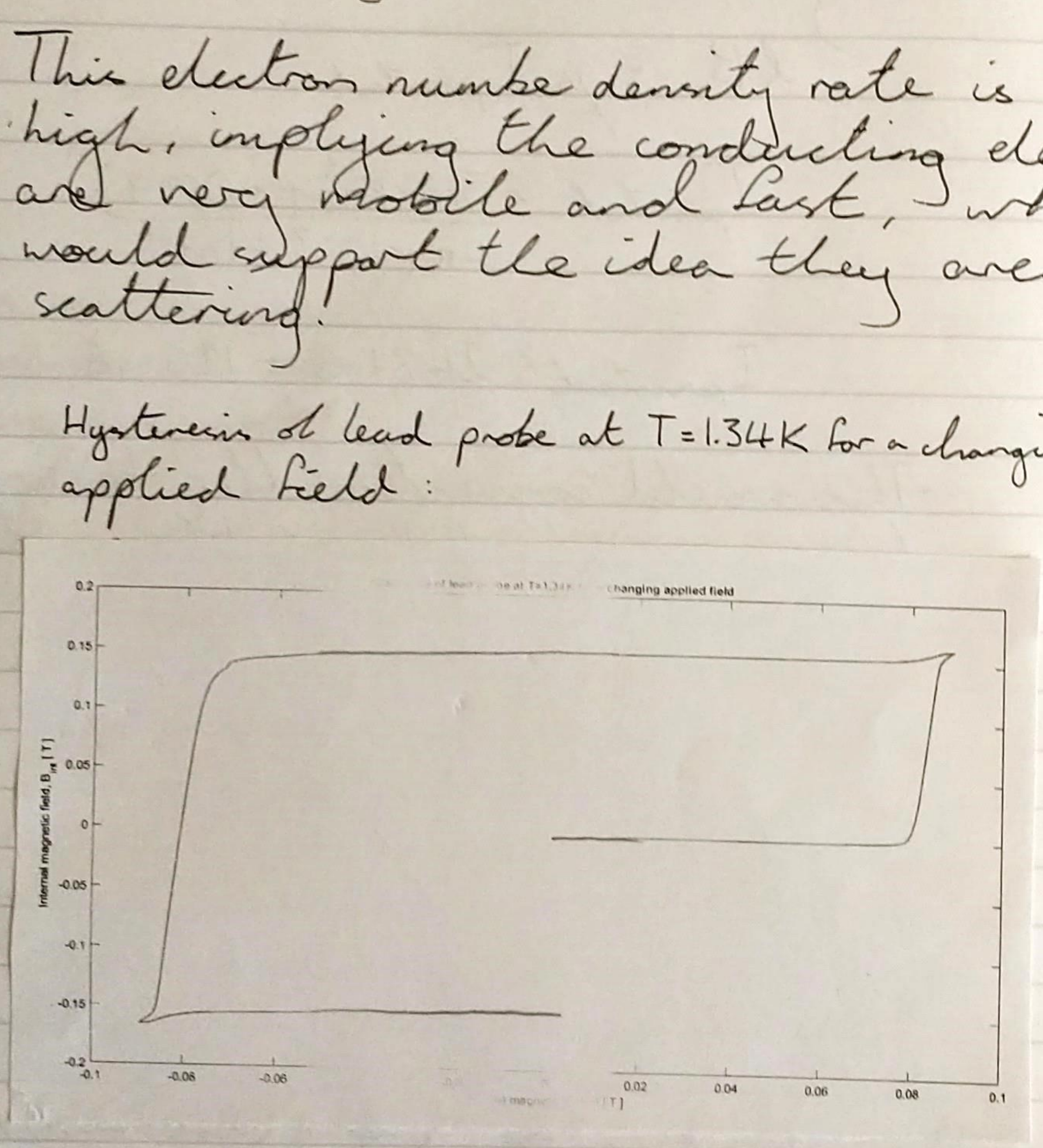
$\therefore$ current density $J = \dfrac{I}{V_{\text{current region}}}$

$$J = 5.015 \times 10^{13} \pm 0.253 \times 10^{13} \ Am^{-3}$$

$\therefore$ electron number density : $n = 3.13 \times 10^{32} \pm 0.16 \times 10^{32} \ m^{-3}s^{-1}$.

- This electron number density rate is very high, implying the conducting electrons are very mobile and fast, which would support the idea they aren't scattering!

Hysteresis of lead probe at T=1.34K for a changing applied field:



- For this plot, the core was heated to 10K, the magnetic field was reduced to zero, then the sample was cooled to 1.34K.

- Residual field, $B_r^{(2)} = 0.152 \pm 0.002$ T.
- Critical field, $B_c^{(2)} = 0.079 \pm 0.003$ T.

Again, the persistant current can be found as

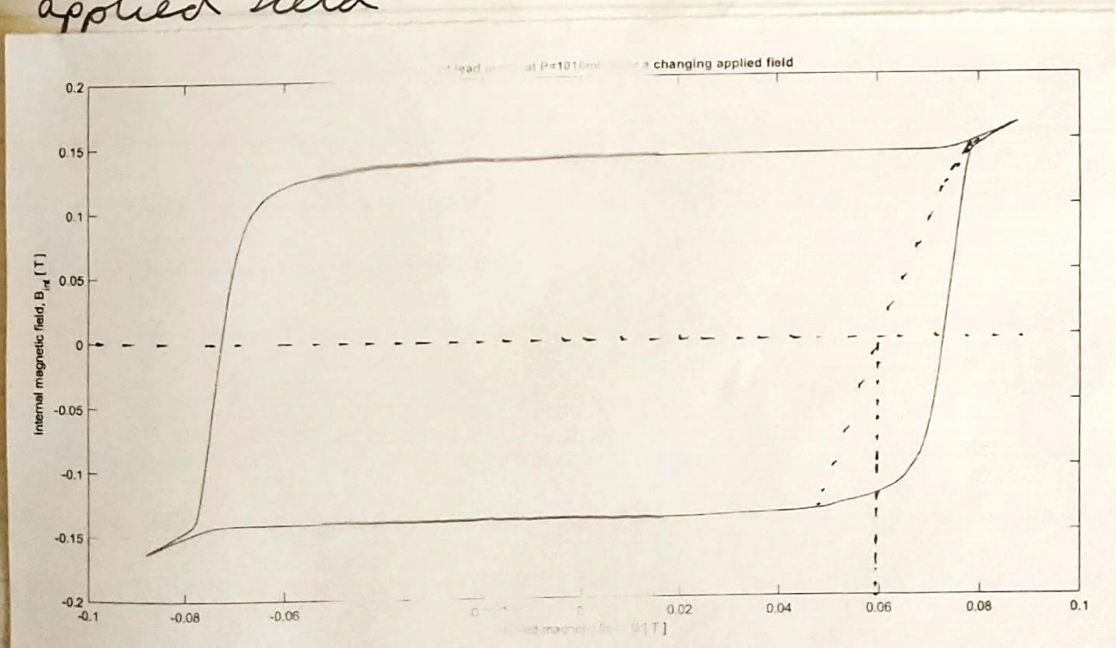
$$I^{(2)}_{persistant} = \frac{l B_r^{(2)}}{\mu_0}$$

$$I^{(2)}_{persistant} = 3810 \pm 50 \text{ A}.$$

- Returning to our first hysteresis loop ($T = 4.2$K), we have

$$B_r^{(1)} = 0.099 \pm 0.005 \text{ T},$$
$$B_c^{(1)} = 0.044 \pm 0.005 \text{ T}.$$

Hysteresis of lead probe at P=1010mbar for a changing applied field:



· The cone was repressurised with warm helium gas from above the hysteresis loop.

$B_r^{(3)} = 0.138 \pm 0.002 \, T,$

- $B_c^{(3)}$ is poorly defined on this hysteresis loop.

$$\therefore I^{(3)}_{\text{persistent}} = \frac{l B_r^{(3)}}{\mu_0}$$

$$= 3459 \pm 50 \text{ A}$$

- Finding the estimates for B_0 and T_c such that

$$B_c = B_0\left(1 - \frac{T^2}{T_c^2}\right)$$

$$\therefore \frac{B_c^{(1)}}{B_c^{(2)}} = \frac{T_c^2 - T_1^2}{T_c^2 - T_2^2}$$

$$\therefore T_c^2\left(\frac{B_c^{(1)}}{B_c^{(2)}} - 1\right) = T_2^2 \frac{B_c^{(1)}}{B_c^{(2)}} - T_1^2$$

$$T_c^2 = \frac{T_2^2 B_c^{(1)} - T_1^2 B_c^{(2)}}{B_c^{(1)} - B_c^{(2)}}$$

$$T_c = \sqrt{\frac{(1.34)^2 \times 0.044 - 4.2^2 \times 0.079}{0.044 - 0.079}}$$

$$= 6.13 \pm 3.9 \text{ K}$$

· For the 3rd plot, it doesn't ever pass through the origin because it starts initially above the superconducting phase with a non-zero ambient field. Thus, when it becomes superconducting, it already has a persistant field, so traverses the hysteresis loop without being able to reach the origin.

$$B_0 = \frac{B_c^{(i)}}{1 - \frac{T_i^2}{T_c^2}}$$

$$\therefore \overline{B_0} = \frac{1}{2}\sum_{i=1}^{2} \frac{B_c^{(i)}}{1 - \frac{T_i^2}{T_c^2}}$$

$$\overline{B_0} = 0.0829 \pm 0.0079 \text{ T}$$

· Using T_c and $\overline{B_0}$, we can calculate the temperature the 3rd graph was plotted at:

- estimated $B_c^{(3)} = 0.06 \pm 0.01$ T.

$$\therefore T_3 = T_c\sqrt{1 - \frac{B_c^{(3)}}{B_0}}$$

$$T_3 = 3.22 \pm 0.54 \text{ K}$$

· As seen in section 5 of the lab script, as $T \to 0$, the electron pair binding energy 2Δ can be expressed as

$$2\Delta = U_n - U_s = \frac{B_c^2}{2\mu_0}.$$

· For tin, we can take an average of our previously aquired values,

$$\cancel{\bar{B}_c^{(tin)} = \frac{1}{2}(30.7 + 28)\ mT}$$

$$\cancel{= 29.35 \pm}$$

$$\bar{B}_c^{(tin)} = \frac{\left(\frac{30.7}{0.5} + \frac{28}{3}\right)}{\frac{1}{0.5} + \frac{1}{3}}$$

$$\bar{B}_c^{(tin)} = 30.3 \pm 0.5\ mT.$$

$$\therefore U_n - U_s = 365.3\ J \pm 8.5.$$

- We want to find a description for the system's entropy along the superconducting phase boundary:

- Along the boundary, the Gibbs energy of the superconducting and normal states equalise.

$$\therefore dG_s = dG_n$$

$$S_s\, dT + M_s\, dB = S_n\, dT + \overset{\sim 0}{M_n}\, dB$$

$$\therefore \quad S_s - S_n = \left(\frac{\partial B_c}{\partial T}\right)_{\substack{\text{along phase}\\ \text{boundary}}} (-M_s)$$

- Because the superconducting state perfectly excludes any applied critical field, we have

$$M_s = -\frac{B_c}{\mu_0}.$$

$$\boxed{\therefore \quad S_s - S_n = \frac{B_c}{\mu_0}\left(\frac{\partial B_c}{\partial T}\right)_{\substack{\text{phase}\\ \text{boundary}}}}$$

- At the critical temperature (assuming the phases are still in equilibrium), we see that $B_c = 0$.

$$\therefore \text{ Latent heat, } L_s = T\Delta S$$

$$= \frac{T_c B_c}{\mu_0} \frac{\partial B_c}{\partial T}$$

$$L_s = 0.$$

- However, for all $T < T_c$, B_c is finite, so

$$L_s = \frac{T B_c}{\mu_0}\left(\frac{\partial B_c}{\partial T}\right)_{\substack{\text{phase}\\ \text{boundary}}}.$$

- The change in heat capacities between the two states is

$$\Delta C = C_s - C_n$$

$$= T\left(\frac{\partial S_s}{\partial T} - \frac{\partial S_n}{\partial T}\right)_B$$

$$= T\frac{\partial}{\partial T}\left(\frac{B_c}{\mu_0}\left(\frac{\partial B_c}{\partial T}\right)_{\text{boundary}}\right)_B$$

$$= \frac{T}{\mu_0}\left(\left(\frac{\partial B_c}{\partial T}\right)^2 + B_c\frac{\partial^2 B_c}{\partial T^2}\right)$$

$$\boxed{\Delta C = \frac{T}{2\mu_0}\left(\frac{d^2(B_c^2)}{dT^2}\right).}$$

- In the limit $T \to T_c$, $B_c \to 0$, this goes to

$$\Delta C = \frac{T_c}{\cancel{2}\mu_0}\left(\left(\frac{\partial B_c}{\partial T}\right)^2 + 0\right)$$

$$\boxed{\Delta C = \frac{T_c}{\cancel{2}\mu_0}\left(\frac{\partial B_c}{\partial T}\right)^2.}$$

· We can try to relate these expressions to the electronic heat capacities in the normal and superconducting phases. If we take $C_n = \alpha T^3 + \gamma T$, and (due to the superconducting energy gap),

$$S_s \to 0, \quad \frac{dS_s}{dT} \to 0$$

$$\therefore \quad \frac{C_s}{T} \to 0.$$

· At $T = 0$, we can say that

$$\left(\frac{C_s - C_n}{T}\right)_{T \to 0} = 0 - \lim_{T \to 0} \frac{C_n}{T}$$

$$= \lim_{T \to 0} \left\{ -\gamma - \alpha T^2 \right\}$$

$$= -\gamma$$

$$= -\frac{\Delta C}{T_0}$$

$$\left(\frac{C_s - C_n}{T}\right)_{T \to 0} = -\frac{1}{2\mu_0} \frac{d^2 B_0^2}{dT^2}$$

$$\boxed{\cancel{\left(\frac{C_s - C_n}{T}\right)_{T \to 0} \to -\frac{1}{2\mu_0} \left(\frac{B_0}{T}\right)^2 \frac{d^2}{dT^2}}}$$

$$\left(\frac{C_s - C_n}{T}\right)_{T \to 0} = \frac{1}{\quad}$$

$$\cancel{\left(\frac{C_s - C_n}{T}\right)_{T_c} \Rightarrow B_\cdot \to 0}$$

$$\cancel{\left(\frac{C_s - C_n}{T}\right)_{T_c}}$$

$$\boxed{\left(\frac{C_s - C_n}{T}\right)_{T\to 0} = -\gamma = -\frac{2}{\mu_0}\left(\frac{B_0}{T_c}\right)^2}$$

$$\left(\frac{C_s - C_n}{T}\right)_{T_c} \simeq \left.\frac{\Delta C}{T}\right|_{T_c}$$

$$\simeq \left.\frac{T_c}{T}\,\frac{1}{\cancel{2}\mu_0}\left(\frac{\partial B_c}{\partial T}\right)^2\right|_{T\to T_c}$$

$$\simeq \frac{1}{\cancel{2}\mu_0}\left(\frac{\partial B_c}{\partial T}\right)^2$$

$$B_c = B_0\left(1 - \frac{T^2}{T_c^2}\right)$$

$$\therefore \frac{\partial B_c}{\partial T} = -\frac{2}{T}\,\frac{B_0 T^2}{T_c^2} \simeq \frac{2B_0}{T}\left(1 - \frac{T^2}{T_c^2}\right) - \frac{2B_0}{T}$$

$$\therefore \left.\frac{\partial B_c}{\partial T}\right|_{T=T_c} = -\frac{2B_0}{T_c}$$

$$\boxed{\therefore \left(\frac{C_s - C_n}{T}\right)_{T_c} \simeq -\frac{4}{\mu_0}\left(\frac{B_c}{T_c}\right)^2 \simeq -2\gamma.}$$

- We can use the results we've experimentally obtained to determine a value for γ:

$$\gamma = \frac{2}{\mu_0}\left(\frac{B_0}{T_c}\right)^2$$

$$\cancel{= \frac{2}{4\pi\times10^{-7}\,Hm^{-1}}\Big(}$$

$$\cancel{B_0^{(tin)} = 29.4 \pm 3.5 \ mT}$$
$$\cancel{T_c^{(tin)} = 3.72 \pm 0.13 \ K}$$

$$\gamma = \frac{2}{4\pi\times10^{-7}\,Hm^{-1}}\left(\frac{29.4 \times 10^{-3}\,T}{3.72\,K}\right)^2$$

$$\cancel{\gamma = 99.4 \pm 17.4 \ Jm^{-3}K^{-2}}.$$

~~- This value for the electronic specific heat capacity agrees with the literature value of $\gamma = 110 \ Jm^{-3}K^{-2}$ within its error bounds.~~

$$B_0^{(tin)} = 30.6 \pm 0.5 \ (\times10^{-3}\,T)$$
$$T_c^{(tin)} = 3.67 \pm 0.04 \ K$$

$$\therefore \gamma = 110.6 \pm 3.07 \ (Jm^{-3}K^{-2})$$

- This value almost perfectly agrees with the literature value for $\gamma^{(tin)} = 110\,Jm^{-3}K^{-2}$.

Summary:

In pure metal samples (e.g SN pure tin and lead) superconducting phases can be achieved by sufficiently lowering the sample's temperature, to allow electrons to bind together in Cooper pairs.

We demonstrated this by measuring the resistance of a SN tin sample whilst lowering the pressure in a helium cryostat. We observed a phase transition in the conductivity at $T_c \simeq 3.63K$, at which point the resistance became negligeable. We were then able to demonstrate that a magnetic field alters the critical temperature in the sample, and demonstrate a linear trend in $B_c \sim T^2$.

Using a different probe, we then demonstrated the Meissner effect in tin, by showing the expulsion of magnetic flux from the sample in the superconducting phase. Again, we were able to categorise the phase transition in terms of $B_c \sim T^2$, to get $B_0 = 28 \pm 3mT$, $T_c = 3.73 \pm 0.07K$.

We also demonstrated how a changing magnetic field can produce persistent currents in lead, and how they produce hysteresis of the internal magnetic flux.

Finally, we used some simple thermodynamic arguments to propose a form of the superconducting phase boundary, and determine the electron specific heat capacity of tin, which we measured as $\gamma = 110.6\ Jm^{-3}K^{-2}$.