Departement Informatik

Theoretische Informatik

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https://courses.ite.inf.ethz.ch/theoInf20

Exercises – Sheet 3

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Exercise 7

Prove that there exist at most finitely many random prime numbers.
Hint: You can use the prime number theorem in your proof. **10 points**

Exercise 8

Design a finite automaton for each of the following languages (using a graphic representation) and determine the class $\mathrm{Kl}[q]$ for every state q of your automaton.

(a) $L_1 = \{\, w \in \{a,b\}^* \mid (2 \cdot |w|_a - |w|_b + 1) \bmod 5 \in \{2,4\} \,\}$,

(b) $L_2 = \{\, aaxb \mid x \in \{a,b\}^* \text{ and } x \text{ contains } bb \text{ as a subword} \,\}$.

10 points

Exercise 9

Design a finite automaton for each of the following languages (using a graphic representation) and provide a short informal justification of their correctness.

(a) $L_1 = \{\, x^k \mid x \in \{ab, bb\},\ k \in \mathbb{N} \,\}$,

(b) $L_2 = \{\, w \in \{0,1\}^* \mid |w| \text{ is even and } w = \mathrm{Bin}(n) \text{ for some } n \in \mathbb{N} \text{ divisible by } 3 \,\}$.

10 points

Bonus Exercise 1

Use the Kolmogorov-complexity method to prove that there exists $d \in \mathbb{N} - \{0\}$ such that

$$\mathrm{Prim}(k) \geq \frac{k}{d \cdot \log_2 k \cdot (\log_2 \log_2 k)^{5/4}}\,,$$

for infinitely many natural numbers $k \in \mathbb{N}$. Provide a concrete value of d. **10 bonus points**

Submission: Friday, October 9, by 11:15 at the latest, either into the boxes in room CAB F 17.1 or as a clearly legible PDF via e-mail directly to the respective teaching assistant.