SPUR GEAR DESIGN

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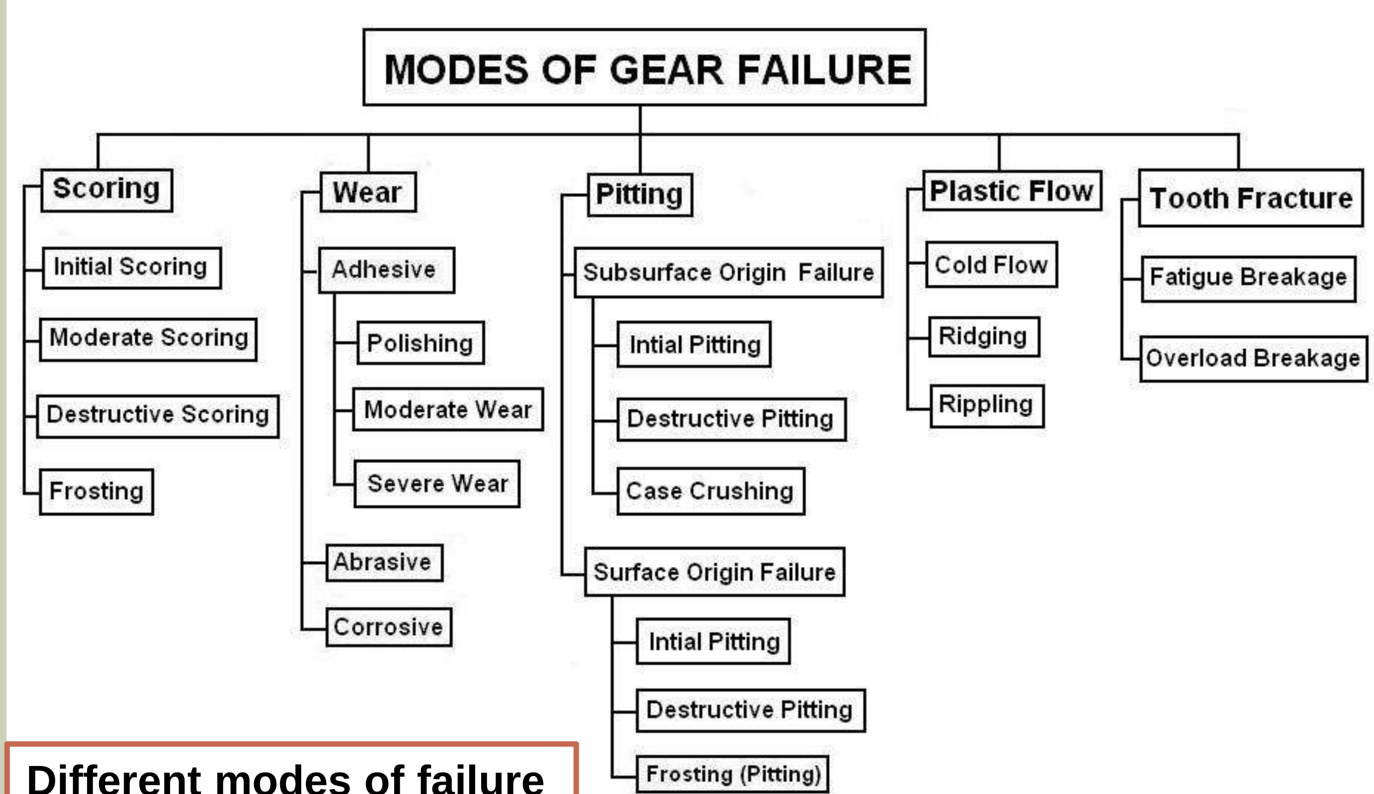
Module 1
GEAR FAILURE

- ❑ Abrasive Wear
- ❑ Corrosive Wear
- ❑ Pitting of Gears
- ❑ Scoring

GEAR TOOTH FAILURE

Gear failure can occur in various modes. In this chapter details of failure are given. If care is taken during the design stage itself to prevent each of these failure a sound gear design can be evolved. The gear failure is explained by means of flow diagram in Fig.



Different modes of failure

ABRASIVE WEAR

Abrasive wear is the principal reason for the failure of open gearing and closed gearing of machinery operating in media polluted by abrasive materials. Examples are mining machinery; cement mills; road laying, building construction, agricultural and transportation machinery, and certain other machines. In all these cases, depending on the size, shape and concentration of the abrasives, the wear will change. Abrasive wear is classified as mild and severe.

MILD ABRASION

Mild abrasion is noticed when there is ingress of fine dust particles in lubricating oil which are abrasive in nature. Since abrasive is very fine, the rate of metal removal is slow. It takes a long time for p

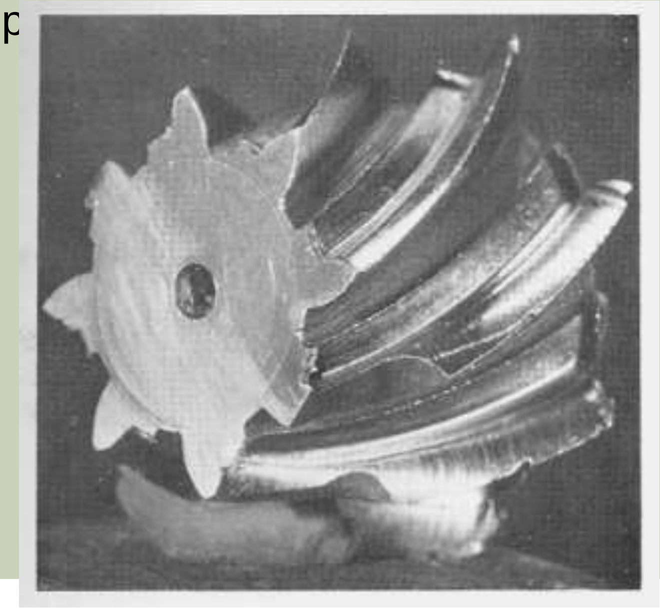


Fig (a) Mild abrasion

ABRASIVE WEAR

The surface appears as though it is polished. A spiral bevel pinion with mild abrasion is shown in Fig. (a) Mild abrasive wear is faced in cement mills, ore grinding mills. Fine dust particles entering the lubricating medium cause three body abrasions. The prior machining marks disappear and surface appears highly polished as shown in Fig.(b). Noticeable wear occurs only over a long time. Sealing improvement and slight pressurization of the gear box with air can reduce the entry of dust particles and decrease this wear.

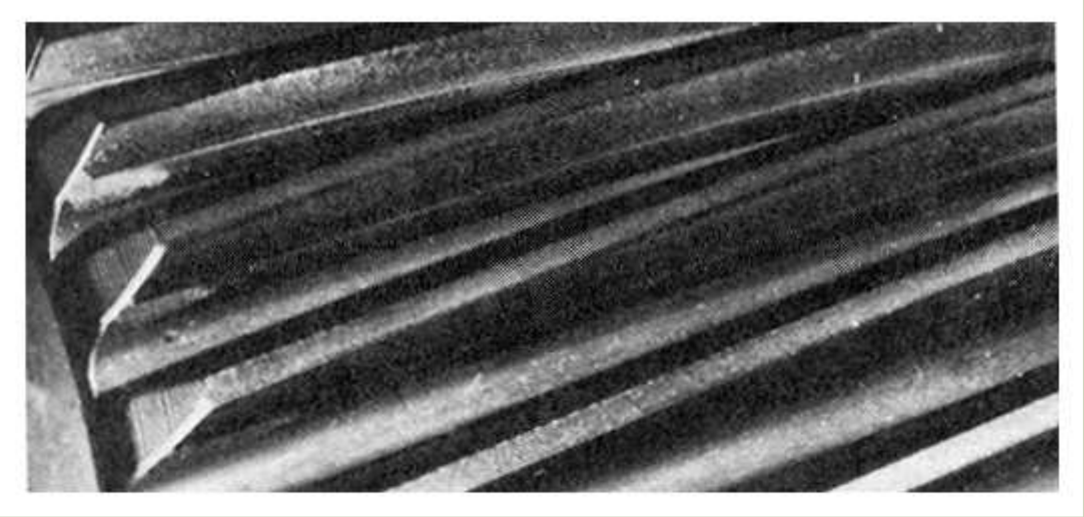


Fig. (b) Mild abrasion

ABRASIVE WEAR

- Cause:
- Foreign particles in the lubricant like dirt, rust, weld spatter or metallic debris can scratch the tooth surface.

- Remedy:
- Provision of oil filters.
- Increasing surface hardness.
- Use of high viscosity oils.

ABRASIVE WEAR

SEVERE ABRASION

This wear occurs due to ingress of larger abrasive particles in the lubricating medium and higher concentration of the particles. The particles will plough a series of groove on the surface in the direction of sliding on the gear tooth as seen in the Fig. (c). High rate of wear in this case will quickly reduce the tooth thickness. Thinned tooth may later on fracture leading to total failure.

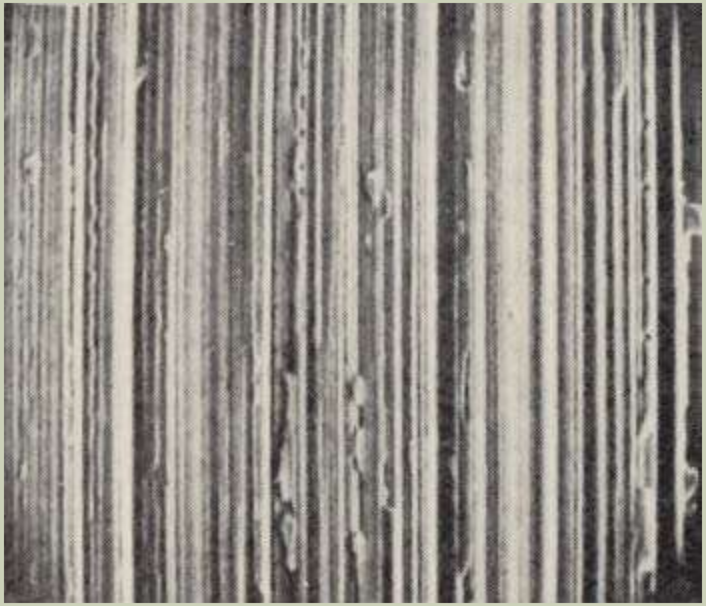
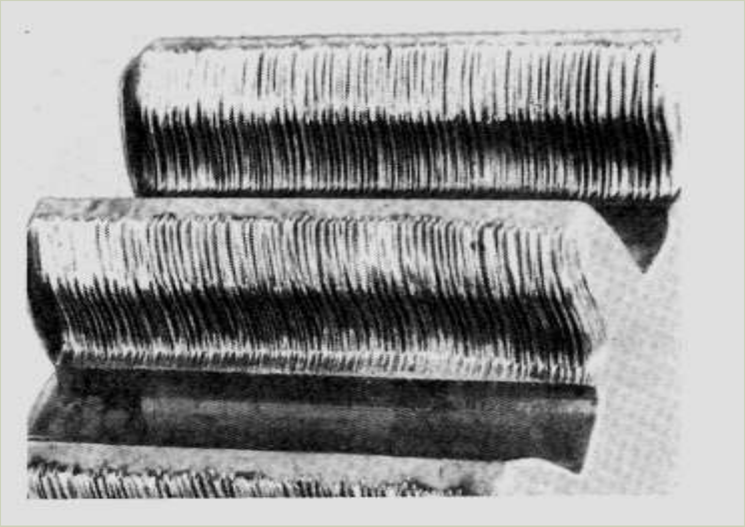


Fig. (c) Severe abrasion

CORROSIVE WEAR

Corrosive wear is due to the chemical action of the lubricating oil or the additives. Tooth is roughened due to wear and can be seen in the Fig.(d1). Chemical wear of flank of internal gear caused by acidic lubricant is shown in Fig. (d2)



(d1)

(d2)

Fig. d Corrosive wear

CORROSIVE WEAR

- Cause: Corrosive of the tooth is caused by corrosive elements such as
- Extreme pressure additives present in lubricating oils.
- Foreign materials.

- Remedies:
- Providing complete enclosure for the gears free from external contamination.
- Selecting proper additives.
- Replacing the lubricating oil at regular intervals.

PITTING OF GEARS

Pitting is a surface fatigue failure of the gear tooth. It occurs due to repeated loading of tooth surface and the contact stress exceeding the surface fatigue strength of the material. Material in the fatigue region gets removed and a pit is formed. The pit itself will cause stress concentration and soon the pitting spreads to adjacent region till the whole surface is covered. Subsequently, higher impact load resulting from pitting may cause fracture of already weakened tooth. However, the failure process takes place over millions of cycles of running. There are two types of pitting, initial and progressive.

PITTING OF GEARS

INITIAL / INCIPIENT PITTING

Initial pitting occurs during running-in period wherein oversized peaks on the surface get dislodged and small pits of 25 to 50 μm deep are formed just below pitch line region. Later on, the load gets distributed over a larger surface area and the stress comes down which may stop the progress of pitting.

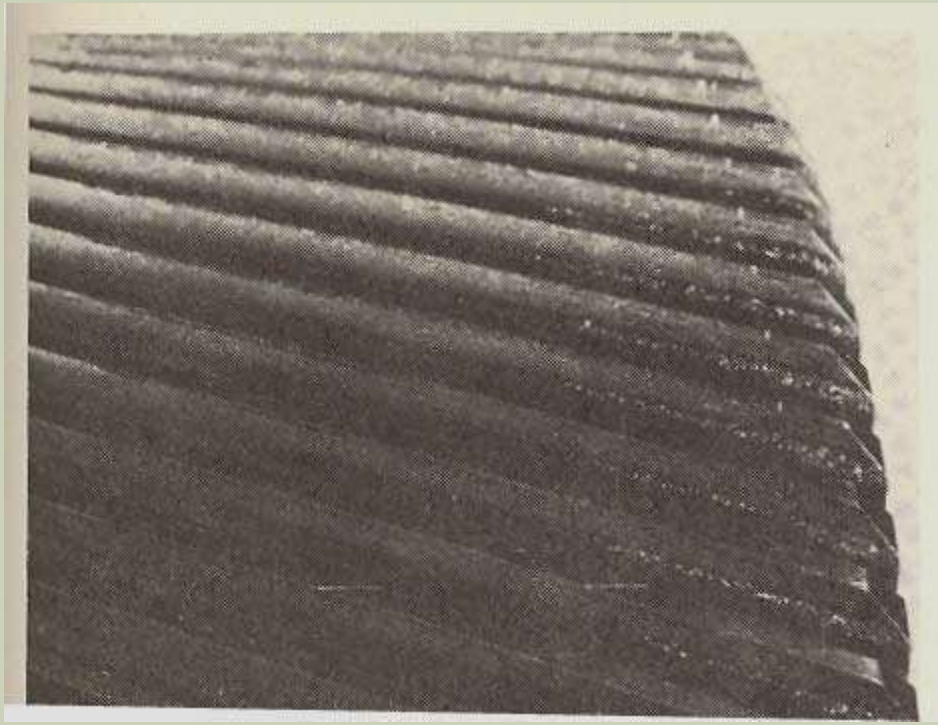


Fig. (e) Initial pitting

PITTING OF GEARS

In the helical gear shown in Fig. (e). pitting started as a local overload due to slight misalignment and progressed across the tooth in the dedendum portion to mid face. Here, the pitting stopped and the pitted surfaces began to polish up and burnish over. This phenomenon is common with medium hard gears. On gears of materials that run in well, pitting may cease after running in, and it has practically no effect on the performance of the drive since the pits that are formed gradually become smoothed over from the rolling action. The initial pitting is non-progressive.

PITTING OF GEARS

INITIAL PITTING:

- Cause:
  - Errors in tooth profile.
  - Surface irregularities.
  - Misalignment.

- Remedies:
  - Precise machining of gears.
  - Correct alignment of gears.
  - Reducing the dynamic loads.

PITTING OF GEARS

PROGRESSIVE OR DESTRUCTIVE PITTING

During initial pitting, if the loads are high and the corrective action of initial pitting is unable to suppress the pitting progress, then destructive pitting sets in. Pitting

spreads all over the tooth length. Pitting leads to higher pressure on the unpitted surface, squeezing the lubricant into the pits and finally to seizing of surfaces.

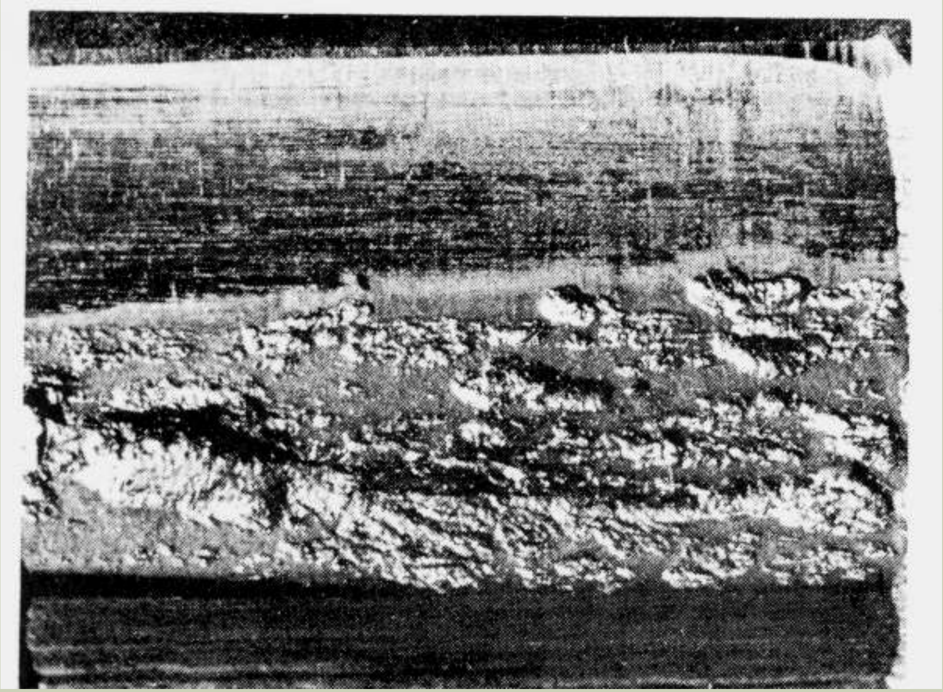


Fig. (f) Tooth surface destroyed by extensive pitting

PITTING OF GEARS

Pitting begins on the tooth flanks near the line along the tooth passing through the pitch point where there are high friction forces due to the low sliding velocity. Then it spreads to the whole surface of the flank. Tooth faces are subjected to pitting only in rare cases. Fig. (f) shows how in destructive pitting, pitting has spread over the whole tooth and weakened tooth has fractured at the tip leading to total failure

Fig. (g) Whole tooth is destroyed by extensive pitting

PITTING OF GEARS

DESTRUCTIVE PITTING:

- Cause:
  - Load on the gear tooth exceeds the surface endurance strength of the material.
- Remedies:
  - Designing the gears in such a way that the wear strength of the gear tooth is more than the sum of static and dynamic loads.
  - The surface endurance strength can be improved by increasing the surface hardness.

SCORING

Scoring is due to combination of two distinct activities: First, lubrication failure in the contact region and second, establishment of metal to metal contact. Later on, welding and tearing action resulting from metallic contact removes the metal rapidly and continuously so far the load, speed and oil temperature remain at the same level. The scoring is classified into initial, moderate and destructive.

INITIAL SCORING

Initial scoring occurs at the high spots left by previous machining. Lubrication failure at these spots leads to initial scoring or scuffing as shown in Fig. (h). Once these high spots are removed, the stress comes down as the load is distributed over a larger area. The scoring will then stop if the load, speed and temperature of oil remain unchanged or reduced. Initial scoring is non-progressive and has corrective action associated with it.

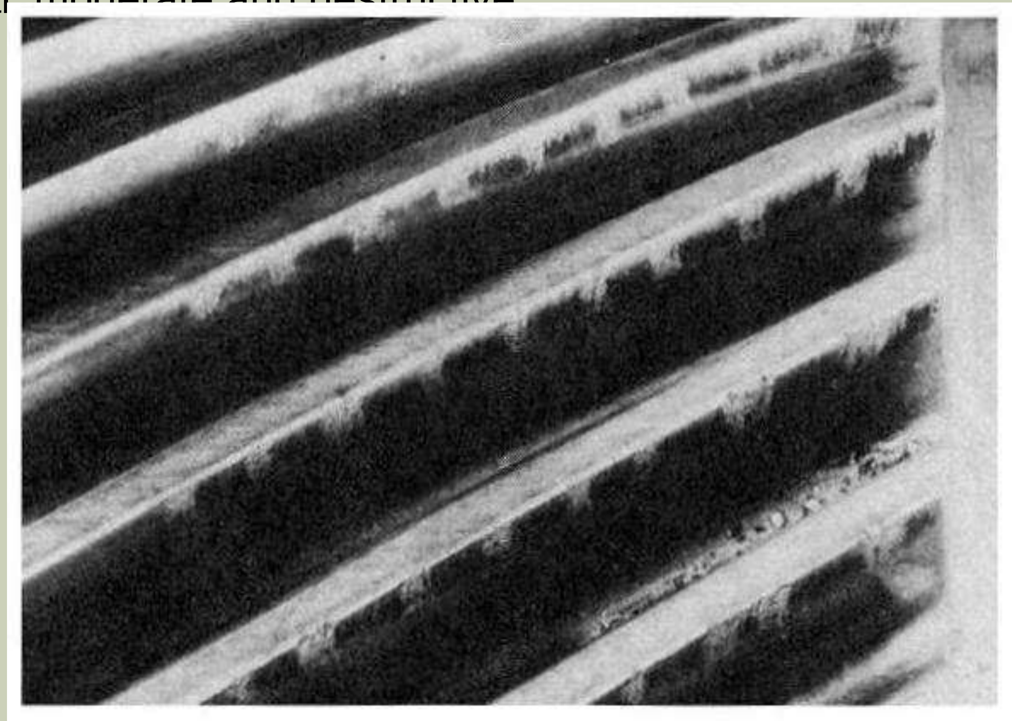


Fig. (h) Initial scoring

SCORING

MODERATE SCORING

After initial scoring if the load, speed or oil temperature increases, the scoring will spread over to a larger area. The Scoring progresses at tolerable rate. This is called moderate scoring as shown in Fig. (i).

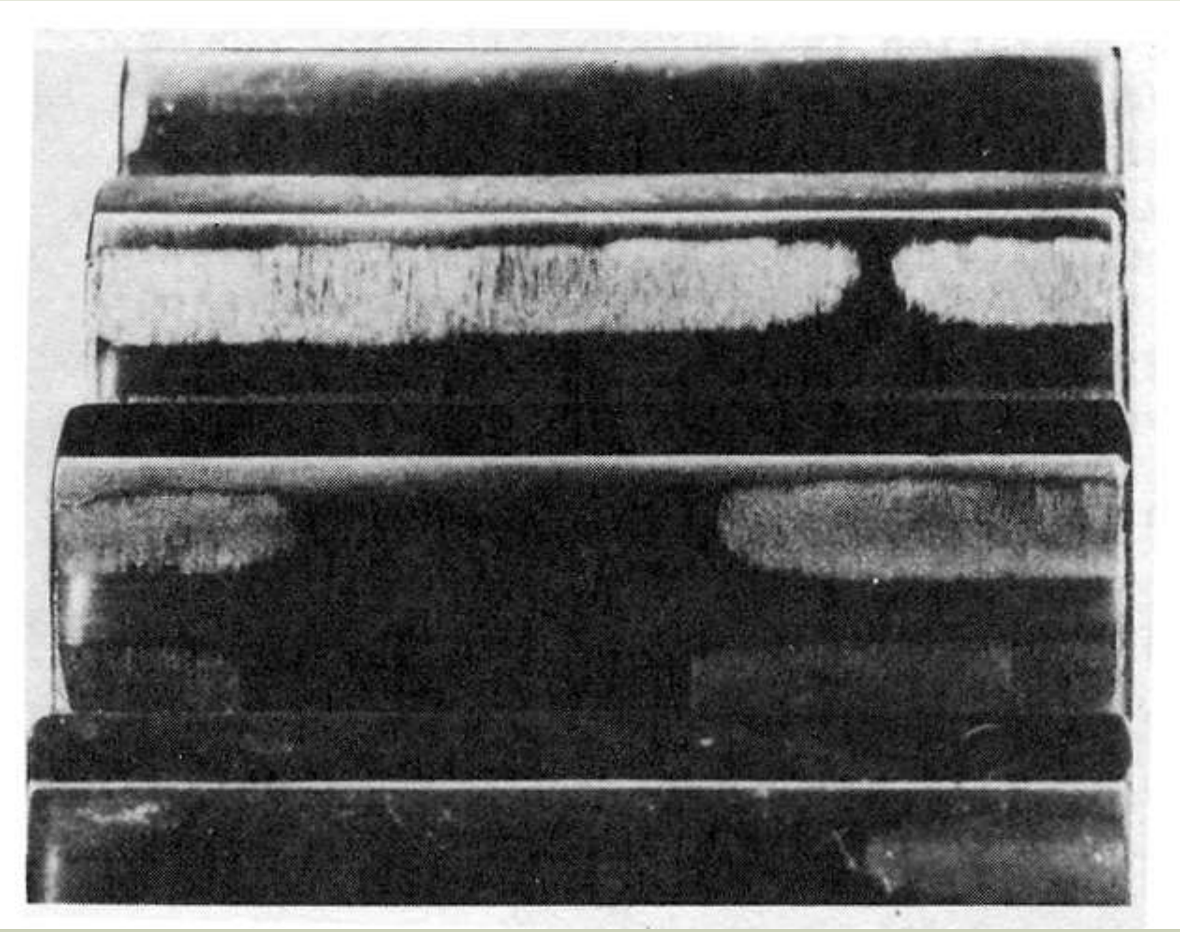


Fig. (i) Moderate scoring

SCORING

DESTRUCTIVE SCORING

After the initial scoring, if the load, speed or oil temperature increases appreciably, then severe scoring sets in with heavy metal torn regions spreading quickly throughout as shown in Fig.(j). Scoring is normally predominant over the pitch line region since elastohydrodynamic lubrication is the least at that region. In dry running surfaces may seize.

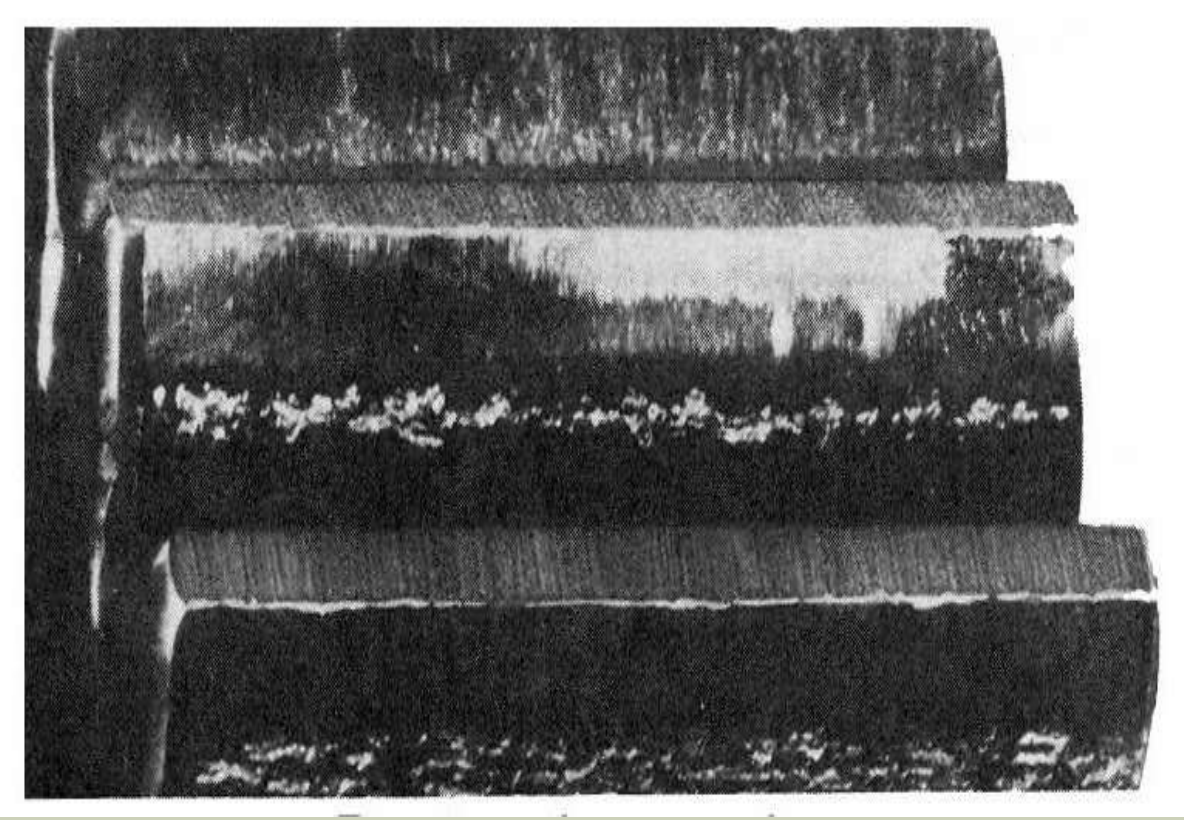


Fig. (j) Destructive scoring

SCORING

- Cause:
- Excessive surface pressure.
- High Surface Speed.
- Inadequate supply of lubricant result in the breakdown of the oil film.

- Remedies:
- Selecting the parameters such as surface speed, surface pressure and the flow of lubricant in such a way that the resulting temperature at the contacting surfaces is within permissible limits.

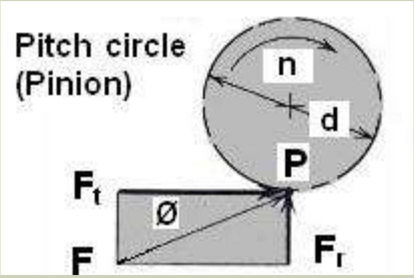
- The bulk temperature of the lubricant can be reduced by providing fins on the outside surface of the gear box and a fan for forced circulation of air over the fins.

Module 2 –
SPUR GEAR DESIGN

- Beam Strength Of Gear Tooth
- Wear Strength Of Gear Tooth

TOOTH FORCE ANALYSIS

SPUR GEAR – TOOTH FORCE ANALYSIS



As shown in Fig

The normal force F can be resolved into two components;

a tangential force F_t which does transmit the power and
radial component F_r which does no work but tends to push the gears apart.

They can hence be written as,

$F_t = F \cos Ø$ (7. 1)

$F_r = F \sin Ø$ (7.2)

From eqn. (7.2),

$F_r = F_t \tan Ø$ (7.3)

TOOTH FORCE ANALYSIS

The pitch line velocity V, in meters per second, is given as

$$V = \frac{\pi d n}{6000} \qquad (7.4)$$

$$W = \frac{F_t V}{1000} \qquad (7.5)$$

where d is the pitch diameter of the gear in millimeters and n is the rotating speed in rpm and W power in kW.

GEAR - TOOTH STRESSES

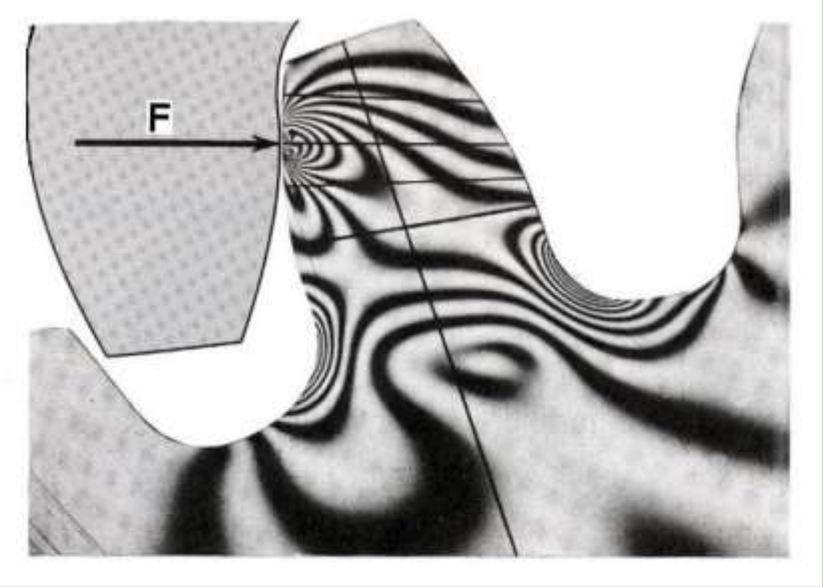


Photo-elastic Model of gear tooth

Stresses developed by Normal force in a photo-elastic model of gear tooth as per Dolan and Broghammer are shown in the Fig. The highest stresses exist at regions where the lines are bunched closest together. The highest stress occurs at two locations:

A. At contact point where the force F acts

B. At the fillet region near the base of the tooth.

BEAM STRENGTH OF GEAR TOOTH

The accurate stress analysis of a gear tooth for a particular application is a complex problem because of the following reasons:

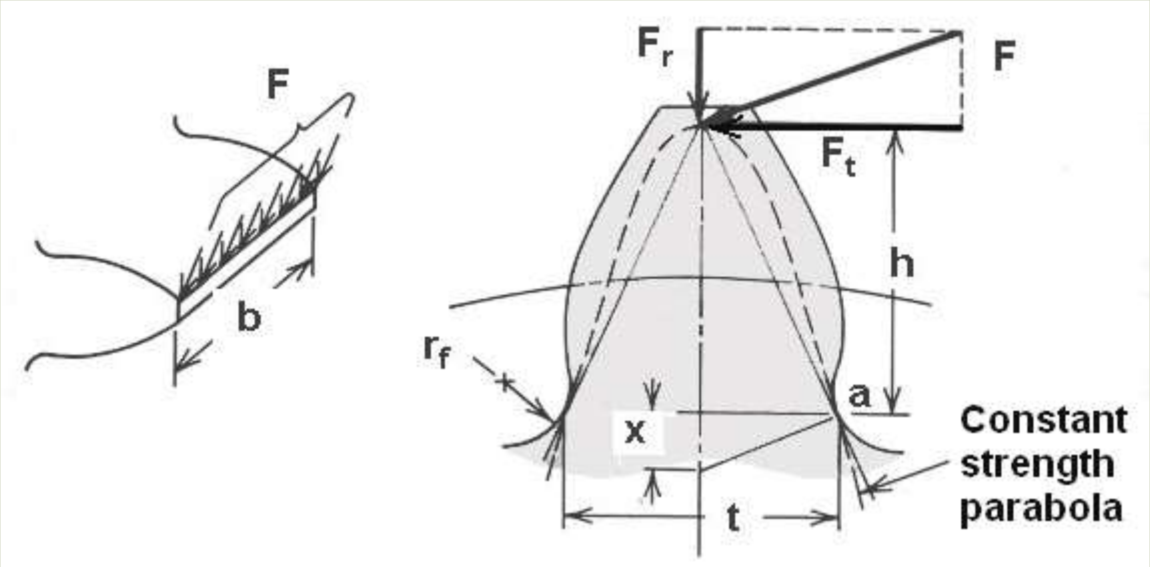
- There is continuous change in the point of application of load on the tooth profile.
- The magnitude and direction of applied load also change.
- In addition to static load, the dynamic load due to inaccuracy of the tooth profile, error in machining and mounting, tooth deflection, acceleration and stress concentration also act on the tooth which are all difficult to model mathematically.

The analysis of bending stresses in gear tooth was done by WILFRES LEWIS. The lewis equation is considered as the basic equation in the design of gears.

In the lewis analysis, the gear tooth is consider as a cantilever beam

BEAM STRENGTH OF GEAR TOOTH

Wilfred Lewis, in a paper titled, “The investigation of the strength of gear tooth” published in 1892, derived an equation for determining the approximate stress in a gear tooth by treating it as a cantilever beam of uniform strength. The beam strength calculation is based upon the following assumptions:



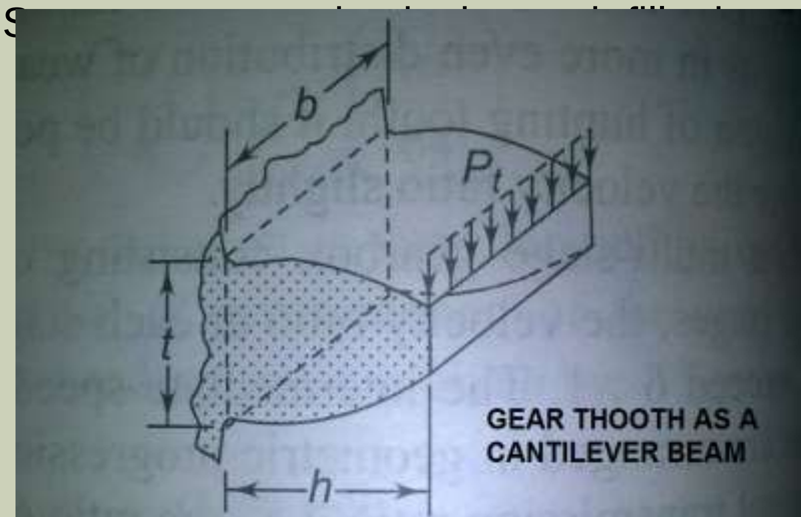
Gear tooth as cantilever beam

BEAM STRENGTH OF GEAR TOOTH

Lewis considered gear tooth as a cantilever beam with static normal force F applied at the tip.

Assumptions made in the derivation are:

1. The full load is applied to the tip of a single tooth in static condition.
2. The radial component is negligible.
3. The load is distributed uniformly across the full face width.
4. Forces due to tooth sliding friction are negligible.
5. S gligible.

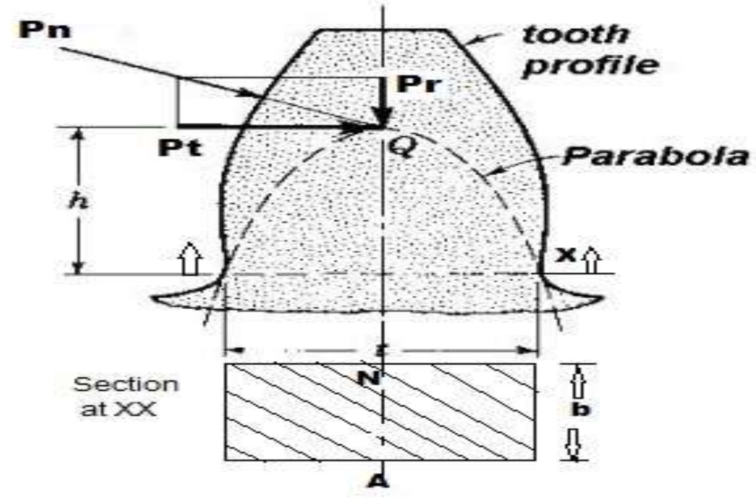


Here force is denoted my P

BEAM STRENGTH OF GEAR TOOTH

The cross-section of the tooth varies from free end to the fixed end. Therefore , A parabola is constructed within the tooth profile. Parabola is beam of uniform strength. The stress at any cross-section of beam is uniform or same.

The weakest section of the gear tooth is at the section XX, where parabola is tangent to the tooth profile.



Gear tooth as Parabolic Beam

BEAM STRENGTH OF GEAR TOOTH

At the section XX,

$M_b = P_t \times h$

$I = (1/12)\, b \times t^3$

$Y = t / 2$

The bending stresses are given by,

$\sigma_b = M_b \times y / I$

$= (p_t \times h)(t/2) / [(1/12)bt^3]$

Rearranging the terms,

$P_t = b\, \sigma_b\, (t^2/6h)$

Multiplying the numerator and denominator of the right-hand side by m,

$P_t = m\, b\, \sigma_b\, (t^2/6hm)$

BEAM STRENGTH OF GEAR TOOTH

Defining a factor Y,
$Y=(t^2/6hm)$

The equation is rewritten as,
$P_t = mb\sigma_b Y$ .... (i)

Y is called the lewis form factor.

Eq.(i) gives relationship between the tangential force(P_t) and the corresponding stress σ_b. When stresses reaches the permissible magnitude of bending stresses, the corresponding force(P_t) is called the beam strength.

Therefore the beam strength(S_b) is maximum value of the tangential force that the tooth can transmit without bending failure.
Replacing P_t by S_b,

$S_b = mb\sigma_b Y$ .... (ii)

S_b = beam strength of gear tooth(N)
σ_b = permissible bending stress (N/mm^2)

BEAM STRENGTH OF GEAR TOOTH

Number of teeth	$\phi = 20^\circ$ a = 0.8m* b = m	$\phi = 20^\circ$ a = m b = 1.25m	$\phi = 25^\circ$ a = m b = 1.25m	$\phi = 25^\circ$ a = m b = 1.35m+
12	0.335 12	0.229 60	0.276 77	0.254 73
13	0.348 27	0.243 17	0.292 81	0.271 77
14	0.359 85	0.255 30	0.307 17	0.287 11
15	0.370 13	0.266 22	0.320 09	0.301 00
16	0.379 31	0.276 10	0.331 78	0.133 63
17	0.387 57	0.285 08	0.342 40	0.325 17
18	0.395 02	0.293 27	0.352 10	0.335 74
19	0.401 79	0.300 78	0.360 99	0.345 46
20	0.407 97	0.307 69	0.369 16	0.354 44
21	0.413 63	0.314 06	0.376 71	0.362 76
22	0.418 83	0.319 97	0.383 70	0.370 48
24	0.428 06	0.330 56	0.396 24	0.384 39
26	0.436 01	0.339 79	0.407 17	0.396 57
28	0.442 94	0.347 90	0.416 78	0.407 33
30	0.449 02	0.355 10	0.425 30	0.416 91
34	0.459 20	0.367 31	0.439 76	0.433 23
38	0.467 40	0.377 27	0.451 56	0.446 63
45	0.478 46	0.390 93	0.467 74	0.465 11
50	0.484 58	0.398 60	0.476 81	0.475 55
60	0.493 91	0.410 47	0.490 86	0.491 77
75	0.503 45	0.422 83	0.505 46	0.508 77
100	0.513 21	0.435 74	0.520 71	0.526 65
150	0.523 21	0.449 30	0.536 68	0.545 56
300	0.533 48	0.463 64	0.553 51	0.565 70
Rack	0.544 06	0.478 97	0.571 39	0.587 39

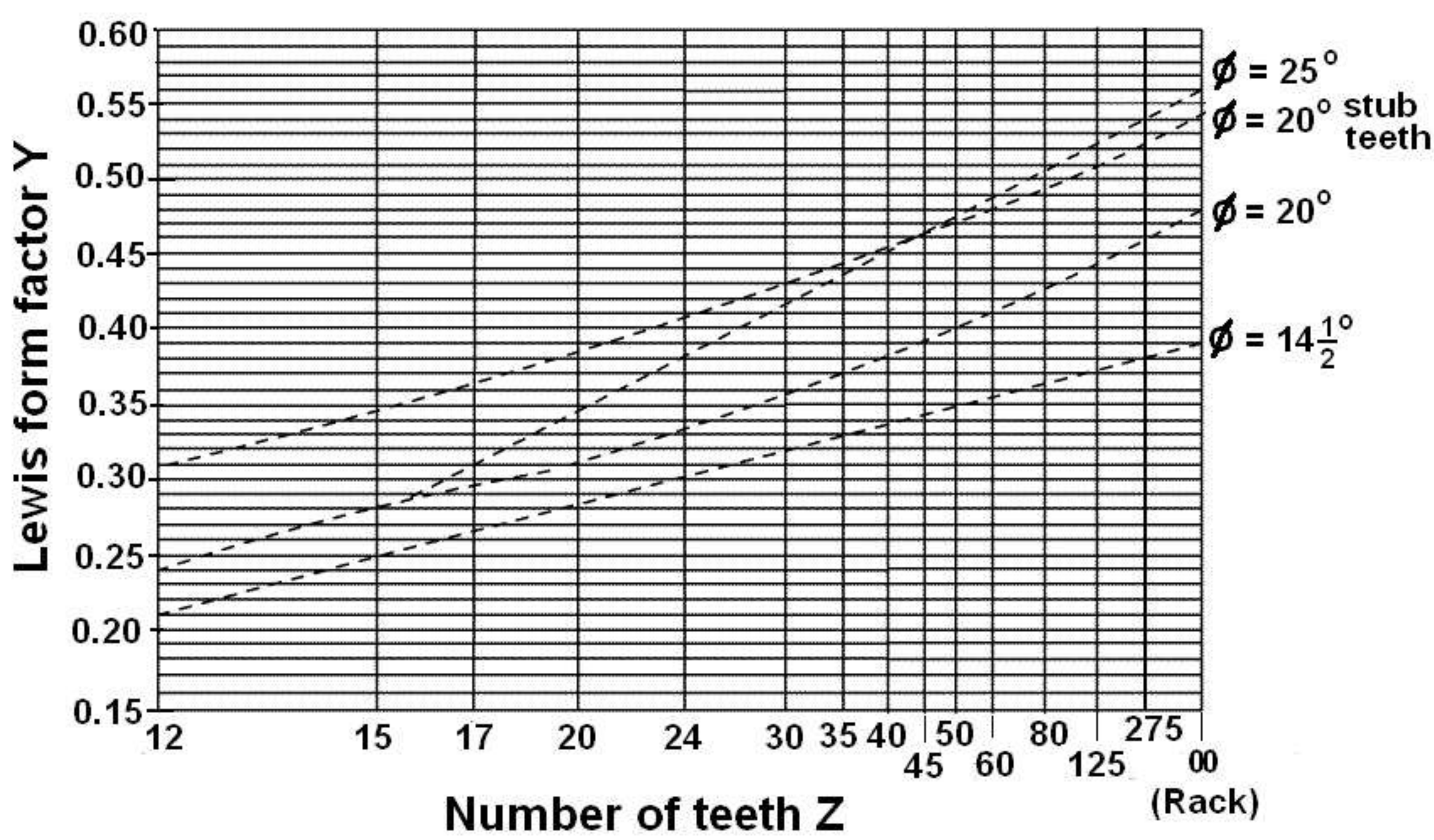
* Stub teeth + Large fillet

Eq.(Ii) is known as lewis equation. Table for values of lewis form factor is given below

In order to avoid the breakage of gear tooth due to bending, the bending strength should be more than the effective force between the meshing teeth.

$S_b >= P_{eff}$

BEAM STRENGTH OF GEAR TOOTH



BEAM STRENGTH OF GEAR TOOTH

The Lewis equation indicates that tooth bending stress varies with the following:

(1) Directly with load,
(2) Inversely with tooth width b,
(3) Inversely with tooth size p or m,
(4) Inversely with tooth shape factor y or Y.

Drawbacks of Lewis equation are:

- The tooth load in practice is not static. It is dynamic and is influenced by pitch line velocity.
- The whole load is carried by single tooth is not correct. Normally load is shared by teeth since contact ratio is near to 1.5.
- The greatest force exerted at the tip of the tooth is not true as the load is shared by teeth. It is exerted much below the tip when single pair contact occurs.
- The stress concentration effect at the fillet is not considered.

BEAM STRENGTH OF GEAR TOOTH

The modified Lewis equation for bending stress is,

$P_t = K_v m b \sigma_b Y$ ……………..(iii)

Where K_v is known as velocity factor and is given by Barth's equation below for known pitch line velocity V in m/s and is given by,

$$K_v' = \frac{6}{6+V}$$

………………(iv)

Eqn. (4) is used for cut or milled teeth or for gears not carefully generated.

$$K_v' = \frac{50}{50+(200V)^{0.5}}$$

………………(v)

Eqn. (5) is used for hobbed and shaped gears.

$$K_v' = \left[\frac{78}{78+(200V)^{0.5}}\right]^{0.5}$$

………………(vi)

Eqn. (6) is used for high-precision shaved or ground teeth.

The modified Lewis equation given in eqn.(3) is used when fatigue failure of the gear teeth is not a problem and a quick estimate is desired for more detailed analysis

WEAR STRENGTH OF GEAR TOOTH

WEAR STRENGTH

Due to rolling and sliding actions of the gear teeth, the following types of surface destructions (wear) may occur:

Abrasive wear. Scratching of the tooth surface due to the presence of foreign materials in the lubricant is called *abrasive wear*.

Corrosive wear. Chemical reactions on the surface of a gear cause *corrosive wear.*

Pitting. Repeated application of the stress cycle, known as *pitting,* cause fatigue failure.

Scoring. Inadequate lubrication between metal-to metal contact cause *scoring.*

It is observed from the results of various experiments that abrasion, corrosive, the scoring are caused by improper lubrication, whereas pitting usually occurs because of repeated application of Hertz contact stress on the portion of a gear tooth which has relatively little sliding motion compared to rolling motion. Clearly, the spur gear will have pitting near the pitch lien where motion is almost all of rolling.

WEAR STRENGTH OF GEAR TOOTH

The failure of the gear tooth due to pitting occurs when the contact stress between two meshing teeth exceed the surface endurance strength of the material.

Pitting is a surface fatigue failure, characterized by small pits on the surface of the gear tooth.

To avoid this type of failure, properties of gear tooth and surface properties should be selected such a way that the wear strength of the gear tooth is more than the effective load between the meshing teeth.

The analysis of wear strength of gear tooth is done by Earle Buckingham. Buckingham 's equation gives the wear strength of the gear tooth. which is based on Hertz theory of contact stresses.

WEAR STRENGTH OF GEAR TOOTH

When two cylinders are pressed together as shown in fig.(a), the contact stress is given by,

$$\sigma_c = 2*P/\pi*b*l \qquad \text{................(1.1)}$$

and

$$b = \{ [\, 2P(1-\mu^2)(1/E_1 + 1/E_2)] / \pi\, l(1/d_1 + 1/d_2) \}^{1/2} \qquad \text{................(1.2)}$$

where,

σ= maximum value of compressive stress (N/mm^2)
P=force pressing the two cylinders together (N)
b= half width of deformation (mm)
l= axial length of the cylinder (mm)
d_1,d_2= diameter of two cylinders (mm)
E_1,E_2= moduli of elasticity of 2 cylinders (N/mm^2)
μ = Poisson's ratio

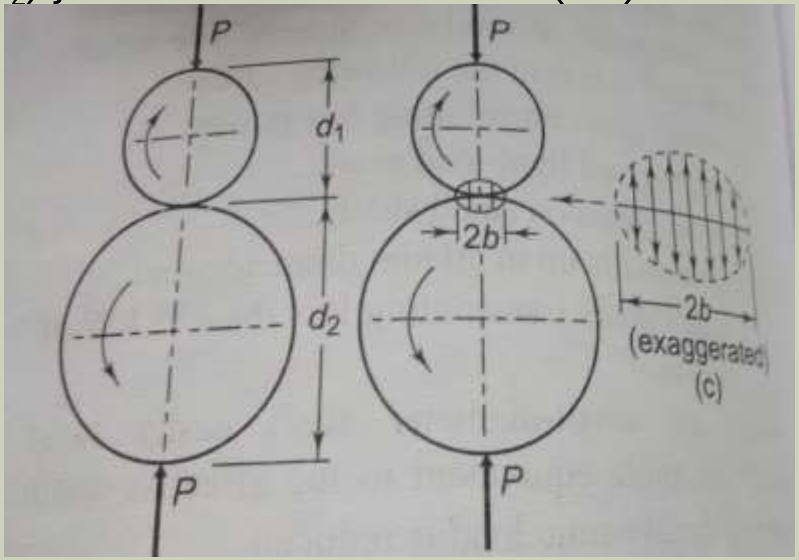


fig. contact stresses

WEAR STRENGTH OF GEAR TOOTH

Due to deformation under the action of load P, a rectangular surface of width (2b) and length (l) is formed between the two cylinders. The elliptical stress distribution across the width (2b) is shown in fig.

Substituting eq.(ii) into(i) and squaring both sides,

$\sigma_c^2 = [1/\pi(1-\mu^2)]*(P/l)*\{(1/r_1 + 1/r_2) / (1/E_1 + 1/E_2)\}$ ....(1.3)

Where, r_1, r_2= radii of two cylinders

$\sigma_c^2 = 0.35*(P/l)*\{(1/r_1 + 1/r_2) / (1/E_1 + 1/E_2)\}$ ....(1.4)

The above equation of the contact stress is based on the following assumptions:

- The cylinders are made of isotropic materials.
- The elastic limit of the material is not exceeded.
- The dimensions r_1, r_2 are very large when compared to the width (2b) of the deformation.

WEAR STRENGTH OF GEAR TOOTH

Fig. shows the contact between two meshing teeth at the pitch point

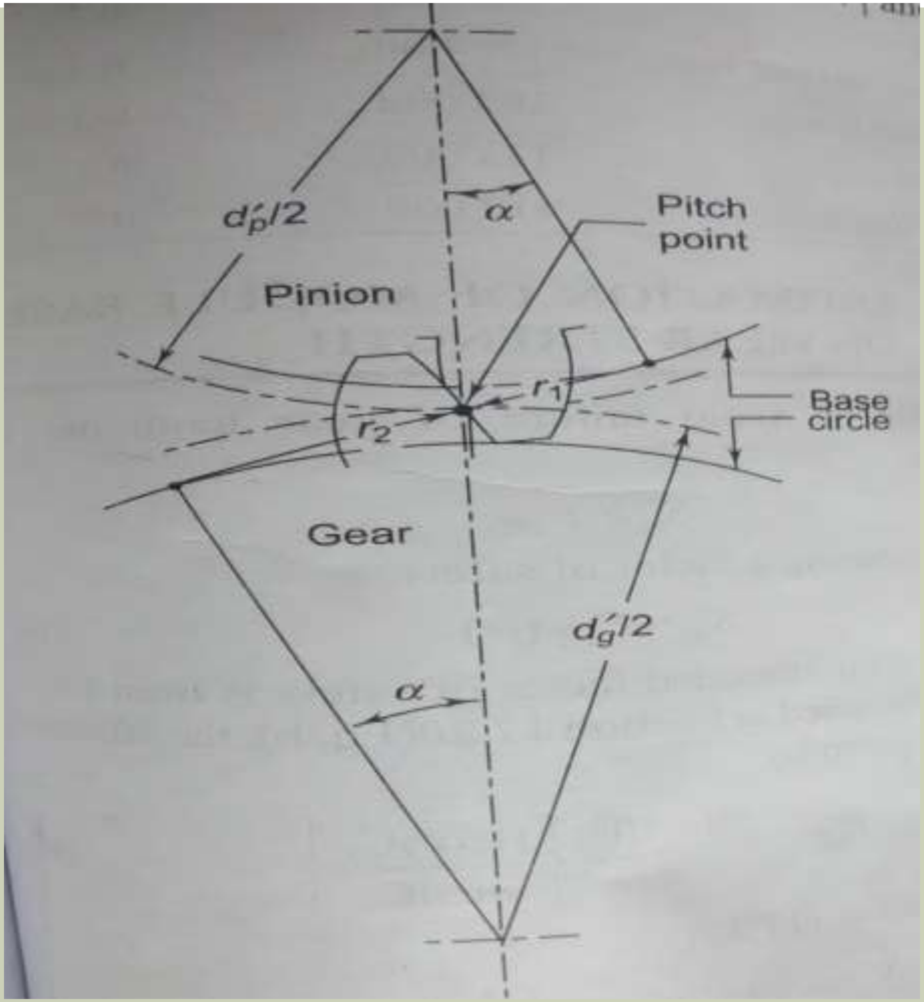


Fig. contact between two meshing teeth at pitch point

WEAR STRENGTH OF GEAR TOOTH

r_1, r_2 are to be replaced by radii of curvature at the pitch point. Therefore,

$r_1 = d_p' * \sin\alpha /2$ $\qquad$ $r_2 = d_g' * \sin\alpha /2$

There are some reasons for taking the radii of curvature at the pitch point.

- The wear on the gear tooth generally occurs at or near the pitch line.
- When only one pair of teeth carries the entire load, contact occurs at the pitch point.
- The dynamic load is imposed on the gear tooth near the pitch line area.

$(1/r_1 + 1/r_2) = 2/\sin\alpha\ [1/d_p' + 1/d_g']$ ....(1.5)

A ratio factor Q is defined as,

$Q = 2*z_g/(z_g+z_p)$ ....(1.6)

WEAR STRENGTH OF GEAR TOOTH

Substituting, $d_p' = m\ z_p$ and $d_g' = m\ z_g$,

$$Q = 2\ d_g'/(d_g' + d_p')$$

Therefore,

$$[1/d_p' + 1/d_g'] = (d_g' + d_p')/d_g' * d_p'$$

$$= 2/\ Q*d_p' \qquad \text{...............(1.7)}$$

From eq.(v) and (vii),

$$(1/r_1 + 1/r_2) = 2/\sin\alpha*Q*d_p' \qquad \text{..............(1.8)}$$

The force acting along the pitch line in fig.(b) is P_N,

$$P = P_N = P_t\ /\ \cos\alpha \qquad \text{.............(1.9)}$$

The axial length of the gears is the face width b,i.e. $l=b$

Therefore,

$$\sigma_c^2 = 1.4*P_t\ /\ b*Q*d_p'*\sin\alpha*\cos\alpha*\ (1/E_1 + 1/E_2) \qquad \text{............(1.10)}$$

WEAR STRENGTH OF GEAR TOOTH

a load stress factor K is defined as,

$$K= \sigma_c^{2} * \sin\alpha * \cos\alpha * (1/E_1 + 1/E_2) /1.4 \qquad \text{..............(1.11)}$$

Substituting the above eq. Into (1.1),

$$P_t = b*Q*d_p'*k \qquad \text{..............(1.12)}$$

This equation gives a relationship between the tangential force P_t and the corresponding contact stress σ_c .

The tangential force is increase with the increasing contact stress. Pitting occurs when the contact stress reaches the magnitude of the surface endurance strength. The corresponding value of P_t is called wear strength.

WEAR STRENGTH OF GEAR TOOTH

Wear strength is maximum value of the tangential force that tooth can transmit without pitting failure.

Replacing P_t by S_w,

$$S_w = b*Q*d_p'*k \qquad \text{.............(1.13)}$$

Where,

S_w= wear strength of the gear tooth (N)
σ_c= surface endurance strength of the material (N/mm^2)
equation (1.13) is known as Buckingham's equation for wear

The ratio factor for internal gears is defined as,

$$Q = 2*z_g/(z_g - z_p) \qquad \text{.............(1.14)}$$

WEAR STRENGTH OF GEAR TOOTH

Example for find out load-stress factor K,

The expression for the load-stress factor K can be simplified when both the gears are made of steel with a 20° pressure angle.

In the special case,

$E_1=E_2= 206000\ N/mm^2$ (for steel)

$\alpha= 20°$

according to G Niemann,

$$\sigma_c= 0.27\ (BHN)\ kgf/m^2$$
$$= 0.27(9.81)\ (BHN)\ N/mm^2$$

Where BHN is Brinell Hardness Number.

WEAR STRENGTH OF GEAR TOOTH

$K = \sigma_c^2 * \sin\alpha * \cos\alpha * (1/E_1 + 1/E_2) / 1.4$

$= (0.27*9.81)^2 (BHN)^2 * \sin(20) * \cos(20) * (2/206000) / 1.4$

$= 0.156 (BHN/100)^2$

Or

$K = 0.16 (BHN/100)^2$

The above equation is applicable only when both the gears are made of steel with 20° pressure angle.

In order to avoid failure of the gear tooth due to pitting, the wear strength should be more than the effective force between the meshing teeth.

THANK YOU