ETH zürich
Departement Informatik

Theoretische Informatik

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Exercises – Sheet 5

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Exercise 13

Using the pumping lemma for regular languages, show that the following languages are not regular.

(a) $L_1 = \{wbw^{\mathsf{R}} \mid w \in \{a,b\}^*\}$, where w^{R} denotes the reversal of w.

Remark: For any word $a = a_1a_2 \dots a_n$, where $a_i \in \Sigma$ for $i \in \{1,2,\dots,n\}$, the reversal of w is the word $w^{\mathsf{R}} = w_n w_{n-1} \dots w_1$. (Definition 2.4 in the German textbook.)

(b) $L_2 = \{w \in \{0,1\}^* \mid |w|_0 \neq |w|_1\}$.

10 points

Exercise 14

Using the Kolmogorov complexity argument, show that the following languages are not regular.

(a) $L_1 = \{0^{\binom{2n}{n}} \mid n \in \mathbb{N}\}$ over the alphabet $\Sigma = \{0\}$,

(b) $L_2 = \{ww \mid w \in \{0,1\}^*\}$.

10 points

(please turn over)

Bonus Exercise 2

The pumping lemma for regular languages has been stated in the lecture as follows: For every regular language L over an alphabet Σ, there exists a constant $n_0 \in \mathbb{N}$ such that every word $w \in \Sigma^*$ with $|w| \geq n_0$ can be expressed as $w = yxz$, where

(i) $|yx| \leq n_0$,

(ii) $|x| \geq 1$, and

(iii) either $\{yx^kz \mid k \in \mathbb{N}\} \subseteq L$ or $\{yx^kz \mid k \in \mathbb{N}\} \cap L = \emptyset$.

In the literature, one often finds a weaker version of the pumping lemma which does not consider all words $w \in \Sigma^*$ with $|w| \geq n_0$, but only the words $w \in L$ with $|w| \geq n_0$, and which uses the following conditions instead of (i) and (iii):

(i′) $|x| \leq n_0$

and

(iii′) $\{yx^kz \mid k \in \mathbb{N}\} \subseteq L$,

respectively.

(a) Show that there exists a nonregular language L that satisfies the conditions (i′), (ii), and (iii′) of the weaker pumping lemma. (For such a language, the weaker pumping lemma cannot thus be used to show its nonregularity.)

(b) One can also state a less weakened pumping lemma using the conditions (i), (ii), and (iii′). Show that there exists a nonregular language L' that satisfies these three conditions.

10 bonus points

Submission: Friday, October 23, by 11:15 at the latest, either into the boxes in room CAB F 17.1 or as a clearly legible PDF via e-mail directly to the respective teaching assistant.