ETH zürich
Departement Informatik

Theoretische Informatik

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`https://courses.ite.inf.ethz.ch/theoInf20`

Exercises – Sheet 7

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Exercise 17

Let w_i be the i-th word over Σ_{bool} with respect to the canonical order and let M_i be the i-th Turing machine in the canonical order. We consider the languages

$$L_1 = \{w \in \{0,1\}^* \mid w = w_{3i+1} \text{ for some } i \in \mathbb{N} \text{ and } M_i \text{ does not accept } w\}$$

and

$$L_2 = \{w \in \{0,1\}^* \mid w = w_i \text{ for some } i \in \mathbb{N} \text{ and } M_{3i+1} \text{ does not accept } w\}.$$

Prove that one of the two languages L_1 and L_2 is not recursively enumerable and explain why an analogous proof does not work for the other language. **10 points**

Exercise 18

For any infinite language $L \subseteq \{0,1\}^*$, describe how a not recursively enumerable subset of L can be obtained and justify your approach. **10 points**

Exercise 19

A frog is jumping along the integer number line. It starts at the point $u \in \mathbb{Z}$ and jumps over the same distance $s \in \mathbb{Z} - \{0\}$ in the same direction (a negative distance denotes a jump to the left) every night, while it stays on the number it has reached and sleeps during the day.

You want to catch the frog, but know neither the direction nor the speed of the frog nor the number at which it starts in the first evening. To catch the frog, you cannot just position yourself somewhere and wait until the frog arrives—after all, you cannot see it at night, moreover, it could be just jumping in the other direction and never come by at your position. Instead, you must choose a number every day and check if the frog is located there. If it sleeps at that position, you can just catch it.

What strategy allows you to choose the visited numbers in a way that guarantees you to catch the frog in finite time? **10 points**

Submission: Friday, November 6, by 11:15 at the latest, as a clearly legible PDF via e-mail directly to the respective teaching assistant.