

$$B_r^{(3)} = 0.138 \pm 0.002 T$$

$B_c^{(3)}$ is poorly defined on this hysteresis loop.

$$\therefore I_{\text{pointant}}^{(3)} = \frac{l B_r^{(1)}}{\mu_0}$$

$$= 3459 \pm 50 \text{ A}$$

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Finding the estimates for B_o and T_c such that

$$B_c = B_o \left(1 - \frac{T^2}{T_c^2} \right)$$

$$\therefore \frac{B_c^{(1)}}{B_c^{(2)}} = \frac{T_c^2 - T_1^2}{T_c^2 - T_2^2}$$

$$\therefore T_c^2 \left(\frac{B_c^{(1)}}{B_c^{(2)}} - 1 \right) = T_2^2 \frac{B_c^{(1)}}{B_c^{(2)}} - T_1^2$$

$$T_c^2 = \frac{T_2^2 B_c^{(1)} - T_1^2 B_c^{(2)}}{B_c^{(1)} - B_c^{(2)}}$$

$$T_c = \sqrt{\frac{(1.34)^2 \times 0.044 - 4.2^2 \times 0.079}{0.044 - 0.079}}$$

$$= 6.13 \pm 3.9 \text{ K}$$

For the 3rd plot, it doesn't ever pass through the origin because it starts initially above the superconducting phase with a non-zero ambient field. Thus, when it becomes superconducting, it already has a persistent field, so traverses the hysteresis loop without being able to reach the origin.

$$B_0 = \frac{B_c^{(i)}}{1 - \frac{T_i^2}{T_c^2}}$$

$$\therefore \bar{B}_0 = \frac{1}{2} \sum_{i=1}^2 \frac{B_c^{(i)}}{1 - \frac{T_i^2}{T_c^2}}$$

$$\bar{B}_0 = 0.0829 \pm 0.0079 \text{ T}$$

Using T_c and \bar{B}_0 , we can calculate the temperature the 3rd graph was plotted at:

- Estimated $B_c^{(3)} = 0.06 \pm 0.01 \text{ T}$.

$$\therefore T_3 = T_c \sqrt{1 - \frac{B_c^{(3)}}{B_0}}$$

$$T_3 = 3.22 \pm 0.54 \text{ K}$$

As seen in section 5 of the lab script, as $T \rightarrow 0$, the electron pair binding energy 2Δ can be expressed as

$$2\Delta = U_n - U_s = \frac{B_c^2}{2\mu_0}$$

For tin, we can take an average of our previously acquired values,

$$\overline{B}_c^{(tin)} = \frac{1}{2} (30.7 + 28) \text{ mT}$$

$$= 29.35 \pm$$

$$\overline{B}_c^{(tin)} = \frac{\left(\frac{30.7}{0.5} + \frac{28}{3} \right)}{\frac{1}{0.5} + \frac{1}{3}}$$

$$\overline{B}_c^{(tin)} = 30.3 \pm 0.5 \text{ mT}$$

$$\therefore U_n - U_s = 365.3 \text{ J} \pm 8.5$$

We want to find a description for the system's entropy along the superconducting phase boundary.

Along the boundary, the Gibbs energy of the superconducting and normal states equalise.

$$\therefore dG_s = dG_n$$

$$S_s dT + M_s dB = S_n dT + M_n dB$$

$$\therefore S_s - S_n = \left(\frac{\partial B_c}{\partial T} \right)_{\text{along phase boundary}} (-M_s)$$

- Because the superconducting state perfectly excludes any applied critical field, we have

$$M_s = -\frac{B_c}{\mu_0}$$

$$\therefore S_s - S_n = \frac{B_c}{\mu_0} \left(\frac{\partial B_c}{\partial T} \right)_{\text{phase boundary}}$$

- At the critical temperature (assuming the phases are still in equilibrium), we see that $B_c = 0$.

$$\therefore \text{Latent heat, } L_s = T \Delta S$$

$$= \frac{T_c B_c}{\mu_0} \frac{\partial B_c}{\partial T}$$

$$L_s = 0$$

- However, for all $T < T_c$, B_c is finite, so

$$L_s = \frac{T B_c}{\mu_0} \left(\frac{\partial B_c}{\partial T} \right)_{\text{phase boundary}}$$

- The change in heat capacities between the two states is

$$\Delta C = C_s - C_n$$

$$= T \left(\frac{\partial S_s}{\partial T} - \frac{\partial S_n}{\partial T} \right)$$

$$= T \frac{\partial}{\partial T} \left(\frac{B_c}{\mu_0} \left(\frac{\partial B_c}{\partial T} \right)_{\text{boundary}} \right)$$

$$= \frac{T}{\mu_0} \left(\left(\frac{\partial B_c}{\partial T} \right)^2 + B_c \frac{\partial^2 B_c}{\partial T^2} \right)$$

$$\Delta C = \frac{T}{2\mu_0} \left(\frac{d^2(B_c^2)}{dT^2} \right)$$

In the limit $T \rightarrow T_c$, $B_c \rightarrow 0$, this goes to

$$\Delta C = \frac{T_c}{2\mu_0} \left(\left(\frac{\partial B_c}{\partial T} \right)^2 + 0 \right)$$

$$\Delta C = \frac{T_c}{2\mu_0} \left(\frac{\partial B_c}{\partial T} \right)^2$$

We can try to relate these expressions to the electronic heat capacities in the normal and superconducting phases. If we take $C_n = \alpha T^3 + \gamma T$, and (due to the superconducting energy gap),

$$S_s \rightarrow 0, \frac{dS_s}{dT} \rightarrow 0$$

$$\therefore \frac{C_s}{T} \rightarrow 0.$$

At $T=0$, we can say that

$$\left(\frac{C_s - C_n}{T} \right)_{T \rightarrow 0} = 0 - \lim_{T \rightarrow 0} \frac{C_n}{T}$$

$$= \lim_{T \rightarrow 0} \left\{ -\gamma - \alpha T^2 \right\}$$

$$= -\gamma$$

$$= -\frac{\Delta C}{T}.$$

$$\left(\frac{C_s - C_n}{T} \right)_{T \rightarrow 0} = -\frac{1}{2\mu_0} \frac{d^2 B_0^2}{dT^2}$$

$$\boxed{\left(\frac{C_s - C_n}{T} \right)_{T \rightarrow 0} \rightarrow -\frac{1}{2\mu_0} \left(\frac{B_0}{T} \right)^2 \frac{d^2}{dT^2}}$$

$$\left(\frac{C_s - C_n}{T} \right)_{T \rightarrow 0} = \frac{1}{2\mu_0}$$

$$\left(\frac{C_s - C_n}{T} \right)_{T_c} \Rightarrow B_s \rightarrow 0$$

$$\left(\frac{C_s - C_n}{T} \right)_{T_c}$$

$$\boxed{\left(\frac{C_s - C_n}{T} \right)_{T \rightarrow 0} = -\gamma = -\frac{2}{\mu_0} \left(\frac{B_0}{T_c} \right)^2}$$

$$\left(\frac{C_s - C_n}{T} \right)_{T_c} \approx \left. \frac{\Delta C}{T} \right|_{T_c}$$

$$= \left. \frac{T_c}{T} \frac{1}{\mu_0} \left(\frac{\partial B_c}{\partial T} \right)^2 \right|_{T \rightarrow T_c}$$

$$\approx \left. \frac{1}{\mu_0} \left(\frac{\partial B_c}{\partial T} \right)^2 \right|_{T=T_c}$$

$$B_c = B_0 \left(1 - \frac{T^2}{T_c^2} \right)$$

$$\therefore \frac{\partial B_c}{\partial T} = -\frac{2}{T} \frac{B_0 T^2}{T_c^2} = \frac{2B_0}{T} \left(1 - \frac{T^2}{T_c^2} \right) - \frac{2B_0}{T_c^2}$$

$$\therefore \left. \frac{\partial B_c}{\partial T} \right|_{T=T_c} = -\frac{2B_0}{T_c}$$

$$\boxed{\therefore \left(\frac{C_s - C_n}{T} \right)_{T_c} = -\frac{4}{\mu_0} \left(\frac{B_0}{T_c} \right)^2 \approx -2\gamma}$$

- We can use the results we've experimentally obtained to determine a value for γ .

$$\gamma = \frac{2}{\mu_0} \left(\frac{B_0}{T_c} \right)^2$$

$$= \frac{2}{4\pi \times 10^{-7} \text{ Hm}^{-1}}$$

$$\frac{B_0^{(\text{tin})}}{T_c^{(\text{tin})}} = 29.4 \pm 3.5 \text{ mT K}^{-1}$$

$$\gamma = \frac{2}{4\pi \times 10^{-7} \text{ Hm}^{-1}} \left(\frac{29.4 \times 10^{-3} \text{ T}}{3.72 \text{ K}} \right)^2$$

$$\gamma = 99.4 \pm 17.4 \text{ Jm}^{-3} \text{ K}^{-2}$$

- This value for the electronic specific heat capacity agrees with the literature value of $\gamma = 110 \text{ Jm}^{-3} \text{ K}^{-2}$ within its error bounds.

$$\frac{B_0^{(\text{tin})}}{T_c^{(\text{tin})}} = 30.6 \pm 0.5 \times 10^{-3} \text{ T K}^{-1}$$

$$\frac{B_0^{(\text{tin})}}{T_c^{(\text{tin})}} = 3.67 \pm 0.04 \text{ K}$$

$$\therefore \gamma = 110.6 \pm 3.07 \text{ (Jm}^{-3}\text{K}^{-2}\text{)}$$

- This value almost perfectly agrees with the literature value for $\gamma^{(\text{tin})} = 110 \text{ Jm}^{-3} \text{ K}^{-2}$.

Summary:

In pure metal samples (e.g. 5N pure tin and lead) superconducting phases can be achieved by sufficiently lowering the sample's temperature, to allow electrons to bind together in Cooper pairs.

We demonstrated this by measuring the resistance of a 5N tin sample whilst lowering the pressure in a helium cryostat. We observed a phase transition in the conductor at $T_c \approx 3.63\text{K}$, at which point the resistance became negligible. We were then able to demonstrate that a magnetic field alters the critical temperature in the sample, and demonstrate a linear trend in $B_c \sim T^2$.

Using a different probe, we then demonstrated the Meissner effect in tin, by showing the expulsion of magnetic flux from the sample in the superconducting phase. Again, we were able to categorise the phase transition in terms of $B_c \sim T^2$ to get $B_0 = 28 \pm 3\text{mT}$, $T_c = 3.73 \pm 0.07\text{K}$.

We also demonstrated how a changing magnetic field can produce persistent currents in lead, and how they produce hysteresis of the integral magnetic flux.

Finally, we used some simple thermodynamic arguments to propose a form of the superconducting phase boundary, and determine the electron specific heat capacity of tin, which we measured as $\gamma = 110.6 \text{ J m}^{-3}\text{K}^{-2}$.