A Unified Approach to the Assembly Condition of Epicyclic Gears

Article *in* Journal of Mechanical Design · September 1998

DOI: 10.1115/1.2829172

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A Unified Approach to the Assembly Condition of Epicyclic Gears

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The paper gives a general method for determining the assembly condition of epicyclic gears, irrespective of the structure or gear type. According to the method, an associated mechanism is considered, having a single arm carrier and a split planet. The assembly condition of the gear train is satisfied when obtaining identical superposition of the teeth of the two half-planets after rotating the arm to the next position. By writing simple kinematic relations between some partial transmitting ratios, general formulae are obtained which can be applied to specific epicyclic gear sets as functions of teeth numbers. The same approach allows the determination of the necessary angles between the arms of the carrier, or the offset angles between the wheels of the compound planets, as alternative solutions for the case in which equidistant assembly of identical planets is not possible.

Introduction

The research work on epicyclic gear train analysis and synthesis has concerned many authors in the past and consequently there exist a wealth of literature on this subject. However, the problem of designing an epicyclic gear set with two central wheels and a number of planets which are to be disposed about the orbit circle, has not been systematically investigated.

In their works, Love (1936) and Kojevnikov (1954) have proposed analytical relations for checking the possibility of equidistant assembling of epicyclic gear trains having a sun gear, an annulus and single wheel planets (Fig. 1(a)). If $\mathbf{z}_1$ and $\mathbf{z}_2$ are the number of teeth in the sun wheel and in the annulus, and $\mathbf{z}_3$ the number of teeth in the planet, with $\mathbf{n}$ the number of planets, the assembly condition for this epicyclic train was found to be:

$$(z_1 + z_2)/n = A \tag{1}$$

where **A** is an integer. One can observe that the procedure by which the above relation is obtained is less obvious for an odd number of teeth of the planet.

In the case of identical and equidistantly disposed compound planet gear in Fig. 1(b) with $\mathbf{z}_{31}$ and $\mathbf{z}_{32}$ the teeth number of the planets, the relation which must hold in order to make possible the gear assembly is according to Alexandru et al. (1984):

$$(z_1 \cdot z_{32} + z_2 \cdot z_{31})/n = A_1 \cdot z_{32} + A_2 \cdot z_{31} \tag{2}$$

with $\mathbf{A}_1$ and $\mathbf{A}_2$ integers.

Proposed Method

The purpose of this paper is to derive some general formulae on the basis of which the assembly condition of any epicyclic train can be obtained. For ease of presentation, the actual case of the gear train shown in Fig. 1(b) will be studied.

According to the procedure, an associated mechanism with a single arm carrier and the planet split into two separate wheels that can independently rotate is considered. In order for the two wheels $\mathbf{z}_{31}$ and $\mathbf{z}_{32}$ to have determinate motions when the arm **0** is turned, the sun and the annulus must be kept fixed to the frame of reference as shown in Fig. 2. Thus, from a reference position, by rotating the arm with an angle γ the wheels **31** and **32** will rotate relative to the arm and against each other with an angle $\Delta\varphi_3$. The angles by which wheels **31** and **32** have rotated relative to the arm are:

$$\varphi_{31} = \frac{\gamma}{i^0_{1-31}} \quad \text{and} \quad \varphi_{32} = \frac{\gamma}{i^0_{1-32}} \tag{3}$$

where i^0_{1-31} and i^0_{2-32} are the transmitting ratios between the sun wheel and the wheel **31**, and between the annulus and the wheel **32** respectively, when the arm **0** is fixed. By analogy with the "basic ratio", defined by Levai (1968) as the ratio of the angular-velocities relative to the arm of the two central gears, i^0_{1-31} and i^0_{2-32} will be nominated "partial basic ratios". The angle by which wheel **31** rotates against its pair wheel **32** can be expressed as:

$$\Delta\varphi_3 = \varphi_{31} - \varphi_{32} = \gamma\left(\frac{1}{i^0_{1-31}} - \frac{1}{i^0_{2-32}}\right). \tag{4}$$

Since $\Delta\varphi_3$ can exceed 2π radians, it is sufficient to operate with a reduced angle i.e.,

$$\delta\varphi_3 = 2\pi \cdot Frac\left(\frac{\gamma}{2\pi}\left|\frac{1}{i^0_{1-31}} - \frac{1}{i^0_{2-32}}\right|\right) \tag{5}$$

where $Frac(\ldots)$ is the fractional part of the expression in parentheses.

Without impairing the generality of the approach, the initial position of the wheels **31** and **32** can be chosen so that two teeth, each marked with a tick line, are aligned as shown in Fig. 3. The possible favorable relative disposition of the wheels **31** and **32** after the arm has been rotated with the angle γ, that lead to a similar superposition of no matter what two teeth are shown in Figs. 4 and 5. Beneath each of the four cases expressions of the corresponding angles $\delta\varphi_3$ in term of the angular pitches $\tau_{31} = 2\pi/z_{31}$ and $\tau_{32} = 2\pi/z_{32}$ of the wheels are given. Generalizing the above observations brings to an analytical formulation of the condition of assembling two identical planets at an angle γ:

$$|\delta\varphi_3| = |A_1 \cdot \tau_{31} \pm A_2 \cdot \tau_{32}| \tag{6}$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are integers. By analyzing numerous combinations of teeth numbers $\mathbf{z}_{31}$ and $\mathbf{z}_{32}$ the author has come to the

Contributed by the Power Transmission & Gearing Committee for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received Feb. 1997. Revised Apr. 1997. Associate Technical Editor C. Gosselin.

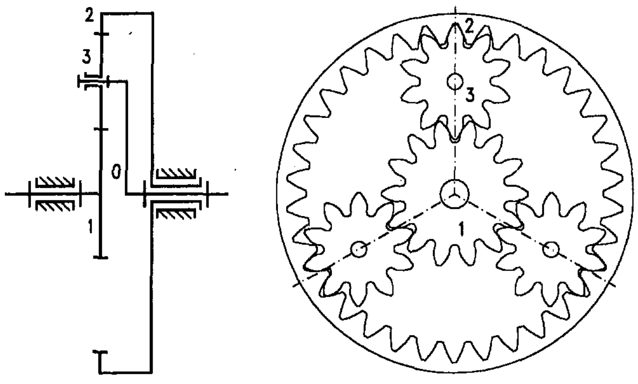

Fig. 1a Simple equidistantly disposed planet gear trains

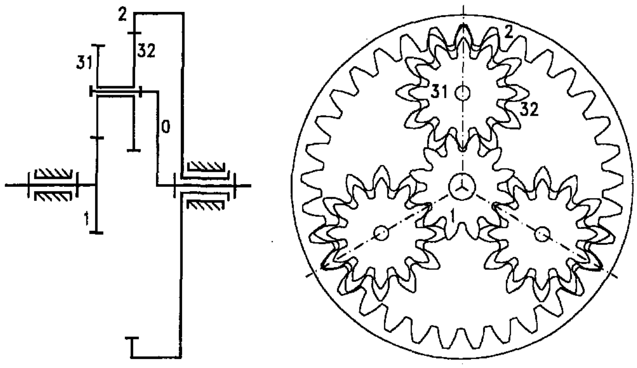

Fig. 1b Compound equidistantly disposed planet gear trains

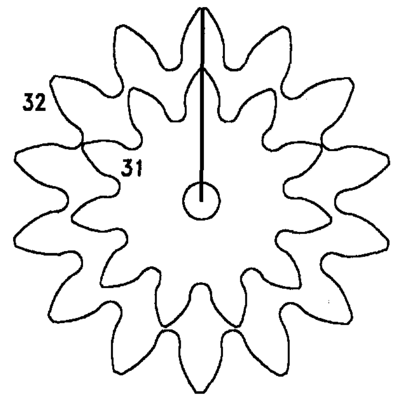

Fig. 3 The reference position of the wheels 31 and 32 of the planet

conclusion that for checking the assembly condition of a given epicyclic gear *it is sufficient to consider values of* $\mathbf{A}_1$ and $\mathbf{A}_2$ only in the domains:

$$0 \leq A_1 < z_{31}/D_3 \quad \text{and} \quad 0 \leq A_2 < z_{32}/D_3 \tag{7}$$

where $\mathbf{D}_3$ is the greatest common divider of $\mathbf{z}_{31}$ and $\mathbf{z}_{32}$. Inequalities (7) are useful for computer implemented algorithms designed to check the assembly condition of a given epicyclic train, particularly when numerous variants are tested bringing to a welcome gain in CPU time.

Equating $\delta\varphi_3$ in relations (5) and (6) and substituting the expressions of the angular pitches τ_{31} and τ_{32}, the condition of assembling the two identical planets in Fig. 2 at an angle γ is obtained:

$$Frac\left(\frac{\gamma}{2\pi}\left|\frac{1}{i^0_{1-31}} - \frac{1}{i^0_{2-32}}\right|\right) = \left|\frac{A_1}{z_{31}} \pm \frac{A_2}{z_{32}}\right|. \tag{8}$$

Because π is a nonrational number, it is obvious that relation (8) holds only if γ is of the form $\gamma = 2\pi \cdot B/B_0$ with B and B_0 integers. For the mechanism under consideration, i^0_{1-31} and i^0_{2-32} can be easily expressed as teeth number ratios i.e.,:

$$i^0_{1-31} = -z_{31}/z_1 \quad \text{and} \quad i^0_{2-32} = z_{32}/z_2. \tag{9}$$

By substituting the above expressions of the partial basic ratios into the general formula (8), and for $\gamma = 2\pi/n$ (which corresponds to **n** identical planets to be disposed equidistantly around the orbit circle) after simple algebraic manipulations the assembly condition becomes:

$$(z_1 \cdot z_{32} + z_2 \cdot z_{31})/n = |A_1 \cdot z_{32} \pm A_2 \cdot z_{31}|. \tag{10}$$

By not applying $Frac(..)$—case in which inequalities (7) no longer hold—a relation similar to the known one (2) has been obtained. The equivalence of these two relations still applies even though $\mathbf{A}_1$ and $\mathbf{A}_2$ in (2) can have both positive and negative values. This is less intuitive bearing in mind the classical procedure employed for determining the assembly condition of epicyclic gears.

A split annulus mechanism as shown in Fig. 6 can also be considered. In this case the sun wheel **1** and reference half-annulus **2′** are fixed. In the initial position, the teeth of the mobile half-annulus 2 and those of the fixed one, 2′, are considered exactly superimposed. By rotating the arm **0** with an angle $\gamma = 2\pi/n$, the half-annulus **2** will rotate relative to **2′** with the angle:

$$\varphi_2 = \frac{2\pi}{n}(1 - 1/i^0_{1-2}). \tag{11}$$

The above expression is obtained by considering $\omega_1 = 0$ in Willis's general formula (see Levai, 1968):

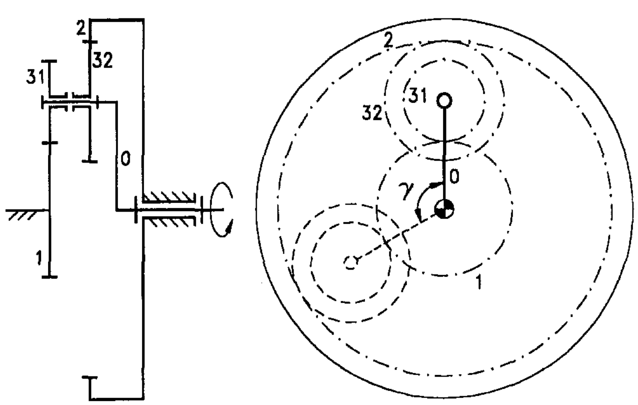

Fig. 2 Single arm carrier associated mechanism with the planet split into 2 separate wheels

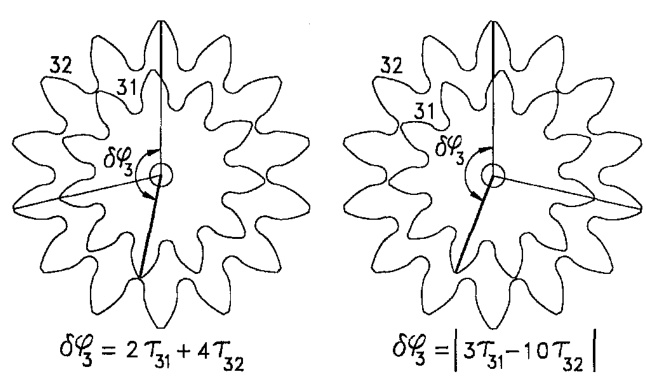

Fig. 4 The relative angle of the wheels 31 and 32 resulting as combination of both τ_{31} and τ_{32}

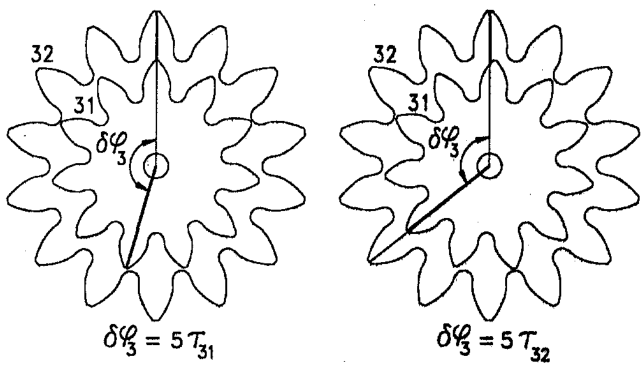

Fig. 5 The relative angle of the wheels 31 and 32 resulting as function of τ_{31} or τ_{32} only

$$\frac{\omega_1 - \omega_0}{\omega_2 - \omega_0} = \frac{-\gamma}{\varphi_2 - \gamma} = i^0_{1-2} \tag{12}$$

where i^0_{1-2} is the basic ratio of the epicyclic train and ω_1 and ω_2 are the angular speeds of the sun gear and annulus respectively. For the mechanism under consideration this basic ratio can be further expressed in term of teeth numbers i.e.:

$$i^0_{1-2} = -\frac{z_{31}}{z_1} \cdot \frac{z_2}{z_{32}} \tag{13}$$

which, substituted into relation (11) yields:

$$\Delta\varphi_2 = \varphi_2 - 0 = \frac{2\pi}{n}\left(1 + \frac{z_1 \cdot z_{32}}{z_{31} \cdot z_2}\right). \tag{14}$$

The assembly condition reduces, in this case, to $\Delta\varphi_2 = A \cdot \tau_2$ and by substituting the angular pitch of the annuls $\tau_2 = 2\pi/z_2$ becomes:

$$(z_1 \cdot z_{32} + z_2 \cdot z_{31})/n = A \cdot z_{31}. \tag{15}$$

This is a particular case of the previous relation (10) obtained for $A_1 = 0$, which obviously confirms only part of the favorable teeth number combinations.

Similarly, if the sun wheel **1** would be considered split in an associated mechanism, the corresponding relation obtained would be:

$$(z_1 \cdot z_{32} + z_2 \cdot z_{31})/n = A \cdot z_{32} \tag{16}$$

which is also less general then the (10) given formula.

This brings us to an important conclusion, that in all cases where a compound planet exists, the method must be applied with the aid of an associated mechanism obtained by splitting this compound planet into two separate wheels.

Equidistant Assembling of Complex Epicyclic Gears

The assembly condition of the gear train in Fig. 7 in which **n** planets are equally spaced, can also be derived. An associated mechanism with the compound planet split into the component wheels will be first considered. From the general formula (8) results:

$$Frac\left(\frac{1}{n}\left|\left(-\frac{z_{31}}{z_1}\right)^{-1} - \left(-\frac{z_{32}}{z_2}\right)^{-1}\right|\right) = \left|\frac{A_1}{z_{31}} \pm \frac{A_2}{z_{32}}\right| \tag{17}$$

with $\mathbf{A}_1$ and $\mathbf{A}_2$ satisfying inequalities (9). After minor transformations relation (17) becomes:

$$Frac(|z_1 \cdot z_{32} - z_2 \cdot z_{31}|/n) = |A_1 \cdot z_{32} \pm A_2 \cdot z_{31}|. \tag{18}$$

Alternatively, if the split planet would be the simple one noted **4**, the general formula (8) becomes:

$$Frac\left(\frac{1}{n}\left|\left(\frac{z_{31}}{z_1} \cdot \frac{z_4}{z_{32}}\right)^{-1} - \left(\frac{z_4}{z_2}\right)^{-1}\right|\right) = \left|\frac{A_1}{z_4} \pm \frac{A_2}{z_4}\right| \tag{19}$$

and finally

$$Frac(|z_1 \cdot z_{32} - z_2 \cdot z_{31}|/n) = A \cdot z_{31} \tag{20}$$

which is a less general case obtained by considering $\mathbf{A}_1 = 0$ in the above relation (18).

An even more complex epicyclic gear train is shown in Fig. 8 (Levai, 1968). For the two compound planets disposed in series, two independent relations are deduced based on the same formula (8) as follows:

$$Frac\left(\frac{1}{n}\left|\left(\frac{z_{31}}{z_1}\right)^{-1} - \left(\frac{z_{42}}{z_2} \cdot \frac{z_{34}}{z_{43}}\right)^{-1}\right|\right) = \left|\frac{A_1}{z_{31}} \pm \frac{A_4}{z_{34}}\right| \tag{21}$$

for the compound planet **31–34**, and:

$$Frac\left(\frac{1}{n}\left|\left(-\frac{z_{31}}{z_1} \cdot \frac{z_{43}}{z_{34}}\right)^{-1} - \left(\frac{z_{42}}{z_2}\right)^{-1}\right|\right) = \left|\frac{A_3}{z_{43}} \pm \frac{A_2}{z_{42}}\right| \tag{22}$$

for the planet **42–43**. After simple algebraic manipulations these two relations become:

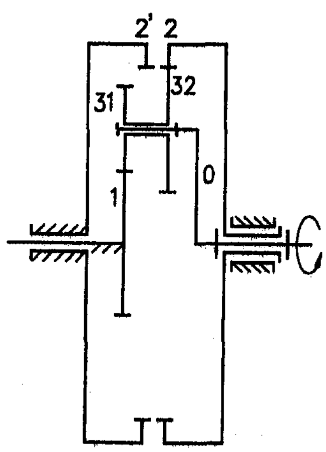

Fig. 6 Single arm carrier associated mechanism with split annulus

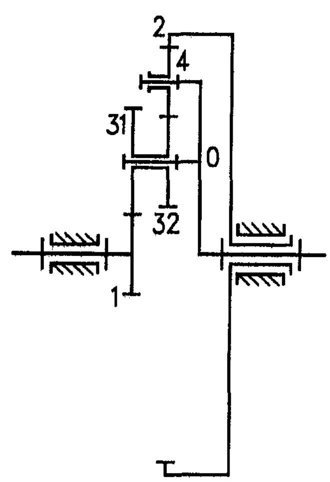

Fig. 7 Epicyclic gear train with two planets (simple and compound) disposed in series

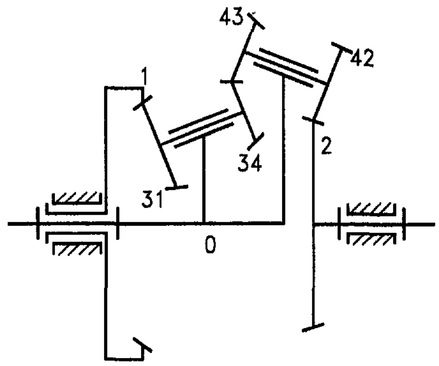

Fig. 8 Epicyclic gear train with two compound planets in series

$$Frac\left(\frac{|z_1 \cdot z_{42} \cdot z_{34} - z_2 \cdot z_{31} \cdot z_{43}|}{n \cdot z_{42}}\right) = |A_1 \cdot z_{34} \pm A_4 \cdot z_{31}|$$

$$Frac\left(\frac{|z_1 \cdot z_{42} \cdot z_{34} + z_2 \cdot z_{31} \cdot z_{43}|}{n \cdot z_{31}}\right) = |A_3 \cdot z_{42} \pm A_2 \cdot z_{43}| \quad (23)$$

and if at least one of them holds, the **n** pairs of planets can be assembled equally spaced.

Nonstandard Solutions to the Assembly Condition

There are cases in which equidistant assembling of **n** identical planets is not possible. For the simple epicyclic gear train in Fig. 1(*a*) the only solution is to alter the spacing of the planets, as in the case analyzed by Gradu and Langenbek (1995). The referred authors also have shown that the load distribution between the teeth of different planets remain no longer even. In order to diminish this phenomenon it becomes necessary the planet asymmetry to be kept to a minimum.

In case of epicyclic gears containing compound planets, it is possible to maintain the equal spacing of the arms of the carrier when relation (8) does not hold by using different planets with the component wheels solidly fixed at different angular offsets.

An example will be further considered for a gear train of the type in Fig. 1(*b*), with $\mathbf{z}_1 = 17$, $\mathbf{z}_2 = 40$, $\mathbf{z}_{31} = 12$, $\mathbf{z}_{32} = 15$ and $\mathbf{n} = 3$, for which the assembly condition (10) does not hold.

Relation (8) permits determining a proper set of angles γ_{12} (between the arms of the carrier supporting planets 1 and 2) and γ_{13} (between the arms supporting planets 1 and 3) such as the assembly of the gear train to be feasible. It has been mentioned in connection with relation (8) that any of the angles γ must be of the form $2\pi \cdot B/B_0$ with B and B_0 integers i.e,:

$$\gamma_{12} = 2\pi \cdot \frac{B_{12}}{B_0} \quad \text{and} \quad \gamma_{13} = 2\pi \cdot \frac{B_{13}}{B_0} \quad (24)$$

Imposing the planet disposition asymmetry to be minimum, implies than B_0 must be as large as possible, which from relations (8) and (9) results into:

$$B_0 = z_1 \cdot z_{32} + z_2 \cdot z_{31} = 17 \cdot 15 + 40 \cdot 12 = 735. \quad (25)$$

Further it is necessary that B_{12} must be the closest neighboring integer of B_0/n and $\mathbf{B}_{13}$ the closest neighboring integer of $2 \cdot B_0/n$ supplementary satisfying the equation:

$$Frac\left(\frac{B_{12}}{z_{31} \cdot z_{32}}\right) = \left|\frac{A_1}{z_{31}} \pm \frac{A_2}{z_{32}}\right| \quad \text{and}$$

$$Frac\left(\frac{B_{13}}{z_{31} \cdot z_{32}}\right) = \left|\frac{A_1}{z_{31}} \pm \frac{A_2}{z_{32}}\right| \quad (26)$$

The corresponding values of B_{12} and B_{13} satisfying the above conditions are:

$$B_{12} = Round(735/3) - 5 = 240 \quad \text{and}$$

$$B_{13} = Round(2 \cdot 735/3) - 10 = 480, \quad (27)$$

for which the corresponding values of the angles γ_{12} and γ_{13} are:

$$\gamma_{12} = 2\pi \cdot 240/735 = 117.551 \text{ deg} \quad \text{and}$$

$$\gamma_{13} = 2\pi \cdot 480/735 = 235.102 \text{ deg} \quad (28)$$

The remaining angle will be:

$$\gamma_{31} = 360 - 235.102 = 124.898 \text{ deg}. \quad (29)$$

When designing an epicyclic gear set with more than three planets, it might be useful to consider other further neighboring values when determining the integers B_{12}, B_{13} etc. such as the remaining angle to be the same close to $2\pi/n$.

Having for example, the complicated case of the mechanism in Fig. 1(*b*), it will be easy to solve a similar problem for the epicyclic gear in Fig. 1(*a*) with simple planets only.

For epicyclic gear trains containing compound planets which can not be equidistantly assembled, an alternative solution exist further detailed for the same numerical example. The solution is to manufacture non identical planets with the component wheels solidly fixed at different angular offsets. This solution has the advantage of not affecting the load distribution symmetry between the **n** planets, nor complicating the balancing of the planet-carrier ensemble.

Relation (5) can be used for calculating the necessary offset angles between the component wheels **31** and **32** of the planets. The 1st planet will be kept as reference with two teeth superimposed as in Fig. 3 (the corresponding offset angle is therefore $\delta\varphi_{3I} = 0$). The following 2nd and 3rd planets must be manufactured with the wheel **31** and **32** rotated relative to each other at the angles:

$$\delta\varphi_{3II} = 2\pi \cdot Frac\left(\frac{1}{3}\left|-\frac{z_1}{z_{31}} - \frac{z_2}{z_{32}}\right|\right)$$

$$= 2\pi \cdot Frac\left(\frac{1}{3} \cdot \frac{735}{12 \cdot 15}\right) = 130 \text{ deg} \quad (30)$$

and

$$\delta\varphi_{3III} = 2\pi \cdot Frac\left(\frac{2}{3}\left|-\frac{z_1}{z_{31}} - \frac{z_2}{z_{32}}\right|\right)$$

$$= 2\pi \cdot Frac\left(\frac{2}{3} \cdot \frac{735}{12 \cdot 15}\right) = 260 \text{ deg} \quad (31)$$

Since the profiles of the wheels **31** and **32** are periodical with period τ_{31} and τ_{32} respectively, the above angles $\delta\varphi_{3II}$ and $\delta\varphi_{3III}$ can be equivalent to much smaller angles. For the second planet, Fig. 9 illustrates how a calculated angle $\delta\varphi_{3II}$ (re-noted $\delta\varphi_3$) can be replaced with smaller angles. From the figure results:

$$\delta\varphi_{3\_1} = Frac(\delta\varphi_3/\tau_{31}) \cdot \tau_{31} = \delta\varphi_3 - 4 \cdot \tau_{31}$$

$$\delta\varphi_{3\_2} = Frac(\delta\varphi_3/\tau_{32}) \cdot \tau_{32} = \delta\varphi_3 - 5 \cdot \tau_{32}. \quad (32)$$

In searching for a more convenient angle $\delta\varphi_3$ the following relations can be considered as alternative to those in relation (32):

$$\delta\varphi_{3\_1'} = -[1 - Frac(\delta\varphi_3/\tau_{31})] \cdot \tau_{31} = \delta\varphi_3 - 5 \cdot \tau_{31}$$

$$\delta\varphi_{3\_2'} = -[1 - Frac(\delta\varphi_3/\tau_{32})] \cdot \tau_{32} = \delta\varphi_3 - 6 \cdot \tau_{32}. \quad (33)$$

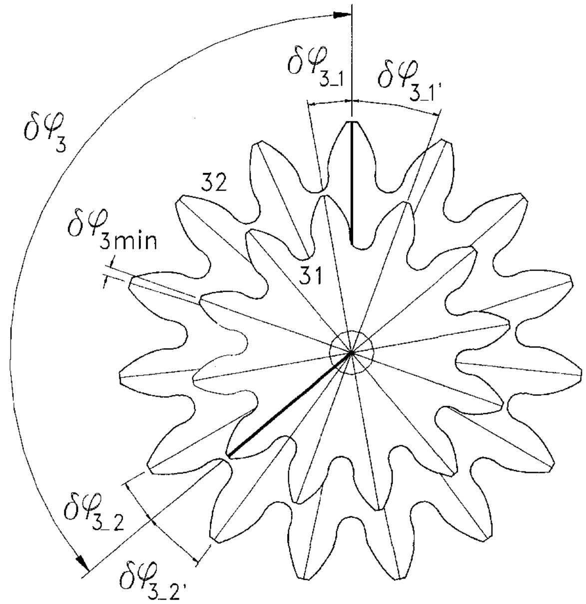

Fig. 9 Schematic for determining a smaller angle equivalent to $\delta\varphi_3$

In relations (32) and (33) the sign and therefore the sense of relative rotation of the component wheels **31** and **32** must be kept consistent in order to obtain appropriate results.

Figure 9 shows that the smallest offset angle $\delta\varphi_{3\min}$ can be determined as a combination of a an integer number of angular pitches τ_{31} and τ_{32} i.e,:

$$\delta\varphi_{3\min} = 3 \cdot \tau_{32} - 2 \cdot \tau_{31}. \tag{34}$$

Determining the minimum possible angle $\delta\varphi_{3\min}$ might be of some importance if the offset of the two component gears **31** and **32** is obtained by plastically torsioning identically manufactured planets.

Conclusions

General formulae for checking the assembly condition of epicyclic gear trains have been proposed employing associated mechanisms having a single arm carrier and the corresponding planet split into two separate wheels. Since these general formulae involve "partial basic ratios" of the epicyclic gear which can be further expressed as teeth-number ratios, the method can be applied irrespective of the type of gearing: internal or external of bevel or spur gears.

The same approach permits the determination of the assembly requirements for the cases in which the tooth numbers do not permit equally spaced assembling of identical manufactured planets. The solution in this case was to alter the equidistant disposition of the planets or (if it is the case) to use nonidentical compound planets with the wheels solidly fixed at different offset angles.

Acknowledgments

The comments and suggestions upon the manuscript of Robert C. Fraser from the Design Unit of the University of Newcastle upon Tyne are acknowledged. Thanks are also extended to Horia Bradau from "Transylvania" University of Brasov Romania for providing an AutoLISP program used in generating the involute profiles appearing in various figures throughout the paper.

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