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$\sigma(x) = \frac{1}{1+\exp(-x)}$ se debe probar que se cumple

$$\frac{\partial}{\partial x}\sigma(x) = \sigma(x)(1-\sigma(x))$$

$$\Rightarrow \frac{\partial}{\partial x}\sigma(x) = \frac{\partial}{\partial x}\frac{1}{1+\exp(-x)} = \frac{\partial}{\partial x}(1+\exp(-x))^{-1}$$

$$\frac{\partial}{\partial x}(1+\exp(x))^{-1} = -[1+\exp(x)]^{-2}\cdot[-\exp(-x)]$$

$$-[1+\exp(-x)]^{-2}[-\exp(-x)] = \frac{\exp(-x)}{1+\exp(-x)} = \frac{1}{1+\exp(-x)}\cdot\left[\frac{\exp(-x)}{1+\exp(-x)}\right]$$

Por fracciones parciales

$$\Rightarrow \frac{1}{1+\exp(-x)}\cdot\left[\frac{1+\exp(-x)}{1+\exp(-x)} - \frac{1}{1+\exp(-x)}\right] = \frac{1}{1+\exp(-x)}\cdot\left[1 - \frac{1}{1+\exp(-\ldots}\right.$$

Se puede reescribir la ecuación de la sig forma

$$\frac{1}{1+\exp(-x)}\cdot\left[1-\frac{1}{1+\exp(-x)}\right] = \sigma(x)[1-\sigma(x)]$$