ETH zürich
Departement Informatik

Theoretische Informatik

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https://courses.ite.inf.ethz.ch/theoInf20

Exercises – Sheet 2

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Exercise 4

(a) Let $w_n = (101)^{4^{3n^2}} \in \{0,1\}^*$, for all $n \in \mathbb{N} - \{0\}$. Provide a best possible upper bound on the Kolmogorov complexity of w_n, expressed in terms of the length of w_n.

(b) Provide an infinite sequence of natural numbers $y_1 < y_2 < y_3 < \cdots$ such that there exists a constant $c \in \mathbb{N}$ satisfying

$$K(y_i) \leq \log_2 \log_3 \log_2(y_i) + c\,,$$

for all $i \geq 1$.

10 points

Exercise 5

Prove that, for all $n \in \mathbb{N}$ and $i < n$, there are at least $2^n - 2^{n-i}$ natural numbers x in the interval $[2^n, 2^{n+1} - 1]$ such that $K(x) \geq n - i$. **10 points**

Exercise 6

We consider the language

$$L = \{1^i 0^j 1^k \mid i + j = 2k \text{ and } i, j, k \in \mathbb{N}, k \geq 1\}\,.$$

Let x_n be the n-th word in L with respect to the canonical order. Prove that there exists a constant $c \in \mathbb{N}$ such that, for all $n \in \mathbb{N}$,

$$K(x_n) \leq 2 \cdot \log_2(|x_n|) + c\,.$$

10 points

Submission: Friday, October 2, by 11:15 at the latest, either into the boxes in room CAB F 17.1 or as a clearly legible PDF via e-mail directly to the respective teaching assistant.