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On the Emergence of Superconductivity and Hysteresis in a Cylindrical Type I Superconductor

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Abstract—One-dimensional vortex-free solutions of the system of Ginzburg–Landau equations (the so-called precursor states) are studied. These states describe the emergence of superconductivity in a long cylindrical type I superconductor, which was initially in the supercooled normal state in a magnetic field, and are formed upon subsequent reduction of the external field. The precursor states are responsible for the magnetic hysteresis in type I superconductors (for which $\kappa < \kappa_c$, where $\kappa_c(R)$ is the critical value of the parameter κ in the Ginzburg–Landau theory, which is a function of radius). The range of fields is determined in which precursor states exist along with the Meissner state (and a hysteresis is possible) in the dependence of the cylinder radius R and parameter κ. *© 2002 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

The emergence (and degradation) of the superconducting state in a magnetic field in superconductors of various geometry was investigated on the basis of the Ginzburg–Landau macroscopic theory of superconductivity [1] by many authors [2–22]. Among other things, it was found that, for type II superconductors (with $\kappa > 1$) in the form of a long cylinder of radius R, a decrease in the external axial magnetic field H leads to the emergence of superconductivity from the normal state through a second-order phase transition in a certain field $H = H_2(m, \kappa, R)$ [4], when a small nonzero value of the modulus of the complex order parameter $\Psi = \psi e^{i\Theta}$ appears for the first time (ψ is the modulus, Θ is the phase, and m is the total number of vortices in the superconductor). In the case of large R and m, the field $H_2(m, \kappa, R)$ coincides with the field $H_{c3} = 1.69H_{c2}$ [3, 4] of emergence of surface superconductivity, where $H_{c2} = \phi_0/2\pi\xi^2$, ϕ_0 being the flux quantum and ξ being the coherence length. Recent numerical investigations [17–22] of one-dimensional (depending only on the radius) solutions of nonlinear Ginzburg–Landau equations for finite values of $\psi \sim 1$ in the case of the cylindrical geometry revealed that these solutions (with $\psi \sim 1$) exhibit a complex dependence on the parameters of the problem (m, κ, R, H). We can mention, for example, the existence of several branches of solutions in type II superconductors [18], the jumpwise rearrangement of these solutions upon a transition through the critical values of parameters [19, 21], the complex shape of the interface $S_{\text{I–II}}(\kappa, R)$ separating type I and II superconductors (this boundary or, which is the same, the critical value of $\kappa_c(R)$ depends on the cylinder radius [21] and does not coincide with the simple value of $\kappa_0 = 1/\sqrt{2}$ typical of the contact between two semi-infinite metallic superconducting (s) and normal (n) phases [1, 2]), and hysteresis phenomena in type II superconductors, associated with the existence of the "depressed" branch of solutions [22].

Among other things, it was noted in [22] that the superconducting state ($m = 0$) emerging in a supercooled (in the magnetic field) normal cylindrical sample of a type I superconductor is described by a special solution (a precursor) which preceeds complete expulsion of the field from the bulk of the sample and a jumpwise transition of the cylinder to the Meissner state. In this work, these solutions (precursors) are studied in greater detail. Since precursor states in type I superconductors (and hysteresis phenomena accompanying them) are manifested most strongly in the vicinity of the $S_{\text{I–II}}$ interface separating type I and II superconductors on the (κ, R) plane [21], we will henceforth consider the general case of arbitrary values of κ, which will allow us to describe the behavior of solutions upon a transition through the boundary $S_{\text{I–II}}$, which depends essentially on parameters R and κ.

Obviously, small-radius cylinders can contain only vortex-free states with $m = 0$. Here, we will confine our analysis to a detailed study of the properties of precisely such states. Among other things, it will be proved that the shape of the magnetization hysteresis loop for a cylinder with $m = 0$ is determined to a considerable extent by parameters R and κ. This can be used, in principle, for experimental determination of these parameters in mesoscopic superconductors. Some details of the picture, which will be obtained below, were earlier unknown and may be useful for discussing experiments with small-size (mesoscopic) superconductors [23–29].

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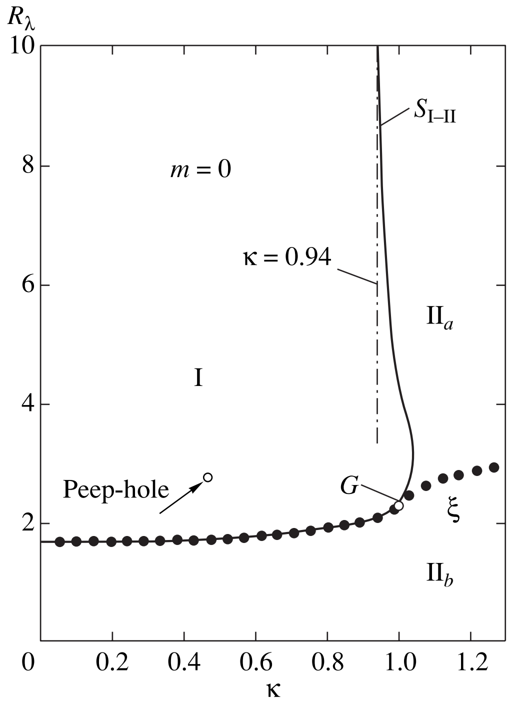

Fig. 1. Critical curves $S_{\mathrm{I-II}}$ and ζ dividing the plane (R_λ, κ; $m = 0$) into three regions. In the field-increase regime, the degradation of the superconducting state in region I occurs through a first-order jump from the Meissner (M) state, $\psi \approx 1$, to the normal (n) state, $\psi \equiv 0$. In region II_a, the M state is first transformed jumpwise into the superconducting e state, which is destroyed finally through a second-order transition. In region II_b, superconductivity degradation (in the field-increase regime) occurs gradually (second-order phase transition), without jumps. In the field reduction regime, the restoration of the M state in region II_a occurs through the e and d states and is accompanied by jumps and hysteresis. In region II_b, no hysteresis effects or jumps are possible. In region I (in the field reduction mode), the superconducting p state emerges from the supercooled n state through a second-order phase transition, after which a jump to the M state occurs and a hysteresis loop is formed. At point G ($k = 1$, $R_\lambda = 2.28$), the critical curves $S_{\mathrm{I-II}}$ and ζ merge into one curve. Below the ζ curve, no hysteresis loop is formed. For $\kappa > 3.5$, curve ζ attains the constant value $R_\zeta \approx 3.6$. For $\kappa \ll 1$, we have $R_\zeta \approx 1.69$.

2. EQUATIONS

The Ginzburg–Landau macroscopic theory of superconductivity [1] leads to a system of two nonlinear equations for the order parameter Ψ and the vector potential A of a magnetic field. The equation for Ψ contains the coherence length ξ and the equation for A, the magnetic field penetration depth λ ($\lambda = \kappa\xi$, where κ is the parameter of the Ginzburg–Landau theory). These two lengths are equivalent, and any of them can be chosen as a unit of measurement. In this work, we take λ as the unit of measurements. We will study the vortex-free states ($m = 0$); in this case, we can set the phase Θ equal to zero and assume that the order parameter is a real-valued quantity, $\Psi = \psi$. In this case, the Ginzburg–Landau equations for an infinitely long cylinder in an axial external magnetic field H in the cylindrical system of coordinates (r, φ, z) can be written in the following dimensionless form:

$$\frac{d^2U}{d\rho^2} - \frac{1}{\rho}\frac{dU}{d\rho} - \psi^2 U = 0, \tag{1}$$

$$\frac{d^2\psi}{d\rho^2} + \frac{1}{\rho}\frac{d\psi}{d\rho} + \kappa^2(\psi - \psi^3) - \frac{U^2}{\rho^2}\psi = 0. \tag{2}$$

Here, $\rho = r/\lambda$ is the dimensionless radial coordinate, $U(\rho)$ is the dimensionless potential of the magnetic field,

$$A = \frac{\lambda\phi_0}{2\pi\lambda^2}\frac{U}{\rho}, \quad B = \frac{1}{r}\frac{d(rA)}{dr}, \quad b = \frac{1}{\rho}\frac{dU}{d\rho},$$

where $b = B/H_\lambda$ is the dimensionless field in the superconductor and $H_\lambda = \phi_0/2\pi\lambda^2$ is the unit of measurement of the field.

The boundary conditions to Eqs. (1) and (2) have the form

$$U\big|_{\rho = 0} = 0, \quad \frac{dU}{d\rho}\bigg|_{\rho = R_\lambda} = h_\lambda, \tag{3}$$

$$\frac{d\psi}{d\rho}\bigg|_{\rho = 0} = 0, \quad \frac{d\psi}{d\rho}\bigg|_{\rho = R_\lambda} = 0. \tag{4}$$

Here, $R_\lambda = R/\lambda$ and $h_\lambda = H/H_\lambda$. The magnetic moment M of the cylinder (or the magnetization per unit volume) can be found from the formula

$$\bar{b} = h_\lambda + 4\pi M_\lambda, \quad \bar{b} = B_{av}/H_\lambda,$$

where B_{av} is the average magnetic field in the superconductor and $M_\lambda = M/H_\lambda$.

Obviously, solutions $U(\rho)$ and $\psi(\rho)$ to Eqs. (1)–(4) depend on three parameters: κ, R_λ, and h_λ. In order to find self-consistent solutions to Eqs. (1)–(4), use was made of the iterative procedure described in greater detail in [17]. This method is equivalent to the analogous numerical procedures used earlier [5–8]. However, in contrast to [5–8], where solutions were determined, as a rule, for several randomly distributed values of parameters κ, R, and h, we carried out a more detailed and systematic study of the solutions in a wide range of variation of the parameters κ, R_λ, and h_λ, which allowed us to discover some interesting features that had remained unnoticed in a less detailed analysis. Some of the results of this study will be described below.

3. NUMERICAL RESULTS

In order to obtain a compact description of the results of numerical calculations, we will first consider the plane of the variables (κ, R_λ). Each point on this plane corresponds to a solution $\psi(\rho; h_\lambda)$ and $U(\rho; h_\lambda)$ to the system (1)–(4). A certain idea concerning the properties of this solution (for given κ and R_λ) can be grasped from an analysis of the dependence of the order parameter ψ_0 at the center of the cylinder as a function of the magnetic field, $\psi_0 = \psi(0; h_\lambda)$. Similar information on the properties of the solution at the point (κ, R_λ) is contained in the dependence of the magnetic moment $-4\pi M_\lambda$ of the system on the external field. It is convenient to imagine mentally that any point on the (κ, R_λ) plane contains a "hole" through which the dependence of ψ_0 (and $-4\pi M_\lambda$) on the field h_λ can be "seen." Comparing these dependences, we can establish that there exist three regions on the (κ, R_λ) plane with qualitatively different behavior of $\psi_0(h_\lambda)$ as well as $M_\lambda(h_\lambda)$. These regions are denoted by I, II_a, and II_b in Fig. 1.

The meaning of division of the plane (κ, R_λ) into separate regions will be clarified below. However, before making a commentary on Fig. 1, we note that the superconducting state (at temperature $T < T_c$) can be obtained in two different ways: either in zero external magnetic field at the initial instant with a subsequent increase in H (field amplification regime) or by reducing a strong magnetic field H in which the metal was initially in the normal state (field reduction regime). These two regimes generally correspond to different solutions for the same value of the field h_λ. While seeking the solutions to Eqs. (1)–(4) corresponding to the field amplification mode, the trial function for the order parameter at the beginning of the iterative procedure was specified in the form $\psi(\rho) \sim 1$. Solutions in the field reduction mode correspond to the initial trial function $\psi(\rho) \sim 0.01 \ll 1$. Figures 2–4 show examples of the dependences $\psi_0(h_\lambda)$ and $-4\pi M_\lambda(h_\lambda)$, while examples of coordinate dependences of the solutions $\psi(\rho)$ and $b(\rho)$ emerging in different regimes are presented in Fig. 5.

If a hole is made on the line $R_\lambda = 6$ at point $\kappa = 0.95$ (in region I), we can see that the solutions appearing in the amplification mode for different fields h_λ correspond to a stable Meissner state (with $\psi_0 \approx 1$). These solutions correspond to the solid line in Fig. 2a. As the field attains the value h_1, the Meissner solution becomes absolutely unstable, and the cylinder passes jumpwise to the normal state ($\psi \equiv 0$). Since the order parameter ψ_0 in the Meissner state remains finite up to the jump point h_1, the value of the critical field $h_1(\kappa, R_\lambda)$ cannot be determined from the linearized theory [4, 9], which can be applied only if the condition $\psi_0 \ll 1$ is satisfied (see [22] for details). The jump in the order parameter ψ_0 at point h_1 is denoted by δ_1. For $h > h_1$, there are no other solutions except the normal solution ($\psi \equiv 0$).

If we now seek solutions in the field reduction mode (for $h_\lambda < h_1$), the normal solution ($\psi \equiv 0$) remains stable (relative to small spatial perturbations) up to the point h_p at which a small ($\psi_0 \ll 1$) nucleus of the superconducting state appears (precursor, or p state). In the interval of fields $\Delta_n = h_1 - h_p$, there exist two stable (in the above sense) solutions: the Meissner state and the supercooled (in a magnetic field) normal state. Obviously, the supercooled normal state is metastable since the free energy is lower in the Meissner state (with $\psi \approx 1$). Since the amplitude of the emerging p state is small, the value of the critical field $H_p(\kappa, R_\lambda)$ can be found from the linearized theory [4, 9], whence it follows that $H_p = \phi_0/2\pi\xi^2 \equiv H_{c2}$ for $R_\lambda \gg 1$.

Upon a further decrease in the field ($h_\lambda < h_p$), the amplitude of the emerging superconducting p state increases (see the dashed curve in Fig. 2a) up to point h_r at which the restoration of the Meissner state occurs jumpwise (the amplitude of the jump is δ_r). In the field interval $\Delta_p = h_p - h_r$, there exist two stable (relative to small perturbations) superconducting states: the Meissner (M) state and the p state. For $h_\lambda < h_r$, only one stable Meissner state exists. The precursor state (as well as the supercooled n state) is metastable; it describes the possibility of a hysteresis in the field reduction mode (the hysteresis loop is indicated by arrows). The total width of the hysteresis loop is

$$\Delta_{pn} = \Delta_p + \Delta_n = h_1 - h_r.$$

It should be emphasized that the field h_r and amplitude ψ_r at the transition point cannot be determined with the help of the linearized theory [4, 9] (see the discussion of this problem in [22]).

A similar pattern emerges in an analysis of magnetization ($-4\pi M_\lambda$) as a function of the field (Fig. 2b). In this case also, we have the supercooled n state (in the field interval Δ_n); the hysteresis loop, $\Delta_{pn} = \Delta_p + \Delta_n = h_1 - h_r$, associated with the existence of the n and p states; and the magnetization jump (δ_r) during the restoration of the M state. Such a pattern is typical of type I superconductors.

If we make a hole at the point $R_\lambda = 6$, $\kappa = 0.98$ lying in the II_a region in Fig.1, the emerging pattern is essentially different (Fig. 2e). Here, the Meissner state in the field amplification regime also becomes absolutely unstable in the field $h_1(\kappa, R_\lambda)$, but the jump δ_1 occurs not to the state with $\psi \equiv 0$ (as in type I superconductors), but to a special superconducting state with a suppressed order parameter, viz., the e (edge-suppressed) state typical of type II superconductors [18, 21, 22]. For the e state, the amplitude of the order parameter ψ_0 decreases smoothly upon an increase in h_λ and vanishes completely in the field $h_\lambda = h_2(\kappa, R_\lambda)$. For $R \gg 1$, the field $H_2(\kappa, R)$ coincides with $H_{c2} = \phi_0/2\pi\xi^2$. In the field interval $\Delta_e = h_2 - h_1$, the dependence $\psi_0(h_\lambda)$ has a smoothly decreasing tail corresponding to the e state.

In the field-reduction mode, the superconducting e state appears again in the field h_2 (Fig. 2e) and continues

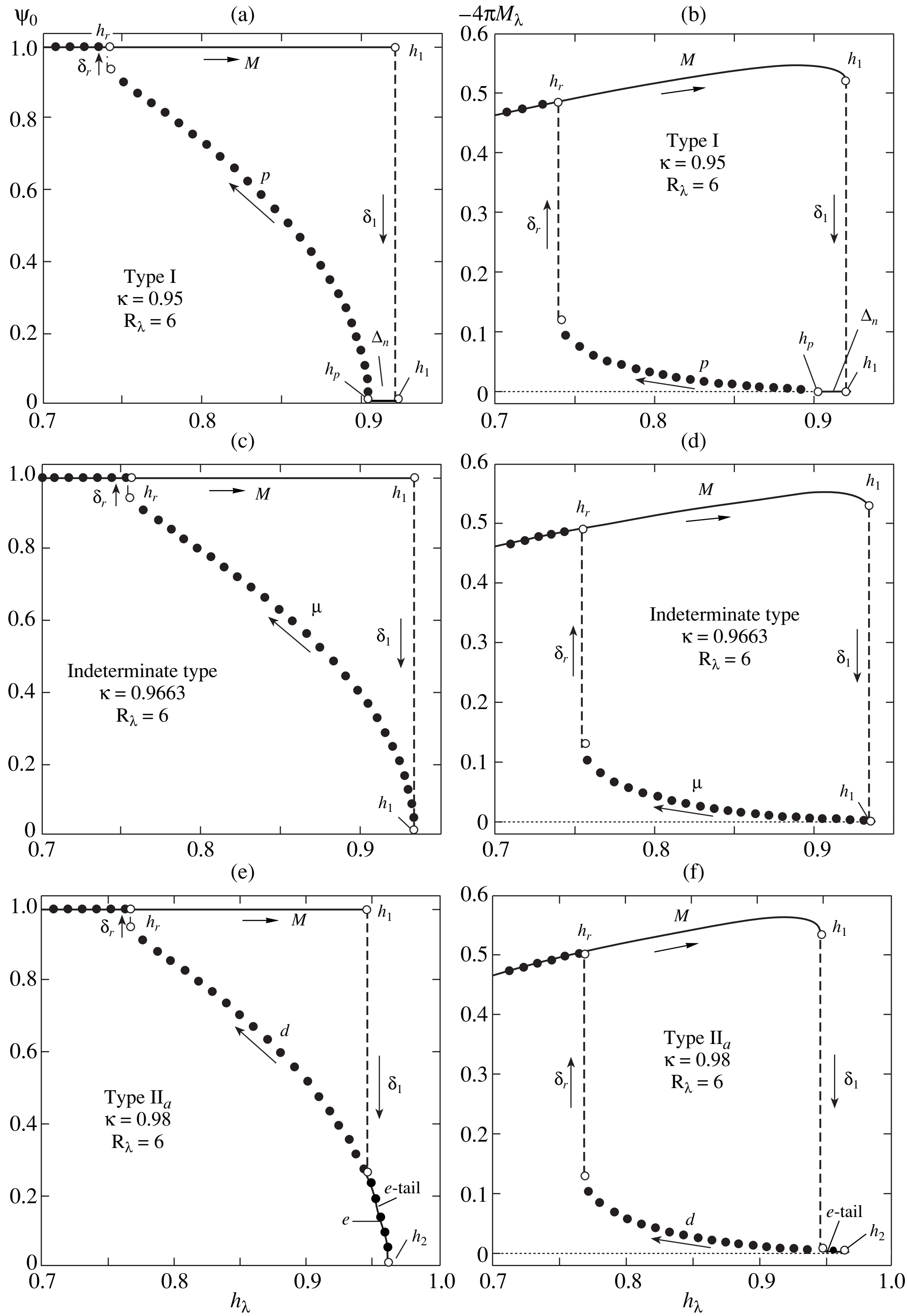

Fig. 2. Various types of superconducting states (M is the Meissner state, p is the precursor, μ is the marginal state, e is the edge-suppressed state, and d is the depressed state) existing near the $S_{\text{I–II}}$ curve in Fig. 1 for $R_\lambda = 6$. The dependences of the amplitude of states (ψ_0) and magnetization ($-4\pi M_\lambda$) on the field h_λ are plotted for different values of κ. Solid curves correspond to the field amplification mode, and dotted curves to the field reduction mode. The jumps δ_1 between states occur at points h_1 in the field amplification mode, and jumps δ_r take place at points h_r in the field reduction mode. The precursor state (p) emerges at point h_p (Figs. 2a and 2b) from the supercooled normal state ($\Delta_n = h_1 - h_p$ is the width of the supercooling region).

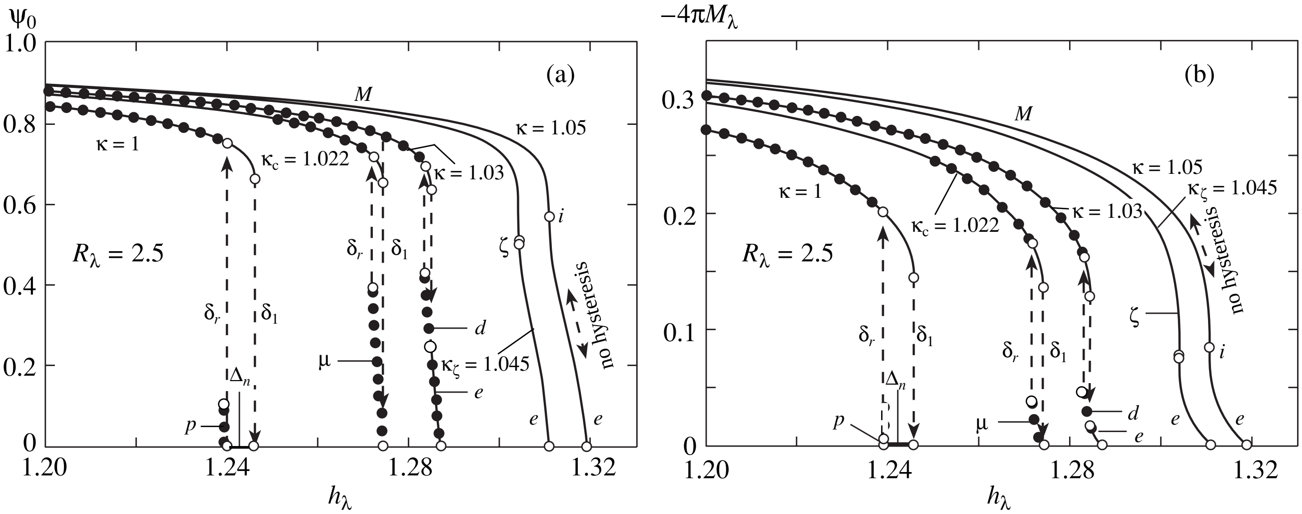

Fig. 3. Dependence of the amplitude of solutions (ψ_0) and magnetization ($-4\pi M_\lambda$) in different states (notation is the same as in Fig. 2) on field h_λ for $R_\lambda = 2.5$ and several values of κ (figures on the curves). Solid curves correspond to the field amplification mode, and dotted curve, to the field reduction mode. For $\kappa = 1$, the solutions (M, p, and n) belong to region I in Fig. 1. For $\kappa_c = 1.022$, the precursor state possesses the maximum amplitude $\psi_r(h_r) = 0.3855$, $h_r = 1.2716$ (μ state). Solutions with $\kappa = 1.03$ lie in region II$_a$ (see Fig. 1); here, there are two singular points, h_1 and h_r, at which the jumps δ_1 and δ_r occur and a hysteresis loop may be formed. For $\kappa = \kappa_\zeta = 1.045$, these two singularities merge at the point $h_\zeta = 1.3037$, where $dM_\lambda/dh_\lambda = \infty$. For $\kappa > \kappa_\zeta$, the values of $dM_\lambda/dh_\lambda < \infty$, and there is no hysteresis (type i curves).

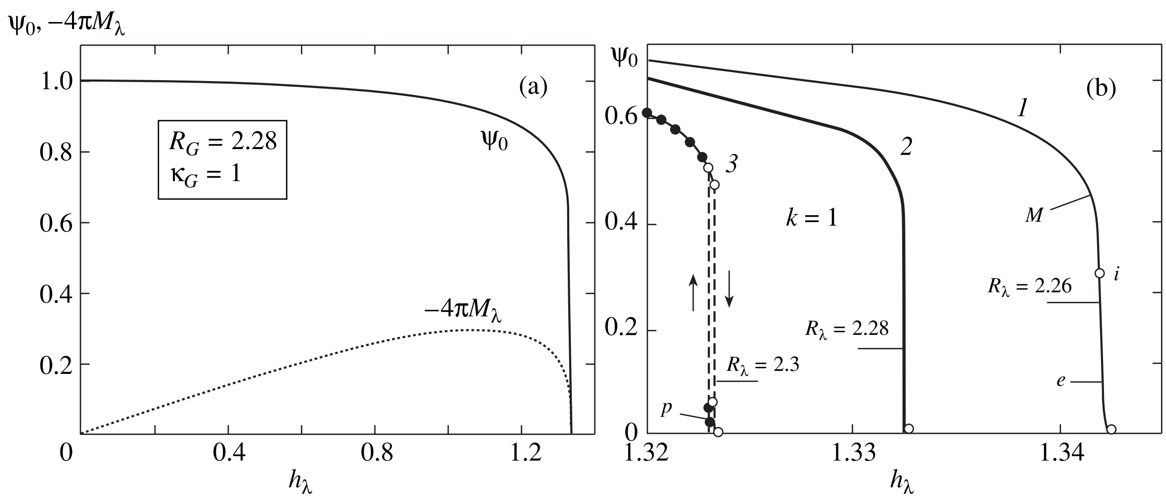

Fig. 4. (a) Dependence of the order parameter ψ_0 (solid curve) and magnetization $-4\pi M_\lambda$ (dotted curve) on the field h_λ at the critical point G ($\kappa_G \approx 1$, $R_G \approx 2.28$). (b) The behavior of $\psi_0(h_\lambda)$ (on a magnified scale) near the points of termination of solutions for R_λ = 2.30 ($R_\lambda > R_G$, region I) (curve *1*), 2.28 ($R_\lambda \approx R_G$) (*2*), and 2.26 ($R_\lambda < R_G$, region II$_b$ in Fig. 1) (*3*). It can be seen that, for $R_\lambda < R_G$, the nonhysteretic curve has the point of inflection i with a finite derivative and with a tail of e states, while the p state and a hysteresis loop appear for $R_\lambda > R_G$.

to exist up to point h_1. In the field interval $\Delta_e = h_2 - h_1$, in both regimes (field enhancement and reduction), there exists a unique e solution, and no hysteresis is possible. As the field decreases further ($h_\lambda < h_1$), the e solution is smoothly transformed into a new d (depressed) state [22] which is preserved up to the point $h_r(\kappa, R_\lambda)$ at which the Meissner solution is restored jumpwise (with jump amplitude δ_r). In the field interval $\Delta_d = h_1 - h_r$, there exist two stable solutions (M and d states); for this reason a hysteresis loop of width Δ_d may be formed.

The magnetic moment $-4\pi M_\lambda(h_\lambda)$ exhibits a similar behavior (Fig. 2f). In this case, the magnetization curve also has a tail, the supercooled n state is absent, and there exists a hysteresis loop associated with the d state, which can be treated as the supercooled e state. Such a pattern is typical of type II superconductors.

As the parameter κ approaches the critical line $S_{\text{I–II}}$ separating type I and II superconductors (solid curve in Fig. 1, which can be denoted by $\kappa_c(R_\lambda)$), the field intervals $\Delta_n = h_1 - h_p$ (in which the supercooled normal state is possible in type I superconductors) and $\Delta_e = h_2 - h_1$ (where the magnetization curve for type II superconductors acquires a tail) decrease and vanish exactly at the critical line. In this case (for $R_\lambda = 6$ and $\kappa_c = 0.9663$), the solution depicted in Figs. 2c and 2d appears. In this case, one cannot tell the type of superconductor (indeterminate type). The precursor solution can be termed marginal (μ) in this special case; it emerges without preliminary formation of the supercooled normal state. All marginal μ states lie on the critical curve $S_{\text{I–II}}$ in Fig. 1. The amplitude of the marginal p state at the jump point, $\psi_r = \psi_0(h_r, \kappa_c)$, is the largest among other p states existing in region I.

Figure 3 shows what happens near the critical curve $S_{\text{I–II}}$ for a smaller radius of the cylinder, $R_\lambda = 2.5$. It can be seen that (in accordance with Fig. 1) the p solution in region I has the maximum amplitude ψ_r on the critical curve $S_{\text{I–II}}$ ($\kappa_c = 1.022$, $\psi_r = 0.3855$, $h_r = 1.2716$; this is a marginal μ state). For $\kappa < \kappa_c$, the amplitude ψ_r decreases rapidly ($\psi_r = 0.095$ for $\kappa = 1$ and $\psi_r = 0.001$ for $\kappa = 0.9$). This allows us to state that, for $\kappa < \kappa_c(R_\lambda)$, the Meissner superconducting state (with $\psi_0 \approx 1$) is restored from the supercooled normal state (existing in the interval Δ_n), as a rule, through a "nearly first-order" phase transition (the jump δ_r occurs from the p state with $\psi_r \ll 1$).

For $\kappa > \kappa_c(R_\lambda)$ (in region II$_a$), the supercooled normal state and the corresponding hysteresis are absent, but (see the curve $\kappa = 1.03$) the e and d superconducting states are formed (e branch in the field amplification mode and d branch in the field reduction mode) and the hysteresis associated with the simultaneous existence of superconducting d and M states becomes possible. For $\kappa > 1.03$, the field interval $\Delta_d = h_1 - h_r$ in which d solutions exist and hysteresis is possible (see Figs. 2e and 2f) decreases and vanishes for $\kappa_\zeta = 1.045$. At the point $\kappa = \kappa_\zeta$, the jumps between the branches also disappear ($\delta_1 = \delta_r = 0$) and $dM_\lambda/dh_\lambda = \infty$ at this point (Fig. 3b). For $\kappa > \kappa_\zeta$ (in region II$_b$), the magnetization curve displays only the point of maximum descent (inflection point i) with a finite value of the derivative dM_λ/dh_λ (see the curve with $\kappa = 1.05$). In this case, the superconducting solutions (M and e) corresponding to the field reduction and enhancement modes merge into a single branch, and hysteresis is impossible; however, the magnetization curve has two distinguishable regions: in front of the point of inflection (M state) and behind it (e state).

The critical values of κ_ζ corresponding to different values of R_λ are depicted in Fig. 1 by the dotted curve ζ. Above this curve (in region II$_a$), hysteresis is possible (d solutions exist), while, below this curve (in region II$_b$), hysteresis is absent. At point G ($R_G \approx 2.28$ for $\kappa \approx 1$), the critical curves $S_{\text{I–II}}$ and ζ merge into one; for $R_\lambda < R_G$, there exists a single critical curve above which (in region I) the processes of superconductivity degradation (and restoration) are accompanied by first-order phase transitions (with jumps δ_1 and δ_2), while, below this curve (in region II$_b$), smooth second-order phase transitions take place. Thus, for a small radius of the cylinder, all type I superconductors (with $\kappa < \kappa_c(R_\lambda)$) become in fact type II superconductors.

Ginzburg [30], who noted that a type I superconductor with a small radius (with $\kappa \ll 1$) behaves in a magnetic field as a type II superconductor, arrived at the same conclusion (on the basis of different considerations). Consequently, point G can be referred to as a Ginzburg bicritical point. On the (κ, R_λ) plane, two critical curves, $S_{\text{I–II}}$ and ζ, converge at point G in contrast to the Landau tricritical point, at which three critical curves corresponding to supercooled, equilibrium, and superheated states converge on the plane of parameters (H, R) (see, for example, [7]). The lower part of curve ζ (lying below point G) determines the radius of the cylinder for which a type I superconductor becomes a nonhysteretic type II superconductor.

Figure 4a shows the dependence of the order parameter ψ_0 (and magnetization $-4\pi M_\lambda(h_\lambda)$) on field h_λ at point G ($\kappa = 1$, $R_\lambda \approx 2.28$). It can easily be verified that hysteretic d states (which are present in Fig. 3a for $R_\lambda = 2.5$) are not observed any longer (since point G lies on the nonhysteretic ζ curve). For the same reason, hysteretic p states (to be more precise, μ states since point G lies on the critical curve $S_{\text{I–II}}$), as well as the e states, must also be absent. As a result, the dependence $\psi_0(h_\lambda)$ at point G must have the form of a single-valued nonhysteretic curve consisting only of the M states, but with a vertical tangent line at the transition point, where ψ_0 vanishes.

All this is illustrated in Fig. 4b, showing on a magnified scale the dependence $\psi_0(h_\lambda)$ in the immediate vicinity of point G ($R_\lambda = 2.28$, curve *2*) as well as at points $R_\lambda = 2.3 > R_G$ (curve *1*, region I) and $R_\lambda = 2.26 < R_G$ (curve *3*, region II$_b$).

Figures 1–4 illustrate the behavior of vortex-free ($m = 0$) solutions to the Ginzburg–Landau equations as functions of parameters κ, R_λ, and h_λ. Figure 5 shows the behavior of the self-consistent solutions $\psi(x)$ and $b(x)$ as functions of the coordinate $x = r/R$. Figure 5a illustrates the form of the order parameter $\psi(x)$ for p solutions (precursors of transition to the Meissner state in region I in Fig. 1) for $R_\lambda = 6$. The marginal μ solution lies on the critical curve $S_{\text{I–II}}$ (with $\kappa_c = 0.9663$), where it attains the maximum amplitude ψ_r at the jump point ($h_r = 0.7548$, $\psi_r = 0.9441$), after which it becomes unstable and is transformed into the M solu-

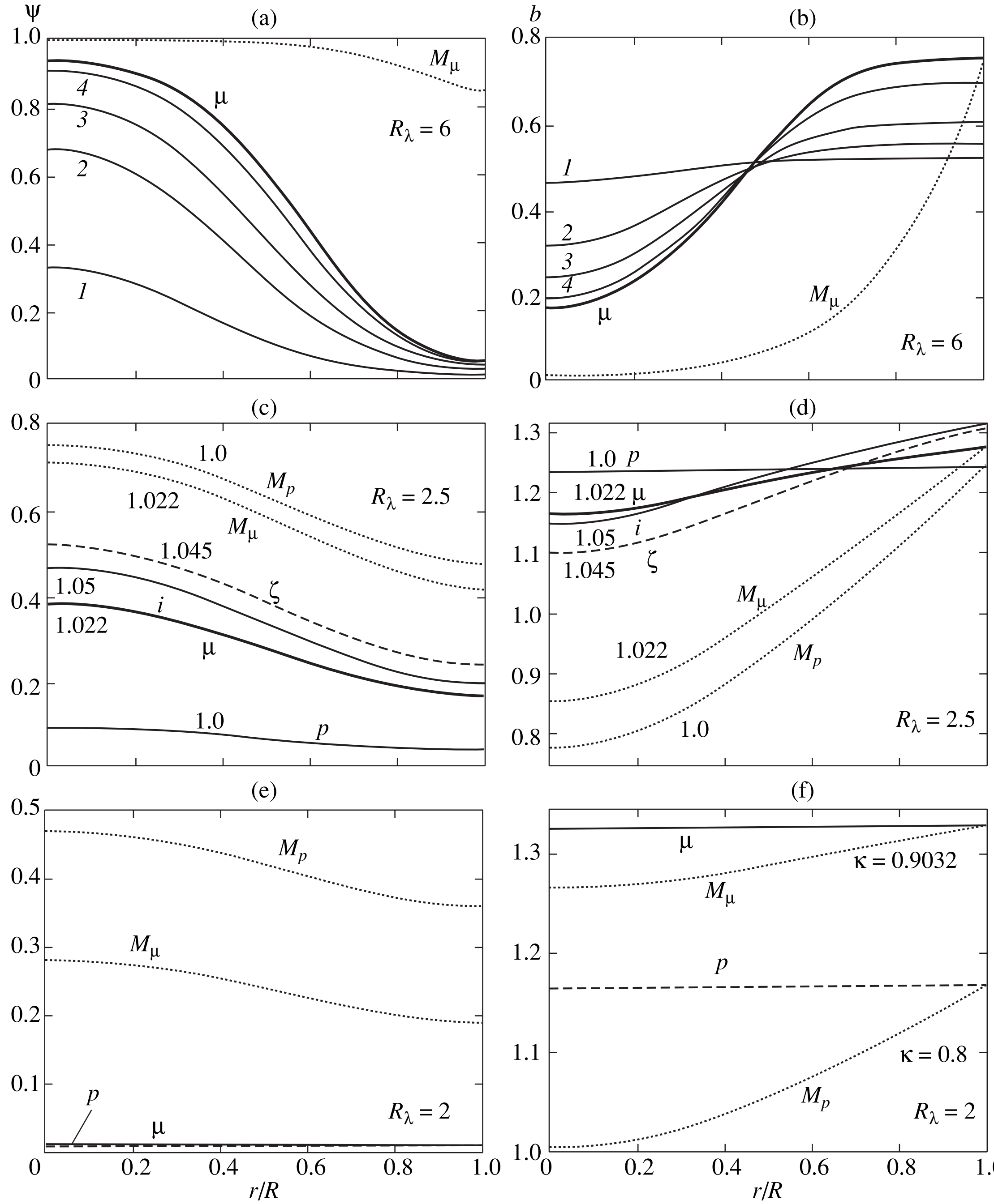

Fig. 5. Examples of the coordinate dependences of solutions $\psi(x)$ and $b(x)$ ($x = r/R$). (a, b) $R_\lambda = 6$. Precursor solutions (p) exist in region I along with the Meissner (M) solutions. Curve μ describes the marginal solution emerging for the critical value $\kappa_c = 0.9663$ (see Figs. 2c and 2d) and having the maximum amplitude $\psi_r = 0.9441$ in the field $h_r = 0.7548$. For $h_\lambda = 0.7547 < h_r$, a jump to the M state occurs. As the value of k decreases to $\kappa < \kappa_c$, the amplitude of the p solution decreases rapidly at the jump point ($\psi_r = \psi(h_r)$): $\kappa = 0.9$, $h_r = 0.6966$, $\psi_r = 0.9135$ (curve *1*); $\kappa = 0.8$, $h_r = 0.6044$, $\psi_r = 0.8150$ (curve *2*); $\kappa = 0.75$, $h_r = 0.5538$, $\psi_r = 0.6784$ (curve *3*); and $\kappa = 0.72$, $h_r = 0.5193$, $\psi_r = 0.3321$ (curve *4*); for $\kappa = 0.71$, $h_r = 0.5057$ and $\psi_r < 1 \times 10^{-4}$. Meissner solutions at the transition points h_r have the form $\psi(x) \approx 1$; these solutions are similar to M_μ and are not shown in the figure. Figures (c, d) ($R_\lambda = 2.5$) and (e, f) ($R_\lambda = 2$) are explained in the text.

tion (dotted curve). As we move from the boundary $S_{\text{I–II}}$ deeper into region I, the amplitude of the p solutions decreases rapidly (Fig. 5a shows only the p solutions at the points of their transformation into M solutions; the corresponding M solutions are not shown).

Figure 5b shows similar dependences for the magnetic field distribution over the cylinder radius, $b(x) = B(x)/H_\lambda$.

Figure 5c illustrates the behavior of the order parameter $\psi(x)$ for $R_\lambda = 2.5$. Here, the marginal μ solution corresponds to $\kappa = 1.022$ ($h_r = 1.2716$, $\psi_r = 0.3855$), the jump from the μ state occurring to the corresponding Meissner M_μ state. The p solution in region I (for $\kappa = 1.0$ and $h_r = 1.2393$) and the corresponding M_p state are also shown. In addition, the ζ solution lying (see Fig. 1) at the boundary between regions II$_a$ and II$_b$ (for $\kappa_\zeta = 1.045$ and $h_\lambda = 1.3037$, where the derivative $dM_\lambda/dh_\lambda \longrightarrow \infty$) is also given, as well as the i solution (see Fig. 3) from region II$_b$ (for $\kappa = 1.05$ at the point $h_\lambda = 1.311$, where the magnetization $M_\lambda(h_\lambda)$ has a point of inflection i with a finite value of the derivative dM_λ/dh_λ (see Fig. 3)). In states of the type ζ and i, hysteresis is impossible in view of single-valuedness of these solutions (see Fig. 3). Solutions for the field $b(x)$ in the case when $R_\lambda = 2.5$ are shown in Fig. 5d.

Solutions $\psi(x)$ and $b(x)$ for $R_\lambda = 2.0$ (region I) are depicted in Figs. 5e and 5f. Here, the μ state corresponds to $\kappa_c = 0.9032$ ($h_r = 1.3301$); the p state existing for $\kappa = 0.8$ (at point $h_r = 1.1666$) is also shown. It can be seen from Figs. 5a, 5c, and 5e that the amplitude of p states decreases rapidly with increasing distance from the boundary $S_{\text{I–II}}$ (in this case, the supercooled n state is transformed onto the superconducting Meissner state through a "nearly first-order" phase transition). It can be seen, by the way, that the amplitude of the p state always attains its maximum at the midpoint of the cylinder; i.e., the superconducting state emerges in the bulk of the cylinder and not at its surface. In this connection, see [31–37], where the emergence of superconductivity in plane-parallel plates is interpreted in a different manner.

4. CONCLUSIONS

In this study, the main attention is paid to an analysis of the behavior of cylindrical type I superconductors (with small values of parameter κ) in an arbitrary magnetic field H. We have obtained self-consistent solutions to the Ginzburg–Landau equations for vortex-free states ($m = 0$) typical of cylinders with small radii R. The boundary $S_{\text{I–II}}$ separating the regions of behavior of the order parameter and magnetization of the cylinder, typical of type I and II superconductors, has been determined. It is shown that the field dependence of magnetization $-4\pi M_\lambda(h_\lambda)$ in type I superconductors exhibits a hysteresis loop associated with the existence of a supercooled normal phase and with the emergence (in the field-reduction regime) of special p states (precursors, see Fig. 3) preceding the complete expulsion of the field from the sample and a transition of the cylinder to the Meissner state in the field $h_r(\kappa, R_\lambda)$. This Meissner state is destroyed through a first-order jump (in the field amplification regime) in the field $h_1(\kappa, R_\lambda)$ and is restored also jumpwise (in the field reduction regime) in the field $h_r(\kappa, R_\lambda)$. In the vicinity of the boundary $S_{\text{I–II}}$, the amplitude of the emerging superconducting p state may become large ($\psi \approx 1$), which can in principle be detected in experiments. With increasing distance from the boundary $S_{\text{I–II}}$ to the bulk of region I, the amplitude of the p state decreases rapidly; for this reason, the restoration of the Meissner state ($\psi \approx 1$) for most type I superconductors (with $\kappa < \kappa_c(R)$) occurs through a "nearly first-order" phase transition (jumpwise from the p state with $\psi_r \ll 1$). The shape of the magnetization hysteresis loop, its location, and the sites and magnitudes of the jumps (see Figs. 2–4) strongly depend on κ (for given R and temperature T), which can be used, in principle, for the experimental determination of the parameters κ, R, and T (see also [20] in connection with the inclusion of temperature dependence).

Here, we confine our analysis to these qualitative remarks concerning the possible relation to experiments since, to our knowledge, no direct observations of hysteretic (and other) phenomena in long mesoscopic cylinders have been reported. Such experiments were mainly conducted on superconducting mesoscopic discs of various shapes [23–29]; the discussion of the results in the framework of the Ginzburg–Landau theory can be found in [10–16]. It turns out, however, that many theoretical results weakly depend on the choice of the sample geometry. Consequently the predictions obtained in the special case of cylindrical geometry may be of a more general significance and should be borne in mind in discussing the results of specific experiments.

Finally, let us clarify why the boundary $S_{\text{I–II}}$ between type I and II superconductors does not coincide with the generally accepted criterion $\kappa_0 = 1/\sqrt{2}$ [1]. This discrepancy can be explained by several factors. First, in [1], an unbounded superconductor is considered, while we are dealing with a superconducting cylinder of a finite radius. Second, in our case, the superconductor borders on a vacuum, while in [1] the contact of two semi-infinite s and n metals is considered. Third, we distinguish between two types of superconductors from the form of the dependence $-4\pi M_\lambda(h_\lambda)$ (i.e., from the presence or absence of a tail on the magnetization curve), while superconductors in [1] are divided into two groups according to a different criterion, i.e., according to the sign of the free energy $\sigma(\kappa)$ on the interface between the s and n phases (in this case, $\sigma = 0$ for $\kappa_0 = 1/\sqrt{2}$ [1]). Thus, the noncoincidence of our boundary $S_{\text{I–II}}$ with the value $\kappa_0 = 1/\sqrt{2}$ is due to the difference in the formulation of the problem.

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