

Exercises – Sheet 8

Zürich, November 6, 2020

Exercise 20

- (a) Justify informally the correctness of the following statement:

Let L be a language over an alphabet Σ . If $L \in \mathcal{L}_{\text{RE}}$ and $L^c \in \mathcal{L}_{\text{RE}}$, then $L \in \mathcal{L}_{\text{R}}$.

- (b) We consider the language

$$L_{U,\lambda} = \{\text{Kod}(M) \mid \lambda \in L(M)\}.$$

Prove that $(L_{U,\lambda})^c \notin \mathcal{L}_{\text{RE}}$.

10 points

Exercise 21

Prove the following statements by providing an explicit reduction and showing its correctness.

(a) $L_{\text{H}} \leq_{\text{R}} (L_{\text{diag}})^c,$

(b) $(L_{\text{diag}})^c \leq_{\text{R}} L_{\text{H}}.$

10 points

Exercise 22

Prove that the language

$$L_{\text{reach}} = \{\text{Kod}(M)\#0^i \mid i \in \mathbb{N}, M \text{ has at least } i+1 \text{ states and there exists a word } x \text{ such that the } i\text{-th state of } M \text{ is reached at least once during the computation of } M \text{ on } x\}$$

is not recursive by providing an explicit many-one-reduction and proving its correctness (see Def. 5.24 in the textbook).

10 points

Submission: Friday, November 13, by 11:15 at the latest, as a clearly legible PDF via e-mail directly to the respective teaching assistant.