Model of PMSM

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1 Introduction

Model of Permanent Magnet Synchronous Motor (PMSM) in Matlab/Simulink. Frequency converter is assumed ideal. Equations from [?]

2 Space Vectors

2.1 Three phase supply

In a *three phase supply*, the phase voltages are:

$$\begin{aligned} v_a(t) &= \hat{V}_a \cos(\omega t + \phi_a) \\ v_b(t) &= \hat{V}_b \cos(\omega t - \frac{2\pi}{3} + \phi_b) \\ v_c(t) &= \hat{V}_c \cos(\omega t - \frac{4\pi}{3} + \phi_c) \end{aligned} \tag{1}$$

During normal operation, $\hat{V}_a \simeq \hat{V}_b \simeq \hat{V}_c$ while ϕ_a, ϕ_b and ϕ_c are small.

2.2 Symmetric system:

For a *symmetric system*:

$$\begin{aligned} v_a(t) &= \hat{V} \cos(\omega t) \\ v_b(t) &= \hat{V} \cos\left(\omega t - \frac{2\pi}{3}\right) \\ v_b(t) &= \hat{V} \cos\left(\omega t - \frac{4\pi}{3}\right) \end{aligned} \tag{2}$$

We have a positive sequence.

Property of symmetric system:

$$v_a(t) + v_b(t) + v_c(t) = 0 \tag{3}$$

This removes one dof since one of the component always can be expressed in the other two.

2.3 Space vector

Therefore an equivalent two-phase system is possible, with 2 perpendicular axes denoted α and β:

$$\vec{v}_s(t) = v_\alpha(t) + jv_\beta(t) = v_a(t) + e^{j\frac{2\pi}{3}}v_b(t) + e^{j\frac{4\pi}{3}}v_c(t) \tag{4}$$

The phases are contributing in 3 different directions. The subscribe "s" indicates that the space vector is expressed in stator coordinates or the stator-fixed reference frame.

Property of space vector It is found that the space vector $\vec{v}_s(t)$ rotates with the mechanical angular frequency ω_{syn} henceforth called the synchronous speed and with the electrical angular speed $\omega_e = \omega = \frac{p}{2}\omega_{syn}$ equal to the supply speed:

$$\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} = \begin{bmatrix} \hat{V}\cos(\omega t + \phi) \\ \hat{V}\cos\left(\omega t - \frac{2\pi}{3} + \phi\right) \\ \hat{V}\cos\left(\omega t - \frac{4\pi}{3} + \phi\right) \end{bmatrix} \tag{5}$$

corresponds the the space vector:

$$\vec{v}_s(t) = \frac{3}{2}\hat{V}e^{j(\omega t + \phi)} \tag{6}$$

2.4 Transformation of Real-Valued Vectors

In case of digital implementation, it is necessary to use real-valued vectors denoted v_s and corresponding to $\vec{v}_s$:

$$\vec{v}_s = v_\alpha + jv_\beta \Leftrightarrow v_s = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \tag{7}$$

The three-phase / two-phase transformation (4) can be expressed using transformation matrice:

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}}_{T_{32}} \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} \tag{8}$$

Transforming instead from two phase to three phase:

$$\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{2}{3} & 0 \\ -\frac{1}{3} & \frac{1}{\sqrt{3}} \\ -\frac{1}{3} & -\frac{1}{\sqrt{3}} \end{bmatrix}}_{T_{32}} \begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} \tag{9}$$

3 Synchronous Machines

3.1 Synchronous Machines

Synchronous refers to the rotation of the rotor which is synchronous with the stator space vector.

Speed definitions when multipole machine:

$\omega = \omega_e$:	supply frequency and electrical speed
ω_{syn}:	synchronous speed
ω_r:	electrical rotor speed
ω_m:	fixed mechanical rotor speed

$$\omega_{syn} = \frac{\omega}{p/2} \tag{10}$$

$$\omega_r = \frac{p}{2}\omega_m \tag{11}$$

where p is the number of poles.

3.2 Dynamic Model for the PMSM

3.2.1 Flux and current

Considering the stator circuit, with resistance R_s of the stator winding equal in all three phases and $\vec{v}_s$ the stator voltage space vector in the stator coordinates:

$$\vec{v}_s - R_s\vec{i}_s - \frac{d\vec{\Psi}_s}{dt} = 0 \tag{12}$$

where $\vec{i}_s$ and $\vec{\Psi}_s$ are the space vectors for stator current and stator flux linkage, respectively. The stator flux is given by

$$\vec{\Psi}_s = L_s\vec{i}_s + \vec{\Psi}_R \tag{13}$$

where L_s is the stator inductance and $\vec{\Psi}_R$ is the rotor flux linkage produced by permanent magnets. The rotor flux axis is displaced by the flux angle θ relative to the α axis. Hence, the rotor flux vector is given by

$$\vec{\Psi}_R = \psi_R e^{j\theta} \tag{14}$$

The flux modulus ψ_R is constant, so for a assumed constant L_s,

$$\frac{d\vec{\Psi}_s}{dt} = L_s\frac{d\vec{i}_s}{dt} + j\omega_r\psi_R e^{j\theta} \tag{15}$$

where $\omega_r = \dot{\theta}$. Now, eq.12 can be written as

$$L_s\frac{d\vec{i}_s}{dt} = \vec{v}_s - R_s\vec{i}_s - j\omega_r\vec{\Psi}_R \tag{16}$$

$$\frac{d\vec{\Psi}_R}{dt} = j\omega_r\vec{\Psi}_R \tag{17}$$

This equation can be represented by the dynamic equivalent circuit depicted in fig.1

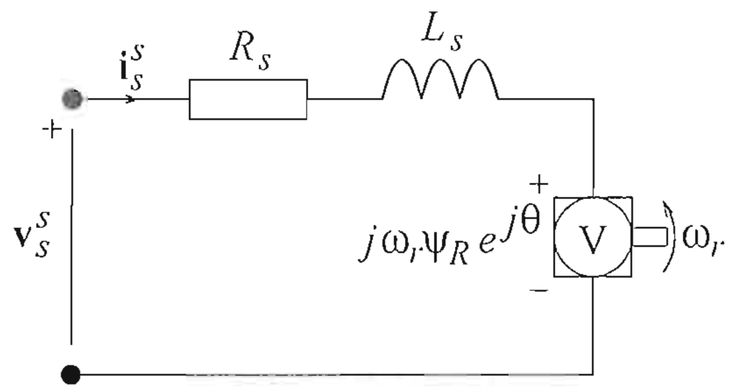

Figure 1: Dynamic equivalent circuit for the synchronous machine

3.2.2 Torque production

Active power is developed at two locations in the equivalent circuit (fig. 1): in the stator resistance and in the rotor emf. The former give copper losses, while the latter is converted to mechanical power. As shown in appendix A, the active power is:

$$P = \frac{2}{3}\Re\{\vec{v}_s\,(\vec{i}_s)^*\} \tag{18}$$

using eq.(16) the electric power P_e to the shaft is:

$$P = \frac{2}{3}\Re\left\{j\omega_r\vec{\Psi}_R\,(\vec{i}_s)^*\right\} = -\frac{2\omega_r}{3}\Im\left\{\vec{\Psi}_R\,(\vec{i}_s)^*\right\} = \frac{2\omega_r}{3}\Im\left\{(\vec{\Psi}_R)^*\,\vec{i}_s\right\} \tag{19}$$

Because power = torque * angular speed and $\omega_r = \frac{p}{2}\omega_m$, the electromagnetic torque is:

$$\tau_{em} = \frac{P_e}{\omega_m} = \frac{pP_e}{2\omega_r} = \frac{p}{3}\Im\left\{(\vec{\Psi}_R)^*\,\vec{i}_s\right\} \tag{20}$$

4 Field Oriented Control of PMSM

4.1 Synchronous Coordinates

4.1.1 Transformation to Synchronous Coordinates

The general definition of the dq transformation, and its reversal, the $\alpha\beta$ transformation, are given by:

$$\vec{v} = e^{-j\theta_e}\vec{v}_s \qquad (dq \text{ transformation}) \tag{21}$$

$$\vec{v}_s = e^{j\theta_e}\vec{v} \qquad (\alpha\beta \text{ transformation}) \tag{22}$$

where

$$\theta_e = \int \omega dt \tag{23}$$

At constant stator angular speed, the transformation removes the rotation of the vector, as observing the space vector from a coordinate system rotating with the stator frequency ω.

A space vector in synchronous coordinates (without any superscripts) has its components with subscripts d and q:

$$\vec{v} = v_d + jv_q \tag{24}$$

dq and $\alpha\beta$ transformations: The dq and $\alpha\beta$ transformations can now be expressed using transformation matrices:

$$\begin{gathered}\vec{v} = v_d + jv_q = e^{-j\theta_e}\vec{v}^s = (\cos\theta_e - j\sin\theta_e)(v_\alpha + jv_\beta) \\ \Rightarrow v = \begin{bmatrix} v_d \\ v_q \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta_e & \sin\theta_e \\ -\sin\theta_e & \cos\theta_e \end{bmatrix}}_{T_{dq}(\theta_e)} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}\end{gathered} \tag{25}$$

And for the $\alpha\beta$ transformation:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix}}_{T_{\alpha\beta}(\theta_e)} \begin{bmatrix} v_d \\ v_q \end{bmatrix} \tag{26}$$

4.1.2 Dynamic Systems in Synchronous Coordinates

The time derivative of a space vector $\vec{y}_s$ in stator coordinates can be transformed in synchronous coordinates as follows:

$$\frac{d\vec{y}_s}{dt} = \frac{d(e^{j\theta_e}\vec{y})}{dt} = e^{j\theta_e}\left(j\omega\vec{y} + \frac{d\vec{y}}{dt}\right) \tag{27}$$

In the Laplace domain, going from stator to synchronous coordinates the following substitution is thus made:

$$s \to s + j\omega \tag{28}$$

Complex-values impedance of an inductor in synchronous coordinates:

$$\vec{Z}(s) = (s + j\omega)L \tag{29}$$

4.1.3 PMSM Model in Synchronous Coordinates

We can use eq. (28), i.e., $s \to s + j\omega$, to immediately obtain the differential equations in synchronous coordinates corresponding to eqs. (16)-(17):

$$L_s\frac{d\vec{i}}{dt} = \vec{v} - (R_s + jL_s\omega)\,\vec{i} - j\omega_r\psi_R e^{j(\theta-\theta_e)} \tag{30}$$

Perfect Field Orientation: $\theta_e = \theta$, then $\omega = \omega_r$ and eq. (30) reduces to

$$L_s\frac{d\vec{i}}{dt} = \vec{v} - (R_s + jL_s\omega)\,\vec{i} - j\omega_r\psi_R \tag{31}$$

and the torque becomes

$$\tau_{em} = \frac{p}{3}\Im\left\{\psi_R\,\vec{i}\right\} = \frac{p}{3}\psi_R i_q \tag{32}$$

A Instantaneous Power

From the well known relation $P = \Re\{\bar{V}\bar{I}^*\}$ (where superscript "*" indicates complex conjugate) for single-phase rms-value-scaled phasors $\bar{V}$ and $\bar{I}$, it may be conjectured that the instantaneous power in a three-phase system is proportional to $\Re\{\vec{v}(\vec{i})^*\}$:

$$\vec{v}(\vec{i})^* = \left(v_a + e^{j2\pi/3}v_b + e^{j4\pi/3}v_c\right)\left(i_a + e^{j2\pi/3}i_b + e^{j4\pi/3}i_c\right)^* \tag{33}$$

Observing that $e^{j4\pi/3} = e^{-j2\pi/3}$, and assuming that $i_a + i_b + i_c = 0$:

$$\begin{aligned}\vec{v}(\vec{i})^* &= v_a i_a + v_b i_b + v_c i_c + \underbrace{\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)}_{e^{j2\pi/3}}(v_a i_c + v_b i_a + v_c i_b) \\ &\quad + \underbrace{\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)}_{e^{-j2\pi/3}}(v_a i_b + v_b i_c + v_c i_a) && (34) \\ &= v_a i_a + v_b i_b + v_c i_c - \frac{1}{2}\left(v_a \underbrace{(i_b + i_c)}_{-i_a} + v_b \underbrace{(i_a + i_c)}_{-i_b} + v_c \underbrace{(i_a + i_b)}_{-i_c}\right) \\ &\quad + j\frac{\sqrt{3}}{2}(v_a(i_c - i_b) + v_b(i_a - i_c) + v_c(i_b - i_a)) && (35) \\ &= \frac{3}{2}\left(v_a i_a + v_b i_b + v_c i_c + j\frac{1}{\sqrt{3}}(v_a(i_c - i_b) + v_b(i_a - i_c) + v_c(i_b - i_a))\right) && (36)\end{aligned}$$

Thus, the instantaneous *active* power is given by

$$P = \frac{2}{3}\Re\{\vec{v}\,(\vec{i})^*\} = v_a i_a + v_b i_b + v_c i_c \tag{37}$$