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MATHEMATICAL MODELLING AND COMPUTER SIMULATIONS AS AN AID TO GEARBOX DIAGNOSTICS

WALTER BARTELMUS

Wroclaw University of Technology, Pl. Teatralny 2, 50-051 Wroclaw, Poland

The paper deals with mathematical modelling and computer simulation as a tool for aiding gearbox diagnostic inference. The results of computer simulations and results obtained by laboratory rigs and field practice are compared. For investigation by computer simulations different factors are taken into consideration. The factors are divided into four groups: design factors, production technology factors, operational factors and change of condition factors. Using computer simulation and taking these factors into account: design, production technology, operation and condition change factors lead to DPTOCC inferring diagnostic information of the gearing system condition. The model for a system with a one-stage gearbox with torsional vibration is presented here as also are models for systems with a one-stage gearbox where both torsional and lateral vibration are taken into consideration and a system with a two-stage gearbox.

1. INTRODUCTION

The modelling of gearbox dynamic behaviour belongs to the fundamental problems of mechanical system modelling. The problem has been the subject of many papers [1–11]. The model presented by Muller in [7] has received a lot of attention in Poland. It was discussed, for example, in [8–11]. The model is a two-parameter (stiffness and damping) one in which the inertia of two wheels has been reduced to one mass equivalent to a one-stage gearbox.

The author of the present paper has found that more sophisticated models are needed to describe the gearbox dynamics properly [12–22] as presented in papers [1–5]. Mathematical modelling and computer simulation can be applied to gearbox dynamic examinations to support diagnostic signal evaluation for diagnostic inference. This is the main aim of the presented research. The computer simulation is based on a mathematical model presented in [12] and [19–21]. General information, on gearing, needed for the computer simulation of gearbox behaviour is given in [19] and [20]. Some results of computer simulations supporting diagnostic inference were presented in [12–22]. The papers show that mathematical modelling and computer simulation enable the detailed investigation of the dynamic properties of a gearing system. All the basic factors such as: design, production technology, operation and change of the gearing system condition, which have a bearing on the vibration generated by a gearset, can be investigated. Using computer simulation and taking these factors into account: design, production technology, operation and condition change factors lead to DPTOCC inferring diagnostic information of the gearing system condition. The causes of vibration in gearboxes are mainly tooth errors, which together with a gearing deflection show the gearing condition and the vibration is an indication of them. The computer simulation results are referred to the laboratory rig investigation results presented in [23] and to the field measurements reported in [24–27]. As mentioned above, the vibration of a gearbox indicates whether there are tooth errors in it. The errors appear at the production stage and during change of condition. The nature of the gear wheel interaction is such that non-linear phenomena occur caused by friction, inter-tooth backlash, impact-like inter-tooth forces and periodic changes in tooth stiffness. As a result, inter-tooth forces may exceed the force values, which follow from the gearbox system's rated moment. Mathematical description allowing one to include these phenomena in the equation of motion is given in [19–20]. The inter-tooth forces increase dramatically in unstable conditions. A one-stage gear system operates in resonance conditions and is unstable when the gearbox system's mesh frequency is equal to its natural frequency. In such conditions the inter-tooth forces are two times or more bigger than the rated forces. The phenomenon of resonance has not been investigated fully for gearbox systems but some considerations are given in [13, 19, 22].

Computer simulations reveal that conditions similar to those occurring at resonance may result as errors (pitting, scuffing of teeth flanks and failure of bearings) increase during the service of a gearbox system. In the present paper, current developments in gearbox modelling are presented with reference to the previous papers by the author. It is shown that a flexible coupling and an error mode random parameter have an influence on gearbox stability (tooth separation). An error mode is described by several parameters, i.e. maximum error value, shape of error plot and random error fluctuation depth.

2. MODELLING OF GEARBOX SYSTEM

Muller's [7] one-stage gearbox model is shown in Fig. 1. It is a two-parameter (stiffness and damping) model. The inertia of the two gear wheels is reduced to one mass. The motion of the mass is equivalent to the relative motion of the two gear wheels. The motion is caused by the relative motion of springs (having different length) in contact with the mass. The motion of the springs with velocity v(m/s) is equivalent to the pitch-line velocity of the wheels. As one can see in the model shown in Fig. 1, the motion of the mass has no influence on the instantaneous change of v as in actual gearboxes. This weakness of the Muller (1979) model, and no possibility of building multistage gearbox models, made the author seek a new model. It is more convenient to use a model with the rotary motion of the wheels and torsional vibration, and thus overcome the weakness of the Muller model. The torsional vibration is also given in models in [1–4] and in [6]. The simplest model of this kind is shown in Fig. 2. This kind of model is considered in [6]. The rotational equations of motion may be expressed by statements according to Newton's second law:

$$I_{1p}\ddot{\varphi}_1 = M_1 - r_1(F + F_t) + M_{zt1} \tag{1}$$

$$I_{p2}\ddot{\varphi}_2 = r_2(F + F_t) - M_2 - M_{zt2}.$$

To get closer to reality, a more sophisticated model is considered. The model is shown in Fig. 3. The mathematical model of the system's torsional vibration is given by the following equations:

$$I_s\ddot{\varphi}_1 = M_s(\dot{\varphi}_1) - (M_1 + M_{1t}), \qquad I_{1p}\ddot{\varphi}_2 = M_1 + M_{1t} - r_1(F + F_t) + M_{zt1}$$

$$I_{2p}\ddot{\varphi}_3 = r_2(F + F_t) - M_2 - M_{zt2}, \qquad I_m\ddot{\varphi}_4 = M_2 - M_r. \tag{2}$$

The values of forces and moments are given by

$$M_1 = k_1(\varphi_1 - \varphi_2), \quad M_2 = k_2(\varphi_3 - \varphi_4), \quad M_{1t} = C_1(\dot{\varphi}_1 - \dot{\varphi}_2). \tag{3}$$

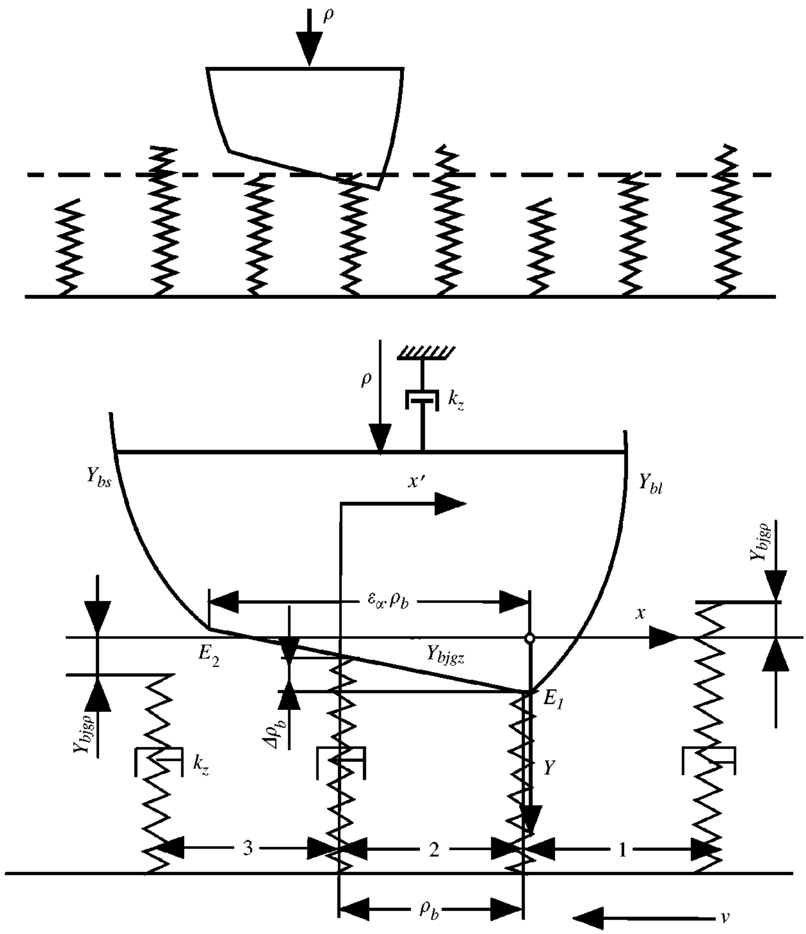

Figure 1. One-mass, two-parameter model of gearbox by Muller (1979).

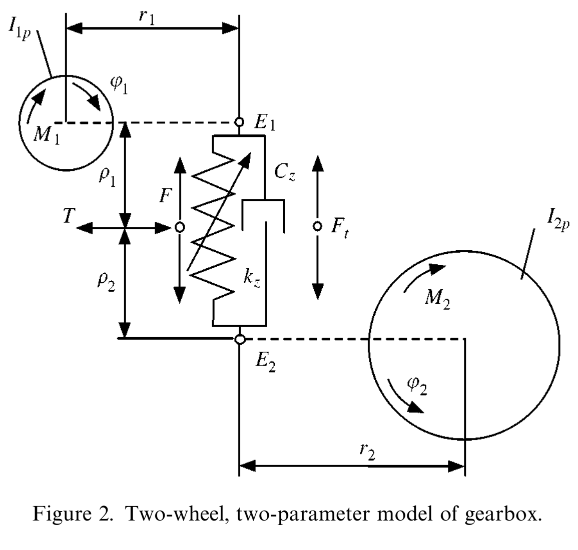

Figure 2. Two-wheel, two-parameter model of gearbox.

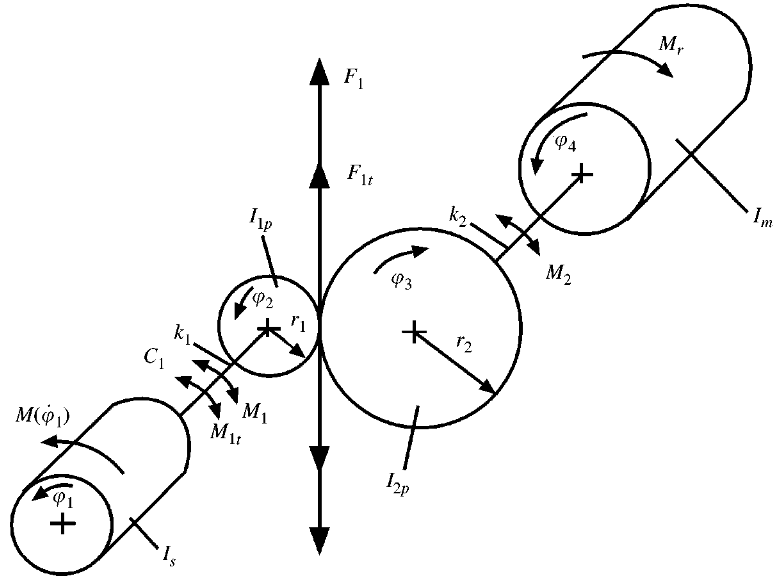

Figure 3. System with one-stage gearbox with: $M_s(\dot{\varphi})$—electric motor driven moment characteristic; M_1, M_2—moments of shaft stiffness; I_s, I_m—moments of inertia for electric motor and driven machine; M_{1t}—damping moment of clutch/coupling; C_1—damping coefficient of coupling; F, F_t—stiffness and damping inter-tooth forces; k_1, k_2—stiffness of shafts.

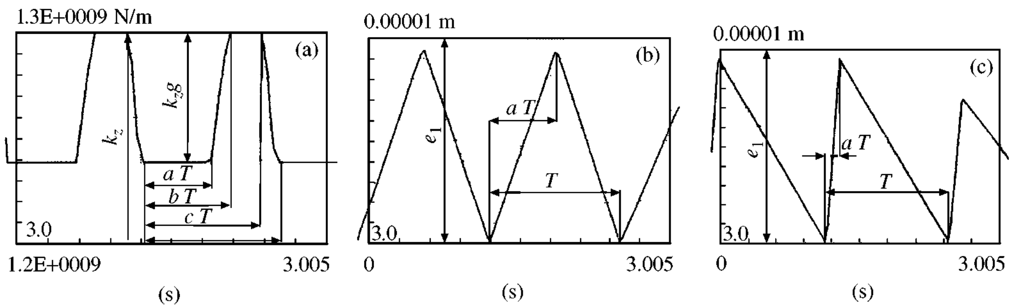

Figure 4. (a) Stiffness function for gearing. (b) Error function for error mode $E(0.5, 10, 0.3)$. (c) Error function for error mode $E(0.1, 10, 0.3)$.

Assuming defining functions: $\min(a, b)$ if $a < b$ then $\min = a$ if not $\min = b$, $\max(a, b)$ if $a > b$ then $\max = a$ if not $\max = b$, one may write a statement for the inter-tooth force

$$F = k_z(p_{om}, g)(\max(r_1\varphi_2 - r_2\varphi_3 - l_u + E(p_{om},a,e,r), \min(r_1\varphi_2 - r_2\varphi_3 + l_u + E(p_{om},a,e,r),0))). \tag{4}$$

The expression in equation (4) should be interpreted as if the expressions are given in Pascals: the first value of $\min(a, b)$ is counted as the next $\max(a, b)$, according to the rules given above.

$$F_t = C_z(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_3) \tag{5}$$

where φ, $\dot{\varphi}$, $\ddot{\varphi}$ are rotation angle, angular velocity, angular acceleration; $M_s(\dot{\varphi})$ the electric motor driving moment characteristic; M_1, M_2 the moments transmitted by shaft stiffness; I_s,

I_m the moments of inertia for electric motor and driven machine; M_{1t} the damping moment of clutch/coupling; C_1 the damping coefficient of coupling; F, F_t the stiffness and damping inter-tooth forces; k_1, k_2 the stiffness of shafts; M_{zt1}, M_{zt2} the inter-tooth moments of friction, $M_{zt1} = T\rho_1$; $M_{zt2} = T\rho_2$ where T is the inter-tooth friction force (Fig. 2), where $k_z(p_{om}, g)$ is the gearing stiffness function, obtained according to considerations given in [7] and [28]; (recently, FE models [5, 11] have been developed for assessing gearing stiffness and the related static transmission error [5]) r_1, r_2 the gear base radii; l_u the inter-tooth backlash; $E(p_{om}, a, e, r)$ the error mode function, where $p_{om} = frac(\varphi_2 z_1/(2\pi))$; a, e, r are the parameters of error function; z_1 the number of teeth in pinion. A full description of the model is given in [12] and after modification—in [19, 20]. For a given pair of teeth the value of an error is random and it can be denoted by

$$e(\text{random}) = [1 - r(1 - l_i)]e \tag{6}$$

where e is the maximum error value; r the coefficient of error scope, range (0–1); l_i the random value, range (0–1).

A symbolic description of an error characteristic (error mode) is $E(a; e; r)$; parameters of the error mode have been described earlier above; a - is given in Fig. 4(b) and (c), it has range (0–1), and it indicates the position of the maximum error value on the line of action. As an example, an error mode for $E(0.5; 10; 0.3)$ is given in Fig. 4(b) and for $E(0.1; 10; 0.3)$ in Fig. 4(c).

In Fig. 4 the value of the stiffness $k_z(p_{om}, g)$ and the value of the error function $E(a; e; r)$ are given as a function of time from 3.0 to 3.005 s and a time range is divided into ten equal parts as are the value of $k_z(p_{om}, g)$ for a range of 1.2–1.3E9 N/m and the value of $E(a; e; r)$ for the range 0–0.00001 m.

As follows from the above discussion, the gearing's dynamic properties depend on several factors. The factors may be divided, according to [20, 27], into: 1—design factors; 2—production technology factors; 3—operation factors; 4—change of condition factors, DPTOCC factors.

Numerical solutions of the differential equations are obtained by means of CSSP (Continuous System Simulation Program) given in [29], using the England procedure of integration. The procedure is a general procedure of the Runge–Kutta type and it assures the stability of integration, even in the case of discontinuity, as well as enabling error estimation and the automatic change of the integration step. CSSP is now available for Windows [30].

Some properties of gearbox condition vibration signals cannot be studied when using the system when only torsion vibration is taken into consideration. Change of gearing condition caused by increased friction between teeth has to be investigated when besides torsion vibration lateral vibrations are also considered. As it is given in paper [16] inter-tooth forces are transmitted through the bearings to the housing of a gearbox. These forces act along the length of an action line E_1E_2, Fig. 2. Friction forces T are perpendicular to the line of action, see Fig. 2. The change of inter-tooth friction forces may be caused by tooth scuffing. The increased intensity of teeth scuffing causes the increase of a friction coefficient. The computer simulation studies have to be done to find vibration signal properties for supporting diagnostic inference, when tooth scuffing is the problem. There is also a need to study the gearbox systems with multiple stages. The model of the system with gearbox with multiple stages will be presented with the model taking into consideration torsion and lateral vibration. The influence of torsion vibration between stages will be investigated. In papers [1–4, 6] the clutch damping C_1 (N sm), a very important factor, is not taken into consideration.

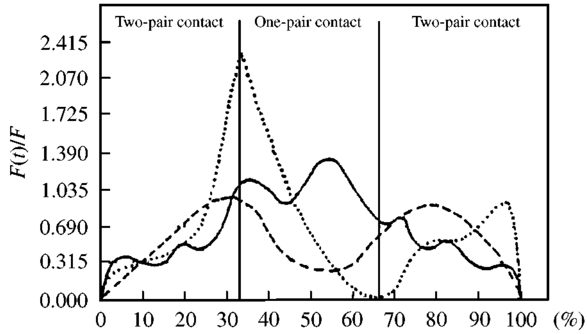

Figure 5. Inter-tooth force measurements at over resonance, under resonance and resonance, $F(t)$—current inter-tooth force value, F—constant, rated inter-tooth force value, according to [23]: – – –, over resonances; ——, under resonsnce; · · · · ·, resonance.

3. COMPARISON OF RESULTS OF COMPUTER SIMULATION TO RESULTS OF MEASUREMENTS ON REAL GEARBOXES

The results of computer simulations were compared with the results obtained from laboratory tests [23] and field tests [24–27]. The results obtained by Rettig [23] are shown in Fig. 5.

A dynamic factor is presented as ratio $K_d = F(t)/F$, in % of length, along the line of action, where $F(t)$ is the current inter-tooth force, F the constant, rated inter-tooth force. A gear system may run under resonance, at resonance and over resonance. The gear system operates at resonance when the gearing's meshing frequency is equal to its natural frequency. The computer simulation results presented in [13] are shown in Figs 6–8. Figure 6 shows a result of computer simulations for the under-resonance operation of the gearing. One can easily notice similarities (cf. Figs 5 and 6) between the measurements and the computer simulation results for the under-resonance run. Figure 7 shows a result of computer simulations for a case when the gearing runs at resonance. One can notice there a distinct peak for one meshing period and the same is observed in the case of the measurements. Figures 5–8 show the influence of an operational factor, i.e. the rotation of the gearbox, on the signal generated by the gearing. The meshing of the gearing depends also on the design and on a change in the condition due to tooth errors. Figure 8 shows a result of computer simulations for the gearing operating in unstable conditions due to tooth errors, [13]. In Fig. 8 the period of meshing is denoted by T (0.0–100%). T_n stands for a natural vibration period.

Figure 9(a) and (b) shows plots of K_d *vs* error e(μm) and error mode, $E(a, e, r)$. The error modes are denoted by symbols a, b, c: a for $E(0.1, 10, 0)$; b for $E(0.5, 10, 0)$; c for $E(0.5, -10, 0)$. The results are for $r = 0$ and for $C_1 = 0$, where r is a measure of error variation, C_1 is a damping coefficient of the clutch. One can see that K_d increases linearly to a certain value of e and above this value the condition is unstable. Similar results are presented in [26, 27] where a linear relationship for the acceleration signal *vs* gearing errors, determined on the basis of field measurements, can be found. The presented computer simulation results are compared with the results presented in [24, 25] where the diagnostic signal was obtained by synchronous summation. An example of a vibration signal, shown as a function of time t (s), is given in Fig. 10. The signal indicates a broken tooth in the gearing. The acceleration plots for the broken tooth, obtained by computer simulations, are shown in Fig. 11. The presented results of computer simulations can be considered as idealised results of synchronous summation.

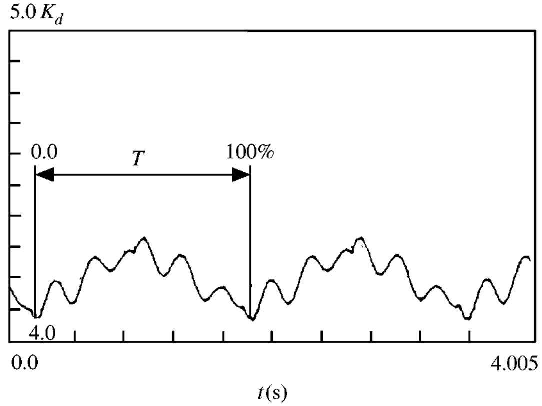

Figure 6. Function of K_d for under-resonance operation of gearing, obtained by computer simulation for error mode $E(0.5, 10, 0)$, according to [13].

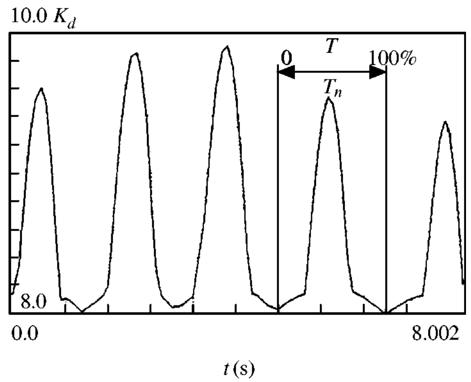

Figure 7. Function of K_d for resonance operation of gearing, obtained by computer simulation for error mode $E(0.5, 10, 0)$, according to [13].

For computer simulations results given in Fig. 9 were obtained for the error mode $E(a, e, r)$ where $r = 0$ and for $C_1 = 0$. The influence of random inter-tooth errors on the vibration generated by the gearbox system (Fig. 3) for different clutch damping coefficients: $C_1 = 0$ and $C_1 > 0$ is given in Fig. 12. The random inter-tooth error is described by the error mode $E(a, e, r)$, where e(random) is given by equation (6). The author's previous studies on gearing dynamics were for the error mode described by two parameters $E(a, e)$ and $C_1 = 0$. According to [13], Fig. 9 an increase in inter-tooth error e(μm) causes a linear increase of dynamic factor K_d, but only to a certain value of e. At this value a non-linear effect caused by unstable running (teeth separation) is observed. Dynamic factor K_d is defined as mentioned as a ratio of the current inter-tooth force to the rated inter-tooth force. The value of e at which unstable running (teeth separation effect) is observed depends also on the value of parameter 'a' in error mode $E(a, e)$. The unstable running of a gearbox occurs when $K_d > 2$. Also parameter r, which specifies the relative range of the inter-tooth force fluctuation, has a bearing on the instability. Figure 12 shows period (1) in which gear rotation increases from 0 to 107 rad/s. No external load is applied to the gearbox system; the gearbox system is loaded only by the system's inertia forces. In period (2) the gearbox system runs without any load. In period (3) the external load increases from 0 to the rated load. In period (4) the gearbox system runs under an external stable rated load. In the

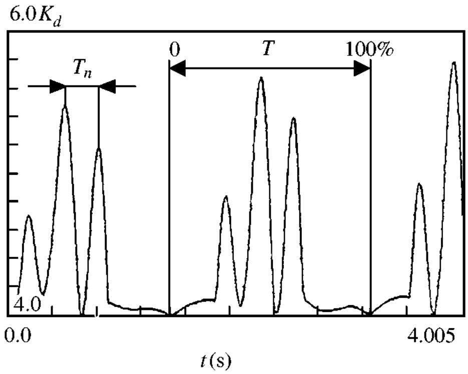

Figure 8. Function of K_d for unstable operation of gearing, obtained by computer simulation for error mode $E(0.5, 15, 0)$, according to [13].

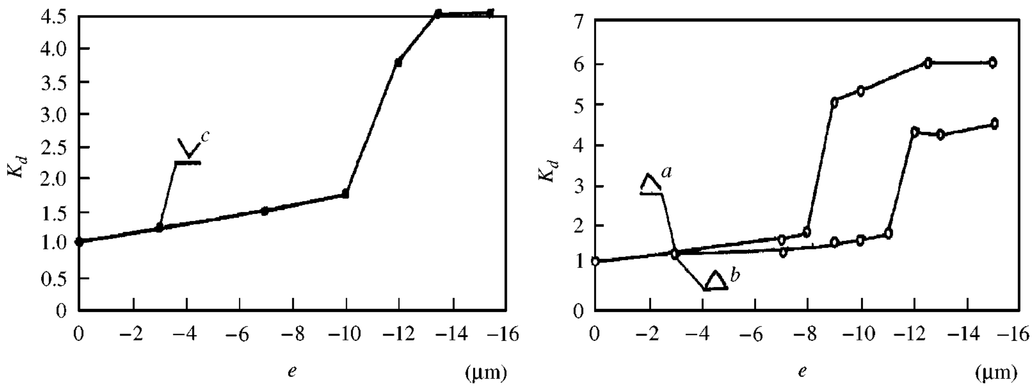

Figure 9. Coefficient K_d as a function of inter-tooth error and error mode: a—$E(0.1, e, 0)$, b—$E(0.5, e, 0)$, c—$E(0.5, -e, 0)$, according to [13].

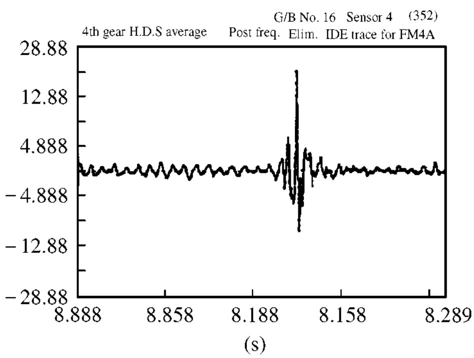

Figure 10. Diagnostic signal for broken tooth, obtained by synchronous summation of signals, according to [24].

present paper, period (4) of the gearbox system run is the focus of attention. In conditions when the error mode is given by $E(0.5, 12, 0.3)$ and $C_1 = 0$, the system shown in Fig. 3 runs stably [$K_d = 1.8$, Fig. 12(a)], for the (4)th period of a gearbox run. If the error mode is changed to $E(0.5, 15, 0.3)$ and $C_1 = 0$ the running of the gearbox system will be unstable, as

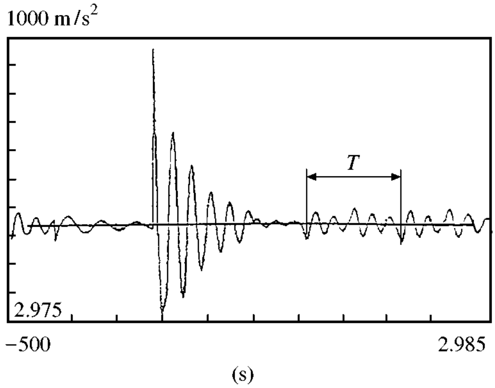

Figure 11. Zoom of acceleration for broken tooth, obtained by computer simulation, according to [14].

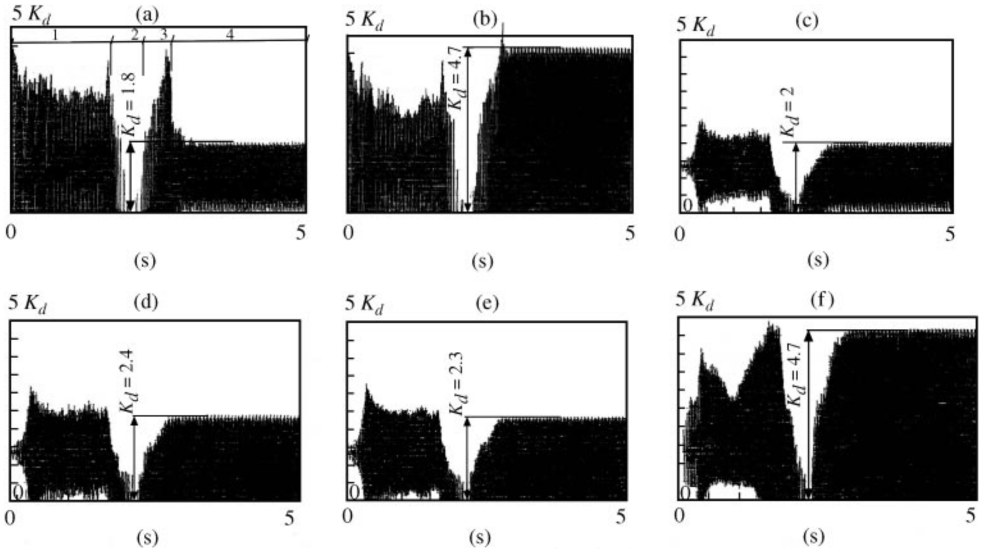

Figure 12. (a) Plot of K_d for error mode $E(0.5, 12, 0.3)$, stiffness coefficient $C_1 = 0$. (b) Plot of K_d for error mode $E(0.5, 15, 0.3)$, stiffness coefficient $C_1 = 0$. (c) Plot of K_d for error mode $E(0.5, 15, 0.3)$, stiffness coefficient $C_1 = 1000$. (d) Plot of K_d for error mode $E(0.5, 20, 0.3)$, stiffness coefficient $C_1 = 1000$. (e) Plot of K_d for error mode $E(0.5, 20, 0.15)$, stiffness coefficient $C_1 = 1000$. (f) Plot of K_d for error mode $E(0.1, 20, 0.3)$, stiffness coefficient $C_1 = 1000$ [19].

shown in Fig. 12(b), for a constant rated speed and a constant external load. Bearing in mind that the coupling damping coefficient has an influence on the gearbox system's dynamics, the plot of K_d for a run at $E(0.5, 15, 0.3)$ and $C_s = 1000$ N m s will be like that shown in Fig. 12(c). The plots for gearing conditions described by $E(0.5, 20, 0.3)$, $E(0.5, 20, 0.15)$ and $C_1 = 1000$ N m s are shown in Fig. 12(d) and (e). The gearbox system runs then with slight instability: $K_d = 2.4$ and 2.3. If parameter a is decreased from 0.5 to 0.1 at error mode $E(0.1, 20, 0.3)$ and $C_1 = 1000$ N m s, the plot of factor K_d is like that shown in Fig. 12(f), where dynamic factor $K_d = 4.7$. If parameter e is increased further, the value of the dynamic factor does not change. Comparing Fig. 12 to Fig. 9 we may conclude that under real conditions the influence of the errors on the diagnostic signal will be as shown in Fig. 12, not like in Fig. 9.

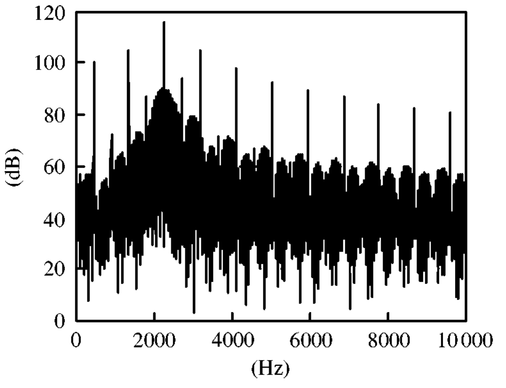

Figure 13. Spectrum of the signal for one tooth stiffness change $0.8k_z$.

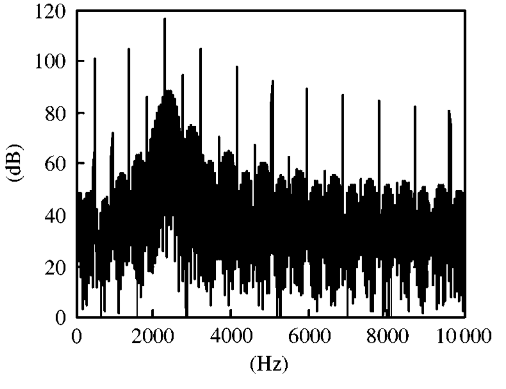

Figure 14. Spectrum of the signal for one tooth stiffness change $0.6k_z$.

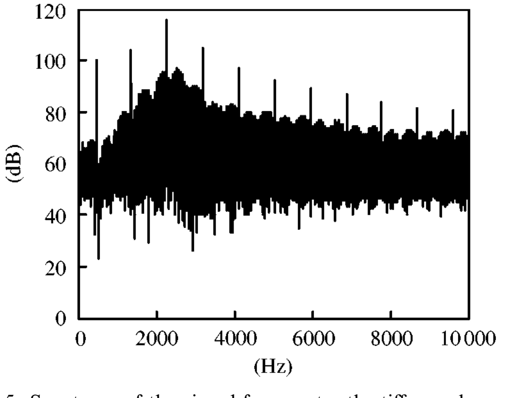

Figure 15. Spectrum of the signal for one tooth stiffness change $0.2k_z$.

One of the ways of vibration signal presentation is presenting it in the form of a spectrum as shown in Figs 13–16. These results were given in [18] and they show the change of the vibration spectrum with the change of one tooth stiffness. The change of stiffness simulates the increase of a tooth fracture.

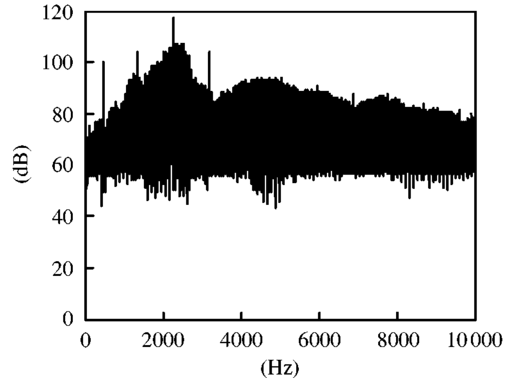

Figure 16. Spectrum of the signal for one tooth stiffness change $0.0k_z$.

Figures 13–16 show the increase of wideband noise of unidentified components whose intensity increases with increase of a tooth fracture from $0.8k_z$ to $0.0k_z$, where k_z is a maximum value of tooth stiffness. The investigations on the influence of the tooth fracture on the vibration spectrum using mathematical modelling and computer simulations are also given in [6] where similar results were obtained as in Figs 13–16.

Further investigations have shown that components, which look like a wideband noise consist of components between which the frequency spacings are equal to the two shaft speeds.

4. DEVELOPMENT OF MODELS WITH TORSIONAL AND LATERAL VIBRATIONS

As was mentioned in Section 1, there is a need for developing models with torsional and lateral vibration, and models of multistage gearboxes. This section gives an introduction to the problem.

If we use the model of a gearbox system given in Fig. 17, the equations of motion take the form

$$\begin{aligned} &I_s\ddot{\varphi}_1 = M(\dot{\varphi}_1) - M_1 - M_{1t}, && I_{1p}\ddot{\varphi}_2 = M_1 + M_{1t} - r_1(F + F_t) + M_{zt1}, \\ &m_{p1}\ddot{y}_3 = F + F_t - F_g - F_{gt}, && I_{2p}\ddot{\varphi}_4 - r_2(F + F_t) - M_2 - M_{zt2}, \\ &m_{p2}\ddot{y} = F + F_t - F_d - F_d - F_{dt} && I_m\ddot{\varphi}_6 = M_2 - M_r. \end{aligned} \tag{7}$$

y describes vertical motion and m the masses of wheels.

Moments and forces are given by the following set of statements:

$$\begin{aligned} &M_1 = k_1(\varphi_1 - \varphi_2), && M_{1t} = C_1(\dot{\varphi}_1 - \dot{\varphi}_2) \\ &F_1 = k_z(r_1\varphi_2 - r_2\varphi_4), && F_t = C_z(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_4) \\ &F_g = k_{yg}y_3, && F_{gt} = C_{yg}\dot{y}_3 \\ &F_d = k_{yd}y_5, && F_{dt} = C_{yd}y_5 \\ &M_2 = k_2(\varphi_4 - \varphi_6). && \end{aligned} \tag{8}$$

In the set of equations (7) and in the set of statements (8) the meaning of some values is the same as was given before. The meaning of the rest is given in Fig. 17. In Fig. 17 the directions of body motions are clearly defined.

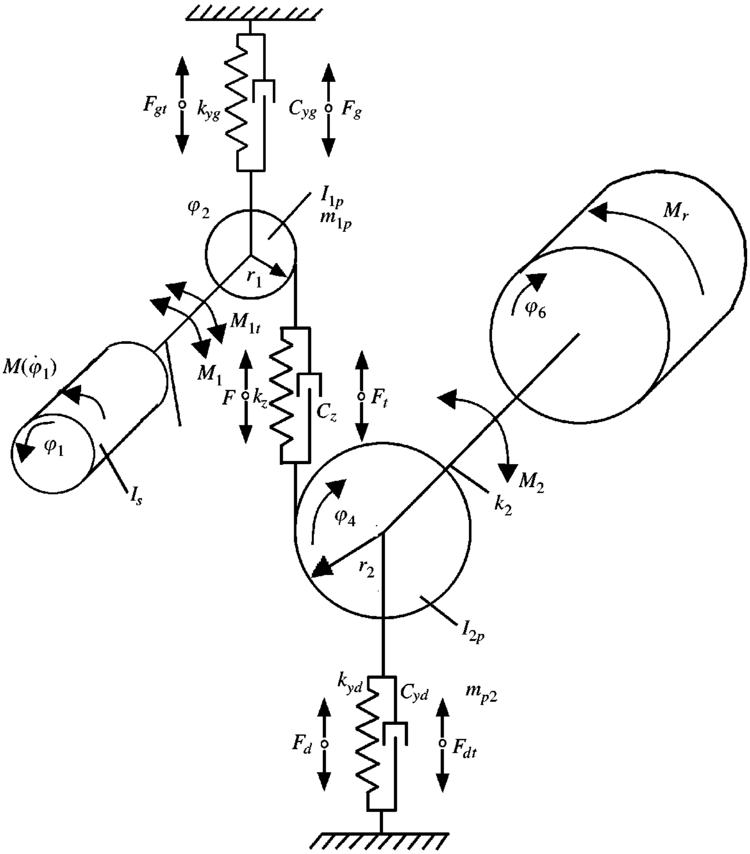

Figure 17. Gearbox system, with six degrees of freedom, driven by electric motor moment $M_s(\dot\varphi_1)$ and loaded with external moment M_r; system consists of: rotor inertia I_s, gear inertia I_{1p}, I_{2p}, gear masses m_{1p}; m_{2p}, driven machine inertia I_m, gearing stiffness k_z and damping C_z, gearing stiffness force F and damping force F_t, gearing friction force T_1, internal moments in first shaft M_1; M_{1t} (M_{1t} coupling damping moment), internal moment in second shaft M_2, internal stiffness and damping forces of upper and lower support (F_g, F_{gt}; F_d; F_{dt}), stiffness and damping parameters of upper and lower supports (k_{yg}, k_{yd}, C_{yg}, C_{yd}).

The gearbox system given in Fig. 17 may be used for investigating the influence of gear supports which are given by their parameters k_{yd}, k_{yg}, C_{yg}, C_{yd}, which are coefficients of stiffness and damping. To investigate the influence of inter-tooth friction, a more complicated model is needed in which lateral vibration may be investigated in two perpendicular directions; for these investigations the proper model is given in Fig. 18. The equations of motion for the system given in Fig. 18 are as follows:

$$\begin{aligned} &I_s\ddot\varphi_1 = M(\dot\varphi_1) - M_1 - M_{1t}, && I_{1p}\ddot\varphi_2 = M_1 + M_{1t} - r_1(F + F_t) + M_{zt1} \\ &m_{p1}\ddot y_3 = F + F_t - F_g - F_{gt}, && m_{p1}\ddot x_4 = T - F_1 - F_{1t} \\ &I_{2p}\ddot\varphi_5 = r_2(F + F_t) - M_2 - M_{zt2}, && m_{p2}\ddot y_6 = F + F_t - F_d - F_{dt} \\ &m_{p2}\ddot x_7 = T - F_p - F_{pt}, && I_m\ddot\varphi_8 = M_2 - M_r. \end{aligned} \tag{9}$$

x and y describe horizontal and vertical motion, m the masses of the wheels/gears. Values of moments and forces are given by the following set of statements:

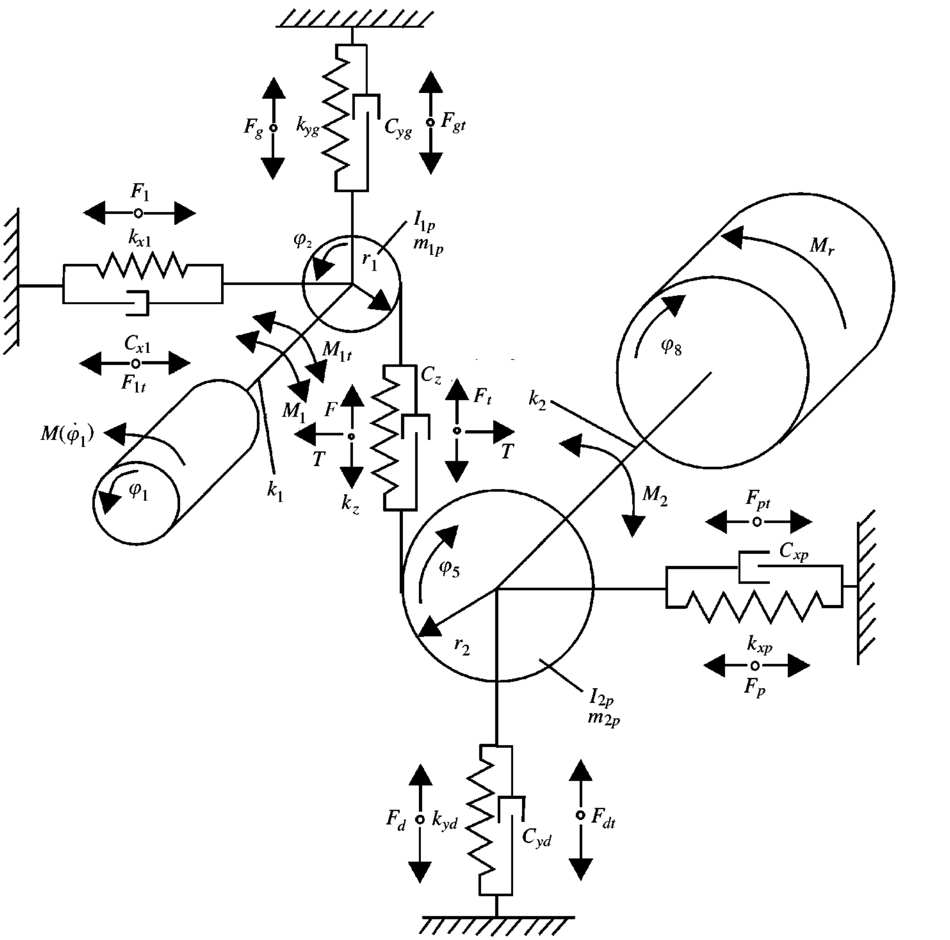

Figure 18. Gearbox system, with eight degrees of freedom, driven by electric motor moment $M_s(\dot{\varphi}_1)$ and loaded with external moment M_r; the system consists of: rotor inertia I_s, gear inertia I_{1p}, I_{2p}, gear masses m_{1p}; m_{2p}, driven machine inertia I_m, gearing stiffness k_z and damping C_z, gearing stiffness force F and damping force F_t, gearing friction force T, internal moments in first shaft M_1; M_{1t} (M_{1t} clutch damping moment), inner moment in second shaft M_2, internal stiffness and damping forces of upper and lower supports (F_g, F_{gt}, F_d, F_{dt}), stiffness and damping parameters of upper and lower supports (k_{yg}, k_{yd}, C_{yg}, C_{yd}) internal stiffness and damping forces of right and left supports (F_p, F_{pt}; F_1; F_{1t}), stiffness and damping parameters of upper and lower supports (k_{xp}, k_{x1}, C_{xp}, C_{x1}).

$$
\begin{aligned}
M_1 &= k_1(\varphi_1 - \varphi_2), & M_{1t} &= C_w(\dot{\varphi}_1 - \dot{\varphi}_2)\\
F_1 &= k_z(r_1\varphi_2 - r_2\varphi_4), & F_t &= C_z(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_5)\\
F_g &= k_{yg}y_3, & F_{gt} &= C_{yg}\dot{y}_3\\
F_d &= k_{yd}y_5, & F_{dt} &= C_{yd}\dot{y}_6\\
F_1 &= k_{x1}x_4, & F_{1t} &= C_{x1}\dot{x}_4\\
F_p &= k_{xp}x_7, & F_{pt} &= C_{xp}\dot{x}_7\\
M_2 &= k_2(\varphi_5 - \varphi_8). & &
\end{aligned}
\tag{10}
$$

Using the model given in Fig. 18 we may investigate the influence of inter-tooth friction force T on the diagnostic signal received from the bearing support positions. There is also a need to investigate systems with gears in which there are more than one stage. In Fig. 19

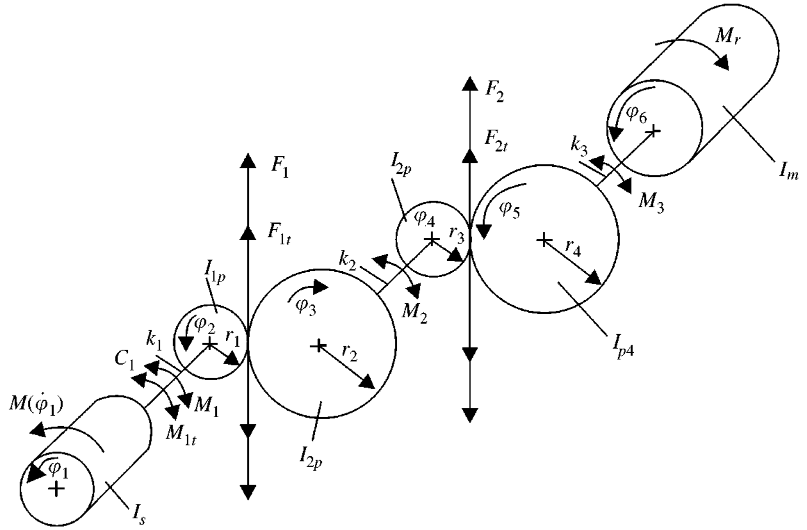

Figure 19. Two-stage gearing system with six torsional degrees of freedom, electric motor moment $M_s(\dot{\varphi}_1)$ and external load moment M_r; system consists of: rotor inertia I_s, gear inertia of first stage I_{1p}, I_{2p}, gear inertia of second stage I_{3p}, I_{4p}, driven machine inertia I_m, gearing stiffness k_z and damping C_z, gearing stiffness forces F_1, F_2, and damping force F_{1t}, F_{2t}, internal moments in first shaft M_1; M_{1t} (M_{1t} coupling damping moment), internal moment in second and third shafts M_2 and M_3.

a system is given with two gear stages. For this system only torsional vibration is considered. For the system given in Fig. 19 the equations of motion may be written in the following form:

$$\begin{aligned} &I_s\ddot{\varphi}_1 = M(\dot{\varphi}_1) - M_1 - M_{1t}, && I_{1p}\ddot{\varphi}_2 = M_1 + M_{1t} - r_1(F_1 + F_{1t}) + M_{zt11},\\ &I_{2p}\ddot{\varphi}_3 = r_2(F_1 + F_{1t}) - M_2 - M_{zt12}, && I_{3p}\ddot{\varphi}_4 = M_2 - r_3(F_2 + F_{2t}) + M_{zt21}\\ &I_{4p}\ddot{\varphi}_5 = r_4(F_2 + F_{2t}) - M_3 - M_{zt22}, && I_m\ddot{\varphi}_6 = M_3 - M_r. \end{aligned} \tag{11}$$

Moments and forces can be written in the following form:

$$\begin{aligned} &M_1 = k_1(\varphi_1 - \varphi_2), && M_{1t} = C_1(\dot{\varphi}_1 - \dot{\varphi}_2)\\ &F_1 = k_{z1}(r_1\varphi_2 - r_2\varphi_4), && F_{1t} = C_{z1}(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_5)\\ &M_2 = k_2(\varphi_3 - \varphi_4), && F_2 = k_{z2}(r_3\varphi_4 - r_4\varphi_5)\\ &F_{2t} = C_{z2}(r_3\dot{\varphi}_2 - r_4\dot{\varphi}_5), && M_3 = k_3(\varphi_5 - \varphi_6). \end{aligned} \tag{12}$$

5. IDENTIFICATION OF INFLUENCE OF FIRST-STAGE TORSION VIBRATION ON VIBRATION OF SECOND STAGE OF GEARBOX BY MODULATION COMPONENTS

A two-stage gearing system with six torsional degrees of freedom is given in Fig. 19. The equations of motion are given by the set of equations (11). To use these equations for computer simulation, inter-tooth forces given by expression (4) have to be included. The set of equations (11) is modified as given in Section 2. The result of computer simulation, given in the form of a spectrum, is presented in Fig. 20. In Fig. 20 mashing frequencies and their harmonic components for the first gear stage are denoted as f_{z1}; $2f_{z1}$; $3f_{z1}$ and for the second

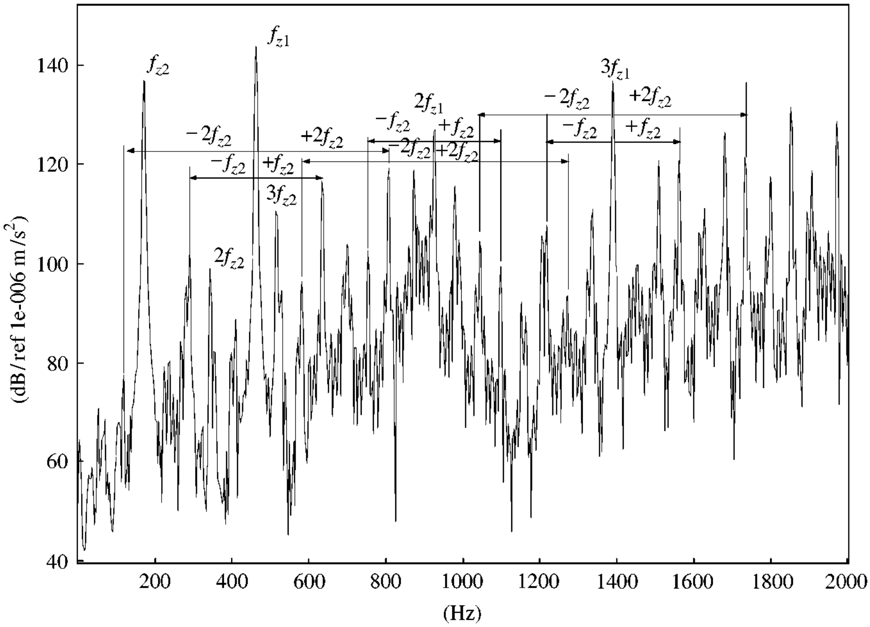

Figure 20. Vibration spectrum of the signal obtained by computer simulation using the model given in Fig. 19; f_{z1}; $2f_{z1}$; $3f_{z1}$—meshing frequency and its components for first stage of gearbox and for second stage: f_{z2}; $2f_{z2}$; $3f_{z2}$; modulation components: $-f_{z2}$; $+f_{z2}$ and $-2f_{z2}$; $+2f_{z2}$.

stage are denoted as f_{z2}; $2f_{z2}$; $3f_{z2}$. Modulation components are denoted as $-f_{z2}$; $+f_{z2}$ and $-2f_{z2}$; $+2f_{z2}$. Figure 20 shows that there is an influence on vibration between stages. This numerical experiment cannot be done using the model given in [7]. There are problems in formulating the equations of motion for a two-stage gearbox using the rules given in [7].

6. CONCLUSIONS

The paper is a review of the current possibility of using mathematical modelling and computer simulation for investigating the dynamic properties of gearbox systems. It is shown that there is no need to reduce the inertia of two wheels into one mass equivalent to a one-stage gearbox as suggested by Muller [7].

For investigations of the dynamic behaviour of a gearbox system many factors have to be taken into consideration. The factors are grouped into four groups: design, production technology, operation, change of condition.

Using computer simulation and keeping in mind the factors mentioned: design, production technology, operation and condition change factors lead to DPTOCC inference diagnostics of the gearing system condition.

The system with a one-stage gearbox may be considered in full; properties which may be gained from torsional vibration were investigated. More detailed description of these properties can be found in the cited literature. It ought to be stressed that the properties of a one-stage gearbox can only be properly treated if the gearbox is included in a system which consists of an electric motor, a coupling, and a driven machine, Fig. 3. It is not addressed in some papers, for example [6]. Taking into consideration an error mode given by symbolic description $E(a, e, r)$ it is seen in Fig. 9 that increasing or decreasing the 'e' value means that the tooth separation holds (starts) for different 'e' and is also a function of the *a* parameter. Figure 12 gives evidence that the *r* parameter, describing the range of random change of *e* value, also has an influence on the dynamic properties. Besides this dramatic influence, the clutch stiffness also has an influence. Comparing results of computer simulation to results of measurements on real gearboxes (see Figs 5, 6 and 7, 10 and further compare 10 to 11) one can come to the conclusion of their good consistency. It ought to be mentioned that in models given in [1–4] and [6] the coupling damping is not taken into consideration.

Summing up, the inter-tooth forces increase dramatically in unstable conditions. A one-stage gear system can operate under resonance conditions and is unstable when the gearbox system's mesh frequency is equal to its natural frequency. In such conditions the inter-tooth forces are two times or more greater than the rated forces. Computer simulations reveal that conditions similar to those occurring at resonance may result as errors (pitting, scuffing of tooth flanks and failure of bearings) increase during the service of a gearbox system. It is shown that a flexible coupling and an error mode random parameter have an influence on gearbox stability (tooth separation).

Figures 13–16 show a dramatic influence of tooth fracture on the vibration spectrum generated by gearing. This is also presented in [6].

Propositions for further models with lateral and torsional vibration are given. It is hoped that phenomena connected with inter-tooth friction will be investigated using the model with lateral vibration, Fig. 18. The model for a system with a two-stage gearbox, taking into consideration torsional vibration, is also given in Fig. 19. Using this model the influence of vibration from a second stage on the vibration of the first stage is given by showing modulation of the meshing components of the first stage.

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