ETH zürich
Departement Informatik

Theoretische Informatik

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https://courses.ite.inf.ethz.ch/theoInf20

Exercises – Sheet 9

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Exercise 23

Prove that $(L_{\text{empty}})^{\complement} \leq_{\text{m}} (L_{\text{diag}})^{\complement}$. **10 points**

Exercise 24

Let M be a 1-tape Turing machine that always halts. Prove that there exists an equivalent 2-tape Turing machine A such that, for some constant c and for all n:

$$\text{Time}_A(n) \leq \frac{\text{Time}_M(n)}{2} + \frac{13n}{12} + c.$$

Hint: The 2-tape TM A can simulate 12 cells of the input or working tape of M in a single cell. **10 points**

Exercise 25

We consider a 1-tape Turing machine that always halts and whose working alphabet Γ contains exactly $k+1$ symbols for some $k \geq 2$. We identify these symbols with the numbers in $\{0, 1, 2, \ldots, k\}$, where $0 = ¢$ and $k = ␣$. The given 1-tape-TM meets the property that the content of its working tape is always sorted in ascending order and contains exactly one symbol 0, i.e., it has the form $01^{i_1}2^{i_2}\ldots(k-1)^{i_{k-1}}kkk\ldots$ for some $i_1, \ldots, i_{k-1} \in \mathbb{N}$ at any moment. It further holds that $\text{Space}_M(n) = s(n)$ for some arbitrary function $s(n)$. Prove that $\text{Time}_M(n) \in \mathcal{O}(n \cdot (s(n))^k)$. **10 points**

Submission: Friday, November 20, by 11:15 at the latest, as a clearly legible PDF via e-mail directly to the respective teaching assistant.