

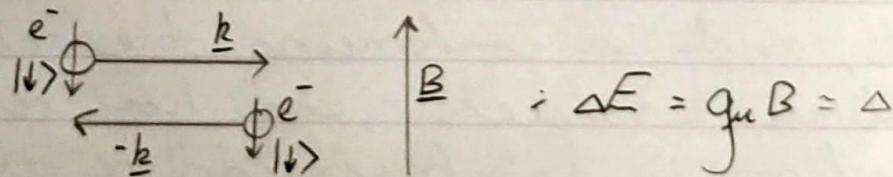
16/11/20 SSOS - Superconductivity

Objective

In this experiment, we will aim to demonstrate that superconductivity occurs in metals at low temperatures, and we will aim to determine the behaviour of these type I superconductors in response to external perturbations (e.g. an applied magnetic field).

Due to current world events, we are doing this lab remotely. Thus, we are using precollected data, but will still be following the data analysis procedures described in the lab script, "SSO8, Superconductivity" 2013.

- We can attempt to describe why an applied magnetic field may suppress superconductivity. If the electrons in the bound pair are both anti-aligned with the field, they gain an energy each,



If the increase in energy per electron is $\sim \Delta$, the electrons will have sufficient energy to become unbound, and scatter, suppressing the superconductivity.

Under high current densities, we might expect superconductivity to fail due to the creation of its own magnetic field. We would

also see the Fermi energy increase as $E_F \sim J^{2/3}$, which would increase the energy of electrons at the Fermi level, meaning they require less thermal energy to overcome their binding energy.)

- 3.1.1 - We are also asked to calculate the conversion between revolutions of the cryostat's exhaust gas meter to the volume of liquid ${}^4\text{He}$ lost:

$$1 \text{ revolution} \equiv 50 \text{ L of } {}^4\text{He} \text{ gas.}$$

$$740 \text{ L of } {}^4\text{He} \text{ gas} \equiv 1 \text{ L of liquid } {}^4\text{He}$$

$$\therefore 1 \text{ revolution} \equiv \frac{50}{740} \text{ L of liquid } {}^4\text{He}$$

$$1 \text{ revolution} \equiv 0.0676 \text{ L of liquid } {}^4\text{He}$$

loss (3sc)

- 3.4 - The data recording program records the integral of the voltage readings. It is important any DC offset is removed, but high frequency AC noise shouldn't be an issue as over the measurement time, it should integrate to an average of 0.

- Having completed the preliminary precautions we can begin with the experiment proper, starting with a measurement of the superconducting phase transition for tin.

4.1 Observation of the normal-superconducting transition using the resistivity & 4-wire

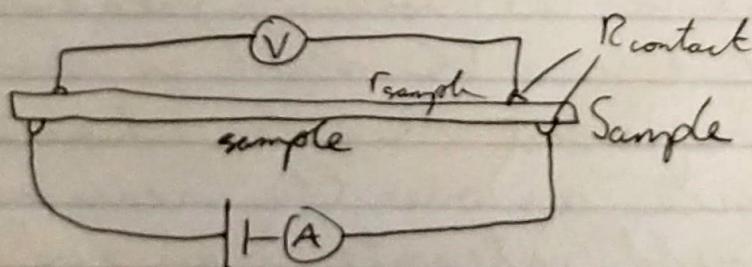
- The 4-wire method is preferable to a 2-wire measurement method, as the contact resistance of the leads to the sample won't affect the measurement of its resistance.

In a 2-wire method, the in-line measurement of the current/voltage will change based on the contact resistance.

$$\text{e.g.: } R_{\text{measured}} = R_{\text{sample}} + R_{\text{contact}}$$

For the 4-wire method, as the voltage measurement is in parallel solely with the sample, we get

$$\begin{aligned} \frac{1}{R_{\text{measured}}} &= \frac{1}{R_{\text{sample}}} + \frac{1}{R_{\text{contact}}} \\ &\approx \frac{1}{R_{\text{sample}}} \quad \text{for } R_{\text{sample}} \ll R_{\text{contact}} \end{aligned}$$



- It is necessary to reverse the current through the sample and measure the potential difference both times, as if the voltage contacts aren't aligned with the current, the measured voltage will take the

odd function

from $V = \text{const.} + f(B+I)$, where B is the ambient magnetic field. Thus, reversing the current allows us to take 2 measurements and cancel out the effects of the magnetic field:

$$\bar{V} = \frac{V(I) + V(-I)}{2}$$

- We can check to see if the current is heating the sample by taking many voltage readings at a fixed current. If there is no sample heating, Ohm's law will apply and the resistance will be constant.

Preliminary Measurements:

- There were two resistance measurements made at high temperature for the tin sample. The data was given as

T [K]	V+ [V]	V- [V]	I = 7.687 mA
300	5.86	-5.85	
77	1.102	-1.095	

$$\begin{aligned}\therefore R(300K) &= \frac{V_+ - (V_-)}{2I} \\ &= 762 \Omega \text{ (3st)}\end{aligned}$$

$$R(77K) = 143 \Omega \text{ (3st)}$$

- We also want the resistance at the highest temperature we are recording actual data from. The temperature has to be

inferred from a pressure measurement,
using an empirical formula,

$$x = \log_{10}(P/1\text{mbar}) ,$$

$$\frac{T}{1K} = 1.24177 + 0.23793x + 0.36207x^2 - 0.33188x^3 + 0.20738x^4 - 0.05294x^5 + 0.00552x^6$$

- Using this gave

	$T[\text{K}]$	$V+ [\times 10^{-3}\text{V}]$	$V- [\times 10^{-3}\text{V}]$
$I=1.002\text{A}$:	4.23	0.0913	-0.0832

$$R(4.23\text{K}) = 8.70 \times 10^{-5} \Omega$$

- If we look at the ratio $R(300\text{K})/R(4.23\text{K})$, we expect that for a pure sample, the ratio will be large. This is because there will be fewer lattice imperfections for electrons to scatter off at low temperatures, leading to a decrease in $R(4.23\text{K})$.

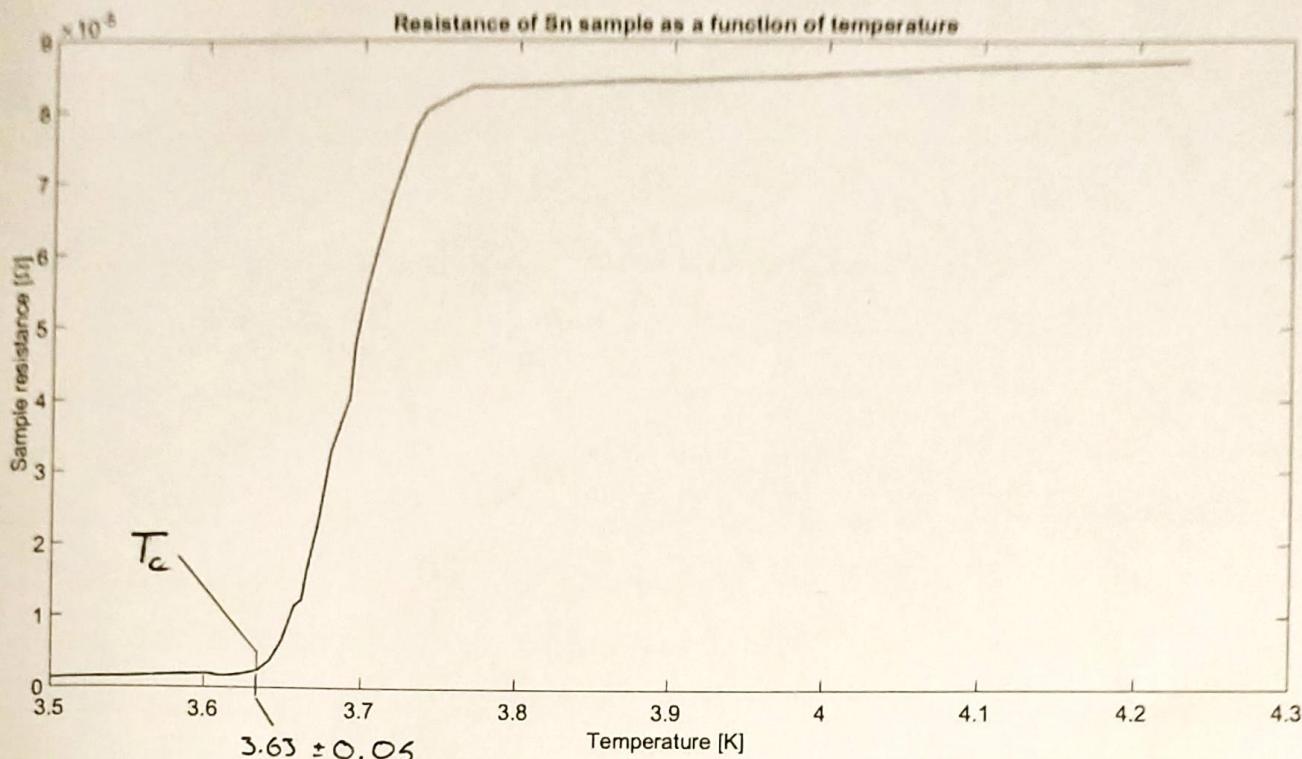
$$\frac{R(300\text{K})}{R(4.23\text{K})} = 8.76 \times 10^6 \Omega \text{ (3st)}$$

The large ratio implies our sample is very pure, and has few lattice imperfections from impurities or internal stress.

Plot of resistance vs.
temperature for the
superconducting
phase transition of
tin (and code used).

- The finite width of the phase transition is due to the finite width of the Fermi distribution's decrease at low temperature. Because of this, as T decreases, fewer electrons have the energy to thermally excite and overcome their pair-binding energy, so the resistance rapidly drops over a finite width until negligible numbers of electrons are conduct scattering.
- We can measure the transition temperature as the temperature at which we begin to leave the superconducting regime.

$$\therefore T_c = 3.63 \pm 0.05 \text{ K}$$



```

1    %% ResistivityDataRemote analysis
2    % This section requires us to determine the resistance of our sample at
3    % different temperatures and constant current. The file has the inputs of
4    % pressure, v+ and v- (voltages for different current bias)
5
6    % SPECIFICALLY, this section will read in the data and convert it into an
7    % appropriate format
8
9    % The below data was taken at current I = 1.002A
10   I = 1.002;
11   %gets the helium vapour pressure in mbar
12   p = ResistivityDataRemote.Heliumpressure;
13   % gets the positive voltage measurement in millivolts, which we will
14   % convert into volts
15   vmax = (1/1000)*ResistivityDataRemote.V;
16   % gets the reversed voltage measurement in millivolts, which we will
17   % convert into volts
18   vmin = (1/1000)*ResistivityDataRemote.Vl;
19
20   % we calculate the average voltage value to better determine the resistance
21   vbar = (vmax - vmin)/2;
22
23   % here we use the empirical formula to change the pressures into a
24   % temperature based on the measurements made by Duriex and Rusby:
25   x = log10(p);
26   T = 1.24177 + 0.23793*x + 0.36207*x.^2 -0.33188*x.^3 +0.20738*x.^4 -0.05294*x.^5 +0.00552*x.^6;
27
28   %% In this section we will compute the resistances and plot them against temperature
29   R = vbar./I;
30
31   p = plot(T,R);
32
33   xlabel("Temperature [K]");
34   ylabel("Sample resistance [\Omega]");
35   title("Resistance of Sn sample as a function of temperature");
36   set(findall(gcf,'-property','FontSize'), 'FontSize',20);
37   p.LineWidth = 2;

```

- After that, we started looking at data recorded at a fixed pressure that demonstrated the loss of superconductivity for large B fields.
- The data was included in files that recorded the shunt voltage and sample voltage. The shunt voltage could be turned into a magnetic field via the relation

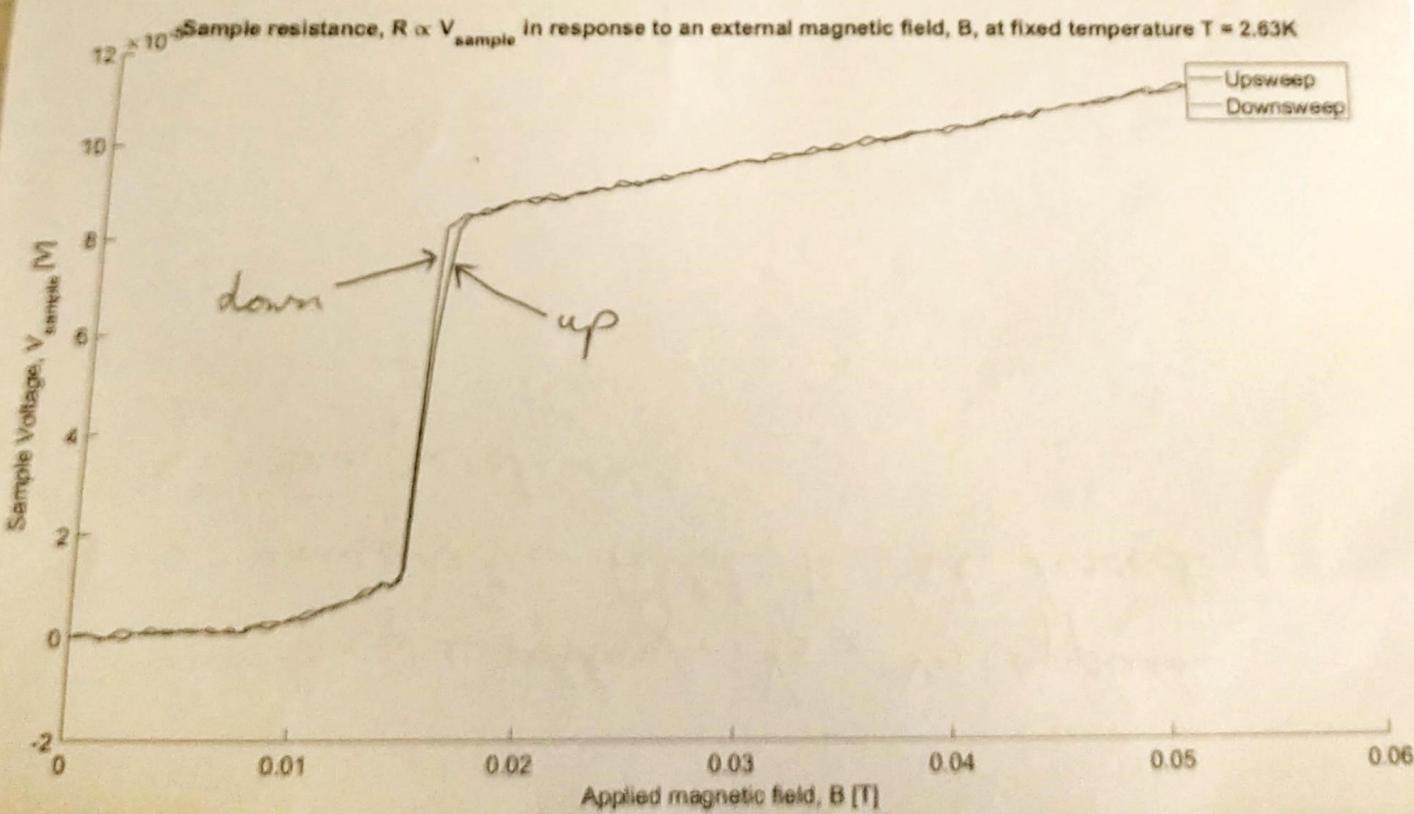
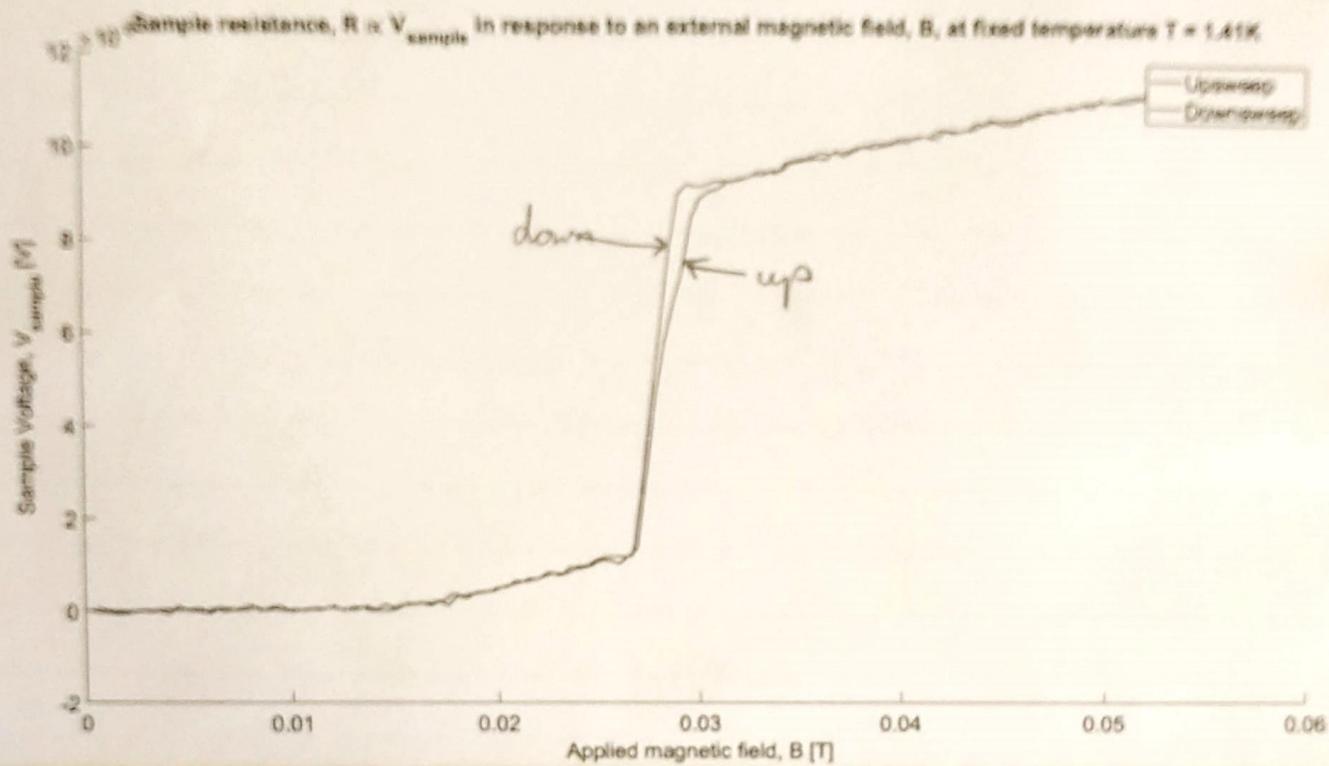
$$B = \frac{0.018 \text{ TA}^{-1}}{0.5 \text{ VA}^{-1}} V_{\text{shunt}}$$

Doing this, we were able to plot the sample voltage against the magnetic field strength for a variety of temperatures.

Sample voltage, V_s , vs. applied magnetic field, B , at fixed temperatures.

$$B = \frac{0.018 \text{ T A}^{-1}}{0.5 \text{ V A}^{-1}} V_{\text{sample}}$$

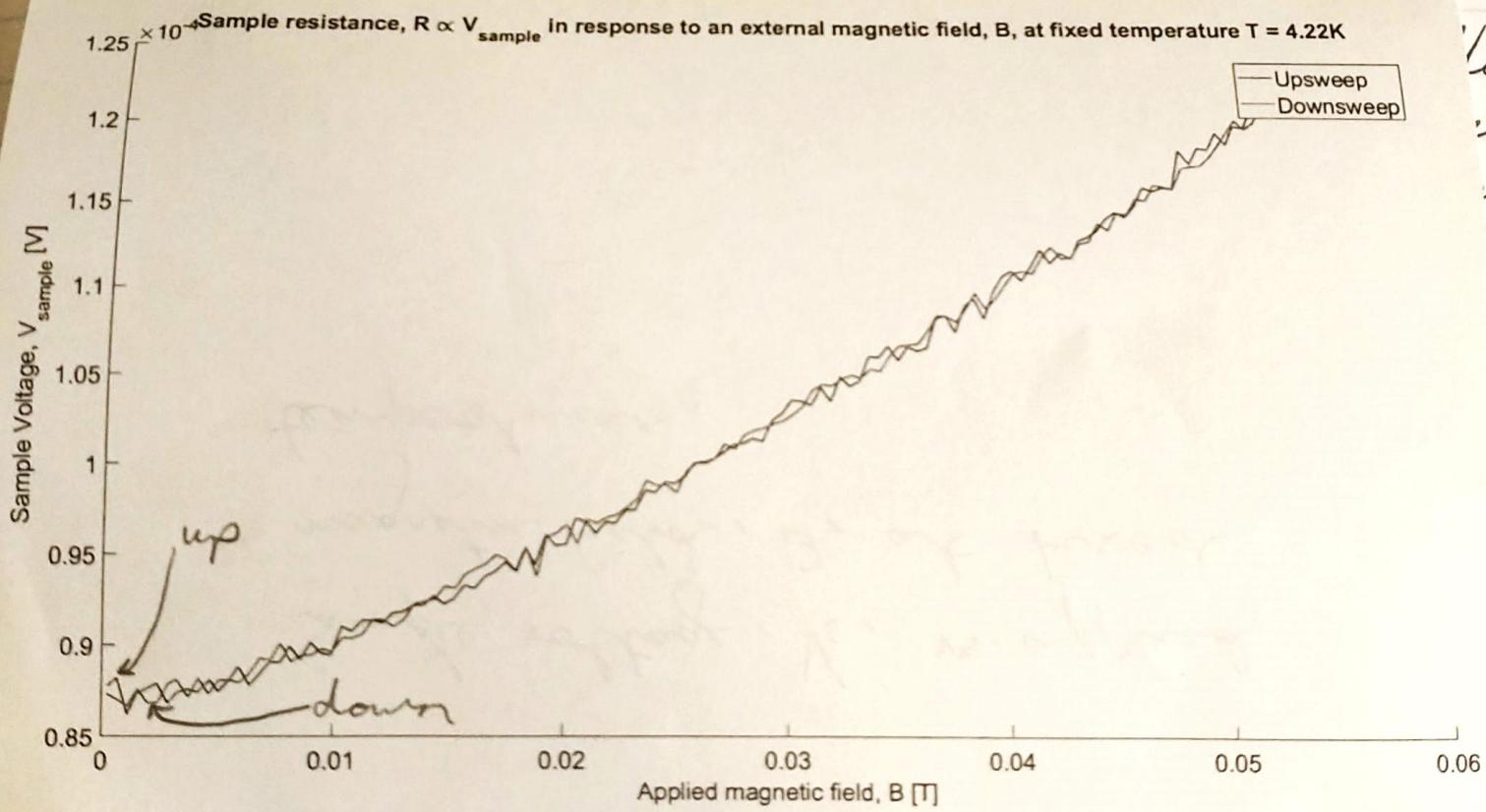
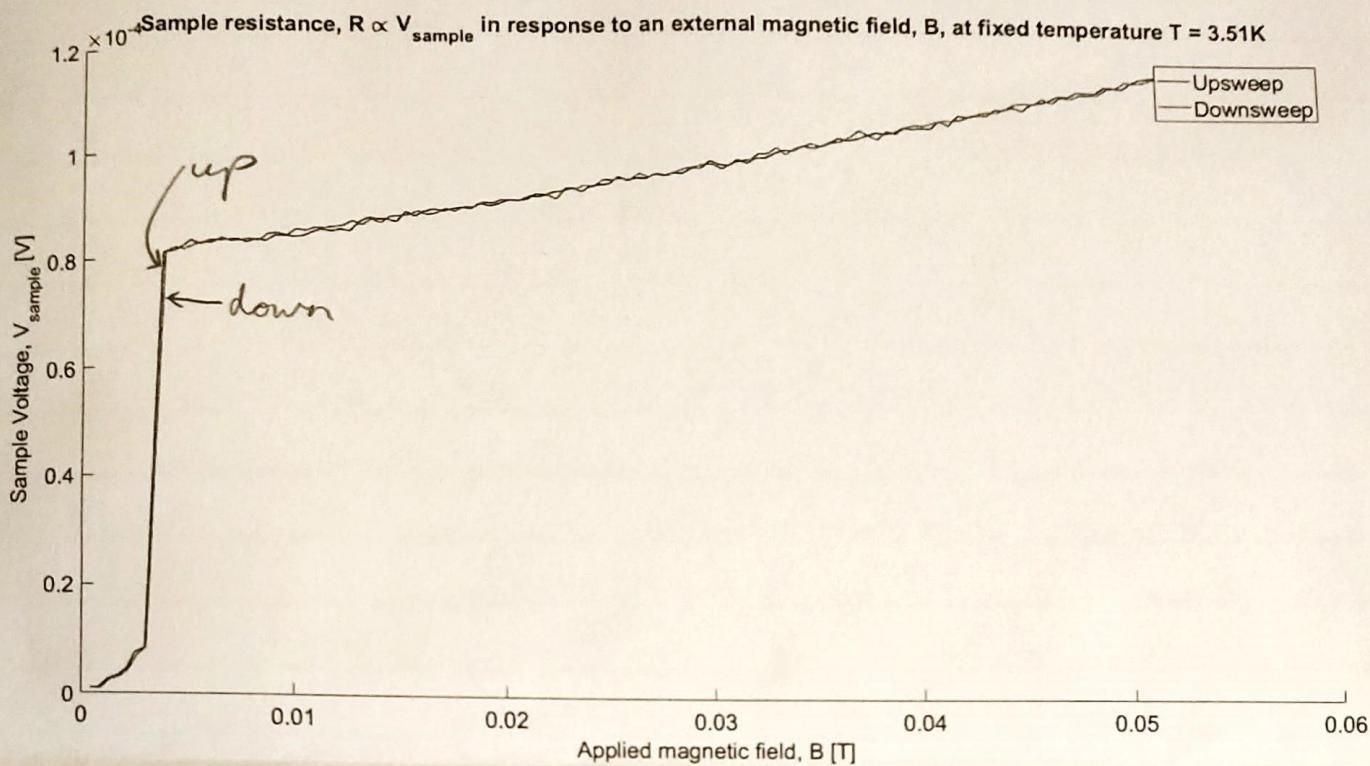
Doing this, we were able to plot the sample voltage against the magnetic field strength for a variety of temperatures.



Sample voltage, V_s , vs. applied magnetic field, B , at fixed temperatures.

- The shape of the transition is very similar to that of $\frac{d}{d}R$ vs. T , as the ~~change~~ increase in energy due to the increased field strength B , for anti-aligned (wrt the field) parallel pairs of electrons, allowing them to overcome their binding energy and begin to scatter. This doesn't happen all at once, but rather electrons furthest from the Fermi surface require stronger fields to overcome the Δ energy cost.
- The down-sweep occurring at lower magnetic fields to the up-sweep could be due to the rotation of atoms in the lattice, in ~~order~~ response to the magnetic field, heating the system. This would lead to more thermal energy being available to the electrons, so ~~they can~~ remain unbound at lower B fields.

$$\frac{d}{d} \mu_B B_{up} \approx \Delta,$$
$$\mu_B B_{down} + k_B T \approx \Delta.$$



Hysteresis in the measurements could be due to

- B_c being defined as a 50% of the extrapolated "normal-state" value is a poor definition as this value could be poorly defined and it isn't related to any predictable quantity.

If instead we define B_c as when the sample voltage / resistance initially begins to increase significantly, this can be estimated by considering the energy requirements of electrons at the Fermi level at finite temperature.

- For the provided temperatures, we then attempted to determine the critical field strength from each plot :

T [K]	B_c [T]
1.41	0.027 ± 0.0005
1.87	0.023 ± 0.001
2.12	0.0205 ± 0.001
2.24	0.019 ± 0.001
2.34	0.0178 ± 0.0005
2.45	0.0165 ± 0.0005
2.63	0.0145 ± 0.0005
2.78	0.0125 ± 0.001
2.91	0.0114 ± 0.0005
3.03	0.0095 ± 0.001
3.13	0.0082 ± 0.0005
3.22	0.0073 ± 0.0005
3.31	0.0062 ± 0.0005
3.37	0.0050 ± 0.0004
3.43	0.0042 ± 0.0005
3.46	0.0036 ± 0.0005

$T [K]$	$B_c [T]$
3.51	0.0031 ± 0.0003
3.55	0.0024 ± 0.0010
3.59	0.0018 ± 0.0005
3.62	0.0013 ± 0.0005

All $v_{\text{sample}} \sim B$ plots
on same axes,
and code used to generate
plots.

Plot of $B_c \sim T^2$

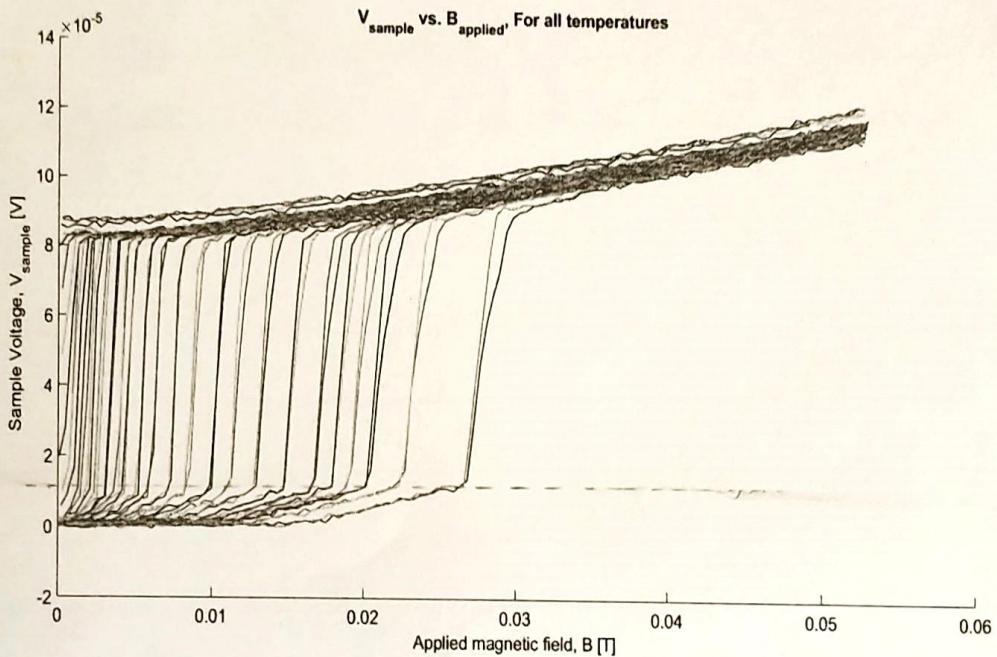
$$B(0) = 30.7 \pm 0.5 \text{ mT}$$

$$B(13.7 \pm 0.3 \text{ K}^2) = 0$$

- If we try and fit our data to the curve

$$B_c = B_0 \left(1 - \frac{T^2}{T_c^2}\right),$$

we get the values:



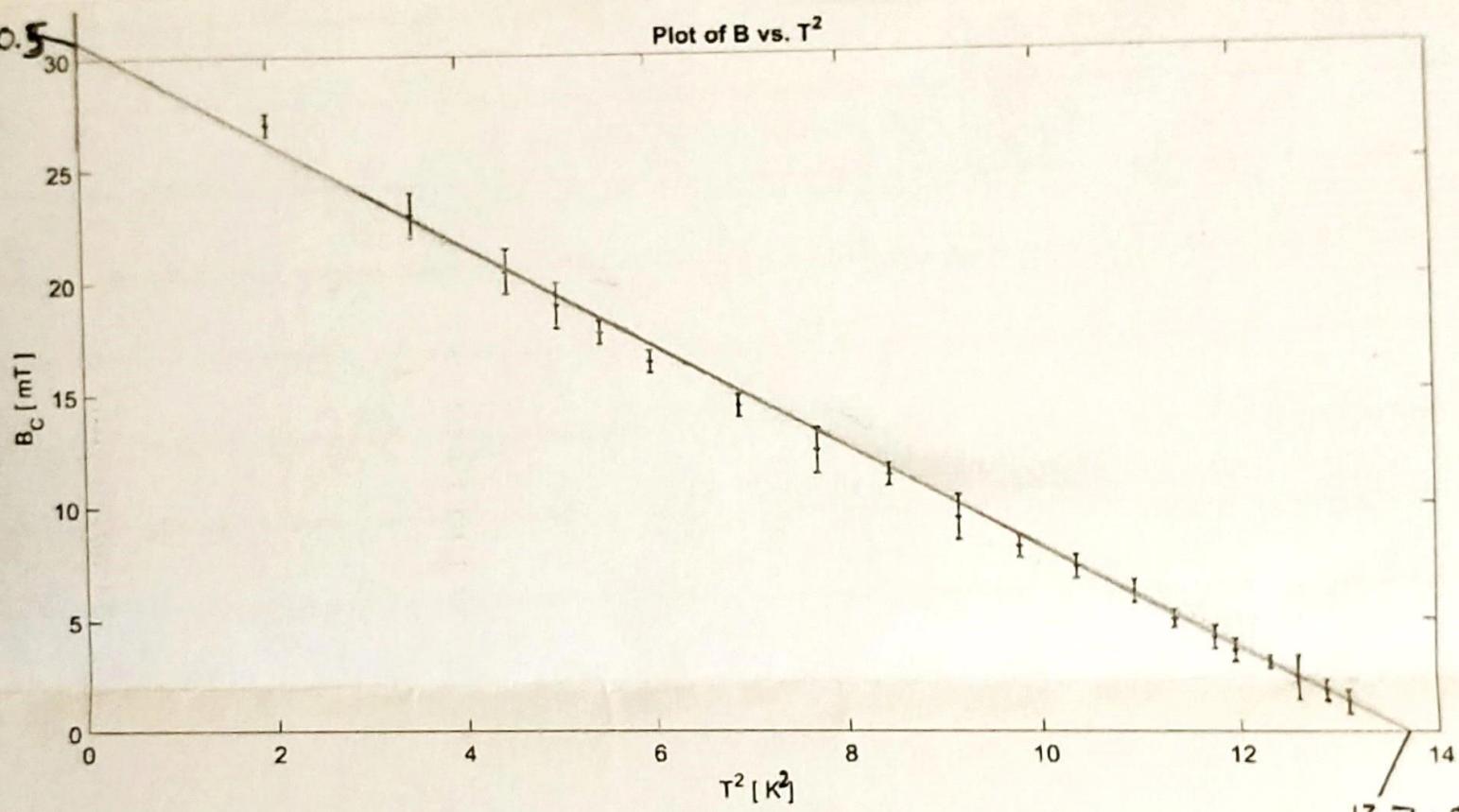
```

1 % This script is for the critical magnetic field strength vs T analysis.
2 % We will need to read in each of the given files (recording time, voltage
3 % and integrated voltage at fixed pressure) and determine the critical
4 % magnetic field strength from each visually via a plot
5
6 % Here we take the pressures in mBar that the data was recorded at
7 pressures = [1010, 890, 630, 610, 595, 570, 560, 540, 520, 500, 473, 450, 430, 400, 370, 330, 290, 250, 210, 170, 130, 92, 74, 58, 43, 21, 3];
8 % next, we'll determine the temperatures these occur at using the formula
9 % from Durieux and Rusty
10 x = log10(pressures);
11 T = 1.24177 + 0.23793*x.^2 - 0.33188*x.^3 + 0.20738*x.^4 - 0.05294*x.^5 + 0.00552*x.^6;
12
13 for i = 1:size(pressures,2)
14     filename = strcat('ProbeA',int2str(pressures(i)), 'mB');
15     data = importfile(filename, 3, 242);
16
17 t = data.t;
18 shuntV = data.shuntV;
19 sampleV = data.sampleV;
20 integratedV = data.integratedV;
21
22 % having read in the data, the shunt voltage can be converted into a
23 % magnetic field measurement via its relation to the current, and that
24 % to the flux density
25 B = (0.018/0.5)*shuntV;
26
27 % this creates a plot of the sample voltage against the shunt voltage (which is proportional to the applied electric field)
28 f = figure();
29 changeIndex = floor(size(shuntV,1)/2);
30 hold on
31 plot(B(1:changeIndex),sampleV(1:changeIndex),'LineWidth',1.5);
32 plot(B(changeIndex+1:end),sampleV(changeIndex+1:end),'LineWidth',1.5);
33 legend('Upsweep','Downsweep');
34 xlabel("Applied magnetic field, B [T]");
35 ylabel("Sample Voltage, V_(sample) [V]");
36 title("Sample resistance, R (propto V_(sample)) in response to an external magnetic field, B, at fixed temperature T = " + num2str(round(T(1),2)) + "K");
37 set(findall(gcf,'-property','FontSize'), 'FontSize', 20);
38 bold off
39 end

```

more

$$B_c = B_0 \left(1 - \frac{T^2}{T_c^2} \right),$$



```

41 % This section will take our critical magnetic field values as a function of temperature
42 % The temperatures at which we could measure the critical magnetic field [Kelvin]
43 - T = [1.41, 1.87, 2.12, 2.24, 2.34, 2.45, 2.63, 2.76, 2.91, 3.03, 3.13, 3.22, 3.31, 3.37, 3.43, 3.46, 3.51, 3.55, 3.59, 3.62];
44
45 % The observed magnetic fields, and their uncertainties [milli Tesla]
46 - Bc = [27, 23, 20.5, 19, 17.8, 16.5, 14.5, 12.5, 11.4, 9.5, 8.2, 7.3, 6.2, 5.0, 4.2, 3.6, 3.1, 2.4, 1.8, 1.3];
47 - uncert = [.5, 1, 1, 1, .5, .5, 1, .5, 1, .5, .5, .4, .5, .5, .3, 1, .5, .5];
48
49 % We want to plot B vs  $T^2$  to get a relation of the form  $B_c = B_0(1 - (T/T_c))$ 
50 - T2 = T.^2;
51 - p = errorbar(T2, Bc, uncert, '+');
52 - xlabel("T^2 [ K ]");
53 - ylabel("B_C [ mT ]");
54 - title("Plot of B vs. T^2");
55 - set(findall(gcf, '-property', 'FontSize'), 'FontSize', 20);
56 - p.LineWidth = 2;
57 - plot(T2, Bc, '+');

```

$$B_0 = (30.7 \pm 0.5) \times 10^{-3} \text{ T},$$

$$T_c^2 = 13.7 \pm 0.3 \text{ K}^2$$

$$\therefore T_c = 3.70 \pm 0.06 \text{ K}$$

4.2 The Meissner Effect in Tin:

- In this section of the lab, the probe being used would have been swapped out for one with a central tin core wrapped with a copper wire, designed to measure the magnetic flux internal to the tin.

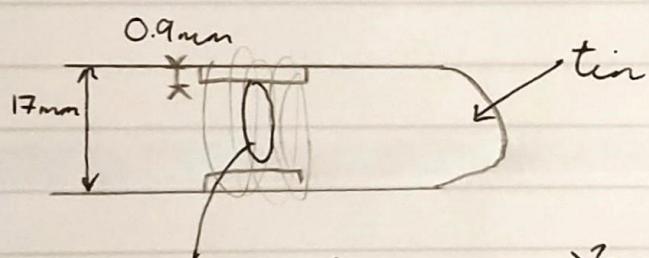


Figure 9
in lab script

$$A = \frac{\pi (17 - 2 \times 0.9)^2}{4} \text{ mm}^2$$

$$= 181.46 \text{ mm}^2 (\text{ss})$$

$$A = 1.8146 \times 10^{-4} \text{ m}^2 (\text{ss})$$

- We can measure the internal magnetic field by considering that a change in internal magnetic flux induces a voltage around the coil.

$$\therefore V_{\text{coil}} = nA \frac{\partial B_{\text{int}}}{\partial t}$$

$$\therefore \int_{t_0}^{t_1} V_{\text{coil}} dt = nA(B(t_1) - B(t_0)).$$

- At the start of each measurement run, the shunt voltage starts from zero, so there is no initially applied magnetic field.

$$\therefore B(t_0) = 0.$$

- The data program records the integrated voltage, $V_{\text{int}} = \int_{t_0}^{t_1} V_{\text{coil}} dt$, so we can say

$$B_{\text{int}}(t) = \frac{1}{nA} V_{\text{int}}(t).$$

- A quick note on procedure, half of the files for this section were .dat files, and half ~~were~~ didn't have any file extension. I converted all of them to .dat files for convenience with the coding.
- We wrote some code to plot the internal magnetic field against the applied magnetic field. Some of the plots for different temperatures are overlaid.

Plots of B_{internal} vs. B_{applied}

for $T = 1.34\text{K}$,

$T = 2.85\text{K}$.

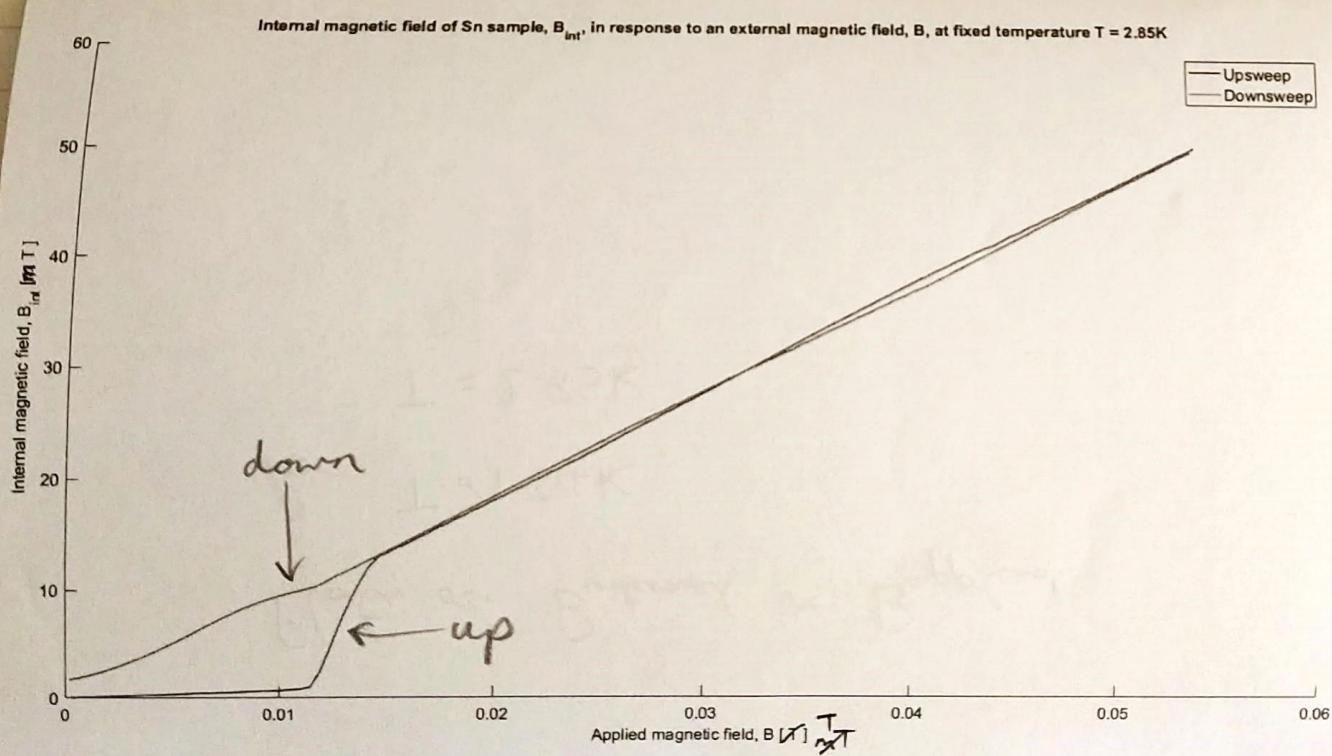
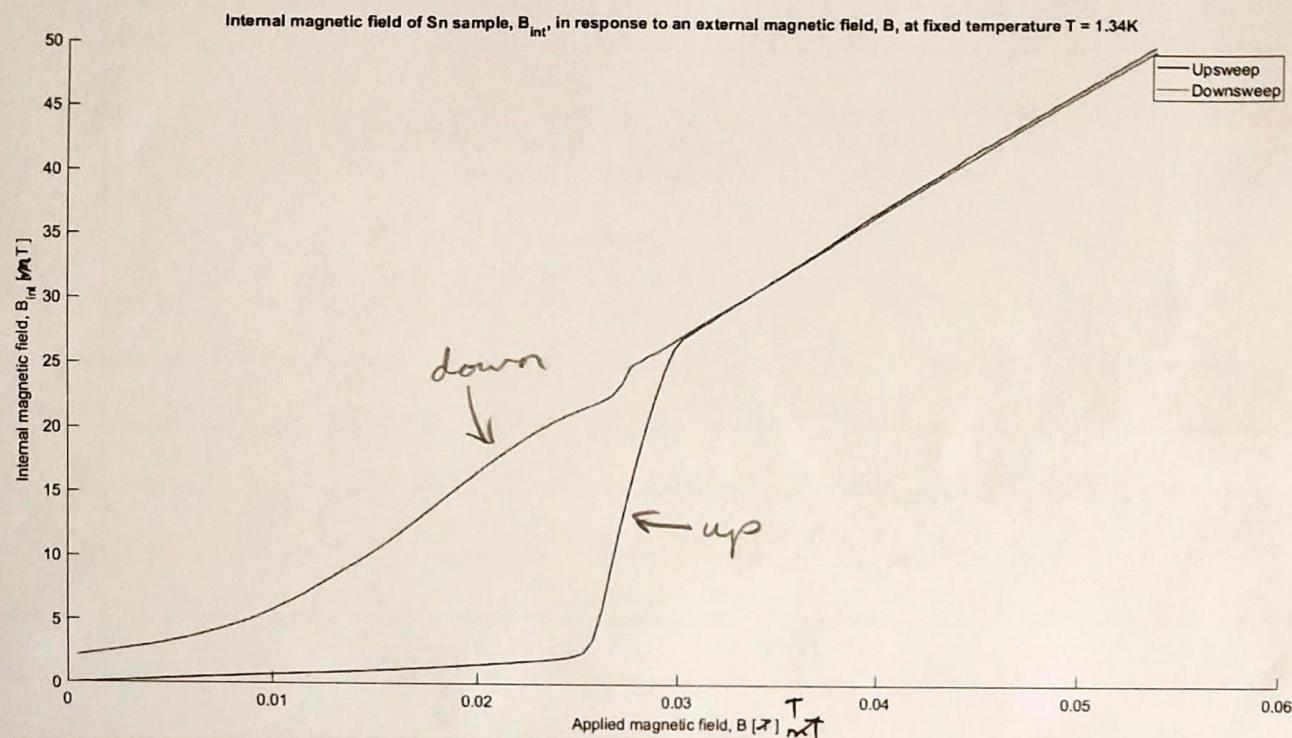
Plots of B_{internal} vs. B_{applied}

for $T = 3.45\text{K}$,

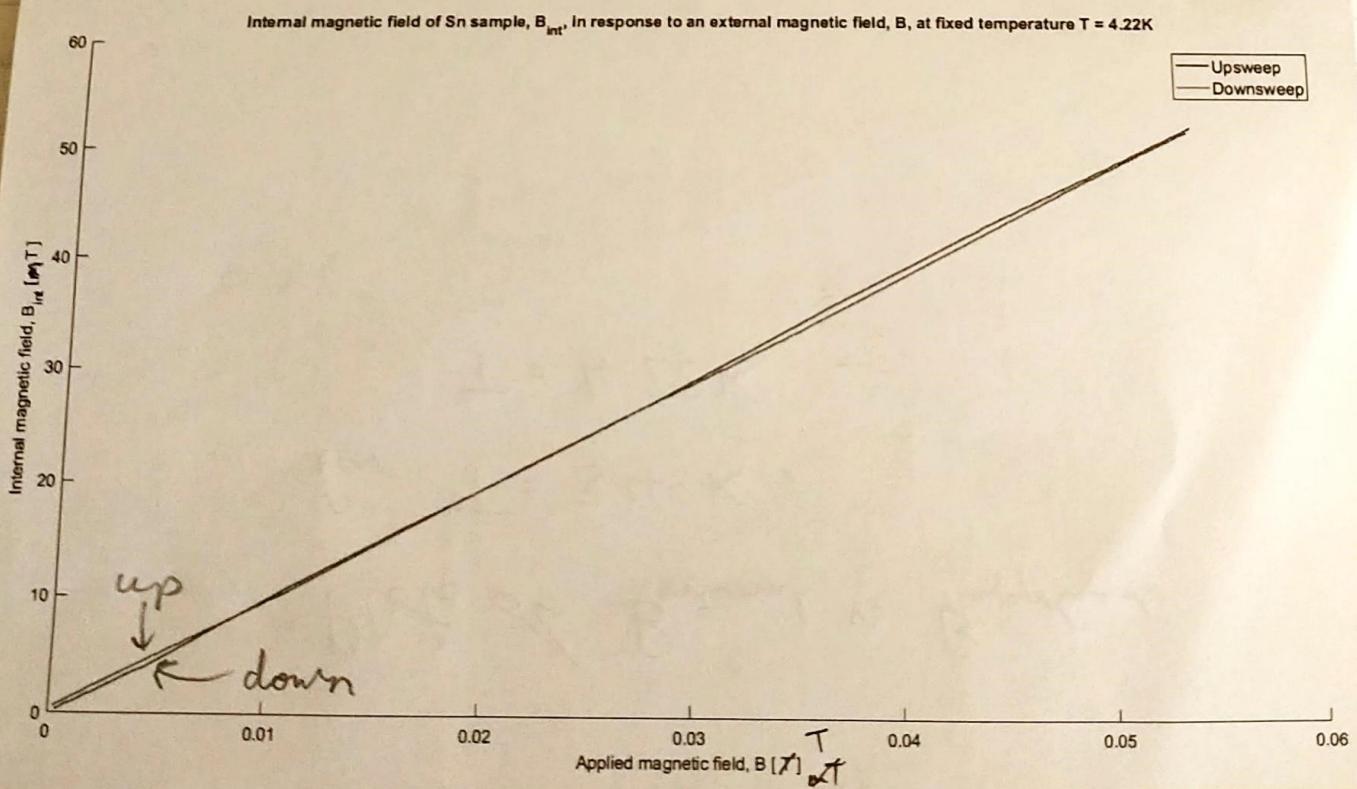
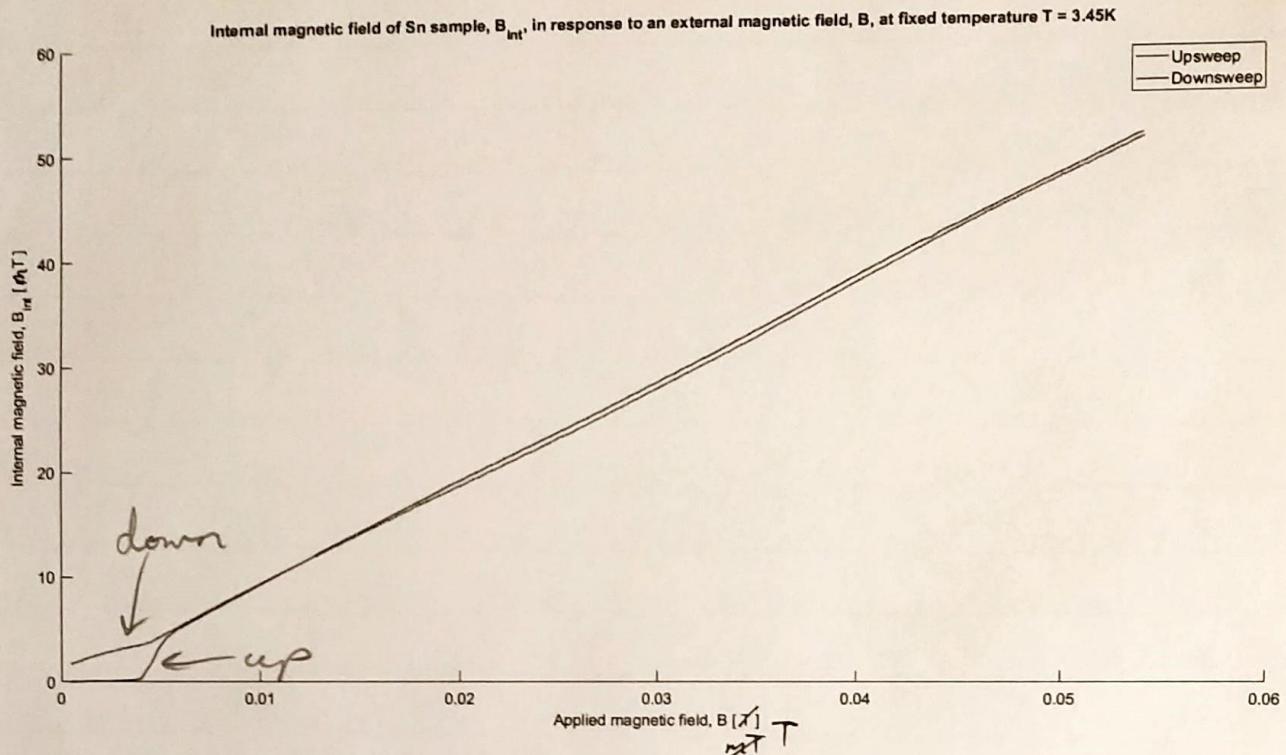
$T = 4.22\text{K}$.

- On the upswing, the tin expels the applied field almost entirely, until the applied field is sufficient to suppress the superconductivity. At this point, the internal flux rapidly increases towards the line where $\cancel{B_{\text{int}} \propto B_{\text{applied}}}$.

This is because the tin can no longer



for
 $T = 2.85K$.



have arbitrarily large currents to oppose the external field, and the currents within it rapidly dissipate. This reduces the amount of expulsion, and at high magnetic fields, hardly any field density is expelled (hence $B_{\text{ext}} \rightarrow B_{\text{applied}}$, which we see).

- If the probe's diameter was larger, we would expect to observe the same critical B_c value, but the increase of $B_{\text{ext}} \rightarrow B_{\text{applied}}$ would occur much quicker, as the larger sample's surface would have a large resistance, so the electrons generating the expelling field would scatter at a faster rate.

- We can determine the critical flux density, B_c , for each temperature by measuring where the up-sweep curve begins to rapidly increase:

$T [K]$	$B_c [\mu T]$
1.34	25.3 ± 0.5
1.86	21.4 ± 0.5
2.1	19.0 ± 0.5
2.24	18.0 ± 0.2
2.48	15.3 ± 0.5
2.64	13.6 ± 0.2
2.76	12.4 ± 0.3
2.85	11.5 ± 0.3
2.94	10.4 ± 0.3
2.99	9.7 ± 0.3

T[K]	Bc [μ T]
3.08	8.6 ± 0.3
3.2	7.1 ± 0.3
3.29	6.1 ± 0.5
3.35	5.4 ± 0.2
3.39	4.6 ± 0.2
3.45	4.05 ± 0.2 4.0 ± 0.3
3.5	3.0 ± 0.2
3.52	3.0 ± 0.3
3.54	2.9 ± 0.2
3.59	1.8 ± 0.5
3.62	1.55 ± 0.2
3.7	<hr/>
3.84	<hr/>
3.88	<hr/>
4.22	<hr/>

} Didn't display
any B expulsion.

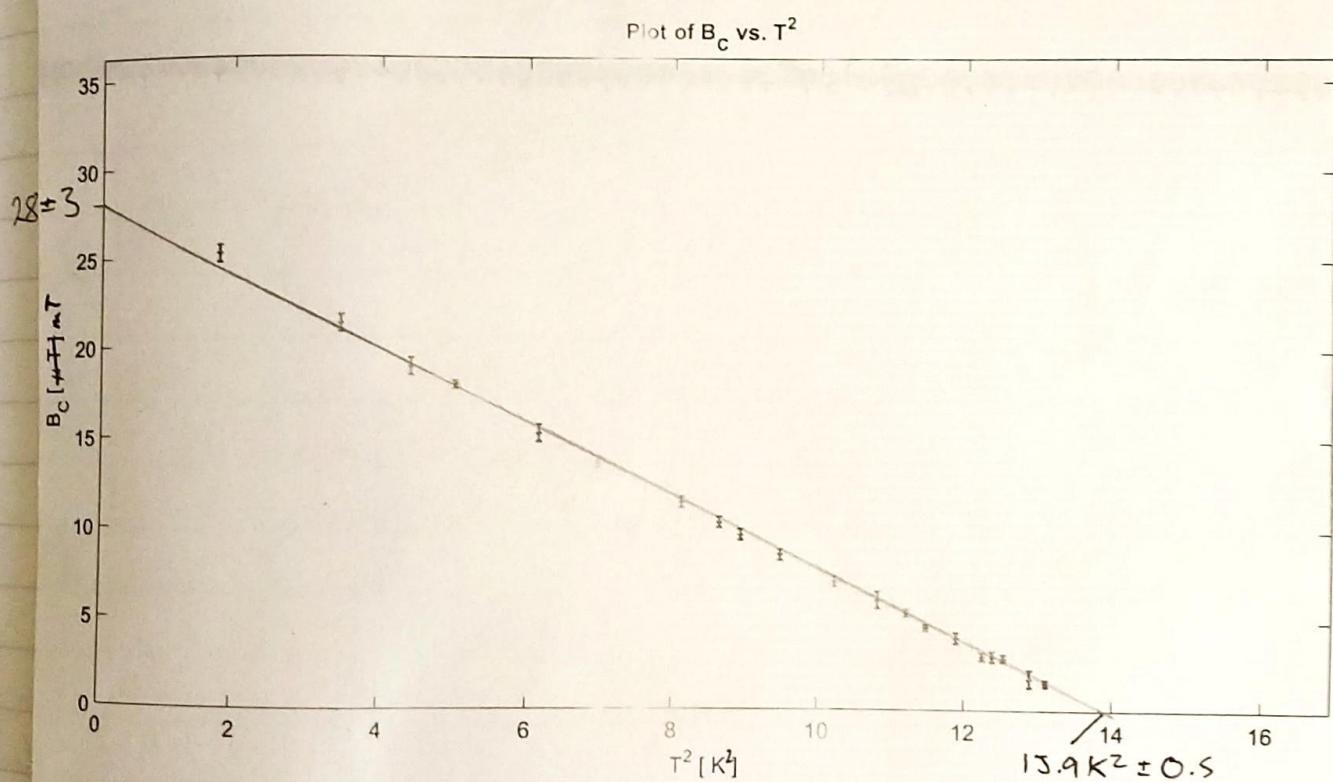
~~A quick note on procedure, we ended up with μ T for B_{internal} because the data in the files for $V_{\text{integrated}}$ has units [Vms], not [V] as is stated in the files. This also gives the expected order of magnitude for our measurements, which is a good sanity check.~~

- Using our plot for $B_c \sim T^2$, we can see it is linear. Using the form

$$B_c = B_0 \left(1 - \left(\frac{T}{T_c} \right)^2 \right),$$

we get $B_0 = 28 \pm 3 \mu\text{T. mT}$.

Code used to generate B_{int} vs. B_{Applied} plots, and to create B_c vs. T^2 plot.



$$B_0 = 28 \pm 3 \text{ mT},$$

$$T_c = 13.9 \pm 0.5 \text{ K}$$

- As we can see, our values for B_0 and T_c agree with the earlier experiment within error bounds.

```

6   % We will start by creating a list of pressures at which measurements were
7   % taken.
8   % pressures = [1010,720,690,590,540,525,490,480,470,440,410,390,360,320,270,230,220,190,165,132,98,80,58,41,20,2];
9   % again, we'll determine the temperatures these occur at using the formula
10  % from Duriex and Rusby
11  % x = log10(pressures);
12  % T = 1.24177 + 0.23793*x + 0.36207*x.^2 - 0.33188*x.^3 + .20738*x.^4 - 0.05294*x.^5 + 0.00552*x.^6;
13  % These variables are the turning number and area of the coil [mm^2]
14  n = 676;
15  A = 181.46;
16  % goes through the file for each of the pressures and gets the data for it.
17  for i = 1:size(pressures,2)
18    filename = strcat('ProbeB',int2str(pressures(i)), 'bar.dat');
19    data = importfile(filename, 3, 242);
20    t = data.t;
21    shuntV = data.shuntV;
22    sampleV = data.sampleV;
23    integratedV = data.integratedV;
24    % having read in the data, the shunt voltage can be converted into a
25    % magnetic field measurement via its relation to the current, and that
26    % to the flux density
27    Bapplied = (0.018/0.5)*shuntV;
28    % we can also convert the integrated voltage into an internal flux
29    % density in the tin probe. Using formula 3 in the lab script, letting
30    % t1 be the start time, and saying that B1 ~= 0, we get
31    Binternal = 1/(n*A) * integratedV;
32    % this creates a plot of the sample voltage against the shunt voltage (which is proportional to the applied electric field)
33    f = figure();
34    % roughly cuts the data in half so we can plot the up and down sweep
35    % separately.
36    changeIndex = floor(size(shuntV,1)/2);
37    hold on
38    plot(Bapplied(1:changeIndex)*10^3,Binternal(1:changeIndex)*10^9,'LineWidth',1.5);
39    plot(Bapplied(changeIndex+1:end)*10^3,Binternal(changeIndex+1:end)*10^9,'LineWidth',1.5);
40    legend('Upsweep','Downsweep');
41    xlabel("Applied magnetic field, B [ \mu T ]");
42    ylabel("Internal magnetic field, B_(int) [ \mu T ]");
43    title("Internal magnetic field of Sn sample, B_(int), in response to an external magnetic field, B, at fixed temperature T");
44    set(findall(gcf,'-property','FontSize'), 'FontSize', 15);
45    hold off
46 end
47 % This section is to test the Bc ~ T^2 relation as described in equation 1 in the lab script
48 % The temperatures at which we could measure the critical magnetic field [Kelvin]
49 T = [1.34,1.86,2.10,2.24,2.48,2.64,2.76,2.85,2.94,2.99,3.08,3.2,3.39,3.35,3.39,3.45,3.5,3.52,3.54,3.59,3.62];
50 % The observed magnetic fields, and their uncertainties [micro Tesla]
51 Bc = [25.3,21.4,19.0,18.15.3,13.6,12.4,11.5,10.4,9.7,8.6,7.1,6.1,5.4,4.6,4.0,3.3,2.9,1.8,1.55];
52 uncert = [.5,.5,.5,.2,.5,.2,.3,.3,.3,.3,.5,.2,.2,.3,.2,.5,.2];
53 % We want to plot B vs T^2 to get a relation of the form Bc = B0(1 - (T/Tc))
54 T2 = T.^2;
55 p = errorbar(T2,Bc,uncert,'+');
56 xlabel("T^2 [ K ]");
57 ylabel("B_C [ \mu T ]");
58 title("Plot of B_C vs. T^2");
59 set(findall(gcf,'-property','FontSize'), 'FontSize', 20);
60 p.LineWidth = 2;
61 %plot(T2,Bc,'+');

```

- If the specimen was instead a classical perfect conductor, we wouldn't observe the Meissner effect. If we consider Ohm's law,

$$j = \sigma E, \quad \sigma \rightarrow \infty,$$

the current density must remain finite to be physical. Thus, as $\sigma \rightarrow \infty$, E must be zero uniformly.

$$\therefore \nabla \times \underline{E} = - \frac{\partial \underline{B}}{\partial t}$$
$$= \underline{0}.$$

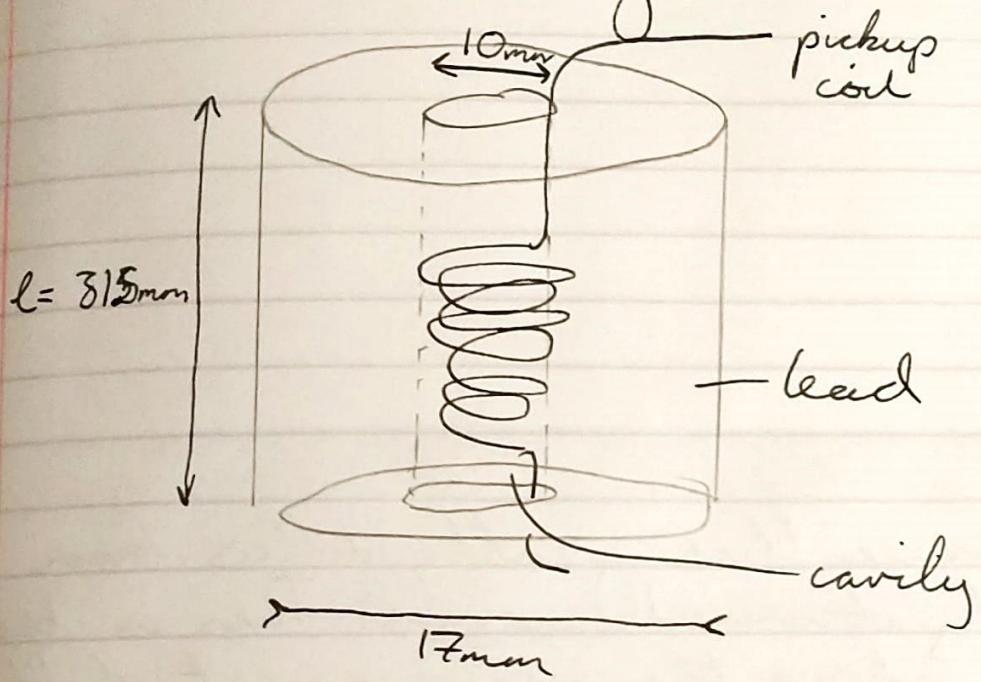
$$\therefore \frac{\partial \underline{B}}{\partial t} = \underline{0},$$

$$\underline{B} = \underline{B}_0 \quad : \text{constant.}$$

- In a perfect conductor, the internal magnetic field would remain constant for all time, regardless of the specimen's temperature or the ambient magnetic field, etc. Thus, there can be no breakdown of this state, which categorises the superconductive phase transition.

4.3 Persistent currents in Lead:

- In this part of the experiment, we've swapped the tin cores for a lead core with a central cylinder bored out of it:



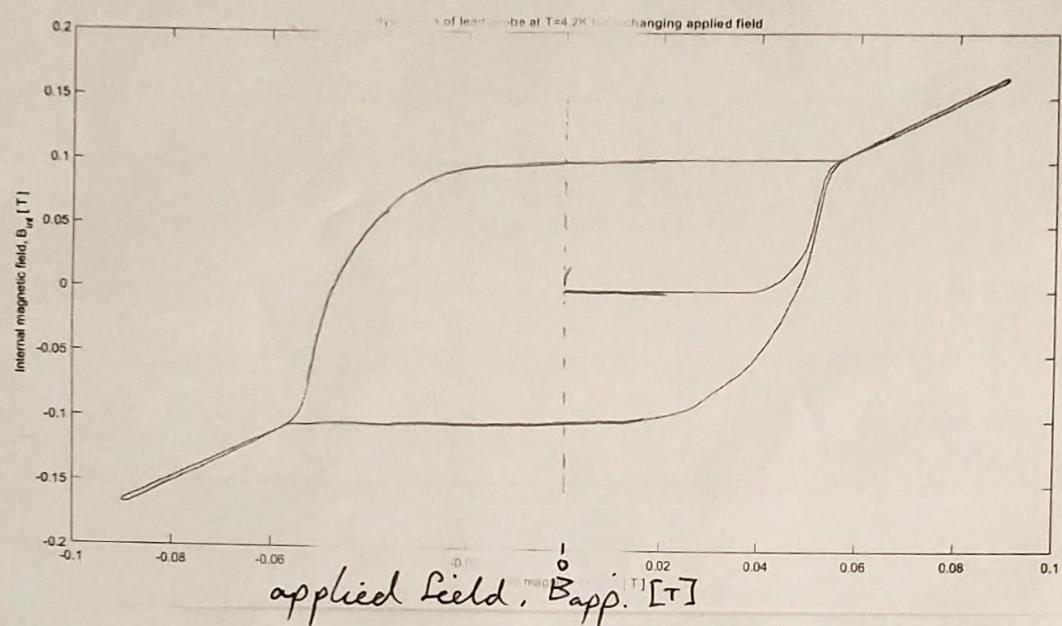
- Taking measurements of the voltage through the search coil, we can convert them into magnetic flux densities again by

$$B_{\text{air}} = \frac{1}{nA} V_{\text{coil (integrated)}},$$

where nA is given in the datalite as
 $nA = 0.075501$ [presumably m^2].

Plotting this against the applied field (calculated from the shunt voltage) we got a hysteresis curve that is shown overleaf.

Hysteresis of lead probe at $T=4.2K$ for a changing applied field:



As we can see, this greatly differs from previously observed curves. This is because we are taking the measurement of flux within a cavity/hole in the lead probe. As this isn't part of the superconductor, we don't see the total expulsion of fields.

Initially, that is what we see, but upon decreasing B beyond B_c , the currents within the superconductor's inner surface become persistent, remaining roughly constant even as the applied field is reduced. This is why the magnetic field we observe in the hole (generated by these persistent currents) remains roughly constant across the superconducting regime. This process occurs again for negative fields too, and the process of creating and destroying the coil's magnetic field creates a hysteresis loop.

In the absence of applied field the magnetic flux density measured in the core is

$$B_{\text{core}} = 0.099 \pm \cancel{0.02} 0.005 \text{ T}$$

Treating the inner surface of the core as a solenoid, and using Ampere's law, we can say that

$$lB = \mu_0 I_{\text{persistent}}$$

$$I_{\text{persistent}} = \frac{31.5 \times 10^{-3} \text{ m} \times 0.099 \text{ T}}{4\pi \times 10^{-7} \text{ H m}^{-1}}$$

$$I_{\text{persistent}} = 2481.6 \pm 125 \text{ A}$$

This would conventionally be a very large current, however the material is in its superconducting phase so a large current is to be expected due to the negligible resistance.

This can be related to the current density by considering the thickness & the layer in which the persistent current is present. This is given as $t_{\text{sheath}} = 5 \times 10^{-8} \text{ m}$

$$\begin{aligned} \therefore V_{\text{current region}} &= l \times \left(\frac{\frac{2}{\cancel{\pi}} \pi (d_{\text{core}} + 2t_{\text{sheath}})^2 - \pi d_{\text{core}}^2}{4} \right) \\ &= 4.948 \times 10^{-11} \text{ m}^3 (4 \text{ sl}) \end{aligned}$$

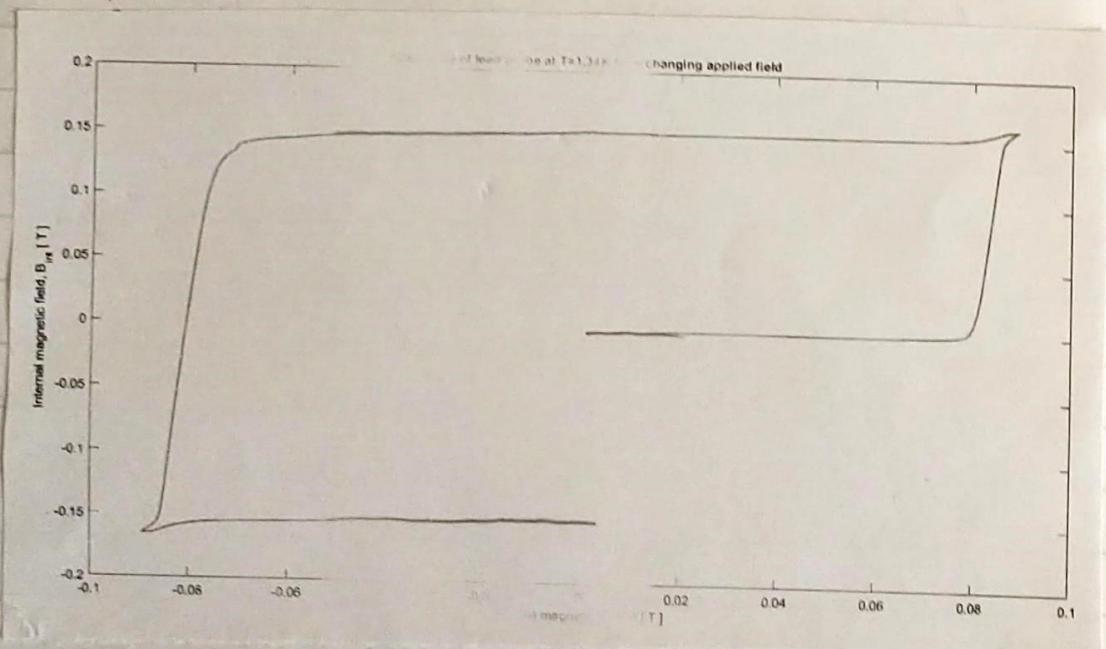
\therefore current density $J = \frac{I}{V_{\text{current region}}}$

$$J = 5.015 \times 10^{15} \pm 0.253 \times 10^{13} \text{ A m}^{-3}$$

\therefore electron number density : $n = 3.13 \times 10^{32} \pm 0.16 \times 10^{32} \text{ m}^{-3} \text{s}^{-1}$

- This electron number density rate is very high, implying the conducting electrons are very mobile and fast, which would support the idea they aren't scattering.

Hysteresis of lead probe at $T = 1.34 \text{ K}$ for a changing applied field :



- For this plot, the core was heated to 10K, the magnetic field was reduced to zero, then the sample was cooled to 1.34K.

-Residual field, $B_r^{(2)} = 0.152 \pm 0.002$ T.

-Critical field, $B_c^{(2)} = 0.079 \pm 0.003$ T.

Again, the persistent current can be found as

$$I_{\text{persistent}}^{(2)} = \frac{l B_r^{(2)}}{\mu_0}$$

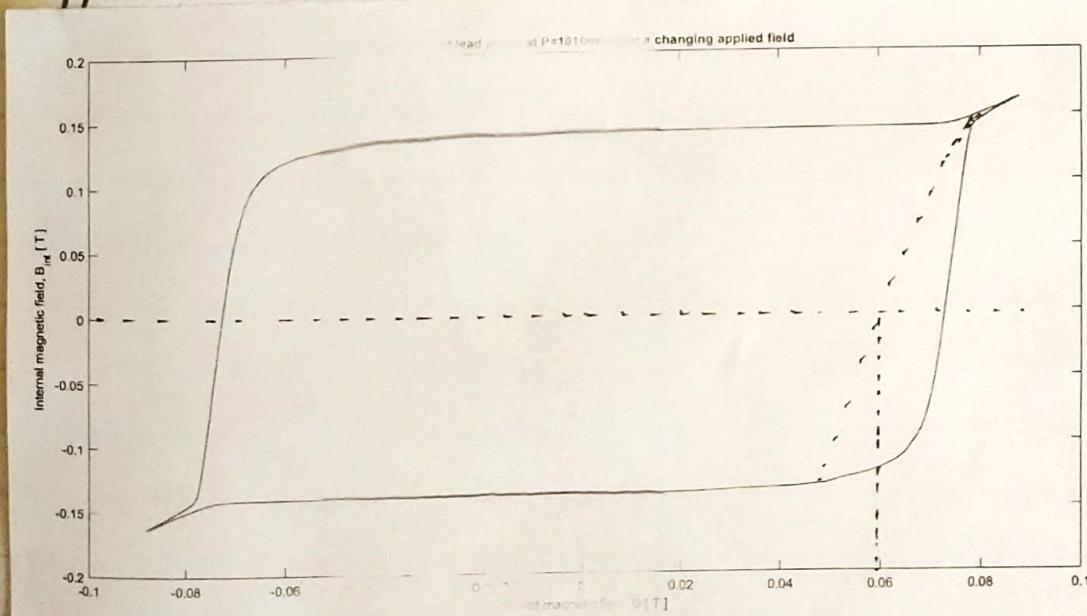
$$I_{\text{persistent}}^{(2)} = 3810 \pm 50 \text{ A}.$$

-Returning to our first hysteresis loop ($T = 4.2\text{K}$), we have

$$B_r^{(1)} = 0.099 \pm 0.005 \text{ T},$$

$$B_c^{(1)} = 0.044 \pm 0.005 \text{ T}.$$

Hysteresis of lead probe at $P = 1010\text{mbar}$ for a changing applied field:



The cone was repressured with warm helium gas from above the hysteresis loop.