ETH zürich
Departement Informatik

Theoretische Informatik

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Exercises – Sheet 10

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Exercise 26

Let $f\colon \mathbb{N} \to \mathbb{N}$ be a monotonically increasing function such that $f(n) \geq n$ for all $n \in \mathbb{N}$.

(a) Prove that NTIME(f) is closed under union.

(b) Let $L \in \text{NTIME}(f)$ and $L' \in \text{TIME}(f)$. Prove that $L - L' \in \text{NTIME}(f)$.

10 points

Exercise 27

(a) Let M be a nondeterministic MTM with $\text{Time}_M(n) \in O(n^2)$ that uses $O(n)$ space during every computation on any word of length n. Prove that $L(M) \in \text{SPACE}(n \log n)$.

(b) Let f be a space-constructible function and let $k \in \mathbb{N}$. Either prove that

$$\text{NSPACE}(f(n)) \cap \text{NTIME}(f(n)^k) \subseteq \text{SPACE}(f(n) \log(f(n)))$$

or explain the difficulty in proving this statement.

10 points

Exercise 28

Let $L \in \text{VP}$ and let A be a polynomial-time verifier for L. Assume that, for every word $w \in L$, there exists a witness x such that $|x| \leq \log_2 |w|$ and A accepts the input (w, x). Prove that $L \in \text{P}$. **10 points**

Submission: Friday, November 27, by 11:15 at the latest, as a clearly legible PDF via e-mail directly to the respective teaching assistant.